

Real-Time Bidding in Online Display Advertising

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February 2017

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Abstract

Display advertising is a major source of revenue for many of the online publishers and content providers. Historically, display advertising impressions have been sold through pre-negotiated contracts, known as *reservation contracts*, between publishers and advertisers. In recent years, a growing number of impressions are being sold in real-time bidding (RTB), where advertisers bid for impressions in real-time, as consumers visit publishers' websites. RTB allows advertisers to target consumers at an individual level using browser cookie information, and enables them to customize their ads for each individual. The rapid growth of RTB has created new challenges for advertisers and publishers on how much budget and ad inventory to allocate to RTB. In this paper, we use a game theory model with two advertisers and a publisher to study the effects of RTB on advertisers' and publishers' strategies and their profits. We show that symmetric advertisers use asymmetric strategies where one advertiser buys all of his impressions in RTB whereas the other advertiser focuses on reservation contracts. Interestingly, we find that while both advertisers benefit from existence of RTB, the advertiser that focuses on reservation contracts benefits more than the advertiser that focuses on RTB. We show that while RTB lowers the equilibrium price of impressions in reservation contracts, it increases the publisher's total revenue. Despite many analysts' belief that, because of being more efficient, RTB will replace reservation contracts in the future, we show that publishers have to sell a sufficiently large fraction of their impressions in reservation contracts in order to maximize their revenue. We extend our model to consider premium consumers, publisher's uncertainty about the number of future visitors, and effectiveness of ad customization.

1 Introduction

Display advertising is a major source of revenue for many of the online publishers and content providers. In 2017, the revenue of this market in the US alone is expected to exceed \$40 billion

(Liu 2016). Historically, display ads have been sold in pre-negotiated contracts, known as *Reservation Contracts*, between publishers and advertisers. In a reservation contract, also referred to as a *Direct Buy*, an advertiser and a publisher agree on the price, targeting criteria, and the number of impressions to be delivered in a certain period of time (e.g, 2 million impressions to male visitors from the state of New York in the month of July for \$8,000). In recent years, a growing number of display advertising impressions are being sold in a new model known as *Real-Time Bidding* (RTB).

In the RTB model, any time a consumer visits a publisher's website, the publisher sends the consumer's information to a central exchange platform (e.g., Google's DoubleClick or OpenX). The exchange platform runs an auction, in real-time, in which the advertisers bid for their ads to be shown to the consumer. The winning ad is sent back to the publisher to be displayed to the consumer along with the content of the website. The entire process, from the time that the consumer clicks on a link, or types in a URL, until the content and the ad are shown to the consumer generally takes less than 100 milliseconds.¹ RTB, introduced in 2009, has quickly become one of the main methods of buying and selling display ads. It currently accounts for over a third of the display advertising market in the US, and is estimated to exceed \$14 billion in 2017 (Fisher and Liu 2016).

A major change in RTB market has been the adoption of *header bidding* technology by publishers over the past couple of years.² Prior to adoption of header bidding, RTB was a remnant auction that enabled publishers to sell their leftover inventory in real-time. Publishers allocated all of their impressions to reservation contracts until those contracts were fulfilled, and then made the leftover available on RTB. Using the header bidding technology, however, publishers make all of their inventory available on RTB. For each impression, after receiving RTB bids, the publisher decides whether to allocate the impression to the highest bidder in RTB or to an advertiser with a reservation contract. In other words, RTB bids compete with reservation contracts, and only if the clearing price in RTB is not sufficiently high, the impression is assigned to a reservation contract.³

An important advantage of RTB for advertisers is that they can target consumers at an indi-

¹<https://static.googleusercontent.com/media/74.125.224.88/en/88/doubleclick/pdfs/what-is-rtb.pdf>

²<https://adexchanger.com/publishers/year-header-bidding-went-mainstream/>

³In header bidding, a publisher places some Java Script code in its website's header. When a page on the website is requested by a consumer, the code reaches out to the supported exchange platforms and advertisers (asking for bids) before the publisher's own inventory of reservation contracts is reached. For more information on header bidding, see <https://martechtoday.com/martech-landscape-what-is-header-bidding-and-why-should-publishers-care-157065>

vidual level using browser cookie information. In other words, advertisers can set their bids, and customize their ads, *after* they observe each consumer’s relevant information. Thus, advertisers typically have higher valuation for impressions in RTB than those in reservation contracts. Furthermore, the fine-grained targeting in RTB results in *unbundling of impressions*, which could improve the efficiency of allocation and increase the social surplus.

On the other hand, the fine-grained targeting in RTB leads to “cherry-picking” of impressions in RTB and negatively affects the average quality of impressions assigned to reservation contracts. As such, RTB lowers the advertisers’ willingness to pay for impressions in reservation contracts, which in turn can lower the publisher’s revenue. The fine-grained targeting may also hurt the publisher’s revenue by *dispersing competition*, i.e., reducing the number of advertisers interested in each impression and making ad auctions thinner (e.g., see Levin and Milgrom 2010; Rafeian and Yoganarasimhan 2017).

The overall impact of RTB on publisher’s revenue and advertisers’ profits is still unclear and has been a topic of discussion among practitioners.⁴ In this paper, we use a game theory model with two advertisers and a publisher to analyze how RTB affects publishers’ and advertisers’ strategies and their profits. We are interested in answering the following research questions:

1. What are the effects of RTB on publishers’ revenue and advertisers’ profits?
2. How should a publisher allocate his ad inventory between reservation contracts and RTB?
3. How should an advertiser allocate his budget between reservation contracts and RTB?

RTB has three main effects on a publisher’s revenue: First, by improving advertisers’ ability to target consumers and customize ads, RTB increases advertisers’ willingness to pay for impressions. For example, an advertiser is willing to pay a higher price for an impression to a consumer that has visited his website before and has shown some interest in some of his products, compared to an average visitor of the same publisher. On the other hand, RTB allows the advertisers to cherry-pick impressions, and therefore, could lower the average quality of impressions assigned to reservation contracts. As such, RTB could lower the per-impression price of reservation contracts, which would

⁴For some examples, see <https://www.exchangewire.com/blog/2012/01/31/to-rtb-or-not-rtb/> and <http://blog.retargeter.com/general/why-publishers-shouldnt-be-afraid-real-time-bidding>

hurt the publisher's revenue. Finally, RTB disperses competition by making ad auctions thinner, which could, again, decrease the publisher's revenue.⁵

We find that the publisher could mitigate the competition dispersion effect of RTB by adopting a sufficiently high reserve price. In particular, we show that when an optimal reserve price is used, offering RTB always increases the publisher's revenue.

Interestingly, we find that the importance of setting a high reserve price in RTB motivates the publisher to lower the price of impressions in reservation contracts. By lowering the price, the publisher could sell more impressions in reservation contracts. Therefore, the publisher can ensure that even if the majority of impressions in RTB are left unsold due to the high reserve price, they can still be allocated to reservation contracts. This result also indicates that the publisher's total revenue is maximized when a sufficiently large fraction of the impressions are sold in reservation contracts.

Finally, in addressing the third research question, we show that symmetric advertisers follow asymmetric strategies regarding the use of RTB. We find that, in equilibrium, one advertiser spends all of his budget in RTB whereas the other advertiser focuses on reservation contracts. We show that both advertisers have higher equilibrium profits when RTB is offered than when it is not. However, interestingly, the advertiser that focuses on reservation contracts benefits more from existence of RTB than the advertiser that focuses on RTB.

After establishing these key insights, we consider several extensions of the model to capture additional features of the market and to assess the robustness of our original findings. First, we consider a segment of *premium consumers* for which both advertisers have high valuation. We show that, in equilibrium, both advertisers bid for premium consumers in RTB, and that all such impressions are sold in RTB. This moderates the asymmetry that we find in advertisers' strategies in the main model. However, advertisers still use asymmetric strategies, and our key insights regarding the publisher's and advertisers' strategies and their profits stay the same. Second, we model *publisher's supply uncertainty* by assuming that the publisher does not know how many visitors it would have in the future, at the time of selling impressions in reservation contracts. We show that under supply uncertainty, RTB increases social surplus, and that both advertisers and the

⁵For example, Chen et al. (2014) show that in a platform with 374 actively participating advertisers, each RTB impression receives only 5 bids on average.

publisher benefit from the increase in social surplus. Third, we consider *ad customization* in RTB. Since advertisers can customize their ads for each individual consumer in RTB, they can increase the probability of conversion, and thus, have higher valuation for the same impression in RTB than in reservation contracts. Interestingly, we show that the effectiveness of ad customization could have a non-monotonic effect on advertisers' profits. In particular, as the probability of conversion due to ad customization increases, the advertisers' equilibrium profits may decrease. We show that our key insights from the main model, regarding the advertisers' and the publisher's strategies and their profits continue to hold in all these extensions.

