Machine Learning for Sensory Signals

Representation Learning in Deep Networks

20-04-2017

Established 1911

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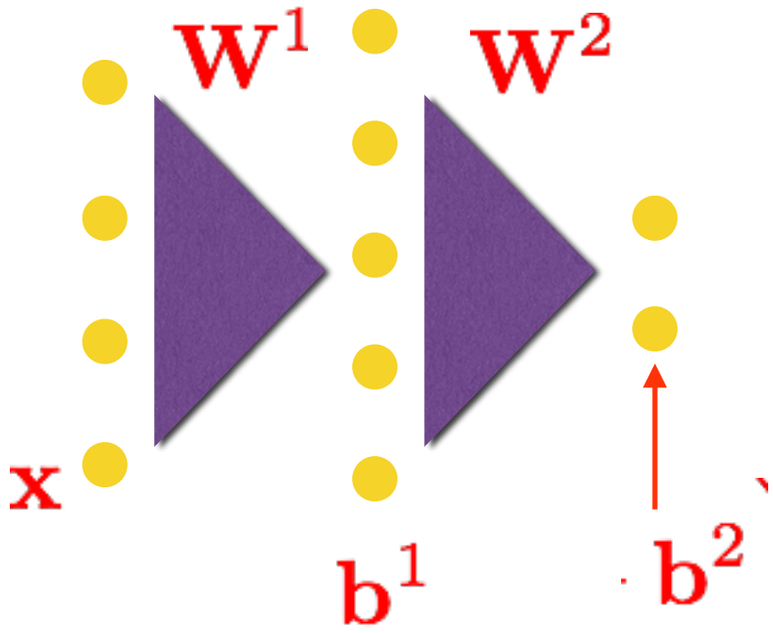
Neural Networks

Multi-layer Perceptron [Hopfield, 1982]



$$\mathbf{v}^2 = \psi\left(\mathbf{W}^2\phi(\mathbf{W}^1\mathbf{x} + \mathbf{b}^1) + \mathbf{b}^2\right)$$

ϕ non-linear function (*tanh*,*sigmoid*)

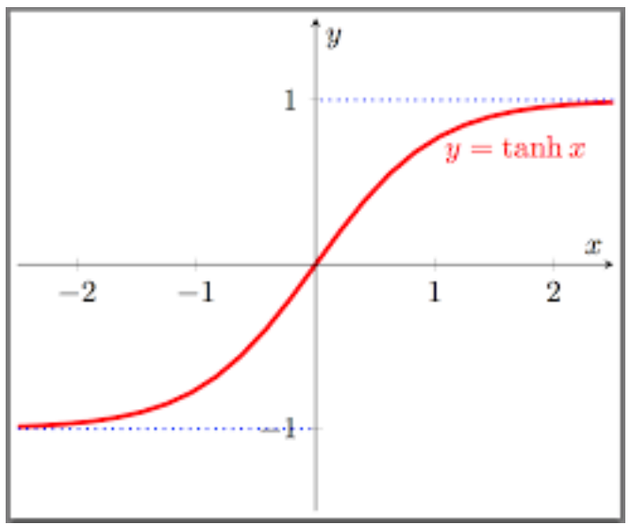
ψ thresholding function

- Useful for classifying non-linear data boundaries - non-linear class separation can be realized given enough data.

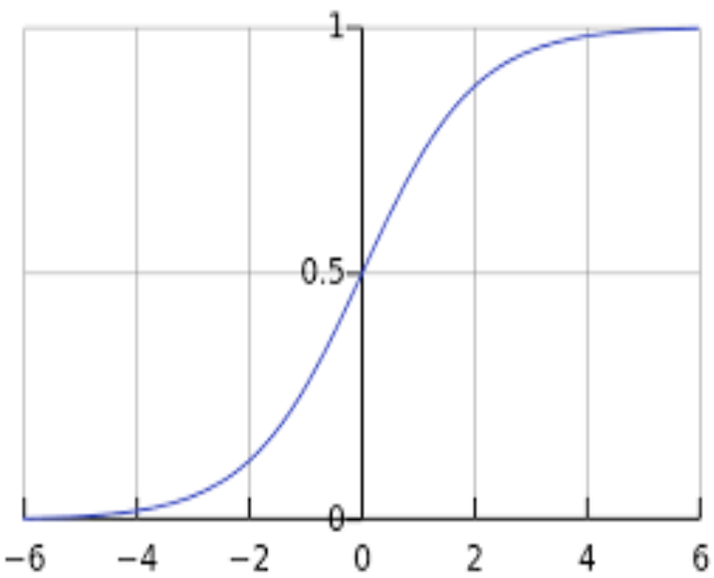
Neural Networks

Types of Non-linearities ϕ

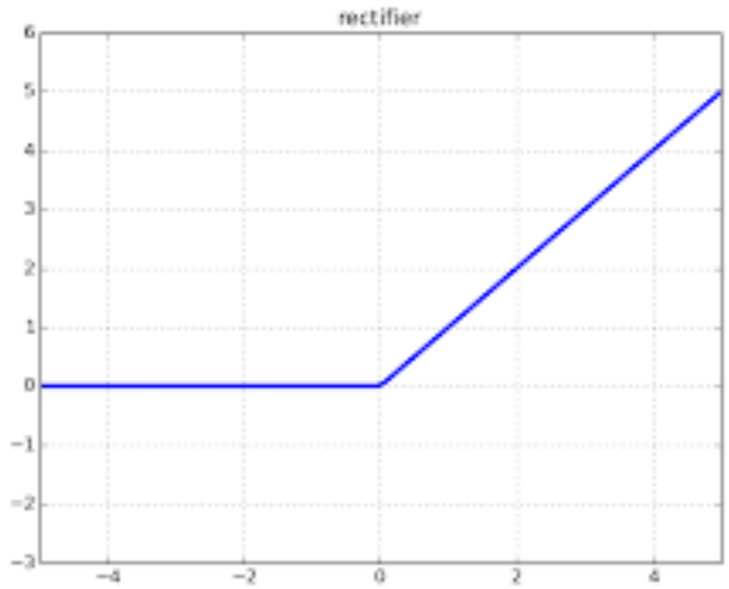
tanh



sigmoid



ReLu



Cost-Function

Mean Square Error

$$J_{MSE} = \sum_{i=1}^{M} ||\mathbf{v}_i - \mathbf{y}_i||^2$$

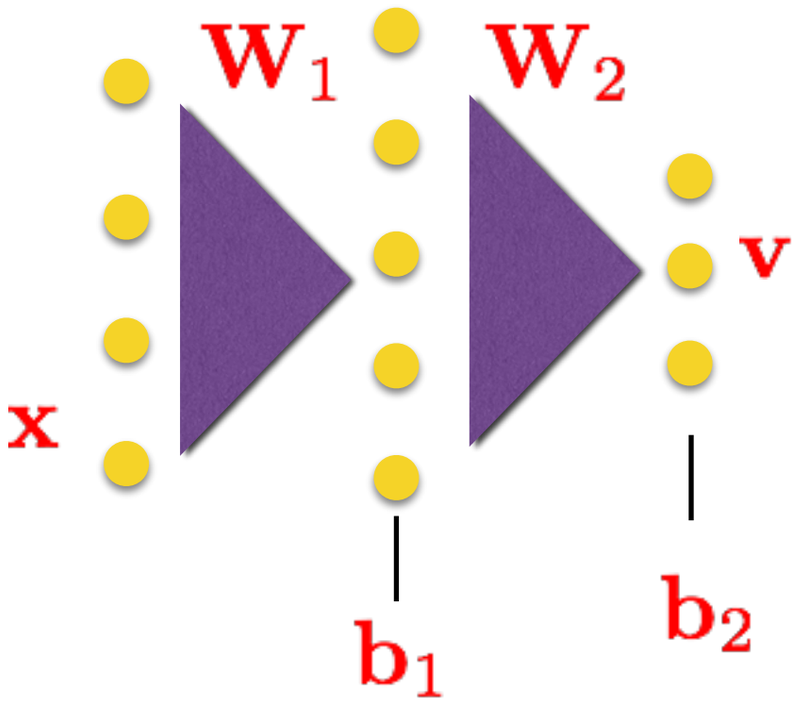
Cross Entropy

$$J_{CE} = -\sum_{i=1}^{M} \mathbf{y}_i^T \log(\mathbf{v}_i)$$

$\mathbf{y}_i$ are the desired outputs

Learning Posterior Probabilities with NNs

Neural networks predict posterior probabilities [Richard, 1991]



$$P(C_i|\mathbf{X}) = \frac{p(\mathbf{X}|C_i)p(C_i)}{p(\mathbf{X})}$$

When DNNs are trained with CE or MSE

$$\mathbf{v}(\mathbf{x}) = \mathcal{E}_{y|\mathbf{X}=\mathbf{x}}[\mathbf{y}]$$

Neural networks estimate conditional expectation of the desired targets given the input

When the targets are discrete classes $\mathbf{y} = [0\ 0\ ..0\ 1\ 0\ ..0]$ conditional expectation is the class posterior !

Richard, Michael D., and Richard P. Lippmann. "Neural network classifiers estimate Bayesian a posteriori probabilities." *Neural computation* 3.4 (1991): 461-483.

Learning Posterior Probabilities with NNs

Choice of target function ψ

- Linear for regression
- Softmax function for classification

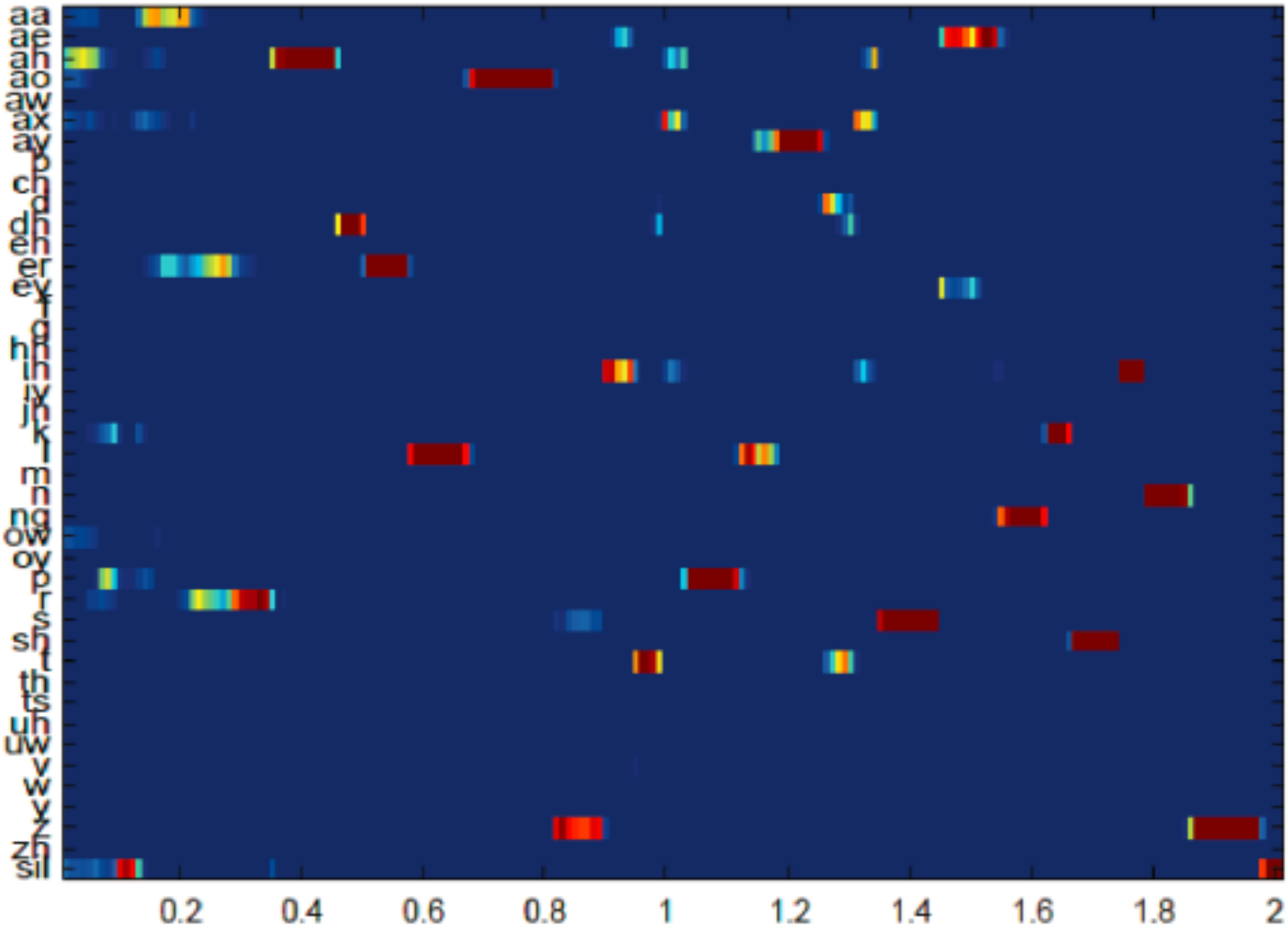
$$\psi(v_i) = \frac{e^{v_i}}{\sum_i e^{v_i}}$$

