Chapter 8

MICRO DATA AND GENERAL EQUILIBRIUM MODELS *

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* We thank Marco Cagetti, John Heaton, Jose Scheinkman, John Taylor, Edward Vytlacil and Noah Williams for comments. Hansen and Heckman gratefully acknowledge funding support by the National Science Foundation.

Handbook of Macroeconomics, Volume 1, Edited by J.B. Taylor and M. Woodford
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Abstract

Dynamic general equilibrium models are required to evaluate policies applied at the national level. To use these models to make quantitative forecasts requires knowledge of an extensive array of parameter values for the economy at large. This essay describes the parameters required for different economic models, assesses the discordance between the macromodels used in policy evaluation and the microeconomic models used to generate the empirical evidence. For concreteness, we focus on two general equilibrium models: the stochastic growth model extended to include some forms of heterogeneity and the overlapping generations model enriched to accommodate human capital formation.

Keywords

general equilibrium models, microeconomic evidence, stochastic growth model, overlapping generations model, calibration

Introduction

An extensive literature in macroeconomics and public finance uses dynamic stochastic general equilibrium models to study consumption savings, capital accumulation, and asset pricing and to analyze alternative policies. Except for a few special cases, the economies studied cannot be analyzed using "paper and pencil" style analysis. It is often difficult to produce general theorems that are true for all parameter values of dynamic general equilibrium models. This is a general feature of non-linear dynamic models in economics as well as in the physical sciences. For such models, knowing which parameters govern behavior is essential for understanding their empirical content and for providing quantitative answers to policy questions. For the numerical output of a dynamic equilibrium model to be interesting, the inputs need to be justified as empirically relevant. There are two sources of information that are commonly used in rationalizing parameter values. One is the behavior of time series averages of levels or ratios of key variables. These time series averages are often matched to the steady-state implications of versions of the models that abstract from uncertainty. The other input is from microeconomic evidence. In this essay we discuss the use of evidence from both sources, concentrating mostly on microeconomic evidence. See King and Rebelo (1999) and Taylor (1999) for extensive discussions of calibrating real-business cycle and staggered contract models, respectively.

It was once believed to be a simple task to extract the parameters needed in general equilibrium theory from a large warehouse of stable micro empirical regularities. Indeed, Prescott (1986) argued that:

> The key parameters of growth models are the intertemporal and intratemporal elasticities of substitution. As Lucas (1980) emphasizes, "On those parameters, we have a wealth of inexpensive available data from census and cohort information, from panel data describing market conditions and so forth".

While this Lucas–Prescott vision of *calibration* offers an appealing defense for building models with microeconomic foundations, implementing it in practice exposes major discrepancies between the micro evidence and the assumptions on which the stylized dynamic models are based. The microeconomic evidence is often incompatible with the macroeconomic model being calibrated. For example, a major finding of modern microeconomic data analysis is that preferences are heterogeneous. For reasons of computational tractability, dynamic general equilibrium model-builders often abstract from this feature or confront it in only a limited way.

This chapter explores the discordance between micro evidence and macro use of it and suggests ways in which it can be diminished. Our chapter raises warning flags about the current use of micro evidence in dynamic equilibrium models and indicates the dangers in, and limitations of, many current practices. It also exposes the weak micro empirical foundations of many widely used general equilibrium modeling schemes. The decision to incorporate micro evidence in an internally consistent manner may alter the structure and hence the time series implications of the model. While steady-state approximations may be useful for some purposes, compositional changes in labor supply or in market demand alter the microeconomic elasticities that are relevant for macroeconomics.

Like several of the other contributions to this Handbook, ours is more of a guide for future research than a summary of a mature literature. Because the micro empirical literature and the macro general equilibrium literature have often moved in different directions, it is not surprising that they are currently so detached. The main goal of this essay is to foster the process of reattachment. Macro general equilibrium models provide a framework within which micro empirical research can be fruitfully conducted. At the same time, dynamic general equilibrium theory will be greatly enriched if it incorporates the insights of the micro empirical literature. The micro foundations of macroeconomics are more fruitfully built on models restructured to incorporate microeconomic evidence. This essay explores three challenges for closing the gap between empirical microeconomics and dynamic macroeconomic theory:

- *Heterogeneity*: Any careful reading of the empirical microeconomics literature on consumption saving and labor supply reveals quantitatively important heterogeneity in agent preferences, in constraints, in dimensions of labor supply and skill, and in human capital accumulation processes. Accounting for heterogeneity is required to calibrate dynamic models to microeconomic evidence.
- *Uncertainty*: Modern macroeconomics is based on models of uncertainty. Aggregating earnings across members in a household and across income types may create a disconnect between uncertainty as measured by innovations in time series processes of earnings and income equations and actual innovations in information. Government or interfamily transfers provide insurance that should be accounted for. Alternative risk components such as risks from changes in health, risks from unemployment and job termination, and risks from changes in family structure, have different degrees of predictability and are difficult to quantify. Measuring the true components of both micro and macro uncertainty and distinguishing them from measurement error and model misspecification remains an empirical challenge that is just beginning to be confronted.
- *Synthesis*: Synthesizing evidence across micro studies is not a straightforward task. Different microeconomic studies make different assumptions, often implicit, about the economic environments in which agents make their decisions. They condition on different variables and produce parameters with different economic interpretations. A parameter that is valid for a model in one economic environment cannot be uncritically applied to a model embedded in a different economic environment. Different general equilibrium models make different assumptions and require different parameters, many of which have never been estimated in the micro literature.

In order to be both specific and constructive, in this essay we limit ourselves to two prototypical general equilibrium models: (a) a stochastic growth model and (b) a perfect foresight overlapping generations model. The first model is sufficiently rich to enable us to explore implications of uncertainty, market structure and some forms of heterogeneity in the preferences and opportunities of microeconomic agents. The second model introduces explicit life-cycle heterogeneity and demographic structures in appealing and tractable ways. We consider a recent version of the second model that introduces human capital formation, heterogeneity in skills, and comparative advantage in the labor market. These attributes are introduced to provide a framework for analyzing labor market policies, to account for a major source of wealth formation in modern economies, and to account for the phenomenon of rising wage inequality observed in many countries.

The plan of this chapter is as follows. We first present two basic theoretical models analyzed in this chapter and the parameters required to implement them. We summarize the main lessons from the micro literature that pertain to each model and their consequences for the models we consider. The models are presented in Sections 1 and 2, respectively, with some accompanying discussion of the relevant micro literature. Section 3 presents further discussion of the micro evidence on intertemporal substitution elasticities.

1. Stochastic growth model

This part of the chapter presents alternative variants of a Brock–Mirman (1972) stochastic growth model and discusses the parameters needed to calibrate them. We explicitly consider the consequences of heterogeneity for the predictions of this model and for the practice of research synthesis. It is often not the median or "representative" preferences that govern behavior asymptotically; rather it is the extreme. The agents with the smallest rates of time preference or smallest risk aversion may dominate the wealth accumulation process, but not the supply of labor. Understanding the source and magnitude of the heterogeneity is required before microeconomic estimates can be "plugged" into macroeconomic models. We also explore the measurement of microeconomic uncertainty needed to quantify the importance of precaution in decision-making and to calibrate equilibrium models with heterogeneous agents.

We use the Brock–Mirman (1972) stochastic growth model as a starting point for our discussion because of its analytical tractability. Moreover, it is the theoretical framework for the real-business cycle models of Kydland and Prescott (1982) and Hansen (1985) and for subsequent multiple consumer extensions of it by Aiyagari (1994), Krusell and Smith (1998) and others. Our use of the stochastic growth model is not meant as an endorsement of its empirical plausibility. Much is known about its deficiencies as a model of fluctuations [e.g., see Christiano (1988), Watson (1993), and Cogley and Nason (1995)] or as a model of security market prices implied by a Lucas–Prescott (1971) type of decentralization [e.g., see Hansen and Singleton (1982, 1983), Mehra and Prescott (1985), Weil (1989), Hansen and Jagannathan (1991), and Heaton and Lucas (1996)]. Nevertheless, the Brock–Mirman model and its extensions provide a convenient and widely used starting point for investigating the difficulties in obtaining plausible parameter configurations from microeconomic data and from aggregate time series data.

1.1. Single consumer model

Suppose that there is a single infinitely-lived consumer. This consumer supplies labor and consumes in each period, evaluating consumption flows using a von Neumann–Morgenstern discounted utility function:

$$E\sum_{t=0}^{\infty}\beta^t U(c_t),$$

where c_t is consumption at date t, U is an increasing concave function and $0 < \beta < 1$ is a subjective discount factor. Labor h_t is either supplied inelastically, or else preferences are modified to incorporate the disutility of work (utility of leisure):

$$E\sum_{t=0}^{\infty}\beta^t U(c_t, h_t).$$

Production takes place according to a two-input production function:

$$c_t + (k_t - \lambda k_{t-1}) = d_t f(k_{t-1}, h_t) \tag{1.1}$$

where k_t is capital and d_t is a technology shock, which is a component of a Markov process $\{x_t\}$. The depreciation rate is $1-\lambda$. Associated with this Markov process is a sequence of information sets $\{I_t\}$. In what follows we sometimes adopt the common and convenient Cobb–Douglas specification of production[1]:

$$f(k, h) = k^{\theta}h^{1-\theta}. \tag{1.2}$$

1.1.1. Parameterizations

We first present the basic utility functions that have been used in empirical work and in many versions of the Brock–Mirman model. We briefly review the micro-econometric evidence on preferences, paying particular attention to the interactions between consumption and labor supply. This evidence is discussed more extensively in Section 3. For convenience, some models abstract from the labor supply decision

[1] In Section 2 we will study deficiencies of this Cobb–Douglas specification. In particular, labor is not homogeneous and an efficiency units assumption to adjust labor to homogeneous units is inconsistent with the evidence from factor markets. Comparative advantage and sectoral choices by agents are key features of modern labor markets.

and use an iso-elastic one-period utility function defined over a single non-durable consumption good[2]:

$$U(c) = \frac{c^{1-\rho} - 1}{1-\rho} \tag{1.3}$$

for $\rho \geqslant 0$. This specification is used in part because, given intertemporal additivity of preferences, it is homothetic and hence leads to simple steady-state characterizations.

To obtain a more interesting model of economic fluctuations, including fluctuations in total or average hours worked, Kydland and Prescott (1982) introduced leisure into the preferences of a Brock–Mirman model [see also King, Plosser and Rebelo (1988a,b) and Cooley and Prescott (1995)]. Most subsequent investigators assume that the one-period utility function can be written in the form

$$U(c, h) = \frac{\left\{c^{\sigma}\left[\psi(h)\right]^{1-\sigma}\right\}^{1-\rho} - 1}{1-\rho} \tag{1.4}$$

where h is hours of work and ψ is decreasing and positive and σ is in the interval (0, 1)[3]. When $\rho = 1$, we obtain the additively-separable model:

$$U(c, h) = \sigma \log(c) + (1-\sigma)\log[\psi(h)].$$

1.1.2. Steady states

With specification (1.4), the marginal rate of substitution between consumption and work is:

$$mrs = \frac{(1-\sigma)\,\psi\prime(h)c}{\sigma\psi(h)} \tag{1.5}$$

and hence is linear in consumption Suppose that there is geometric growth in the technology process $\{d_t\}$. Given the Cobb–Douglas production function (1.2), a steady state exists in which hours worked, the consumption–capital ratio and the implied firm

[2] Some consumption models allow for many goods. For example, many dynamic international trade models follow the tradition of static models and allow that "traded" and "non-traded" goods enter the utility function differently; see, for example, Backus, Kehoe and Kydland (1995) and Stockman and Tesar (1995).

[3] For some dynamic equilibrium models, consumption and labor supply are composites. For example, Kydland and Prescott (1982) have preferences defined over a weighted sum of current and lagged labor supply and Eichenbaum and Hansen (1990) and Hornstein and Praschnik (1994) define consumption as a CES aggregator of the flow of services from durables and non-durables. Auerbach and Kotlikoff (1987) use a CES version of Equation (1.4) in their overlapping generations model.

expenditure share on labor costs are constant. Steady-state calibration proceeds as follows. Steady states and steady-state ratios are measured by time series averages. The production function parameter θ is pinned down by labor's share of output, and the depreciation factor for capital from the steady-state investment–capital ratio. For a given ψ, say $\psi = 1 - h$, the parameter σ may be solved out by equating minus the marginal disutility of work (1.5) with the marginal product of labor. This yields

$$(1-\theta)\frac{c+i}{c} = \frac{-(1-\sigma)\,\psi\prime(h)h}{\sigma\psi(h)},$$

where i is steady-state investment[4].

An important question this theory has to confront is whether the functional forms for preferences of consumption and labor supply used to interpret aggregate time series data as steady states are compatible with microeconomic evidence on the functional form of preferences. The time series evidence measures the fraction of available time an "average" worker spends in market work. The claim in the real-business cycle literature is that per capita leisure has remained relatively constant in the post-war period while real wages have been rising at the same rate as output. However, this stability in average hours worked per capita masks divergent trends for males and females. A central finding from the empirical micro literature is that the time series of the labor supply behavior of men and women is different and neither is close to a stationary time series. [See Pencavel (1986) and Killingsworth and Heckman (1986).]

If preference parameters are to be based on microeconomic evidence, two questions have to be answered. First, do the functional forms found in the micro literature produce growth steady states? Second, given the changes in the composition of the labor force, whose labor elasticities should be used in calibrating a macroeconomic model? The answers to these questions rely in part on the relative quality of the aggregate time series and the microeconomic evidence. Durlauf and Quah (1999) raise serious doubts about the validity of the steady-state approximation as an accurate factual description of modern economies. Note further that the functional form restrictions required for the conjectured steady states apply to a fictitious composite household model of consumption and leisure. In practice the microeconomic evidence is extracted separately for men and women using preference specifications that are

[4] Consideration of household production and the possibility of substituting work time in the home for expenditures on goods lead authors such as Benhabib, Rogerson and Wright (1991), Greenwood and Hercowitz (1991) and Greenwood, Rogerson and Wright (1995) to allow for derived utility functions over consumption and market hours that are somewhat more general than the class of models considered here. Their home production specification introduces technological progress into the "reduced-form" depiction of the preferences for consumption and labor supply and loosens the restrictions needed for the existence of a steady state of the desired form. See Eichenbaum and Hansen (1990) for a similar development for modeling the preferences for durable and non-durable consumption goods.

outside the form given in Equation (1.4). For example, MaCurdy (1983) reports that a specification of male labor supply with

$$u(c, h) = \frac{\left(\sigma \frac{c^{1-\alpha_c}}{1-\alpha_c} - \frac{h^{1+\alpha_h}}{1+\alpha_h}\right)^{1-\rho} - 1}{1-\rho}$$

is consistent with monthly male labor supply data from the US where $\rho = 0.14$, $\alpha_c = 0.66$ and $\alpha_h = 0.16$, and these parameters are precisely estimated. Note in particular that $\alpha_c \neq 1$. The marginal rate of substitution between consumption and work is:

$$mrs = -\frac{h^{\alpha_h}}{\sigma c^{-\alpha_c}},$$

and this empirical specification is not consistent with steady-state growth because $\alpha_c \neq 1$. It is, however, consistent with the well known observation that male hours of work per capita have declined over time.

1.1.3. Micro evidence

Our more detailed discussion of the microeconomic evidence presented in Section 3 establishes the following additional empirical conclusions:

- Most of the responsiveness of labor supply with respect to wage change is due to entry and exit from employment; yet most of the micro evidence for intertemporal labor supply elasticities is presented for continuously working, continuously married prime age males – the demographic group least responsive to wage changes, especially at the extensive margin[5].
- There is evidence that consumption is complementary with male labor supply while the evidence is mixed on the interaction between consumption and female labor supply. At present there are no reliable estimates of this interaction. Yet the difference between male and female labor supply highlights the problem of pooling the labor supply of diverse groups into one aggregate.

[5] Rogerson (1988) designed an aggregate model of labor supply that focuses exclusively on the extensive margin. Individuals are allocated randomly to jobs that require a fixed number of hours. The number of jobs fluctuates over time but not the number of hours per job. Hansen (1985) adapted this framework to the Brock–Mirman stochastic growth model. While these models successfully emphasize the extensive margin, they are not well suited to capture differential labor supply responses between men and women. We discuss this model further in Section 3.

- The elasticity of intertemporal substitution (*eis* $= -1/\rho$) as determined from consumption is usually poorly determined. If constancy across the population is imposed on this elasticity, then there is no strong evidence against the view that this elasticity is slightly *above* minus one. There is, however, evidence that the *eis* varies both with observable demographics and with the level of wealth so that the homothetic iso-elastic form is rejected and an assumption that *eis* is minus one for all demographic groups is *not* consistent with the evidence. The same evidence suggests that low wealth households are relatively more averse to consumption fluctuations than are high wealth households.
- For leisure, the elasticity of intertemporal substitution is between 0.1 and 0.4 for annual hours for men and 1.61 for women. There is evidence that these elasticities are larger for shorter units within a year. Because these labor supply elasticities ignore the entry and exit decision, they provide only part of the information needed to construct the aggregate labor supply curve.

1.2. Multiple agents

Heterogeneity in preferences, discount rates, and risk aversion parameters is found in numerous micro studies. As a step towards achieving a better match between micro economic evidence and dynamic stochastic economics, it is fruitful to explore macro general equilibrium models with explicit heterogeneity. Such models are of considerable interest in their own right and often produce rather different outcomes than their single consumer counterparts. Adding heterogeneity enriches the economic content of macro models, and calls into question current practices for obtaining parameter estimates used in general equilibrium models.

We start with a very simple specification. Consider a large population with J types of agents indexed by j. We abstract from labor supply as in the Brock–Mirman (1972) stochastic growth model and we also ignore human capital accumulation. Instead we suppose initially that labor is supplied inelastically. Following Aiyagari (1994), we adopt the simple version of the Brock–Mirman model in which individual agents confront stochastic productivity shocks $y_{j,t}$ to their labor supply. This scheme produces idiosyncratic shocks in labor income in spite of the presence of a common wage (per unit productivity) and leads to a simple analysis. Later on, we explore complications caused by the addition of the labor supply decision.

1.2.1. Preferences

We follow the common practice of using preferences with a constant elasticity of intertemporal substitution but we let the *eis* and the subjective rate of time discount differ among individuals:

$$E \sum_{t=0}^{\infty} (\beta_j)^t \frac{(c_{j,t})^{1-\rho_j} - 1}{1-\rho_j}, \tag{1.6}$$

where (β_j, ρ_j) differ by consumer type. The evidence discussed both here and in Section 3 documents that such heterogeneity is empirically important.

1.2.2. Labor income

Assume that $\{x_t\}$ is a Markov process governing aggregate shocks and that the productivity for type j at time $t+1$, $y_{j,t+1}$, is a component of a person-specific state vector $s_{j,t+1}$. The probability distribution of $s_{j,t+1}$ given current period state vectors $s_{j,t}$ and x_t is denoted by $F_j(\cdot \mid s_{j,t}, x_t)$. The income of person j at time t is $w_t y_{j,t}$ where w_t is the endogenously determined wage rate at time t. Aggregate or average labor supply corrected for efficiency is given by

$$h_t^a = \frac{1}{J}\sum_{j=1}^{J} y_{j,t},$$

where J is the number of agents in the economy and the individual labor supply is normalized to be unity. In equilibrium, wages satisfy the aggregate marginal product condition

$$w_t = (1-\theta)\, d_t (k_{t-1}/h_t^a)^\theta.$$

1.2.3. Market structure

Depending on the institutional environment confronting agents, different interactions among them may emerge. Market interactions can be limited by informational constraints and the ability to commit to contractual arrangements. Here, we consider a variety of market contexts and their implications for behavior and calibration. We initially explore a complete market model as a benchmark. Suppose consumers trade in a rich array of security markets. We introduce a common sequence of information sets and use I_t to denote information available at date t. Consumers can make state-contingent contracts conditioned on information available in subsequent time periods. Given the ability to make such contracts, we obtain a large array of equilibrium security market prices. Moreover, in studying consumption allocations, we may simplify the analysis by exploiting the implications of Pareto efficiency. Although our interest is in economies with heterogeneous consumers, we begin our exposition of Pareto efficient models by first considering agents with homogenous preferences but heterogeneous endowments.

1.2.4. Preference homogeneity

Suppose initially that consumers have common preferences (β, ρ). Endowments may differ; in this case, preferences aggregate in the sense of Gorman (1953). At a mechanical level, this can be checked as follows. The intertemporal marginal rates of substitution are equated so:

$$m_{t+1,t} = \beta\left(\frac{c_{j,t+1}}{c_{j,t}}\right)^{-\rho}, \qquad \text{thus} \qquad c_{j,t}\left(\frac{m_{t+1,t}}{\beta}\right)^{1/(-\rho)} = c_{j,t+1}.$$

Averaging over the consumption of each type, we find that

$$c_{a,t}\left(\frac{m_{t+1,t}}{\beta}\right)^{1/(-\rho)} = c_{a,t+1},$$

where $c_{a,t}$ denotes consumption averaged across types. We may solve this equation to obtain an alternative expression for the common marginal rate of substitution:

$$m_{t+1,t} = \beta\left(\frac{c_{a,t+1}}{c_{a,t}}\right)^{-\rho}.$$

This result is due to Rubinstein (1974). Under the stated conditions, there exists an aggregate based on preferences that *look like* the common individual counterparts. An alternative way to view this economy is as an example of Wilson's (1968) theory of syndicates. With the marginal rates of substitution equated across consumers, we are led to a solution whereby individual consumption is a constant fraction of the aggregate over time:

$$c_{j,t} = \kappa_j c_{a,t},$$

or equivalently that the allocation *risk-sharing rules* are linear[6].