Findings Inline with Recent Trends in RTB Market

Our findings can explain some of the recent trends in this market:

Importance of reserve price optimization. Our results highlight the importance of reserve price, also referred to as “price floor,” in determining the clearing price of an RTB auction. In particular, we show that the publisher can mitigate the competition dispersion effect of RTB by adopting a sufficiently high reserve price. This finding is consistent with industry reports showing that in the early years, publishers were unsatisfied with RTB due to low clearing prices. However, after reserve prices were increased, publishers achieved revenues that were higher than before RTB.⁶ This has led to development and adoption of advanced technologies, such as dynamic price floors, for reserve price optimization.⁷

Adoption of header bidding. Originally, RTB was a remnant auction. Publishers allocated all of their impressions to reservation contracts until those contracts were fulfilled, and then made the leftover available on RTB. Adoption of *header bidding* affected this order and changed the nature of the competition. Using header bidding, publishers make all of their inventory available for sale on RTB. For each impression, after receiving RTB bids, the publisher decides whether to allocate the impression to RTB or to reservation contracts. Our results indicate that the new model leads to a higher equilibrium revenue for the publisher than the original model. This finding is consistent with industry reports showing that publishers who adopt header bidding sell their impressions at higher prices,⁸ and also explains the fast growth in the percentage of publishers who use header

⁶<http://digiday.com/platforms/what-is-real-time-bidding/>

⁷<http://rubiconproject.com/blog/using-dynamic-price-floors-to-protect-publisher-value/>

⁸<https://adexchanger.com/publishers/year-header-bidding-went-mainstream/>

bidding (from virtually 0% in 2014 to over 70% in 2016).⁹

Persistence of reservation contracts. RTB is a new technology that allocates the impressions more efficiently, i.e., each advertiser gets the impressions that are most valuable to him. As such, many analysts believed that RTB would replace reservation contracts in the future.¹⁰ In contrast, we show that existence of reservation contracts allows the publisher to set a higher reserve price in RTB, and thus extract more revenue. In other words, even when publishers use optimal reserve prices in RTB, their equilibrium revenue when they sell a fraction of their inventory in reservation contracts (and the rest in RTB) is higher than when they sell their entire inventory in RTB. Therefore, our results indicate that reservation contracts will be preserved by publishers. Consistent with our prediction, and despite its rapid growth in the past several years, industry reports show that the growth of RTB relative to total display advertising is slowing down. The spending on RTB, as a fraction of total display advertising in the US, is estimated to be 31%, 34%, 36% and 36% in 2015, 2016, 2017 and 2018, respectively (Fisher and Liu 2016; Liu 2016).

Related Literature

Our work is related to four streams of research in the literature:

Online Advertising Auctions. The increasing prevalence of online advertising auctions has motivated a growing body of theoretical papers in the marketing academic literature. Katona and Sarvary (2010) and Jerath et al. (2011) study advertisers' incentives in obtaining lower versus higher positions in search advertising auctions. Sayedi et al. (2014) investigate advertisers' poaching behavior on trademarked keywords, and their budget allocation across traditional media and search advertising. Desai et al. (2014) analyze the competition between brand owners and their competitors on brand keywords. Lu et al. (2015) and Shin (2015) study budget constraints, and budget allocation across keywords. Zia and Rao (2016) look at the budget allocation problem across search engines. Zhu and Wilbur (2011) and Hu et al. (2015) study the trade-offs involved in choosing between two types of contracts, namely "cost-per-click" and "cost-per-action" contracts. Wilbur and Zhu (2009) find the conditions under which it is in a search engine's interest to allow some click fraud. Cao and Ke (2016) model a manufacturer and retailers' cooperation in search advertising

⁹<http://www.businessinsider.com/header-bidding-gains-momentum-drives-up-publisher-ad-revenue-2016-5>

¹⁰<http://www.slideshare.net/SearchLaboratory/the-future-of-display-advertising-31126811>

and show how it affects intra-brand and inter-brand competition. Amaldoss et al. (2015a) show how a search engine can increase its profits and also improve advertisers' welfare by providing them first-page bid estimates. Berman and Katona (2015) study the impact of search engine optimization, and Amaldoss et al. (2015b) analyze the effect of keyword management costs on advertisers' strategies. Berman (2016) explores the effects of advertisers' attribution models on their bidding behavior and their profits. There are also several empirical papers that study search advertising auctions (e.g., see Yang and Ghose 2010, Rutz and Bucklin 2011, and Yao and Mela 2011). An important difference between search advertising and display advertising markets, which is also the focus of this paper, is that in search advertising the entire inventory is sold in auctions whereas display advertising impressions are sold in both auctions and reservation contracts.

Online Display Advertising. Several papers in marketing literature study various aspects of display advertising. Anand and Shachar (2011), Tucker (2014), Urban et al. (2013), and Hoban and Bucklin (2015) show that ad customization can improve the effectiveness of display advertising. Goldfarb and Tucker (2011) and Lambrecht and Tucker (2013) look at negative effects of ad customization. Lambrecht and Tucker (2013) show that too much ad customization can backfire at the time when consumers begin a product search and their preferences are construed at a high level. Goldfarb and Tucker (2011) show that targeting can negatively affect display ad effectiveness when paired with highly visible creative. Bleier and Eisenbeiss (2015) study the relation between timing and effectiveness of ad customization and show that customization works best when ads are shown on motive-congruent websites. Manchanda et al. (2006) analyze the effect of display advertising on customers' purchase behavior. They show that display advertising has a positive effect on purchase frequency for existing customers, and that this effect is greatest when consumers view a large number of web pages across a variety of websites. Bart et al. (2014) establish the effectiveness of mobile display advertising for high involvement and utilitarian products. Hoban and Bucklin (2015) analyze display ad effects for users at different stages of the purchase funnel and find that display advertising positively affects visitation to the advertiser's website for users in most stages of the purchase funnel, but not for those who previously visited the site without creating an account. Lu and Yang (2016) study the impact of the targeting breadth, which measures the percentage of a consumer's identified interests, on the behaviors and profits of consumers, advertisers, and the platform in behaviorally targeted display advertising. To the best of our knowledge, the

only other paper in marketing literature on real-time bidding is Johnson (2013) in which the author studies privacy policy decisions. In contrast, we provide a market-level analysis by studying the interactions of RTB and reservation contracts, and analyzing advertisers' and publishers' strategies in that regard.

Targeting in Advertising. Our paper also relates to the analytical work on targeting in advertising. Chen et al. (2001) and Iyer et al. (2005) show that improved targeting can increase advertisers' profits by softening competition. From a publisher's perspective, Levin and Milgrom (2010) argue that advertisers' improved targeting can lower the publisher's revenue by dispersing competition and making ad auctions thinner. Bergemann and Bonatti (2011) show that the equilibrium price of advertisements first increases and then decreases as advertisers' targeting capacity increases. In the context of online advertising auctions, Hummel and McAfee (2015) analyze the conditions under which improved targeting increases or decreases the publisher's revenue. Our results also indicate that improved targeting in RTB could potentially lower the publisher's revenue. However, we show that when a sufficiently large fraction of impressions are sold in reservation contracts, the overall effect of improved targeting in RTB on the publisher's revenue is positive.

Real-Time Bidding in Computer Science and Operations Management Literature. There is a growing body of papers in computer science and operations management literature that studies real-time bidding. Muthukrishnan (2009) is the first paper to discuss several research issues in ad exchange markets. Korula et al. (2015) provide a survey of some of the main display advertising problems from a mechanism design point of view. Several papers in operations management and computer science literature study various aspects of bid optimization problem in RTB from an advertiser's point of view (e.g., Chen et al. (2011), Lee et al. (2013), Yuan et al. (2013), and Zhang et al. (2014)). Another set of papers (e.g., Bharadwaj et al. 2010, Radovanovic and Heavlin 2012, Chen et al. 2014 and Balseiro et al. 2014) solve the allocation and revenue maximization problems of a publisher. In contrast, the focus of our paper is on advertisers' and publishers' equilibrium strategies and their equilibrium profits.

The rest of the paper is structured as follows. In Section 2, we present the model. In Section 3, we analyze the game and discuss the results. Section 4 includes several extensions of the main model in which show the robustness of our results and provide new insights. Finally, we conclude the paper in Section 5. All proofs are relegated to the appendix.

2 Model

The model consists of a publisher, two advertisers and a unit of consumers. Consumers are uniformly located on a Hotelling line represented by the $[0, 1]$ interval. The advertisers are located at the two endpoints of the interval: Advertiser 1 at 0, and Advertiser 2 at 1. The Hotelling distance between a consumer and an advertiser corresponds to the mismatch between the consumer's need and the advertiser's offering. For example, the proximity on the Hotelling line could represent the likelihood of purchase if exposed to an ad from that advertiser. As such, advertisers have higher valuation for showing their ads to consumers that are closer to them. The valuation of Advertiser 1 for showing his ad to a consumer located at x is $1 - tx$, and the valuation of Advertiser 2 for showing his ad to the same consumer is $1 - t(1 - x)$, where $t \leq 1$ is the travel cost in the Hotelling model, and corresponds to consumers' heterogeneity in taste.

Advertisers typically know more than publishers about consumers, particularly, regarding consumers' interest in the products that they advertise. Advertisers track consumers on several websites, and regularly purchase tracking information from data brokers, activities that publishers rarely engage in (Yuan et al. 2013). For example, consumers that have previously been searching for vacations in Barbados are very valuable to some travel companies, but a publisher might not even know that those consumers have been visiting his website. As such, we assume that the advertisers know the location of a consumer on the Hotelling line, but the publisher does not. In practice, a publisher may have some information about consumers, e.g., age range, gender, geographic location, etc., and this information could be allowed for targeting in reservation contracts. For example, advertisers may specify gender or geographic location in their reservation contracts. However, the amount of information that the publisher has, and the targeting capability of reservation contracts is much more limited than what advertisers can achieve in RTB. To facilitate exposition, we consider an extreme case¹¹ where no targeting is possible in reservation contracts, and the publisher has no information about the consumers. To reconcile this with reality, we can

¹¹We should note that we get similar results in a model where the publisher has some information about consumers. For example, consider the model where the publisher knows which subinterval $[0, \frac{1}{3})$, $[\frac{1}{3}, \frac{2}{3}]$ or $(\frac{2}{3}, 1]$ a consumer is located in, but does not know the exact location within that subinterval. In this model, the publisher sells all impressions in the first subinterval to Advertiser 1 and all impressions in the third subinterval to Advertiser 2, all at per-impression price $\frac{5}{6}$ in reservation contracts. The remaining impressions (subinterval $[\frac{1}{3}, \frac{2}{3}]$) are sold in an equilibrium very similar to what we find in Proposition 3: the impressions in $[\frac{5}{9}, \frac{2}{3}]$ are sold in RTB to Advertiser 2, and the impressions in $[\frac{1}{3}, \frac{5}{9}]$ are sold in reservation contracts to Advertiser 1.

