STAT 5113: Statistical Inference

Homework 2 – Due February 9, 2018

A. **The following problem *might* be graded.**

1. Consider a sample $X_1, \dots, X_n$ from the $\mathcal{U}nif(0, \theta)$ distribution. The MLE of θ is given by

$$\hat{\theta} = \max_{1 \le i \le n} X_i.$$

   a. Find the cdf of $\hat{\theta}$, and use it to find the pdf of $\hat{\theta}$. [Hint: use the fact that $\max_i x_i \le t$ iff $x_i \le t$ for every i.]

   b. Derive an expression for the bias of $\hat{\theta}$.

   c. Suppose the sample consisted of the following numbers:

6.83	8.85	1.46	7.81	5.89	7.20	6.60	11.98	10.55	8.12	7.59	4.50
10.51	0.18	8.62	9.58	6.89	2.30	7.55	4.12	10.67	1.08	0.53	9.47

   Provide an estimate of θ and of the bias of the estimator.

   d. Using the data provided above, give an estimate of the MSE of $\hat{\theta}$.

B. **The following problem *will* be graded.**

2. As in problem A.3 of Homework 1, consider independent samples

$$X_i \sim \mathcal{N}(\mu_1, \sigma^2), \quad i = 1, \dots, n_1, \qquad Y_j \sim \mathcal{N}(\mu_2, \sigma^2), \quad j = 1, \dots, n_2.$$

   Define the one-sample MLEs

$$\bar{X} = \frac{1}{n_1} \sum_{i=1}^{n_1} X_i \qquad \bar{Y} = \frac{1}{n_2} \sum_{j=1}^{n_2} Y_j$$

$$S_1^2 = \frac{1}{n_1} \sum_{i=1}^{n_1} (X_i - \bar{X})^2 \qquad S_2^2 = \frac{1}{n_2} \sum_{j=1}^{n_2} (Y_j - \bar{Y})^2$$

   The MLEs of the unknown parameters, which you have derived in the previous homework, are

$$\widehat{\mu_1} = \bar{X}$$
$$\widehat{\mu_2} = \bar{Y}$$
$$\widehat{\sigma^2} = \frac{n_1 S_1^2 + n_2 S_2^2}{n_1 + n_2}.$$

   a. Find the (joint) sampling distribution of $\widehat{\mu_1}, \widehat{\mu_2}$, and $\widehat{\sigma^2}$.

   b. Find the bias of the three estimators. Which one is unbiased? Which one is biased?