<u>Unit -2</u>

Vector and Matrices

Physical Quantities

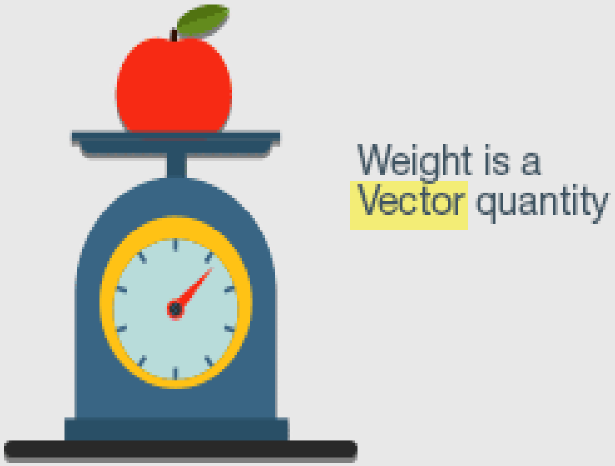
- Scalar
  - Has magnitude only
- Vector
  - Has magnitude and direction

What are Scalars and Vectors?

- A scalar has only magnitude (size):
- 3.044, −7 and 2½ are scalars

- density, charge, pressure, energy, work and power are all scalars.

SCALAR AND VECTOR



SCALAR

A scalar is a quantity that is fully described by a magnitude only. It is described by just a single number. Some examples of scalar quantities include speed, volume, mass and time.

VECTOR

A vector is a quantity that has both a magnitude and a direction. Vector quantities are important in the study of motion. Some examples of vector quantities include force, velocity, acceleration and momentum.

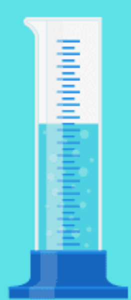
Scalar vs Vector

SCALAR

A scalar quantity has magnitude only.

speed

mass



volume

time

VECTOR

A vector has both magnitude and direction.

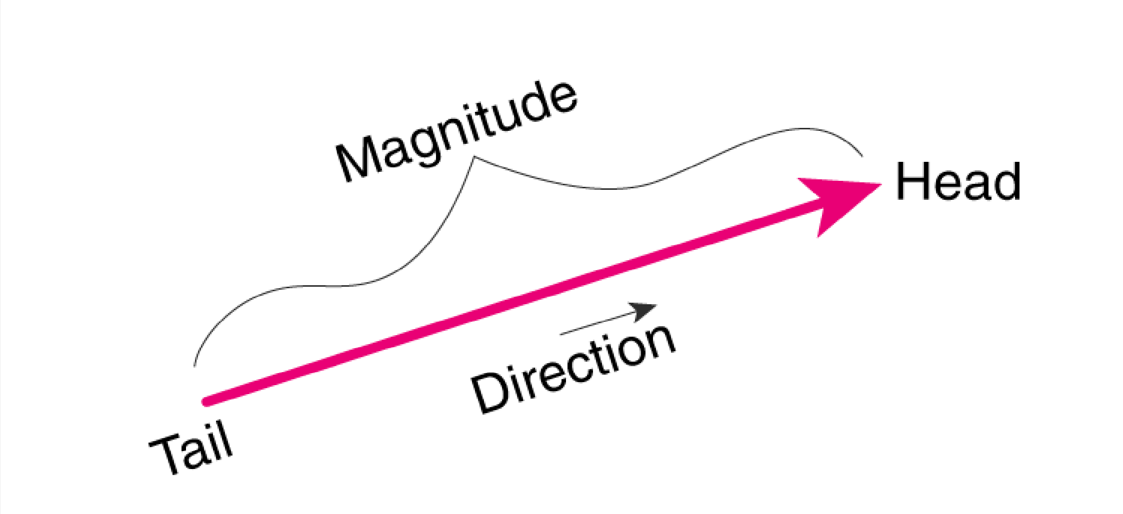
velocity

weight

friction

What are Vectors?

