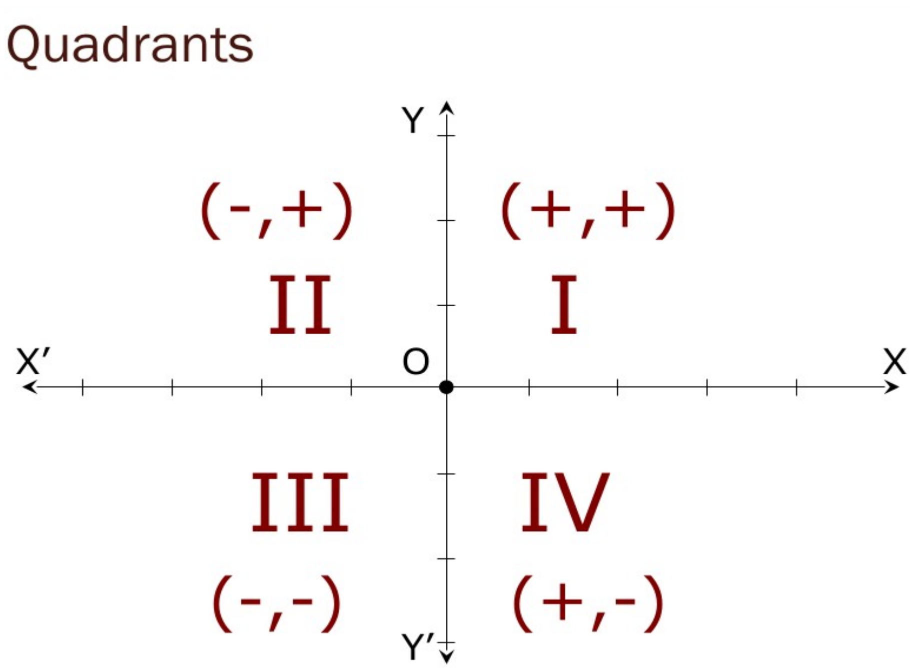
Unit -1 Coordinate Geometry

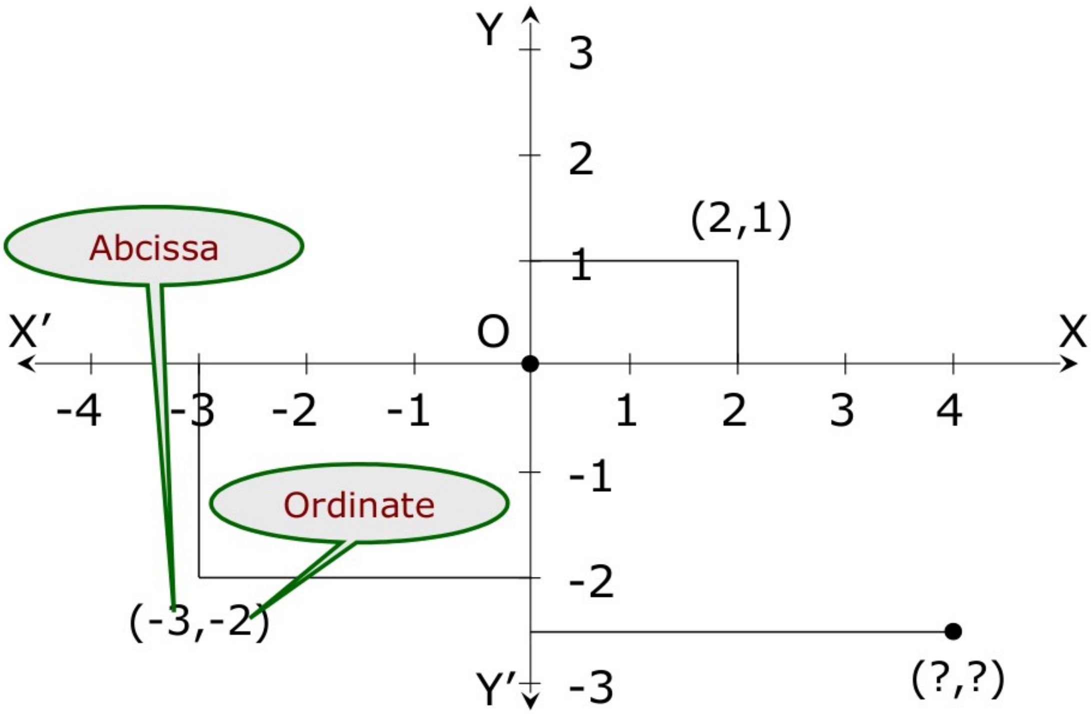
- A system of geometry where the position of points on the plane is described using an ordered pair of numbers.
- The method of describing the location of points in this way was proposed by the French mathematician René Descartes .
- He proposed further that curves and lines could be described by equations using this technique, thus being the first to link algebra and geometry.
- In honor of his work, the coordinates of a point are often referred to as its Cartesian coordinates, and the coordinate plane as the Cartesian Coordinate Plane.

