An Extended Model of Herd Behavior

1 Introduction

Decision making is a vital process in the human society. The process of decision making employs a variety of strategies to solve various problems under various scenarios. One such strategy that has attracted us is the imitative strategy i.e. copying the decision made by another person in a similar scenario. The consequence of using such imitative strategies extensively leads to a set of people following the same behavior which is referred as herd behavior.

A simple model of herd behavior was studied and presented in [1]. As an extension to this model, we try to incorporate game theoretic principles explained in [2]. [2] discusses about imitation in large games by characterizing the players as a distribution of player types and defining different strategies for different player types.

In this article, we try to set up a simple and generic game model with a set of players distributed into multiple types and a set of assets each associated with a payoff. A players strategy is dictated by his type, and his ultimate objective is to choose an asset that maximizes his payoff. Such a game is simulated for multiple rounds to study and characterize formation of herds for different distributions of player types and other parameters that are sensitive to such games later defined by the model. We then proceed to study the effects of such parameters on herd behaviour in the game based on the results generated by the output of the simulation.

2 Basic Model of Herd Behaviour

The Basic Model of Herd Behaviour as characterized in [1] describes a sequential decision model where the decision makers make their decisions looking at the decisions made by the previous decision makers. The author further defines a set of tie-breaking assumptions made by the decision makers while making decisions, and concludes that the model results in an equilibrium that is inefficient.

In the model there is a finite set of assets, represented on a number line, among which one of them, say i*, is associated with a positive payoff z. The payoffs rendered by rest of the assets is 0. Each decision maker, with the number decision makers participating in the model being finite, chooses an asset on the number line in a fixed order. As usual, the objective for each decision maker is to earn the maximum payoff, make the optimal decision and hence effectively choose i*. Further, each decision maker is provided with a signal to an asset with a probability alpha and the asset pointed to by the signal is i* with a probability beta. For example, an alpha of 0.5 and a beta of 0.6, implies each decision maker may or may not get a signal with a probability 0.5 and on the chance that he does get a signal , it points to right choice i* with a probability 0.6.

On receiving a signal, the decision maker can choose to follow the signal or defer it. Deferring a signal would be to choose an asset in the compliment set of the asset pointed to by the signal. A good example of a scenario in which a decision maker would prefer deferring his signal would be when there is a herd already being formed at a particular asset by his predecessors. In this case, it would be natural for the decision maker to follow the herd or imitate , as the mere formation of a herd at an asset implies that there has been enough information encountered by his predecessors (through their signals) to believe that their choice, i.e. the herded choice, is the right choice. Deferring a signal, would also imply loss of possible useful information from the deferred signal for the subsequent decision makers.

On setting up and describing the 'simple model' the paper further studies the rationale behind the yesimitation strategy, the information it sustains, the loss of information as the game proceeds, and the implications of following such a strategy for the decision makers that follow. The main contributions of the paper are that the equilibrium pattern of choices will always be inefficient no matter what the order of decision makers or their signals, and that the probability that none of the decision makers make the right choice is bounded away from zero, for any size of the population.

3 An Extended Model of Herd Behaviour

The model proposed in [1] is basic and does not accommodate more complex scenarios, like that of a repeated play, different player types , etc. In this article , we explore such extensions to the basic model by using a game theoretic approach, study the formation of herds in such game settings and analyse the effect of related parameters on herd formation.

In this extension of the basic model, we consider repeated games with a distributed payoff structure. The model has a finite number of players N, a finite number of assets M, and the game is repeated a finite number of times, NR. The assets are represented on a number line. Each asset is associated with a payoff, as decided by a payoff function. Thus, there is no single right choice. Which implies, the objective of a player in this game would be, to make a choice that would improve upon his current best payoff i.e. a choice that would give him a payoff better than any of his previous ones. This plays a heavy role in making decisions in a repeated game play, as here there is an added chance for the players to learn more about the assets as the game proceeds.

As the payoff is distributed, and players are not aware of the payoffs associated with the assets, in a real life scenario he could call upon some sort of prescient knowledge of the possible distribution from external sources or his own. This knowledge is manifested here as a signal to the player. This signal that each player receives points to a choice from the set of assets. This choice could improve upon his current-best payoff with a probability 1-beta /2 , where beta is the probability with which the choice could be the same as that from which the player gained his current best payoff. Building on which, the player has the probability of getting a signal pointing to a choice that does not improve upon his current-best payoff with the probability 1-beta /2.