Armed with this result, the general equilibrium of this model can be computed as follows. Given the aggregate endowment process and the capital accumulation process, we may solve for the optimal aggregate consumption process. This may be thought of as special case of a Brock–Mirman style stochastic growth model in which the fictitious consumer has preferences that coincide with those of the individual identical agents. The solution to this problem yields the equilibrium processes for aggregate consumption aggregate investment and aggregate capital stock[7]. Notice that we can compute the aggregate quantities without simultaneously solving for the equilibrium prices. It is not necessary to ascertain how wealth is allocated across consumers because we can construct well defined aggregates[8].

[6] The reference to this as a risk-sharing rule is misleading. Consider economies in which endowments of individuals oscillate in a deterministic way, but the aggregate endowment is constant. Then consumption allocations will be constant as implied by the linear allocation rule, but there is no risk.

[7] See also Lucas and Prescott (1971).

[8] The simplicity here is overstated in one important respect. In characterizing the aggregate endowment behavior, we either must appeal to a cross-sectional version of the Law of Large Numbers, or we must keep track of the idiosyncratic state variables needed to forecast individual endowments.

Given our assumption of homothetic preferences, the equilibrium allocation of consumption assigns a constant (over time and across states) fraction of aggregate consumption to each person. Each consumer is endowed with an initial asset stock along with his or her process for consumption endowments. To determine the equilibrium allocation of consumption across people we must solve for the equilibrium valuation of the consumption endowments. With this valuation in hand, the individual consumption assignments are readily deduced from the intertemporal budget constraint. Then we can consider the equilibrium pricing of state-contingent claims to consumption.

Following Rubinstein (1974) and Hansen and Singleton (1982), pricing implications for this economy may be obtained by using the equilibrium consumption vector and forming the equilibrium intertemporal marginal rates of substitution: the equilibrium versions of $\{m_{t+1,t}\}$. From this process we can construct the pricing operator: $\mathcal{P}_t$. Let z_{t+1} represent a claim to consumption at time $t+1$. For instance, z_{t+1} may be the payoff (in terms of time $t+1$ consumption) to holding a security between dates t and $t+1$. For securities with longer maturities than one time period, we can interpret z_{t+1} as the liquidation value of the security (the dividend at time $t+1$ plus the price of selling the security at $t+1$). The consumption claim z_{t+1} may depend on information that is only observed at date $t+1$ and hence is a random variable in the information set I_t. The equilibrium restrictions of our model imply that the price at time t can be expressed as

$$\mathcal{P}_t(z_{t+1}) = E(m_{t+1,t} z_{t+1} | I_t), \tag{1.7}$$

where $\mathcal{P}_t(z_{t+1})$ is the date t price quoted in terms of date t consumption Thus the pricing operator $\mathcal{P}_t$ assigns time t equilibrium prices to these contingent consumption claims in a linear manner[9]. The intertemporal marginal rate of substitution, $m_{t+1,t}$, acts like a state-contingent discount factor. Since it is stochastic, in addition to discounting the future, it captures risk adjustments for securities with uncertain payouts.

For this economy we may extract preference parameters using Euler equation estimation. Let Z_{t+1} denote a vector of one-period (gross) payoffs and Q_t the corresponding price vector. The preference parameter vector (β, ρ) can be identified from the unconditional moment restriction:

$$E\left[\beta\left(\frac{c_{t+1}}{c_t}\right)^{-\rho} Z_{t+1} - Q_t\right] = 0. \tag{1.8}$$

[9] We are being deliberately vague about the domain of this operator. Since $m_{t+1,t}$ is positive, any non-negative payoff can be assigned an unambiguous but possibly infinite value. To rule out value ambiguities that arise when the positive part of the payoff has value $+\infty$ and the negative part $-\infty$, additional restrictions must be imposed on payoffs that depend on properties of the candidate discount factors.

The asset payoffs may consist of multiple security market returns, or they may be synthetic payoffs constructed by an econometrician [10]. We obtain the same preference parameters when the model is estimated using aggregate or individual data on consumption In principle, this provides a way to test the assumed preference specification given the market environment if there is access to micro data. As noted by Rubinstein (1974), Hansen (1987), and in effect, Gorman (1953), this result is special, even under preference homogeneity. It is not applicable to any concave increasing one-period utility function unless the consumers have the same initial wealth.

Given knowledge of the technology parameters, the stochastic process for aggregate capital stocks may be deduced as in Brock and Mirman (1972) by endowing the representative agent with preference parameters and labor supply h_t^a. From this solution we may solve for equilibrium aggregate consumption equilibrium stochastic discount factors (from intertemporal marginal rates of substitution), equilibrium wages (from marginal products), the initial wealth distribution and hence the sharing parameters (the κ_j).

1.2.5. Risk aversion or intertemporal substitution?

By estimating Euler equations, econometricians may identify preference parameters without having to solve the decision problem confronting individual agents. In particular, parameters can be identified without any need to measure microeconomic uncertainty or wealth. Of course if preferences are misspecified by an econometrician, estimated Euler equations will not recover the true preference parameters of individual agents. We now consider a misspecification of particular substantive interest.

The parameter ρ is associated with two conceptually distinct aspects of preferences: risk aversion and intertemporal substitution along certain paths. The link between attitudes towards risk and intertemporal fluctuations in consumption over time is indissolubly embedded in models that simultaneously assume separability over time and over states [see Gorman (1968)] [11]. It is the latter type of separability that is the key assumption in expected utility models. Hall (1988) and Epstein and Zin (1989, 1991) argue that it is fruitful to disentangle attitudes toward risk from intertemporal substitution as they correspond to two different aspects of consumer behavior. Concern about intertemporal substitution comes into play even in economies with deterministic movements in technologies. These considerations led Epstein and Zin (1989) to use

[10] Synthetic payoffs and prices are constructed as follows. Multiply a single gross return, say $1 + r_{t+1}$, by instrumental variables in the conditioning information set I_t. Since this conditioning information is available to the consumer at purchase date t, the price of the scaled return is given by the instrumental variable used in the scaling. By creating enough synthetic securities, unconditional moment condition (1.8) can be made to imititate the conditional pricing relation [see Hansen and Singleton (1982) and Hansen and Richard (1987)].

[11] The parameter ρ also governs precautionary savings or prudence [see Kimball (1990)]. This latter link is readily broken by adopting a more flexible parameterization of expected utility.

a recursive utility formulation due to Kreps and Porteus (1978) in which preferences are represented using "continuation utility" indices, which measure the current period value of a consumption plan from the current period forward. The continuation utility index, $V_{j,t}$, for person j is obtained by iterating on the recursion:

$$V_{j,t} = \left[(1-\beta)\left(c_{j,t}\right)^{1-\rho} + \beta \mathcal{R}_t(V_{j,t+1})^{1-\rho}\right]^{1/(1-\rho)}, \tag{1.9}$$

where $\mathcal{R}_t$ makes a risk adjustment on tomorrow's utility index:

$$\mathcal{R}_t(V_{j,t+1}) = \left\{\left[E\left(V_{j,t+1}\right)^{1-\alpha} \mid I_t\right]\right\}^{1/(1-\alpha)}. \tag{1.10}$$

Observe that the utility index today is homogeneous of degree one in current and future (state-contingent) consumption This specification of preference nests our specification (1.6) with a common ρ provided that $\alpha = \rho$. By allowing α to be distinct from ρ we break the connection between risk aversion and intertemporal substitution. The parameter α is irrelevant in an environment without uncertainty, but the intertemporal elasticity of substitution $(-1/\rho)$ parameter is still important. The parameter α makes an incremental risk adjustment, which is absent from the standard von Neumann–Morgenstern formulation.

Some of our previous analysis carries over directly to this recursive utility formulation. The efficient allocation for individual consumptions and individual utility indices satisfies

$$c_{j,t} = \kappa_j c_{a,t}, \qquad V_{j,t} = \kappa_j V_{a,t}$$

for some numbers κ_j where $V_{a,t}$ is constructed using the process for the representative consumer $\{c_{a,t+k} : k = 0, 1, \ldots\}$ in place of $\{c_{j,t+k} : k = 0, 1, \ldots\}$. With this modification, the procedure we have previously described for solving out Brock–Mirman economies applies for preferences of the type (1.9).

The intertemporal marginal rates of substitution are, however, altered, and this complicates the construction of the one-period stochastic discount factors. With the Kreps–Porteus utility recursion,

$$m_{t,t+1} = \beta\left(\frac{c_{a,t+1}}{c_{a,t}}\right)^{-\rho}\left[\frac{V_{a,t+1}}{\mathcal{R}_t(V_{a,t+1})}\right]^{\rho-\alpha}$$

which now depends on the continuation utility $V_{a,t+1}$. The same formula works if individual consumptions and continuation utilities are used in place of the aggregates. Micro- or macroeconomic estimation procedures based on Euler equations that erroneously assume that $\alpha = \rho$ generally fail to produce a usable estimate of either α or ρ unless they are equal. Even if a risk-free asset is used in estimating the Euler equation (1.8), the intertemporal substitution parameter will not be identified. The presence of risk aversion parameter $\alpha(\neq \rho)$ will alter the Euler equation if there is any uncertainty affecting the consumption decisions of the individual [see Epstein and Zin (1991)]. For this case, a correctly specified Euler equation contains the continuation utility $(V_{j,t+1})$ and its conditional moment [12]. If the risk adjustment is logarithmic ($\alpha = 1$), then a logarithmic version of the Euler equation will recover ρ provided the return on the wealth portfolio is used as the asset return instead of the riskfree return [see Equation 18 in Epstein and Zin (1991)] [13].

Since continuation utilities now enter correctly specified Euler equations, one way to modify the unconditional moment restrictions used in estimation is to solve recursion (1.9) for values of the preference parameters. This solution requires knowledge of the equilibrium consumption process for either individuals or the aggregate. Thus it is no longer possible to separate the estimation of preference parameters from the estimation of the other features of the model as is conventional in the standard Euler equation approach. In particular, it is necessary to specify the underlying uncertainty individuals confront. Given that this explicit computation of the full model is required, there are other, more direct, approaches to estimation than plugging solved continuation utilities into Euler equations in a two-stage procedure [14]. Barsky, Juster, Kimball and Shapiro (1997) pursue an alternative way of measuring risk preferences (independent of intertemporal substitution) that is based on confronting consumers with hypothetical employment gambles. We discuss their work below.

1.2.6. Preference heterogeneity

Even when we restrict ourselves to state-separable power-utility functions, once we allow for heterogeneity in preferences we must modify our aggregation theorem and our method for solving the general equilibrium model. We continue to impose the complete market structure, but drop back to the simple additively-separable preference

[12] Epstein and Zin (1991) present a clever solution to this problem whereby they derive an alternative Euler equation that depends instead on the one-period return on a hypothetical wealth portfolio. In practice it is difficult to construct a reliable proxy that is compatible with the observed consumption data. In their Euler equation estimation using aggregate data, Epstein and Zin (1991) used the value-weighted return on the New York Stock Exchange, but this proxy only covers a component of wealth in the actual economy.

[13] When the risk adjustment is made using the negative exponential counterpart to Equation (1.10), then Euler equation (1.8) continues to apply but with an endogenously determined distorted conditional expectation operator. This risk adjustment is equivalent to inducing a specific form of pessimism. See Hansen, Sargent and Tallarini (1999) for a discussion of this point.

[14] An interesting question is what security data are needed to identify the risk adjustment in the utility index. Epstein and Melino (1995) address this question without imposing parametric restrictions on the risk adjustment. Not surprisingly, the degree of identification depends on the richness of the security market returns used in the investigation. When investors have access to more security markets and the resulting returns are observed by an econometrician, the range of admissible risk adjustments shrinks. Hansen, Sargent and Tallarini (1999) illustrate this point in the parametric context of a permanent income model with an exponential risk adjustment.

specification. It is again convenient to pose the equilibrium problem as an optimal resource allocation problem for the purpose of computing equilibrium quantities. To accomplish this in a world of heterogeneous preferences we use a method devised by Negishi (1960) and refined by Constantinides (1982), Lucas and Stokey (1984) and others.

Using standard Pareto efficiency arguments, and assuming interior solutions, consumers equate their marginal rates of substitution

$$(\beta_j)^t \left(\frac{c_{j,t}}{c_{j,0}}\right)^{-\rho_j} = (\beta_1)^t \left(\frac{c_{1,t}}{c_{1,0}}\right)^{-\rho_1}.$$

For each individual j, we assign a time t Pareto weight $\omega_{j,t}$ with a deterministic equation of evolution:

$$\omega_{j,t} = \left(\frac{\beta_j}{\beta_1}\right) \omega_{j,t-1}. \tag{1.11}$$

Equating marginal rates of substitution we obtain:

$$\omega_{j,t}(c_{j,t})^{-\rho_j} = \omega_{1,t}(c_{1,t})^{-\rho_1}.$$

We may thus characterize Pareto efficient allocations by combining evolution equation (1.11) with the solution to the static deterministic optimization problem:

$$\max_{c_1, c_2, \ldots, c_J} \sum_{j=1}^{J} \omega_j \frac{(c_j)^{1-\rho_j} - 1}{1-\rho_j} \text{ subject to } \frac{1}{J}\sum_{j=1}^{J} c_j = c. \tag{1.12}$$

The solution to problem (1.12) is obtained from the following argument. Let μ denote the common marginal utility across individuals:

$$\omega_j(c_j)^{-\rho_j} = \mu.$$

Then,

$$c_j = \left(\frac{\mu}{\omega_j}\right)^{1/(-\rho_j)}. \tag{1.13}$$

By first averaging this expression over individuals, we may compute μ by solving the non-linear equation

$$c = \frac{1}{J}\sum_{j=1}^{J} \left(\frac{\mu}{\omega_j}\right)^{1/(-\rho_j)}.$$

Plugging the solution for the common marginal utility μ back into Equation (1.13), we obtain the allocation equations:

$$c_j = \phi_j(c;\, \omega), \qquad j = 1, \ldots, J,$$

where ω denotes the vector of Pareto weights. The allocation rules ϕ_j are increasing and must average out to unity. Substituting these rules back into the original objective function we construct a utility function for aggregate consumption

$$U(c;\, \omega) = \sum_{j=1}^{J} \omega_j \left(\frac{\phi_j(c;\, \omega)^{1-\rho_j} - 1}{1 - \rho_j} \right).$$

It is straightforward to verify that $U(c, \omega)$ is concave and strictly increasing. By the Envelope Theorem,

$$\frac{\partial U(c;\, \omega)}{\partial c} = \omega_j \left[\phi_j(c;\omega)\right]^{-\rho_j} = \mu, \tag{1.14}$$

which is the common marginal utility. The "mongrel" function U will generally depend on the Pareto weights in a non-trivial manner.

We may use this constructed utility function to deduce optimal allocations in the following way. Given any admissible initial ω_0, solve the optimal resource allocation problem using the preference ordering induced by the von Neumann–Morgenstern "mongrel" utility function:

$$E \sum_{t=0}^{\infty} (\beta_1)^t\, U(c_t;\, \omega_t)$$

subject to the equation of motion (1.1) and the evolution equation for vector of Pareto weights (1.11). If resources are allocated efficiently, we obtain an alternative (to Gorman) justification of the representative consumer model. This justification carries over to the derived pricing relations as well. Prices may be deduced from the marginal rates of substitution implied by the mongrel preference ordering. This follows directly from the link between individual and aggregate marginal rates of substitution given in Equation (1.14).

This construction justifies a two-step method for computing efficient allocations of resources when preferences are heterogeneous. In the first step we compute a mongrel utility function for a fictitious representative consumer along with a set of allocation rules from a static and deterministic resource allocation problem. The mongrel utility function may be used to deduce equilibrium aggregate consumption and investment rules and equilibrium prices. The static allocation rules may be used for each state and date to allocate aggregate consumption among the individual consumers. These computations are repeated for each admissible choice of Pareto weights. To compute a particular general equilibrium, Pareto weights must be found that satisfy the intertemporal budget constraints of the consumer with equality.

This economy has the following observable implications. First, under discount factor homogeneity, the procedure just described can be taken as a justification for using a representative agent model to study aggregate consumption investment and prices. Microeconomic data are not required to calibrate the model provided that the mongrel preferences used to compute the general equilibrium are not used for welfare analyses. Second, one can use microeconomic and macroeconomic data together to test whether individual consumption data behave in a manner consistent with this model.

If discount factors are heterogenous, and if this economy runs for a long period of time, in the long run the consumer with the largest discount factor essentially does all of the consuming[15]. This follows directly from Equation (1.11). Thus it is the discount factor of the (eventually) wealthiest consumer that should be of interest to the calibrator of a representative agent model, provided that the aim is to confront aggregate time series data and security market data. Since in the US economy, 52% of the wealth is held by 5% of the households, and there is evidence, discussed below, that wealthy families have lower discount rates, this observation is empirically relevant. A research synthesis that uses a *median* or *trimmed mean* discount factor estimated from micro data to explain long-run aggregate time series data would be seriously flawed. This raises the potential problem that the estimated extreme value may be a consequence of sampling error or measurement error instead of genuine preference heterogeneity.

If the aim is to evaluate the impact of macroeconomic policies on the welfare of the person with the *median* discount rate, then the welfare evaluation should be performed outside the representative consumer framework used for matching aggregate time series data. To be accurate it would have to recognize the diversity of subjective discount rates in the population.

With discount factor homogeneity, individual consumption will be a time invariant function of aggregate consumption This occurs because evolution equation (1.11) implies that the Pareto weights are invariant over time. In spite of this invariance, if there is heterogeneity in intertemporal substitution elasticities in an economy with growth it may still be the case that one type of the consumer eventually does most of the consumption because of non-linearity in the allocation rule. To demonstrate this we follow Dumas (1989) and, suppose that we have two types of consumers, both facing a common discount factor, but

$$\rho_1 < \rho_2.$$

This difference in the intertemporal substitution elasticity is sufficient for consumers of type 1 to eventually do most of the consumption in the economy. Simulations are

[15] Lucas and Stokey (1984) use this as a criticism of models with discount factors that are independent of consumption.

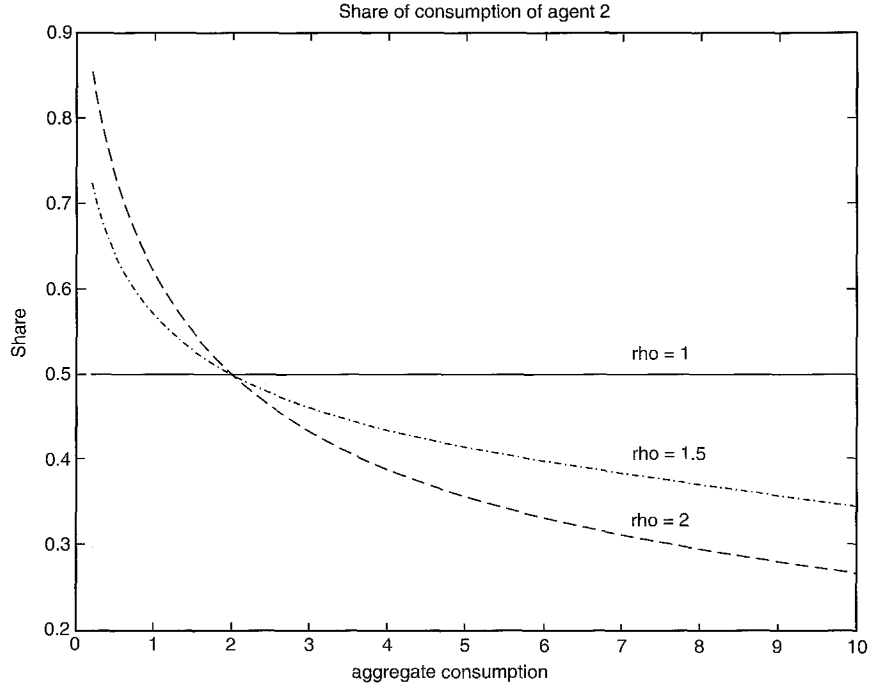

Fig. 1.1. Fraction of aggregated consumption assigned to agent 2 plotted for $\rho_2 = 1$ (solid line), $\rho_2 = 1.5$ (dash-dotted curve), and $\rho_2 = 2$ (dashed curve). In all cases $\rho_1 = 1$. Pareto weights are $\frac{1}{2}$ for both agents.

depicted in Figures 1.1 and 1.2 for a Dumas-style economy. With all consumers facing a common β, to explain the data on aggregate quantities and security market prices, it is again the preferences of the eventually wealthiest consumer that matter and not the preferences of the average or median person.