- A vector has magnitude and direction:

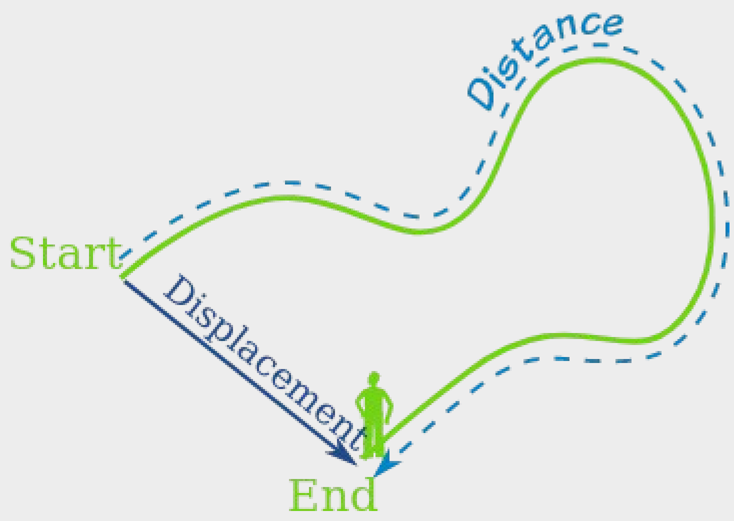


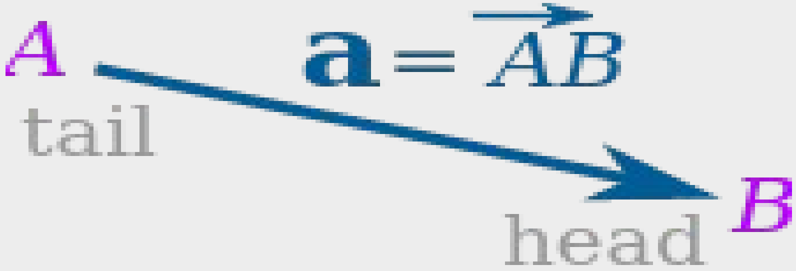
Displacement, velocity, acceleration, force and momentum are all vectors.

What are Vectors?

- Vector is a physical quantity that has both direction and magnitude.
- In other words, the vectors are defined as an object comprising both magnitude and direction. It describes the movement of the object from one point to another.

- Distance is a scalar ("3 km")
- Displacement is a vector ("3 km Southeast")
- You can walk a long distance, but your displacement may be small (or zero if you return to the start).



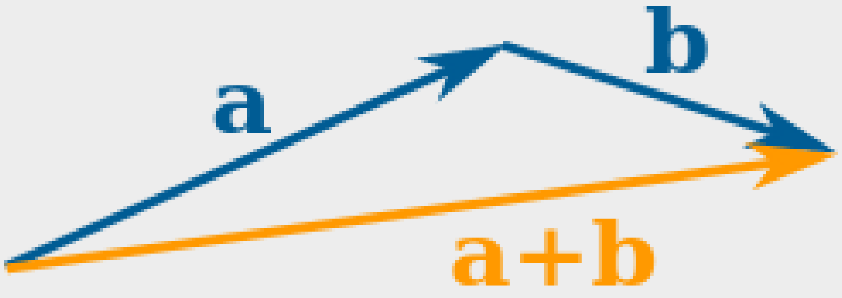
- A vector can be written as the letters of its head and tail with an arrow above it, like this:
- 

- **Using Scalars**

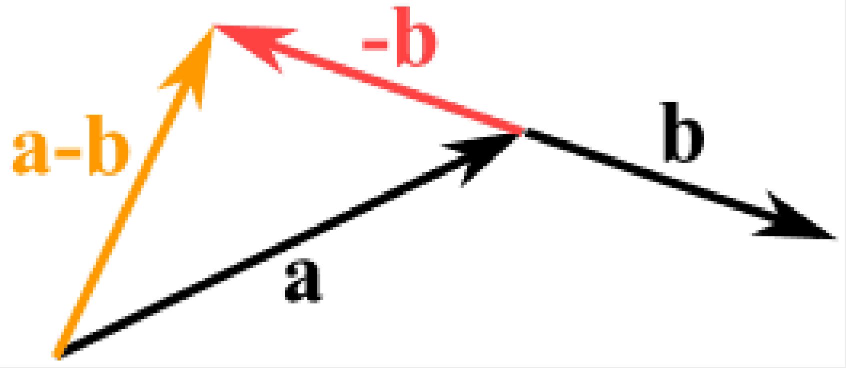
  Scalars are easy to use. Just treat them as normal numbers.