The decision making in this model just as the basic model in [1], is sequential. The player to make the first decision, the next , and so on in the sequence, is chosen at random and no player is repeated with in a round. Each player making a decision, can look at the choices made by the predecessors in the sequence, within the round, but not their payoff. Added to which, he cannot find out the signal received by his predecessors that led them to make their decisions. At the end of each round, which is finite in itself, each player is evaluated for his choices and awarded with the payoff pertaining to their choice. This glimpse into the payoff associated with a choice plays a significant role in a repeated play, as this implies that the player can now remember the choice and its payoff, for the subsequent rounds. Let us say he adds it to his memory of explored assets and their payoffs and let us call this the explored-asset set. He can now intelligently invest on assets in the set complement to his explored-asset set, in the subsequent rounds.

When a choice has to be made, we can hardly brand people to follow the same rules, and react the same way to situations and options they are presented with. Similarly, the players can be grossly seen to fall into the following three types: Optimizers, Imitators and Randomizers.

- The first category of players called the Optimizers are those who gather complete information about the situation before making a decision. Based on all the information present, the player assess the different possible strategies he could adopt along with their expected outcomes, and makes an informed decision. Players of this type could examine their history to gather information about the assets they have already chosen in the previous rounds, choice pointed by the current signal, the choices at which herds are already formed, and choose the option that they expect would give a payoff higher than their current best payoff.
- The next category of players, namely the Imitators, are those kind of players who prefer to use minimum resources in their decision making process even if it results in a sub-optimal choice. To them , the process of making a decision by collecting all the information, is too complicated, and costly compared to the simple act of imitating a player who is already doing that. Thus in the game they can be seem to give a higher priority to a growing herd, instead of their own signal.
- The last category of players, the Randomizers are those who prefer to toss a coin and pick an asset at random in each round of the game rather than optimizing or imitating. These type of players play a key role in such multiplayer games , as they bring a sort of dynamism to the game play as that which would be seen often in a real scenario.

Finally, here a herd is numerically defined by a threshold on the total number of players in the game. A threshold of 5 % in a game with 200 players would imply, when the number of players who decide to invest on the same asset equals or exceeds 10 , a herd is said to have been formed at that asset.

4 A Study on the Extended Model using Simulations

In this section, we further discuss the extended model by running simple simulations of it and present their analysis. The simulations are designed by making simple compromises on the extended model - such as constrained player memory(termed window size here), random assignment of player types - that may at the least introduce only minimal noise to the data which is accounted for in its analysis.

The population in which the game is played can be very variable, say 80% and 15% of the players could be optimizers and imitators respectively, which would fix the randomizers at 5%, or there could 3 times the number of imitators as the optimizers , like in the case of 20% optimizers and 60% imitators,etc.. So, a valid study should include an analysis of the decisions and choices made by the players of different types, their inclinations towards following a herds, when a herd can be expected to form, etc., under different conditions of player distributions, and that of the game such as number of rounds, threshold, etc., they form.

As the model involves a repeated game play, the sequential decision making process is iterated for a finite number rounds (NR) with a fixed threshold (Th), beta value and a finite number of players (N) and assets (M), defined as a part of the input. Thus, an iteration of the game would include assignment of types to players as dictated by the input population distribution, the generation of signals to players before they make a decision, player decision making process, determining the formation of herds based on the threshold value during and after a round and realization of payoffs for the players. The same game is repeated for different population distributions keeping all the other input parameters fixed.

