# 1 Introduction

The beta function is a special function that belongs to the first category of Euler's integrals. The beta function is denoted by the symbol "$B$". $B(x,y)$ refers to the beta function, where $x$ and $y$ are real-valued parameters.
The formula of $B(x,y)$ is[1]:

- $B(x,y) = \frac{(x-1)!(y-1)!}{(x+y-1)!}$ For positive integers

- $B(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}dt$ For positive real numbers

# 2 Properties of Beta Function

- Beta function is symmetric which means its beta value is independent of the order of its parameters: $B(x,y) = B(y,x)$
- $B(x,y) = B(x,y+1) + B(x+1,y)$
- $B(x,y+1) = B(x,y) \cdot [\frac{y}{x+y}]$
- $B(x+1,y) = B(x,y) \cdot [\frac{x}{x+y}]$
- $B(x,y).B(x+y,1-y) = \frac{\pi}{x}\sin \pi y$
- Beta function in terms of Gamma functions as: $B(x,y) = \frac{\Gamma x \Gamma y}{\Gamma(x+y)}$ , When x and y are positive whole number then it follows the form of gamma function.
- The beta function can be extended to a function with more than two arguments: $B(X_1, X_2, ...X_n) = \frac{\Gamma X_1 \Gamma X_2 ... \Gamma X_n}{\Gamma(X_1 + X_2 + ... + X_n)}$

# 3 Domain and Co-Domain

The domains of real numbers are where the beta function is defined. The limitations of the integral function determine the beta function's co-domain. the beta function is defined For real values that are positive and greater than zero. nonetheless, there are other ways to write a beta function, for example,

- $B(x,y) = \int_0^{\frac{\pi}{2}} (\sin \theta)^{2x-1} (\cos \theta)^{2x-1}$ where $x > 0$ and $y > 0$

- $B(x,y) = \int_0^\infty \frac{t^{x-1}}{(1+t)^{x+y}}dt$ where $x > 0$ and $y > 0$

for the variables domain is (0,$\infty$] and based on the beta function's integral limits co-domain can be defined.
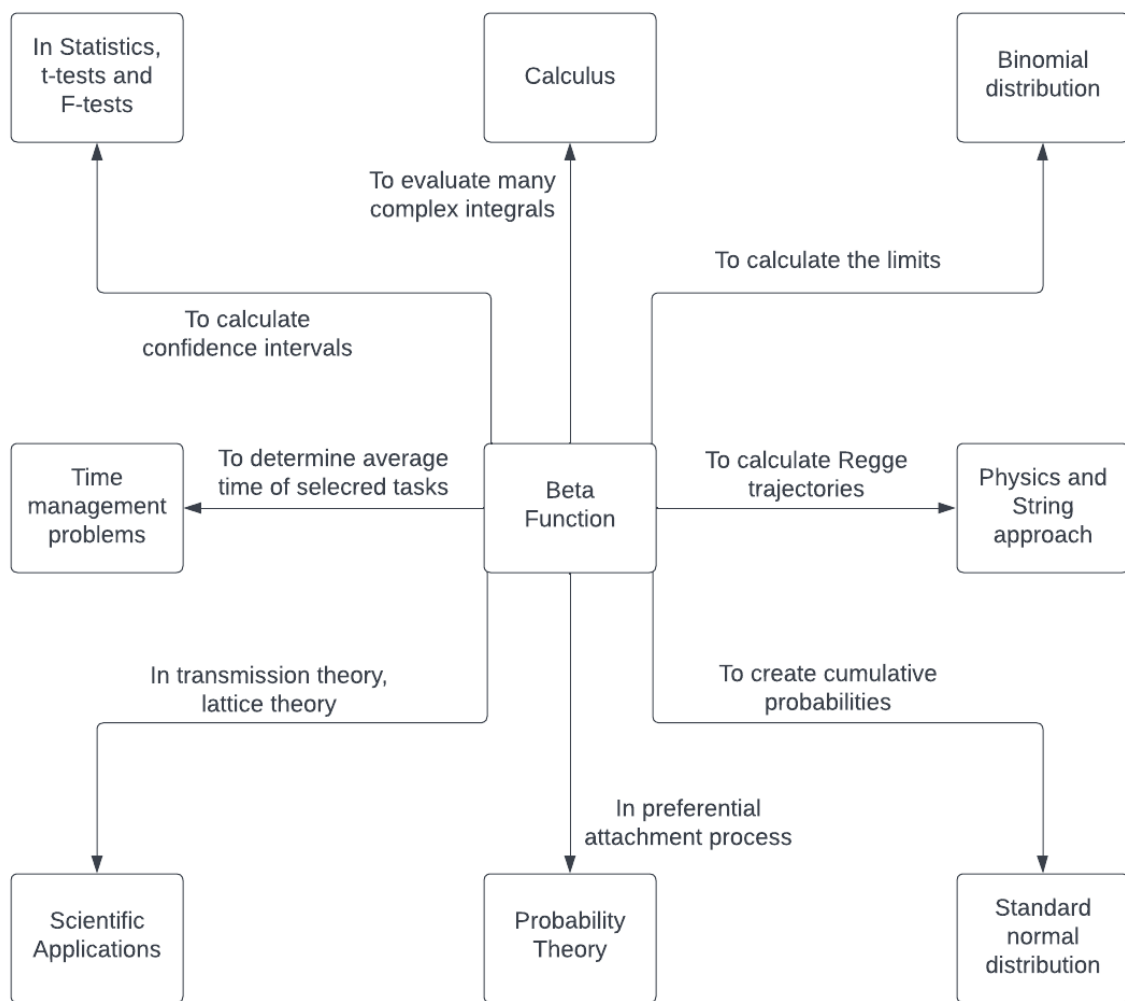
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[1]https://en.wikipedia.org/wiki/Beta_function

# Context of Use Model



Figure 1: Context of use model for a user.