INDEX

05	An urban area has a population of 2 million residents. Three competing trunked mobile networks (systems A, B, and C) provide cellular service in this area. System A has 394 cells with 19 channels each, system B has 98 cells with 57 channels each, and system C has 49 cells, each with 100 channels. Find the number of users that can be supported at 2% blocking if each user averages 2 calls per hour at an average call duration of 3 minutes. Assuming that all three trunked systems are operated at maximum capacity, compute the percentage market penetration of each cellular provider.	16
06	If a transmitter produces 50 watts of power, express the transmit power in units of (a) dBm, and (b) dBW. If 50 watts is applied to a unity gain antenna with a 900 MHz carrier frequency, find the received power in dBm at a free space distance of 100 m from the antenna, what is P (10 km)? Assume unity gain for the receiver antenna	20
07	Determine the path loss of a 900MHz cellular system in a large city from a base station with the height of 100m and mobile station installed in a vehicle with antenna height of 2m. The distance between mobile and base station is 4Km.	22
08	Determine the path loss between base station (BS) and mobile station (MS) of a 1.8GHz PCS system operating in a high-rise urban area. The MS is located in a perpendicular street to the location of the BS. The distances of the BS and MS to the corner of the street are 20 and 30 meters, respectively. The base station height is 20m.	24
09	A mobile is located 5 km away from a base station and uses a vertical λ /4 monopole antenna with a gain of 2.55 dB to receive cellular 3 radio signals. The E-field at 1 km from the transmitter is measured to be V/rn. The carrier frequency used for this system is 900 MHz. (a) Find the length and the gain of the receiving antenna. (b) Find the received power at the mobile using the 2-ray ground reflection model assuming the height of the transmitting antenna is 50 m and the receiving antenna is 1.5m above ground.	27
10	A hexagonal cell within a 4-cell system has a radius of 1.387 km. A total of 60 channels are used within the entire system. If the load per user is 0.029 Erlangs, and λ= call/hour, compute the following for an Erlang C system that has a 5% probability of a delayed call: (a) How many users per square kilometer will this system support? (b) What is the probability that a delayed call will have to wait for more than 10s? (c) What is the probability that a call will be delayed for more than 10 seconds	28

<u>Problem No:</u> 01

<u>Problem Name</u>: If a total of 33 MHz of bandwidth is allocated to a particular FDD cellular telephone system which uses two 25 kHz simplex channels to provide full duplex voice and control channels, compute the number of channels available per cell if a system uses-

(a) 4-cell reuse. (b) 7-cell reuse. (c) 12-cell reuse.

If 1 MHz of the allocated spectrum is dedicated to control channels, determine an equitable distribution of control channels and voice channels in each cell for each of the three systems.

<u>Solution:</u>

Have given,

Total bandwidth = 33 MHz,
= 33,000 kHz

Channel bandwidth = 25 kHz x 2 simplex channels
= 50 kHz / duplex channel

Total available channels = Total Bandwidth / Channel Bandwidth
= 33,000 / 50
= 660 channels

If 1 MHz of the allocated spectrum is dedicated to control channels, i.e.
Control channel bandwidth = 1000 kHz

The number of available control channel = Control channel bandwidth / Channel bandwidth
= 1000 / 50
= 20 channels

(a)

Have given,

Cluster size, N = 4

Total number of channels available per cell = Total available channels / N
=660/4
≈165 channels.

∴ **Equitable distribution of,**

Voice Channel = (Total available channels - The number of available control channel) / N
= (660 – 20) / 4
≈ 160 channels

Control Channel = Total number of channels available per cell – Voice Channel
= 165 – 160
= 5 channels

(b)

Have given,

Cluster size, $N = 7$

Total number of channels available per cell = Total available channels / N

=660/7

≈94 channels.

∴ **Equitable distribution of,**

Voice Channel = (Total available channels - The number of available control channel) / N

= (660 – 20) / 7

≈ 91 channels

Control Channel = Total number of channels available per cell – Voice Channel

= 94 – 91

= 3 channels

(c)

Have given,

Cluster size, $N = 12$

Total number of channels available per cell = Total available channels / N

= 660/12

≈ 55 channels.

∴ **Equitable distribution of,**

Voice Channel = (Total available channels - The number of available control channel) / N

= (660 – 20) / 12

≈ 53 channels

Control Channel = Total number of channels available per cell – Voice Channel

= 55 – 53

= 2 channels

Source Code (Python):

```
# Input cluster sizes as a list, e.g., [4, 7, 12]
cluster_sizes = list(map(int, input("Enter Cluster Sizes with [ ] around Them: ").strip("[]").split()))

# Parameters
bw = 33000        # Total Bandwidth in kHz
sim_ch_bw = 25        # Simplex channel bandwidth in kHz
dup_ch_bw = 2 * sim_ch_bw        # Duplex channel bandwidth in kHz
t_ch = bw / dup_ch_bw         # Total available channels
cc_bw = 1000           # Control channel bandwidth
t_cc = cc_bw / dup_ch_bw          # Number of available control channels

# Calculating results for each cluster size
for N in cluster_sizes:
    # Calculate the desired results for each system use
    ch_per_cell = round(t_ch / N)        # Channels available per cell
    vc = round((t_ch - t_cc) / N)        # Voice channels
    cc = ch_per_cell - vc             # Control channels

    # Print the results
    print(f"For Cluster size N = {N}")
    print("------------------------")
    print(f"Total number of channels available per cell: {ch_per_cell} channels")
    print(f"Voice Channel: {vc} channels")
    print(f"Control Channel: {cc} channels\n")
```

Input:

Enter Cluster Sizes with [ ] around them: [4 7 12].