5 Analysis of Simulation Results

5.1 Effects of Change in Window Size on Herd Behavior

For, Number of Players (N) = 400, Number of Assets (M) = 40,Number of Rounds(NR)=30, Threshold Percentage(Th) = 10 and Beta = 0.6 with -

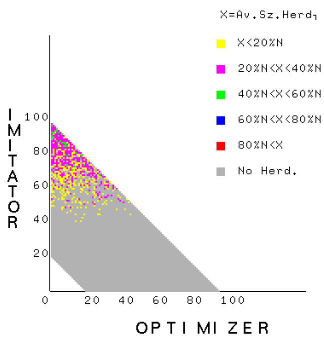


Fig. 1. Window Size 5

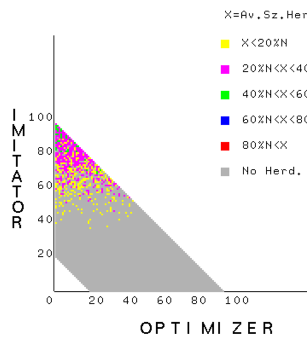


Fig. 2. Window Size 15

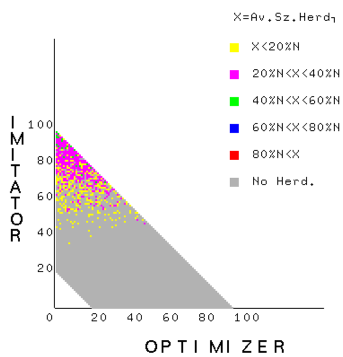


Fig. 3. Window Size 30

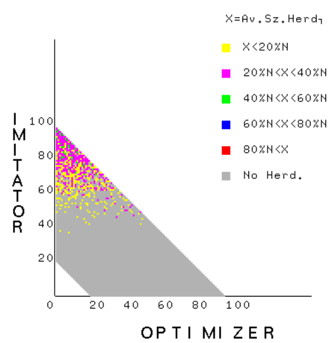


Fig. 4. Window Size 40

The window size represents the players active memory at any point in the game. The memory of the player here implies the information collected about the choices in the game over the past rounds. Naturally, with greater memory and hence a greater W, it is natural to expect the players to make informed decisions towards their objective. It is expected that with the increase in W, there would be more Herd formation towards the higher paying choices for a fixed number of rounds. For all W¿=NR, not much of a difference in the Herd Distribution can be observed, except for the changes due to signal generations, as the excess windows will not even have a chance to be used. With the Optimizers being the players making the maximum use of the windows, as W increases there is higher Herd Behaviour in the distributions with a higher percentage of Optimizers. This is implied from the simple relation that as a greater number of players now make use of the information in their windows, they have a better chance to move towards achieving their objective.

5.2 Effects of Change in Number of Rounds on a Herd Behavior

For, Number of Players (N) = 300, Number fo Assets (M) = 40, Window Size for each Player (W) = 5, Threshold Percentage(Th) = 10 and Beta = 0.6 with -

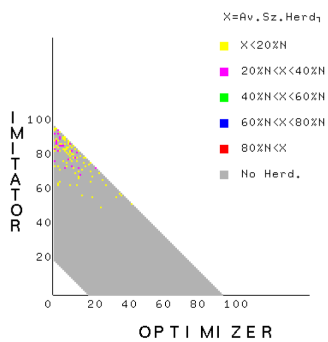


Fig. 5. Number of Rounds = 20

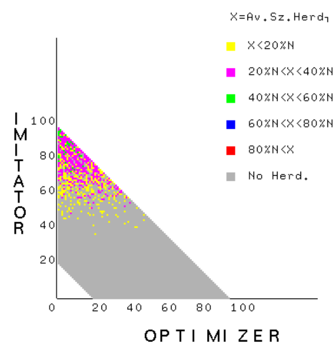


Fig. 6. Number of Rounds = 30

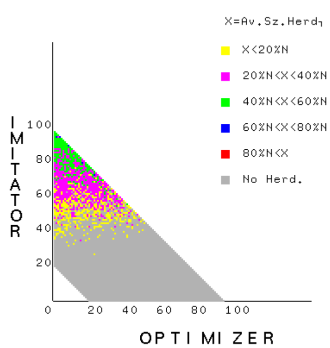


Fig. 7. Number of Rounds = 40

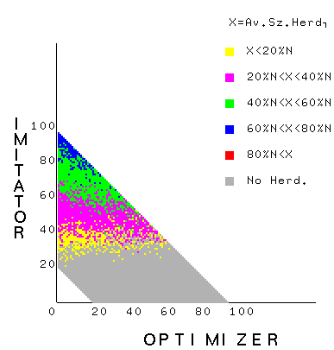


Fig. 8. Number of Rounds = 60

With greater number of rounds of play, the players have a greater opportunity to learn about the payoff distribution. Given that the objective of the players is to improve on their payoffs, with greater number of rounds they have a greater chance at attempting to satisfy the same. It is implied from the previous statement that , with greater NR there is a natural movement of the players