- Softmax produces positive values that sum to 1
- Allows the interpretation of outputs as posterior probabilities

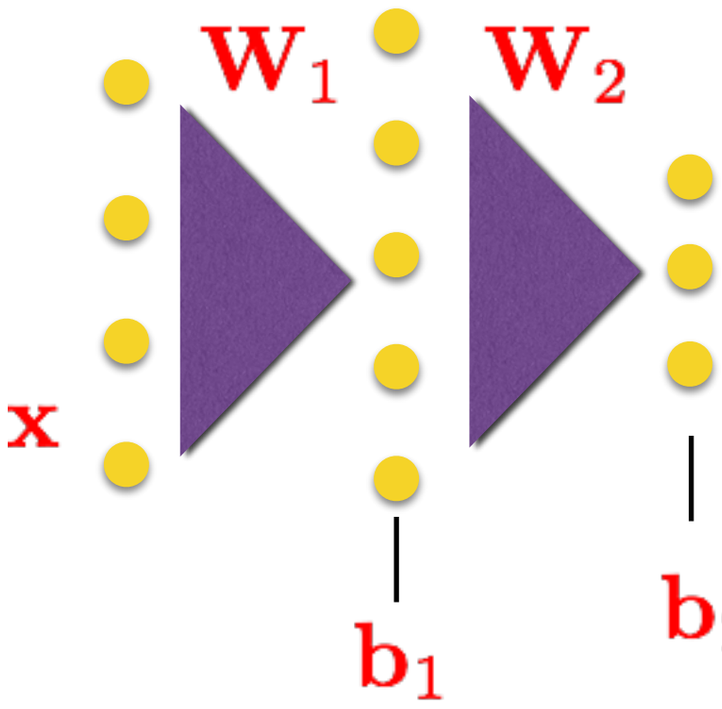
Example - Train a NN to approximate speech classes using sigmoidal non-linearity and softmax target function

Learning Posterior Probabilities with NNs

Example of a speech posteriogram



Parameter Learning

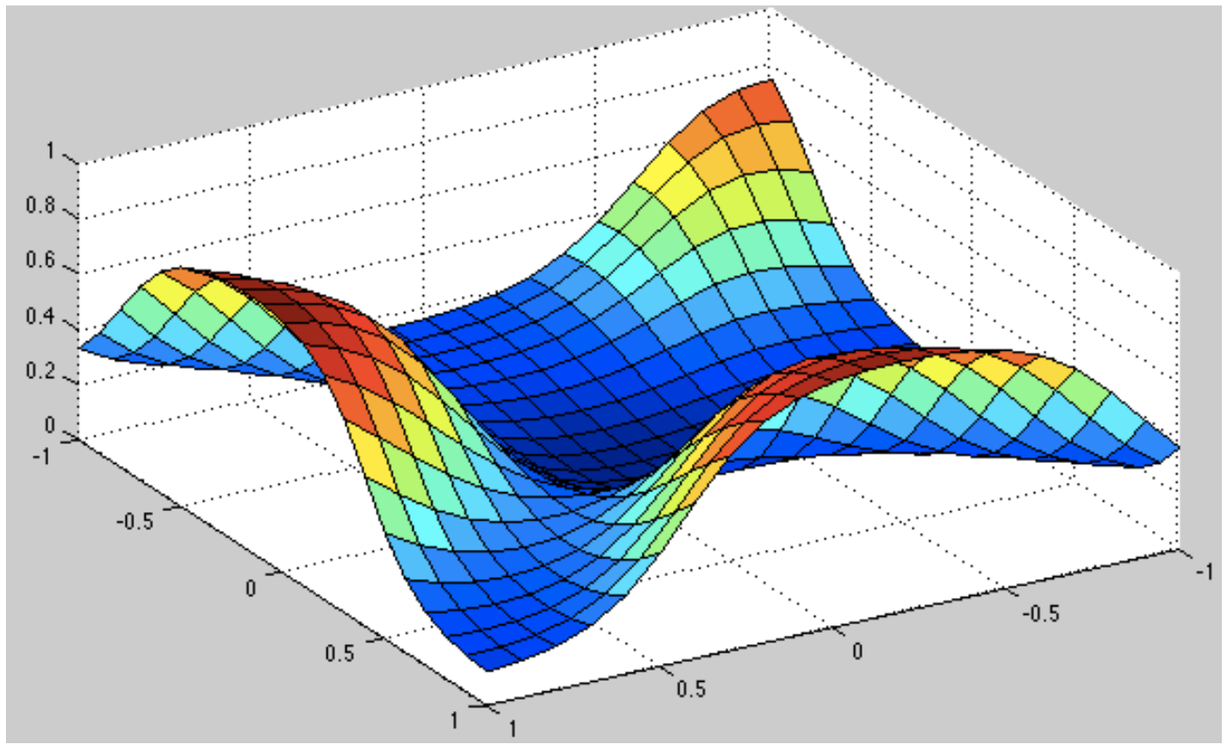


$$\mathbf{v}^2 = \psi\left(\mathbf{W}^2\phi(\mathbf{W}^1\mathbf{x} + \mathbf{b}^1) + \mathbf{b}^2\right)$$

Error function for entire data

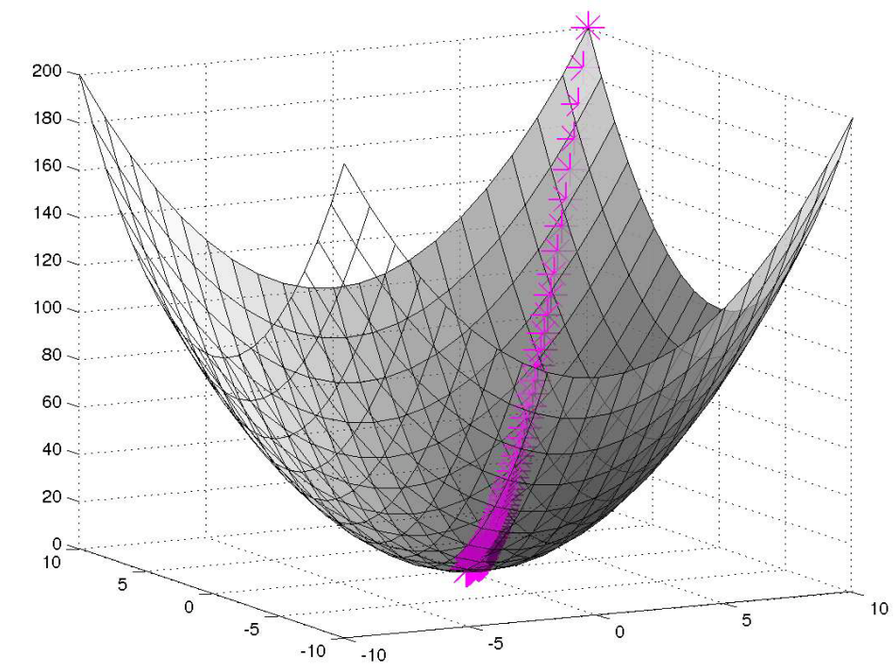
$$J_{MSE} = \sum_{i=1}^{M} ||\mathbf{v}_i - \mathbf{y}_i||^2$$

Typical Error Surface as a function of parameters (weights and biases)



Parameter Learning

Error surface close to a local optima



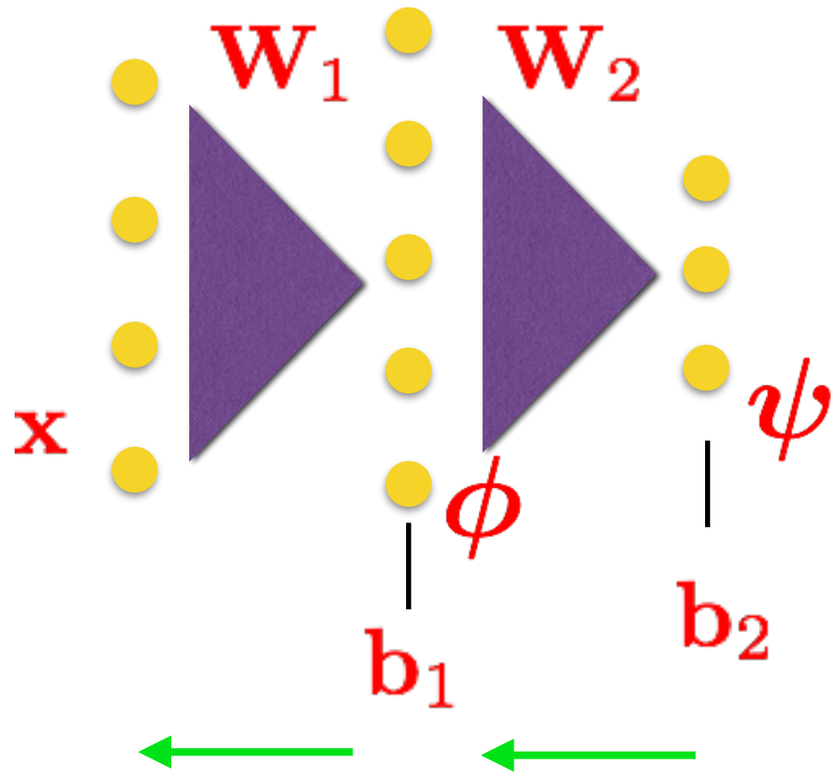
Non-linear nature of error function

- Move in the reverse direction of the gradient

$$\boldsymbol{W}_1^t = \boldsymbol{W}_1^{t-1} - \eta \frac{\partial J}{\partial \boldsymbol{W}_1}$$

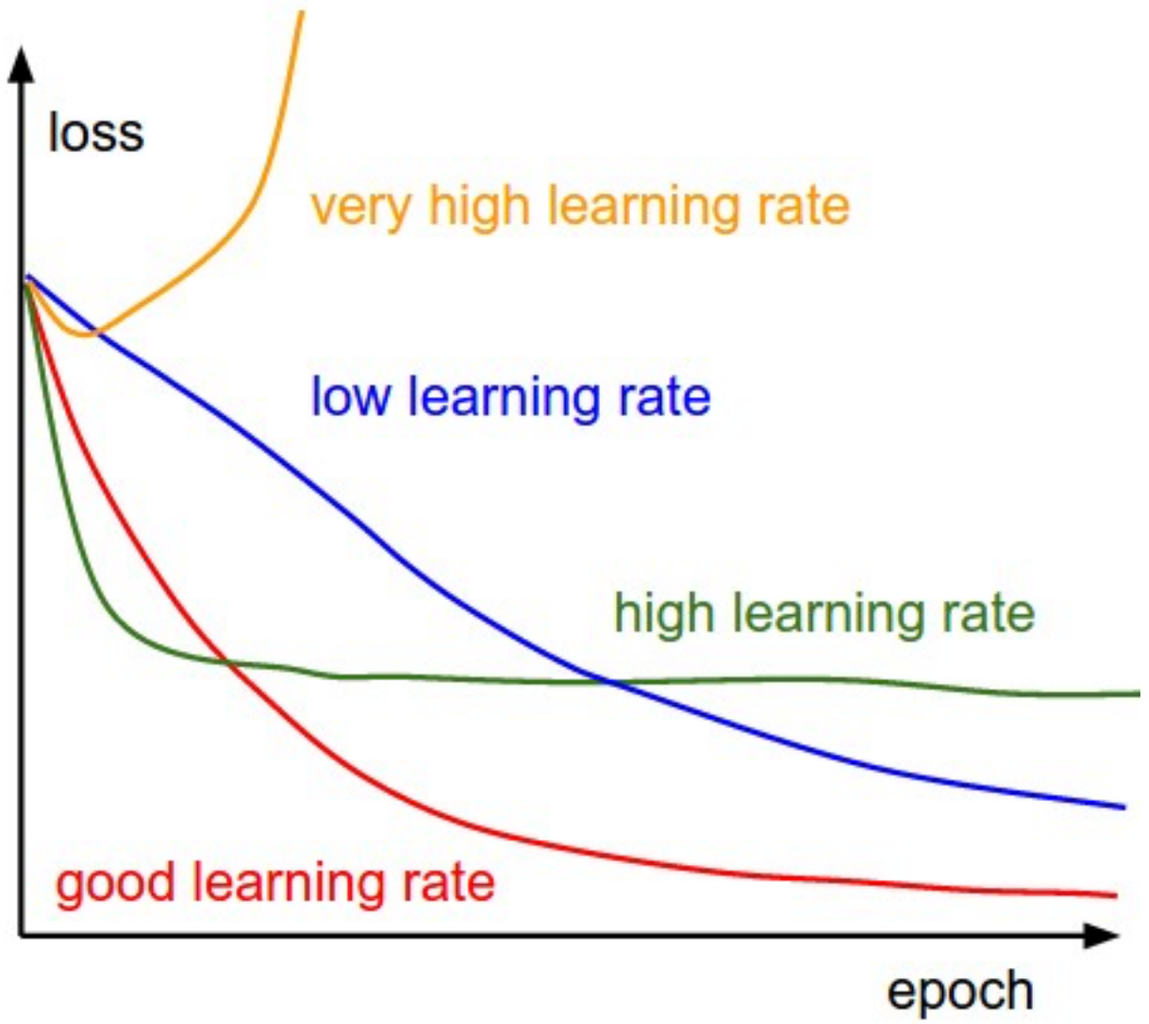
Error back propagation

$$\frac{\partial J}{\partial \mathbf{W}_1} = \frac{\partial J}{\partial \psi} \times \frac{\partial \psi}{\partial \phi} \times \frac{\partial \phi}{\partial \mathbf{W}_1}$$