When we add labor supply into this setup, the lessons for picking parameters are different. Suppose for simplicity, we assume individual preferences are additively separable between consumption and labor supply so that for an individual of type i:

$$U_i(c, h) = \sigma_i \frac{c^{1-\rho_i} - 1}{1-\rho_i} + (1-\sigma_i)\frac{\psi_i(h)^{1-\rho_i} - 1}{1-\rho_i}.$$

Let the preference parameter for the first type satisfy $\rho_1 = 1$ and assume $\rho_1 < \rho_2$. Consider a social planner allocating consumption and hours across individuals in a Pareto efficient manner. Provided that hours can be converted into an efficiency units standard and the hours allocations for each individual are away from corners (assumptions we question in Sections 2 and 3), we can derive an aggregation result for the disutility of aggregate hours using the same techniques just described for aggregate consumption While person 1 eventually does most of the consuming in the

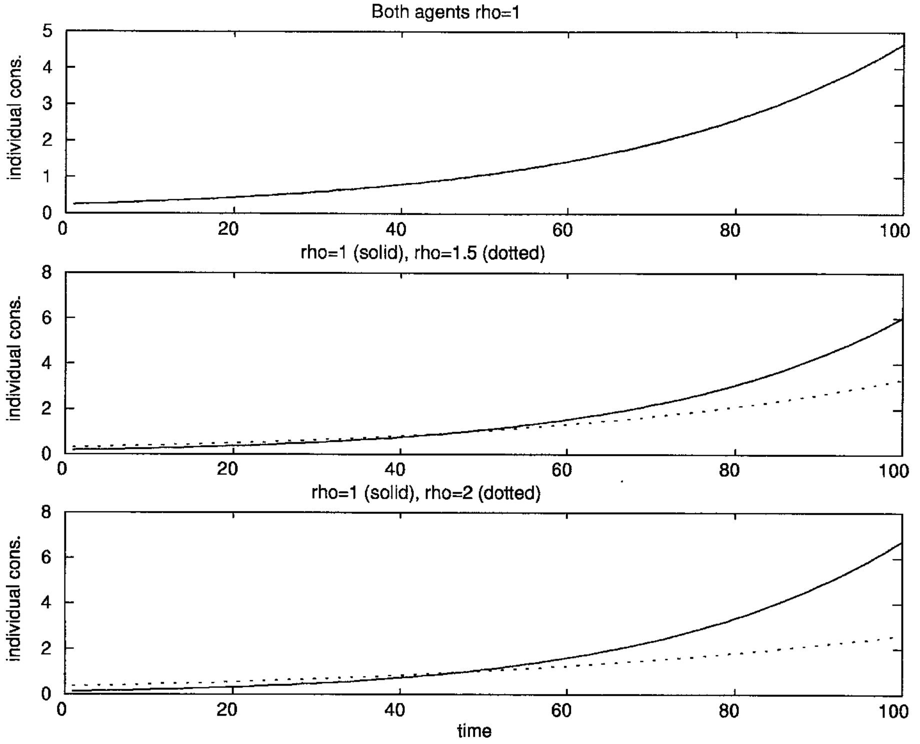

Fig. 1.2. Consumption assigned to each agent when aggregate consumption grows over time. Pareto weights are $\frac{1}{2}$ for both agents; growth rate is 3% per time period. Solid curve: agent 1; dotted curve, agent 2. Dotted and solid curves coincide in the top panel.

economy, the leisure preferences of person 2 will figure prominently in the aggregate (mongrel) preference ordering. Thus to construct the mongrel representative agent for this economy requires that the analyst use the intertemporal consumption elasticity of the rich person, but an intertemporal elasticity for labor supply that recognizes both agent types. In the presence of heterogeneity in preferences, it will sometimes be necessary to apply different weighting schemes across the population for consumption elasticities than for labor supply elasticities in order to construct an aggregate that fits the data[16].

Tests of the complete-market model usually focus on linear allocation rules, whereas accounting for preference heterogeneity leads one to recognize important

[16] In a somewhat different setting, Kihlstrom and Laffont (1979) use preference heterogeneity to build a model in which more risk averse individuals become workers and less risk averse individuals become entrepreneurs.

non-linearities in allocation rules[17]. In models with endowment uncertainty, but heterogeneity in discount rates and intertemporal elasticities of substitution, the efficient allocation is that individual consumption is a deterministic function of aggregate consumption alone. Even this implication can be altered, however, if von Neumann–Morgenstern preferences are replaced with a more general recursive formulation in which [as in Epstein and Zin (1991)] future utilities are adjusted for risk. In this case the evolution of the Pareto weights is stochastic [see Dumas, Uppal and Wang (1997) and Anderson (1998)]. As a consequence, allocation rules depend, not only on current aggregate consumption, but also on the past history of the aggregate state variables (the capital stock and x_t). The deterministic relationship between individual consumption and aggregate allocations will be altered if time non-separabilities are introduced in the form of habit persistence or durability in consumption goods. In these cases, past histories of consumption also enter into the allocation rules. Thus the fix up of mongrel preferences has to be substantially altered when we consider more general preference specifications.

We now present our first bit of evidence on the empirical importance of preference heterogeneity. This evidence demonstrates that our concerns about this issue are not purely aesthetic.

1.2.7. Questionnaire evidence on the scale and distribution of risk aversion

In an innovative paper, Barsky, Juster, Kimball and Shapiro (1997) elicit risk preferences from hypothetical questions administered to a sample of respondents in the University of Michigan Health and Retirement Survey (HRS). The aim of this study was to extract the degree of relative risk aversion without linking it to the elasticity of intertemporal substitution. Respondents were asked about their willingness to participate in large gambles of various magnitudes. For example, adult respondents are asked to imagine themselves as the sole earners in their families and are asked to choose between a job with their current annual family income guaranteed for life versus a prospect with a 50–50 chance of doubling family income and a 50–50 chance of reducing family income by a third. Respondents who take the gamble offered in the first question are then asked if they would take an option with the same gain as offered in the first question, and the same probability of gain, but a greater loss that cuts income in half. Respondents who decline the gamble offered in the first question are offered a second option with the same gain (and probability of gain) as the first option but the loss is reduced to 20 percent.

Answers to these questions enable one to bound the coefficient of relative risk aversion. The results of this hypothetical exercise are reported in Table 1.1 for a variety of demographic groups. The two notable features of this table are: (a) the substantial proportion of risk averse people and (b) the heterogeneity in risk aversion both across demographic groups and within them.

[17] See Attanasio (1999) for a survey of the literature testing for linear allocation rules.

Table 1.1
Risk aversion by demographic groups

Demographic Group	Risk aversion		Interval		Number of Responses
	$[3.76, \infty]$	(2.00,3.76)	(1,2)	(0,1)	
Age under 50 years	58.5	14.4	13.8	13.1	1147
50 to 54 years	61.9	12.0	12.2	13.7	3800
55 to 59 years	66.0	11.5	9.8	12.5	4061
60 to 64 years	69.3	9.5	9.4	11.6	2170
65 to 69 years	66.6	12.0	9.2	12.0	390
Over 70 years	68.3	6.4	9.3	15.8	139
Female	65.1	11.8	11.0	11.9	6448
Male	64.0	11.2	10.7	13.9	5259
White	64.9	12.5	10.7	11.8	8508
Black	66.7	9.1	10.6	13.3	1884
Other	62.3	10.0	13.7	13.7	109
Asian	57.9	10.3	11.1	20.6	126
Hispanic	59.3	9.2	12.6	18.7	1054
Protestant	66.2	11.5	10.8	11.4	7404
Catholic	62.3	10.8	11.4	15.3	3185
Jewish	56.3	13.2	11.1	19.2	197
Other	61.6	14.3	9.6	14.3	900

[a] Source: Barsky, Juster, Kimball and Shapiro (1997), Table III.
The p-value for the hypothesis that the mean risk tolerance is equal across age groups is 0.0001, that it is equal across sexes is 0.015, that it is equal across races is 0.0001, and that it is equal across religions is 0.0001.

There are serious questions about the relationship between actual risk-taking behavior and the responses elicited from questionnaires. In addition, there are serious questions about the magnitude of the gambles in these hypothetical choice experiments. Preferences that exhibit constant relative risk aversion, link the aversion to small gambles to the aversion to large gambles and hence justify calibrating risk aversion to large gambles. Behavior responses to small bets may be different from the responses to large ones, and the bets studied in this survey are indeed substantial. In fact, Epstein and Melino (1995) provide empirical evidence that the risk aversion may be much larger for small gambles than large ones. Nonetheless, the results summarized in Table 1.1 are very suggestive of considerable heterogeneity in the population. We present further evidence on preference heterogeneity in Section 3.

We next consider more general market environments without full insurance, and the empirical challenges that arise in constructing and calibrating general equilibrium models in such environments.

1.3. Incomplete markets

While the multiple consumer, Pareto optimal economy is pedagogically convenient, it assumes the existence of a rather large collection of markets. Moreover, it eliminates many interesting policy questions by assumption such as those having to do with borrowing constraints or adverse selection. We now consider what happens when most of the markets assumed to be open in the Pareto optimal economy are closed down. Following Miller (1974), Bewley (1977), Scheinkman and Weiss (1986), Aiyagari (1994) and Krusell and Smith (1998), among others, we suppose that consumers can only purchase and sell shares of the capital stock and are not permitted to trade claims to their future individual productivities. Moreover, only non-negative amounts of capital can be held. This is an economy with a "borrowing" constraint and other forms of market incompleteness that hold simultaneously. We discuss how these constraints can arise later on.

In an environment with incomplete markets, we can no longer exploit the convenient Pareto formulation. The economy we consider is one in which prices and quantities must be computed simultaneously. Under the efficiency units assumption, the current period wage rate satisfies a standard marginal productivity condition. The gross return to holding capital must satisfy

$$1 + r_{t+1} = \theta d_{t+1}(k_t/h^a_{t+1})^{\theta-1} + \lambda.$$

For the special case of Aiyagari's economy, there is no aggregate uncertainty. As a consequence, the equilibrium rate of return to capital will be riskless. In Krusell and Smith (1998), there is aggregate uncertainty but this uncertainty is sufficiently small so that there is little difference between the risky return on capital and a riskfree security [18]. Given that only non-negative amounts of capital can be held, the familiar consumption Euler equation is replaced by

$$E\left[\beta_j \left(\frac{c_{j,t+1}}{c_{j,t}}\right)^{-\rho_j} (1 + r_{t+1}) \mid I_t\right] \leqslant 1, \tag{1.15}$$

where equality holds when the consumer chooses positive holdings of the capital stock. The familiar interior solution Euler equation no longer characterizes some of the agents in the economy [see, for example, Zeldes (1989)].

[18] As a consequence, like the original Brock–Mirman model, theirs is a poor model of the aggregate return on equity measured using stock market data.

In an attempt to explain the wealth distribution, Krusell and Smith (1998) introduce a form of discount factor heterogeneity modeled as a persistent stochastic process. They do not, however, make specific use of the preference heterogeneity measured from microeconomic consumption studies in their calibration. Furthermore, as we will see, calibrating to microeconomic data in environments with less than full insurance requires more than just extracting preference parameters; it requires measuring the uninsured uncertainty facing agents.

1.3.1. Microeconomic uncertainty

Aiyagari (1994) and Krusell and Smith (1998) attempt to quantify the impact of the precautionary motive for savings on both the aggregate capital stock and the equilibrium interest rate, assuming that the source of uncertainty is in individual labor market productivity. In order to accomplish this task, these analysts require a measure of the magnitude of microeconomic uncertainty, and how that uncertainty evolves over the business cycle. Euler equation estimates of preference parameters must be supplemented by measures of individual uncertainty. This introduces the possibility of additional sources of heterogeneity because different economic agents may confront fundamentally different risks. To calibrate the macroeconomic model it becomes crucial to measure the distribution of individual shocks. The income of person j at time $t+1$ is $w_{t+1}y_{j,t+1}$ and its distribution depends in part on the aggregate state variable x_t. In practice, household income arises from many sources with possibly different risks and people in different occupations face quantitatively important differences in the uncertainty they confront [see the evidence in Carroll and Samwick (1997)]. Aggregating income from all sources or pooling agents in different risk classes is a potentially dangerous practice that may mask the true predictability of the individual components. Aggregates of income sources may not accurately represent the true economic uncertainty facing agents. The persistence in the idiosyncratic shocks and the manner in which aggregate state variables shift the distributions of idiosyncratic shocks are known to have an important impact on consumption allocations in incomplete market models [see Mankiw (1986) and Constantinides and Duffie (1996)]. Aggregating across risk components can alter the measured predictability.

We now present evidence on the time series processes of labor earnings and wage innovations drawing on research by MaCurdy (1982) and Abowd and Card (1989). Hubbard, Skinner and Zeldes (1994) consider measures of uncertainty for other sources of household income. We summarize the econometric evidence on the form of the forecasting equation represented as an ARMA model, and the magnitude of the innovation variance, which is often used as a measure of uncertainty.

1.3.1.1. Estimated processes for wages and earnings. There is agreement among micro studies of nationally based representative samples that differences in the residuals of male log earnings or wage rates from a Mincer earnings function are

Table 1.2
Estimated ARMA processes for residuals from first differenced Mincer log wage or log earnings equations[a]

ARMA	a		m_1		m_2		$E(\varepsilon^2_{i,t})$[b] (innovation variance)	
Log hourly wage rates								
(0,2)	–		−0.484	(17)	−0.066	(2.7)	0.061	(17)
(1,1)	0.122	(2.6)	−0.608	(13)	–		0.061	(16)
Log annual earnings								
(0,2)	–		−0.411	(14)	−0.106	(3.8)	0.054	(15)
(1,1)	0.216	(3.95)	−0.621	(13)	–		0.056	(14)

[a] Source: MaCurdy (1982), Tables 3 and 4.
$\Delta u_{i,t} = a\Delta u_{i,t-1} + \varepsilon_{i,t} + m_1\varepsilon_{i,t-1} + m_2\varepsilon_{i,t-2}$; t-statistics in parentheses.
[b] Computed assuming stationarity of the innovation process.

adequately represented by either an MA(2) process or an ARMA(1,1) process. There is surprisingly little information available about the time series processes of female earnings and wage rates. The representations for the change in log earnings and wage rates for married males that receive the most support in the studies of MaCurdy (1982) and Abowd and Card (1989) are:

$$\Delta u_{i,t} = \varepsilon_{i,t} + m_1\varepsilon_{i,t-1} + m_2\varepsilon_{i,t-2} \tag{1.16}$$

or

$$\Delta u_{i,t} = a\Delta u_{i,t-1} + \varepsilon_{i,t} + m_1\varepsilon_{i,t-1}, \tag{1.17}$$

where

$$\Delta u_{i,t} = u_{i,t} - u_{i,t-1}$$

and $u_{i,t}$ is the residual of a Mincer regression for log earnings or wage rates for person i. (See Section 2.2 for a discussion of Mincer earnings models.) Estimates of the parameters of these models are presented in Table 1.2 [taken from MaCurdy (1982)]. He reports that he is unable to distinguish between these two representations of the time series process of residuals. In Table 1.3 we report MaCurdy's (1982) estimates when the autoregressive unit root specification is not imposed in Equation (1.17). The freely estimated autoregressive coefficients are close to but slightly less than one, and they are estimated with enough accuracy to reject the unit root model using statistical tests.

The analysis of Abowd and Card (1989) is generally supportive of the results reported by MaCurdy (1982), except that MaCurdy finds that the coefficients m_1, m_2 are constant over time whereas Abowd and Card report a rejection of the overall

Table 1.3
Estimated ARMA processes for residuals from first levels of Mincer log wage or log earnings equations[a]

ARMA	a		m_1		m_2		$E(\varepsilon^2_{i,t})$[b] (innovation variance)	
Log hourly wage rates								
(1,2)	0.025	(2.8)	−0.46	(13.4)	−0.053	(1.78)	0.061	(17)
Log annual earnings								
(1,1)	0.026	(2.5)	−0.39	(12.2)	−0.094	(3.13)	0.055	(18.3)

[a] Source: MaCurdy (1982), Tables 5 and 6.
$u_{i,t} = (1-a)\,u_{i,t-1} + \varepsilon_{i,t} + m_1\varepsilon_{i,t-1} + m_2\varepsilon_{i,t-2}$; t-statistics in parentheses.
[b] Computed assuming stationarity of the innovation process.

hypothesis of stationarity for the model. There is no necessary contradiction between the two studies because MaCurdy does not require that the variances of the $\varepsilon_{i,t}$ be constant, nor does he report evidence on the question. However, he uses the assumption of constancy in the innovation variances of earnings processes to report the innovation variances given in the final column of Tables 1.2 and 1.3.

From the vantage point of macroeconomics, time series variation in the innovation variances is of interest, especially the extent that the variances fluctuate over the business cycle. Figures 1.3 and 1.4 demonstrate how the conditional variance of $\Delta u_{i,t}$ changes over the business cycle. In periods of rising unemployment, the innovation variance in log wage equations increases. This evidence is consistent with the notion that microeconomic uncertainty is greater in recessions than booms[19].

As we have previously noted, the models described in this section take households as the decision unit. This led researchers such as Heaton and Lucas (1996) and Hubbard, Skinner and Zeldes (1994) to present estimates of pooled family income processes[20]. They do not report separate estimates of the earnings processes for husbands and wives. In samples for earlier periods, there is evidence of negative covariance between spousal earnings [Holbrook and Stafford (1971)]. In later samples, there is evidence of positive covariance [Juhn and Murphy (1994)][21]. We question whether pooling household earnings processes is a sound practice for extracting components of risk

[19] The evidence for professional and educated workers reported by Lillard and Weiss (1979) and Hause (1980) suggests the presence of a person-specific growth trend. This growth trend is not found to be important in national samples across all skill groups and for the sake of brevity we do not discuss specifications of the earnings functions with this growth trend.

[20] Also, to obtain a better match with their model, Heaton and Lucas (1996) look at individual income relative to aggregate income. In effect they impose a cointegration relation between individual and aggregate log earnings.

[21] However, one should not make too much of this difference. The Holbrook and Stafford study reports a relationship for panel data; the Juhn and Murphy study is for cross-sectional data.

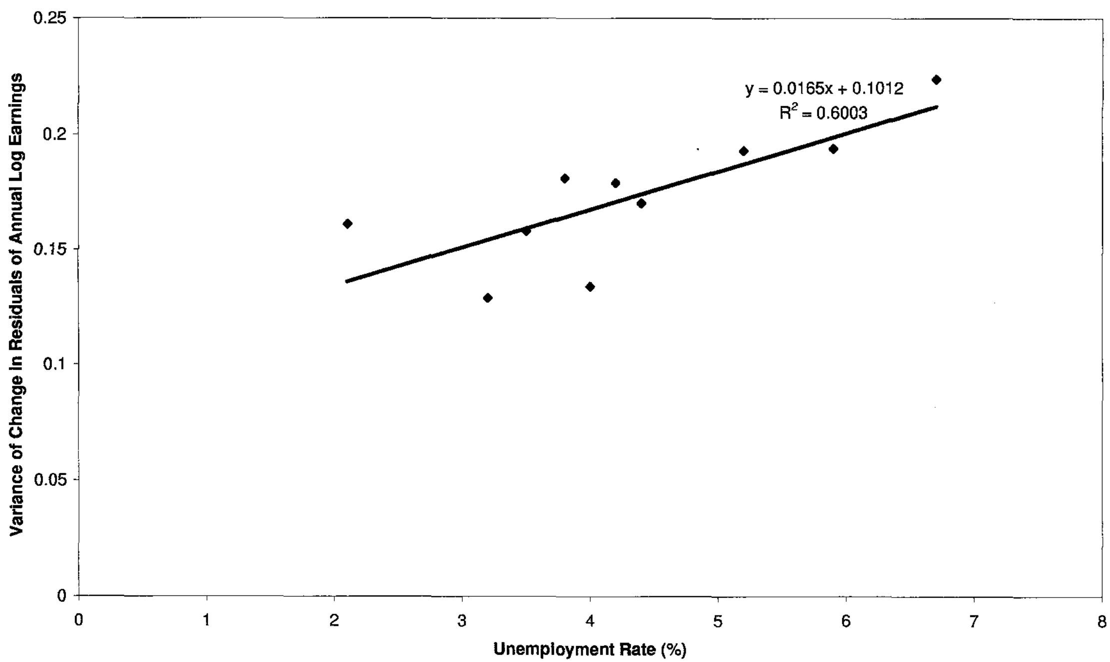

Fig. 1.3. Variance of change in residuals of log earnings plotted against unemployment rate for the Year PSID Data: males, 1969–1979; includes low income oversample. Data from Abowd and Card (1989), Table IV.

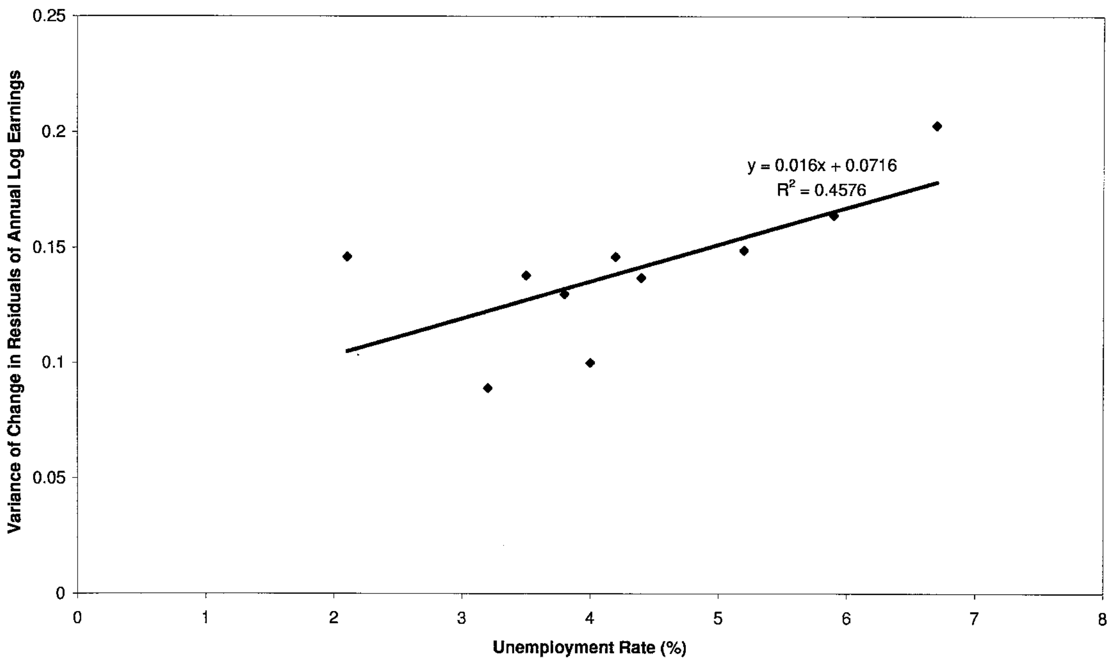

Fig. 1.4. Variance of change in residuals of log earnings plotted against unemployment rate for the Year PSID Data: males, 1969–1979. Data from Abowd and Card (1989), Table V.

facing households in models designed to track the wealth distribution of the economy. Each income source is likely to have its own component of uncertainty. Further research on this topic would be highly desirable.