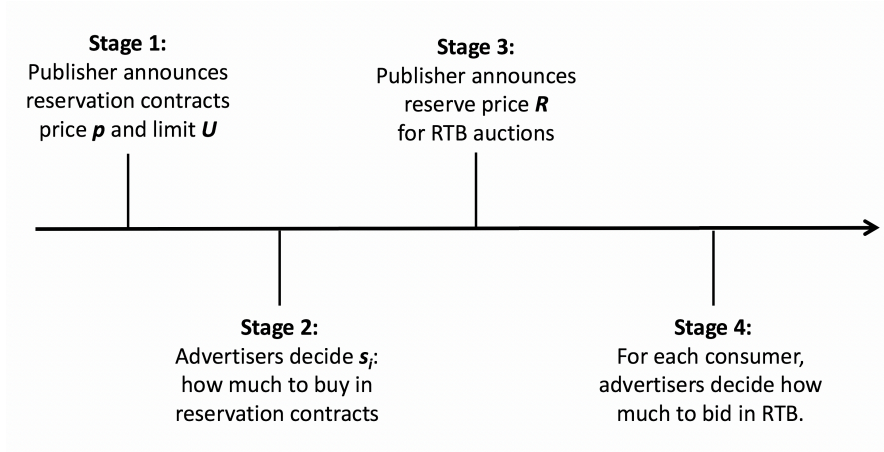


Figure 1: Timeline of The Game

think of the consumers in our model as a segment of all consumers that is already narrowed down by the information (e.g., age range, gender and location) available to the publisher.

We assume that the advertisers are profit maximizers. The profit of Advertiser i is the sum of his valuation for all impressions that he purchases, minus the total cost (what he pays the publisher). The publisher maximizes his revenue, i.e., what he collects from the advertisers. The timing of the game is as follows, and is summarized in Figure 1.

Stage 1: In the first stage, the publisher announces a per-impression price p and a maximum amount U available for sale in reservation contracts.¹²

Stage 2: In the second stage, advertisers decide how much inventory they want to buy, denoted by s_i , in reservation contracts. If the total demand is more than U , i.e., $U < s_1 + s_2$, we assume that the publisher first sells to Advertiser 1, and then offers the leftover, $U - s_1$, to Advertiser 2, i.e., $s_2 \leq U - s_1$.¹³

Stage 3: In the third stage, consumers arrive. The publisher chooses R , and makes each impression

¹²While the fixed price selling is the common practice, in Section 4.4, we propose an alternative selling mechanism for reservation contracts that could potentially improve the publisher's revenue.

¹³We show that other rationing rules, such as one where the advertiser who buys first is selected randomly, or one where the inventory is divided between the two advertisers proportionally, both lead to the same equilibrium outcome.

available on RTB in a second-price auction with reserve price R .¹⁴ Note that since the publisher does not know the consumers' locations, the same reserve price will be used for all impressions.¹⁵

Stage 4: In the fourth stage, advertisers decide how much they want to bid for each impression:

- If the highest bid is greater than or equal to R , the highest bidder wins the impression and pays the maximum of the second highest bid and R .¹⁶
- If the highest bid is less than R , and the demand of reservation contracts is not delivered yet, the impression will be allocated randomly to one of the ads in reservation contracts.
- If the highest bid is less than R and the demand of reservation contracts is already delivered, the impression will be left unallocated.
- Finally, if at any point in time, the amount of inventory sold in RTB becomes $1 - s_1 - s_2$, the publisher allocates the rest of the impressions to reservation contracts; i.e., the publisher ensures that the demand of reservation contracts is always delivered.

Note that we are not incorporating two features of RTB that we discussed in the introduction into the model yet. Specifically, at this point, we do not consider the publisher's supply uncertainty and the effectiveness of ad customization in improving consumers' probability of conversion. However, after establishing the key insights from the main model, in Section 4, we add these features to the model and discuss their roles.

¹⁴In practice, RTB auction is a slight variation of the second-price auction. Publishers set two reserve prices: a hard reserve price R , and a soft reserve price R' , where $R' > R$. If the highest bid is below R , no one wins. If the highest bid is above R' , then the highest bidder pays the maximum of R' and the second highest bid. Finally, if the highest bid is between R and R' , then the highest bidder pays his own bid (i.e., similar to a first-price auction). In this paper, we do not explicitly model the soft reserve price R' . However, we should note that, since the soft reserve price R' never affects the allocation, using the Revenue Equivalence Theorem, we know that the soft reserve price does not affect equilibrium profits of the publisher and the advertisers (see Krishna 2009). In other words, although advertisers may shade their bids in a model with soft reserve price, the equilibrium outcome in terms of profits and allocations are exactly the same with and without the soft reserve price. The hard reserve price, however, does affect equilibrium allocation, and therefore, has an important role in revenue maximization.

¹⁵In practice, publishers set their reserve prices in real-time, after consumers arrive. This is why we assume that R is set in Stage 3, and not in Stage 1. When we look at the publisher's supply uncertainty in Section 4.1, we show that the realized number of visitors, which is unknown in Stages 1 and 2, affects the equilibrium value of R .

¹⁶In practice, the publisher is not committed to assigning the impression to the winner of the RTB auction. In other words, after receiving RTB bids, the publisher may still decide to assign the impression to reservation contracts, even if the winning bid in RTB is above the reserve price. However, we can show that when the reserve price in RTB is optimized, the publisher never assigns an impression for which the RTB bid is above the reserve price to a reservation contract (even when there is no commitment). In other words, while this assumption simplifies the model, it does not affect the equilibrium outcome of the game. A formal proof is available upon request.

3 Analysis

We use backward induction to find subgame perfect equilibria of the game.

Fourth Stage

In the fourth stage, advertisers observe the location of each consumer and decide how much they want to bid. Note that since the unsold impressions in RTB are assigned to reservation contracts, advertisers may still win an impression that they did not bid for. In particular, even if an advertiser does not bid for an impression in RTB, he could still win that impression if the competitor does not win the impression in RTB either, *and* he is “lucky enough” so that the impression is assigned to his reservation contract (rather than his competitor’s). The price of an impression sold through RTB is at least R , and the price of an impression in reservation contracts is p . Therefore, when $R > p$, if the valuation of an advertiser for an impression is not sufficiently high (even if the valuation is higher than R), the advertiser may prefer to wait and win the impression at price p with some probability, than to bid and win the impression at price R .

In equilibrium, each advertiser bids for impressions that are most valuable to him, i.e., impressions that he does not want to risk losing. Let m_1 and m_2 be such that Advertiser 1 bids for and wins the impressions in $[0, m_1]$, and Advertiser 2 bids for and wins the impressions in $[m_2, 1]$. In equilibrium, we must have $m_2 - m_1 \geq s_1 + s_2$, i.e., the number of unsold impressions in RTB should be greater than or equal to the number of impressions sold in reservation contracts. Assuming that the reserve price R is sufficiently high,¹⁷ the expected profit of Advertiser 1 is

$$\pi_1 = \int_0^{m_1} 1 - tx - R \, dx + \int_{m_1}^{m_2} \frac{s_1}{m_2 - m_1} (1 - tx) - p \, dx. \quad (1)$$

The first term on the right-hand side is the expected profit from RTB, and the second term is the expected profit from reservation contracts. Fraction $\frac{s_1}{m_2 - m_1}$ is the probability that an unsold impression in RTB is assigned to reservation contract of Advertiser 1. Similarly, the expected profit of Advertiser 2 is

$$\pi_2 = \int_{m_2}^1 1 - t(1 - x) - R \, dx + \int_{m_1}^{m_2} \frac{s_2}{m_2 - m_1} (1 - t(1 - x)) - p \, dx. \quad (2)$$

¹⁷In the Appendix, we prove that this condition holds.

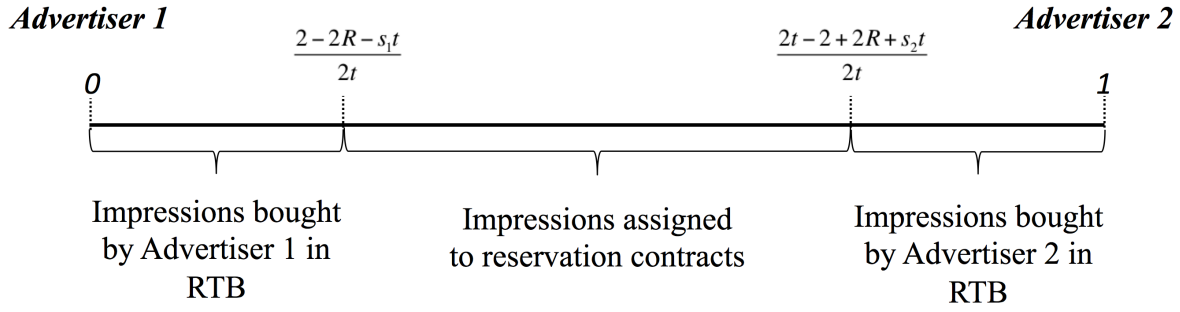


Figure 2: Allocation of impressions to RTB and reservation contracts as presented in Proposition 1.

Maximizing π_1 with respect to m_1 , and π_2 with respect to m_2 give us advertisers' bidding strategies in RTB auctions, summarized in Proposition 1.