Output:

For Cluster size N = 4

Total number of channels available per cell : 165 channels
Voice Cannel : 160 channels
Control Cannel : 5 channels

For Cluster size N = 7

Total number of channels available per cell : 94 channels
Voice Cannel : 91 channels
Control Cannel : 3 channels

For Cluster size N = 12

Total number of channels available per cell : 55 channels
Voice Cannel : 53 channels
Control Cannel : 2 channels

Problem No: 02

Problem Name: If a signal to interference ratio of 15 dB is required for satisfactory forward channel performance of a cellular system, what is the frequency reuse factor and cluster size that should be used for maximum capacity if the path loss exponent is-

(a) $n = 4$. (b) $n = 3$.

Assume that there are 6 co-channels cells in the first tier and all of them are at the same distance from the mobile. Use suitable approximations.

Solution:

Have given,

Minimum Required Signal-to-Noise interference ratio, S/I = 15 dB,

The number of Co-channel interfering cells, i_o = 6

We Know,

Number of cell reuse, $N = i^2 + i*j + j^2$ ………………………………………… (i)

First, let us consider a 7-cell reuse pattern, N = 7 For i=1, j=2

Also,

The Frequency Reuse Factor, Q = D/R

$= \sqrt{(3N)}$ ……………..…………………………… (ii)

= 4.583.

Where,

D = Distance between centers of the nearest Co-channel cells.

R = Radius of the cell.

(a)

Have given,

Path Loss exponent, n = 4

Frequency Reuse Factor, Q = 4.583.

We know,

Signal-to-Noise interference ratio, S/R $= 10 \log (Q^n / i_0)$

$= 10 \log ((4.583)^4 / 6) =$ 18.66 dB.

Since this is greater than the minimum required S/I (18.66 > 15), N = 7 can be used.

(b)

Have given,

Path Loss exponent, n = 3

We know,

Signal-to-Noise interference ratio, S/R $= 10 \log(Q^n / i_0)$ ……………….....…. (iii)

$= 10 \log ((4.583)^3 / 6)$

= 12.05 dB.

Since this is less than the minimum required S/I (12.05 < 15), we need to use a larger N.

Using equation (i), the next possible value of N = 12; For i = j = 2.
The corresponding co-channel ratio is given by equation (ii) as- **Frequency Reuse Factor**, Q = 6.

Using equation (iii) the signal-to-interference ratio, S/I = 15.56 dB.
Since, this is greater than the minimum required S/I (15.56 > 15), N = 12 can be used.

Source Code (Python):

```
import math

def calculate_frequency_reuse_factor(N):
    Q = math.sqrt(3 * N)
    return Q

def calculate_SIR(io, n, Q):
    SI = 10 * math.log10((1 / io) * (Q ** n))
    return SI

def find_optimal_parameters(R_SI, io, N):
    for n in [4, 3]:
        Q = calculate_frequency_reuse_factor(N)
        SI = calculate_SIR(io, n, Q)
        print(f"For n = {n}:")
        print(f"Frequency reuse factor (Q): {Q}")
        print(f"Signal to interference ratio (SI): {SI} dB")

        if SI < R_SI:
            # Adjusting parameters
            i = 2
            j = 2
            N = i ** 2 + i * j + j ** 2
            Q = calculate_frequency_reuse_factor(N)
            SI = calculate_SIR(io, n, Q)
            print("Adjusting parameters:")
            print(f"New Frequency reuse factor (Q): {Q}")
            print(f"New Signal to interference ratio (SI): {SI} dB")
        print()

# Given parameters
R_SI = 15  # Required signal to interference ratio (dB)
io = 6  # Path loss exponent

# Finding optimal parameters
find_optimal_parameters(R_SI, io, 7)
```

Input:

Enter Path Loss exponent with [ ] around them: [4 3].

Output:

For Path Loss Exponent, n = 4

Signal-to-Noise interference Ratio, S/ I: 18.663 dB > 15 dB
Hence, Cluster size, N : 7
Frequency Reuse Factor, Q : 4.583

For Path Loss Exponent, n = 3

Signal-to-Noise interference Ratio, S/I : 15.563 dB > 15 dB
Hence, Cluster size, N : 12
Frequency Reuse Factor, Q : 6.000

Problem No: 03

Problem Name: How many users can be supported for 0.5% blocking probability for the following number of trunked channels in a blocked calls cleared system?

(b) 1, (b) 5, (c) 10, (d) 20, (e) 100.

Assume each user generates 0.1 Erlangs of traffic.