1.3.1.2. Missing risks. Most of the microeconomic evidence is based on samples of annual earnings or average hourly wages for continuously working, continuously married males. Risks associated with long-term job loss due to job displacement, illness or marital disruption are typically ignored. On these grounds the estimated innovation variances to narrowly defined earning processes are likely to *understate* the risks confronted by agents. Many labor force and life-cycle sources of risk are abstracted from in these studies. Carroll (1992), Hubbard, Skinner and Zeldes (1994), and Lillard and Weiss (1997) make this point, and estimate additional components of uncertainty.

1.3.1.3. Statistical decompositions. Statistical decompositions of wage and earnings processes are intrinsically uninformative about the information available to economic agents. As in Friedman (1957), all of the components of Equations (1.16) or (1.17) could be known and acted on by agents. Estimated innovation variances include measurement error components, factors known to the agents and unknown to the econometrician, and true components of uncertainty. On these grounds, the estimates of the variances in empirical earnings equations likely *overstate* the true uncertainty facing agents.

To demonstrate the value of the cross-equation restrictions connecting consumption and earnings in identifying the innovation in earnings, consider the following example based on the permanent income model of consumption Suppose that the first difference of the level of labor income (earnings) e_t evolves according to[22]

$$\Delta e_t = \varphi + \sum_{j=0}^{\infty} \alpha_j \cdot \eta_{t-j},$$

where $\{\eta_t\}$ is a non-degenerate stationary, multivariate martingale difference sequence with a finite second moment. We impose as a normalization that the covariance of η_t is the identity matrix. The α_j are vectors of moving-average coefficients. The martingale difference sequence $\{\eta_t\}$ is adapted to the sequence of information sets available to the consumer. The multiple components are introduced into η_t to capture the multiple sources of uncertainty in, say, household income.[23]

For simplicity, we assume the preferences for consumption are quadratic, as in Flavin (1981) or Hansen (1987), and that the real interest r is constant[24]. Then from those

[22] This specification includes ones used by Friedman in his Ph.D. thesis, see Friedman and Kuznets (1945). In contrast to the processes fit by MaCurdy (1982), this specification is depicted in terms of first differences of income levels instead of first differences of logarithms.

[23] For a related discussion, see Blundell and Preston (1998).

[24] See Hansen (1987) for a general equilibrium interpretation of this model.

analyses we know that the change in consumption from date $t-1$ to t, $c_t - c_{t-1}$, is just the change in the flow of discounted current and future income from t to $t-1$:

$$c_t - c_{t-1} = \left[\sum_{j=0}^{\infty} \left(\frac{1}{1+r} \right)^j \alpha_j \right] \cdot \eta_t. \tag{1.18}$$

Not only is consumption a martingale as noted by Hall (1978), but the composite income–consumption process must be present-value neutral [see Hansen, Roberds and Sargent (1991)]. That is, consumption and income must respect a present-value budget constraint for all realizations of the shock vector η_t. Relation (1.18) respects this constraint by taking account of the fact that the shock η_t alters income in future time periods.

Even in the absence of measurement error in consumption and income, the entire innovation vector η_t cannot necessarily be identified by an econometrician using data on income alone. For instance, if η_t has more than one entry, fitting a univariate time series process to income will not reveal this vector of new information pertinent to the consumer. By looking instead at consumption, η_t can at least be partially identified because consumption is a martingale adapted to the true information set of the consumer. It follows from Equation (1.18) that the first-difference in consumption reveals one linear combination of the shock vector η_t[25].

Suppose now that measured earnings are

$$e_t^* = e_t + \nu_t,$$

where the measurement error $\{\nu_t\}$ is mean zero, finite variance and is independent of the process $\{\eta_t\}$, but the serial correlation properties of the $\{\nu_t\}$ are unrestricted. Then from the autocovariances of $\{\Delta e_t^*\}$ one cannot identify the moving-average coefficients (the α_j) even if $\{\eta_t\}$ is a scalar process. Thus the strategy of estimating the variance of the innovations in information from the covariances of the error processes of measured earnings equations, as used by Carroll (1992), Hubbard, Skinner and Zeldes (1994) and others, fails, because it is impossible to separate the measurement error from earnings innovations. Again consumption data is informative because agents respond to innovations in earnings but not to the measurement error.

Suppose that consumption is also contaminated by measurement error. Thus we write

$$\Delta c_t^* = \left[\sum_{j=0}^{\infty} \left(\frac{1}{1+r} \right)^j \alpha_j \right] \cdot \eta_t + \upsilon_t,$$

where $\{c_t^*\}$ is measured consumption and $\{\upsilon_t\}$ is a measurement error process that is independent of $\{\eta_t\}$ and $\{\nu_t\}$. We may now use the cross covariances between the

[25] Hansen, Roberds and Sargent (1991) use this result and generalizations of it to deduce testable implications of present value budget balance.

$\{\Delta c_t^*\}$ and $\{\Delta e_t^*\}$ to obtain at least partial identification of the income information structure confronting the consumer. This follows from the formula

$$\operatorname{Cov}(\Delta e_{t+k}^*, \Delta c_t^*) = \alpha_k \cdot \left[\sum_{j=0}^{\infty} \left(\frac{1}{1+r}\right)^j \alpha_j\right],$$

which uses the fact that η_t has an identity as its covariance matrix and is valid for $k \geqslant 0$. Given prior knowledge of the constant interest rate r, we may use this formula and formula (1.18) to deduce the variance of the Δc_t:

$$\begin{aligned}\operatorname{Var}(\Delta c_t) &= \left[\sum_{k=0}^{\infty} \left(\frac{1}{1+r}\right)^k \alpha_k\right] \cdot \left[\sum_{j=0}^{\infty} \left(\frac{1}{1+r}\right)^j \alpha_j\right] \\ &= \sum_{k=0}^{\infty} \left(\frac{1}{1+r}\right)^k \operatorname{Cov}(\Delta e_{t+k}^*, \Delta c_t^*).\end{aligned}$$

From these calculations then we may again infer how the true income process responds to a linear combination of the η_t shock vector[26]. If the $\{\eta_t\}$ process is scalar, then we have full identification of the information structure confronting the consumer that is pertinent for the evolution of income. Thus the use of consumption data, even if measured with error, can help to identify the true income uncertainty confronting economic agents.

We conclude this subsection with a brief discussion of the sources of market incompleteness.

1.3.2. Limited commitment and private information

In our incomplete markets model, we made no attempt to justify the form of market incompleteness. Two common justifications include problems of enforceability and observability. Measurements of microeconomic uncertainty are critical ingredients to economies that explicitly account for limited commitment and private information.

1.3.2.1. Limited commitment. Kehoe and Levine (1993) and Kocherlakota (1996) propose the following alternative to the incomplete markets models we have considered thus far. Suppose consumers are permitted to walk away from obligations, but when they do so they are excluded from future participation in markets. Instead they are restricted to use their own backyard storage technologies or simply consume only their

[26] Hansen, Roberds and Sargent (1991) use a closely related argument to show that present-value budget balance has testable implication when consumption is a martingale. They do not, however, explore the ramifications of measurement error in consumption and income.

labor income. As a consequence, in equilibrium, consumers are guaranteed a lower bound on their discounted utilities at each date and state. Consumption allocations are obtained by solving Pareto problems subject to utility lower bounds implied by the utility threat points: the points at which consumers are indifferent between honoring their obligations and defaulting. While there is no default in equilibrium, allocations are altered by the presence of the utility threat points.

Alvarez and Jermann (1999) consider an economy with limited commitment in which consumers have no access to backyard storage technologies and hence are punished by constraining future consumption to equal future income, period by period. They show that the resulting allocations may be decentralized by introducing person-specific solvency or borrowing constraints[27]. Euler equations are replaced by Euler inequalities as in Luttmer's (1996) work on asset pricing when investors face solvency constraints. Alvarez and Jermann (1999) are able to imitate asset pricing features of an economy with solvency constraints, but with different predictions about when agents will be up against financial market constraints. Individual Euler equations linking consumption to asset prices continue to characterize the behavior of some individuals in each time period. When and which consumers are constrained in their ability to borrow can be ascertained by computing the utility threat points, which in turn depend on the microeconomic uncertainty they would be forced to confront in the absence of risk sharing. Thus the same problem of measuring uncertainty discussed for economies with incomplete markets applies to economies in which limited commitment is the only source of financial market frictions.

1.3.2.2. Private information. In the limited commitment economies considered by Kehoe and Levine (1993) and by Alvarez and Jermann (1999), idiosyncratic endowment shocks are publicly observed. Suppose instead that they are only known to the individual agents and not to the public. Again, the options and information available to agents matter. For instance, suppose that a capital accumulation technology is only available to the society as a whole and not to individuals. In this case, individual consumption contracts can be enforced because the agents are unable to privately transfer consumption from one period to the next. There is a substantial body of work on optimal resource allocation subject to incentive constraints that relies on the enforceability of consumption contracts [see Green (1987), Phelan and Townsend (1991), Atkeson and Lucas (1992) among others]. In general, the efficient allocations do not have decentralizations that look like the incomplete market structure previously described. Economies with a simple security market structure are not Pareto efficient even after accounting for the incentive constraints [e.g., see Lucas (1992)]. Of course, security markets could be supplemented by other institutions designed to reduce or

[27] The person-specific nature of the solvency constraint stretches a bit the notion of a decentralized economy. As an alternative, we may view these limited commitment economies as (constrained) efficient benchmarks to which we might compare alternative institutional arrangements.

eliminate the efficiency wedge. In contrast, when a capital accumulation technology is privately available, individual agents can hide their consumption from the public. Thus individual consumption contracts are no longer enforceable. For some special versions of these environments, Allen (1985) and Cole and Kocherlakota (1997) show that the incomplete security market economy we described previously fully decentralizes the Pareto efficient allocations subject to incentive constraints. Even when there is an efficiency wedge, the specification of microeconomic uncertainty is a critical ingredient in both the decentralized economy and in the Pareto efficient economy subject to incentive constraints. The problems of measuring microeconomic uncertainty arise in private information economies as well.

2. Overlapping generations model

2.1. Motivation

The overlapping generations model (OLG), now widely used in macroeconomics and public finance, captures heterogeneity among cohorts, something not captured by the models considered in Section 1. Thus it provides valuable information on cohort-specific consequences of economic policies and of macro disturbances. Influential versions of the OLG model by Auerbach and Kotlikoff (1987) and Fullerton and Rogers (1993) are widely used to evaluate a variety of policy reforms, including tax and social security policies.

The versions of this model that are widely used are based on perfect foresight. A major computational advantage of this form of the OLG model is that it is relatively straightforward to calculate transitional paths for high dimensional specifications, as demonstrated by Greenwood and Yorukoglu (1997). This is in contrast with the stochastic growth model with incomplete markets, where computation with high dimensional state vectors is still a very difficult problem. However, perfect foresight is a strong assumption. Huggett (1996) considers a version of an OLG model with idiosyncratic uncertainty, but he only analyzes steady states.

Most empirical implementations of the OLG model ignore decisions to accumulate human capital by taking skills to be exogenous endowments. On the other hand, human capital is a more substantial component of total wealth than physical capital. For this reason, we present a generalization of the Auerbach–Kotlikoff overlapping generations model due to Heckman, Lochner and Taber (1998) (HLT) that incorporates human capital, in addition to the physical capital that is the centerpiece of the Auerbach–Kotlikoff analysis. We consider the evidence required to empirically implement this and other versions of the OLG model using either micro or macro data.

We use our exposition of this extended version of the OLG model to consider the benefits of introducing richer forms of heterogeneity in skills and human capital production technologies and to consider choices at both the intensive and extensive margins. Schooling decisions are made at the extensive margin. Moreover, investment in schooling and on-the-job training are distinct activities and produce distinct skills, and output is produced by a skill mix supplied by different agents. The following facts motivate our choice of models:

- Empirical evidence in labor economics demonstrates that comparative advantage in factor markets is the rule and that different persons specialize in different skills [see Sattinger (1993)]. Widely-used efficiency units corrections that collapse skill to a single dimension do not explain either cross-section wage distributions or the evolution of wage distributions over time.
- There are substantial cohort effects in earnings functions.
- Heterogeneity in saving behavior and wealth accumulation helps in fitting OLG models to data. Life-cycle models with preference homogeneity in each cohort do not generate enough dispersion in savings to explain the observed capital holdings.

To accommodate these facts, we follow HLT by embedding an extension of the model of Ben Porath (1967) of individual human capital production into a general equilibrium setting. In Ben-Porath's model, in the stochastic growth model discussed in Section 1, and in the Auerbach–Kotlikoff model or the Fullerton–Rogers model, wage inequality can only be generated by differences in amounts of a common skill (the so-called "efficiency units" model). All skill commands the same price. In the model considered by HLT and in this part of our chapter, different levels of schooling enable individuals to invest in different skills through on-the-job training in the post-schooling period. In the aggregate, the skills of different schooling groups are not perfect substitutes[28]. Within schooling groups, however, persons with different amounts of skill are perfect substitutes[29]. For our purposes, this model provides us with a useful platform for integrating empirical results from labor economics into a general equilibrium, macroeconomic model. Before turning to the details of the HLT model, it is fruitful to place their work in the context of current models of earnings determination widely used in the literature.

2.2. Economic models of earnings and human capital investment

Three different frameworks for interpreting wage and earnings functions coexist in the literature on empirical labor economics. Their coexistence in the literature is a constant source of confusion. The only agreement among users of these alternative frameworks is their agreement to call the coefficient on schooling in a log wage equation a "rate of return", although the conditions required to do so are known to be restrictive[30].

[28] This specification is consistent with evidence that the large increase in the supply of educated labor consequent from the baby boom depressed the returns to education. [See Freeman (1976), Autor, Katz and Krueger (1997) and Katz and Murphy (1992).]

[29] This specification accords with the empirical evidence summarized in Hamermesh (1993, p. 123) that persons of different ages but with the same education levels are highly substitutable for each other.

[30] In order to interpret the schooling coefficient as an *internal rate of return*, it is necessary to assume an environment of perfect certainty and to further assume that earnings are multiplicatively separable

The first two frameworks are scalar and vector attribute pricing equations, respectively. The third framework accounts for personal investment and the estimated coefficients on attributes of fitted earnings functions are *not* prices although they are often interpreted to be so.

The most widely used empirical framework for wages in aggregative analysis writes the wage w of person i (w_i) as the product of the price R of labor services denominated in "efficiency units" with the quantity of efficiency units Q_i embodied in the agent:

$$w_i = RQ_i, \tag{2.1}$$

where Q_i is a function of characteristics of individuals such as education and work experience. In this framework, $R = F_{\bar{Q}}(\bar{Q}, \bar{K})$ where F is the aggregate production function and $\bar{K}$ is the aggregate capital stock and $\bar{Q} = \sum_{i=1}^{N} Q_i$ where N is the number of agents working in the economy.

Equation (2.1) underlies the calibrated real-business cycle models of Kydland (1984, 1995) and a variety of microeconomic studies. [See the one-sector model in Heckman and Sedlacek (1990) or the aggregative models of Stokey and Rebelo (1995), Lucas (1988), Uzawa (1965) and Caballe and Santos (1993).] In logs, Equation (2.1) has the strong implication that aggregate shocks operate only through intercepts of log wage equations. This framework cannot explain the well-documented rise in inequality in means among skill classes or increases in variances of log wage equations that are central features of many modern economies [see, e.g. Katz and Murphy (1992)] unless quality changes within skill groups. Since the mean of log wages and the variance of log wages increase greatly over short periods of time for groups with stable skill characteristics, such an explanation of rising wage inequality is implausible. It is tested and rejected in the study of Katz and Murphy (1992).

Real wages respond to aggregate unemployment (see Table 2.1). However, the response to local unemployment rates varies by gender, education and age (see Table 2.2). The wages of unskilled persons are much more cyclically sensitive to local unemployment rates. This evidence does not support a one-dimensional efficiency units model.

In addition, an efficiency units model is inconsistent with the well-documented evidence on comparative advantage in the labor market. [See Heckman and Sedlacek (1985), or Sattinger (1993).] A model in which persons choose sectors based on their comparative advantage in them explains the distribution of US wage data whereas a model with efficiency units does not [Heckman and Sedlacek (1990)]. Finally, the evidence reported in Topel (1986) and Heckman, Layne-Farrar and Todd (1996) demonstrates that the wages of different skill groups respond differently to local labor market shocks. This is inconsistent with efficiency units wage specification (2.1).

in schooling and experience and that all persons of all education levels have the same post-school experience profiles. See Heckman and Klenow (1997) and Heckman, Lochner and Taber (1999) for a derivation.

Table 2.1
Elasticity of real wages with respect to aggregate unemployment: first-differenced panel data

Author	Data Set	Elasticity	(S.E.)
Bils (1985)	NLS young men (whites), 1966–1980 10 changes	–0.089	(0.019)
Rayack (1987)	PSID males (whites), 1968–1980 12 changes	–0.081	(0.016)
Blank (1990)	PSID males (whites), 1969–1982 13 changes, pairwise balanced	–0.081 [a]	(0.043)
Solon, Barsky and Parker (1994)	PSID males, 1967–1987 20 changes	–0.085	(0.022)

[a] Blank (1990) regresses change in real wages on percentage change in real GNP. Her estimate is transformed to unemployment elasticity using an estimated "Okun" coefficient of 0.30.

An alternative specification of the wage equation that meets some of these objections to the efficiency units wage model is the Gorman–Lancaster model of earnings. [Welch (1969), applies this model.] In this framework, the wage of person i is a function of characteristics embodied in the person times their price. Letting A_i be a $J \times 1$-dimensional vector of attributes for person i and R_A a $1 \times J$-dimensional vector of their economy wide prices,

$$w_i = R_A A_i. \tag{2.2}$$

The prices are determined by the aggregate production function using the aggregates

$$\bar{A}_j = \sum_{i=1}^{N} A_{ij} \quad j = 1, \ldots, J \qquad \text{and} \qquad R_j = F_j(\bar{A}_1, \ldots, \bar{A}_J, \bar{K}),$$

where R_j is the jth entry of R_A. In principle, this model can explain rising wage inequality in log earnings if aggregate shocks, or local labor market shocks, do not operate uniformly on the arguments of the production function, $\bar{A}_j$, $j = 1, \ldots, J$, and persons possess more than one type of skill. Equation (2.2) is inconsistent with the evidence on comparative advantage in the labor market, however. Heckman and Scheinkman (1987) demonstrate that for the US economy skill prices R_A are not uniform across different sectors for a variety of different types of sectors. Since prices are not uniform, a model of sectoral self-selection and comparative advantage in the labor market is required. Heckman and Sedlacek (1985, 1990) find that such a model better explains both the cross section and time series of the US wage distribution than do efficiency units models. The usual least squares estimates of wage equations reported in the literature do not estimate skill prices unless analysts account for self selection across sectors where skill prices are different.

Table 2.2
Elasticities of wages, hours, and earnings with respect to state unemployment rates[a]

Category	Hourly wage		Annual hours		Annual earnings	
	Actual (1)	Adjusted (2)	Actual (3)	Adjusted (4)	Actual (5)	Adjusted (6)
1. All	–0.07	–0.08	–0.11	–0.12	–0.18	–0.20
	(0.02)	(0.02)	(0.01)	(0.01)	(0.02)	(0.02)
2. By Gender						
a. Women	–0.06	–0.06	–0.08	–0.09	–0.14	–0.16
	(0.02)	(0.02)	(0.02)	(0.02)	(0.03)	(0.03)
b. Men	–0.08	–0.09	–0.13	–0.15	–0.21	–0.24
	(0.02)	(0.02)	(0.01)	(0.01)	(0.02)	(0.02)
3. By education						
a. <12 years	–0.04	–0.06	–0.14	–0.19	–0.18	–0.25
	(0.03)	(0.02)	(0.04)	(0.03)	(0.05)	(0.04)
b. 12–15 years	–0.09	–0.09	–0.13	–0.13	–0.22	–0.23
	(0.02)	(0.02)	(0.02)	(0.01)	(0.02)	(0.02)
c. 16+ years	–0.01	–0.05	–0.02	–0.06	–0.03	–0.12
	(0.02)	(0.02)	(0.02)	(0.02)	(0.03)	(0.03)
4. By age						
a. Age 16–29	–0.12	–0.13	–0.16	–0.18	–0.28	–0.31
	(0.02)	(0.02)	(0.02)	(0.02)	(0.04)	(0.03)
b. Age 30–44	–0.06	–0.05	–0.10	–0.10	–0.16	–0.15
	(0.02)	(0.02)	(0.01)	(0.01)	(0.03)	(0.03)
c. Age 45-65	–0.03	–0.03	–0.06	–0.07	–0.09	–0.10
	(0.02)	(0.02)	(0.02)	(0.02)	(0.03)	(0.03)
5. By number of employers last year						
a. One	–0.07	–0.07	–0.10	–0.11	–0.16	–0.18
	(0.02)	(0.02)	(0.01)	(0.01)	(0.02)	(0.02)
b. Two or more	–0.14	–0.14	–0.20	–0.21	–0.34	–0.35
	(0.03)	(0.03)	(0.02)	(0.02)	(0.04)	(0.04)

[a] Source: Card (1995).

Standard errors are given in parentheses. Table entries are elasticities of variables indicated in column headings with respect to state unemployment rate. Estimates are based on 51 state observations for 1979, 1982, 1985, 1988, and 1991. Unadjusted data are means of log hourly wages, log annual hours, and log annual earnings for each state-year cell. Adjusted data are means of regression-adjusted wages, hours, and earnings. All models include state and year dummies.

