SOEN 6011 SEP | **Mahavir Patel**

Problem 1 | **40198619**

Function 6 : $B(x, y)$ Beta Function | Date: 05/07/2022

1 Introduction

The beta function is a special function that belongs to the first category of Euler's integrals. The beta function is denoted by the symbol "B". $B(x, y)$ refers to the beta function, where x and y are real-valued parameters.
The formula of $B(x, y)$ is[1]:

- $B(x, y) = \frac{(x-1)!(y-1)!}{(x+y-1)!}$ For positive integers
- $B(x, y) = \int_0^1 t^{x-1}(1-t)^{y-1}dt$ For positive real numbers

2 Properties of Beta Function

- Beta function is symmetric which means its beta value is independent of the order of its parameters: $B(x, y) = B(y, x)$
- $B(x, y) = B(x, y + 1) + B(x + 1, y)$
- $B(x, y + 1) = B(x, y) \cdot [\frac{y}{x+y}]$
- $B(x + 1, y) = B(x, y) \cdot [\frac{x}{x+y}]$
- $B(x, y).B(x + y, 1 - y) = \frac{\pi}{x} \sin \pi y$
- Beta function in terms of Gamma functions as: $B(x, y) = \frac{\Gamma x \Gamma y}{\Gamma(x+y)}$, When x and y are positive whole number then it follows the form of gamma function.
- The beta function can be extended to a function with more than two arguments: $B(X_1, X_2, ...X_n) = \frac{\Gamma X_1 \Gamma X_2 ... \Gamma X_n}{\Gamma(X_1+X_2+...+X_n)}$

3 Domain and Co-Domain

The domains of real numbers are where the beta function is defined. The limitations of the integral function determine the beta function's co-domain. the beta function is defined For real values that are positive and greater than zero. nonetheless, there are other ways to write a beta function, for example,

- $B(x, y) = \int_0^{\frac{\pi}{2}} (\sin \theta)^{2x-1}(\cos \theta)^{2x-1}$ where $x > 0$ and $y > 0$
- $B(x, y) = \int_0^{\infty} \frac{t^{x-1}}{(1+t)^{x+y}} dt$ where $x > 0$ and $y > 0$

for the variables domain is (0,∞] and based on the beta function's integral limits co-domain can be defined.

[1] https://en.wikipedia.org/wiki/Beta_function