Nonlinear Biomedical Physics

Historical Overview:

In a paper, the authors proposed a very simple model of a one-dimensional network on a ring.
With a likelihood p, connections('edges') are chosen at random and connected to another vertex also chosen randomly. With increasing p more and more edges become randomly reconnected and finally for p=1 all connections are random.
When p is only slightly higher than 0 the path length L drops sharply while the clustering coefficient hardly changes. Thus networks with a small fraction of randomly rewired connections combine both high clustering and a small path length, and these networks are called **small-world** networks that are found in the nervous system of C.elegans.

The likelihood that a newly added edge will connect to a vertex depends upon the degree of the vertex. Thus, vertices that have a high degree are more likely to get even more edges.
The network generated by this law is characterized by a degree distribution which can be described by power-law: P(k) = k^(-1/a). The exponent is 3. These networks are called **scale-free**. They have an extremely short path length.

Basics of Modern Network Theory:

1) The Clustering Coefficient C_i of a vertex i with degree k_i is usually defined as the ratio of the number of existing edges(e_i) between neighbors of i, and the maximum possible number of edges between neighbors of i.

   $C_i = 2e_i/(k_i(k_i-1)$.
2) Path length or distance d between two vertices i and j is the minimal number of edges that have to be traveled to go from i to j.

Characteristic path length is the mean of all path lengths between all possible pairs of vertices.

3) Motif: It is a simple subgraph consisting of a small number of vertices connected in a specific way. Triangles are a simple type of motif.

   The average degree K_{nn} of the neighbors of a node with degree k is given by:

   $K_{nn}(k) = \Sigma_{k'} k'P(k'|k)$

   Graphs with positive degree correlation are called assortative and with negative degree correlation are called disassortative. Most of the biological networks are disassortative.

4) Betweenness of a node b is defined as the ratio of all shortest paths between j and k that run through $i(n_{j,k}(i))$, divided by all shortest paths between j and $k(n_{j,k}(i))$.

5) Efficiency: Inverse of the shortest distance between two vertices. The average overall pairwise efficiency is the Global efficiency.

Applications to Neuroscience:

1) Small-world networks showed both a fast system response as well as coherent oscillations, suggesting this type of architecture could be optimal for information processing in neural networks.
2) Various studies showed the small-world architecture of a real nervous system.
3) Small world metric defined as S=(C/C-r)/(L/L-r).Here, C-r and L-r refers to the clustering coefficient and path length of corresponding ensembles of random network for a range of parameter settings.
4) Human brain networks has the characteristics of a small-world with gamma(C/C-r) = 2.36 and lambda(L/L-r) = 1.15 and a small world parameter S = 2.04.

5) A) They applied graph analysis to 28 channel EEG recorded during a 2-back working memory test. EEG filtered in different frequency bands was analyzed with SL and converted to unweighted graphs either as a function of threshold or as a function of degree K(K=5).
   B) During the working memory task, the networks in the group with lower education as compared to the highly educated groups were close to small-world networks as revealed by ba higher C/C-r and a lower L/L-s in the theta, alpha1 alpha2, beta and gamma band.
   C) The group having schizophrenia group were less small-world like and more random compared to controls.
   They were having less clustering coefficient, shorter path length and a lower centrality index of major network hubs.
6) The ratio C/C-r but not L/L-r was found to increase during all sleep stages compared to the awake state.
7) The problem of disconnected points was dealt with defining L as harmonic mean instead of arithmetic mean.