Some Basic Points

- To locate the position of a point on a plane,we require a pair of coordinate axes.
- The distance of a point from the y-axis is called its **x-coordinate,OR abscissa.**
- The distance of a point from the x-axis is called its **y-coordinate,OR ordinate.**
- The coordinates of a point on the x-axis are of the form (x, 0) and of a point on the y-axis are of the form (0, y).



Coordinates



Ex 1: Identify the quadrant

1. (1,0) - X-axis
2. (-1,2) – 2^{nd}
3. (0,5) – Y-axis
4. (-3,-2) – 3^{rd}
5. (5,-3)- 4^{th}
6. (6,7)- 1^{st}

Ex 2: Find the distance between A(7,8) and B(1,0)

$AB^2 = (x_1-x_2)^2 + (y_1-y_2)^2$

$= 36+64$

$=100$

$AB = \sqrt{100}$

$AB = 10$

Ex 3: Find the distance between A(6,4) and B(-1,3)

$AB^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$

$= (6-(-1)) +^2 (4-3)^2$

$= 50$

$AB = \sqrt{50} = 5\sqrt{2}$

Ex 4: A(-2,0) and B(a,8) are two points if AB=10, Then find the value of a.

$AB^2 = (a+2)^2 + (8-0)^2$

$100 = a^2 + 4a + 4 + 64$

$a^2 + 4a - 32 = 0$

$(a+8)(a-4) = 0$

$a = -8$ or $a = 4$

Ex 5: Find the distance between A(b,0) and B(0,a)

$AB^2 = (a-0)^2 + (b-0)^2$

$AB = \sqrt{a^2 + b^2}$

Ex 6: A(0,0) and B(-2,-4)

$AB^2 = (-2-0)^2 + (-4-0)^2 = 20 = 4 \times 5$

$AB = 2\sqrt{5}$

Ex 7: P(0,0) and Q(-1,3) Or Find the distance of (-1,3) from origin

Ans: $PQ = \sqrt{10}$

Ex:8 If the distance between (a,-5) and (2,a) is 13 . find the value of a.

Hint: $(a-2)^2 + (a+5)^2 = (13)^2$

Types of triangle:

- **Equilateral**: **"equal"**-lateral (lateral means side) so they have all **equal sides**
- **Isosceles**: means "equal legs", and we have **two legs**, right? Also i**sos**celes has two equal **"S**ides" .
- **Scalene**: means "uneven" or "odd", so no equal sides.
- **Right angle Triangle**: In a right angled triangle:
  the square of the hypotenuse is equal to
  the sum of the squares of the other two sides.

Ex 8: Identify the triangle formed by following vertices :

a. A(-2,-2) , B(-1,2) and C(3,1)
   AB=BC
   Isosceles triangle

b. A(-3,2) , B(1,2) and C(-3,5)

   $AB^2 = 16$, $BC^2 = 25$ $AC^2 = 9$

   $AB^2 + AC^2 = BC^2$

   Angle A= 90

   Right angled triangle

c. A(2,5) , B(8,5) and C(5,5+3$\sqrt{3}$¿)

   $AB^2 = 36$
   $BC^2 = 9+(3\sqrt{3}$¿$^2 = 9+(27) = 36$
   $AC^2 = 36$
   AB= BC= AC

   Equilateral traingle

d. A(-1,-1) , B(1,5) and C(2,8)
   $AB = 2\sqrt{10}$, $BC = \sqrt{10}$ $AC = 3\sqrt{10}$
   If AB + BC = AC 0r BC +CA= AB
   Points are collinear

e. A(1,4) B(2,3) C(-1,0)

$AB^2 = (1-2)^2 + (4-3)^2 = 2$, $BC^2 = 18$, $AC^{2=}(4+16)=20$

$AB \neq BC \neq AC$

Scalene or right angle

Note: A right angle triangle can be isosceles or scalene.

Section formula

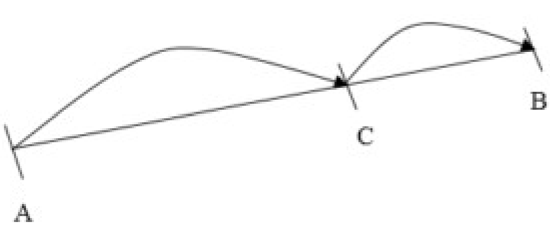
When a point C divides a segment AB in the ratio m:n, we use the section formula to find the coordinates of that point. The section formula has 2 types. These types depend on the position of point C. It can be present between the 2 points or outside the segment.

