Exercise 02

1) A probability mass function is defined as

$\sum_{i=1}^{n} P(i) = 1$ They all add upto 1

$P(i) \geq 0$ for all i Every probability is non negative

Since the given function in question is

$P(i) = \frac{i}{M_n}$ where $M_n = \sum_{K=1}^{n} K$

Here M_n is sum of n positive integer

$M_n = 1+2+3+4+5+6 \cdots n = \frac{n(n+1)}{2}$

Non negativity checking :-

Since $i = \{1 \ldots n)$ is positive

$M_n = \frac{n(n+1)}{2}$ is also positive

$\boxed{\text{Therefore } P(i) \geq 0 \text{ for all } i}$

All add upto of checking

$\sum_{i=1}^{n} P(i) = \sum_{i=1}^{n} \frac{i}{M_n}$

$\frac{1}{M_n} = \sum_{i=1}^{n} i$

$= \frac{1}{M_n} (M_n) = \boxed{1}$

2. Expectation Value

a)

$\because$ x_i is the value of i that diee can show.

$\therefore$ $P(X = i) = \frac{i}{M_n}$

$$E_{P_n}(X) = \sum_{i=1}^{n} x_i P_n(x_i)$$

$$= \sum_{i=1}^{n} i \times \frac{i}{M_n} = \frac{1}{M_n} \sum_{i=1}^{n} i^2$$

As we know the formula for the sum of square

$$i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\frac{1}{M_n} \sum_{i=1}^{n} i^2 = \frac{1}{\frac{n(n+1)}{2}} \times \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{2}{n(n+1)} \times \frac{n(n+1)(2n+1)}{6} = \boxed{\frac{2n+1}{3}}$$

b) for $n = 6$

$$E_{P_n}(X) = \frac{2n+1}{3} = \frac{2(6)+1}{3} = \boxed{4.333}$$