A third specification of the earnings equation is the "Mincer model" that is widely cited as a rationale for wage equations in empirical labor economics. Actually there are two Mincer models that are algebraically similar but that are economically distinct. The first Mincer model (1958) assumes that everyone is alike, the economy is stationary and that persons live forever. However, different levels of schooling are associated with different skill levels. Mincer's model is an equalizing-differentials or arbitrage pricing model for the lifetime permanent wage of a person of schooling S, where r is an externally determined interest rate:

$$\frac{w(S)\,\mathrm{e}^{-rS}}{r} = w(0), \tag{2.3}$$

and $w(0)$ is the benchmark no-schooling wage. Thus $\ln w(S) = \ln[rw(0)] + rS$. Because people are alike, allocations to schooling are demand driven via the aggregate production function defined on the aggregate stocks of skills

$$\bar{S}(j) = N_j S_j \quad j = 1, \ldots, J,$$

where N_j is the number of persons in schooling group j, S_j is the schooling level of persons in schooling group j, $w(S_j) = F_j(\bar{S}(1), \ldots, \bar{S}(J), \bar{K})$, the marginal product of schooling at level j. The evidence against the equalizing differentials model is overwhelming [see, e.g. Murphy and Topel (1987)] and for that reason Equation (2.3) is not useful as a framework for interpreting earnings data.

A second Mincer model (1974) is widely appealed to although it is not well understood. It is an accounting identity that writes, in discrete time, potential earnings at age a as $E(a)$:

$$E(a) = \sum_{j=0}^{a-1} r(j)\,C(j) + E(0), \tag{2.4}$$

where $C(j)$ is the cost of investment in period j, $r(j)$ is the average return on investment in period j, and $E(0)$ is initial earnings potential. Schooling is defined as occurring in periods in which all potential earnings are invested ($C(j) = E(j)$). After schooling, $C(j) < E(j)$. Let $k(j)$ be the fraction of potential earnings invested, $k(j) = 1$ during schooling and $k(j) < 1$ afterward.

Observed earnings at age a, $w(a)$, reflect investment because part of the potential earnings are invested:

$$w(a) = E(a)[1 - k(a)]. \tag{2.5}$$

Mincer's distinction between observed earnings and potential earnings is an important insight especially for young persons. Even if $E(a)$ can be written as the product of prices and attributes as in specifications (2.1) or (2.2), $w(a)$ cannot, and coefficients on attributes do not identify measured attribute prices.

Mincer assumes that schooling is exogenously determined and in post-school periods τ, $0 \leqslant \tau \leqslant \Upsilon$, where $\Upsilon + S = \bar{a}$, the total length of working and schooling life and τ is work experience,

$$k(\tau) = \left(1 - \frac{\tau}{\Upsilon}\right) k(0). \tag{2.6}$$

This relationship is assumed to be the same at all schooling levels. Heckman, Lochner and Taber (1998) test and reject this specification in US data for both males and females. Collecting results, a simple recursion shows that

$$\ln w(\tau) = \alpha_0 + \alpha_1 S + \alpha_2 \tau + \alpha_3 \tau^2, \tag{2.7}$$

where α_1 is the average rate of return to schooling and α_2 and α_3 are functions of $k(0)$, Υ and the average rate of return to post-school investment[31]. Schooling is determined outside the model, and there is no direct relationship between the earnings function and the schooling equation.

The coefficients α_0, α_1, α_2 and α_3 are reduced-form coefficients in the sense of Marschak (1953). They are not the policy invariant structural parameters of Lucas and Sargent (1981). As preferences, technology and policies change, so do these coefficients. Heckman, Lochner and Taber (1998) demonstrate the empirical importance of this point when they estimate a structural earnings equation and test it against the Mincer model. They demonstrate how the coefficients of the Mincer model change in response to changes in interest rates, skill prices and aggregate shocks. In addition, the assumption that $k(0)$ does not depend on S, which is central to Mincer's claim that log earnings profiles are parallel in experience across schooling groups, although not parallel in terms of age, receives limited empirical support in the recent US data [Heckman and Todd (1997)]. At a purely empirical level, Equation (2.7) omits an important interaction between schooling and work experience.

2.2.1. The Ben-Porath framework

The model of Ben Porath (1967) provides a more rigorous formulation of the earnings equation that combines a theory of earnings with a theory of schooling and on-the-job training. It accounts for Mincer's important insight that measured earnings are less than potential earnings so that earnings functions are not just pricing functions for observed characteristics. The Ben-Porath model assumes income maximizing agents who live $\bar{a}$ periods, who face parametric interest rate r and no credit constraints. The

[31] More precisely, $\alpha_1 = r_s$, the "average rate of return to schooling"; $\alpha_2 = r_p k_0(0)[1 + (1/2\Upsilon)] + k_0(0)/\Upsilon$ and $\alpha_3 = -r_p k(0)/2\Upsilon$, where r_p is the average rate of return on all post-school investments. Υ is the length of life spent working after schooling and $k(0)$ is defined in the text. Given α_2, α_3 and Υ, r_p and $k(0)$ are identified from a least squares regression assuming that the error term is orthogonal to the regressors.

model is an efficiency units model that writes the potential wage, E, for a person with human capital H at age a in period t as

$$E(H, a, t) = R(t)H(a),$$

where $R(t)$ is the marginal product of *aggregate* human capital. Versions of this model are widely used in modern growth theory. [See, e.g. Lucas (1988), Stokey and Rebelo (1995), and the references listed below.]

Let $I(a, t)$ denote the proportion of work time spent investing at age a in period t, where 1 is the maximum amount of time available and $D(a, t)$ is the tuition cost of investment or goods paid to receive training, with price $P_D(t)$. Let $F(I(a, t), H(a, t), D(a, t))$ be the production function of human capital for individuals which is assumed to be concave in I and D for fixed H. The net earnings of the individual are

$$w(a, t) = R(t)H(t)[1 - I(a, t)] - P_D(t)D(a, t). \tag{2.8}$$

Ignoring taxes, and for simplicity assuming a stationary economy, the agent maximizes earnings over the life cycle and solves

$$\begin{aligned} &\max_{I(a), D(a)} \int_0^{\bar{a}} \mathrm{e}^{-ra} \{RH(a)[1 - I(a)] - P_D D(a)\} \, \mathrm{d}a \\ &\quad \text{subject to } \dot{H}(a) = F(I(a), H(a), D(a)) - \sigma H(a), \end{aligned} \tag{2.9}$$

and initial condition $H(0)$ where σ is the rate of depreciation on human capital[32]. Let μ be the multiplier associated with the dynamic constraint. In this framework schooling occurs when $I = 1$. If $\sigma = 0$, schooling occurs only once, at the beginning of life, if it occurs at all. [See Weiss (1986).] Schooling ends at age a^* where $I(a^*) = 1$ and

$$\mu(a^*)F_1(I(a^*), H(a^*), D(a^*)) = RH(a^*), \tag{2.10}$$

so if "0" is the beginning of life, a^* is also the length of time spent in school.

The functional form for F that is most commonly used in the literature writes

$$\dot{H} = AI^{\alpha}H^{\beta} - \sigma H. \tag{2.11}$$

In the post-school period, measured earnings are $RH(a)[1 - I(a)] - P_D(a)D(a)$ if the goods costs of investment are subtracted from earnings. Otherwise, measured earnings

[32] Strictly speaking the Ben-Porath model writes $F = F(I(a)H(a), D(a))$, but the more general specification is a minor extension of this technology. This technology is sometimes called a "neutrality" model because human capital accumulation raises the marginal cost of human capital investment and the marginal productivity of human capital investment in the same proportion.

are $RH(a)[1 - I(a)]$. The Ben-Porath model provides a theory of $I(t)$ that is missing in Mincer's analysis[33]. It recognizes the distinction between potential and actual earnings. It tightly links schooling and on-the-job training decisions through the human capital production function and explicitly links those decisions to the earnings function. It is a theory with testable cross-equation restrictions once the functional form of F is specified. It provides a framework for testing the relationship among earnings, schooling and training dynamics and in this regard is a scientific success.

It is important to distinguish the investment behavior generated by the Ben-Porath model from that produced from a learning-by-doing model of the sort developed in Heckman (1971), Shaw (1989) or Altug and Miller (1990, 1998). In the common form of that model, an hour of work experience in any sector of the economy produces the same growth in wages. Investment time and work time in final goods production are bundled. This model is based on an efficiency units assumption that is inconsistent with the evidence on comparative advantage in labor markets in modern economies. In particular, the model abstracts from comparative advantage in the labor market both in the production of output and in the production of skills.

Implicit in the model is a "free lunch" assumption: since learning is uniform across sectors, there is no cost of learning, unlike what is assumed in the Mincer or Ben-Porath models. This is a consequence of the Leontief assumption that bundles work and investment *in the same proportion* in all sectors. Once heterogeneity in learning opportunities across sectors is recognized, and the Leontief assumption is relaxed, the "learning by doing model" becomes similar to the Ben-Porath model and learning becomes a costly activity rather than a free lunch. Cossa, Heckman and Lochner (1998) discuss these points and develop a test between the two specifications.

Estimates of this technology for discrete-time versions of Equation (2.11) are reported in Table 2.3. Until recently, all the estimates reported in the literature were for males and assumed neutrality (so $F = F(IH, D)$). Only Heckman (1976) and Rosen (1976) estimate non-neutral Ben-Porath models. Rosen (1976) imposes a different restriction on the model [$\alpha = \frac{1}{2}$, $\beta = 1$ in the notation of Equation (2.11)]. Heckman (1976) allows α and β to be freely specified. The recent analysis of Heckman, Lochner and Taber (1998) estimates an unrestricted model for males and females for different education and ability groups. See Table 2.4 for a summary of their estimates for each gender and ability group. They are not able to reject the hypothesis that a neutral Ben-Porath model ($\alpha = \beta$) describes the human capital accumulation process for persons of different ability and education groups.

There are several important limitations of the original Ben-Porath framework. First, like model (2.1), the Ben-Porath model is based on an efficiency units assumption for the labor market. There is no room for the operation of comparative advantage in such labor markets. The market affects skill prices identically at all levels of human

[33] Mincer simply postulates the functional form of $k(t) = [I(a)\,RH(a) + P_D D(a)]/RH(a)$, assuming that earnings are net of both types of cost.

Table 2.3
Estimates of the human capital production function (males)[a]

Source	α	β	γ	A	r	σ	Restricted schooling and OJT model?	Labor supply	Synthetic cohorts?
Heckman (1976) two models	0.99 (.003)	−6.69 (.043)	–	45.49 (3.034)	0.10 (imposed)	0.0016 (0.00025)	No	Yes	Yes
US Bureau of the Census (1960) males	0.67 (0.052)	0 (imposed)	–	0.14×10^{-2} (0.04×10^{-2})	0.10 (imposed)	0 (constrained)	No	Yes	Yes
Heckman (1976)	0.812 (0.0225)	α[b] (restricted)	–	1.53 (1.62)	0.176 (0.275)	0.089 (0.068)	No	No	Yes
US Bureau of the Census (1960) males	0.52 (0.07)	α[b] (restricted)	–	17.3 (25.2)	0.196 (0.613)	0.037 (0.90)	No	Yes	Yes
Haley (1976) CPS (1956–1966) aggregates	0.578 (0.012)	α[b] (restricted)	–	0.019–0.04	0.04–0.069 (0.004) (0.003)	0.005–0.04 (0.014) (0.008)	No[c]	No	Yes
Brown (1976)[d] NLS young men	0.56–0.89	α[b] (restricted)	–	[f]	0.33–0.15	0 (imposed)	No[e]	No	No
Rosen (1976) US Census 1960 and 1970	0.5	1 (imposed)	–	$r+\varepsilon$ ($\varepsilon>0$) (see next column)	0.0725 (highschool) 0.0875 (college)	176 (0.275)	No	No	Yes

[a] $H_{t+1} = (1-\sigma)H_t + AI_t^{\alpha}H_t^{\beta}D_t^{\gamma}$.
r, interest rate; standard errors are given in parentheses.
[b] $\alpha=\beta$.
[c] All schooling groups.
[d] Brown makes alternative assumptions about the rate of growth of the price of labor services. See also Rosen.
[e] Only highschool graduates.
[f] Not reported.

Table 2.4
Estimated parameters for human capital production function[a]

Parameter	Estimated value			
	Males		Females	
	Highschool ($S=1$)	College ($S=2$)	Highschool ($S=1$)	College ($S=2$)
α	0.945(0.017)	0.939(0.026)	0.967	0.968
β	0.832(0.253)	0.871(0.343)	0.810	1.000
$A(1)$	0.081(0.045)	0.081(0.072)	0.079	0.057
$H_0(1)^*$	9.530(0.309)	13.622(0.977)	6.696	8.347
$A(2)$	0.085(0.053)	0.082(0.074)	0.082	0.057
$H_0(2)^*$	12.074(0.403)	14.759(0.931)	7.806	9.453
$A(3)$	0.087(0.056)	0.082(0.077)	0.084	0.058
$H_0(3)^*$	13.525(0.477)	15.614(0.909)	8.777	11.563
$A(4)$	0.086(0.054)	0.084(0.083)	0.086	0.058
$H_0(4)^*$	12.650(0.534)	18.429(1.095)	9.689	13.061

[a] Source: Heckman, Lochner and Taber (1998), Table 1.
Human capital production: $H_{a+1}^S = A^S(\theta)(I_a^S)^{\alpha_s}(H_a^S)^{\beta_s} + (1-\sigma)H_a^S$, with $S = 1, 2$. Standard errors are given in parentheses.
[b] Heckman, Lochner and Taber (1999) do not report the standard errors for females.
[c] Initial human capital for person of ability quantile using ability levels from NLSY.

capital since in an efficiency units model, different amounts of human capital represent different amounts of the same skill. Only if agents at different skill levels invest differently in response to common aggregative shocks will means and variances of measured earnings vary across skill groups. [Heckman, Lochner and Taber (1998) present empirical evidence on this issue.] Second, in smooth Ben-Porath problems where F is strongly concave in I, the proportion of time investing, $I(a)$, gradually declines from 1 in the post-school period. Thus, the model predicts that earnings do not jump after the end of schooling but gradually increase from 0, a phenomenon not actually observed in the data. For all of these reasons, the Ben-Porath model is not consistent with the available evidence on life-cycle labor earnings. Heckman, Lochner and Taber (1998) extend the model, decouple the schooling decision from on-the-job training investment, and present a model in which earnings jump to a substantial positive number upon completion of schooling.

An additional problem with the Ben-Porath model is that it makes no distinction between human capital as an input that facilitates subsequent learning and human capital as a direct productive service used to produce market goods. At an intuitive level, schooling serves both purposes but the human capital acquired on the job affects subsequent learning differently than the general human capital acquired at school. We next turn to the framework of Heckman, Lochner and Taber (1998) that solves these problems with the Ben-Porath model and enables them to develop an empirically concordant model of earnings that provides a framework for analyzing labor markets with heterogeneous human capital and comparative advantage.

2.2.2. *The HLT model of earnings, schooling and on-the-job training*

HLT extend the Ben-Porath framework in several ways to account for the central facts of modern labor markets. (1) It distinguishes between schooling capital and job training capital at a given schooling level. Schooling capital is an input to the production of capital acquired on the job but the tight link between schooling and on-the-job training investments embodied in Equation (2.10) is broken. Earnings can now jump after completion of schooling instead of gradually increasing from zero as occurs when the two types of human capital are assumed to be the same. (2) Among persons of the same schooling level, there is heterogeneity both in initial stocks of human capital and in the ability to produce job-specific human capital. (3) Skills produced at different schooling levels command different prices, and wage inequality among persons is generated by differences in skill levels, differences in investment, and differences in the prices of alternative skill bundles. (4) There is a labor–leisure choice in addition to a labor–investment choice [34]. (5) The model is embedded in a general equilibrium setting so that the relationship between capital markets and human capital markets at different skill levels is explicitly developed. All of these extensions recognize heterogeneity in skills across persons and emphasize choices at both the intensive and extensive margins. These features of their work undermine the representative agent paradigm. We now present these extensions in greater depth.

2.3. *Structure of the model*

Assuming perfect certainty, we first derive the optimal consumption on-the-job investment, and schooling choices for a given individual of type θ who takes skill prices as given. We aggregate the model to produce a general equilibrium model. Throughout this section, we simplify the tax code and assume that income taxes are proportional [35]. Retirement is mandatory. In the first portion of the life cycle, a prospective student decides whether or not to remain in school. Once he has left school, he cannot return. He chooses the schooling option that gives him the highest level of lifetime utility.

Define $K^S_{a,t}$ as the stock of physical capital held at time t by a person of age a and schooling level S; $H^S_{a,t}$ is the stock of human capital at time t of type S at age a. The optimal life-cycle problem can be solved in two stages. First condition on schooling and solve for the optimal path of consumption ($C^S_{a,t}$), leisure ($L_{a,t}$) and post-school

[34] Heckman (1975, 1976), Blinder and Weiss (1976) and Ryder, Stafford and Stephen (1976) extend the Ben-Porath model to include labor supply.

[35] Heckman, Lochner and Taber (1998) relax this assumption.

investment ($I^S_{a,t}$) for each type of schooling level, S. Total time is normalized to unity, so work time is $h_{a,t} = 1 - L_{a,t} - I_{a,t}$. Individuals then select among schooling levels to maximize lifetime welfare. Given S, an individual of age a at time t has the value function

$$V_{a,t}(H^S_{a,t}, K^S_{a,t}, S) = \max_{C^S_{a,t}, I^S_{a,t}, L^S_{a,t}} U(C^S_{a,t}, L^S_{a,t}) + \delta V_{a+1,t+1}(H^S_{a+1,t+1}, K_{a+1,t+1}, S), \tag{2.12}$$

where U is strictly concave and increasing and δ is a time preference discount factor. This function is maximized subject to the budget constraint

$$K^S_{a+1,t+1} \leqslant K^S_{a,t}[1 + (1-\tau)r_t] + (1-\tau) R^S_t H^S_{a,t}(1 - I^S_{a,t} - L^S_{a,t}) - C^S_{a,t}, \tag{2.13}$$

where τ is the proportional tax rate on capital and labor earnings, R^S_t is the rental rate on human capital of type S, and r_t is the net return on physical capital at time t. We abstract from all activities of government except for taxation.

On-the-job human capital for a person of schooling level S accumulates through the human capital production function

$$H^S_{a+1,t+1} = A^S(\theta)(I^S_{a,t})^{\alpha_S}(H^S_{a,t})^{\beta_S} + (1-\sigma^S)H^S_{a,t}, \tag{2.14}$$

where the conditions $0 < \alpha_S < 1$ and $0 \leqslant \beta_S \leqslant 1$ guarantee that the problem is concave in the control variable, and σ^S is the rate of depreciation of job-S-specific human capital. This functional form is widely used in both the empirical literature and the literature on human capital accumulation. (See the survey of estimates in Tables 2.3 and 2.4.)

For simplicity, we ignore the input of purchased goods into the production of human capital on the job. For an analysis of post-school investment, this is not restrictive as we can always introduce goods and solve them out as a function of $I^S_{a,t}$, thereby reinterpreting $I^S_{a,t}$ as a goods–time investment composite. HLT explicitly allow for tuition costs of college which we denote by $D^S_{a,t}$. The same good that is used to produce capital and final output is used to produce schooling human capital. After completion of schooling, time is allocated to two activities: on-the-job investment, $I^S_{a,t}$, and work, ($h^S_{a,t} = 1 - I^S_{a,t} - L^S_{a,t}$), both of which must be non-negative. The agent solves a life-cycle optimization problem given initial stocks of human and physical capital, $H^S(\theta)$ and K_0, as well as an ability parameter that governs the production of human capital on the job, $A^S(\theta)$.

$H^S(\theta)$ and $A^S(\theta)$ represent ability to "earn" and ability to "learn", respectively, measured *after* completing school. They embody the contribution of schooling to subsequent learning and earning in the schooling-level-S-specific skills as well as any initial endowments. HLT, Auerbach and Kotlikoff (1987) and Fullerton and Rogers (1993) abstract from short-run credit constraints that are often featured in the literature on schooling and human capital accumulation. Their models are consistent with the evidence presented in Cameron and Heckman (1998a,b) that long-run family factors correlated with income (the θ operating through $A^S(\theta)$ and $H^S(\theta)$) affect schooling, but that short-term credit constraints are not empirically important. Such long-run factors account for the empirically well-known correlation between schooling attainment and family income. The mechanism generating the income–schooling relationship is through family-acquired human capital and not credit rationing. The α and β are also permitted to be S-specific, which emphasizes that schooling affects the process of learning on the job in a variety of different ways.

Conditional on the choice of schooling, the following first-order condition governs the model:

$$U^S_{C_{a,t}} = \delta \frac{\partial V_{a+1,t+1}}{\partial K_{a+1,t+1}}, \tag{2.15}$$

$$U_{L^S_{a,t}} \geqslant \delta \frac{\partial V_{a+1,t+1}}{\partial K_{a+1,t+1}} (1-\tau) R^S_t H^S_{a,t}. \tag{2.16}$$

A strict inequality implies no work – consistent with models of retirement and labor force withdrawal,

$$\frac{\partial V_{a+1,t+1}}{\partial K_{a+1,t+1}} = \frac{\partial V_{a+1,t+1}}{\partial H_{a+1,t+1}} \left(\frac{A^S \alpha_S (I^S_{a,t})^{\alpha_S - 1} (H^S_{a,t})^{\beta_S}}{R^S_t H^S_{a,t} (1-\tau)} \right) \tag{2.17}$$

(marginal return to investment time equals marginal cost);

$$\frac{\partial V_{a,t}}{\partial K_{a,t}} = \delta \frac{\partial V_{a+1,t+1}}{\partial K_{a+1,t+1}} \left[1 + r_t(1-\tau)\right] \tag{2.18}$$

(intertemporal arbitrage in returns on physical capital);

$$\begin{aligned} \frac{\partial V_{a,t}}{\partial H^S_{a,t}} &= \delta \frac{\partial V_{a+1,t+1}}{\partial K_{a+1,t+1}} R^S_t (1 - I^S_{a,t} - L^S_{a,t})(1-\tau) \\ &\quad + \delta \frac{\partial V_{a+1,t+1}}{\partial H^S_{a+1,t+1}} \left[A^S \beta_S (I^S_{a,t})^{\alpha_S} (H^S_{a,t})^{\beta_S - 1} + (1 - \sigma^S) \right] \end{aligned} \tag{2.19}$$

(marginal value of human capital is the return to current and future earnings).

