CSE431/531, Problem Set 4 Due Monday Nov. 3, in class

Note: There are 6 problems with a total of 100 points. You are required to do all the problems.

1. (15 points) In the problem of scheduling all activities discussed in class, someone claims that the algorithm still works correctly if we follow (1) a sorted order of the finishing time of all activities or (2) an arbitrary order. Prove or disprove such a claim for each of the two cases.

2. (20 points) Let P_1 be a convex polygon and P_2 be an arbitrary polygon (not necessary convex) inside P_1. The polygon separation problem is to find another polygon P_3 to separate P_1 and P_2 (*i.e.,* P_3 is inside P_1 and contains P_2 in its interior) and minimize its number of edges. You may assume that P_3 shares a vertex with P_1. Design a greedy algorithm to solve this problem and make your algorithm run as fast as possible. You should justify the correctness of your algorithm.

3. (15 points) Prove that if all the weights in a graph G are distinct, then G has a unique minimal spanning tree.

4. (15 points) Suppose that n files having lengths $L_1, L_2, \cdots, L_n$ are stored on a tape. If the files are stored in the order of $i_1, i_2, \cdots, i_n$, then the time to retrieve file i_k is $T_k = \sum_{j=1}^{k} L_{i_j}$. The average retrieval time is defined as $\frac{1}{n}\sum_{k=1}^{n} T_k$. Design a greedy algorithm for determining the order of the n files on a tape so as to minimize the average retrieval time. Show that your algorithm is optimal by stating and proving the greedy choice property and the optimal substructure property.

5. (15 points) Show how to solve the fractional knapsack problem in $O(n)$ time, where n is the number of items.

6. (20 points) Let $P = \{p_1, p_2, \cdots, p_n\}$ be a set of points on the x axis with each point $p_i, 1 \le i \le n$, represented by its coordinates. Design a greedy algorithm to find a minimum number of intervals with unit length on the x axis to cover the set of points in P, where a point p_i is covered by an interval if its x coordinate falls in the interval. For each interval you need to determine its position (*i.e.,* its starting and ending points). Prove the correctness of your algorithm. Now suppose that the points in P are located on a 2D plane and each interval becomes an axis-aligned unit square. Prove or disprove whether your greedy strategy for the 1-D case can still be extended to the 2-D case. For proving it works, you need to clear state how the greedy strategy is extended to 2D and prove its correctness. For disproving it, you just need to give a counter example.