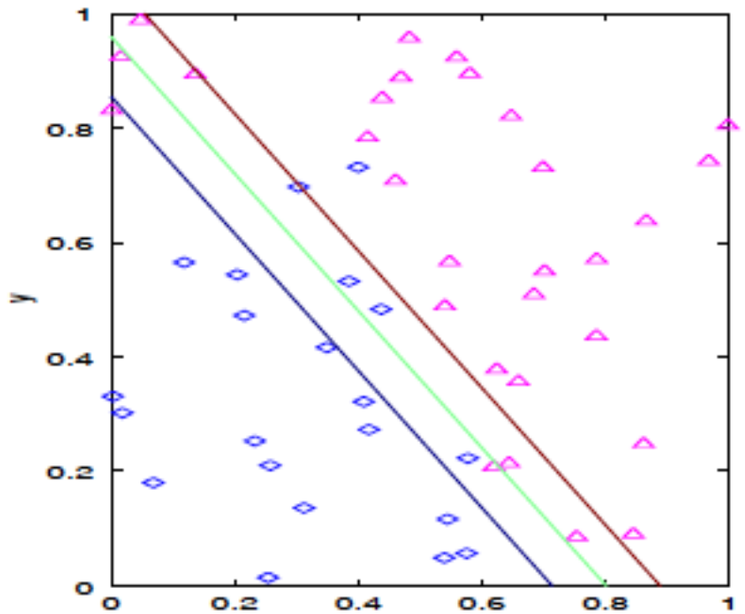


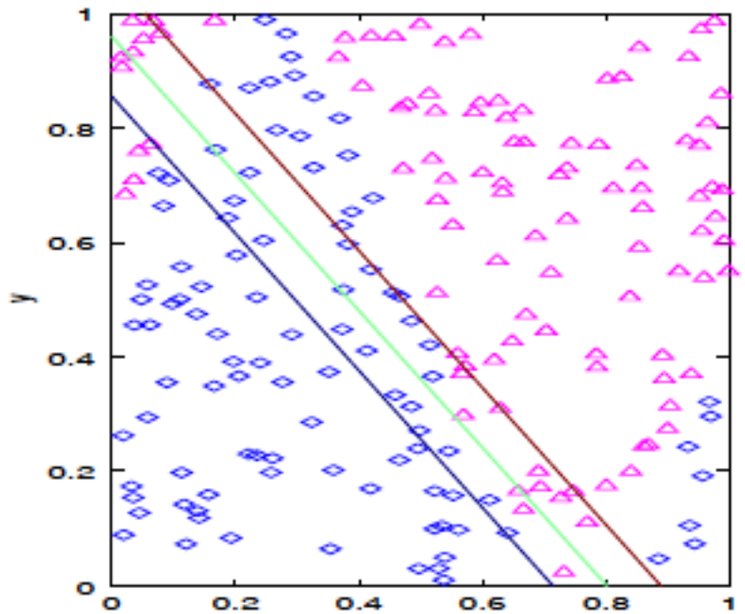
Parameter Learning

- Solving a non-convex optimization.
- Iterative solution.
- Depends on the initialization.
- Convergence to a local optima.
- Judicious choice of learning rate



Underfit

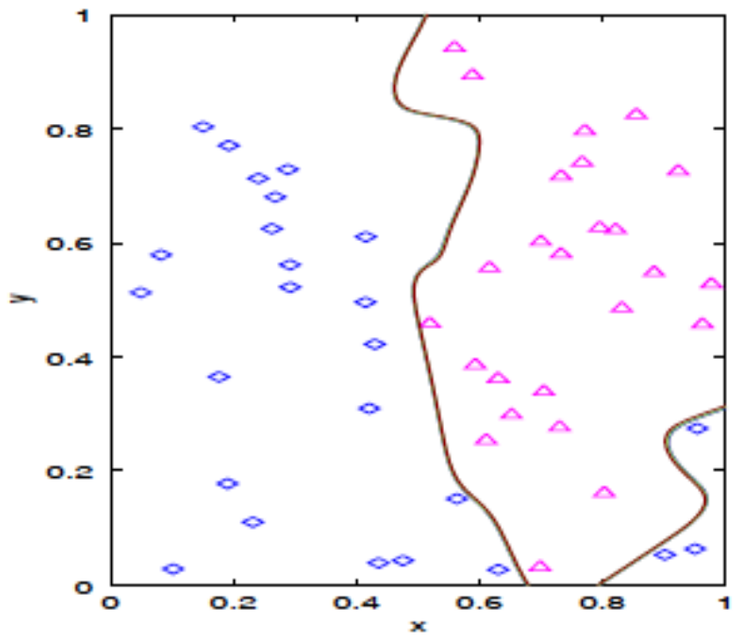


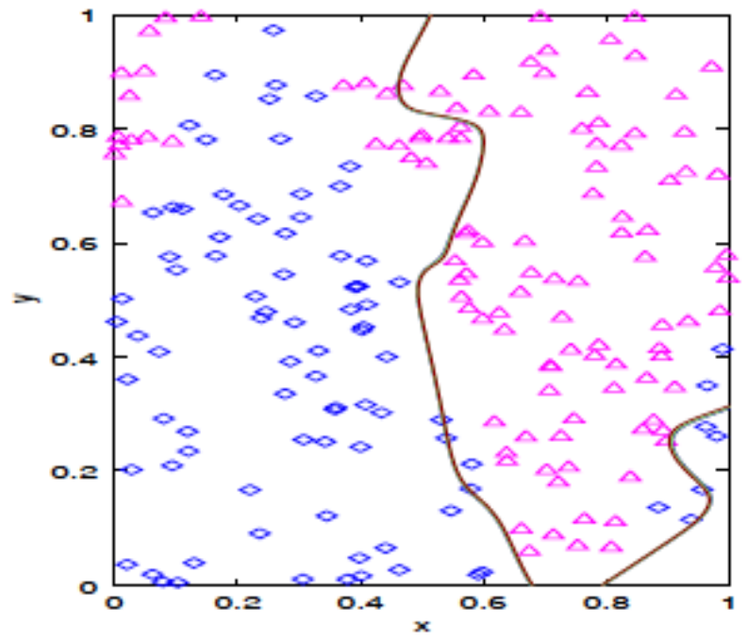


- The model is not able to capture the variability in the data (Linear Model)
- Both the training and testing error are high (15%,20%)
- Try to learn a more complex model – more features, more hidden neurons, decrease regularization
- More data would not help

(Courtesy - Dr D. Vijayasenan, NITK)

Overfit

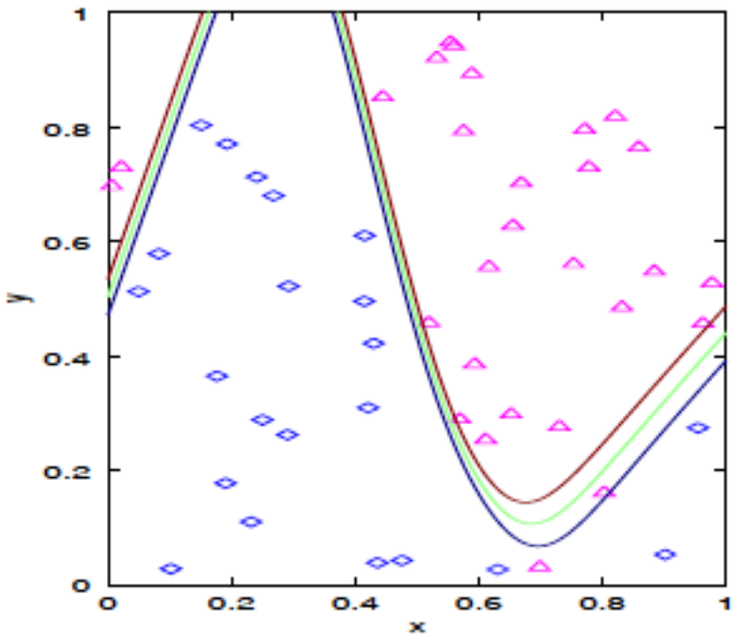


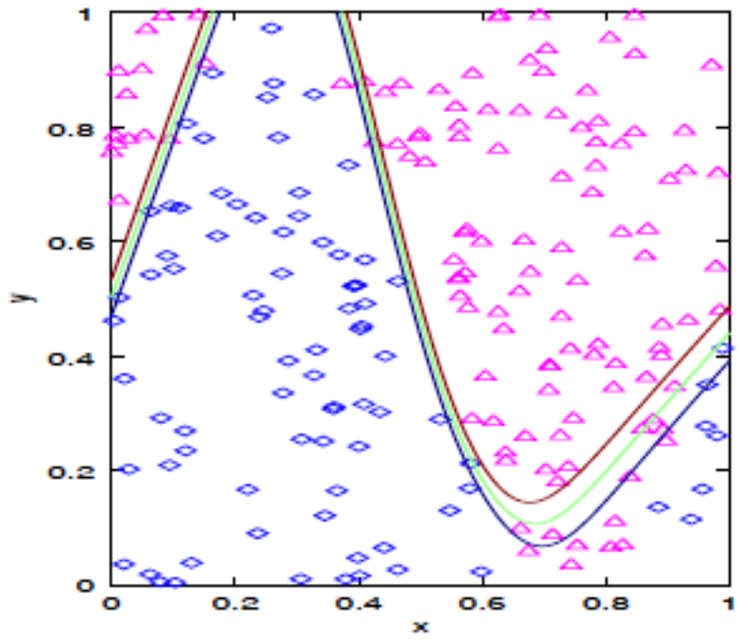


- The model is capturing data as well as accidental variations (100 hidden neurons)
- Training error is too low and testing error is too high (0%, and 16%)
- Try to learn a simpler model – less features, less hidden neurons, increase regularization
- More data would help

(Courtesy - Dr D. Vijayasenan, NITK)

Compromise





- Reasonable training and test errors – $(4\%, 8\%)$
- Appropriate model – capturing only the global characteristics not details

(Courtesy - Dr D. Vijayasenan, NITK)

Summary so far…

- Neural networks as discriminative classifiers
- Need for hidden layer
- Choice of non-linearities and target functions
- Estimating posterior probabilities with NNs
- Parameter learning with back propagation.

Roadmap

- Basics of Machine Learning
- Neural Networks
- Deep Networks
- Representation Learning in Deep Networks
- Applications in Speech Processing and Insights
- Future Research Directions

Need For Deep Networks

Modeling complex real world data like speech, image, text

- Single hidden layer networks are too restrictive.
- Needs large number of units in the hidden layer and trained with large amounts of data.
- Not generalizable enough.