Solution:

Have given,

Blocking Probability, P_B = 0.5%,

Traffic Intensity, A_u = 0.1 Erlangs

We Know,

For Erlangs B, Grade of Service, GOS = P_B

= 0.005

And, Total number of user, U = A / A_u ……………………………....... (i)

Where,

A = Offered Traffic Intensity.

Also,

Table 3.1: Capacity of an Erlang B System.

Number of Channels C	Capacity (Erlangs) for GOS = 0.01	= 0.005	= 0.002	= 0.001
2	0.153	0.105	0.065	0.046
4	0.869	0.701	0.535	0.439
5	1.36	1.13	0.900	0.762
10	4.46	3.96	3.43	3.09
20	12.0	11.1	10.1	9.41
24	15.3	14.2	13.0	12.2
40	29.0	27.3	25.7	24.5
70	56.1	53.7	51.0	49.2
100	84.1	80.9	77.4	75.2

From Table 3.1, we can find the total capacity in Erlangs for the 0.5% GOS for different numbers of channels.

(a)

Have given,

Trunked channels, C = 1

From table 3.1, For C = 1 we obtain, A = 0005

From equation (i), we have-

Total number of user, U = A / A_u

= 0.05 users.

But, actually one user could be supported on one channel. So, U = 1.

(b)

Have given,

Trunked channels, $C = 5$

From table 3.1, For $C = 5$ we obtain, $A = 1.13$ Erlang.

From equation (i), we have-

Total number of user, $U = A / A_u$

$\approx$ 11 users.

(c)

Have given,

Trunked channels, $C = 10$

From table 3.1, For $C = 10$ we obtain, $A = 3.96$ Erlang

From equation (i), we have-

Total number of user, $U = A / A_u$

$\approx$ 39 users.

(d)

Have given,

Trunked channels, $C = 20$

From table 3.1, For $C = 20$ we obtain, $A = 11.10$ Erlang

From equation (i), we have-

Total number of user, $U = A / A_u$

$\approx$ 110 users.

(e)

Have given,

Trunked channels, $C = 100$

From table 3.1, For $C = 100$ we obtain, $A = 80.9$ Erlang.

From equation (i), we have-

Total number of user, $U = A / A_u$

$\approx$ 809 users.

Source Code (Python):

```
import numpy as np

# Parameters
GOS = 0.5 / 100  # Blocking probability (0.5%)
Au = 0.1  # Traffic intensity per user

# From Erlang B chart
A = np.array([0.005, 1.13, 3.96, 11.1, 80.9])  # Offered Traffic Intensity
C = np.array([1, 5, 10, 20, 100])  # Trunked Channels

# Total number of users
U = np.round(A / Au).astype(int)

# Print results
print(f"Grade of Service, GOS = {GOS:.3f}")
print("Trunked Channels, C:")
print(C)
print("From table 3.1, we obtain Offered Traffic Intensity, A For all Channels, C:")
print(A)
print("Total number of users, U")
print("---------------------------")
print(U)
```

Input:

Trunked Channels, C = [1 5 10 20 100];

Output:

Grade of Service, GOS = 0.005

Trunked Channels,	C:	1	5	10	20	100
Offered Traffic Intensity,	A:	0.0050	1.1300	3.9600	11.1000	80.9000
Total number of user,	**U:**	0	11	40	111	809

Problem No: 04

Problem Name: An urban area has a population of 2 million residents. Three competing trunked mobile networks (systems A, B, and C) provide cellular service in this area. System A has 394 cells with 19 channels each, system B has 98 cells with 57 channels each, and system C has 49 cells, each with 100 channels. Find the number of users that can be supported at 2% blocking if each user averages 2 calls per hour at an average call duration of 3 minutes. Assuming that all three trunked systems are operated at maximum capacity, compute the percentage market penetration of each cellular provider.

Solution:

Have given,

Blocking Probability, P_B = 2%,

The average number of call requests per unit time λ = 2.

The average duration of a call, H = 3/60 seconds

There are 2 million residents in the given urban area = 2000000

We Know,

For Erlangs B, Grade of Service, GOS = P_B

= 0.02

And, Traffic Intensity, A_u = λH

= 0.1 Erlangs

Also, Total number of user, U = A / A_u ……………………………....... (i)

Where,

A = Offered Traffic Intensity.

Also,

Table 4.1: Capacity of an Erlang B System.

