ADVANCED MECHANICS

PLANETARY ORBITS

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1 Planetary Motion

The motion of a planets is an n-body problem. However, solving the two-body problem correlating the motion of a particular planet with respect to the sun allows us to arrive at good approximations to understand the orbits of the planet. As part of this project we obtain the trajectory of an earth-like planet about the sun by numerically solving the following equations that were derived analytically:

$$k = GMm \tag{1.1}$$

$$\mathbf{L} = \mathbf{r} \times m\mathbf{v} \tag{1.2}$$

$$c = \frac{L^2}{km} \tag{1.3}$$

$$E = \frac{1}{2}m(\dot{\mathbf{r}})^2 - \frac{k}{r} \tag{1.4}$$

$$a = \frac{c}{1-e^2} \tag{1.5}$$

$$T^2 = \frac{4\pi^2 a^3}{GM} \tag{1.6}$$

$$\omega = \frac{2\pi}{T} \tag{1.7}$$

$$r = a(1 - e\cos\Psi) \tag{1.8}$$

$$\zeta = \omega(t + t_p) = \Psi - e\sin\Psi \tag{1.9}$$

$$r = \frac{c}{1 + e\cos(\phi - \phi_0)} \tag{1.10}$$

The initial conditions for the trajectory of this particular earth-like planet are as follows:

$$\text{Mass of sun} = 1.989 * 10^{30} kg$$

$$\text{Mass of planet} = 5.97219 * 10^{24} kg$$

$$\text{Initial distance from Sun} = 1AU$$

$$\text{Velocity at initial position} = 20kmph$$

2 Results

Plotting the graph results in an elliptical orbit, as to be expected from Kepler's First Law. It is noted that the trajectory as obtained for the above initial conditions places the planet at the Aphelion of its trajectory at $t = 0$. The resultant graphs show the trajectory taken by the planet between 4.7 and 5 earth years from the initial time.
The following parameters are also obtained which helps define our planets trajectory:

$$\text{eccentricity of the elliptical trajectory} = e = 0.55$$

$$\text{Time Period of trajectory} = T = 0.52 \text{ Earth Years}$$

The graphs of the trajectory and Ψ vs ζ are as shown in the following page and the code is attached at the end of the document.