Proposition 1 *Advertiser 1 bids truthfully for impressions to consumers located in $[0, \frac{2-2R-s_1t}{2t}]$, and does not bid for impressions to other consumers. Similarly, Advertiser 2 bids truthfully for impressions to consumers located in $[\frac{2t-2+2R+s_2t}{2t}, 1]$, and does not bid for impressions to other consumers.*

Proposition 1 shows that advertisers cherry-pick when bidding in RTB auctions. In other words, each advertiser only bids for his most valuable segment of consumers. As the reserve price increases, the segment of consumers for which each advertiser bids shrinks. This is an intuitive result and is driven by the fact that as the reserve price increases, the expected payment of each advertiser in an RTB auction increases. Interestingly, Proposition 1 also shows that as the size of Advertiser i 's reservation contract, s_i , increases, Advertiser i participates in fewer RTB auctions. There are two forces that drive this result. First, as s_i increases, the probability that Advertiser i is assigned an unsold impression in RTB increases. As such, the advertiser is more willing to take the risk of not bidding in the RTB auction. Second, buying in RTB negatively affects the expected value of impressions that are allocated to reservation contracts. In other words, when Advertiser i cherry-picks using RTB, he is negatively affecting the average value of his own impressions in reservation contracts. Therefore, as s_i increases, Advertiser i participates in fewer RTB auctions. Proposition 1 implies that Advertiser i bids for an impression in RTB if and only if his valuation is at least $R + \frac{s_i t}{2}$. The result of Proposition 1 is depicted in Figure 2

Third Stage

Given the advertisers' bidding strategies, the publisher sets a reserve price that maximizes his revenue. The publisher sells $s_1 + s_2$ impressions at price p in reservation contracts and $\min(1 - s_1 - s_2, m_1 + 1 - m_2)$ impressions in RTB at price R (assuming that R is sufficiently large). The publisher's revenue can be written as

$$\Pi = (s_1 + s_2)p + R \min(1 - s_1 - s_2, m_1 + 1 - m_2).$$

By using the values of m_1 and m_2 from Proposition 1, we can write the publisher's revenue as

$$\Pi = (s_1 + s_2)p + R \min\left(1 - s_1 - s_2, \frac{2 - 2R - s_1 t}{2t} + 1 - \frac{2t - 2 + 2R + s_2 t}{2t}\right).$$

Maximizing Π with respect to R gives us the optimal reserve price of RTB auctions.

Proposition 2 *For given s_1 , s_2 and t , the optimal reserve price for the publisher is given by*

$$R^* = 1 - \frac{(2 - s_1 - s_2)t}{4}.$$

Proposition 2 shows that as the amount of inventory sold in reservation contracts, $s_1 + s_2$, increases, the publisher increases the reserve price of RTB auctions. This is despite the result of Proposition 1 which shows that as s_i increases, Advertiser i 's willingness to participate in RTB auctions decreases. In other words, given the result of Proposition 1, one would expect the publisher to lower his reserve price as $s_1 + s_2$ increases to compensate for the advertisers' lower interest in RTB auctions. However, Proposition 2 shows the opposite.

Intuitively, as $s_1 + s_2$ increases, the publisher can allocate more unsold inventory of RTB auctions to reservation contracts. This allows the publisher to set a higher reserve price. If the unsold impressions in RTB auctions could not be "repurposed," as is the case in (non-RTB) second-price auctions in many other contexts, the optimal reserve price would have been $1 - \frac{t}{2}$. This is the same as R^* for $s_1 = s_2 = 0$. Since reservation contracts enable the publisher to repurpose the unsold impressions of RTB auctions, their existence allows the publisher to set a higher reserve price, and ultimately extract more revenue in RTB.

Second Stage

In the second stage, the advertisers have to decide how many impressions they want to buy in reservation contracts. In Equations (1) and (2), if we replace the values of R and m_i with those from Propositions 1 and 2, the profit of Advertiser i can be written as

$$\pi_i = \frac{\left(5s_i^2 + s_j^2 - 6s_i s_j - 20s_i - 4s_j + 4\right)t + 32(1-p)s_i}{32}$$

where $j = 3 - i$ is the index of the other advertiser. The second derivative of π_i with respect to s_i is $\frac{5t}{16} > 0$ which implies that π_i is a convex function of s_i .

There are two effects that make Advertiser i 's profit a convex function of s_i . First, as an advertiser buys more impressions in reservation contracts (i.e., s_i increases), according to Proposition 2, the optimal reserve price in RTB increases. Therefore, as s_i increases, impressions in reservation contracts become more appealing than before compared to the impressions in RTB. Second, since impressions are cherry-picked in RTB, as s_i increases, the marginal expected value of Advertiser i for an impression in reservation contracts increases. In other words, every impression that moves from RTB to reservation contracts (as s_i increases) is a “cherry” that was being picked in RTB before, and is more valuable than all previous impressions assigned to reservation contracts. Therefore, as s_i increases, Advertiser i 's incentive to buy impressions in reservation contracts also increases.

Since π_i is a convex function of s_i , an advertiser either buys no impressions in reservation contracts (when the price p is too high), or buys the maximum available quantity. The condition on price p is presented in Lemma 1.

Lemma 1 *Suppose that quantity $U - s_j$ is available for Advertiser i to buy in reservation contracts. If $p > \frac{5Ut+32-20t-11s_jt}{32}$, Advertiser i does not buy any impressions in reservation contracts; otherwise, he buys all of the available impressions, $U - s_j$.*

Lemma 1 shows that as the amount of inventory available for reservation contracts, $U - s_j$, increases, Advertiser i 's willingness to pay per impression in reservation contracts increases. As discussed before, this is driven by the fact that the advertiser's profit is a convex function of the amount of inventory that he buys in reservation contracts.

Lemma 1 also shows that as the size of the competitor's reservation contract, s_j , increases,

Advertiser i 's willingness to pay for impressions in reservation contracts decreases. There are two reasons for this effect. The first reason is that, as s_j increases, there are fewer impressions available for Advertiser i , and due to the convexity Advertiser i 's profit in s_i , Advertiser i 's willingness to pay decreases. However, this is not the only reason. In fact, if we replace U with $U + s_j$ so that the number of impressions available for Advertiser i is kept constant, as s_j increases, Advertiser i 's willingness to pay for impressions in reservation contracts still decreases.

This is a counter-intuitive effect because an increase in s_j leads to a higher reserve price in RTB, which, intuitively, should motivate Advertiser i to buy more impressions in reservation contracts; but Lemma 1 shows the opposite. This is because as s_j increases, according to Proposition 1, Advertiser j bids in fewer RTB auctions. Therefore, many of the impressions that are valuable to Advertiser j , thus not valuable to Advertiser i , will be left unsold in RTB. This reduces the expected valuation of Advertiser i for the unsold impressions in RTB, i.e., impressions that are assigned to reservation contracts. As such, Advertiser i 's willingness to pay per impression in reservation contracts decreases as the size of the reservation contract of Advertiser j , s_j , increases.

First Stage

Each advertiser's willingness to pay per impression in reservation contract increases as the number of impressions in the contract increases. Therefore, for a given total quantity U , the publisher's revenue is maximized if all of the impressions are sold to one advertiser. The publisher sets the price of reservation contracts, p , such that the advertisers are indifferent between buying nothing and buying all of the available inventory U . In equilibrium, one advertiser, which we assume without loss of generality is Advertiser 1, buys ad inventory U in reservation contracts, and Advertiser 2 buys nothing.

Proposition 3 *The publisher makes $U = \frac{2}{3}$ impressions available for reservation contracts at price $p = 1 - \frac{25t}{48}$, and sets the reserve price of RTB auctions to $R^* = 1 - \frac{t}{3}$. Advertiser 1 buys all of the U impressions in reservation contracts, and does not bid in RTB. Advertiser 2 only bids for consumers who are located in $[\frac{2}{3}, 1]$. The publisher's total revenue is $1 - \frac{11t}{24}$, Advertiser 1's profit is $\frac{t}{8}$, and Advertiser 2's profit is $\frac{t}{18}$.*

Proposition 3 summarizes how RTB affects advertisers' and publishers' selling and buying strate-

gies in equilibrium. In Corollary 1, we use the result of Proposition 3 to show that the publisher's revenue under the current selling mechanism is higher than two alternatives.

Corollary 1 *Given the publisher's equilibrium revenue in Proposition 3, we have:*

- *The publisher's revenue when selling $U = \frac{2}{3}$ of the impressions in reservation contracts and the rest in RTB is higher than when selling all of the impressions in RTB.*
- *The publisher's revenue when he makes all of the impressions available on RTB, and allocates the impressions that do not receive sufficiently high RTB bids to reservation contracts, is higher than when he allocates all of the impressions to reservation contracts until reservation contracts are fulfilled, and makes the leftover available on RTB.*

The results of Proposition 3 and Corollary 1 have several interesting implications for advertisers and publishers:

1. *Future of Display Advertising.* Since RTB is a new technology that allocates impressions to advertisers more efficiently, many analysts believe that RTB will replace reservation contracts in the future. In contrast, Corollary 1 shows that, since the publisher uses reservation contracts to increase the reserve price of RTB auctions, RTB will not replace reservation contracts in equilibrium. In fact, existence of reservation contracts allows the publisher to achieve a higher total revenue.

Proposition 3 shows that two third of the impressions are sold in reservation contracts in equilibrium. While this number is close to industry reports as of 2017, we should note that in the extensions that we consider in Section 4, depending on the model parameters, this number might change. In particular, we show that as the publisher's uncertainty about the number of his future visitors increases (Section 4.1), as the number of premium consumers increases (Section 4.2), or as the effectiveness of ad customization increases (Section 4.3), the fraction of impressions that are sold in reservation contracts decreases.