Erlang B Traffic Table

Maximum Offered Load Versus B and N

B is in %

N/B	0.01	0.05	0.1	0.5	1.0	2	5	10	15	20	30	40
1	.0001	.0005	.0010	.0050	.0101	.0204	.0526	.1111	.1765	.2500	.4286	.6667
2	.0142	.0321	.0458	.1054	.1526	.2235	.3813	.5954	.7962	1.000	1.449	2.000
3	.0868	.1517	.1938	.3490	.4555	.6022	.8994	1.271	1.603	1.930	2.633	3.480
4	.2347	.3624	.4393	.7012	.8694	1.092	1.525	2.045	2.501	2.945	3.891	5.021
5	.4520	.6486	.7621	1.132	1.361	1.657	2.219	2.881	3.454	4.010	5.189	6.596
9	1.826	2.302	2.558	3.333	3.783	4.345	5.370	6.546	7.551	8.522	10.58	13.05
10	2.260	2.803	3.092	3.961	4.461	5.084	6.216	7.511	8.616	9.685	11.95	14.68
16	5.339	6.250	6.722	8.100	8.875	9.828	11.54	13.50	15.18	16.81	20.30	24.54
17	5.911	6.878	7.378	8.834	9.652	10.66	12.46	14.52	16.29	18.01	21.70	26.19
18	6.496	7.519	8.046	9.578	10.44	11.49	13.39	15.55	17.41	19.22	23.10	27.84
19	7.093	8.170	8.724	10.33	11.23	12.33	14.32	16.58	18.53	20.42	24.51	29.50
20	7.701	8.831	9.412	11.09	12.03	13.18	15.25	17.61	19.65	21.64	25.92	31.15
56	33.49	36.13	37.46	41.23	43.32	45.88	50.54	56.06	60.98	65.94	77.00	90.97
57	34.27	36.95	38.29	42.11	44.22	46.82	51.55	57.14	62.14	67.18	78.43	92.64
94	64.25	68.07	69.98	75.41	78.43	82.17	89.10	97.53	105.3	113.2	131.2	154.3
95	65.08	68.93	70.85	76.33	79.37	83.13	90.12	98.63	106.4	114.4	132.6	155.9
100	69.27	7~.25	75.24	80.91	84.06	87.97	95.24	104.1	112.3	120.6	139.7	164.3

N is the number of servers. The numerical column headings indicate blocking probability B in %. Table generated by Dan Dexter

From Table 4.1, we can find the total capacity in Erlangs for the 2% GOS for different numbers of channels.

For System-A

Have given,

Number of channels per cell used in the system, $C = 19$

From table 4.1, For $C = 19$ and $GOS = 0.02$ we obtain, $A = 12$ Erlangs

From equation (i), we have-

Total number of user, $U = A / A_u$

$= 120$ users.

*Since there are 394 cells, the total number of' subscribers that can be supported by **System A** is equal to 120 x 394 = 47280.*

Since, the percentage market penetration = 47280/2000000 = 2.36%

For System-B

Have given,

Number of channels per cell used in the system, $C = 57$

From table 4.1, For $C = 57$ and $GOS = 0.02$ we obtain, $A = 45$ Erlangs

From equation (i), we have-

Total number of user, $U = A / A_u$

$= 450$ users.

*Since there are 98 cells, the total number of' subscribers that can be supported by **System B** is equal to 450 x 98 = 44,100.*

Since, the percentage market penetration = 44100/2000000 = 2.205%

For System-C

Have given,

Number of channels per cell used in the system, $C = 100$

From table 4.1, For $C = 100$ and $GOS = 0.02$ we obtain, $A = 88$ Erlangs

From equation (i), we have-

Total number of user, $U = A / A_u$

$= 880$ users.

*Since there are 49 cells, the total number of' subscribers that can be supported by **System C***

is equal to 880 x 49 = 43,120.

Since, the percentage market penetration = 43,120/2000000 = 2.156%

Therefore, total number of cellular subscribers that can be supported by these three systems are (47280 + 44100 + 43120) = 134500 users.

The market penetration of the three systems combined is equal to 134500/2000000 = 6.725

Source Code (Python):

```
blocking_p = 2 / 100
lamda = 2
H = 3 / 60
Au = lamda * H

# System A
channel_a = 19
cell_A = 394
A = 12
Ua = A / Au
subscriber_A = Ua * cell_A
percentage_market_penetration_for_A = (subscriber_A / 2000000) * 100

# System B
channel_b = 57
cell_B = 98
Ab = 45
Ub = Ab / Au
subscriber_B = Ub * cell_B
percentage_market_penetration_for_B = (subscriber_B / 2000000) * 100

# System C
channel_c = 100
cell_C = 49
Ac = 88
Uc = Ac / Au
subscriber_C = Uc * cell_C
percentage_market_penetration_for_C = (subscriber_C / 2000000) * 100

Total_number_of_subscriber = subscriber_A + subscriber_B + subscriber_C
Market_penetration_for_three_system = (Total_number_of_subscriber / 2000000) * 100

print("For system A:")
print("Number of users in System A:", Ua)
print("Total number of subscriber in system A:", subscriber_A)
print("Percentage market penetration for A:", percentage_market_penetration_for_A)

print("\nFor system B:")
print("Number of users in System B:", Ub)
print("Total number of subscriber in system B:", subscriber_B)
print("Percentage market penetration for B:", percentage_market_penetration_for_B)

print("\nFor system C:")
print("Number of users in System C:", Uc)
print("Total number of subscriber in system C:", subscriber_C)
print("Percentage market penetration for C:", percentage_market_penetration_for_C)
print("\nTotal number of subscribers for all three systems:", Total_number_of_subscriber)
print("Market penetration for all three systems:", Market_penetration_for_three_system)
```

Input:

Trunked Channels, C = [19 57 100];

Output:

For system A:

Total number of users for system A: 47280
Percentage market penetration for System A: 2.364%

For system B:

Total number of users for system B: 44100
Percentage market penetration for System B: 2.205%