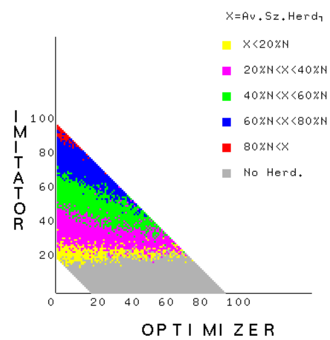
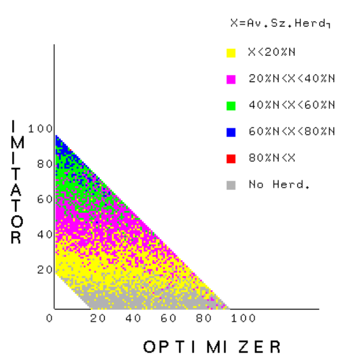


Fig. 9. Number of Rounds = 100

towards the higher payoffs. In other words, as more players converge towards the optimal solution, there is a higher chance of Herd Behaviour and the formation of bigger Herds.

5.3 Effects of Change in Herd Threshold on a Herd Behavior

For, Number of Players (N) = 400, Number fo Assets (M) = 40, Number of Rounds(NR) = 30, Window Size for each Player (W) = 5 and Beta = 0.6 with -



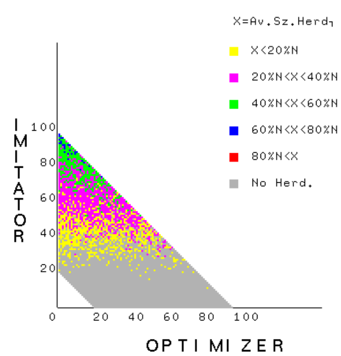


Fig. 10. Threshold Percentage = 5 **Fig. 11.** Threshold Percentage = 7

The threshold value defines the necessary number of players to accumulate at a choice for it to be called a herd. As the threshold increases the probability of Herd formation decreases as the number of choices at which the threshold is crossed, decreases considerably. With a smaller threshold value, it can easily seen that this value is surpassed early in the repeated game play. This implies that

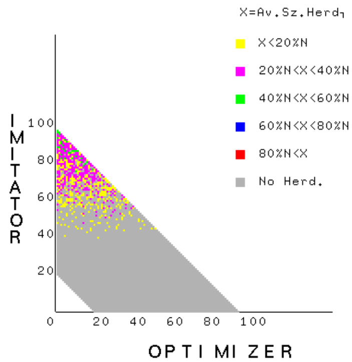


Fig. 12. Threshold Percentage = 10

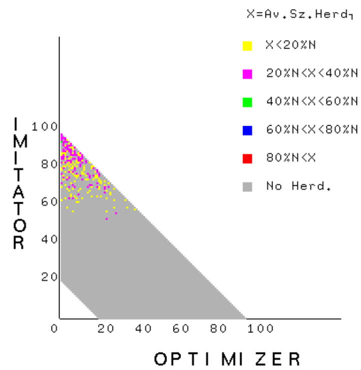


Fig. 13. Threshold Percentage = 12

the very first herd is formed earlier in the game for lower Thresholds. Owing to the imitators priority to follow herds already formed as their rationale in making a decision , the formation of large herds can be found. As expected, the graph for a larger Threshold shows the formation of smaller herds.

