<u>Unit 2</u>

Cross product of two vectors:

Example:

Find the cross product of the given two vectors:

$\vec{X} = 5\vec{i} + 6\vec{j} + 2\vec{k}$ *and* $\vec{Y} = \vec{i} + \vec{j} + \vec{k}$

Solution:

Given:

$\vec{X} = 5\vec{i} + 6\vec{j} + 2\vec{k}$

$\vec{Y} = \vec{i} + \vec{j} + \vec{k}$

To find the cross product of two vectors, we have to write the given vectors in determinant form. Using the determinant form, we can find the cross product of two vectors as:

$$\vec{X} \times \vec{Y} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 5 & 6 & 2 \\ 1 & 1 & 1 \end{vmatrix}$$

By expanding,

$\vec{X} \times \vec{Y} = (6-2)\vec{i} - (5-2)\vec{j} + (5-6)\vec{k}$

Therefore,

$\vec{X} \times \vec{Y} = 4\vec{i} - 3\vec{j} - \vec{k}$

Question 2: Find the cross product of two vectors $\overrightarrow{a} = (3,4,5)$ and $\overrightarrow{b} = (7,8,9)$

Solution:

The cross product is given as,

$$a \times b = \begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 5 \\ 7 & 8 & 9 \end{matrix}$$

$= [(4\times9)-(5\times8)]\,\hat{i} - [(3\times9)-(5\times7)]\hat{j} + [(3\times8)-(4\times7)]\,\hat{k}$

$= (36-40)\hat{i} - (27-35)\hat{j} + (24-28)\,\hat{k} = -4\hat{i} + 8\hat{j} - 4\hat{k}$

Answer: $\therefore\ \overrightarrow{a} \times \overrightarrow{b} = \mathbf{-4\hat{i} + 8\hat{j} - 4\hat{k}}$

<u>Unit 3</u>

Applications of Matrix:

Real-Life Example

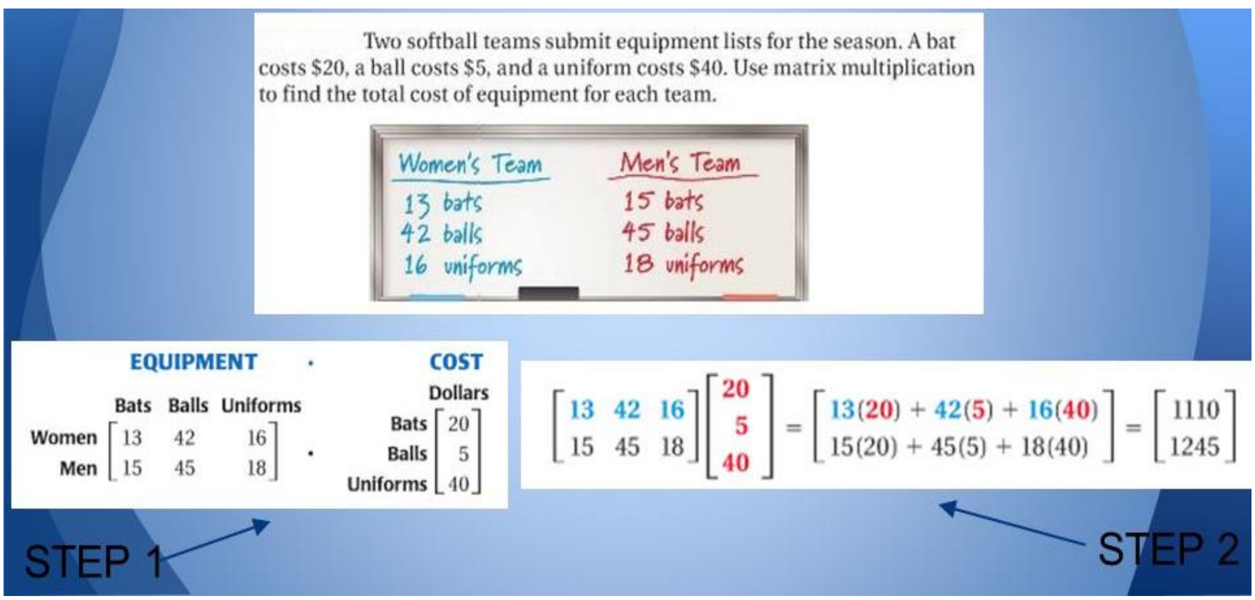
Let's say you're in avid reader, and in June, July, and August you read fiction and non-fiction books, and magazines, both in paper copies and online. You want to keep track of how many different types of books and magazines you read, and store that information in matrices. Here is that information, and how it would look in matrix form:

June			July			August		
	Paper	Online		Paper	Online		Paper	Online
Fiction	2	4	Fiction	3	2	Fiction	1	3
Non-Fiction	3	1	Non-Fiction	1	1	Non-Fiction	2	3
Magazines	4	5	Magazines	5	3	Magazines	4	6
Matrix Form:	$\begin{bmatrix} 2 & 4 \\ 3 & 1 \\ 4 & 5 \end{bmatrix}$		Matrix Form:	$\begin{bmatrix} 3 & 2 \\ 1 & 1 \\ 5 & 3 \end{bmatrix}$		Matrix Form:	$\begin{bmatrix} 1 & 3 \\ 2 & 3 \\ 4 & 6 \end{bmatrix}$	

a) The senior class play was performed on three different evenings. The attendance for each night is shown below. Adult tickets sold for \$3.50 and students for \$2.50. Use matrix multiplication to determine how much money was taken in each night.

Night	Adults	Students
Thursday	420	300
Friday	400	450
Saturday	510	475

$$\begin{bmatrix} 420 & 300 \\ 400 & 450 \\ 510 & 475 \end{bmatrix} \cdot \begin{bmatrix} 3.50 \\ 2.50 \end{bmatrix} = \begin{array}{cl} \begin{bmatrix} \$2220 \\ \$2525 \\ \$2972.50 \end{bmatrix} & \begin{array}{l} \text{Thursday} \\ \text{Friday} \\ \text{Saturday} \end{array} \end{array}$$



<u>Rank of Matrix :</u> The "rank" of a matrix A, RK(A), is the number of rows or columns, n, of the largest $n \times n$ square submatrix of A for which the determinant is nonzero.

1. Find the rank of the matrix $\begin{pmatrix} 1 & 5 \\ 3 & 9 \end{pmatrix}$

Solution:

Let A= $\begin{pmatrix} 1 & 5 \\ 3 & 9 \end{pmatrix}$

Order of A is 2×2 ∴ $\rho(A) \leq 2$

Consider the second order minor

$$\begin{vmatrix} 1 & 5 \\ 3 & 9 \end{vmatrix} = -6 \neq 0$$

There is a minor of order 2, which is not zero. ∴$\rho(A) = 2$

2. Find the rank of the matrix $\begin{pmatrix} -5 & -7 \\ 5 & 7 \end{pmatrix}$

Solution:

Let A= $\begin{pmatrix} -5 & -7 \\ 5 & 7 \end{pmatrix}$

Order of A is 2×2 $\therefore \rho(A) \leq 2$

Consider the second order minor

$\begin{vmatrix} -5 & -7 \\ 5 & 7 \end{vmatrix} = 0$

Since the second order minor vanishes, $\rho(A) \neq 2$

Consider a first order minor $|-5| \neq 0$

There is a minor of order 1, which is not zero

$\therefore \rho(A) = 1$

3. Find the rank of the matrix $\begin{pmatrix} 0 & -1 & 5 \\ 2 & 4 & -6 \\ 1 & 1 & 5 \end{pmatrix}$

Solution:

Let A= $\begin{pmatrix} 0 & -1 & 5 \\ 2 & 4 & -6 \\ 1 & 1 & 5 \end{pmatrix}$

Order Of A is 3x3

$\therefore \rho(A) \leq 3$

Consider the third order minor $\begin{vmatrix} 0 & -1 & 5 \\ 2 & 4 & -6 \\ 1 & 1 & 5 \end{vmatrix} = 6 \neq 0$

There is a minor of order 3, which is not zero

$\therefore \rho(A) = 3$.

4. Find the rank of the matrix $\begin{pmatrix} 5 & 3 & 0 \\ 1 & 2 & -4 \\ -2 & -4 & 8 \end{pmatrix}$

Solution:

Let A= $\begin{pmatrix} 5 & 3 & 0 \\ 1 & 2 & -4 \\ -2 & -4 & 8 \end{pmatrix}$

Order Of A is 3x3

$\therefore \rho\ (A) \leq 3$

Consider the third order minor $\begin{vmatrix} 5 & 3 & 0 \\ 1 & 2 & -4 \\ -2 & -4 & 8 \end{vmatrix} = 0$

Since the third order minor vanishes, therefore $\rho(A) \neq 3$

Consider a second order minor $\begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix} = 7 \neq 0$

There is a minor of order 2, which is not zero.

$\therefore \rho(A) = 2.$