For system C:

Total number of users for system C: 43120
Percentage market penetration for System C: 2.156%

For all three systems:

Total number of users of all three system: 134500
Percentage market penetration for all three System: 6.725%

Problem No: **05**

Problem Name: A certain city has an area of 1,300 square miles and is covered by a cellular system using a 7-cell reuse pattern. Each cell has a radius of 4 miles and the city is allocated 40 MHz of spectrum with a full duplex channel bandwidth of 60 kHz. Assume a GOS of 2% for an Erlang B system is specified. If the offered traffic per user is 0.03 Erlangs, compute-

a) The number of cells in the service area,
b) The number of channels per cell,
c) Traffic intensity of each cell,
d) The maximum carried traffic,
e) The total number of users that can be served for 2% GOS,
f) The number of mobiles per channel, and
g) The theoretical maximum number of users that could be served at one time by the system.

Solution:

(a)

Have given,

Total coverage area = 1300 miles

Cell radius = 4 miles

We know,

The area of a cell (hexagon) can be shown to be $2.5981R^2$

Thus each cell covers $2.5981 \times (4)^2 = 41.57$ sq km.

Hence, the total number of cells, N_c = 1300/41.57 = 31 cells

(b)

Have given,

Allocated spectrum	= 40, 000,000 Hz
Channel width	= 60,000 Hz
Frequency reuse factor, N	= 7 cells

We know,

The total number of channels per cell, C = Allocated spectrum / (Channel width × N)
= 40, 000,000 / (60,000 × 7)
= 95 channels/cell

(c)

Have given,

From (b) No, C = 95

And, GOS = 0.02

From the table 4.1 (Erlang B chart) For C = 95 and GOS = 0.02, we have- Traffic intensity per cell, A = 84 Erlangs/cell

(d)

Have given,

From (a), Number of cells = 31 cells

From (c), Traffic intensity per cell = 84 Erlangs/cell

We Know,

Maximum carried traffic = Number of cells $\times$ Traffic intensity per cell

$= 31 \times 84 = 2604$ Erlangs.

(e)

Have given,

Traffic per user, A_u = 0.03 Erlangs

From (d), Total traffic, A = 2604 Erlangs.

We Know,

Total number of users, $U = A / A_u$ =

2604 / 0.03

= 86,800 users.

(f)

Have given,

Allocated spectrum = 40, 000,000 Hz

Channel width = 60,000 Hz

From (e), Number of users, U = 86,800 users.

We Know,

Number of channels = Allocated Spectrum / Channel Width

= 40, 000,000/60,000

$\approx$ 666

Number of mobiles per channel = Number of users/Number of channels

= 86,800 / 666

$\approx$ 130 mobiles/channel

(g)

Have given,

From (b) No, C = 95 channels/cell

From (a), the total number of cells, N_c= 31 cells.

From (e) Total number of users, U = 86,800 users.

We Know,

The theoretical maximum number of served mobiles is the number of available channels in the system (all channels occupied) $= C \times N_c$

$= 95 \times 31 = 2945$ users,

Which is $(2945/86,800) \times 100 = 3.4\%$ of the customer base.

Source Code (Python):

```
import math
# Given values
area = 1300
radius = 4
each_cell_covers = math.floor(2.5981 * radius ** 2)  # in square kilometers
print('(a)')
number_of_cells = math.floor(area / each_cell_covers)
print("Number of cells:", number_of_cells)

# (b)
allocated_spectrum = 40000
channel_width = 60
frequency_reuse_factor = 7
print('(b)')
number_of_channel_per_cell = math.floor(allocated_spectrum / (channel_width * frequency_reuse_factor))
print("Number of channels per cell:", number_of_channel_per_cell)

# (c)
print('(c)')
traffic_intensity_per_cell = 83.13  # from erlang chart B
print("Traffic intensity per cell:", traffic_intensity_per_cell)

# (d)
print('(d)')
maximum_carried_traffic = number_of_cells * traffic_intensity_per_cell
print("Maximum carried traffic:", maximum_carried_traffic)

# (e)
traffic_per_user = 0.03
print('(e)')
total_number_of_user = maximum_carried_traffic / traffic_per_user
print("Total number of users:", total_number_of_user)

# (f)
number_of_channels = number_of_channel_per_cell * frequency_reuse_factor
print('(f)')
number_of_mobile_per_channel = math.floor(total_number_of_user / number_of_channels)
print("Number of mobiles per channel:", number_of_mobile_per_channel)
# (g)
print('(g)')
theoretical_maximum_number_of_user_that_could_be_served = number_of_cells * number_of_channel_per_cell
print("Theoretical maximum number of users that could be served:",
      theoretical_maximum_number_of_user_that_could_be_served)
```

Output:

(a) Total number of cells, Nc : 31 cells

(b) The total number of channels per cell, C : 95 channels/cell

(c) Traffic intensity per cell, A : 84 Erlangs/cell

(d) Maximum carried traffic : 2604 Erlangs

(e) Total number of users, U : 86800 users

(f) Number of mobiles per channel : 130 mobiles/channel

(g) Theoretical maximum no of user that could be served: 2945 users

<u>Problem No:</u> 06

<u>Problem Name:</u> If a transmitter produces 50 watts of power, express the transmit power in units of

a) dBm and b) dBW.

