Machine Learning for Sensory Signals

Dimensionality Reduction

09-02-2017

Principal Component Analysis

- First M eigenvectors of data covariance matrix

$$S = \frac{1}{N}\sum_{n=1}^{N}(\mathbf{x}_n - \bar{\mathbf{x}})(\mathbf{x}_n - \bar{\mathbf{x}})^T$$

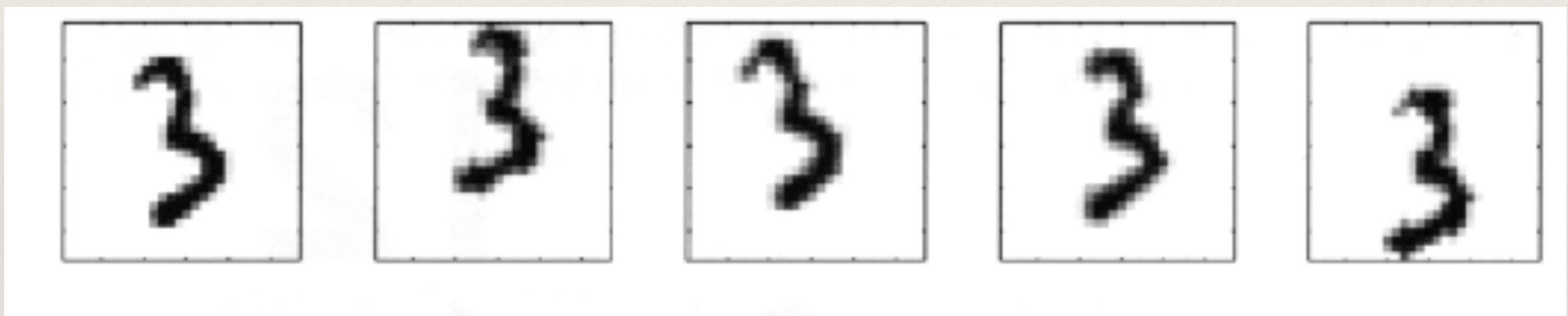
- Residual error from PCA

$$J = \sum_{i=M+1}^{D} \lambda_i$$

PRML - C. Bishop (Sec. 12.1)