The two types are:

1. Internal Section Formula
2. External Section Formula

Internal Section Formula

Also known as the Section Formula for Internal Division. When the line segment is divided internally in the ration m:n, we use this formula. That is when the point C lies somewhere between the points A and B.



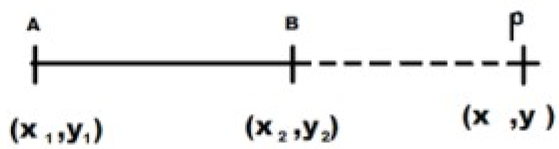
The Coordinates of point C will be,

$$\mathbf{P(x,y)} = \left(\frac{mx2+nx1}{m+n}, \frac{my2+ny1}{m+n}\right)$$

Breaking it down, the x coordinate is $(mx_2+nx_1)/(m+n)$ and the y coordinate is $(my_2+ny_1)/(m+n)$

Section Formula for External Division

When the point P lies on the external part of the line segment, we use the section formula for the external division for its coordinates.



A point on the external part of the segment means when you extend the segment than its actual length the point lies there. Just as you see in the diagram above. The section formula for external division is,

$$P(x,y)= \left(\frac{mx2-nx1}{m-n}, \frac{my2-ny1}{m-n}\right)$$

Breaking it down, the x coordinate is$(mx_2-nx_1)/(m-n)$and the y coordinate is$(my_2-ny_1)/(m-n)$

Midpoint Formula

When we need to find the coordinates of a point that lies exactly at the center of any given segment we use the midpoint formula.

The midpoint formula is,

$$P=\left(\frac{x1+x2}{2}, \frac{y1+y2}{2}\right)$$

Breaking it down, the x-coordinate is **$(x_1+x_2)/2$** and the y-coordinate is **$(y_1+y_2)/2$**

Solved Examples for You

Question 1: The point P divides the line segment AB joining points A(2,1) and B(-3,6) in the ratio 2:3 then find P.

Answer : Given that A(2,1)=(x1,y1), B(-3,6)=(x2,y2)

Point P divides the segment AB in the ratio 2:3, hence m=2, n=3

Since it isn't mentioned in the question that the point divides the segment externally we use the section formula for internal division,

Formula: $P=\{[(mx_2+nx_1)/(m+n)],[(my_2+ny_1)/(m+n)]\}$

Substituting all the known values,

$=\{[(2(-3)+3(2))/(2+3)],[(2(6)+3(1))/(2+3)]\}$

$=[(-6+6/5), (12+3/5)] =(0/5, 15/5)$

Implies, P =(0,3)

Question 2: Z (4, 5) and X(7, – 1) are two given points and the point Y divides the line-segment ZX externally in the ratio 4:3. Find the coordinates of Y.

Answer : Given that, Z(4,5)=(x1,y1), X(7,-1)=(x2,y2)

Point Y divides the segment ZX in the ratio 4:3, hence m=4, n=3

Since it is mentioned in the question that the point Y divides the segment externally we use the section formula for external division,

Formula: $Y=\{[(mx_2-nx_1)/(m-n)],[(my_2-ny_1)/(m-n)]\}$

Substituting the known values,

$=\{[(4(7)-3(4))/(4-3)],[(4(-1)-3(5)/(4-3)]\}$

$=\{(28-12)/1,(-4-15)/1\} =\{16,-19\}$

The coordinates for the point Y are (16,-19)

Question 3: Find the midpoint of segment AB where A(2,3) and B(6,7).

Answer: Given that, A(2,3)=(x1,y1), B(6,7)=(x2,y2)

Formula: $P=\{(x_1+x_2)/2,(y_1+y_2)/2\}$

Substituting the known values,

P={(2+6)/2,(3+7)/2} ={8/2,10/2} =(4,5)

The Midpoint of the segment AB is (4,5).