  Example: 3 kg + 4 kg = 7 kg

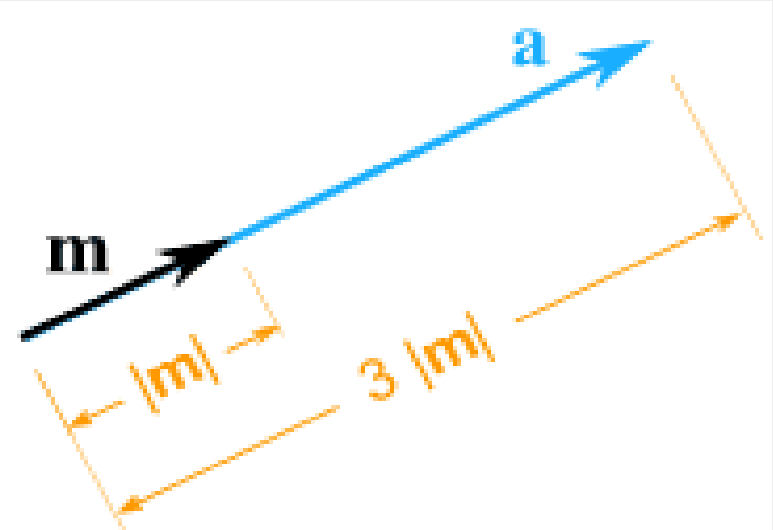
- **Using Vectors**
- We can add two vectors by joining them head-to-tail:



- We can subtract one vector from another:
- first we reverse the direction of the vector we want to subtract,then add them as usual:



- We can multiply a vector by a scalar (called "scaling" a vector):
- Example: multiply the vector m = (7,3) by the scalar 3



- vector scaling a = 3m = (3×7,3×3) = (21,9)
- It still points in the same direction, but is 3 times longer
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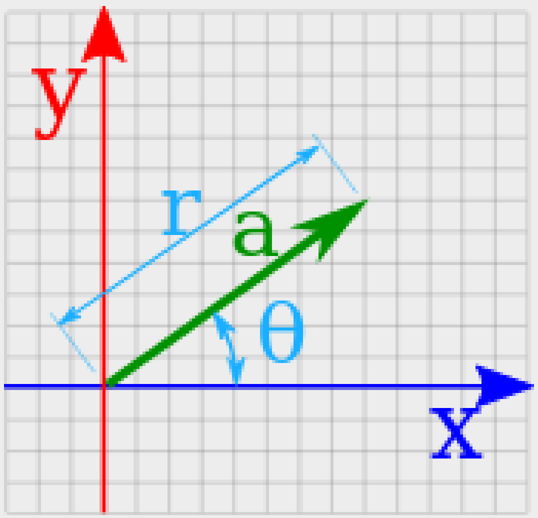
- **Polar or Cartesian**

  A vector can be in:
- magnitude and direction (Polar) form,
- or in x and y (Cartesian) form