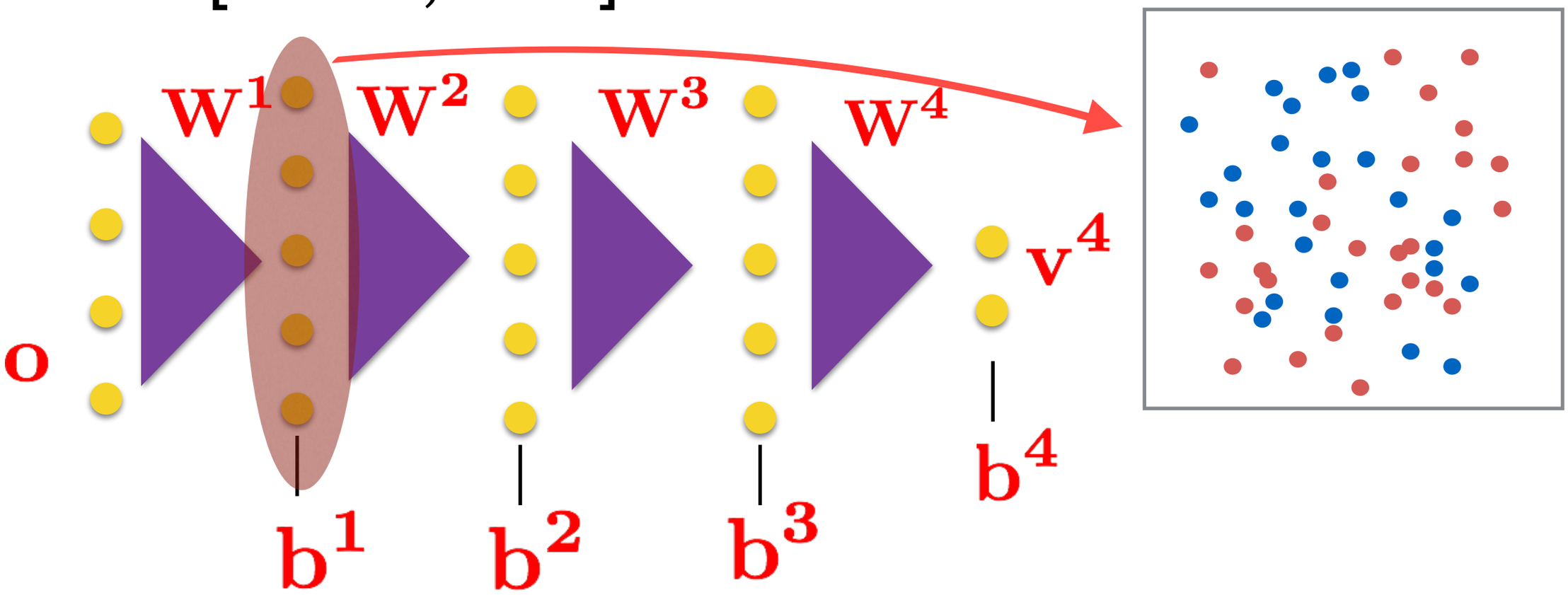
Networks with multiple hidden layers - deep networks

(Open questions till 2005)

- Are these networks trainable ?
- How can we initialize such networks ?
- Will these generalize well or over train ?

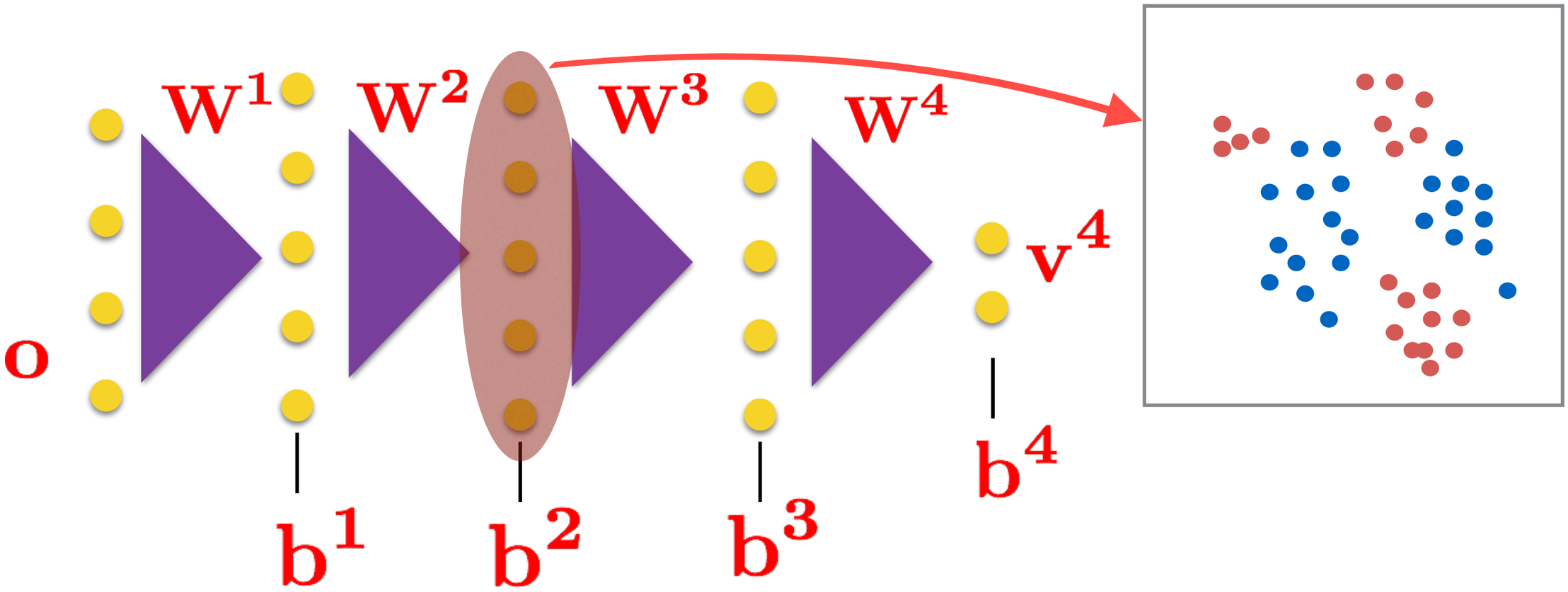
Deep Networks Intuition

Neural networks with multiple hidden layers - Deep networks [Hinton, 2006]



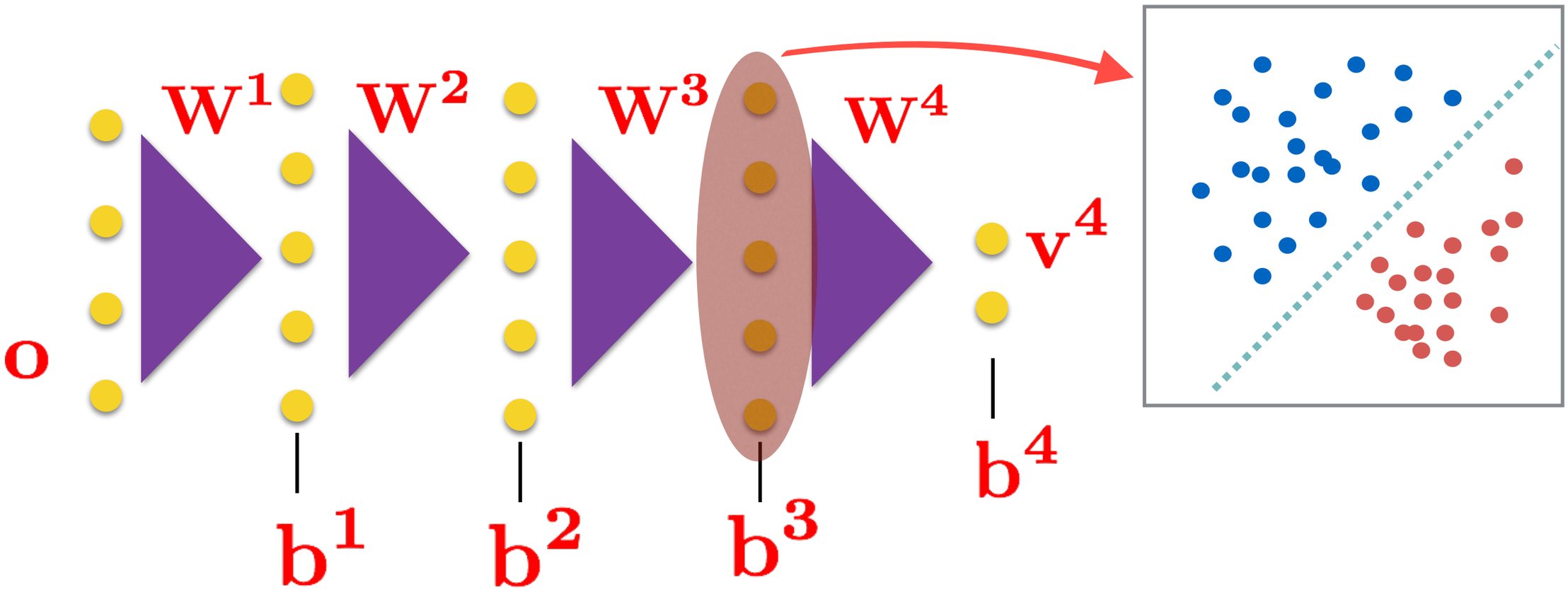
Deep Networks Intuition

Neural networks with multiple hidden layers - Deep networks



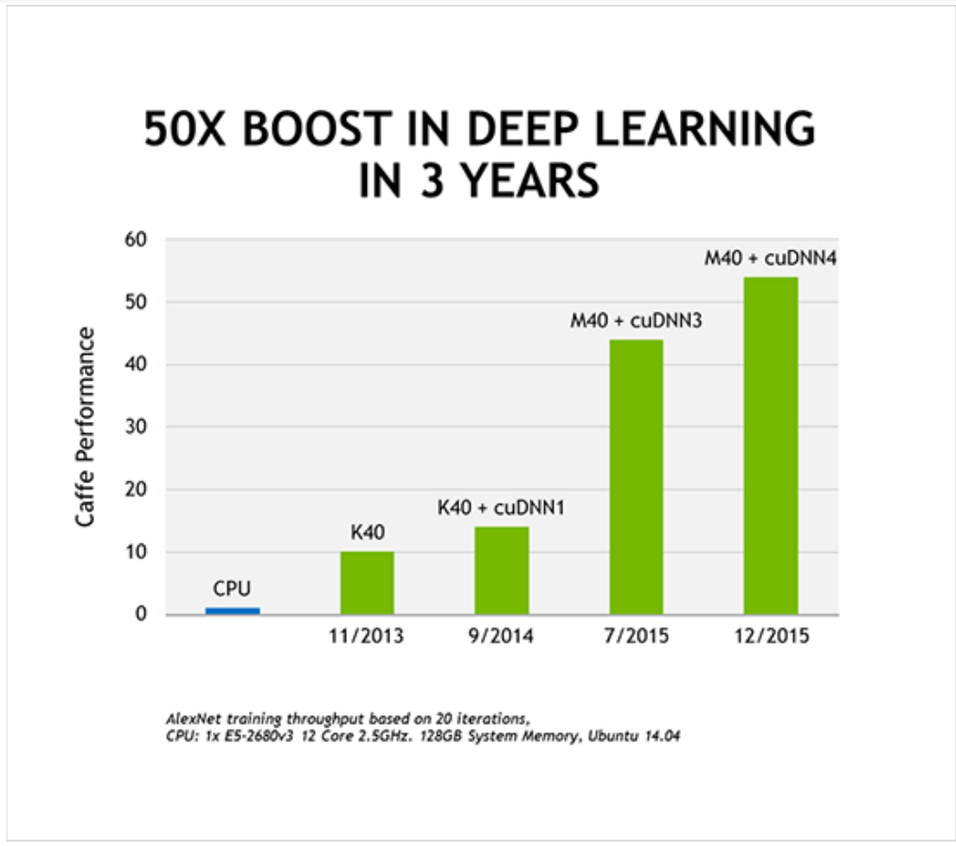
Deep Networks Intuition

Neural networks with multiple hidden layers - Deep networks



Deep networks perform hierarchical data abstractions which enable the non-linear separation of complex data samples.