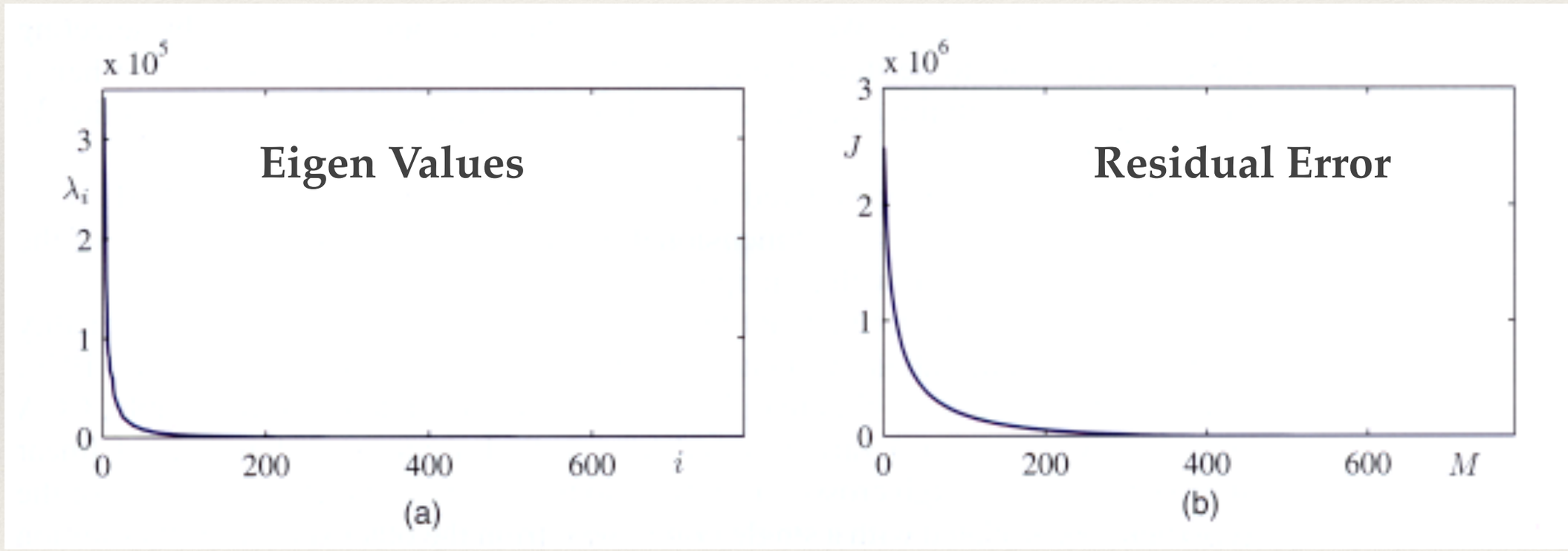
PCA

Handwritten digits used for PCA training…



PRML - C. Bishop (Sec. 12.1)

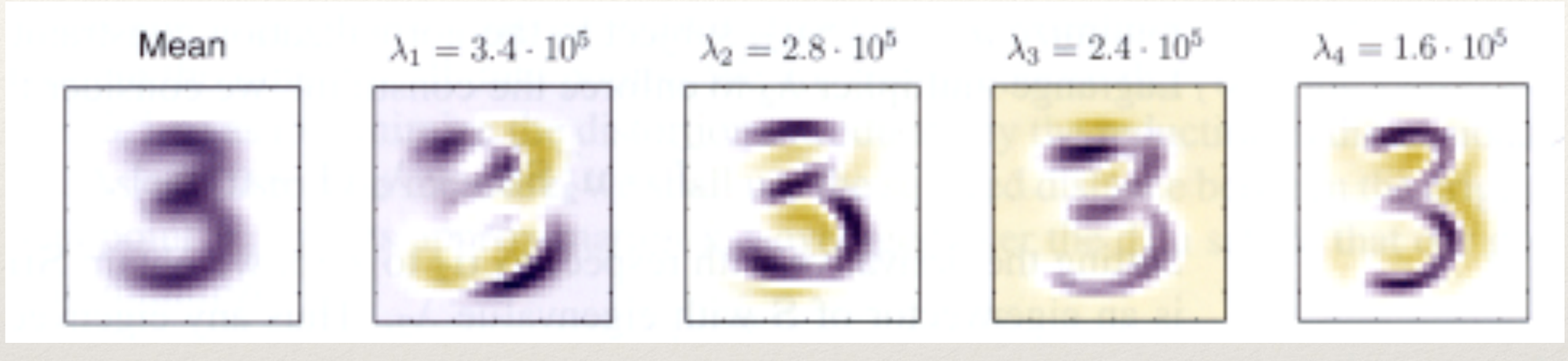
PCA



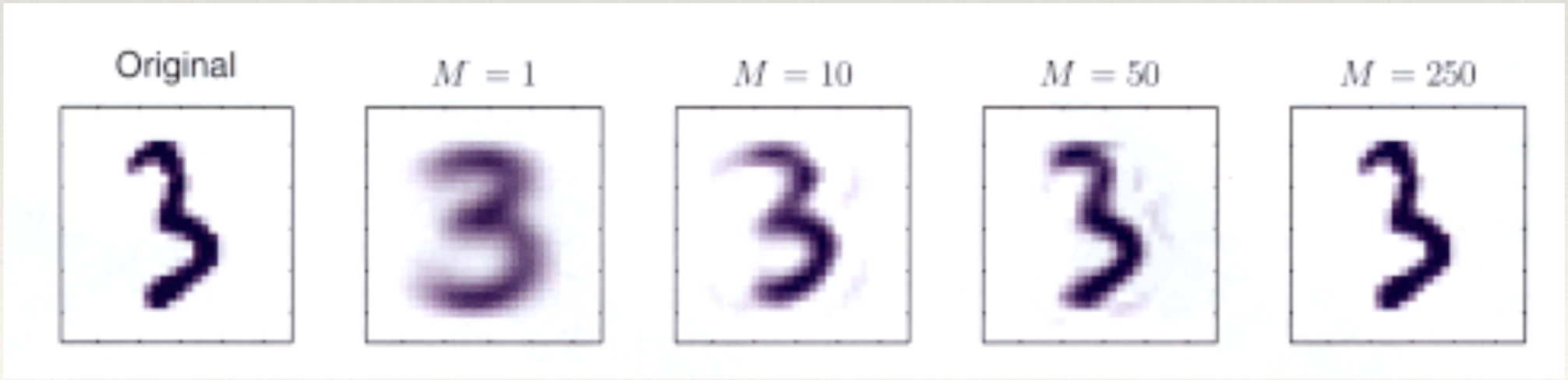
PRML - C. Bishop (Sec. 12.1)