At the end of the endogenously determined working life, the final term, which is the contribution of human capital to earnings, has zero marginal value.

At the beginning of life, agents choose the value of S that maximizes lifetime utility:

$$\hat{S} = \operatorname{Argmax}_S \left[V^S(\theta) - \varepsilon^S \right], \tag{2.20}$$

where $V^S(\theta)$ is now the value of schooling at level S inclusive of D^S, the discounted direct cost of schooling; and ε^S represents non-pecuniary benefits. Discounting of

V^S is back to the beginning of life to account for different ages of completing school. Tuition costs are permitted to change over time so that different cohorts face different environments for schooling costs. Given optimal investment in physical capital, schooling, investment in job-specific human capital, and consumption one can compare the path of savings. For a given return on capital and rental rates on human capital, the solution to the S-specific optimization problem is unique given concavity of the production function (2.14) in terms of $I_{a,t}^S$, $(0 < \alpha_S < 1)$, given the restriction that human capital be self-productive, but not too strongly $(0 \leqslant \beta_S \leqslant 1)$, given that investment is in the unit interval $(0 \leqslant I_{a,t}^S \leqslant 1)$, and given concavity of U in terms of C and L[36].

The choice of S is unique almost surely if ε^S is a continuous random variable. The dynamic problem is of split-endpoint form. The initial condition for human and physical capital is given and optimality implies that investment is zero at the end of life. For any terminal value of H^S and K^S, one can solve backward to the initial period and obtain the implied initial conditions[37]. One can iterate until the simulated initial condition equals the prespecified value.

The prices of skills and capital are determined as derivatives of an aggregate production function. In order to compute rental prices for capital and the different types of human capital, it is necessary to construct aggregates of each of the skills. At the micro level, agents specialize in choosing one skill among a variety of possible skills. At the macro level, different skills associated with different schooling levels have different effects on aggregate output. They are not perfect substitutes. Given the solution to the individual's problem for each value of θ and each path of prices, one can use the distribution of θ, $G(\theta)$, to construct aggregates of human and physical capital. The population at any time is composed of $\bar{a}$ overlapping generations, each with an identical ex ante distribution of heterogeneity, $G(\theta)$.

Human capital of type S is a perfect substitute for any other human capital of the same schooling type, whatever the age or experience level of the agent, but it is not perfectly substitutable with human capital from other schooling levels. Cohorts differ from each other only because they face different price paths and policy environments within their lifetimes. Assuming perfect foresight [as used in Auerbach and Kotlikoff (1987)], let c index cohorts, and denote the date at which cohort c is born by t_c. Their first period of life is $t_c + 1$. Let P_{t_c} be the vector of paths of rental prices of physical and human capital confronting cohort c over its lifetime from time $t_c + 1$ to $t_c + \bar{a}$. The rental rate on physical capital at time t is r_t. The rental rate on human capital is R_t^S. The choices made by individuals depend on the prices they face, P_{t_c}, their type, θ, and hence their endowment and their non-pecuniary costs of schooling, ε^S. Let $H_{a,t}^S(\theta, P_{t_c})$

[36] Heckman, Lochner and Taber (1998) use a present value formulation in their model because consumption is separable from investment in their setup.

[37] One can solve this problem numerically using the method of "shooting" or the methods described by Santos (1999) or Judd (1998).

and $K^S_{a,t}(\theta, P_{t_c})$ be the amount of human and physical capital possessed, respectively, and let $I^S_{a,t}(\theta, P_{t_c})$ be the time devoted to investment by an individual with schooling level S, at age a, of type θ, in cohort c.

By definition, the age at time t of a person born at time t_c is $a = t - t_c$. Let $N^S(\theta, t_c)$ be the number of persons of type θ, in cohort c, of schooling level S. For simplicity suppose that retirement is mandatory, at age a_R, although this is not strictly required. In this notation, the aggregate stock of employed human capital of type S at time t is cumulated over the non-retired cohorts in the economy at time t:

$$\bar{H}^S_t = \sum_{t_c = t - a_R}^{t-1} \int H^S_{t-t_c,t}(\theta, P_{t_c}) \left[1 - I^S_{t-t_c,t}(\theta, P_{t_c}) - L^S_{t-t_c,t}(\theta, P_{t_c})\right] N^S(\theta, t_c)\, \mathrm{d}G(\theta),$$

where $a = t - t_c$, $S = 1, \ldots, \bar{S}$, where $\bar{S}$ is the maximum number of years of schooling. The aggregate *potential* stock of human capital of type S is obtained by setting $I^S_{t-t_c,t}(\theta, P_{t_c}) = 0$ and $L^S_{t-t_c,t}(\theta, P_{t_c}) = 0$ in the preceding expression:

$$\bar{H}^S_t(\text{potential}) = \sum_{t_c = t - \alpha_R}^{t-1} \int H^S_{t-t_c,t}(\theta, P_{t_c}) N^S(\theta, t_c)\, \mathrm{d}G(\theta).$$

Allowing for human capital investment on the job produces a model of endogenous utilization rates. The aggregate capital stock is the capital held by persons of all ages:

$$\bar{K}_t = \sum_{t_c = t - a_R}^{t-1} \sum_{s=1}^{\bar{S}} \int K^s_{t-t_c,t}(\theta, P_{t_c}) N^s(\theta, P_{t_c})\, \mathrm{d}G(\theta).$$

2.3.1. Equilibrium conditions under perfect foresight

To close the model, it is necessary to specify the aggregate production function $F(\bar{H}^1_t, \ldots, \bar{H}^{\bar{S}}_t, \bar{K}_t)$ which is assumed to exhibit constant returns to scale. The equilibrium conditions require that marginal products equal pre-tax prices $R^S_t = F_{\bar{H}^S_t}(\bar{H}^1_t, \ldots, \bar{H}^{\bar{S}}_t, \bar{K}_t, t)$, $S = 1, \ldots, \bar{S}$ and $r_t = F_{\bar{K}_t}(\bar{H}^1_t, \ldots, \bar{H}^{\bar{S}}_t, \bar{K}_t, t)$. For their two-skill economy, HLT specialize the production function to

$$F(\bar{H}^1_t, \bar{H}^2_t, \bar{K}_t) = a_3 \left\{ a_2 \left[a_1 (\bar{H}^1_t)^{\nu_1} + (1 - a_1)(\bar{H}^2_t)^{\nu_1}\right]^{\nu_2/\nu_1} + (1 - a_2)\bar{K}_t^{\nu_2} \right\}^{1/\nu_2}. \tag{2.21}$$

This specification is general enough to accommodate all of the models used in the applied general equilibrium literature based on OLG models. When $\nu_1 = \nu_2 = 0$, the technology is Cobb–Douglas. Auerbach and Kotlikoff (1987) assume efficiency units so different labor skills are perfect substitutes ($\nu_1 = 1$). In addition, they assume a Cobb–Douglas aggregate technology relating human capital and physical capital

($v_2 = 0$). When $v_2 = 0$, the model is consistent with the constancy of capital's share irrespective of the value of v_1. Heckman, Lochner and Taber (1998) estimate Equation (2.21) and report that $v_2 = 0$, but $v_1 = 0.306$ so the elasticity of substitution between skill groups is 1.41 where $\overline{H}_1$ is the aggregate stock of high school human capital and $\overline{H}_2$ is the aggregate stock of college human capital. They document that (a) the two skills cannot be aggregated into a composite efficiency unit; (b) the stock of skilled human capital is increasing over time so that a growth steady state is implausible for modern economies as a factual description of a state of affairs; and, (c) that even in principle a growth steady state does not exist because the elasticities of substitution among skill groups are not unity.

2.3.2. *Linking the earnings function to prices and market aggregates*

The earnings at time t for a person of type θ and age a from cohort c with human capital $H^S(a, t, c)$ are

$$W^S(a, t, c) = R_t^S H_{a,t}^S(\theta, P_{t_c}) \left[1 - I_{a,t}^S(\theta, P_{t_c}) - L_{a,t}^S(\theta, P_{t_c})\right]. \tag{2.22}$$

They are determined by aggregate rental rates (R_t^S), individual endowments $H_{a,t}^S(\theta, P_{t_c})$ and individual investment decisions ($I_a^S(\theta, P_{t_c})$) and leisure decisions $L_a^S(\theta, P_{t_c})$. The last three components depend on agent expectations of future prices. Different cohorts facing different price paths will invest differently and have different human capital stocks. This insight rationalizes the evidence on cohort effects in earnings reported by MaCurdy and Mroz (1995) and Beaudry and Green (1997) among others. An essential idea introduced in the HLT paper, which is absent from currently used specifications of earnings equations in labor economics, is that *utilized* skills and not potential skills determine earnings[38]. The utilization rate is an object of choice linked to personal investment and labor supply decisions and is affected both by individual endowments and aggregate skill prices. As the quantity of aggregate skill is changed, so are aggregate skill prices. This affects schooling decisions, investment decisions, labor supply decisions, measured wages, and savings decisions.

2.4. *Determining the parameters of OLG models*

This section discusses how to choose the parameters of the OLG model just presented and various other versions of the model. Many of the problems that arise in determining the parameters of this model also arise in determining the parameters of the stochastic growth model. A Cobb–Douglas assumption for physical capital and a composite of labor is an appropriate description for the US economy where the constancy of capital's share is a well-established empirical regularity. However, the evidence from Europe

[38] This idea is central to the Becker–Chiswick (1966) and Mincer (1974) models.

is much less clear cut. The elasticity of substitution among the various skill groups is not infinite (as in the efficiency units model) nor is it one (as in Cobb–Douglas specifications).

A major problem that arises in the homogeneous preference form of this class of models is the inability to reproduce the aggregate capital–output ratio and the distribution of wealth holdings from a pure life-cycle model. See, for example, the discussion in Auerbach and Kotlikoff (1987), Hubbard, Skinner and Zeldes (1994) or Huggett (1996). As noted in Section 1, about half of the physical wealth is held by the top 5% of the families. While Huggett (1996) finds that the Gini coefficient for physical wealth is improved by accounting for earnings uncertainty, the resulting wealth distribution still misses on both tails. For instance, when the utility curvature parameter $\rho = 1.5$, the top 5% of wealth distribution only accounts for about a third of the total wealth (depending on the magnitude of the borrowing constraint), and the fraction of people with zero or negative wealth is too large. One appealing way to resolve this problem is to introduce preference heterogeneity beyond that induced by the overlapping generations. Persons with low rates of time preference or low risk preferences may account for the bulk of the life-cycle saving[39]. What is cause and what is effect, however, has not been sorted out in the literature. Do people endowed with low subjective discount rates simply choose to accumulate more wealth, or do subjective discount rates change as people become wealthier?

The ideal data set for the purposes of estimating a general equilibrium model with heterogeneous skills would combine micro data on firms, data on the earnings of workers, their life-cycle consumption and wealth holdings, and macro data on prices and aggregates. With such data, one could estimate all the parameters of our model and the distribution of wages, wealth, and earnings. Using the micro data joined with aggregate prices, one could estimate the parameters of the micro model. Using the estimated micro functions, it would be possible to construct aggregates of human capital that could be used in determining the output technology. The estimated aggregates should match measured empirical aggregates and, when inserted in aggregate technology, should also reproduce the market prices used in estimation.

Two practical obstacles prevent implementation of this approach. (1) Analysts typically do not have information on individual consumption linked to labor earnings and labor supply. (2) The data on market wages do *not* reveal skill prices, as is evident from the distinction between R_t^S and $W^S(a, t, c)$ in Equation (2.22). Since prices cannot be directly equated with wages, it is apparently not possible to estimate aggregate stocks of human capital to use in determining aggregate technology. These obstacles led Heckman, Lochner and Taber (1998) to propose and implement an alternative

[39] Other ingredients may also work. In a partial equilibrium context, Carroll (1992) claims that small probabilities of bad income shocks can better account for wealth holdings of low income consumers. Hubbard, Skinner and Zeldes (1994) offer another explanation based in part on uncertainty from multiple sources of income.

for assembling information from different data sets to check the consistency of the constructed model with the available micro and macro data. They require that the econometric procedures used to produce the micro-based parameters employed in their model, including the implicit assumptions made about the economic environment and expectations, recover the parameters estimated from synthetic micro data sets generated by the model. In estimating skill-specific human capital production functions, HLT demonstrate that it is necessary to account for heterogeneity in ability, in the technology required to produce skills, and in endowments.

3. Micro evidence

3.1. Introduction

This part of the chapter presents additional evidence from the microeconomic literature on the parameter values that are required to implement the dynamic general equilibrium (DGE) models analyzed in Sections 1 and 2. Several conceptually distinct labor supply and consumption demand elasticities are presented. We discuss the problem of research synthesis and issue a warning against uncritical use of the existing micro evidence in standard general equilibrium models. We also present further evidence on preference heterogeneity. We start by defining a variety of conceptually different elasticities that are frequently confused in both the micro and macro literatures.

3.2. Defining elasticities

When considering the estimation of parameters for ultimate use in a DGE model, it is important to keep track of exactly what is being held constant (the "conditioning variables") in the process of estimation in order to ascertain whether the parameter being estimated corresponds to the parameter required in a general equilibrium model. We illustrate this point by considering a simple model of consumption and labor supply, but our discussion applies much more generally. We derive three frequently estimated elasticities typically formulated for a model in which an agent sells labor on a spot market without any transactions costs or fixed costs of employment. Although these elasticities are often referred to by the same name, they correspond to different choices of conditioning variables and hence distinct conceptual experiments. We then consider aggregate labor supply response measures that account for heterogeneity and dichotomous work–no work decisions. In later sections we present some parameter estimates of these elasticities taking care to specify which elasticity is being estimated.

Suppose that in a given period t a person chooses current non-durable consumption c_t and hours of market work $h_t = T - l_t$. Preferences are intertemporally additive but the within-period utility function $U(c_t, h_t)$ is not. In what follows we separate the within-time-period decision of how to allocate consumption and leisure given current period net expenditures from the intertemporal savings decision. This leads us to define "total net expenditure" or "net dissavings" $e_t = p_t c_t - w_t h_t$, where p_t is the time t price of consumption and w_t is the time t wage rate. These prices can be denominated in any convenient unit of account, including time t dollars. Deciding how much to consume and how much to work for a given amount of net dissavings is one part of the decision problem confronting the consumer.

In a model with a more fully specified security market, the link between net dissavings and the market opportunities for investment is clearer. As we discussed in Section 1, it is conventional in the RBC literature to consider a wide array of alternative security markets. Following Heckman (1974) the impact of the allocation of e_t over time is conveniently determined by the life-cycle evolution of the shadow price λ_t of the net dissavings: the marginal utility of income. Changing the specification of the security market environment alters the evolution of $\{\lambda_t\}$. For instance, suppose that the consumer/investor has access to a one-period risk-free security with rate of return r_{t+1}. Then the marginal utility of income satisfies the stochastic difference equation:

$$\lambda_t = \beta(1 + r_{t+1}) E(\lambda_{t+1} \mid I_t), \tag{3.1}$$

where β is the subjective discount factor, provided there is no binding borrowing constraint [MaCurdy (1983)]. Under the permanent income restriction that $\beta(1 + r_{t+1}) = 1$, this results in the familiar conclusion that the marginal utility of income is a martingale [MaCurdy (1978), Hall (1978)]. Including risky securities at the same time imposes further restrictions on the evolution of $\{\lambda_t\}$. When the rate of return is a risky asset, Equation (3.1) becomes

$$\lambda_t = \beta E\left[(1 + r_{t+1})\lambda_{t+1} \mid I_t\right].$$

Including additional risky securities we obtain

$$\lambda_t = \beta E[(1 + r^j_{t+1})\lambda_{t+1} \mid I_t]$$

for $j = 1, \ldots, J$ where J is the number of securities. The presence of short sale constraints may convert these arbitrage equalities into strict inequalities. In the limiting complete market case, the ratio $\beta(\lambda_{t+1}/\lambda_t)$ is the same for all consumers and is equal to the market stochastic discount factor for pricing single-period securities described in Equation (1.7) in Section 1 [Altug and Miller (1990, 1998)]. We next consider Frisch demand functions that condition on the shadow price λ_t. While a multiplier λ_t can be defined for all environments, its interpretation is more complicated in environments with borrowing or short sale constraints.

3.2.1. Frisch demands

Assuming interior solutions for consumption and hours ($c_t > 0$ and $T > h_t > 0$), and Equation (3.1), or some generalization consistent with no corner solutions in intertemporal financial transfers, the household optimal consumption and hours of work satisfy two first-order conditions:

$$U_c(c_t, h_t) = \lambda_t p_t, \tag{3.2}$$

$$U_h(c_t, h_t) = -\lambda_t w_t. \tag{3.3}$$

If the utility function $U(c_t, T - l_t)$ is strictly concave in consumption and leisure, we can invert Equations (3.2) and (3.3) to give the Frisch (or λ-constant) consumption and labor supply functions (where now we drop the t subscripts):

$$c = c(p, w, \lambda), \tag{3.4}$$

$$h = h(p, w, \lambda). \tag{3.5}$$

From the integrability conditions, these functions are homogeneous of degree zero in the price, the wage and the inverse of λ; symmetric ($c_w = -h_p$); and satisfy negativity (which implies $c_p < 0$ and $h_w > 0$).

The parameters of $c(\cdot)$ and $h(\cdot)$ have been the subject of intensive empirical investigation and we review this empirical literature in Sections 3.3 and 3.4. A common assumption in both the empirical literature in microeconomics and in the macroeconomic literature is that consumption and labor supply are additively separable within the period; this is equivalent to assuming that $c_w = h_p = 0$. We do not invoke this assumption here and we note below that the micro evidence speaks against it.

The case in which an individual or household chooses not to work: $h = 0$, is also of considerable interest. In the absence of fixed costs for entry and exit, a person chooses not to work if:

$$U_c(c, 0) = \lambda p, \qquad U_h(c, 0) < -\lambda w,$$

that is, if the reservation value of leisure at zero hours of work is greater than the market wage. We say more about this case in Section 3.2.4.

Associated with the Frisch consumption and labor supply functions are the Frisch (or λ-constant) price and wage elasticities:

$$\begin{aligned} \varphi(p, w, \lambda) &= \frac{\partial \ln c}{\partial \ln p} = c_p(p, w, \lambda)\frac{p}{c}, \\ \theta(p, w, \lambda) &= \frac{\partial \ln h}{\partial \ln w} = h_w(p, w, \lambda)\frac{w}{h}. \end{aligned} \tag{3.6}$$

These elasticities consider changes in demands and supplies for a particular good when its own price is changed but other prices are held constant. For instance, θ, the *Frisch (or λ-constant) elasticity of labor* with respect to the nominal wage, holds the nominal price of consumption constant and hence also measures hours response to increases in the real wage. As we will see, the conditioning on λ gives both elasticities an intertemporal character.

To construct the *intertemporal elasticity for consumption* we consider

$$\eta(p, w, \lambda) = \frac{\partial \ln c}{\partial \ln \lambda} = c_\lambda (p, w, \lambda) \frac{\lambda}{c}.$$

There is an obvious counterpart for labor supply. Assuming no binding borrowing or short sale constraints, the change in λ can be thought of as arising from a change in the interest rate or any other factor that alters λ through forward-looking relation (3.1). In contrast to η, φ is defined for a change that holds λ fixed. For this reason, φ sometimes is referred to as an intertemporal elasticity of substitution for consumption When the utility function U is additively separable between consumption and hours, then φ and η coincide, but in general they do not. Instead, by the homogeneity of degree zero of the Frisch demand functions, we have the relation

$$c_p (p, w, \lambda) \frac{p}{c} + c_w (p, w, \lambda) \frac{w}{c} = c_\lambda (p, w, \lambda) \frac{\lambda}{c} = \eta(p, w, \lambda),$$

or

$$\varphi (p, w, \lambda) + c_w (p, w, \lambda) \frac{w}{c} = \eta(p, w, \lambda).$$

Thus, the intertemporal elasticity η can also be viewed as the consumption response when both the price and wage change proportionately.

3.2.2. *Other demand functions*

Conditioning on the multiplier λ provides a convenient way of estimating the parameters required to implement many of the models discussed in Sections 1 and 2. To apply this strategy requires that there be no binding constraints on transferring resources over time. Within-period elasticities can be defined even if Euler inequalities rather than Euler equations characterize consumer behavior. Assuming equalities hold, the within-period elasticities can be derived by substituting total net expenditures $e = pc - wh$ for λ, by inverting the following expression:

$$e = pc (p, w, \lambda) - wh (p, w, \lambda) = \psi (p, w, \lambda) \Rightarrow \lambda = \mu (p, w, e). \tag{3.7}$$

Substituting in the Frisch consumption and labor supply functions we obtain the within-period *uncompensated* or *Marshallian* consumption and labor supply functions:

$$c = c [p, w, \mu (p, w, e)] = c^* (p, w, e), \tag{3.8}$$

$$h = h [p, w, \mu (p, w, e)] = h^* (p, w, e). \tag{3.9}$$

This derivation is more restrictive than necessary because it assumes that Frisch demands exist. An alternative derivation that stresses the more general nature of these demand functions maximizes within-period utility subject to a period-specific budget constraint that may be augmented beyond (or below) current period earnings. Under this interpretation, e is the supplement to current earnings that governs within-period choices and e may be either exogenously or endogenously determined[40].