2. *Price of Impressions in Reservation Contracts.* Since advertisers can cherry-pick impressions in RTB, impression assigned to reservation contracts become less valuable when RTB exists. In other words, RTB negatively affects advertisers' willingness to pay for impressions in reservation contracts. A few papers in computer science and operations management literature

(e.g., see Ghosh et al. 2009 and Balseiro et al. 2014) tackle this problem by proposing methods to maintain the quality of impressions assigned to reservation contracts, e.g., by not making all of the impressions available in RTB.¹⁸

In contrast, we show that the publisher's optimal strategy is to allow cherry-picking in RTB, and to set a lower price for impressions in reservation contracts. In particular, as we show in Proposition 3, the price of impressions in reservation contracts when RTB is available, $1 - \frac{25t}{48}$, is lower than when it is not, $1 - \frac{t}{2}$. Interestingly, despite the reduction in the price of impressions in reservation contracts, Proposition 3 shows that the publisher's total revenue increases with RTB. In other words, we show that as long as the reserve price in RTB is set optimally, the high price of impressions in RTB compensates for the low price of impressions in reservation contracts such that the publisher's total revenue when RTB exists is higher than when it does not.

3. *Advertisers' Asymmetric Strategies.* Since advertisers know that buying in reservation contracts increases the reserve price of RTB, they may want to avoid reservation contracts even if the price is lower than their expected valuation. However, for a sufficiently low price of impressions in reservation contracts, advertisers use asymmetric strategies where Advertiser 1 buys all of his ad inventory in reservation contracts, and, therefore, is not affected by the higher reserve price in RTB. Advertiser 2 buys the inventory at a high reserve price in RTB, but, since RTB allows for cherry-picking, still benefits from existence of RTB.

We should note that the result that Advertiser 1 does not bid in RTB is moderated under two of the extensions that we consider in Section 4. In particular, when there is a segment of premium consumers for which both advertisers have high valuation (Section 4.2), we show that both advertisers bid in RTB for premium consumers. Furthermore, if we consider publisher's supply uncertainty (Section 4.1), again, both advertisers participate in RTB auctions. Nonetheless, in both extensions, the advertisers still use asymmetric strategies, and the key insights of Proposition 3 continue to hold.

4. *Advertisers' and Publisher's Profits.* Since RTB increases the overall efficiency of allocation,

¹⁸For example, Ghosh et al. (2009) address this issue by directly bidding in RTB auctions on behalf of contract-buying advertisers, randomly varying the bid in order to get a representative/fair allocation in reservation contracts.

all parties benefit from its existence. Proposition 3 shows that RTB improves the expected revenue of the publisher from $1 - \frac{t}{2}$ to $1 - \frac{11t}{24}$, and the expected profits of Advertisers 1 and 2 from 0 to $\frac{t}{8}$ and $\frac{t}{18}$, respectively. Interestingly, Advertiser 1 who does not use RTB benefits more from existence of RTB than Advertiser 2 who only uses RTB.

5. *Header Bidding.* As discussed in the introduction, header bidding changes the order in which the publisher allocates impressions to reservation contracts and RTB. Before header bidding, the publisher allocated all of his impressions to reservation contracts until those contracts were fulfilled; then, made the leftover available on RTB. With header bidding, the publisher receives bids on all of his impressions, and then decides whether to allocate each impression to RTB or reservation contracts. Corollary 1 shows that the publisher's revenue under the new mechanism, i.e., with header bidding, is higher than the old mechanism. This finding is consistent with the trend in RTB market that shows a rapid adoption of header bidding by publishers in 2015 and 2016.

4 Extensions

In the previous section, we used a simple model to discuss the effects of RTB on advertisers' and publishers' strategies. In this section, we consider several extensions of the model to capture additional features of the market and to assess the robustness of our original findings.

In developing our model, we made a few simplifying assumptions to facilitate the exposition of our key results. For example, thus far we have assumed that the publisher knows the exact number of future visitors at the time of selling impressions in reservation contracts. In Section 4.1, we relax this assumption and show the robustness of our results. Furthermore, we show that under supply uncertainty, RTB increases social surplus, and that both advertisers and the publisher benefit from the increase in social surplus.

In the preceding analysis, we examined a market where consumers are located on a Hotelling line. In Section 4.2, we consider two additional segments of consumers: "premium consumers," which both advertisers highly value, and "unsusceptible consumers," for which both advertisers have low valuation. We find that both advertisers bid for impressions to premium consumers in RTB, and that all such impressions are sold in RTB. However, advertisers do not bid for impressions

to unsusceptible consumers, and all those impressions are assigned to reservation contracts. We show that the advertisers still use the asymmetric strategies discussed in Proposition 3 for buying the impressions to consumers on the Hotelling line.

In Section 4.3, we analyze the ad customization (also referred to as “personalization” in the literature) effect of RTB. Since advertisers can customize their ads for each individual consumer in RTB, they can increase the probability of conversion given impression, and therefore, would have higher willingness to pay for the same impression in RTB than in reservation contracts. In Section 4.3, we assess the robustness of our results in presence of ad customization. Interestingly, we also find that the advertisers’ profits may decrease as their ad customization becomes more effective (i.e., as the probability of conversion due to ad customization increases).

Finally, in Section 4.4, motivated by the growth of *Programmatic Reservation Contracts*, we propose a new selling mechanism for impressions in reservation contracts. We show that if the impressions in reservation contracts are sold simultaneously, e.g., in an auction, rather than sequentially, which is the common practice, the publisher achieves a higher revenue. The additional revenue is due to the negative externalities that reservation contracts impose on RTB by increasing the optimal reserve price of RTB. We also show that our findings in Proposition 3 continue to hold under the new selling mechanism.

4.1 Publishers’ Supply Uncertainty

In practice, publishers do not know how many visitors their website will have in the future. As such, publishers are uncertain about the size of their supply when selling impressions in reservation contracts. Before RTB, this uncertainty, along with the fact that reservation contracts include penalties for underdelivery, made the decision of how many impressions to sell in reservation contracts a major challenge for many publishers, and publishers were often left with unsold impressions. RTB was introduced as a mechanism to sell the leftover inventory in real-time. In this section, we study how RTB affects advertisers’ and publishers’ strategies under supply uncertainty.

We assume that the supply of impressions (e.g., number of visitors) is $1 + D$ where D is either 0 or $d \geq 0$, each with probability $\frac{1}{2}$. The size of the supply (i.e., the value of D) is unknown in Stages 1 and 2, during the sale of impressions in reservation contracts. We assume that the penalty of underdelivery in reservation contracts is sufficiently high so that the publisher does not

sell more than 1 unit in reservation contracts. Since the publisher can change the reserve price in real-time in response to changes in the number of visitors, we assume that the supply is realized at the beginning of Stage 3, before the publisher sets the reserve price. Proposition 4 summarizes the equilibrium strategies under supply uncertainty.

Proposition 4 *The publisher makes $U = \frac{4(d+1)}{3(d+2)}$ units available for reservation contract at price $p = 1 - \frac{25t}{48}$. Advertiser 1 buys all of the U units in reservation contracts. The publisher's expected revenue is $\frac{1}{64} \left(\frac{16(d+1)t}{3(d+2)} + 16(d+2)(2-t) \right)$, the expected profit of Advertiser 1 is $\pi_1 = \frac{1}{16}(d+2)t$, and the expected profit of Advertiser 2 is $\pi_2 = \frac{d(9d+16)+16t}{144(d+2)}$. When the supply is $1+d$, the publisher sets the reserve price of RTB to $R = 1 - \frac{(3d+4)t}{6(d+2)}$, and when the supply is 1, the reserve price is set to $R = 1 - \frac{(d+4)t}{6(d+2)}$. Advertiser 1 bids for consumers located in $[0, \frac{1-R}{t} - \frac{U}{2(1+D)}]$, and Advertiser 2 bids for consumers located in $[\frac{R+t-1}{t}, 1]$.*

Proposition 4 shows the robustness of our findings in Proposition 3. In particular, it shows that advertisers use asymmetric strategies where one advertiser buys most of his impressions in reservation contracts and the other advertiser buys all of his impressions in RTB. Proposition 4 also shows that under supply uncertainty, RTB increases the social surplus, and that both advertisers and the publisher benefit from this increase.

4.2 Premium Consumers and Unsusceptible Consumers

In this section, in addition to the main segment of consumers with mass 1 that we considered before, we consider two new segments. First, we consider a segment of *premium consumers* with mass P for which both advertisers have high valuation. This segment represents consumers that are identified by both advertisers as having high probability of conversion. We assume that the value of each advertiser for an impression to a consumer in this segment is 1. Second, we consider a segment of *unsusceptible consumers* with mass Q for which both advertisers have low valuation. This segment corresponds to consumers who are identified by both advertisers as having low probability of conversion (e.g., because they use ad blockers, or have already purchased the product). We assume that the value of each advertiser for an impression to a consumer in this segment is 0. As before, the advertisers know their valuation for each consumer. However, the publisher does not know the advertisers' valuations, and, specifically, does not know if a consumer is in premium

segment or unsusceptible segment. Proposition 5 shows that the existence of these segments does not affect any of our findings in the main model.

Proposition 5 *The publisher makes quantity $U = \frac{2}{3} + Q$ available for reservation contracts at price $p = \frac{48-25t}{72Q+48}$, and sets the reserve price of RTB at $R^* = 1 - \frac{t}{3}$. Advertiser 1 buys all of the U units in reservation contracts. Both advertisers bid truthfully for RTB impressions to premium consumers, and all those impressions are sold in RTB at price 1. Advertiser 1 does not bid for non-premium impression in RTB, and Advertiser 2 only bids for non-premium impressions to consumers who are located in $[\frac{2}{3}, 1]$. The publisher's total revenue is $1 + P - \frac{11t}{24}$, Advertiser 1's profit is $\frac{t}{8}$ and Advertiser 2's profit is $\frac{t}{18}$.*

When advertisers' valuations for a segment of consumers are sufficiently high, both advertisers bid for those impressions in RTB. In the extreme case where advertisers' valuations are equal, advertisers' surplus will be zero. Therefore, the publisher would be the only one to benefit from existence of RTB for premium consumers. In practice, however, since the valuations are not the same, advertisers also gain positive surplus.