Deep Networks



- Are these networks trainable ?
  - Advances in computation and processing
  - Graphical processing units (GPUs) performing multiple parallel multiply accumulate operations.
  - Large amounts of supervised data sets

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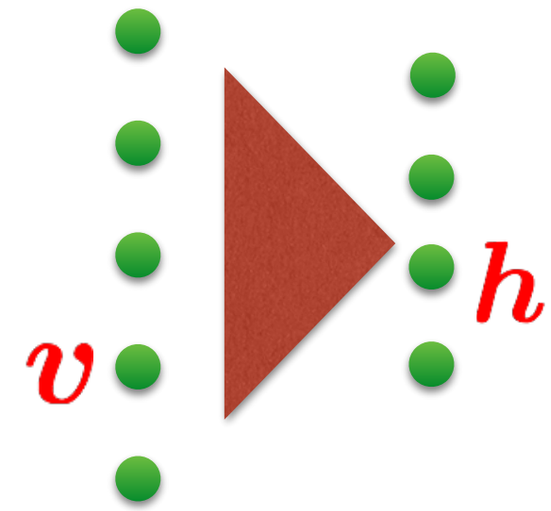
Deep Networks Initialization



Initializing large networks with Restricted Boltzmann Machine (RBM)

- Gaussian Bernoulli RBM - Gaussian visible layer and Bernoulli hidden layer.
- Define an energy function

$$E(\mathbf{v}, \mathbf{h}) = 0.5(\mathbf{v} - \mathbf{a})^T(\mathbf{v} - \mathbf{a}) - \mathbf{b}^T\mathbf{h} - \mathbf{h}^T\mathbf{W}\mathbf{v}$$

- Define joint probability density $P(\mathbf{v}, \mathbf{h}) = \dfrac{e^{-E(\mathbf{v},\mathbf{h})}}{Z}$
- Estimate the parameters $a,\ b,\ W$ by maximizing the joint density

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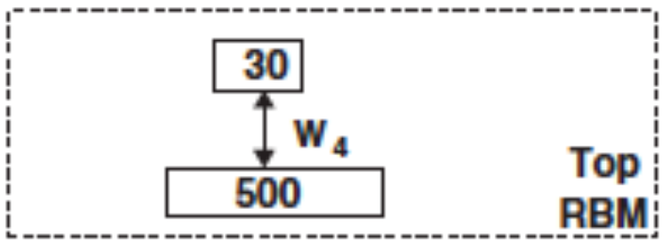
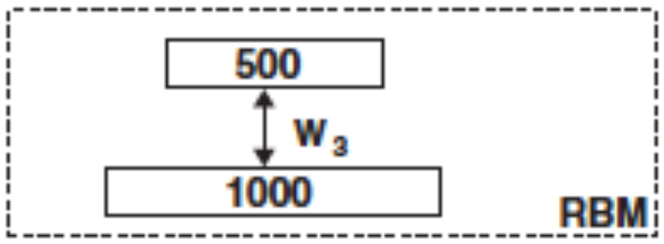
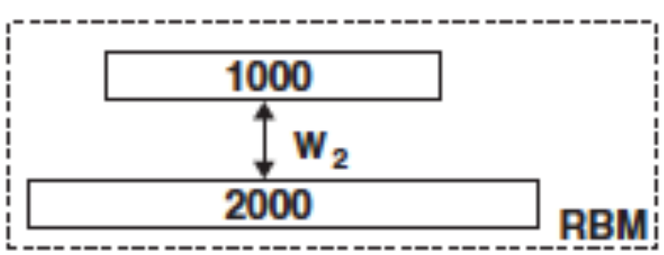
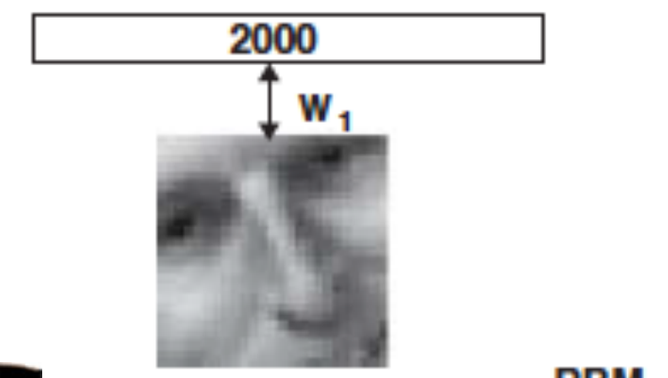
Restricted Boltzmann Machine









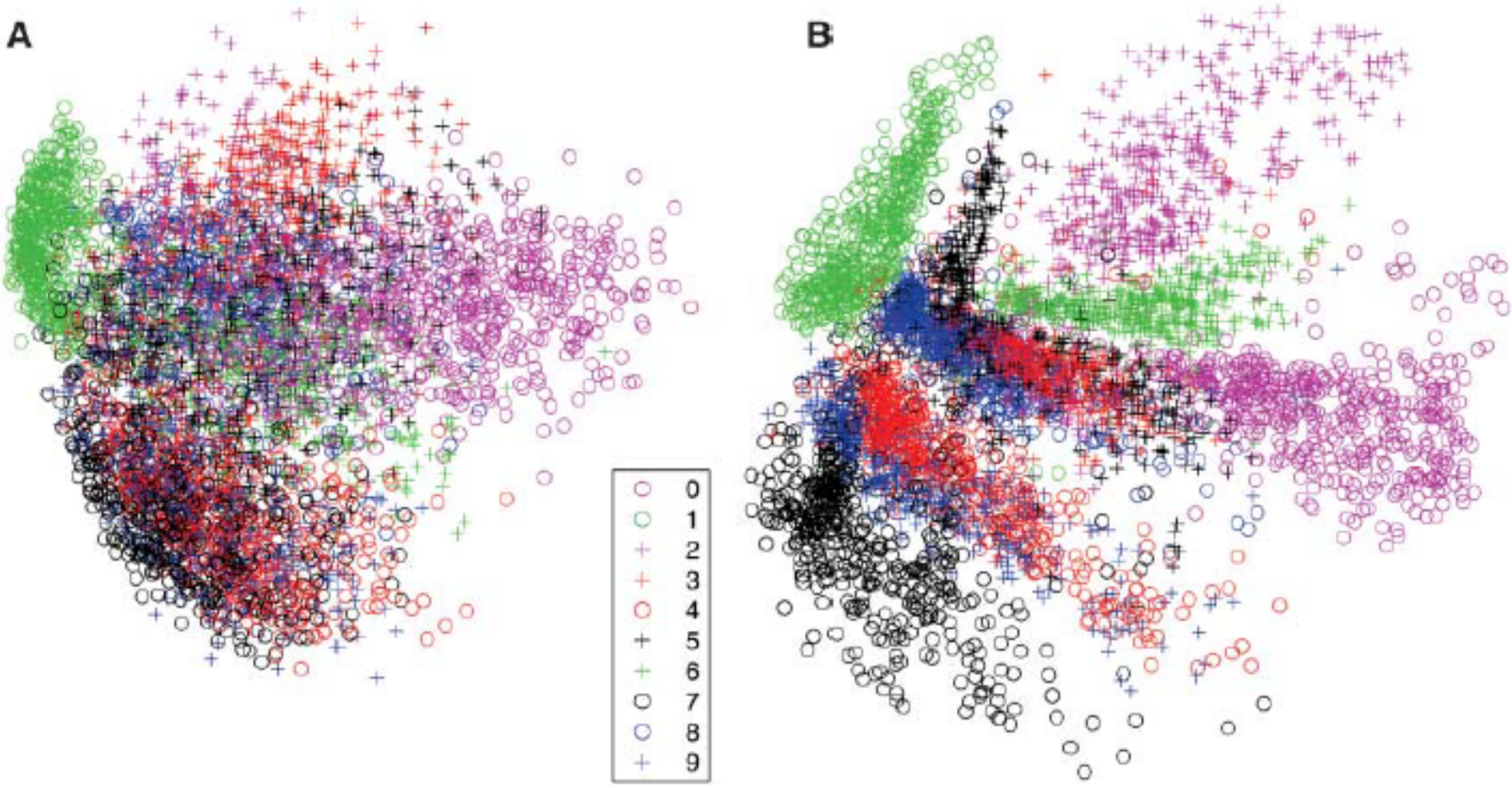
It can be shown that probability of hidden unit being is the sigmoid function

$$P(h_i = 1|\mathbf{v}) = \frac{1}{1 + e^{-(w_i^T \mathbf{v} + b_i)}}$$

RBM layers can be stacked one above the other in a hierarchical fashion.

Earliest application was in dimensionality reduction [Hinton, 2006].

Restricted Boltzmann Machine



PCA **RBM**

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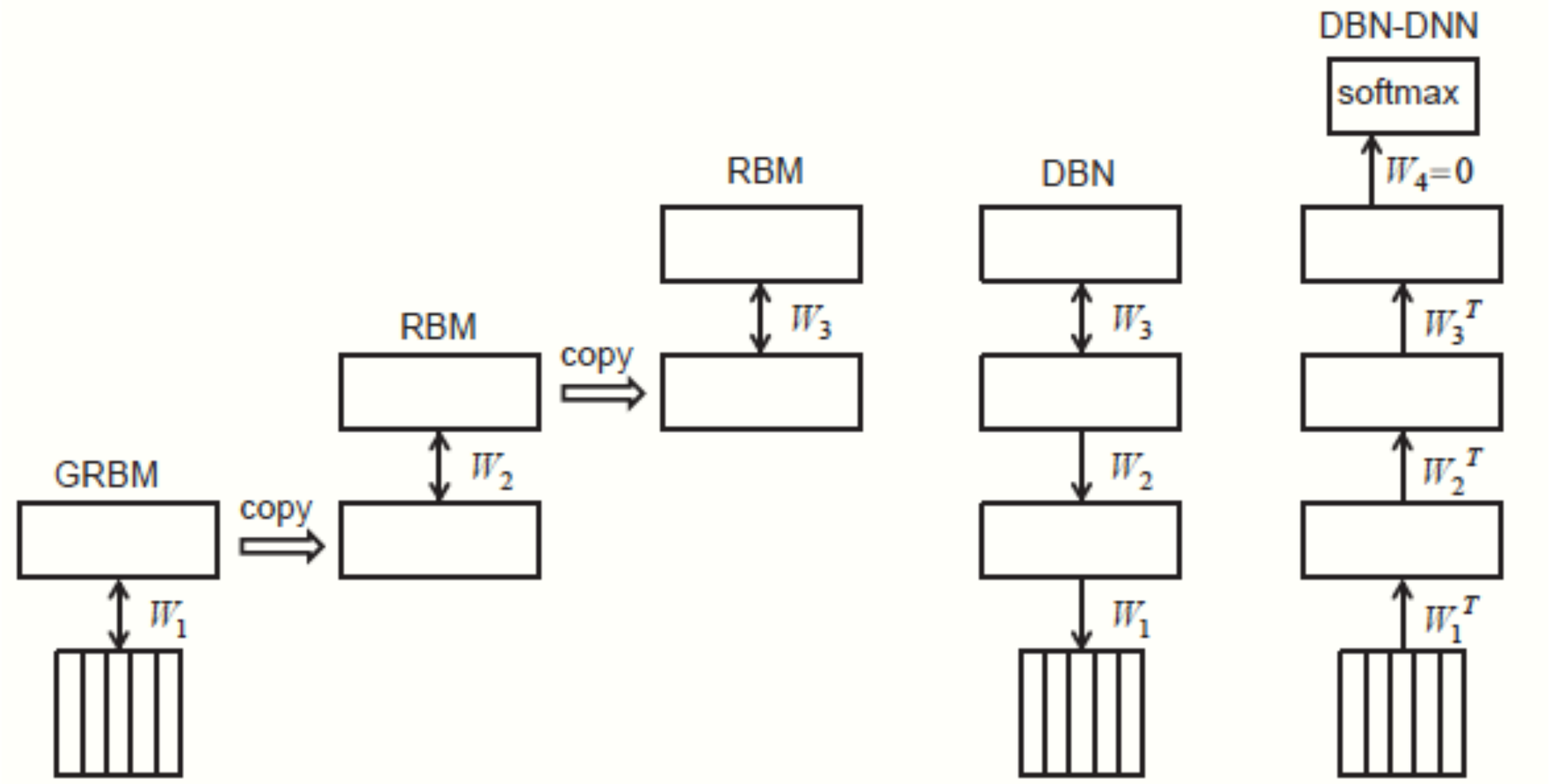
DBN for DNN Initialization



[Hinton, 2013]

Summary so far…

- Deep Neural networks as extensions of NNs.
- Initiation behind multiple hidden layers
- Initialization with Restricted Boltzmann Machine
  - RBMs for dimensionality reduction

Roadmap

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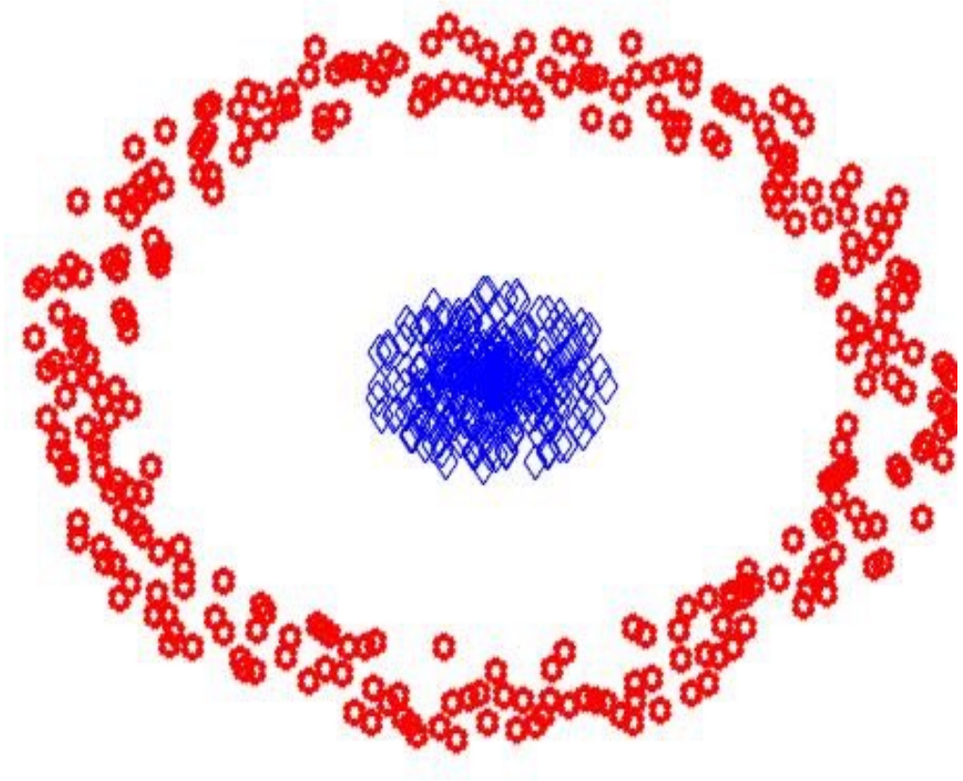
Deep Networks

- Will the networks generalize with deep networks
    - DNNs are quite data hungry and performance improves by increasing the data.
    - Generalization problem is tackled by providing training data from all possible conditions.
        - Many artificial data augmentation methods have been successfully deployed
- Providing the state-of-art performance in several real world applications.