PCA - Reconstruction

Eigenvectors



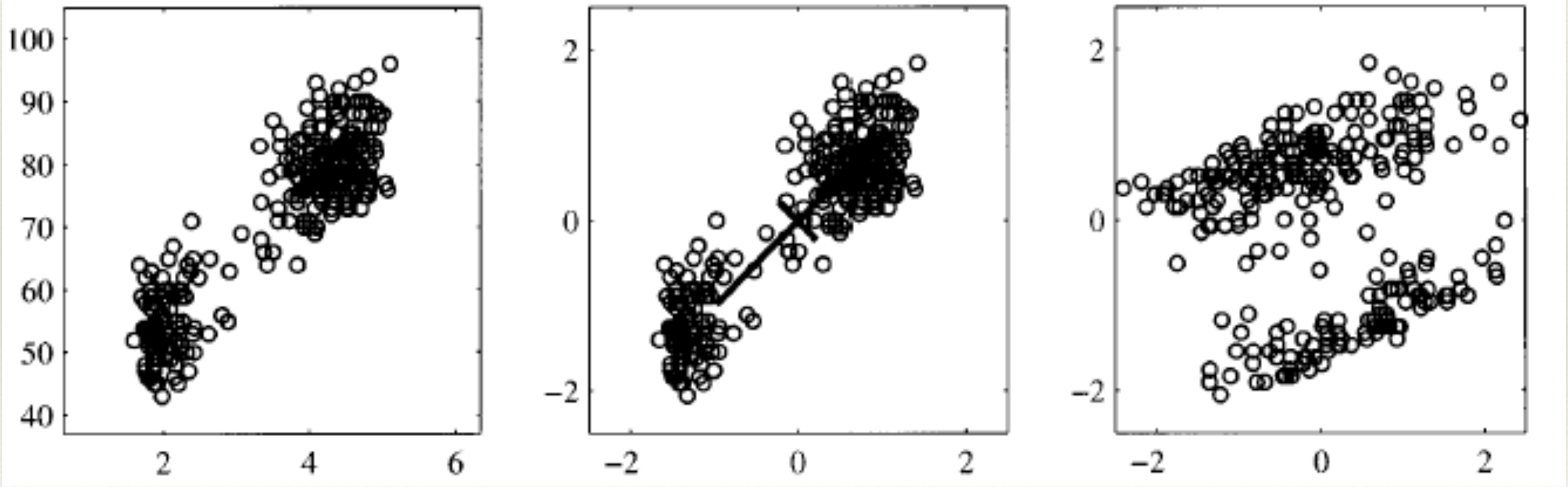
PCA - Reconstruction



PRML - C. Bishop (Sec. 12.1)

Whitening the Data

Original Data **Mean Removed data** **Whitened data**



PRML - C. Bishop (Sec. 12.1)

Linear Discriminant Analysis

Find a linear transform $f(\mathbf{x}) = \mathbf{w}^T\mathbf{x}$ with a criterion which maximizes the class separation

- Maximize the between class distance in the projected space while minimizing the within class covariance

