3. Laplace Distribution and MLE.

Given the Leplace distribution with density: ②

$$f(x;\theta) = \frac{1}{2} e^{-|x-\theta|}$$

a) Log-likelihood for n samples $x_1, \ldots x_n$:

$$l(\theta) = \sum_{i=1}^{n} \log\left(\frac{1}{2} e^{-|x_i-\theta|}\right)$$

$$= -n\log(2) - \sum_{i=1}^{n} |x_i - \theta|$$

So the log likelihood is:

$$l(\theta) = \text{constant} - \sum_{i=1}^{n} |x_i - \theta|$$

b) MLE?

To maximize the likelihood, minimize:

$$\sum_{i=1}^{n} |x_i - \theta|$$

This is minimized when θ is the median ~~(x_1)~~ of the sample, so

$$\hat{\theta}_{MLE} = \text{median}(x_1, \ldots x_n).$$

→ If n is odd, The median is unique.

→ If n is even, the MLE is not unique. (any value between the two middle nos. works)

x———x