Representation Learning in Deep Networks

- The input data representation is one of most important components of any machine learning system.

Cartesian Coordinates



Polar Coordinates

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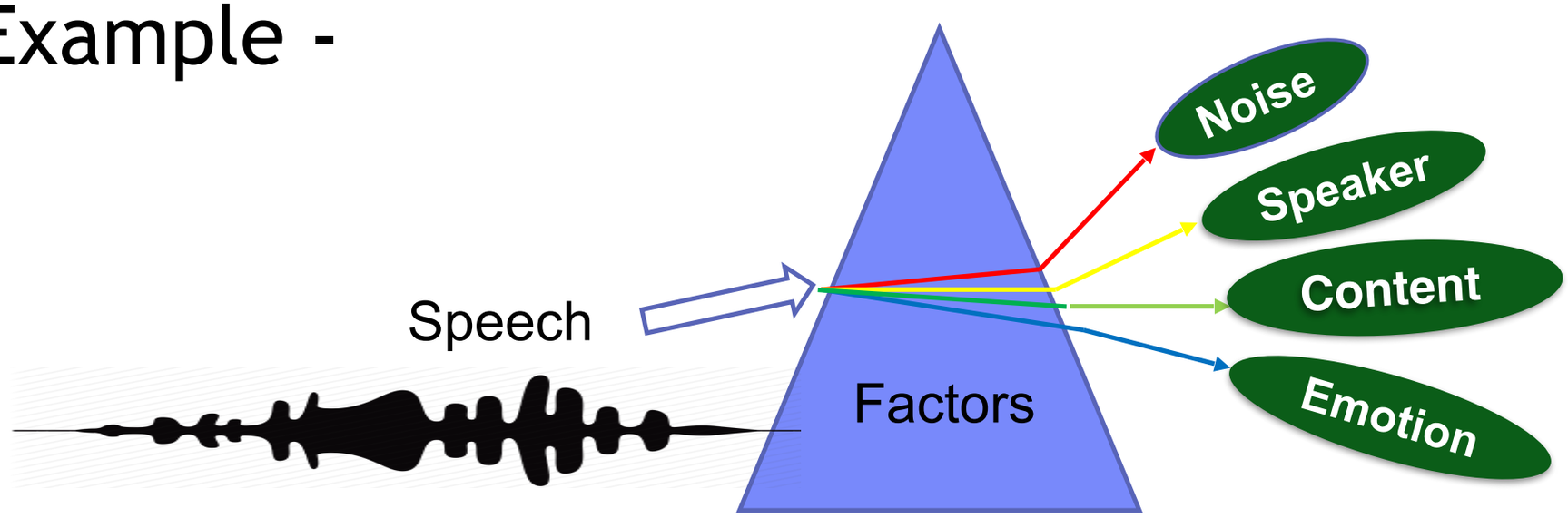
Representation Learning in Deep Networks

- The input data representation is one of most important components of any machine learning system.
  - Extract features that enable classification while suppressing factors which are susceptible to noise.
- Finding the right representation for real world applications - substantially challenging.
  - Example -

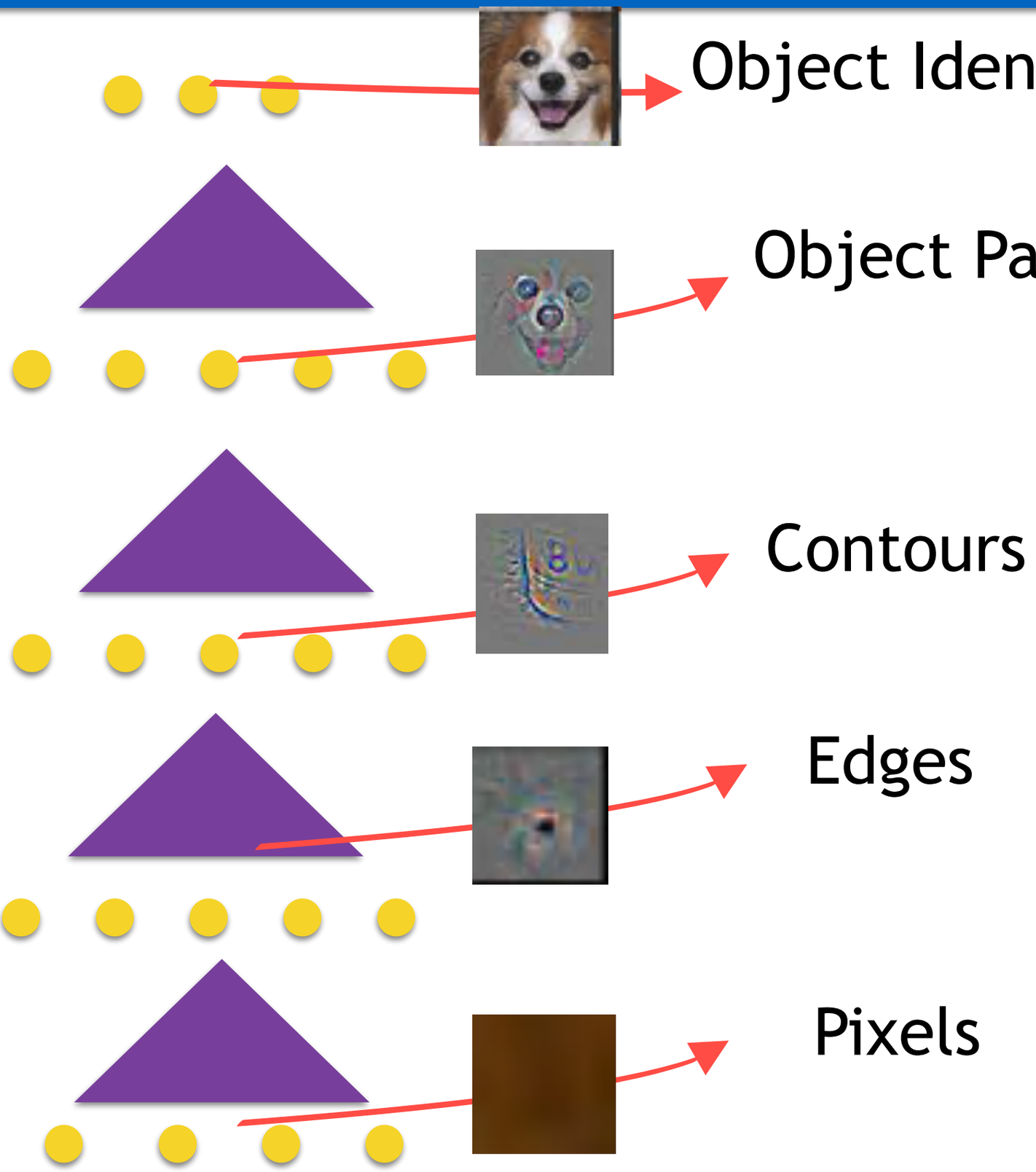


Representation Learning in Deep Networks

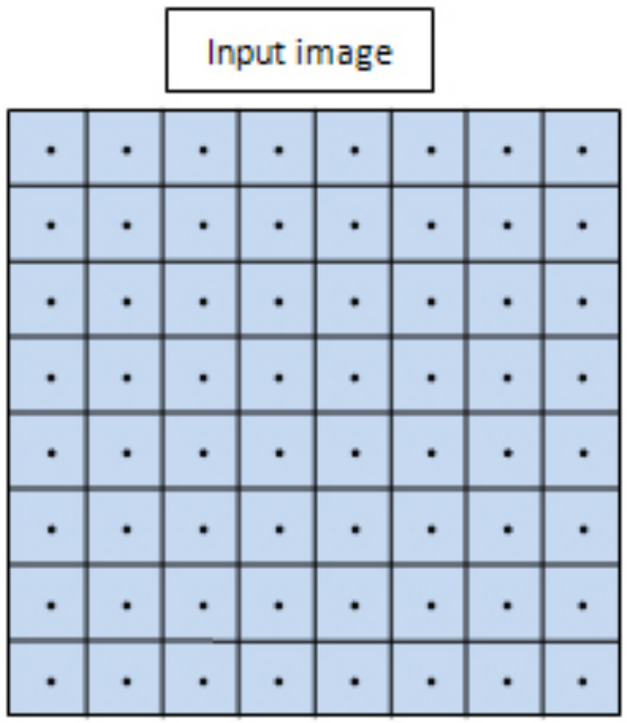
- The input data representation is one of most important components of any machine learning system.
  - Extract factors that enable classification while suppressing factors which are susceptible to noise.
- Finding the right representation for real world applications - substantially challenging.
  - Deep learning solution - build complex representations from simpler representations.
  - The dependencies between these hierarchical representations are refined by the target.

Representation Learning in Deep Networks

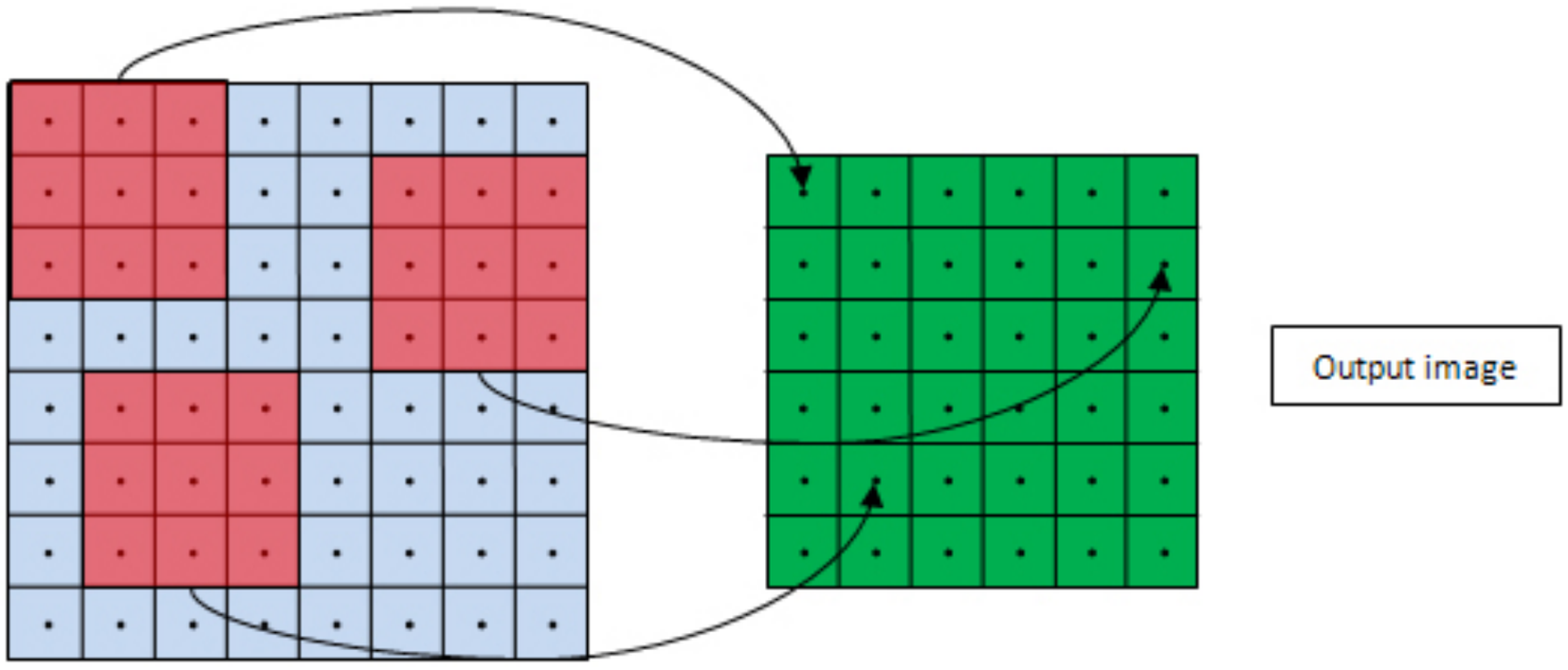
[Zeiler, 2014]



