Homework 2

STAT 5333 (Spring 2021)

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Problem 2.2

Here, X = true status (1 = disease, 2 = no disease), Y = diagnosis (1 = positive, 2 = negative) and $\pi_i = P(Y = 1|X = i) \quad \forall i \in \{1, 2\}$

(a) Sensitivity = True Positive Rate = $P(Y = 1|X = 1) = \pi_1$.
Specificity = True Negative Rate = $P(Y = 2|X = 2) = 1 - P(Y = 1|X = 2) = 1 - \pi_2$

(b) Here, $P(X = 1) = \gamma$

$$P(X = 1|Y = 1) = \frac{P(Y = 1|X = 1)P(X = 1)}{P(Y = 1|X = 1)P(X = 1) + P(Y = 1|X = 2)P(X = 2)} = \frac{\gamma\pi_1}{\gamma\pi_1 + \pi_2(1 - \gamma)}$$

(c) Here, $\gamma = 0.01$, $\pi_1 = 0.86$ and $\pi_2 = 1-$Specificity $= 1 - 0.88 = 0.12$. The pobability of truely having cancer given positive test result is

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p1 = 0.86; p2 = 1-0.88; gamma = 0.01
((gamma*p1)/((gamma*p1)+p2*(1-gamma)))
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## [1] 0.06750392
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Problem 2.10

Let, X = race of murderer (1 = black, 2 = white) and Y = race of victim (1 = black, 2 = white)

(a) The conditional distributions refer to $X|Y$. Here, $P(X = 1|Y = 1) = 0.91$ and $P(X = 2|Y = 2) = 0.83$

(b) Odds Ratio,

$$\theta = \frac{P(X = 1, Y = 1)}{P(X = 1, Y = 2)}\frac{P(X = 2, Y = 2)}{P(X = 2, Y = 1)} = \frac{P(X = 1|Y = 1)}{P(X = 1|Y = 2)}\frac{P(X = 2|Y = 2)}{P(X = 2|Y = 1)} = \frac{0.91}{1 - 0.83}\frac{0.83}{1 - 0.91} = \boxed{49.36}$$

Hence, the odds of a black man killed by a black murderer is 49.36 times higher than odds of being killed by a white murderer. Hence, this kind of murders are highly dependent on racial line.

(c) The probability that the victim was white, given that a murderer was white

$$P(Y = 2|X = 2) = \frac{P(X = 2|Y = 2)P(Y = 2)}{P(X = 2|Y = 1)P(Y = 1) + P(X = 2|Y = 2)P(Y = 2)} = \frac{0.83 \times P(Y = 2)}{0.09 \times P(Y = 1) + 0.83 \times P(Y = 2)}$$

So, we need either $P(Y = 1)$ or $P(Y = 2)$ to find the probability above.