# Homework 1

STAT 5333 (Spring 2021)

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## Problem 1.2

(a) Nominal (b) Ordinal (c) Ordinal (d) Nominal (e) Nominal (f) Ordinal

## Problem 1.4

(a) Probability mass function, $P(Y = y) = \frac{n!}{y!(n-y)!}\pi^y(1-\pi)^{n-y}$. Here, $\pi = 0.5$ and $n = 2$

```
p = 0.5
n = 2
dbinom(0:n,n,p)
```

```
## [1] 0.25 0.50 0.25
```

Also, mean, $\mu = n\pi = 2 \times 0.5 = 1$ and standard deviation, $\sigma = \sqrt{n\pi(1-\pi)} = \frac{1}{\sqrt{2}}$

(b)

```
p = 0.6
dbinom(0:n,n,p)
```

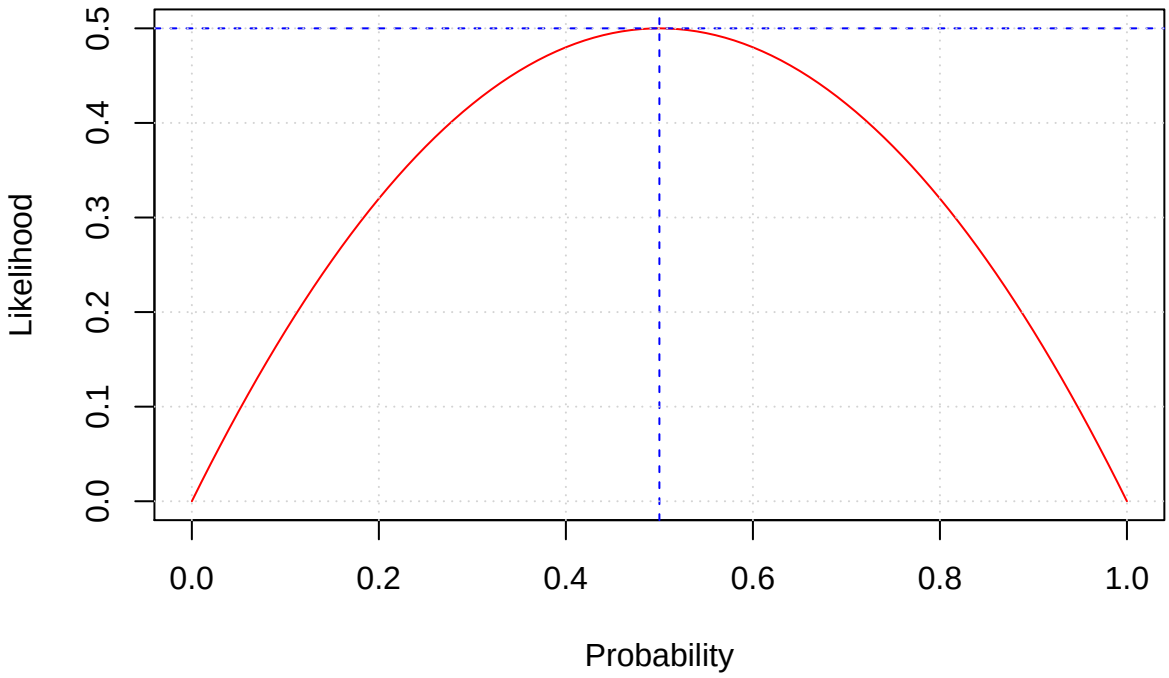
```
## [1] 0.16 0.48 0.36
```

```
p = 0.4
dbinom(0:n,n,p)
```

```
## [1] 0.36 0.48 0.16
```

(c) Likelihood function, $L(\pi) = P(Y = 1|\pi) = \frac{2!}{1!1!}\pi^1(1-\pi)^1 = 2\pi(1-\pi)$

```
p = seq(0,1,0.01)
L = 2*p*(1-p)
plot(p,L,type="l", col="red", xlab="Probability", ylab="Likelihood")
abline(v=0.5, col="blue", lty = 2)
abline(h=0.5, col="blue", lty = 2)
grid()
```




(d) The maximum value of $L(\pi)$ is 0.5 at $\pi = 0.5$, which is the ML estimate.

## Problem 1.6

(a) Here, $n = n_1 + n_2 + n_3$. Hence, $n_3 = n - n_1 - n_2$ if we know $n_1$ and $n_2$.

(b) All (=10=$\frac{5!}{2!3!}$) possible observations $(n_1, n_2, n_3)$ with $n = 3$ are (0,0,3), (0,1,2), (0,2,1), (0,3,0), (1,0,2), (1,1,1), (1,2,0), (2,0,1), (2,1,0), (3,0,0)

(c) Multinomial probability of $(n_1, n_2, n_3) = (1, 2, 0)$ with $(\pi_1, \pi_2, \pi_3) = (0.25, 0.50, 0.25)$ is

```
dmultinom(x=c(1,2,0),prob=c(0.25,0.5,0.25))
```

```
## [1] 0.1875
```

(d) $n_1$ has binomial distribution with $n_1 \sim Bin(n, \pi_1)$.

$$P(n_1 = k) = \frac{n!}{k!(n-k)!}\pi_1^k(1-\pi_1)^{n-k} = \frac{n!}{k!(n-k)!}\pi_1^k(\pi_2+\pi_3)^{n_2+n_3}$$