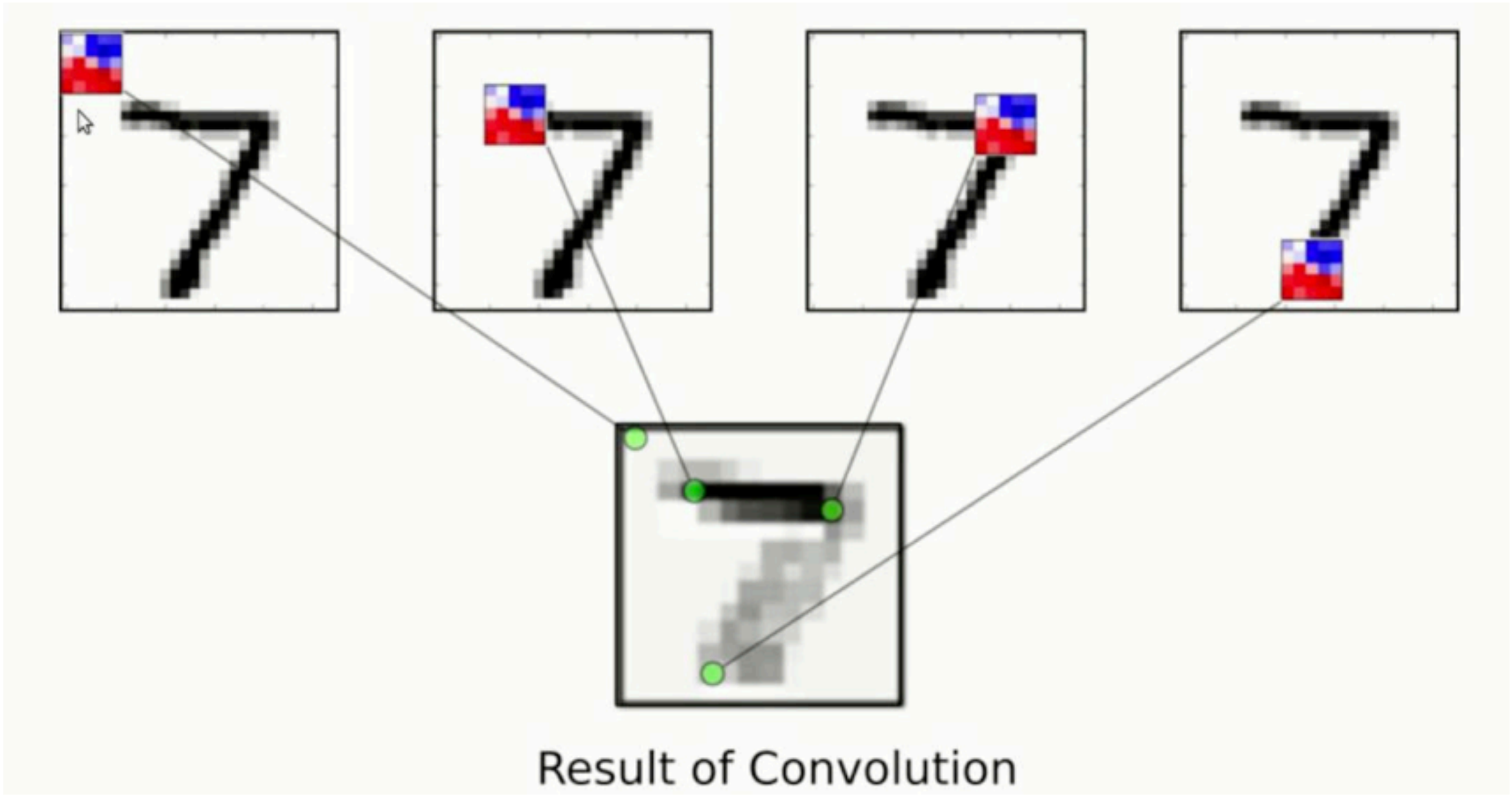
Convolution Operation



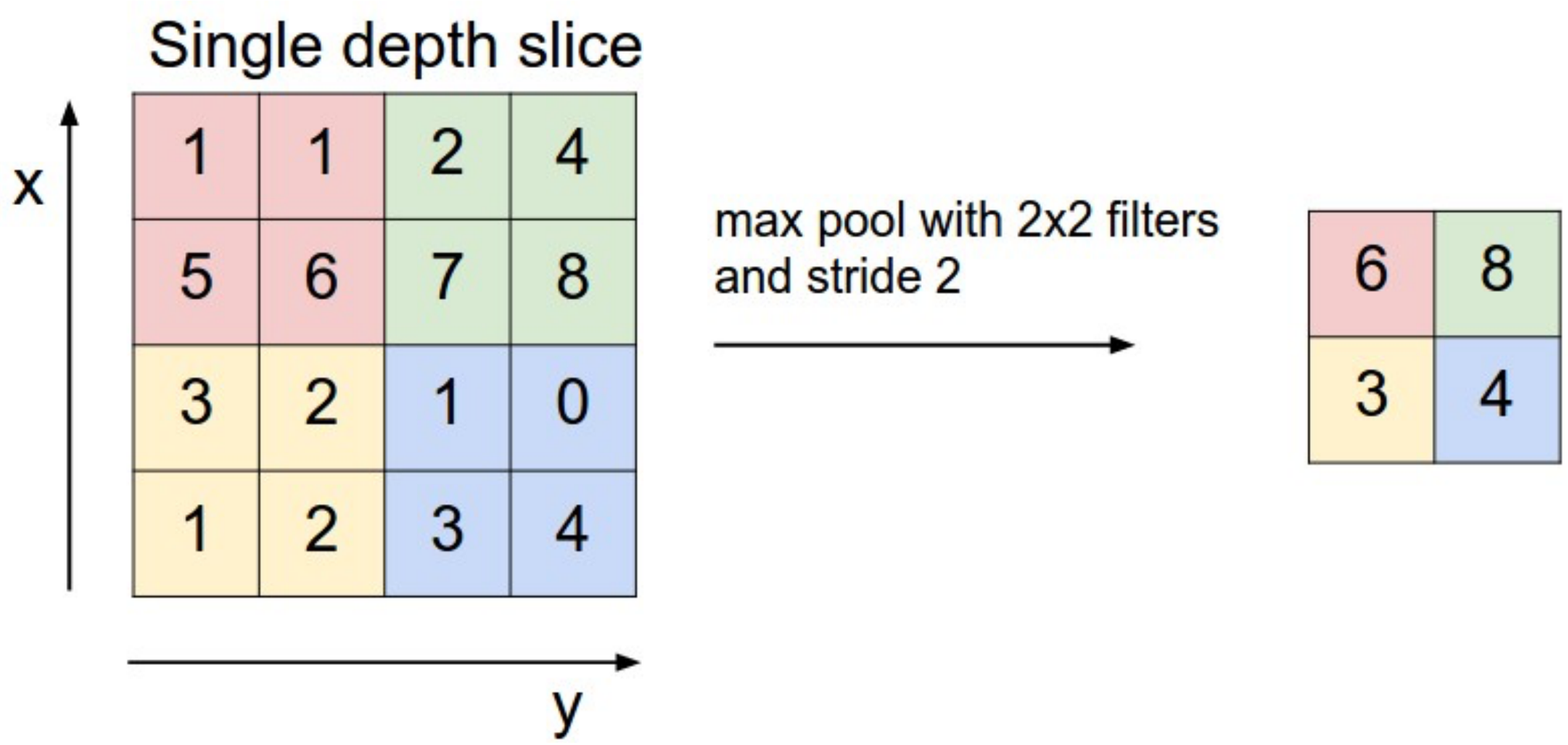
Weight sharing



Convolution Operation

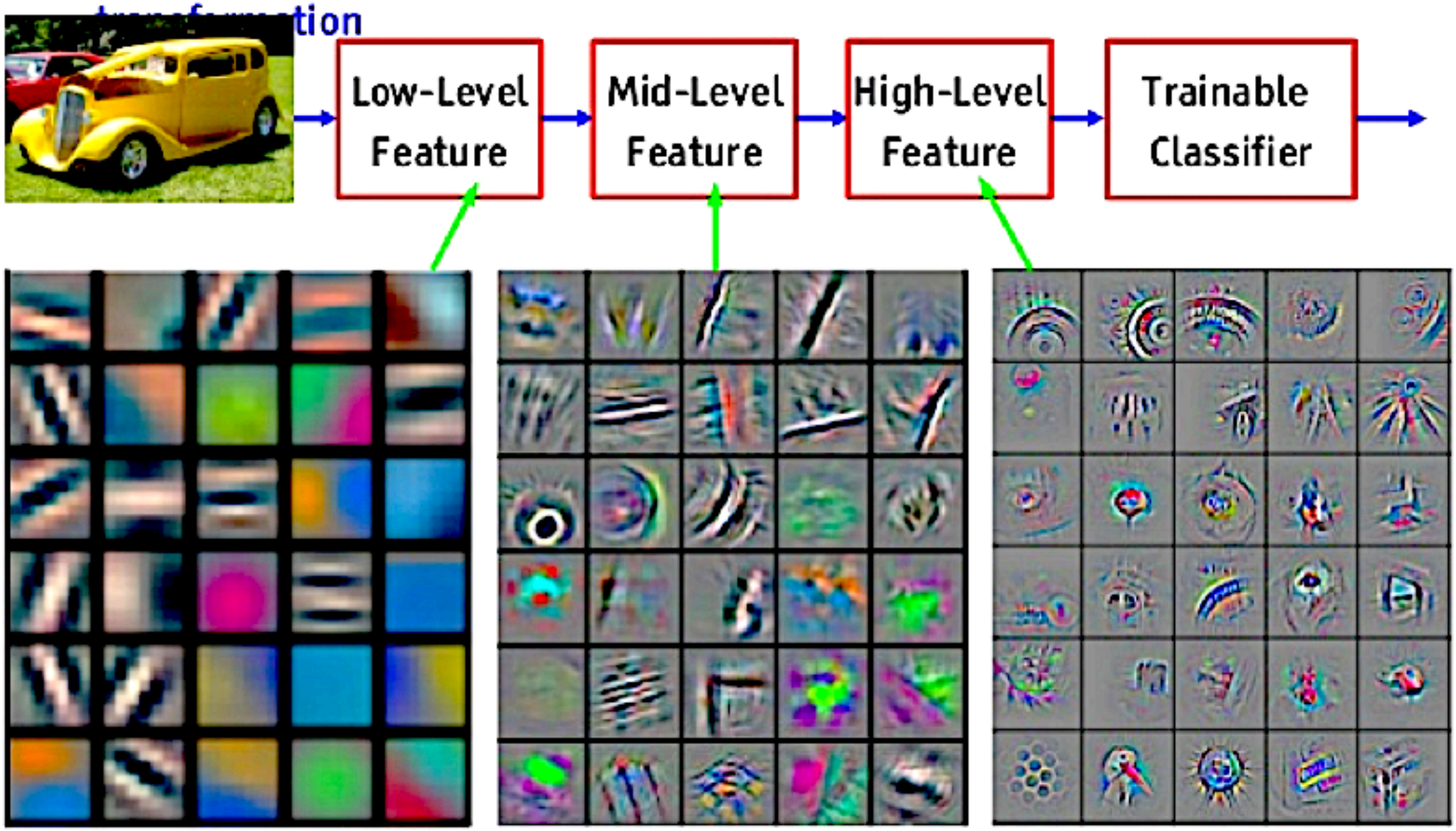


Max Pooling Operation



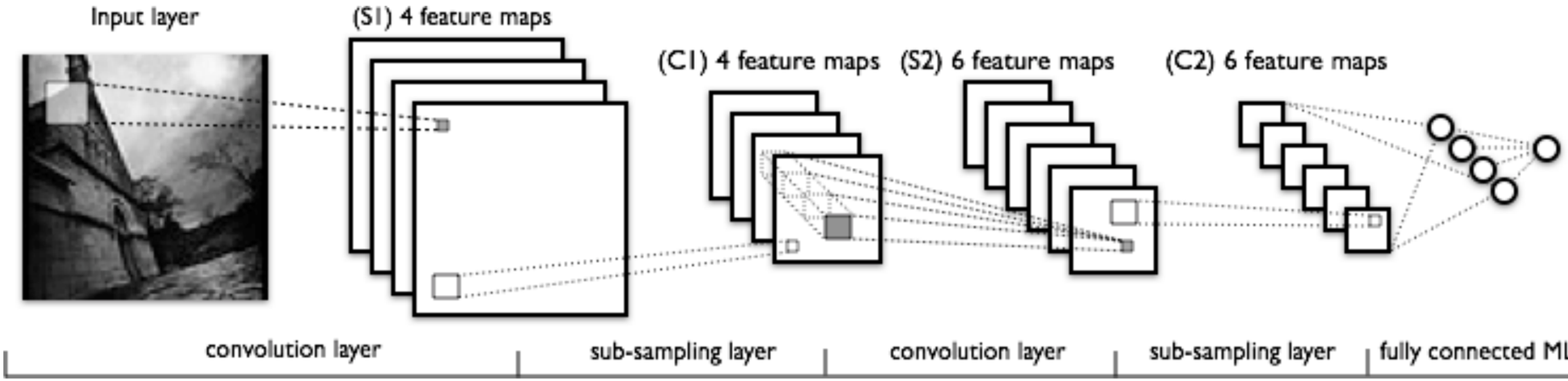
Deep Convolutional Networks

- It's deep if it has more than one stage of non-linear feature transformation



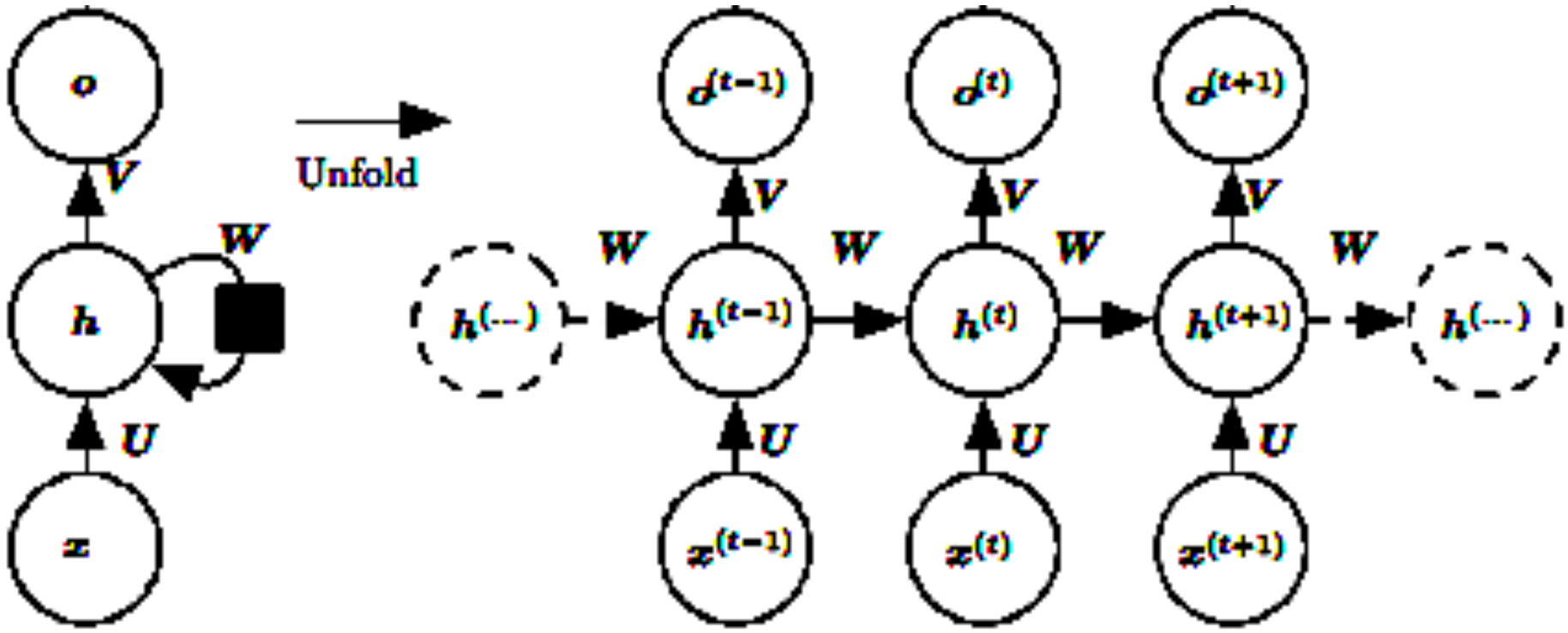
Feature visualization of convolutional net trained on ImageNet from [Zeiler & Fergus 2013]

Deep Convolutional Networks



- Multiple levels of filtering and subsampling operations.
- Feature maps are generated at every layer.