3 Graphs and Code

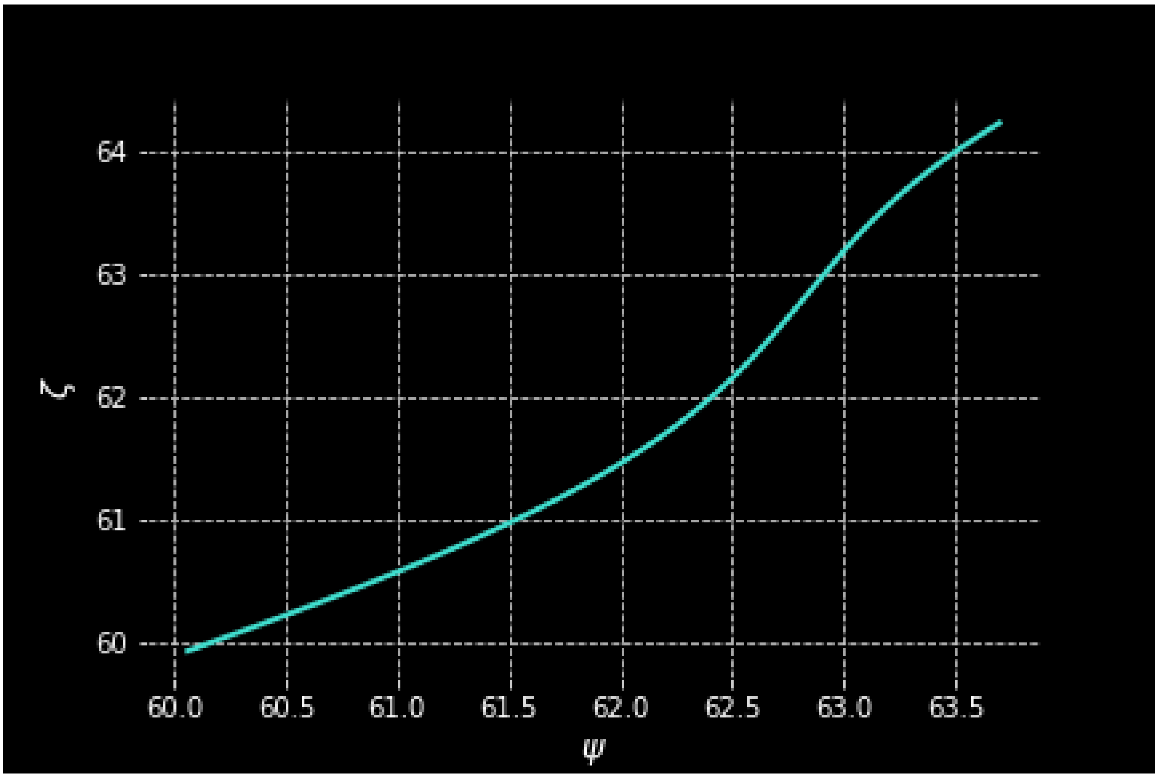

Figure 1: Ψ vs ζ between 4.7 and 5 earth years

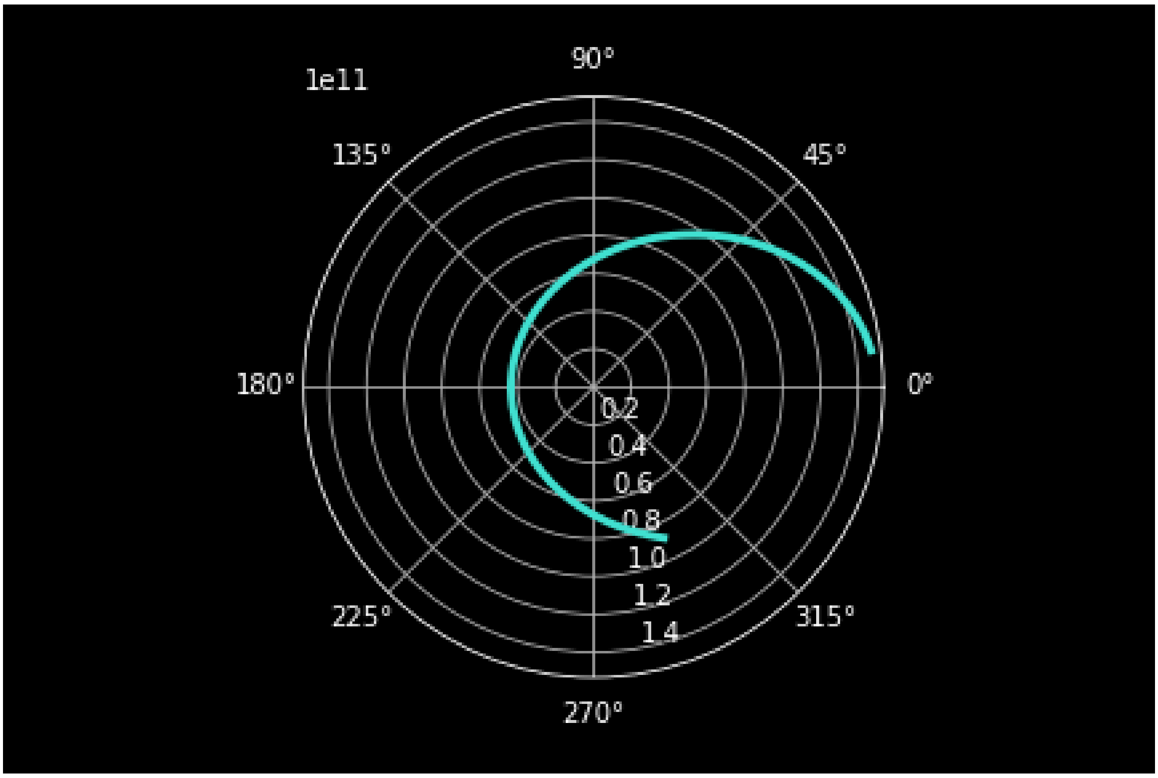

Figure 2: Trajectory of the Planet between 4.7 and 5 earth years(scale of length is in meters)

```python
#import packages
import numpy as np
import matplotlib.pyplot as plt

#constants
G = 6.67430*(10**(-11)) #Gravitational constant
Ms = 1.989*(10**30) #Mass of sun
Me = 5.97219*(10**24) #Mass of earth
day = 60*60*24 #in seconds
year = 60*60*24*365 #in seconds

#inital conditions
r0 = 149587871*1000 #meters #from distance from sun
v0 = 20*1000 #meters/sec #velocity at perihelion
ti = 4.7*year #time to start plotting in seconds
tf = 5*year #time to start plotting in seconds

#single calculations
k = G*Ms*Me
L = Me*v0*r0
c = (L**2)/ (k*Me)
E = 0.5*Me*(v0**2) - k/r0
e = np.sqrt((E*2*c/k) + 1)
a = c/(1 - e**2)
T = np.sqrt(4*(np.pi**2)*(a**3)/(G*Ms))
w = 2*np.pi/T
psi_0 = np.arccos((1 - (r0/a))/e)
phi_0 = np.arccos(((c/r0) - 1)/e)
tp = (psi_0 - e*np.sin(psi_0))/w

#defining t
tstep = 1000
t = np.linspace(ti, tf, tstep)

#defining zeta
z = np.zeros(len(t))
for i in range(len(t)):
    z[i] = w*(t[i] + tp)

#finding psi

psi = np.zeros(len(t))

def f(zeta):
    return zeta - e*np.sin(zeta) - z[i]

def fd(zeta):
    return 1 - e*np.cos(zeta)
```

```python
def newtonraphson(zeta, i):