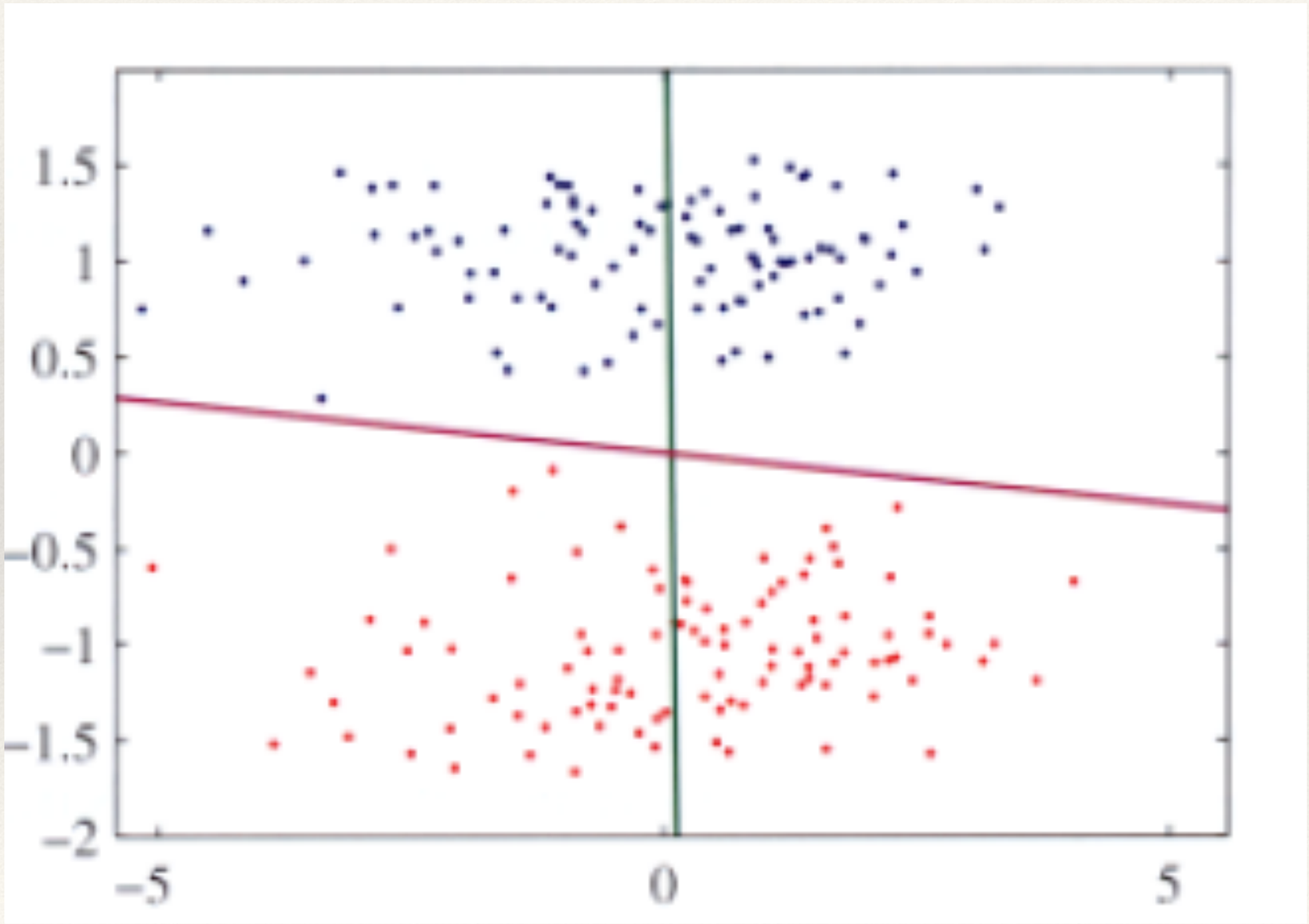
$$J = \frac{\mathbf{w}^T \boldsymbol{S}_b \mathbf{w}}{\mathbf{w}^T \boldsymbol{S}_w \mathbf{w}}$$

$$\boldsymbol{S}_b = \sum_{k=1}^{K} N_k (\mathbf{m}_k - \boldsymbol{m})(\mathbf{m}_k - \boldsymbol{m})^T \quad \boldsymbol{S}_w = \sum_{k=1}^{K} \sum_{n \in C_k} (\mathbf{x}_n - \boldsymbol{m}_k)(\mathbf{x}_n - \boldsymbol{m}_k)^T$$

- Generalized Eigenvalue problem
- Eigenvectors of $\boldsymbol{S}_w^{-1}\boldsymbol{S}_b$

PRML - C. Bishop (Sec. 4.1.4, Sec. 4.1.6)

PCA versus LDA



PCA

LDA

PRML - C. Bishop (Sec. 4.1.4, Sec. 4.1.6)