Question 4: In what ratio the line joining A(5,12) and B(2,9) is divided by a point P(3,10)

X= $\frac{mx2+nx1}{m+n}$

3= $\frac{2m+5n}{m+n}$

3m+3n=2m +5n

m=2n

m:n =2:1

Question 5: In what ratio the line joining A(1,-3) and B(3,5) is divided by a point P(6,17)

Slope or Gradient of a straight line passing through points (x1,y1) and (x2,y2)

M= $\frac{y2-y1}{x2-x1}$

Question 6 :find the slope for the line joining these points

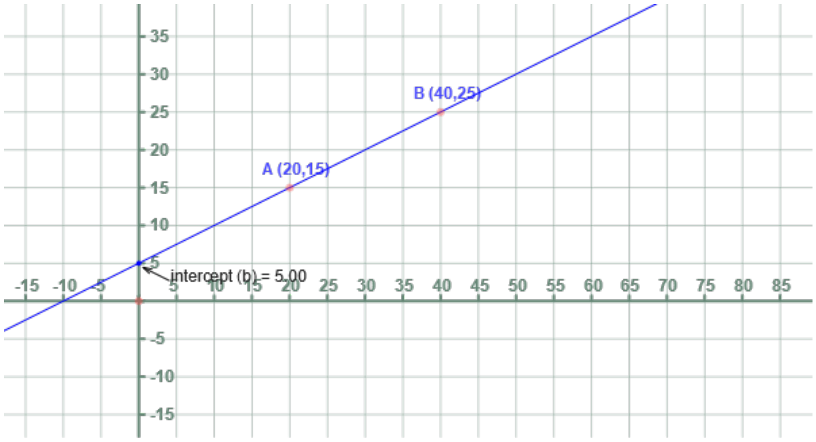
i)A(2,3) and B(7,6)

ii)A(0,7) and B(5,-2)

Intercepts of a line on the axis

The x-intercept of a line is the point at which it crosses x axis.

The y-intercept of a line is the point at which it crosses y axis.



Equation of a line making intercept a on x-axis and b on y-axis :

$$\frac{x}{a} + \frac{y}{b} = 1$$

Equation of a line joining two given points

Suppose a straight line passes through the points A(x_1, y_1) and B(x_2, y_2) with another point on the line P(x, y).

The equation of a line through A(x_1, y_1) and B(x_2, y_2) is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

Ex: Find the equation of the line joining points A(−2,3) and B(1,4)

(x1, y1) = (-2,3) and (x2, y2) = (1, 4)

$$\frac{y - 3}{4 - 3} = \frac{x - (-2)}{1 - (-2)}$$

$$\text{or } \frac{y - 3}{1} = \frac{x + 2}{1 + 2}$$

$$\text{or } y - 3 = \frac{x + 2}{3}$$

$$\text{or } 3y - 9 = x + 2$$

$$\text{or } 3y = x + 11 \text{ or } y = \frac{1}{3}x + \frac{11}{3}$$

Equation :x-3y+11=0

Ex: (3,-5) and (4,7)

Equation : 12x-y=41

Equation of a line with slope m and passing through a given point

The equation of a line through A(x_1, y_1) with gradient m is

$y – y_1 = m(x – x_1)$

Ex: find the equation of line having slope $\frac{1}{2}$ and passing through (5,4)

(x1, y1) = (5, 4) ,m = 1/2

y - 4 = (1/2)(x - 5)

2(y - 4)= x-5

2y-8 = x-5

x-2y+3=0

Ex: find the equation of line having slope $\frac{1}{3}$ and passing through (-2,7)

Equation: x-3y+23=0

Equation of a line with slope m and making intercept c on y- axis:

y=mx+c

EX: A line makes intercept $\frac{5}{2}$ on y-axis and its slope is $\frac{3}{4}$ find its equation