Recurrent Networks

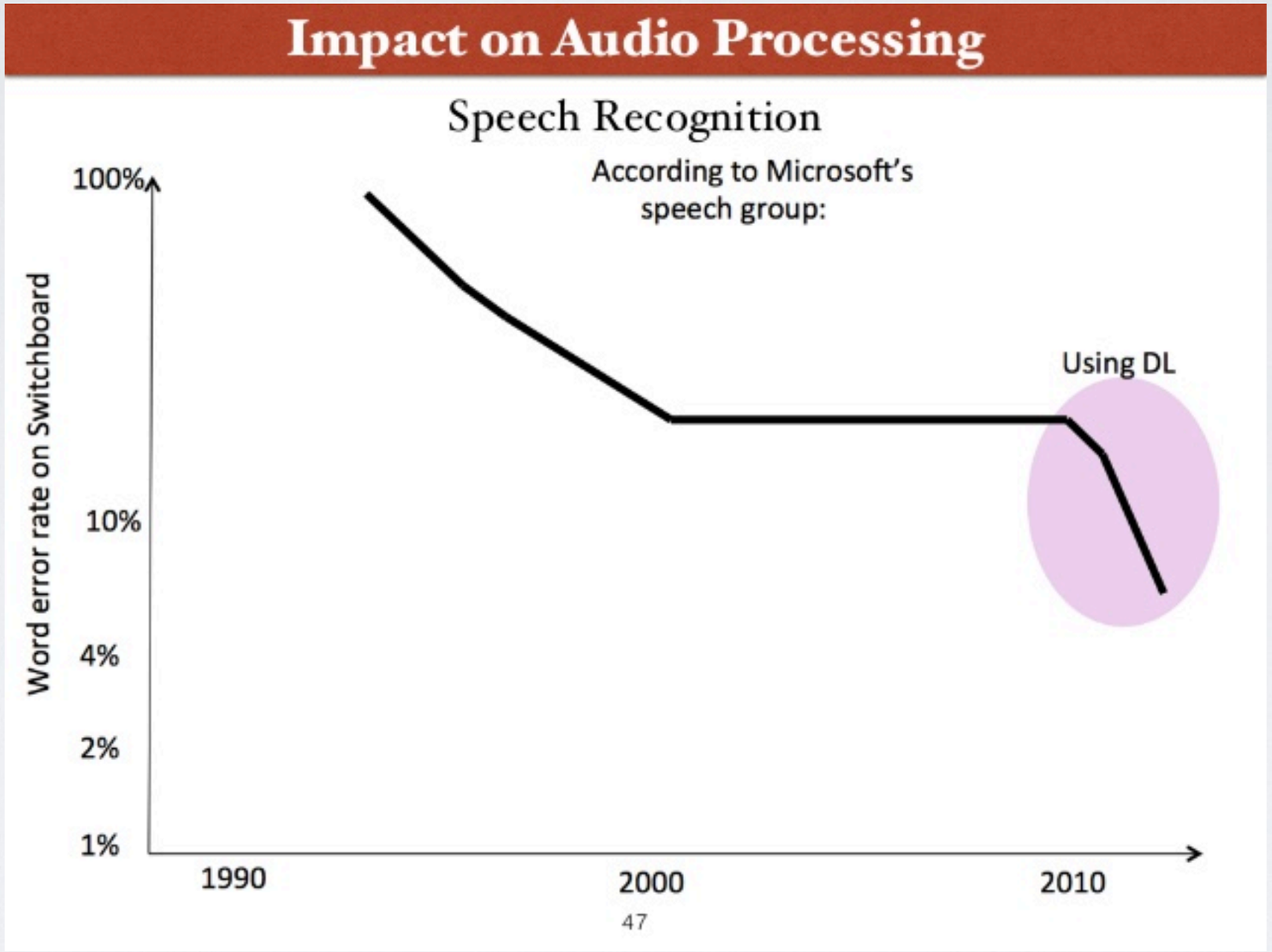


Feedback

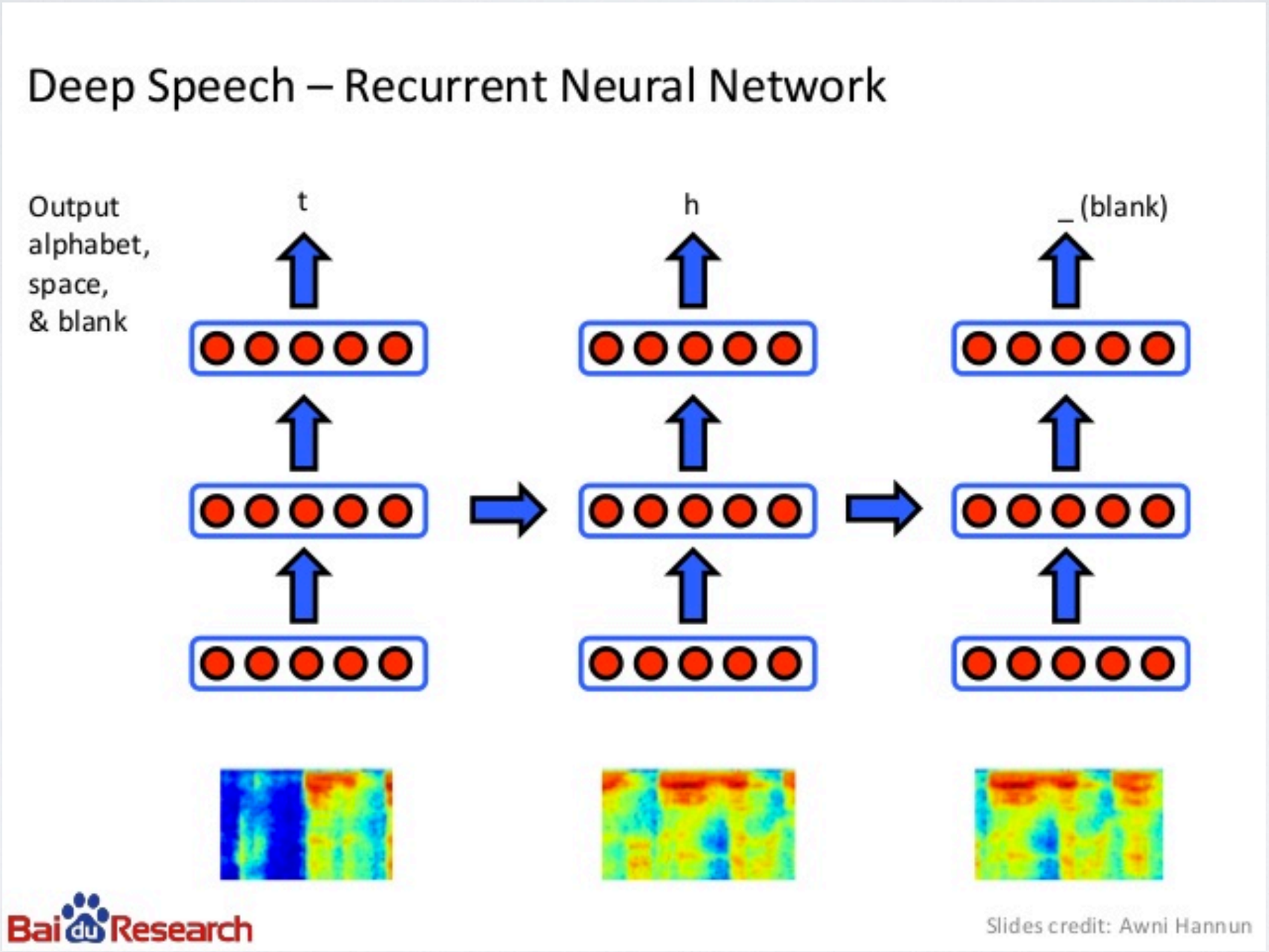
Summary so far…

- Identifying the right representation from data
- Uncovering the representation learnt in hidden layers
- Convolution and max-pooling operation
  - Convolutional neural networks (CNN)
- RNNs and dropouts.

Speech Recognition



Speech Recognition



Open Source Tools for Deep Learning

Theano

http://deeplearning.net/software/theano/

TensorFlow

https://www.tensorflow.org/

Caffe

http://caffe.berkeleyvision.org/

Kaldi

http://kaldi-asr.org/

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