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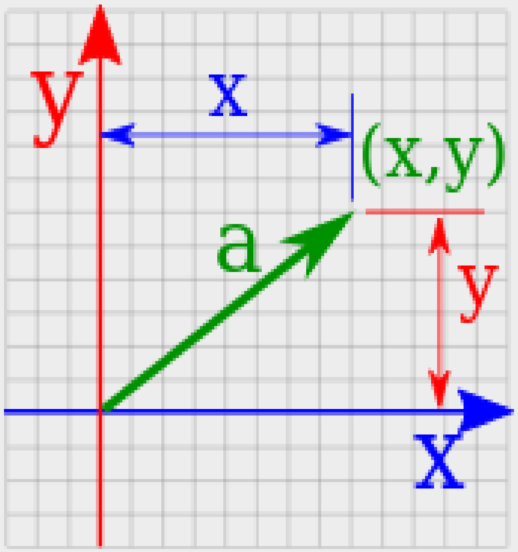
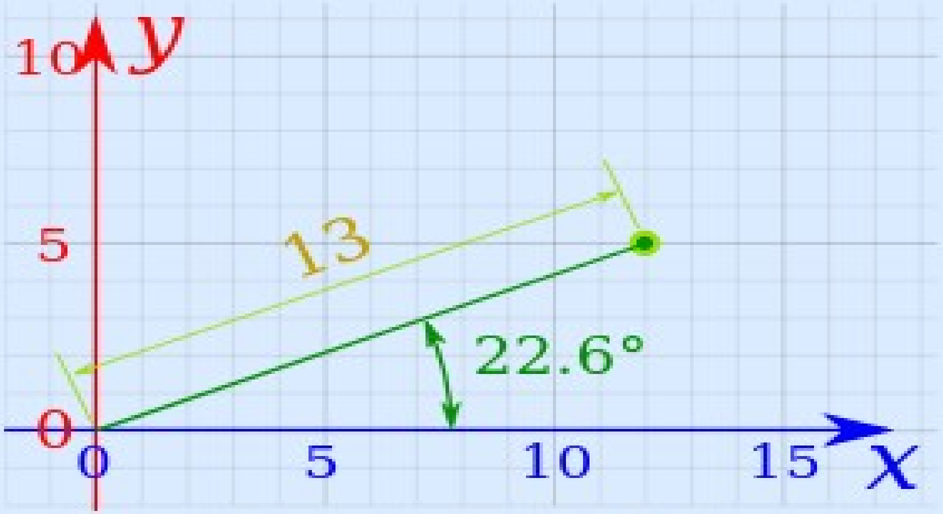
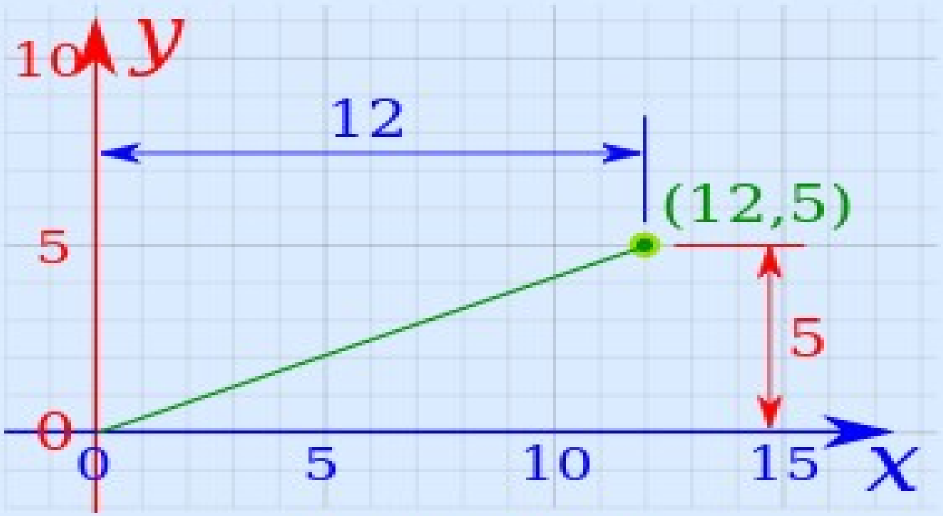


In Polar (magnitude and direction) form:



The vector **13 at 22.6°**

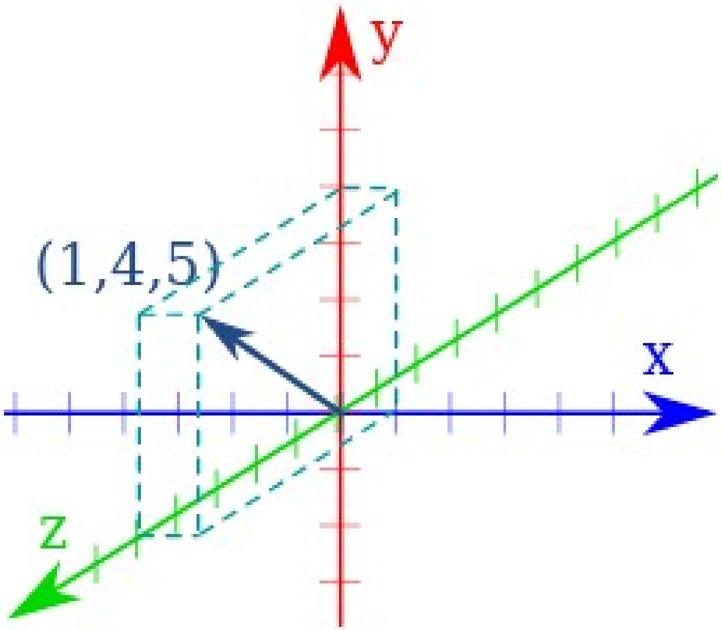
Is approximately **(12,5)** In Cartesian (x,y) form:



The vector **(12,5)**

More Than 2 Dimensions

Vectors also work perfectly well in 3 or more dimensions:



The vector (1,4,5)

List of Numbers

So a vector can be thought of as a **list numbers**:

- 2 numbers for 2D space, such as (4,7)
- 3 numbers for 3D space, such as (1,4,5)

Types of Vectors

- Zero Vector
- Unit Vector
- Position Vector
- Co-initial Vector
- Like and Unlike Vectors
- Co-planar Vector
- Collinear Vector
- Equal Vector
- Displacement Vector
- Negative of a Vector

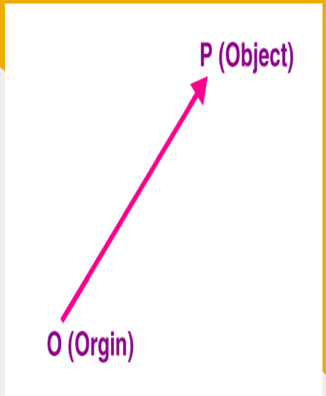
Zero vector

- When the starting point and the finish point of a vector coincide with each other, it is known as a zero vector or null vector.
- The magnitude of such vectors is zero, and they, in particular, do not represent any direction.