        h = f(zeta)/fd(zeta)
        while abs(h) >= 0.0001:
            zeta = zeta - h
            h = f(zeta)/fd(zeta)

        psi[i] = zeta

for i in range(len(t)):
    newtonraphson(psi_0, i)

#finding r from psi
r = np.zeros(len(t))
for i in range(len(t)):
    r[i] = a*(1-e*np.cos(psi[i]))

#finding phi from r
phi = np.zeros(len(t))
for i in range(len(t)):
    if i!=0 and r[i] > r[i - 1]:
        phi[i] =  + np.arccos((c/r[i] - 1)/e) + phi_0
    else:
        phi[i] = - np.arccos((c/r[i] - 1)/e) + phi_0

#plotting
fig, ax = plt.subplots(subplot_kw={'projection': 'polar'})
fig.set_facecolor("black")
ax.set_facecolor("black")
ax.set_rlabel_position(280)
ax.tick_params(axis="both",colors="White", labelsize = 10)
for spine in ax.spines.values():
    spine.set_edgecolor("White")
ax.plot(phi,r, color="turquoise", linewidth = 3)
fig.savefig("planetary_orbit.png")

fig, ax1= plt.subplots()
fig.set_facecolor("black")
ax1.set_facecolor("black")
ax1.set_ylabel("$\zeta$",color="white", size = 13)
ax1.set_xlabel("$\psi$",color="white", size = 13)
ax1.tick_params(axis="both",colors="White", labelsize = 10)
for spine in ax.spines.values():
    spine.set_edgecolor("White")
ax1.grid(color = "lightgrey", linestyle = "--")
ax1.plot(z, psi, color="turquoise", linewidth = 2)
fig.savefig("planetary_orbit2.png")
```