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A. Q1:

$\omega_p = 0{,}2\pi$, $\omega_s = 0{,}4\pi$ $A = 60\,dB \rightarrow$ Blackman

$\Delta\omega = \omega_s - \omega_p = 0{,}2\pi$

$\longrightarrow \frac{11\pi}{M} = \Delta\omega \Rightarrow \frac{11\pi}{M} = 0{,}2\pi \longrightarrow M = 55$

$\alpha = \frac{M-1}{2} = \frac{54}{2} = 27$

$\text{cut-off frequency} = \omega_c = \frac{\omega_p + \omega_s}{2} = \frac{0{,}6\pi}{2} = 0{,}3\pi$

$R_p = 0{,}25$

$$\begin{cases} R_p = 0{,}004 \\ A_s = 71\,dB \end{cases}$$

← چون پنجره در یک فیلتر ضرب شده یک تضعیف ایجاد می‌کند، چون بلکمن $A = 74\,dB$ است، A فیلتر ما به سمت $-74\,dB$ می‌رود

← باندگذر ما $\Delta\omega = 0{,}2\pi$ و $0{,}3\pi = \omega_c$، چون باید مال پنجره متفاوت است ریپل ممکن است متفاوت شود. چون باندگذر کوچکتر می‌شود، ریپل را تحت تاثیر می‌گذارد

B. Q1:

$\omega_p = 0{,}2\pi$, $\omega_s = 0{,}4\pi \longrightarrow \Delta\omega = \omega_s - \omega_p = 0{,}2\pi$

$A = 60\,dB \longrightarrow M = \frac{60 - 7{,}95}{2{,}285\Delta\omega} + 1 = 37{,}25 \longrightarrow M = 38$ $\rightarrow$ FIR-GLP Type I

$\alpha = 19$

$\beta = 0{,}1102(A_s - 8{,}7) = 0{,}1102(60 - 8{,}7) = 5{,}65326$

← چون FIR-GLP Type I شده فیلتر خوبی است

A_s بدست آمده دقیقاً $60\,dB$ است که خواسته مسئله را برآورده کرده است

C:

Rectangular $\frac{1{,}8\pi}{M} = 0{,}2\pi \longrightarrow M = 9$

$A_s = 18\,dB \longrightarrow$ زیاد peak approximate error

$R_p = 2 \longrightarrow$ ریپل خیلی زیاد

← چون $A_s = -18\,dB$ است، بعضی از صداهایی که انتظار نداریم در خروجی ظاهر می‌شود.

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: A : Q_2

$\omega_p = 0{,}2\pi$, $\omega_s = 0{,}4\pi$ $\qquad T_d = 2$

$R_p = 1dB$, $A_s = 20dB$

$\Omega_p = \frac{2}{T} \tan\left(\frac{\omega_p}{2}\right) = \tan\left(\frac{0{,}2\pi}{2}\right) = \tan(0{,}1\pi) = 0{,}32491$

$\Omega_s = \frac{2}{T} \tan\left(\frac{\omega_s}{2}\right) = \tan(0{,}2\pi) = 0{,}72654$

Butterworth : $|H_c(j\Omega)|^2 = \frac{1}{1+\left(\frac{\Omega}{\Omega_c}\right)^{2N}} \longrightarrow \boxed{N=4}$

$$N = \frac{\log\left[\left(10^{\frac{R_p}{10}}-1\right)/\left(10^{\frac{A_s}{10}}-1\right)\right]}{2\log_{10}\left(\frac{\Omega_p}{\Omega_s}\right)}$$

$$H_a(s) = \frac{\Omega_c^N}{\underset{\substack{LHP\\ Poles}}{\prod}(s-p_k)}$$

$$\begin{cases} \Omega_c = \dfrac{\Omega_p}{\sqrt[2N]{10^{R_p/10}-1}} = \dfrac{0{,}32491}{\sqrt[8]{10^{0{,}1}-1}} = 0{,}38469 \\ \Omega_c = \dfrac{\Omega_s}{\sqrt[2N]{10^{\frac{A_s}{10}}-1}} = \dfrac{0{,}72654}{\sqrt[8]{10^2-1}} = 0{,}40907 \end{cases}$$

$$H_a(s) = \frac{0{,}0219}{(s^2+0{,}7108s+0{,}1480)(s^2+0{,}2944s+0{,}1480)}$$

$$H(z) = H_a\left(\frac{2}{T}\cdot\frac{1-z^{-1}}{1+z^{-1}}\right) = H_a\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$$

$$\rightarrow H(z) = \frac{0{,}0082(1+z^{-1})^4}{(1-0{,}9167z^{-1}+0{,}2352z^{-2})(1-1{,}1814z^{-1}+0{,}5918z^{-2})}$$