If 50 watts is applied to a unity gain antenna with a 900 MHz carrier frequency,

c) Find the received power in dBm at a free space distance of 100 m from the antenna,

d) What is P (10 km)?

Assume unity gain for the receiver antenna.

<u>Solution:</u>

Have given,

Transmitter power, $P_t = 50$ W

Carrier frequency, $f_c = 900$ MHz

(a)

We know,

Transmitter power, $P_t(\text{dBm}) = 10 \log[P_t(\text{mW})/(1\text{mW})]$

$= 10 \log [50 \text{ x } 10^3]$

$= 47.0$ dBm

(b)

We know,

Transmitter power, $P_t(\text{dBW}) = 10 \log[P_t(\text{W})/(1\text{W})]$

$= 10 \log [50]$

$= 17.0$ dBW

(c)

If 50 watts is applied to a unity gain antenna with a 900 MHz carrier frequency,

Have given,

Transmitter Gain, G_t	$= 1$
Receiver Gain, G_r	$= 1$
Wave length λ	$= c / f = 1 / 3$ m
The T-R separation distance, d	$= 100$m
The system loss factor, L	$= 1$

We know,

The received power, $P_r = (P_t \times G_t \times G_r \times \lambda^2) / (4\pi^2 \times d^2 \times L)$

$= (50 \times 1 \times 1 \times (1/3)^2) / ((4\pi)^2 \times 100^2 \times 1)$

$= 3.5 \times 10^{-3}$ mW

Received power, $P_r(\text{dBm}) = 10 \log[P_r(\text{mW})]$

$= 10 \log[P_r(3.5 \times 10^{-3})]$

$= -24.5$ dBm

(d)

Have given, d_o= 10 km
=10000m

We Know,

The received power at 10 km can be expressed in terms of dBm, we have

$$\therefore P_r(10\text{ km}) = P_r(100) + 20\log[d / d_o]$$
$$= P_r(100) + 20\log[100 / 10000]$$
$$= -24.5 - 40 =$$
$$-64.5\text{ dBm}$$

Source Code (Python):

```
 # given
import math

Pt = 50  # Transmitter Power
fc = 900  # Carrier Frequency

# a
PtdBm = 10 * math.log10(Pt * 1e3)
print('Transmitted Power: %.1f dBm' % PtdBm)

# b
PtBW = 10 * math.log10(Pt)
print('Transmitted Power: %.1f dBW' % PtBW)

# received power
Gt, Gr, lam, d, L = 1, 1, (1 / 3), 100, 1
Pr = (Pt * Gt * Gr * (lam ** 2)) / (((4 * math.pi) ** 2) * (d ** 2) * L)
PrdBm = 10 * math.log10(Pr * 1e3)
print('Received Power: %.1f dBm' % PrdBm)

# Pr(10Km)
Pr10Km = PrdBm + 20 * math.log10(100 / 10000)
print('Received Power: %.1f dBm' % Pr10Km)
```

Output:

(a) Transmitter power,P_t in dBm : 47 dBm

(b) Transmitter power,P_t in dBW : 17 dBW

(c) Received power,P_r in dBm : -24.54 dBm

(d) Received power,P_r at 10km in dBm : -64.54 dBm

Problem No: 07

Problem Name: Determine the path loss of a 900MHz cellular system in a large city from a base station with the height of 100m and mobile station installed in a vehicle with antenna height of 2m. The distance between mobile and base station is 4 km.

Solution:

Have given,

The frequency, f_c = 900 MHz (150 MHz to 1500MHz)

The effective transmitter (base station) antenna height, h_{te} = 100m

The effective transmitter (mobile) antenna height, h_{re} = 2m

T-R separation distance, d = 4 km

Now, The correction factor for effective movile antenna height, $a(h_{re}) = 3.2 (\log 11.75 h_{re})^2 - 4.97$ dB for fc ≥ 300 MHz

From Okumura-Hata Model we know,

The path loss in urban areas is given by

$$L_{50}(urban)(\text{dB}) = 69.55 + 26.16\log f_c - 13.82\log h_{te} - a(h_{re}) + (44.9 - 6.55\log h_{te})\log d$$

$$= 69.55 + 26.16 \times 2.954 - 13.82 \times 2 - 1.045 + (44.9 - 13.1) \times 0.6 = 137.3 \text{ dB}$$

Source Code (Python):

```
# given
import math

Pt = 50  # Transmitter Power
fc = 900  # Carrier Frequency

# a
PtdBm = 10 * math.log10(Pt * 1e3)
print('Transmitted Power: %.1f dBm' % PtdBm)

# b
PtBW = 10 * math.log10(Pt)
print('Transmitted Power: %.1f dBW' % PtBW)

# received power
Gt, Gr, lam, d, L = 1, 1, (1 / 3), 100, 1
Pr = (Pt * Gt * Gr * (lam ** 2)) / (((4 * math.pi) ** 2) * (d ** 2) * L)
PrdBm = 10 * math.log10(Pr * 1e3)
print('Received Power: %.1f dBm' % PrdBm)

# Pr(10Km)
Pr10Km = PrdBm + 20 * math.log10(100 / 10000)
print('Received Power: %.1f dBm' % Pr10Km)
```

Input:

```
hte = 100;    % Effective transmitter (base station) antenna height in meter
hre = 2;      % Effective receiver (mobile) antenna height in meter
fc = 900;     % Frequency in MHz
d = 4;        % T-R separation distance in kilometer
```

Output:

The path loss in urban areas, L_p = 137.29 dB.