On the other hand, when advertisers' valuations for a segment of consumers are sufficiently low, advertisers do not bid for those impressions in RTB. Therefore, all those impressions are assigned to reservation contracts. As such, the per-impression price of reservation contracts in presence of unsusceptible consumers is lower than in the main model. The advertiser that buys the impressions in bulk in reservation contracts gets the Q impressions in unsusceptible segment, but since he pays a lower price for impressions in reservation contracts, his profit remains unaffected.

We should also note that by using a Hotelling line in the main model, we implicitly assumed that advertisers' valuations for impressions are negatively correlated. As in most papers that use the Hotelling model, this assumption is motivated by the facts that firms are differentiated in their offerings and consumers are heterogenous in their tastes. This assumption also allowed us to capture the competition dispersion effect of RTB, which is known to be an issue in online advertising markets (Levin and Milgrom 2010; Rafeian and Yoganarasimhan 2017). In practice, however, advertisers' valuation may be positively correlated for some consumers. Proposition 5 shows the robustness of our results from Proposition 3 when advertisers' valuations are positively

correlated for some consumers (premium consumers and unsusceptible consumers).¹⁹

To summarize, for premium consumers, since advertisers' valuations are positively correlated, both advertisers bid in RTB, and the impressions are sold at a price higher than the reserve price. For the main segment of consumers, however, since advertisers' valuations are negatively correlated, one advertiser picks his impressions in RTB and leaves the rest for the other advertiser. In this case, the publisher has to use a relatively high reserve price in RTB to optimize his revenue. Finally, the impressions to unsusceptible consumers are assigned to reservation contracts; however, since their presence lowers the price of impressions in reservation contracts, they do not affect the publisher or the advertisers' equilibrium profits.

4.3 Ad Customization

In the main model, we only considered the fine-grained targeting effect of RTB. In practice, advertiser can also customize (also referred to as personalize) their ads when using RTB. This allows the advertisers to increase the probability of conversion given an impression. We assume that the value of an advertiser for a customer with distance x , when customizing the ad using RTB, is $1 - \alpha tx$ (instead of $1 - tx$ in the main model) where $\alpha < 1$. As α decreases, the advertisers' ability to influence consumers by customizing their ads increases. When α is 0, the customization is so strong such that each consumer perceives the advertiser as being located at the place as himself on the Hotelling line (i.e., perceived perfect match). Note that α only affects advertisers' valuations for impressions that are sold in RTB; the advertisers' valuation for impressions in reservation contracts remain the same as before.

Proposition 6 *The publisher makes $U = \frac{2\alpha}{2\alpha+1}$ units available for reservation contracts at price $p = \frac{32\alpha+16-t(4\alpha(\alpha+5)+1)}{32\alpha+16}$. Advertiser 1 buys all of the U units in reservation contracts. The publisher's expected revenue is $\frac{\alpha(16-t(2\alpha+9))+8}{16\alpha+8}$, the expected profit of Advertiser 1 is $\pi_1 = \frac{t\alpha}{8}$, and the expected profit of Advertiser 2 is $\pi_2 = \frac{t\alpha}{2(2\alpha+1)^2}$. The publisher sets the reserve price of RTB to $R = \frac{2\alpha+1-t\alpha}{2\alpha+1}$. Advertiser 1 does not bid in RTB, and Advertiser 2 bids for consumers located in interval $[\frac{2\alpha}{2\alpha+1}, 1]$.*

¹⁹We could also confirm that when advertisers' valuations are independent draws from uniform $U[0, 1]$ distribution, the equilibrium outcome is similar to that of Proposition 3. The publisher sets the price of impressions in reservation contracts to $\frac{7}{18}$ (slightly less than $\frac{1}{2}$, the price that it would set if RTB did not exist) and sets the reserve price of RTB to $\frac{3}{4}$ (which is higher than $\frac{1}{2}$, the optimal reserve price if all impressions were sold in RTB). Advertiser 1 buys $\frac{3}{4}$ of the inventory in reservation contracts, and Advertiser 2 bids for impressions for which his valuation is at least $\frac{3}{4}$ in RTB.

Proposition 6 shows that as customization becomes stronger, the publisher sells fewer impressions in reservation contracts. Interestingly, although customization is not possible in reservation contracts, the publisher increases the price of reservation contracts as customization becomes stronger. This is because customization increases the advertisers' willingness to pay, and hence the reserve price, in RTB auctions. As the reserve price in RTB increases, the publisher could also set a higher price for impressions in reservation contracts.

Another interesting finding in Proposition 6 is the effect of customization on the advertisers' profits. Intuitively, one would expect the advertisers' profits to increase as they become better in ad customization. However, Proposition 6 shows that as customization becomes stronger (i.e., α decreases), Advertiser 1's equilibrium profit decreases and Advertiser 2's equilibrium profit first increases and then decreases.

As α decreases, the price of reservation contracts, p , increases, and the number of units sold in reservation contracts, U , decreases. Both of these effects lower the expected profit of Advertiser 1. Furthermore, since Advertiser 1 does not bid in RTB auctions, he does not benefit from stronger customization. Therefore, as customization becomes stronger (i.e., α decreases), the expected profit of Advertiser 1 decreases.

As α decreases, Advertiser 2's valuation for each RTB impression increases. Furthermore, since lower α leads to lower U , the number of impressions that Advertiser 2 buys in RTB, $1 - U$, also increases. Both of these forces positively affect the expected profit of Advertiser 2. However, as α decreases, the publisher increases the reserve price of RTB auctions. This negatively affects the expected profit of Advertiser 2. In the most extreme case, where $\alpha = 0$, Advertiser 2's valuation for all impressions is 1. In this case, the publisher sets the reserve price to 1 which leaves Advertiser 2 with 0 profit. Overall, as customization becomes stronger (i.e., α decreases), the expected profit of Advertiser 2 first increases, and then decreases.

4.4 A Different Mechanism for Selling Impressions in Reservation Contracts

In the main model, we considered a selling mechanism where the publisher sets a price p and a limit U for reservation contracts. The contract is first offered to one advertiser (which we assumed without loss of generality is Advertiser 1), where he can buy as many units as he wants. Then, the rest is offered to the other advertiser (Advertiser 2). We showed that for limit $U = \frac{2}{3}$ if price

p is more than $1 - \frac{25t}{48}$, both advertisers buy 0 in reservation contracts. In fact, the publisher's revenue, using this selling mechanism, is maximized when the price is $p = 1 - \frac{25t}{48}$. While this selling mechanism is common in practice, in this section, we show that there is room for improvement.

Using Proposition 2, we know that as the number of impressions sold in reservation contracts increases, the reserve price of RTB increases. In other words, an advertiser's expected profit in RTB decreases when the competitor buys impressions in reservation contracts. Due to this negative externality, advertisers are willing to pay a higher price for impressions in reservation contracts *if* they know that by not doing so, their competitor would buy those impressions. As such, the publisher could use the *threat* of giving the impressions to competitor to increase the price of impressions in reservation contracts.

When the price of impressions in reservation contracts is set to $p = 1 - \frac{25t}{48}$, Advertiser 1 is indifferent between the two outcomes: one in which he buys all U impressions in reservation contracts, and one in which neither advertiser buys any impressions in reservation contracts. If the price is set to $p > 1 - \frac{25t}{48}$, Advertiser 1 uses backward induction. He knows that if he does not buy, then the impressions will be offered to Advertiser 2 at that price. In that case, since Advertiser 2 would already know that Advertiser 1 has not bought any impressions in reservation contracts, he would not buy any impressions either. Knowing this, Advertiser 1 refuses to buy any impressions in reservation contracts at price $p > 1 - \frac{25t}{48}$. Therefore, the publisher's revenue would be maximized at $p = 1 - \frac{25t}{48}$ when reservation contracts are offered sequentially.

However, if impressions in reservation contracts are offered simultaneously to both advertisers, e.g., in an auction, so that each advertiser knows that if he does not win the reservation contract, his competitor does, advertisers would pay a higher price for impressions in reservation contracts. In other words, each advertiser has to decide between these two outcomes: one in which he buys all U impressions in reservation contracts, and one in which his competitor buys all U impressions in reservation contracts. Therefore, as we show in Proposition 7, advertisers pay a higher price for impressions in reservation contracts in equilibrium.

Proposition 7 *If the publisher sells the impressions in reservation contracts in a second price auction, both advertisers bid $1 - \frac{5t}{12}$ per impression in equilibrium. Advertiser 1 wins the reservation contract at price $1 - \frac{5t}{12}$ per impression, and does not bid in RTB. Advertiser 2 only bids in RTB using*

the same strategy as in Proposition 3. The expected total profit of each advertiser is $\pi_1 = \pi_2 = \frac{t}{18}$, and the expected revenue of the publisher is $1 - \frac{7t}{18}$.

Proposition 7 shows that the publisher can increase his revenue by selling the impressions in reservation contracts in an auction. The same revenue gain could be achieved if the publisher offers the contract at a higher price to Advertiser 1, but commits to lower the price for Advertiser 2 so that Advertiser 1 knows that if he does not buy the impressions, Advertiser 2 would buy. Therefore, due to the negative externalities, Advertiser 1 would pay a higher price to buy the impressions in reservation contract.

Also, it is interesting to note that Advertiser 1's advantage over Advertiser 2, in the main model, disappears when reservation contracts are sold in auctions. In particular, since Advertiser 1 has a right of first purchase in the main model, he earns a higher profit, $\pi_1 = \frac{t}{8}$, than Advertiser 2, $\pi_2 = \frac{t}{18}$. However, when reservation contracts are sold in auction, both advertisers have the same profit, $\pi_1 = \pi_2 = \frac{t}{18}$.