Under either interpretation, the parameters of $c^*(\cdot)$ and $h^*(\cdot)$ can be estimated using only cross-section data. However, they do not recover all the parameters of the original functions $c(\cdot)$ and $h(\cdot)$ except under very stringent assumptions. In particular, because they condition on the within-period allocation of expenditures, they are uninformative on interperiod substitution unless strong functional form assumptions are maintained. On the other hand, those parameters that are identified from Equations (3.8) and (3.9) are robust to misspecifying the asset market structure used to characterize the evolution of the marginal utility of income. For example, these relationships are valid whether or not the consumer is up against a borrowing constraint.

A third labor supply function can be derived directly by equating the intratemporal marginal rate of substitution between hours and consumption to the real wage. By forming ratios of Equations (3.2) and (3.3), we eliminate the marginal utility of income λ. Solving for labor supply in terms of the price of consumption, the wage and consumption gives:

$$h = h^{**}(p, w, c), \tag{3.10}$$

where now we condition labor supply responses on consumption Alternatively, we could solve for consumption by conditioning on the wage and hours. Since these demand functions are readily derived from the marginal rate of substitution first-order condition, they are known as m-demand/supply functions [see Browning (1998)]. They are valid whether or not the intertemporal Euler equations are binding. As noted in Section 1, these demand relations are also among those used by macroeconomists to calibrate parameters from steady-state relations using time series averages.

The two conditional labor supply wage elasticities associated with Equations (3.10) and (3.9) generally differ from each other and from the corresponding Frisch elasticity given in Equation (3.6). Assuming that there are no binding constraints connecting transfers among periods, the three demand functions are connected by the identities

$$h(p, w, \lambda) \equiv h^*[p, w, \psi(p, w, \lambda)] \equiv h^{**}[p, w, c(p, w, \lambda)] \tag{3.11}$$

so that the wage effects are related by differentiating with respect to w:

$$\begin{aligned} h_w(p, w, \lambda) &= h_w^*(p, w, e) + h_e^*(p, w, e)\,\psi_w(p, w, \lambda) \\ &= h_w^{**}(p, w, c) + h_c^{**}(p, w, c)\,c_w(p, w, \lambda). \end{aligned} \tag{3.12}$$

40 Sometimes, analysts condition on full income $(e + wT)$ and not e. Assuming normality of leisure, the wage effect holding $e + wT$ constant is greater than or equal to the wage effect holding e constant since to hold $e + wT$ fixed while w goes up one must reduce e. Full income-constant (or Becker) labor supply functions overstate the Marshallian (or e-constant) labor supply functions.

The first equation states that the Frisch (or λ-constant) wage responses for labor supply are equal to the within-period wage response holding resource flow constant plus an intertemporal net savings response that accounts for how savings is altered by the wage change. The second equation shows that the Frisch wage effect can be decomposed into a within-period effect of wages on hours holding current consumption constant plus an intertemporal response of consumption to wages.

From cross-section data on consumption expenditures and labor supply we can estimate $h^*(p, w, e)$ and $h^{**}(p, w, c)$ assuming either exogeneity of the conditioning variables or access to valid instruments for the endogenous regressors. Each demand equation can be used to bound the intertemporal Frisch response $h_w(p, w, \lambda)$.

More precisely, from Equation (3.7) we have

$$\psi_w(p, w, \lambda) = pc_w(p, w, \lambda) - wh_w(p, w, \lambda) - h.$$

The sign of $\psi_w(\cdot)$, the effect of wages on borrowing, or dissaving, is ambiguous, if as is widely assumed, consumption and labor supply are Frisch complements (so $c_w(\cdot) > 0$). However, it is unlikely that the cross effect on the right hand side will outweigh the other two terms. In a period of high wages it seems plausible that savings increase, rather than decrease or remain constant. Thus $\psi_w < 0$. In this case, $h_w(\cdot) \geqslant h_w^*(\cdot)$ since $h_e^*(\cdot)$ is negative if leisure is a normal good. The m-supply response $h_w^{**}(p, w, c)$ gives an upper bound for the Frisch response $h_w(\cdot)$ if leisure and consumption are normal (so that $h_c^{**}(\cdot) < 0$) and consumption and labor supply are complements (so $c_w(\cdot) > 0$). Thus $h_w^{**} < h_w < h_w^*$ so we can bound h_w from the cross-sectional relationships. Obviously if consumption and leisure are additively separable within periods, the m-supply response function is the Frisch response. In general, we must take care to specify exactly what is being held constant (the marginal utility of income, total net expenditure or consumption) in the empirical study we examine when we use an estimated wage elasticity The empirical literature presents estimates of all of these elasticities and more, as we discuss below, and often does not distinguish among them.

3.2.3. An example

To make the discussion of Section 3.2.2 more concrete, consider the following utility function which, as we have previously noted, is sometimes used in DGE models:

$$U(c, h) = \frac{\left[c^{\sigma}(T-h)^{(1-\sigma)}\right]^{(1-\rho)} - 1}{(1-\rho)}; \qquad 0 < \sigma < 1, \quad \rho > 0, \tag{3.13}$$

where T is the time available for work. The conditions on the admissible parameter values are produced by monotonicity and strict concavity. The associated Frisch consumption and labor supply functions are:

$$\ln c = \alpha_c + \beta_c \ln p + \gamma_c \ln w + (\beta_c + \gamma_c) \ln \lambda, \tag{3.14}$$

$$\ln l = \ln (T - h) = \alpha_h + \beta_h \ln p + \gamma_h \ln w + (\beta_h + \gamma_h) \ln \lambda, \tag{3.15}$$

where

$$\beta_c = \frac{(\sigma\rho - \sigma - \rho)}{\rho}, \qquad \gamma_c = -\frac{(1-\sigma)(1-\rho)}{\rho},$$
$$\beta_h = -\frac{\sigma(1-\rho)}{\rho}, \qquad \gamma_h = \frac{\sigma(1-\rho)-1}{\rho}.$$

The monotonicity and concavity conditions on the utility function imply that β_c and γ_h are both negative. Note as well that

$$(\beta_c + \gamma_c) = (\beta_h + \gamma_h) = -\frac{1}{\rho} < 0,$$

so that both consumption and leisure are normal goods (that is, increases in lifetime wealth lead to decreases in the marginal utility of income and consequent increases in both consumption and leisure). When $\rho = 1$, the utility function U is additively separable in consumption and leisure:

$$U(c, h) = \sigma \ln(c) + (1-\sigma)\ln(T-h),$$

and as a consequence, $\gamma_c = \beta_h = 0$ and $\gamma_h = \beta_c = -1$.

The Frisch labor supply wage elasticity is given by:

$$\frac{\partial \ln h}{\partial \ln w} = -\gamma_h \frac{(T-h)}{h}. \tag{3.16}$$

The Frisch elasticity for consumption holding the wage constant and the marginal utility of income constant, is given by β_c; and the elasticity of intertemporal substitution is given by $-1/\rho$, which is the coefficient on $\ln \lambda$ in Equation (3.14). This latter elasticity can alternatively be viewed as the *elasticity of intertemporal substitution holding the real wage constant*. This may be seen by rewriting the consumption function as

$$\ln c = \alpha_c + (\beta_c + \gamma_c) \ln p + \gamma_c \ln \frac{w}{p} + (\beta_c + \gamma_c) \ln \lambda. \tag{3.17}$$

Both $\ln p$ and $\ln \lambda$ now have a common coefficient:

$$\beta_c + \gamma_c = -\frac{1}{\rho},$$

which shows that the two elasticities are the same.

The consumption elasticities β_c and $\beta_c + \gamma_c$ result from two different conceptual experiments, as reflected by their different conditioning variables. They coincide only in very special cases. When the consumer does not care about non-work time (leisure) ($\sigma = 1$) only the consumption equation (3.14) is relevant and $\gamma_c = 0$. In this case both consumption elasticities coincide and are given by $-1/\rho$. This is the usual definition for an iso-elastic consumption utility function. Alternatively, when $U(c, h)$ is additively separable ($\rho = 1$), γ_c is again zero and $\beta_c = -1$. When consumption and labor supply are not additively separable within the period we have two distinct Frisch intertemporal substitution elasticities for consumption one holding the current wage constant (β_c) and one holding the real wage constant ($\beta_c + \gamma_c$).

The utility function given in Equation (3.13) is very restrictive since it confounds intertemporal substitution conditions – a high value for ρ implies a high propensity to substitute across time – and within-period substitution possibilities – the cross elasticities (β_h and γ_c) are positive if and only if $\rho > 1$. When $\rho > 1$, market work and consumption are (Frisch) complements; that is, a rise in the wage leads to an increase in both consumption and labor supply; market goods substitute for the home production foregone when someone works. As discussed in the previous subsection, we can also derive within-period labor supply functions that condition on either net dissaving or consumption For the utility function (3.13), the e-constant demand function is not very illuminating but the c-constant function is given by

$$\ln(T-h) = \ln\left(\frac{\sigma}{1-\sigma}\right) + \ln(c) - \ln\left(\frac{w}{p}\right). \tag{3.18}$$

This equation clearly demonstrates that we cannot generally identify the preference parameters for the intertemporal allocation from within-period information. Indeed, in this case the parameter ρ cannot be identified from the c-constant function and thus none of the parameters of the Frisch consumption and labor supply functions can be deduced. While parameter σ can be inferred from this intratemporal relation, this particular choice of functional form implicitly imposes the requirement that at a fixed real wage, consumption and leisure move together – a prediction that is at odds with the evidence surveyed in Section 3.3 [41].

3.2.4. The life-cycle participation decision

The discussion so far assumes that solutions for consumption and leisure are interior. Such an assumption is congenial to the representative agent approach to macroeconomics but is grossly inconsistent with the microeconomic evidence on labor supply summarized by Pencavel (1986) and Killingsworth and Heckman (1986). Summarizing the Ph.D. research of Coleman (1984), Heckman (1984) noted that even

[41] In Section 1 [Equation (1.4)] we described an extension of this functional form that macroeconomists sometimes use because it still accommodates steady state calibration.

for prime age males, variations in employment contribute about 50% of the total variation in person hours over the business cycle. A central finding of the modern labor supply literature summarized in Heckman (1978, 1993) and Blundell and MaCurdy (1999) is that most of the curvature in the labor supply–wage relationship comes from choices at the extensive (entry–exit) margin.

Accounting for entry and exit decisions forces analysts to introduce heterogeneity among agents. It is implausible that all agents either work in a period or do not work in a given period. Some mechanism must be introduced to account for why some agents work while others do not. In the macro literature building on Rogerson (1988), an assumption of fixed costs of work or some source of non-convexity is introduced so that it is optimal for individual agents to work either full time or not at all. The mechanism used to allocate work across people is a lottery embedded in a complete contingent claims market. [See Rogerson (1988) or the survey in Hansen and Prescott (1995).] Ex ante identical persons get different draws from the lottery. Draws are independent over time. Under additive separability in consumption and leisure, the winners of the lottery are those denied work; they get the same consumption bundle as workers but enjoy more leisure than workers. Rogerson (1988) shows how lotteries can be priced in a complete market RBC model of the sort considered in Section 1 [42]. Under conditions presented in his paper, a decentralized mechanism exists and a competitive equilibrium can be supported by the pricing system.

This description of the employment allocation mechanism strains credibility and is at odds with the micro evidence on individual employment histories. Heckman and Willis (1977), Heckman (1982) and Clark and Summers (1979) document that employment indicator variables for persons are highly correlated over time – contrary to the Bernoulli assumption implicit in Rogerson (1988) and Hansen and Prescott (1995). Over long stretches of time, some people work all of the time while others never work. Heckman (1982) explicitly tests and rejects the Bernoulli assumption. This persistence in employment status remains even after controlling for commonly observed characteristics such as education, age and work experience. Persistence in the employment or non-employment state is a central feature of the micro data. At a minimum, individual-specific lotteries with outcomes strongly correlated over time are required to account for the micro data on employment – histories complicating the analysis of equilibrium lottery pricing. The heterogeneity in employment experience among persons of the same apparent demographic and productivity characteristics suggests that problems with adverse selection and moral hazard are likely to render the competitive lotteries of the sort analyzed by Rogerson (1988) infeasible.

The micro literature explains cross-section heterogeneity in employment experiences, and the persistence of employment status over time, by introducing temporally invariant person-specific unobserved heterogeneity. The evidence in Heckman and

[42] Hansen and Prescott (1995) extend Rogerson's model to account for non-separabilities in consumption and leisure. This produces leisure-dependent consumption allocations.

Willis (1977) and Heckman (1982) indicates that such invariant components account for much of the persistence in employment status over time. Invariant unobservables can be introduced into all three types of demand functions analyzed in Section 3.2.2.

Within the Frisch framework, if we abstract from any fixed costs of work or other non-convexities, a person will not work in period t if

$$U_c(c_t, 0) = \lambda_t p_t \tag{3.19}$$

and

$$U_h(c_t, 0) < -\lambda_t w_t. \tag{3.20}$$

We can solve out for the virtual wage, w_t^*, that makes inequality (3.20) an exact equality at zero hours of work and use w_t^* in the Frisch consumption demands. Thus, unless there is contemporaneous additive separability, estimation of the consumption demand equation will depend on virtual wages which equal actual wages if a person works. Similarly, in the absence of contemporaneous additive separability, the employment decision will depend on p_t as well as on λ_t and w_t. To see this, solve the first-order condition for consumption (3.19) for $c_t(\lambda_t, p_t;\, h_t = 0)$ and substitute into inequality (3.20). This produces an equation characterizing life-cycle employment that depends on p_t, among other factors.

The employment decision is discontinuous in terms of w_t; below or at w_t^*, persons will not work; above w_t^*, persons work at age t. Introduce a person-specific time-invariant random variable ε to account for heterogeneity in the population. This is unobserved by the econometrician. Instead the econometrician observes a vector X of demographic characteristics that partially predict ε. Associated with this random variable is a distribution of types in the population. Let $d_t = 1$ if a person is employed at time t; $d_t = 0$ otherwise. Accounting for heterogeneity, ε, which for simplicity is assumed to be scalar, we may write this inequality as

$$\begin{aligned} &d_t = 0 \text{ if } U_h(c_t(\lambda_t(\varepsilon), p_t;\, h_t = 0), 0;\, \varepsilon) \leqslant -\lambda_t(\varepsilon) w_t, \\ &d_t = 1 \text{ otherwise,} \end{aligned}$$

or more succinctly

$$d_t = 1[U_h(c(\lambda_t(\varepsilon), p_t;\, h_t = 0), \varepsilon) > -\lambda_t(\varepsilon) w_t],$$

where $1(\cdot)$ is the indicator function and where we note that $\lambda_t(\varepsilon)$ does not depend on w_t if the inequality is strict[43]. However, standard results in consumer theory demonstrate

[43] The assumption that ε is scalar is only a simplifying device. It is not strictly required. Note that $\lambda_t(\varepsilon)$ is a function of ε and initial assets as well as prices and wages in periods where persons work. For simplicity of notation we only exhibit the dependence on ε.

that $\lambda_t(\varepsilon)$ depends on ε and current and future prices and wages for periods in which persons work and consume. The set of ε values for which $d_t = 0$ for wage w_t is thus given by

$$\underline{\varepsilon}_t = \{\varepsilon \mid U_h[c_t(\lambda_t(\varepsilon), p_t; h_t = 0), 0; \varepsilon] \leqslant -\lambda_t(\varepsilon) w_t\},$$

which depends implicitly on all prices and wages over the life cycle in periods outside of t in which the consumer works and consumes as well as initial endowments. Let the boundary of the set $\underline{\varepsilon}_t$ for persons with potential market wage w_t be

$$B(\underline{\varepsilon}_t) = \{\varepsilon \mid U_h[c_t(\lambda_t(\varepsilon), p_t; h_t = 0), 0; \varepsilon] = -\lambda_t(\varepsilon) w_t\}.$$

Assume, for simplicity, that prices and wages are independent of ε and demographic characteristics X do not enter preferences directly, except through their effect on ε. These assumptions simplify the notation and are easily relaxed. Then, in period t, the proportion not working is

$$\Pr(d_t = 0 \mid X) = \int_{\underline{\varepsilon}_t} \mathrm{d}F(\varepsilon \mid X),$$

where $F(\varepsilon \mid X)$ is the conditional distribution of ε. The life-cycle wage response of participation is

$$\frac{\partial \Pr(d_t = 0 \mid X)}{\partial w_t} = f(B(\underline{\varepsilon}_t) \mid X) \frac{\partial B(\underline{\varepsilon}_t)}{\partial w_t}, \tag{3.21}$$

where we are assuming that there is only one boundary point, an upper boundary point, and that the distribution is continuous at that point with density f:

$$\left.\frac{\mathrm{d}F(\varepsilon \mid X)}{\mathrm{d}\varepsilon}\right|_{\varepsilon = B(\underline{\varepsilon}_t)} = f(B(\underline{\varepsilon}_t) \mid X).$$

For more general boundary sets, we require a notion of the density added or subtracted as the boundary is changed by the wage[44]. Aggregating over age, wage and X groups, as in Section 2, produces the aggregate proportion of people who do not work.

[44] We require evaluation of the limit

$$\lim_{\Delta w_t \to 0} \frac{\int_{\underline{\varepsilon}_t(w_t + \Delta w_t)} \mathrm{d}F(\theta \mid X) - \int_{\underline{\varepsilon}_t(w_t)} \mathrm{d}F(\theta \mid X)}{\Delta w_t},$$

where we make the dependence of $\underline{\varepsilon}_t$ on w_t explicit. This expression is easily generalized to allow for vector ε.

Aggregate labor supply is constructed accounting for both the choice at the intensive margin and choice at the extensive margin. See Heckman (1978) or Pencavel (1986) for more details. Aggregate employment–wage parameters combine preferences and distribution parameters and cannot be directly compared with the interior solution elasticities unless the distribution of taste parameters is accounted for[45].

A similar analysis can be performed for the e-constant and c-constant demand functions[46]. In each case we can define a reservation wage and can define a set of ε values such that for persons with a given $e_t(\varepsilon)$ or $c_t(\varepsilon)$, respectively, and for the other prices and X variables, persons work or do not work in period t. In general e_t and c_t depend on ε, just as λ_t depends on ε. Employment and non-employment proportions are determined by integration of $\mathrm{d}F(\varepsilon \mid X)$ over the appropriate sets. We note, however, that all three interpretations of the leisure demand function produce the same aggregate employment elasticity provided that all three exist[47]. We now turn to the empirical evidence on these elasticities starting with the *eis* for consumption.

3.3. Consumption estimates

In this section we present estimates from micro data of the parameters of preferences for consumption Given our discussion in the previous subsection, it will be clear that this will sometimes require us to also consider labor supply although the great majority of consumption studies assume within-period additivity between consumption and labor supply. We discuss labor supply estimates in the next subsection.

Here we shall concentrate on two aspects of these estimates. The first is the conditioning variables used – in particular whether within-period additivity is assumed

[45] Our analysis generalizes to an environment of uncertainty. For a model of perfect insurance, the analysis in the text applies without any modification. For a model of less than perfect insurance, λ_t is replaced by the derivative of the value function with respect to current assets, and variation in w_t is taken holding any effect of the current wage on future values constant.

[46] Thus for the c-constant functions the set of ε values for which persons with consumption $c_t(\varepsilon)$ and w_t, p_t who do not work is

$$\varepsilon_t^c = \left\{ \varepsilon \mid \frac{U_h(c_t(\varepsilon), 0; \varepsilon)}{U_c(c_t(\varepsilon), 0; \varepsilon)} < -\frac{w_t}{p_t} \right\},$$

and $\Pr(d_t = 0 \mid X) = \int_{\underline{\varepsilon}_t^c} \mathrm{d}F(\varepsilon \mid X)$. For the e-constant functions, the ε set for non workers with earnings $e_t(\varepsilon)$ is

$$\varepsilon_t^e = \left\{ \varepsilon \mid \frac{U_h(e_t(\varepsilon)/p_t, 0; \varepsilon)}{U_c(e_t(\varepsilon)/p_t, 0; \varepsilon)} < -\frac{w_t}{p_t} \right\},$$

and the probability of non-participation is $P(d_t = 0 \mid X) = \int_{\underline{\varepsilon}_t^e} \mathrm{d}F(\varepsilon \mid X)$. Again, $c_t(\varepsilon)$ and $e_t(\varepsilon)$ depend in general on initial endowments, assets and prices in periods with interior solution demand functions.