Problem No: 08

Problem Name: Determine the path loss between base station (BS) and mobile station (MS) of a 1.8GHz PCS system operating in a high-rise urban area. The MS is located in a perpendicular street to the location of the BS. The distances of the BS and MS to the corner of the street are 20 and 30 meters, respectively. The base station height is 20m.

Solution:

Have given,

The frequency, f_c = 1.8 GHz (0.9 to 2 GHz)

The effective transmitter (base station) antenna height, h_b = 20m

T-R separation distance, $d = \sqrt{(20^2 + 30^2)}$ = 0.036 km

From Okumura-Hata Model we know,

The path loss in a high-rise urban areas with Perpendicular Street to the location of the Base Station is given by-

$$L_p = 135.41 + 12.49 \log f_c - 4.99 \log h_b + [46.84 - 2.34 \log h_b] \log d$$

$$= 135.41 + 12.49 \times \log(1.8) - 4.99 \times \log 20 + [46.84 - 2.34 \log 20] \times \log 0.036$$

$$= 68.91 \text{ dB}$$

Source Code (Python):

```
import math

# Given values
fc = 1.8  # Frequency (GHz)
hb = 20  # Height of the base station antenna (meters)
d = math.sqrt(20 ** 2 + 30 ** 2) / 1000  # Distance between the base station and mobile station (kilometers)

print('Distance: %.4f Km' % d)

# Calculate path loss
PathLoss = 135.41 + (12.49 * math.log10(fc)) - (4.99 * math.log10(hb)) + ((46.82 - 2.34 * math.log10(hb)) * math.log10(d))

print('PathLoss: %.4f dB' % PathLoss)
```

Input:

```
hb = 20;     % Effective transmitter (base station) antenna height in meter fc
= 1.8;     % Frequency in GHz
```

Output:

The path loss in a high-rise urban areas, L_p = 68.91 dB

Problem No: 09

Problem Name: A mobile is located 5 km away from a base station and uses a vertical λ /4 monopole antenna with a gain of 2.55 dB to receive cellular 3 radio signals. The E-field at 1 km from the transmitter is measured to be V/m. The carrier frequency used for this system is 900 MHz.

a) Find the length and the gain of the receiving antenna.
b) Find the received power at the mobile using the 2-ray ground reflection model assuming the height of the transmitting antenna is 50 m and the receiving antenna is 1.5m above ground.

Solution:

Have given,

Frequency of operation, f = 900 MHz
Gain of antenna, G = 1.8 = 2.55 dB

(a)

We Know,

Wave length,

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{900\times10^6} = 0.333 \text{ m.}$$

Length of the antenna, $L = \lambda/4 = 0.333/4 = 0.0833 \text{ m} = 8.33 \text{ cm.}$

And, Gain of antenna, G = 2.55 dB.

(b)

Have given,

T-R separation distance, d = 5 km
E-field at a distance of 1 km, E_o = 10^{-3} V/m
Transmitter distance d_o = 1km
Transmitting antenna height, h_t = 50m
Receiving antenna height, h_r = 1.5m
Wave length, λ = 0.333

We Know,

Since $d \gg \sqrt{h_t h_r}$, the electric field is given by

$$E_R(d) \approx \frac{2E_0 d_0}{d} \frac{2\pi h_t h_r}{\lambda d} \approx \frac{k}{d^2} \text{ V/m}$$

$$= \frac{2 \times 10^{-3} \times 1 \times 10^{3}}{5 \times 10^{3}} \left[\frac{2\pi(50)(1.5)}{0.333(5 \times 10^{3})} \right]$$