We should mention that changing the selling mechanism in practice might be challenging for publishers for two reasons. First, advertisers need to change their equilibrium strategies for the publisher to gain a higher revenue. For example, if advertisers (temporarily) keep using the same strategies as in Proposition 3 after the mechanism changes, the publisher's revenue from reservation contracts would (temporarily) drop to 0. In general, this is a common obstacle whenever a seller wants to change his selling mechanism.²⁰ Second, since reservation contracts are traditionally negotiated in person, implementing a new mechanism, in particular a simultaneous auction, might be challenging from an operational point of view. However, given that a growing number of impressions in reservation contracts are being bought and sold by computer programs (Fisher and Liu 2016), we expect the implementation of the proposed mechanism to become easier in the future.

²⁰For example, Varian and Harris (2014) mention this as one of the reasons that Google did not change its search advertising auctions from GSP to VCG.

5 Conclusion

Real-time bidding in display advertising is a new and growing model for buying and selling advertising space online. In this paper, we study the implications of RTB on advertisers' and publishers' strategies and their profits. Our theoretical analysis offers useful insights on several issues of managerial significance.

Implications for Publishers. RTB has several advantages for publishers. First, since it allows for fine-grained targeting and ad customization, RTB increases advertisers' willingness to pay for impressions. Second, since RTB unbundles the impressions in display advertising, it increases the overall efficiency of ad allocation, and improves social surplus in the market. Third, since impressions are sold in real-time, RTB resolves publishers' supply uncertainty by allowing them to sell the impressions that they could not anticipate.

RTB also has some negative effects on publishers' revenue. First, RTB could lower publishers' revenue by making ad auctions thinner, i.e., dispersing competition. In fact, RTB auctions are known to have relatively few bids per impression. Second, since RTB allows for cherry-picking of impressions in real-time, it negatively affects the quality of impressions assigned to reservation contracts. As such, advertisers' willingness to pay for impressions in reservation contracts declines, which in turn lowers publishers' revenue from reservation contracts. Our results have the following implications for publishers.

- Despite RTB's higher efficiency in ad allocation, and advertisers' higher willingness to pay for RTB impressions, publishers should not phase out reservation contracts. Reservation contracts allow publishers to set higher reserve prices in RTB, and achieve higher total revenue.
- Since advertisers can cherry-pick impressions in RTB, their willingness to pay for impressions assigned to reservation contracts decreases. Publishers should not fight this cherry-picking. Instead, they should lower the price of impressions in reservation contracts so that advertisers still buy impressions in reservation contracts, and increase the reserve price of RTB auctions to achieve a higher total revenue.
- As advertisers become more effective in ad customization, i.e., the probability of conversion due to ad customization increases, publishers should lower the amount of inventory allocated

to reservation contracts and increase the reserve prices of RTB.

Implications for Advertisers. RTB has several advantages for advertisers as well. First, RTB allows advertisers to only bid, and pay, for impressions that they want. Second, advertisers can customize their ads, and thus increase the probability of conversion, for each consumer in RTB. Third, since RTB allows advertisers to bid only on impressions to consumers in their target market, advertisers can differentiate on the impressions they bid on, and thus, can avoid head-on competition for all impressions.

RTB could also negatively affect advertisers' profits. Advertisers typically have to pay a higher price for impressions in RTB. Furthermore, since impressions are cherry-picked in RTB, advertisers who rely on reservation contracts receive lower-quality impressions on average. Our results have the following implications for advertisers.

- Advertisers use asymmetric strategies. An advertiser could focus on reservation contracts by purchasing large quantities of impressions at low prices (lower than before RTB), or focus on RTB by cherry-picking impressions at high prices. While both advertisers benefit from existence of RTB, the advertiser that focuses on reservation contracts benefits more.
- An advertiser who focuses on reservation contracts should still participate in RTB. In particular, when there is a segment of premium consumers that both advertisers highly value, even an advertiser who focuses on reservation contracts should bid for impressions to premium consumers in RTB. Furthermore, in case of publisher's supply uncertainty, an advertiser that focuses on reservation contracts should also participate in RTB, but should only bid on impressions that he *highly* values, i.e., impressions for which the valuation is sufficiently higher than the reserve price.
- An advertiser who focuses on RTB should bid on all impressions in RTB for which his valuation is above the reserve price, and should not buy impressions in reservation contracts.

Future Research. Our work is a first step towards studying the impact of real-time bidding in display advertising on publishers' and advertisers' strategies and their profits. There are several opportunities for further research. For example, we do not model budget constraints in this paper. Modeling budget constraints and understanding their effects on advertisers' adoption of RTB could

lead to valuable insights. Next, advertisers and publishers often use agencies for buying and selling RTB impressions. It would be interesting to investigate how the insertion of an intermediary, which may introduce agency considerations, affects our insights. Exchange platforms such as OpenX and Google's DoubleClick are trying to consolidate the market by also acting as advertisers' and publishers' agents. It would be interesting to understand whether exchange platforms have the same incentives as third-party agents or not. Future work can also study the competition between exchange platforms, and how it affects advertisers' and publishers' strategies.

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Proofs

Proof of Proposition 1

Assume that Advertiser 1 buys the impressions in interval $[0, m_1]$ and Advertiser 2 buys the impressions in $[m_2, 1]$. First consider an equilibrium (if exists) in which the demand for auctions is less than or equal to $1 - s_1 - s_2$. In such equilibrium the profit of each advertiser is:

- $\pi_1 = m_1 \left((1 - t \frac{m_1}{2}) - R \right) + s_1 \left((1 - t \frac{m_1 + m_2}{2}) - p \right)$
- $\pi_2 = (1 - m_2) \left(1 - t \frac{1 - m_2}{2} - R \right) + s_2 \left(1 - t \left(1 - \frac{m_1 + m_2}{2} \right) - p \right)$

Each advertiser tries to set m_i in a way that leads to his maximum profit, which leads to $m_1 = \frac{2-2R+s_1t}{2t}$ and $m_2 = \frac{2t-2+2R+s_2t}{2t}$. If $R \geq 1 - \frac{t}{4}(2 - s_1 - s_2)$, total sales in RTB, i.e., $m_1 + 1 - m_2$, will be less than or equal to $1 - s_1 - s_2$, so the above equilibrium exists. Setting the reserve price below $1 - \frac{t}{4}(2 - s_1 - s_2)$ leads to other equilibria; however, it is dominated because the publisher cannot sell more than $1 - s_1 - s_2$ in RTB, and therefore, cannot make more than $R(1 - s_1 - s_2)$ in RTB.

Proof of Proposition 2

Given Proposition 1, the revenue of the publisher from the auction is given by $R \min(\frac{4-4R-(s_1+s_2)t}{2t}, 1 - s_1 - s_2)$ which is maximized at $R = 1 + \frac{(s_1+s_2-2)t}{4}$.

Proof of Lemma 1

Substituting R and m_i values from Propositions 1 and 2, the profit of Advertiser i is:

$$\pi_i = \frac{1}{32} (t(-6s_i s_j + 5(s_i - 4)s_i + s_j^2 - 4s_j + 4) - 32(p - 1)s_i)$$

where $j = 3 - i$ is the index of the other advertiser. The second derivative of π_i with respect to s_i is $\frac{5t}{16} > 0$ which implies that π_i is a convex function of s_i . Therefore, the optimal value of s_i is either 0 or $U - s_j$. Using basic algebra, it is easy to see that if $5Ut + 32 - 20t - 11s_j t < 32p$ then Advertiser i prefers 0 to $U - s_j$.

Proof of Proposition 3

Using the results of Proposition 1, 2 and Lemma 1, the revenue of the publisher is

$$Up + (1 - U)R = \frac{1}{32}(t((4 - 3U)U - 16) + 32).$$

This function is maximized at $U = \frac{2}{3}$. The optimum price of reservation contracts is, $p = 1 - \frac{25t}{48}$, and the optimum reserve price of RTB auctions is $R = 1 - \frac{t}{3}$. The publisher's total revenue is $1 - \frac{11t}{24}$, Advertiser 1's profit is $\frac{t}{8}$ and Advertiser 2's profit is $\frac{t}{18}$.

Proof of Corollary 1

If the publisher does not sell any impressions in reservation contracts, his revenue would be the same as when $U = 0$. From proof of Proposition 3, we know that the publisher's revenue when $U = 0$ is $\Pi = 1 - \frac{t}{2}$, which is lower than the publisher's revenue when U is optimally set to $\frac{2}{3}$.

If the publisher allocates the first U impressions to reservation contracts, assuming that consumers along the Hotelling line arrive uniformly, the expected value of each advertiser for each impression in reservation contracts would be $1 - \frac{t}{2}$. The remaining $1 - U$ impressions will be made available on RTB. Using Proposition 2, it is easy to see that the optimal reserve price in RTB is $1 - \frac{t}{2}$, and all of those impressions will be sold at the reserve price. Therefore, the publisher's total revenue is

$$\Pi = U(1 - \frac{t}{2}) + (1 - U)(1 - \frac{t}{2}) = 1 - \frac{t}{2}$$

which is again smaller than the revenue of the publisher in Proposition 3.