5.4 Effects of Change in Beta on a Herd Behavior

For, Number of Players (N) = 400, Number fo Assets (M) = 40, Number of Rounds(NR) = 30, Window Size (W) = 5 and Threshold(Th) = 10 with -

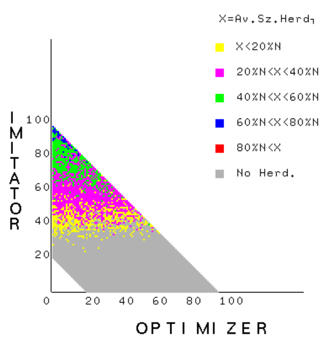


Fig. 14. Beta = 0.0

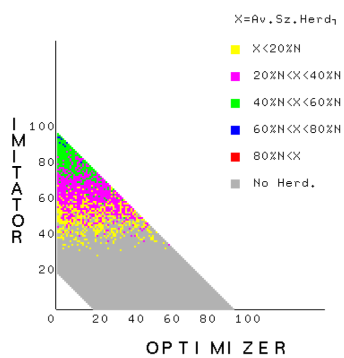


Fig. 15. Beta = 0.3

The Beta value determines the dynamic signals pointing to choices that may promise a better payoff for the player is generated for each player at each round. It implies that the player may get a signal for a choice he has already seen with a

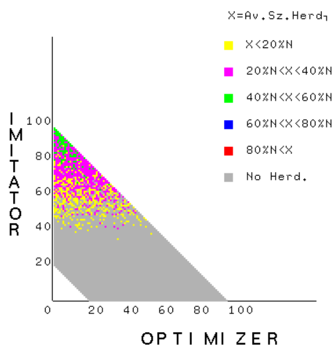


Fig. 16. Beta = 0.5

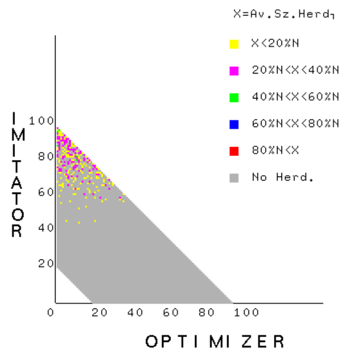


Fig. 17. Beta = 0.7

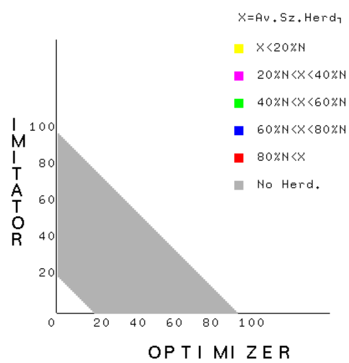


Fig. 18. Beta = 1.0

probability of Beta. Beta takes the values from [0,1] and as the value increases, the formation of herd decreases as the probability with which the players experiment with the choices not seen before decreases. For a lower Beta value, the player have a higher chance to move towards achieving their objective. This behavior can be clearly seen from the graphs. When beta is one, the signal is generated to a choice already seen by the player and hence does not aid the player in improving his current best payoff. Hence a players deviation from his initial choice is minimal thus contributing to a lesser chance of a herd behaviour.

5.5 Effects of Change in Number of Players on a Herd Behavior

Here a progressively diminishing pattern of Herd Behaviour is seen as in the number of players increased , for the same setting of the number of assets, For, Number fo Assets (M) = 40, Number of Rounds(NR) = 30, Window Size for each Player (W) = 5, Threshold(Th) = 10 and Beta = 0.6 with -

