

Note: There are 6 problems with a total of 100 points. You are required to do all the problems.

- (15 points) In the Selection algorithm discussed in class, we partition elements into groups of size 5 each. Is it possible to achieve an $O(n)$ -time algorithm by partitioning elements into groups of size 3, 4, 6, 7, 9, and 11? Justify your answer by giving a detailed analysis of the running time of the selection algorithm for each of the 6 different sizes.
- (20 points) In class we have learned how to solve the closest pair problem for 2D points by using Divide-and-Conquer (DaC) strategy. Extend the algorithm to 3D points by still using the DaC strategy. You should make the running time of your algorithm as fast as possible.
- (20 points) Let a_1, \dots, a_n be n distinct real numbers, and w_1, \dots, w_n be a set of n positive weights with $w_1 + \dots + w_n = 1$. The weighted median of the set $\{a_1, \dots, a_n\}$ is the number a_k for which $\sum_{i:a_i < a_k} w_i < \frac{1}{2}$ and $\sum_{i:a_i > a_k} w_i \leq \frac{1}{2}$. (a) Prove that such an a_k always exists. (b) Give a $\Theta(n)$ worst-case running time algorithm computing the weighted median.
- (15 points) Given a set $S = \{a_1, \dots, a_n\}$ of n unsorted real numbers and a real value B , design an $O(n^2)$ -time algorithm to determine whether there exist three distinct numbers a_i, a_j and a_k in S such that $a_i + a_j + a_k = B$.
- (15 points) Given an array $A = \{a_1, \dots, a_n\}$ of n unsorted numbers, design an $O(n \log n)$ -time algorithm for reporting the number of inversions in A . An inversion in A is a pair of numbers a_i and a_j such that $i < j$ but $a_i \geq a_j$.
- (15 points) In the Strassen's matrix multiplication algorithm, we have

$$\begin{aligned}
 p_1 &= (a - c)(s + t) &= as + at - cs - ct \\
 p_2 &= (b - d)(u + v) &= bu + bv - du - dv \\
 p_3 &= (a + d)(s + v) &= as + dv + av + ds \\
 p_4 &= a(t - v) &= at - av \\
 p_5 &= (a + b)v &= av + bv \\
 p_6 &= (c + d)s &= cs + ds \\
 p_7 &= d(u - s) &= du - ds
 \end{aligned}$$

Write the followings in terms of p_i 's:

$$as + bu = ???$$

$$at + bv = ???$$

$$cs + du = ???$$

$$ct + dv = ???$$