Proof of Proposition 4

Let m_1 and m_2 be such that the first advertiser bids for consumers in $[0, m_1]$ and Advertiser 2 bids for consumers located in $[m_2, 1]$. Suppose that the supply is $1 + D$ where $D = 0$ or $D = d$. First consider an equilibrium (if exists) in which the demand for auctions is less than or equal to $1 + D - s_1 - s_2$. In such equilibrium the profit of each advertiser is:

- $\pi_1 = (D + 1)m_1 \left(-\frac{m_1 t}{2} - R + 1\right) + s_1 \left(-\frac{1}{2}t(m_1 + m_2) - p + 1\right)$
- $\pi_2 = (D + 1)(1 - m_2) \left(-\frac{1}{2}(1 - m_2)t - R + 1\right) + s_2 \left(-t \left(\frac{1}{2}(-m_1 - m_2) + 1\right) - p + 1\right)$

Each advertiser tries to set m_i in a way that leads to his maximum profit, which leads to

$$m_1 = \frac{1 - R}{t} - \frac{s_1}{2(D + 1)}$$

and

$$m_2 = \frac{s_2}{2D + 2} + \frac{R + t - 1}{t}.$$

If $R \geq \frac{-2D(t-2)+t(s_1+s_2-2)+4}{4(D+1)}$, total sales in RTB, i.e., $(m_1 + 1 - m_2)(1 + D)$, will be less than or equal to $1 + D - s_1 - s_2$, so the above equilibrium exists. Setting the reserve price below $\frac{-2D(t-2)+t(s_1+s_2-2)+4}{4(D+1)}$ leads to other equilibria; however, it is dominated because the publisher cannot sell more than $1 + D - s_1 - s_2$ in RTB, and therefore, cannot make more than $R(1 + D - s_1 - s_2)$ in RTB.

The revenue of the publisher from the auction is given by $R \min((1 + D)(m_1 + 1 - m_2), 1 + D - s_1 - s_2)$, and is maximized at

$$R = \frac{-2d(t - 2) + t(s_1 + s_2 - 2) + 4}{4(d + 1)}.$$

Substituting R and m_i from above, the profit of Advertiser i can be written as:

$$\pi_i = \frac{-64(d + 1)(p - 1)s_i + 5(d + 2)s_i^2 t - 2s_i t(3(d + 2)s_j + 20(d + 1)) + t((d + 2)s_j^2 - 8(d + 1)s_j + 4(d + 1)(d + 2))}{64(d + 1)}$$

where $j = 3 - i$ is the index of the other advertiser. The second derivative of π_i with respect to s_i is $\frac{5(d+2)t}{32(d+1)} > 0$ which implies that π_i a convex function of s_i . Therefore, the optimal value of s_i is

either 0 or $U - s_j$. Using basic algebra, it is easy to see that if

$$5U(d+2)t < 64(d+1)(p-1) + t(11(d+2)s_j + 40(d+1))$$

then Advertiser i prefers 0 to $U - s_j$. The publisher sets the price such that Advertiser 1 is indifferent between buying U units in reservation contracts, and buying 0. Advertiser 1 buys U units in reservation contracts, and Advertiser 2 buys 0. The expected revenue of the publisher, hence, can be written as

$$Up + (1 + D - U)R_D = \frac{1}{64} \left(-\frac{3(d+2)tU^2}{d+1} - 16(d+2)(t-2) + 8tU \right)$$

where R_D is the reserve price conditional on supply $1+D$. This function is maximized at $U = \frac{4(d+1)}{3(d+2)}$. The optimum price of reservation contracts is, $p = 1 - \frac{25t}{48}$. Using the equilibrium values of U and p , we can calculate other parameters as functions of t and d . The optimum reserve price of RTB auctions when supply is $1+d$ is $R = 1 - \frac{(3d+4)t}{6(d+2)}$, and when the supply is 1 is $R = 1 - \frac{(d+4)t}{6(d+2)}$. The publisher's expected total revenue is

$$\frac{1}{64} \left(\frac{16(d+1)t}{3(d+2)} - 16(d+2)(t-2) \right).$$

Advertiser 1's profit is

$$\pi_1 = \frac{1}{16}(d+2)t,$$

and Advertiser 2's profit is

$$\pi_2 = \frac{(d(9d+16) + 16)t}{144(d+2)}.$$

Proof of Proposition 5

Consider an impression to a premium consumer. If an Advertiser wants to bid on any impression in RTB, he would bid on this impression because it has the highest valuation. We also know that an advertiser may not bid for an impression on which he has valuation above reserve price *only if* he knows that the competitor will not bid for that impression either (so that he could still win that impression through reservation contracts with positive probability). Therefore, in equilibrium, both advertisers bid on all premium impressions. These impression will be sold at price 1.

On the other hand, advertisers do not bid on impressions to unsusceptible consumers. As such, all those impressions are assigned to reservation contracts. This lowers the equilibrium price of impressions in reservation contracts such that Advertiser 1 remains indifferent between buying and not buying; i.e.,

$$p = (1 - \frac{25t}{48}) \frac{\frac{2}{3}}{Q + \frac{2}{3}} = \frac{48 - 25t}{72Q + 48}$$

The advertisers' strategies for other impressions remain the same as those in Proposition 3.

Proof of Proposition 6

Assume that the first advertiser buys the interval $[0, m_1]$ and the other buys $[m_2, 1]$. First consider an equilibrium (if exists) in which the demand for auctions is less than or equal to $1 - s_1 - s_2$. In such equilibrium the profit of each advertiser is:

- $\pi_1 = m_1 \left((1 - \alpha t \frac{m_1}{2}) - R \right) + s_1 \left((1 - t \frac{m_1 + m_2}{2}) - p \right)$
- $\pi_2 = (1 - m_2) \left(1 - \alpha t \frac{1 - m_2}{2} - R \right) + s_2 \left(1 - t(1 - \frac{m_1 + m_2}{2}) - p \right)$

Each advertiser tries to set m_i in a way that leads to his maximum profit, which leads to $m_1 = -\frac{2R + s_1 t - 2}{2t\alpha}$ and $m_2 = \frac{2R + s_2 t - 2}{2t\alpha} + 1$. If $R \geq \frac{1}{4}(2t\alpha(s_1 + s_2 - 1) + t(-(s_1 + s_2)) + 4)$, total sales in RTB, i.e., $m_1 + 1 - m_2$, will be less than or equal to $1 - s_1 - s_2$, so the above equilibrium exists. Setting the reserve price below $\frac{1}{4}(2t\alpha(s_1 + s_2 - 1) + t(-(s_1 + s_2)) + 4)$ leads to other equilibria; however, it is dominated because the publisher cannot sell more than $1 - s_1 - s_2$ in RTB, and therefore, cannot make more than $R(1 - s_1 - s_2)$ in RTB.

Given m_1 and m_2 , the revenue of the publisher from the auction is given by $R \min(m_1 + 1 - m_2, 1 - s_1 - s_2)$ which is maximized at

$$R = \frac{1}{4}(2t\alpha(s_1 + s_2 - 1) + t(-(s_1 + s_2)) + 4).$$

Substituting R and m_i values from above, the profit of Advertiser i is:

$$\pi_i = \frac{-4\alpha \left(8(p - 1)s_i + t(s_i^2 + s_i(2s_j + 3) + (s_j - 1)s_j) \right) + t(5s_i^2 - 6s_i s_j + s_j^2) + 4t\alpha^2(s_i + s_j - 1)^2}{32\alpha}$$

where $j = 3 - i$ is the index of the other advertiser. The second derivative of π_i with respect to s_i is $\frac{t(4(\alpha - 1)\alpha + 5)}{16\alpha} > 0$ which implies that π_i is a convex function of s_i . Therefore, the optimal value of

s_i is either 0 or $U - s_j$. Using basic algebra, it is easy to see that if

$$Ut(4(\alpha - 1)\alpha + 5) < 4\alpha(8p + 2t\alpha + 3t - 8) + s_2t(11 - 4(\alpha - 1)\alpha)$$

then Advertiser i prefers 0 to $U - s_j$.

Using the above expressions, the revenue of the publisher could be written as

$$Up + (1 - U)R = \frac{5tU^2 - 4t(3(U - 2)U + 4)\alpha^2 + 4\alpha(t(U - 5)U + 8)}{32\alpha}.$$

This function is maximized at $U = \frac{2\alpha}{2\alpha+1}$. Using the optimum value U , we can calculate other parameters of the models as functions of t and α . The optimum price of reservation contracts is,

$$p = \frac{32\alpha + 16 - t(4\alpha(\alpha + 5) + 1)}{32\alpha + 16},$$

and the optimum reserve price of RTB auctions is $R = \frac{-t\alpha+2\alpha+1}{2\alpha+1}$. The publisher's total revenue is $\frac{\alpha(16-t(2\alpha+9))+8}{16\alpha+8}$, Advertiser 1's profit is $\pi_1 = \frac{t\alpha}{8}$ and Advertiser 2's profit is $\pi_2 = \frac{t\alpha}{2(2\alpha+1)^2}$. Finally, note that the derivative of π_2 with respect to α is $\frac{\partial\pi_2}{\partial\alpha} = \frac{t-2t\alpha}{2(2\alpha+1)^3}$ which is positive if and only if $\alpha < \frac{1}{2}$. In other words, the profit of Advertiser 2 increases as α decreases (i.e., customization becomes stronger) if and only if $\alpha > \frac{1}{2}$.

Proof of Proposition 7 First, note that, using Proposition 3, the profit of the firm that does not win the impressions in reservation contract is $\frac{t}{18}$. Therefore, when bidding for impressions in reservation contracts, Advertiser i 's willingness to pay per impression, say w , is such that his total profit is at least $\frac{t}{18}$. In other words, we have

$$U(1 - \frac{Ut}{2} - w) = \frac{t}{18}$$

which, given that $U = \frac{2}{3}$, gives us $w = 1 - \frac{5t}{12}$. Since the impressions in reservation contracts are sold in a second price auction, both advertisers bid truthfully. The impressions are sold at $p = 1 - \frac{5t}{12}$ and each advertiser's profit is $\frac{t}{18}$.