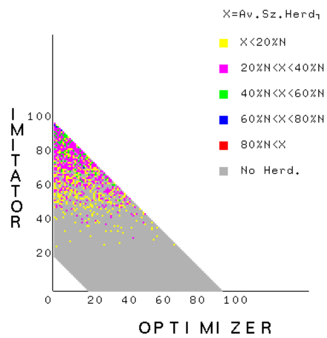


Fig. 19. Number of Players = 200

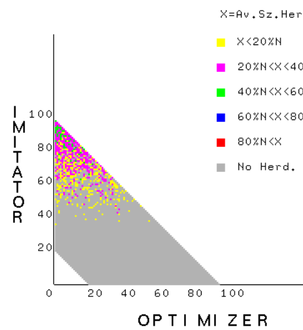


Fig. 20. Number of Players = 300

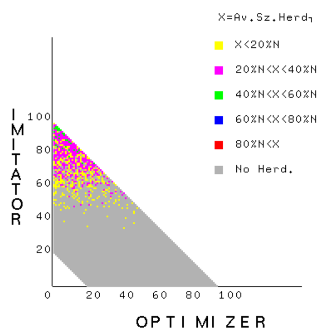


Fig. 21. Number of Players = 400

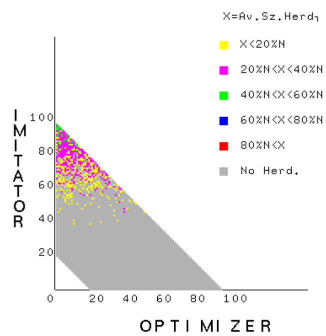


Fig. 22. Number of Players = 500

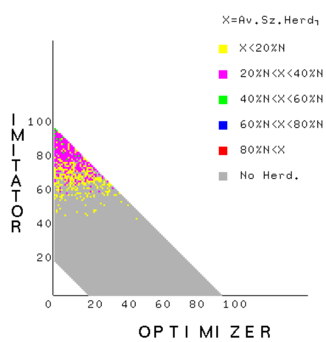


Fig. 23. Number of Players = 700

the herd defining threshold and other parameters, is seen. This behaviour can be explained by an argument similar to the one employed for the explaining the effect of increasing threshold on herd formation. For smaller values of N in a setting with the same threshold percentage, the first ever herd formed is formed in the earlier rounds. Larger number of players inevitably implies a higher number of players to accumulate at a choice for it to be called a herd.

5.6 Effects of Change in Number of Assets on a Herd Behavior

For, Number of Players (N) = 400, Number of Rounds(NR)=30, Window Size(W) = 5, Threshold Percentage(Th) = 10 and Beta = 0.6 with -

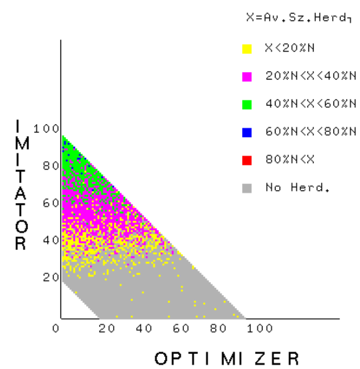


Fig. 24. Number of Assets = 20

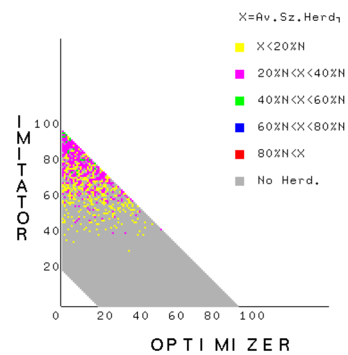


Fig. 25. Number of Assets = 30

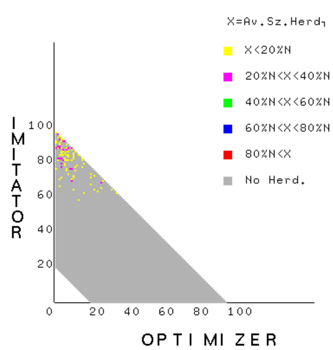


Fig. 26. Number of Assets = 40

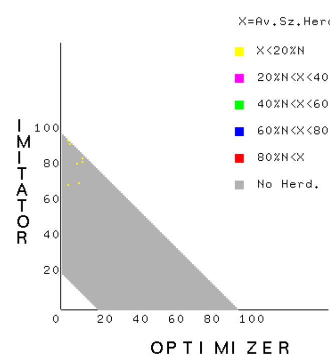


Fig. 27. Number of Assets = 50

As the number of assets increases, the players have a larger space to choose from. A good example of how the decision made by the players may be affected by an availability of more options would be, a scenario of opinion polling in a district during elections. In a region previously only contested by two major political parties, say Party A and Party B, when set foot into by a new political party, say Party C, gives an alternative option for the voters previously not interested in either of the parties but were mandated to choose anyway. Similarly with more choices i.e. an increase in the number of assets M , the players previously a part of a growing herd now have a choice to move away from it. This causes a distribution of the players on the choice space, thus less likely to form larger herds or even herds in general.

6 Conclusion

References

[1] Abhijit V. Banerjee (1992): *A* Simple Model of Herd Behaviour. The Quarterly Journal of Economics 107(3),pp. 797817.

[2] Soumya Paul, R. Ramanujam : *I*mitation in Large Games. In proceedings of the First International Symposium on Games, Automata, Logics and Formal Verification (GandALF), Electronic Proceedings in Theoretical Computer Science, 2010.