$$= 113.1 \times 10^{-6} \text{ V/m.}$$

Here, Effective Aperture

$$A_e = \frac{\lambda^2}{4\pi} G$$

$= 0.016 \text{ m}^2$

Now, the received power at a distance d can be obtained using

$$P_r(d) = P_d A_e = \frac{|E|^2}{120\pi} A_e$$

$= ((113.1 \times 10^{-6})^2 \times 0.016)/337$

$= 5.4 \times 10^{-13}$ W

$= -122.68$ dbW

$= -92.68$ dBm

Source Code (MATLAB):

```
import math

# Parameters
f = 900  # Frequency in MHz
g = 2.55  # Gain of antenna in dB

# Question (a)
gain = 10 ** (g / 10)  # Gain (linear scale)
lambda_ = (3 * 10**8) / (f * 10**6)  # Wavelength in meters
L = lambda_ / 4  # Antenna length

# Print results for (a)
print("For (a)")
print("---------")
print(f"Length of the antenna: {L:.3f} m")
print(f"Gain of the antenna: {gain:.1f} = {g:.2f} dB\n\n")

# Question (b)
d = 5000  # T-R separation distance in meters
E0 = 10**-3  # Electric field in V/m
d0 = 1000  # Reference transmitter distance in meters
ht = 50  # Transmitting antenna height in meters
hr = 1.5  # Receiving antenna height in meters
```

```
Er_d = (2 * E0 * d0 * 2 * math.pi * ht * hr) / (lambda_ * d**2)  # Electric field at distance
d
Ae = (gain * lambda_**2) / (4 * math.pi)  # Effective aperture
Pr_d = (Er_d**2 / (120 * math.pi)) * Ae  # Received power at distance d
Pr_dB = 10 * math.log10(Pr_d)  # Received power in dBW

# Print results for (b)
print("For (b)")
print("---------")
print(f"Electric Field, Er(d): {Er_d:.9f} V/m")
print(f"Effective Aperture, Ae: {Ae:.3f} m^2")
print(f"Received power at 5 km distance Er(5 km): {Pr_dB:.3f} dBW")
```

Input:

```
f = 900;            % Frequency in MHz
g = 2.55;           % Gain of antenna in dB

d = 5000;           % T-R separation distance
E0 = 10^-3;         % Electric-field
d0 = 1000;          % Transmitter distance
ht = 50;            % Transmitting antenna height, ht (m)
hr = 1.5;           % Receiving antenna height, hr (m)
```

Output:

For (a)

Length of the antenna, L : 0.083 m
Gain of the antenna, G : 1.8 = 2.55dB

For (b)

Electric Field, $E_r(d)$: 0.000113098 v/m
Effective Aperture, A_e : 0.016 m^2
Received power at 5 km distance $E_r(5\text{ km})$: -122.679 dbW

<u>Problem No</u>: 10

<u>Problem Name:</u> A hexagonal cell within a 4-cell system has a radius of 1.387 km. A total of 60 channels are used within the entire system. If the load per user is 0.029 Erlangs, and λ= call/hour, compute the following for an Erlang C system that has a 5% probability of a delayed call

a) How many users per square kilometer will this system support?
b) What is the probability that a delayed call will have to wait for more than 10s?
c) What is the probability that a call will be delayed for more than 10 seconds?

<u>Solution:</u>

Have given,

Cell radius, R = 1.387 km

Area covered per cell is $2.598 \times (1.387)^2$ ≈ 5 sq km

Number of cells per cluster,n = 4

Total number of channels, N = 60

Therefore, number of channels per cell = 60 / 4 = 15 channels. From Erlang C chart, for 5% probability of delay with C = 15,

Traffic intensity, A = 9.0 Erlangs.

(a)

Have given,

Traffic per user, A_u = 0.029 Erlangs.

We know,

The number of users, U = A / A_u

= 9.0/0.029

= 310 users

The number of users per square km = 310 users /5 sq km

= 62 users /sq km

(b)

Have given,

Wave length, λ = 1 call/hour

Holding time, H = A_u / λ

= 0.029 hour

= 104.4 seconds.

Time, t = 10s

We know,

The conditional probability that a delayed call will have to wait for more than t seconds is
$P_r[\text{delay} > t \mid \text{delay}] = \exp(-(C-A)t/H)$
$= \exp(-(15-9)10/104.4)$
$= 56.29\ \%$

(c)

Have given,

The probability of delayed call, $P_r[\text{delay} > 0] = 5\ \% = 0.05$

We know,

Probability that a call is delayed more than 10 seconds,

$$P_r[\text{delay} > 10] = P_r[\text{delay} > 0] \times P_r[\text{delay} > t \mid \text{delay}]$$
$$= 0.05 \times 0.5629$$
$$= 2.81\ \%$$

Source Code (Python):

```
import math
# Parameters
R = 1.387  # Cell Radius in km
n = 4  # Number of cells
N = 60  # Total number of channels
area = round(2.5981 * R**2)  # Area covered per cell (rounded)
C = N / 4  # Number of channels per cell
A = 9  # Traffic intensity from Erlang C chart with GOS = 0.05 and Au = 0.029

# Question (a)
Au = 0.029  # Traffic per user
U = math.floor(A / Au)  # Total number of users
U_per = round(U / area)  # Number of users per square km
print(f"(a) Number of users per square km: {U_per} users/sq km\n")

# Question (b)
lambda_ = 1  # Arrival rate in hours
H = (Au / lambda_) * 3600  # Holding time in seconds
t = 10  # Time delay in seconds (assumed to be 10s as per the MATLAB code)
Prb = math.exp((-(C - A) * t) / H)  # Probability of delay
print(f"(b) The probability that a delayed call will have to wait: {Prb * 100:.2f}%\n")

# Question (c)
Prc = 0.05 * Prb * 100  # 5% probability of a delayed call
print(f"(c) The probability that a call will be delayed: {Prc:.2f}%\n")
```

Output:

(a) Number of users per square km	: 62 users/sq km
(b) The probability that a delayed call will have to wait	: 56.29%
(c) The probability that a call will be delayed	: 2.81%