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$$\omega_n = \frac{2}{\pi}\tan^{-1}\left(\frac{\Omega_c \cdot 2}{2}\right) = \frac{2}{\pi}\tan^{-1}(\Omega_c)$$: B . Q2

poles: $\Omega_c e^{+j\frac{\pi}{2N}(2k+N-1)}$

$$N = \frac{\log\left[(10^{\frac{R_p}{10}}-1)/(10^{\frac{A_s}{10}}-1)\right]}{2\log\left(\frac{\Omega_p}{\Omega_s}\right)} = 3,8 \longrightarrow 4$$

$$\Omega_c = \frac{\Omega_p}{\sqrt[2N]{10^{\frac{R_p}{10}}-1}} = 0,3847$$

$$\omega_n = \frac{2}{\pi}\tan^{-1}(0,3847) = 0,2338$$

$$H(z) = \frac{0,0081684\,(z+1)^4}{(z^2-0,9167z+0,2352)(z^2-1,1813z+0,5918)}$$

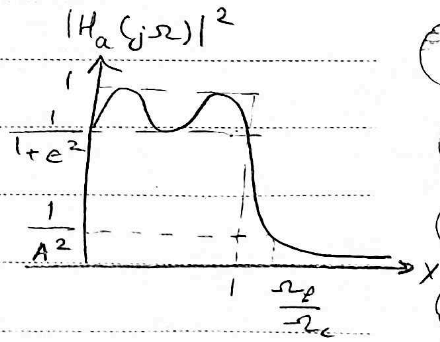
Chebyshev: chebyshev-I filter have equiripple response in the passband, while chebyshev-II filter have equiripple response in the stopband. Butterworth filters have monotonic response in both bands. : C : Q2

→ Chebyshev filters provide lower order than butterworth filters for the same specifications.

chebyshev-I: $|H_a(j\Omega)|^2 = \dfrac{1}{1+\epsilon^2 T_N^2\left(\frac{\Omega}{\Omega_c}\right)}$

Passband ripple factor ← ϵ

order of filter ← N



$$T_N(x) = \begin{cases} \cos(N\cos^{-1}(x)), & 0 \leq x \leq 1 \\ \cosh(\cosh^{-1}(x)), & 1 < x < \infty \end{cases} \qquad x = \frac{\Omega}{\Omega_c}$$

$\epsilon = \sqrt{10^{0,1R_p}-1}$, $A = 10^{A_s/20}$

$\Omega_c = \Omega_p$, $\Omega_r = \dfrac{\Omega_s}{\Omega_p}$

$g = \sqrt{(A^2-1)/\epsilon^2}$, $N = \left\lceil \dfrac{\log\left[g+\sqrt{g^2-1}\right]}{\log\left[\Omega_r+\sqrt{\Omega_r^2-1}\right]} \right\rceil$

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+ A Chebyshev-II filter is related to the Chebyshev-I filter through a simple transformation. It has monotone passband and an equiripple stopband, which implies that this filter has both poles and zeros in the s-plane. Therefore the group delay characteristics are better (and the phase response more linear) in the passband than Chebyshev-I

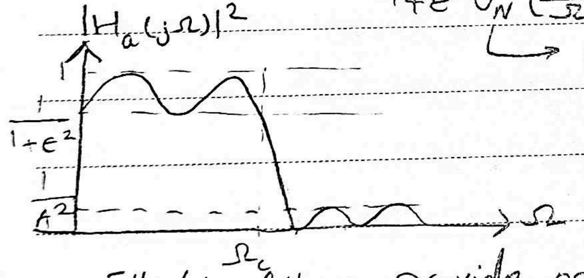
$$|H_a(j\Omega)|^2 = \frac{1}{1+[\epsilon^2 T_N^2(\frac{\Omega_c}{\Omega})]^{-1}}$$

+ Elliptic low pass filters exhibit equiripple behavior in the passband as well as in the stopband. They are similar in magnitude response characteristics to the FIR equiripple filters. Therefore elliptic filters are optimum filters in that they achieve the minimum order N for the given specifications (or alternately, achieve the sharpest transition band for the given order N)

These filters are very difficult to analyze and to design.

$$|H_a(j\Omega)|^2 = \frac{1}{1+\epsilon^2 U_N^2(\frac{\Omega}{\Omega_c})}$$

U_N ↳ N-th order Jacobian elliptic function.



→ Elliptic filters provide optimal performance in the magnitude-squared response but have highly nonlinear phase response in the passband (which is undesirable)

Butterworth filters, have maximally flat magnitude response and require a higher-order N (more poles) to achieve the same stopband specification. They exhibit a fairly linear phase response in their passband. The Chebyshev filters have phase characteristics that lie somewhere in between. In practical applications, we do consider Butterworth as well as Chebyshev filters, in addition to elliptic filters. The choice depends on both the filter order and phase characteristics.

filter order ↳ influence processing speed and implementation complexity

phase characteristics ↳ Controls distortion

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for the 1st signal pairs linear convolution and circular - : 5 : Q3
-convolution are the same beause $\longrightarrow$ $L = 4+4 = 8$

$$P = 8+4 = 12$$

$$\Delta = L+P-1 = 19$$

$$\Delta < N = 21$$

but for the 2nd signal pairs $\rightarrow$ $L = 7+5 = 12$

$$P = 11+5 = 16$$

$$\Delta = 12+16-1 = 27$$

$$\Delta > N = 21$$

$\Delta - N = 27 - 21 = 6 \longrightarrow$ 6 نمونه نادرست خواهیم داشت