m = ¾, c = 5/2

y = (3/4)x + 5/2

4y = 3x +10

3x-4y+10 =0

Ex: A line makes intercept 3 on y-axis and its slope is 2 find its equation

General form of equation of line

$Ax+By+C=0$

Slope = $-\frac{A}{B}$

x-intercept = $-\frac{C}{A}$

y-intercept = $-\frac{C}{B}$

Ex: find intercepts and slope

i) $\mathbf{2x+3y=4}$

$2x+3y-4=0$

Slope = $-\frac{2}{3}$

X-intercept= 2

y-intercept= $\frac{4}{3}$

ii) $\mathbf{x=y+2}$

$x-y-2=0$

Slope = 1

X-intercept= 2

y-intercept= -2

iii) $\mathbf{5x+2y+3=0}$

iv) $\frac{x}{a} + \frac{y}{b} = 1$

Hint : $bx+ay-ab=0$

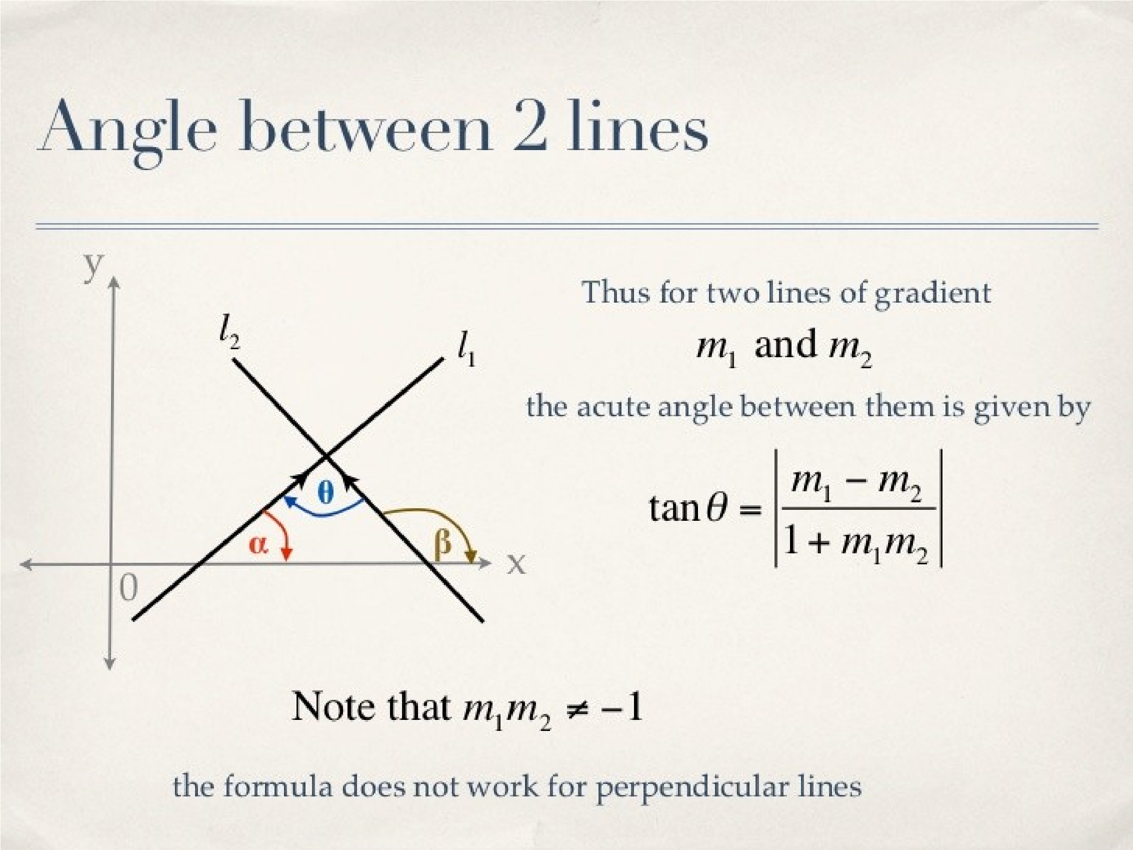
$Ax+By+C=0$

v) $\mathbf{y-11 = 0}$

$Ax+By+C=0$

$A=0,\ B=1\ ,\ C=-11$

Angle between two lines



If m1=m2 two lines are parallel

If m1.m2=-1 two lines are perpendicular

Ex: Find the angle between the lines 5x-2y+5 =0 and 3x-7y+3=0

m1 = 5/2 m2 = 3/7

Now, let's find the angle θ using the formula:

tan(θ) = |(m1 - m2) / (1 + m1 * m2)|

tan(θ) = |((5/2) - (3/7)) / (1 + (5/2) * (3/7))|

tan(θ) = |(35 - 6) / (14 + 15)| tan(θ) = |29 / 29|

tan(θ) = |29/29| = 1

θ = 45 degrees

So, the angle between the two lines is 45 degrees.

Ex: Check whether given lines are parallel or perpendicular?

3x+4y+2=0 $m_1 = -\frac{3}{4}$

16x+12y-7=0 $m_2 = \frac{-3}{4}$

Parallel

Ex: Find the value of k, if slope of the given line 4x-ky-7=0 is 3.

Slope =3

K= $\frac{4}{3}$

Ex : Find the equation of a line perpendicular to the line joining (3,2) and (4,0) and passing through (5,7)

Slope of the line passing through (3,2) and (4,0).

m = (y2 - y1) / (x2 - x1)

m = (0 - 2) / (4 - 3) = -2

Slope of the perpendicular line = -1 / (-2) = 1/2

The point-slope form is given by:

y - y1 = m(x - x1)

Using (x1, y1) = (5,7) and slope m = 1/2, we get:

y - 7 = (1/2)(x - 5)

2(y - 7)= x-5

2y -14 = x - 5

x – 2y +9 =0