Unit vector

- Vectors having a value of exactly one are known as a unit vector.
- Unit vectors are very important, and note that if two vectors are unit vectors, they are not specifically equal.
- They might have the same magnitude but can differ in their direction.

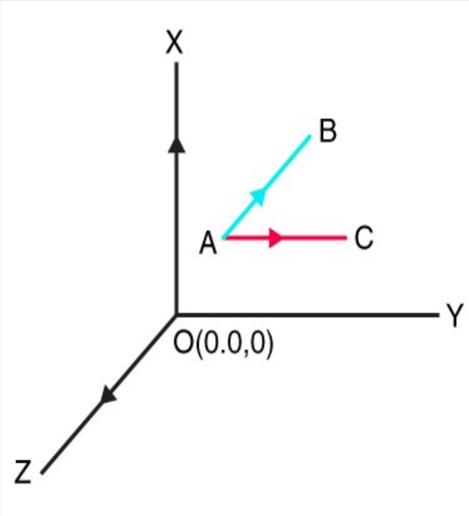
Position Vector



Position vectors are known to determine the position of any vector.

A position vector is nothing but a point on any vector which tells the position of that vector in a plane

Co-initial Vectors



Vectors are expressed as co-initial vectors

if they have the same origin point.

This implies that the point of origin

is common for these types of vectors, and then they may scatter in different directions.

- For example, let us consider two vectors PQ and PR; they are called co- initial vectors due to the fact they have the same beginning point, i.e., P.
-

Like Vectors

When two or more vectors share the same direction, they are known as like vectors.

Unlike Vectors

When two or more vectors travel in different directions, they are termed as unlike vectors.

Coplanar Vectors

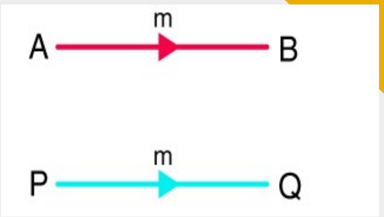
Coplanar vectors are vectors (three or more) that lie in the same plane.

Collinear Vectors

These are also referred to as parallel vectors because they lie in the parallel line concerning their magnitude and direction.

Equal Vectors

Vectors having the same magnitude and the same directions are known as equal vectors.



Displacement Vector

The vector KL represents a displacement vector if a point is moved (displaced) from the position K to L.

Negative of a Vector

Let us assume that vector K has a magnitude ‘p’ and is in a certain direction, now let us suppose that another vector L is present having the same magnitude ‘p’ but travels in exactly the opposite direction of K. Thus, L is referred to as the negative of a vector K. K = -L Therefore, the negative of any vector is another vector with the same magnitude but opposite in direction.

Vector Properties

Let $\vec{u},\ \vec{v},\ \vec{w}\ \in \mathbb{R}^n$ be three vectors.
Let $c,\ d \in \mathbb{R}$ be two scalars.

- $\vec{u} + \vec{v} = \vec{v} + \vec{u}$
- $(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$
- There exists a zero vector $\vec{0} = \langle 0, 0, \ldots, 0 \rangle \in \mathbb{R}^n$ such that $\vec{v} + \vec{0} = \vec{v}$
- There exists a negative vector $-\vec{v}$ such that $\vec{v} + (-\vec{v}) = \vec{0}$
- $(cd)\,\vec{v} = c\,(d\vec{v})$
- $(c + d)\,\vec{v} = c\vec{v} + d\vec{v}$
- $c\,(\vec{v} + \vec{u}) = c\vec{v} + c\vec{u}$
- $1\vec{v} = \vec{v}$
- $0\vec{v} = \vec{0}$

Operations on Vectors

Vector Addition

If u = <u1   u2> and v = <v1 v2>

Then u + v = <u1 +v1   u2 +v2>

Scalar Multiplication: k u = < ku1   ku2>

Vector Subtraction

If u = <u1   u2> and v = <v1 v2>

Then u - v = u + (-v) = <u1 -v1   u2 -v2>

Dot Multiplication

u. v = u1.v1 + u2.v2

Operations on Vectors

Vector Addition and Scalar Multiplication:

Given that $\vec{a} = \langle 1, 5 \rangle$ and $\vec{b} = \langle -7, 3 \rangle$, what are the components of $2\vec{a} + 2\vec{b}$ and $2(\vec{a} + \vec{b})$?

$$\boxed{2\vec{a} + 2\vec{b} = \langle -12, 16 \rangle}$$

$2(\vec{a} + \vec{b})$

$$\vec{a} + \vec{b} = \langle 1, 5 \rangle + \langle -7, 3 \rangle$$

$$= \langle 1 + (-7), 5 + 3 \rangle$$

$$= \langle -6, 8 \rangle$$

$$2(\vec{a} + \vec{b}) = 2\langle -6, 8 \rangle$$

$$\boxed{2(\vec{a} + \vec{b}) = \langle -12, 16 \rangle}$$

Scalar Multiplication of a vector:

$$k\langle c, d \rangle = \langle kc, kd \rangle$$

Addition of two vectors:

$$\langle c, d \rangle + \langle m, n \rangle = \langle c + m, d + n \rangle$$

Distributive property of Scalar Multiplication over vector addition:

$$k(\vec{a} + \vec{b}) = k\vec{a} + k\vec{b}$$

$$2(\vec{a} + \vec{b}) = 2\vec{a} + 2\vec{b}$$

$$= \langle -12, 16 \rangle$$

Operations on Vectors

Two vectors $\vec{A}$ and $\vec{B}$ are given in the component form as $\vec{A} = 5\hat{i} + 7\hat{j} - 4\hat{k}$ and $\vec{B} = 6\hat{i} + 3\hat{j} + 2\hat{k}$. Find $\vec{A} + \vec{B}$, $\vec{B} + \vec{A}$, $\vec{A} - \vec{B}$, $\vec{B} - \vec{A}$

Solution

$$\vec{A} + \vec{B} = (5\hat{i} + 7\hat{j} - 4\hat{k}) + (6\hat{i} + 3\hat{j} + 2\hat{k})$$
$$= 11\hat{i} + 10\hat{j} - 2\hat{k}$$
$$\vec{B} + \vec{A} = (6\hat{i} + 3\hat{j} + 2\hat{k}) + (5\hat{i} + 7\hat{j} - 4\hat{k})$$
$$= (6 + 5)\hat{i} + (3 + 7)\hat{j} + (2 - 4)\hat{k}$$
$$= 11\hat{i} + 10\hat{j} - 2\hat{k}$$
$$\vec{A} - \vec{B} = (5\hat{i} + 7\hat{j} - 4\hat{k}) - (6\hat{i} + 3\hat{j} + 2\hat{k})$$
$$= -\hat{i} + 4\hat{j} - 6\hat{k}$$
$$\vec{B} - \vec{A} = \hat{i} - 4\hat{j} + 6\hat{k}$$

Operations on Vectors

Norm or Length of Vector

The norm of a vector $\vec{v}$ is the length or magnitude of $\vec{v}$, and it is denoted: $||\vec{v}||$

If $\vec{v} = \langle v_1,\ v_2,\ v_3, ..., v_n \rangle \in \mathbb{R}^n$, then we calculate the norm of $\vec{v}$ as:

$$\boxed{||\vec{v}|| = \sqrt{v_1^2 + v_2^2 + \cdots + v_n^2}}$$

Ex : Find the norm of Vector <u> = <1 3   -2>

Solution : $||\vec{u}|| = \sqrt{1^2 + 3^2 + (-2)^2} = \sqrt{14}$

Operations on Vectors

Ex: If $\langle 1,2,2\rangle + k\langle 2,1,2\rangle = \langle 5,4,6\rangle$, find k.

$$\langle 5,4,6\rangle = \langle 1,2,2\rangle + k\langle 2,1,2\rangle$$

$$\langle 5,4,6\rangle = \langle 1,2,2\rangle + \langle 2k,k,2k\rangle$$

$$\langle 5,4,6\rangle = \langle 1+2k,\ 2+k,\ 2+2k\rangle$$

Comparing both sides we get

1+2k = 5,

2+k = 4

And 2 + 2k =6 , we get k = 2

Ex: **If u = < 1   3 -5> and v = <2   -1 4> then find u.v**

u.v = 2 + (-3) + (-20) = -21