[47] Note that the e- and c-constant demands can exist when the Frisch demands do not.

or not, and whether the wage or labor supply is held fixed. The second is accounting for heterogeneity. Examples of observable heterogeneity include household composition, health status, age, and cohort (year of birth). To illustrate how we might incorporate heterogeneity consider the second-order approximation to the Euler equation for the iso-elastic (constant relative risk aversion) utility function, assuming within-period additivity between consumption and labor supply [see, for example, Browning and Lusardi (1996), Equation (2.4)]:

$$\Delta \ln c_{t+1} = \frac{1}{\rho}\left[\ln\left(\beta_{t+1,t}\right) + r_{t+1} + \tfrac{1}{2}\sigma_t^2\right] + u_{t+1}, \tag{3.22}$$

where r_{t+1} is a (lending, net of tax) real rate of interest[48]. The coefficient $\beta_{t+1,t}$ is the time preference discount factor between periods t and $t+1$, $-1/\rho$ is the *eis* (ρ is the CRRA parameter), and σ_t^2 is the variance of Euler equation error conditioned on time t information. When the rate of return is riskless, σ_t^2 is proportional to the conditional variance in consumption growth; it captures the precautionary motive and usually depends on factors such as the uncertainty associated with, for example, future income and health as well as the insurance possibilities open to the agent and the income realization and level of assets in period t. Now suppose that the discount factor is given by

$$\beta_{t+1,t} = \beta_0 \exp\left(\beta_z \Delta z_{t+1} + \varepsilon_\beta\right), \tag{3.23}$$

where β_0 is the baseline discount factor. The variable z_t is a demographic variable that changes the utility of consumption in period t. For example, it is generally believed that children increase consumption so that if z is a dummy variable for the presence of children then β_z will be positive. The (zero mean) variable ε_β captures (unobservable) variations in the discount factor. Similarly, let

$$\tfrac{1}{2}\sigma_t^2 = \theta y_t + \varepsilon_\sigma, \tag{3.24}$$

where y_t is the level of a variable such as income or assets in period t. The variable ε_σ captures (fixed) differences in future risk and insurance arrangements. For example, for tenured university professors this is virtually zero while it may be quite high for young workers. Unlike its counterpart for the discount factor, this variable is unlikely to have zero mean. Combining Equations (3.22) to (3.24) we have the structural Euler equation:

$$\Delta \ln c_{t+1} = \frac{1}{\rho}(\ln\beta_0 + \varepsilon_\beta + \varepsilon_\sigma) + \frac{1}{\rho}r_{t+1} + \frac{1}{\rho}\beta_z\Delta z_{t+1} + \frac{1}{\rho}\theta y_t + u_{t+1} \tag{3.25}$$

and an associated reduced form

$$\Delta \ln c_{t+1} = \alpha_0 + \alpha_r r_{t+1} + \alpha_z \Delta z_{t+1} + \alpha_y y_t + u_{t+1}. \tag{3.26}$$

Note that in this form lagged levels variables (in this case, y_t or variables correlated with the permanent differences in discount factors ε_β and expected consumption

[48] If agents cannot carry forward debt at this rate then there is an extra non-negative Lagrange multiplier term in this equation [see Browning and Lusardi (1996)].

variance ε_σ) may be correlated with consumption growth – this causes obvious problems in choosing variables for orthogonality conditions to identify the model. If we can recover consistent estimates of the parameters in Equation (3.26) then we can recover some of the parameters of Equation (3.25). In particular we can identify the *eis* ($= -1/\rho$) and the observable variation in the discount factor β_z. We cannot, however, identify the mean discount factor β_0 without further analysis. We turn now to some micro-based estimates of the parameters of the Euler equation.

Most empirical consumption Euler equations do not condition on either the wage or labor supply. This implicit assumption that consumption and labor supply are additively separable within the period is very convenient but probably unwarranted. Evidence from consumption studies against the additive form will be presented below but there is also other evidence. For example, using family expenditure data, Browning and Meghir (1991) show that demand patterns depend significantly on male and female labor force status in an intuitively plausible way (for example, the budget shares of transport, clothing, and eating out are all rising with labor supply). If this is the case then preferences over non-durables are not (weakly) separable from labor supply and consequently preferences over some non-durable composite commodity cannot be additively separable from labor supply. Note that since this result is independent of the normalization of the utility function, this rules out any separable utility function, which unfortunately includes many functional forms assumed in the DGE literature.

In Table 3.1 we present evidence from consumption Euler equation estimates based on micro data that maintain within-period additivity between goods and leisure[49]. Almost always, the principal focus of these papers is on tests of the orthogonality conditions implied by the standard additive model so that either anticipated income growth or lagged income is also included on the right hand side. We restrict attention to those papers that explicitly report all of the effects of demographics. Most papers assume an iso-elastic utility function so that the equation estimated is similar to that given in Equation (3.26), usually without the lagged income term. The exceptions are Hall and Mishkin (1982) who use a quadratic utility function and Attanasio and Browning (1995) who use a generalization of the iso-elastic form.

Table 3.1 reveals that most investigators find significant observable heterogeneity in the discount factor [the α_z in Equation (3.26)]. A typical example is Zeldes (1989) who finds that the elasticity of food consumption with respect to "needs" (effectively, the number of adult equivalents) is about 0.24 with a standard error of 0.03. Lawrance (1991) also allows for variation in the intercept with the *level* of presample income (ostensibly to capture differences in the discount factor that are correlated with lifetime wealth), education, and race. In terms of Equations (3.23) and (3.24) these factors could capture the variation due to ε_β or ε_σ (or even a propensity to be liquidity constrained). Lawrance assumes that only the discount factor varies across the population and identifies the baseline discount factor by estimating the mean of

[49] We include Attanasio and Browning (1995) since they present results with and without the separability assumption so it provides a bridge between the two sets of results.

Table 3.1
Consumption studies assuming separability from labor supply

Authors	Data set	Preferences, commodity	Age	ln(needs)	child	adult	Other	*eis*
Hall and Mishkin (1982)	PSID 1968–1975	Quadratic, Food	3.89 (1.3)		167 (6.4)	242 (8.7)	Age^2	–
Zeldes (1989)	PSID 1968–1982	Iso-elastic, Food	–0.004 (0.002)	0.23 (0.06)			Fixed effects	–0.43 (1.22)
Lawrance (1991)	PSID 1974–1982	Iso-elastic, Food		0.29 (0.04)			Time dummies levels of demographics	–0.73 (0.70)
Keane and Runkle (1992)	PSID 1975–1982	Iso-elastic, Food	–0.002 (0.0002)				Fixed effects	–0.44 (0.14)
Shea (1995)	PSID 1981–1987	Iso-elastic, Food		0.29 (0.14)			Time dummies	–1.56 (1.4)
Attanasio and Browning (1995)	FES 1970–1986	Generalised iso-elastic, Nondurables	–0.14 (0.07)	0.24 (0.37)	–0.04 (0.08)	0.05 (0.14)	Household composition, age, age^2	Range –0.25 to –0.43
Lusardi (1996)	CEX and PSID 1980–1987	Iso-elastic, Nondurables	7×10^{-5} (3×10^{-4})	0.11 (0.02)				

the expected consumption variance under a normality assumption. Although the point estimates sometimes indicate large variations in the discount factor with presample income levels, these are not generally statistically significant (perhaps because of the correlation between income and education). Her preferred estimates [Lawrance (1991), Table 4] suggest that bottom income decile households have annual discount factors that are about two percentage points lower than top decile households and also that households with a college educated head have discount rates that are about two percentage points lower than otherwise comparable households. Even if one rejects the scale of her estimates because it relies on a normality assumption, her evidence of heterogeneity in discount rates is still valid.

There are few attempts to capture unobserved heterogeneity in the papers listed in Table 3.1. The only examples of explicit allowance for unobserved heterogeneity is the inclusion of a fixed effect in the Euler equation in Zeldes (1989) and Keane and Runkle (1992). In terms of Equation (3.26) these pick up the means (over time) of ε_β or ε_σ. Neither of the two papers finds that the fixed effects are statistically significant. This is at odds with the Lawrance evidence but it may simply reflect the problem of trying to estimate preference heterogeneity with a fixed effect which can include measurement error components as well.

It is difficult to credibly estimate subjective discount factors (that is, the parameters and distribution of ε_β in Equation (3.23)) from Euler equations. Yet the mean and distribution of the discount factor are, potentially, important elements in any DGE model. Where, then, can we look for estimates of the distribution of discount factors based on micro data? Recently two different approaches have been tried. Gourinchas and Parker (1996) use a mixture of calibration and estimation on the US CEX data to estimate the discount factor for groups characterized either by their education or their occupation. Their method exploits the changing relative strengths (suggested by theory) of the precautionary motive and the life-cycle (saving for retirement) motive as agents age. Their results depend on the shapes of the income and expenditure paths over the life-cycle and retirement "needs" rather than on period to period changes as in an Euler equation approach. They find that high school and college graduates have a discount factor of 0.96 and 0.97, respectively, when the real rate of interest is set to 3% and there is assumed to be no uncertainty in the return on capital. Although the estimates are sensitive to modelling assumptions (particularly the choice of the real rate used in the model) the use of lifetime income and expenditure patterns is a distinct improvement on the use of Euler equations to estimate the discount factor.

An even more promising approach is that of Samwick (1997) who uses wealth holdings at different ages as observed in the USA in 1992 to infer the underlying distribution of discount factors. For example, a household that is close to retirement and has a low wealth to permanent income ratio is inferred to have a high discount rate since it will necessarily have lower consumption in the retirement period. Samwick finds that the distribution has three components. First, about 70% of households are characterized by an approximately normal distribution centered on 5% with a standard deviation of about five percentage points (so that most agents in this group have discount rates of between −5% and 15%). Then there is a small group of wealthy households (about 5% of the total) who have large wealth and consequently large negative discount rates (below −15%). Finally there is a group (about one quarter of the total) who have very low wealth and consequently very high discount factors (above 20%). Although the numerical values of the estimates may not be robust to the modelling choices that Samwick makes (concerning initial assets, other forms of wealth, income processes, etc.), the evidence of heterogeneity in discount rates is likely to be robust and his approach merits further investigation.

Consider next the other important parameter, the *eis*. As can be seen from the final column of Table 3.1, many of these studies report an estimate of the coefficient on the real interest rate (= −*eis*) although these are usually not very well determined[50]. All of the PSID studies listed in Table 3.1 use cross-section variation in the real rate (due to differences in marginal tax rates) to aid in identification. As is typical of the entire microeconomic consumption and labor supply literature, most investigators do not introduce prices in their models and instead include time dummies that "wipe out" the real interest rate coefficient, so that it is open to question whether these estimates are genuinely estimating an intertemporal substitution effect. Attanasio and Browning (1995) use quarterly data from 17 years which gives a great deal of time series variation in the real rate of return to capital (the quarterly rate varies from −8.6% to 2.4%). They also estimate a more general form of the iso-elastic form that allows for an *eis* that varies with the level of consumption (here seen as a proxy for lifetime wealth) and demographics. They find that there is significant variation in the *eis* with demographics and also that the iso-elastic form is rejected against the more general form. Thus the *eis* is not constant over the population even when demographics are held constant. It is an open question as to whether this is due to everyone having the same generalized iso-elastic utility function or whether it reflects everyone having iso-elastic utility with unobserved variation in the parameters that is correlated with lifetime wealth (or some form of misspecification).

In Table 3.2 we present results on consumption for those studies that allow for non-separabilities between consumption and labor supply. Since the effects of other demographics are generally similar to those for the separable case, we concentrate here on the effects of non-separabilities between consumption and labor supply. One thing to note here is that most authors are more interested in qualitative results (for example, "is there excess sensitivity?" or "is consumption additively separable from male labor supply?") than in precise estimation of specific parameters. This is reflected

[50] One thing to bear in mind is that if we use different commodities then we should expect different estimates of the *eis*, see Atkeson and Ogaki (1996) and Browning and Crossley (1997). The latter show that if goods are additively separable within the period then the *eis* for a sub-component of non-durables (for example, food) is equal to the *eis* for non-durables as a whole multiplied by the (uncompensated) expenditure elasticity for the sub-component (this is an exact version of Pigou's Law that income elasticities are proportional to own price elasticities if preferences are additive). Thus we would expect that the *eis* in studies that use food as the consumption good should be, say, half of the *eis* in studies that use a broader measure.

in the fact that many of the estimates are rather implausible. The table also reflects the wide gulf between calibrators and micro-econometricians in that there are no Euler equation estimates of the most widely used functional form found in the DGE models that do not assume additivity between consumption and labor supply.

Of the five studies that present separability tests for male labor supply, three – Attanasio and Weber (1993), Blundell, Browning and Meghir (1994) and Attanasio and Browning (1995) – report significant complementarities between consumption and male labor supply. Moreover, the effects are quite large; for example, Attanasio and Browning (1995) have consumption falling 28% if an anticipated change from full-time work to no work takes place [51]. However, two other studies, Browning, Deaton and Irish (1985) and Meghir and Weber (1996), find that consumption is additively separable from male labor supply. The former study conditions on the real wage rather than labor supply, so that it is estimating a different parameter. The specification in Meghir and Weber (1996) is very different from the others used; they model three non-durable goods (food at home, transport, and services) rather than a single composite; they allow for interactions between the wife's labor force status and various parameters; they condition on other quantities ("food out", clothing, and fuel) and they use the panel aspect of the CEX. Although this evidence on the separability between consumption and male labor supply is a bit ambiguous, taken as a whole, it supports the notion that consumption and male labor supply are Frisch complements.

The evidence concerning female labor supply is even more mixed. Given the evidence that demands do not appear to be separable from female labor supply within the period [see, for example, Browning and Meghir (1991)], given the presumed presence of costs of going to work (including day care for children since this is included in "services"), and given the possibility of substituting home production for market purchases, it might be supposed that we would find strong complementarities between consumption (strictly, market purchases of non-durables) and female labor supply. However, this is not the case. Of the five studies that allow for non-separabilities between consumption and female labor supply, two [Altug and Miller (1990) and Blundell et al. (1994)] find them to be Frisch complements (but, for the former, only if it is assumed that male labor supply is additively separable from both consumption and female labor supply, which is contrary to the male labor supply results reported in the last paragraph); two others [Attanasio and Weber (1995) and Attanasio and Browning (1995)] find weak evidence in favor of substitutability and one [Meghir and Weber (1996)] finds very strong evidence of non-separabilities (which cannot be characterized in the usual way).

The papers listed in Table 3.2 also present estimates of the *eis* for consumption The largest effects are found in Blundell et al. (1994) [52]. They permit the *eis* to vary with the level of consumption labor force status, and demographics. Generally, the variation

[51] This is interpreted as being due to non-separabilities. A plausible alternative is that households in which the husband is out of work may be liquidity constrained and this explains the low consumption.

[52] Note, however, that they include a dummy variable for the post-1980 period and this "sharpens up" their results considerably.

Table 3.2
Consumption studies allowing for non-separabilities with labor supply

Authors	Data	Labor supply conditioning good	Other conditioning variables	Comment
Browning, Deaton and Irish (1985)	UK FES, 1970–1976, annual, quasi-panel, nondurables	Male real wage	Household composition, male occupation	No evidence of nonseparability between consumption and male labor supply. *eis* (holding male real wage constant) badly determined.
Altug and Miller (1990)	PSID, 1967–1980, annual, panel, food	Female labor supply	Household size	Either food consumption additively separable from male and female leisure or male leisure additively separable from food and female leisure (but not both). In latter case, food and female labor supply are complements. *eis* (holding both leisures constant) badly determined.
Attanasio and Weber (1993)	UK FES, 1970–1986, quarterly, quasi-panel, nondurables	Male and female labor supply	Seasonals, male occupation, household composition	Consumption and male labor supply complements but point estimate implausible (consumption in unemployment falls to one third of consumption in employment). Weak evidence of complementarity between consumption and female labor supply [Coefficient = 0.22 (se = 0.16)]. *eis* (holding both labor supplies constant) = −0.78 (standard error = 0.28).
Blundell, Browning and Meghir (1994)	UK FES, 1970–1986, quarterly, quasi-panel, six nondurable goods	Male and female labor supply	Age, household composition	*eis* depends on consumption level (rejects iso-elastic form) and demographics. For fixed demographics *eis* varies from −2.9 (first decile) to −0.96 (ninth decile). Mean *eis* = −1.21 (se = 0.13)

continued on next page

Table 3.2, *continued*

Authors	Data	Labor supply conditioning good	Other conditioning variables	Comment
Attanasio and Weber (1995)	US Consumer Expenditure Survey, 1980-1990, quarterly, quasi-panel, nondurables	Female labor supply	Household composition	Consumption and female labor supply are substitutes. Going from full-time to out of employment, consumption rises by 26% (standard error cannot be computed from reported estimates). *eis* (holding female labor supply constant) = −0.34 (se = 0.28)
Attanasio and Browning (1995)	UK FES, 1970–1986, quarterly, quasi-panel, nondurables	Male and female labor supply	Household composition	Consumption and male labor supply complements [full-time to unemployment gives a 28% fall in consumption (se = 11%)]. Consumption and female labor supply substitutes [full-time to unemployment gives a 7% rise in consumption (se = 4%)]. *eis* (holding male and female labor supply constant) depends significantly on level of consumption and labor supply.
Meghir and Weber (1996)	US CEX, 1980–1991, quarterly, panel, three nondurable goods	Male and female labor supply	Education, race, tenure, household composition, age, region, area of residence	Male labor supply additively separable from consumption. Female labor supply highly correlated with consumption of three goods. First/third quartile *eis* for individual goods: food, −0.79/+0.17; transport, −1.54/−0.48; services, −1.48/−0.42.

with consumption is greater than the variation with the other observable factors and quite high values (in absolute value) are found for some high consumption households; the range in the sample is from −2.9 (the first decile) to −0.96 (the ninth decile). If this dependence on consumption actually reflects persistent heterogeneity in the *eis*, then this may go some way to rationalizing the different portfolios that high consumption and low consumption households hold.

Four main conclusions concerning consumption can be drawn from these studies:

- The intertemporal allocation of consumption varies significantly with variations in demographics. In particular, consumption increases with household size.
- There is consistent evidence that consumption is complementary with male labor supply. The estimates suggest large decreases in consumption in response to an anticipated change from full-time work to no work.
- The evidence is mixed on the interaction between consumption and female labor supply. At present we believe that there are no reliable estimates of this interaction.
- The elasticity of intertemporal substitution (*eis*) is usually poorly determined. Two studies, however, find significant variation in the *eis* with demographics, labor force status, and the level of consumption, with quite high elasticities for some high consumption households. If constancy is imposed on the *eis*, then one characterization of the literature is that there is no strong evidence against the view that the *eis* for non-durables (holding labor supply constant) is a bit less than −1 and the *eis* for food is −0.5.

The first finding is problematic for calibrating infinitely-lived agent models using microeconomic data. The dynasty models are not designed for dealing with changes in household size over the life cycle. The second two findings, taken together, demonstrate that separate treatment of male and female labor supply is needed if microeconomic evidence is to be used in calibration. Yet, as noted in Section 1 of this chapter, the convention in the macroeconomics literature is to pool genders to exploit the rough constancy in hours per head across both groups and to calibrate preference and production parameters using the constancy of aggregate person hours per capita.

We next turn to the estimates of labor supply elasticities.

3.4. Labor supply

The literature on labor supply offers an interesting contrast with the literature on consumption just surveyed. There are only a handful of intertemporal labor supply estimates which we summarize here. In contrast, there is a vast predecessor literature on static labor supply models. The difficulty in using this literature arises from the lack of a clear framework for interpreting the estimates. Much of the literature proceeds by estimating a relationship between hours and wages, holding constant demographics and some measure of wealth or unearned income, but *not* holding fixed the marginal utility of expenditure, as is required to isolate the *eis*. In this section we show that the static literature provides a lower bound for the *eis* for labor supply.

3.4.1. Labor supply estimates

Table 3.3 presents the estimates of the Frisch labor supply elasticity for males, females, and the aggregate per capita labor supply of households. As is typical of the entire published literature, most of these estimates are based on hours worked within a year for continuously married prime age males who work some time in each year for long stretches of time – typically six to ten years. Only Heckman and MaCurdy (1980) estimate the Frisch labor supply elasticity for married females. In all of the male *micro* samples, observations with no earnings in the year are excluded. The estimated elasticities for males are small but are consistent with each other. For married females, the Frisch elasticity is much larger. Only Ghez and Becker (1975) examine whether the Frisch elasticity depends on demographics; they find that the Frisch elasticity declines with educational attainment for males. For black males, the Frisch elasticity is negative. Ghez and Becker (1975) also report evidence that suggests that male and female

Table 3.3
Estimates of elasticity of intertemporal substitution in labor supply

Authors	Estimate	Frisch labor supply elasticity	Dependence on demographics	
MaCurdy (1981)	Life cycle labor supply for males	[0.10, 0.40]	None	Assumed
Altonji (1986)	Life cycle labor supply for males	[0, 0.35] (for labor supply)	None	Assumed
Heckman and MaCurdy (1982)	Life cycle labor supply for females	(1.61)[a] (labor supply)	None	Assumed
Lucas and Rapping (1970)[b]	US aggregate time series (males and females); manhours (person)	1.40 (labor supply (short run)	None	Assumed
Ghez and Becker (1975)	Males	0.39 (whites) −0.106 (nonwhites)	Whites: Blacks:	0.49 (Grade School) 0.36 (high school) 0.30 (college) roughly constant

[a] This number is obtained from that reported by Heckman and MaCurdy (1982) who estimate a demand for leisure by multiplying their *eis* for leisure by 4 to obtain the implied elasticity of labor supply $H = T - L$:

$$\frac{\partial \ln H}{\partial \ln W} = -\frac{L}{T-L}\frac{\partial L}{\partial W}\left(\frac{W}{L}\right), \text{where, on average, } \frac{L}{(T-L)} \doteq 4 \text{ in their sample.}$$

[b] The interpretation of the Lucas and Rapping parameter as a Frisch parameter is based on MaCurdy (1985). Their estimate is for aggregate person hours and includes entry and exit. For this reason, their estimated elasticity is higher than the estimates that use workers who are continuously working. See Heckman (1978).

leisure time are direct substitutes in within-period household utility. Abowd and Card (1989) report very large and statistically imprecisely determined estimates of the Frisch elasticity for males. We do not discuss their study in detail because of the imprecision of their estimates. One interesting finding implicit in the Abowd and Card (1989) study is that the Frisch elasticity increases the smaller the time unit of labor supply that is analyzed: the Frisch elasticity is smallest on biennial labor supply observations and largest for six month intervals. This suggests that an instantaneously additively-separable model is inappropriate and that time is more substitutable over shorter intervals than longer ones. Further suggestive evidence on this point is given by MaCurdy's (1983) very high reported elasticities for monthly hours of work.

It is important to note that none of these elasticities account for the participation response [that is, none compute Equation (3.21) or an elasticity version of it][53]. They are all taste-constant responses to wages whereas the participation elasticity includes a component that arises from changing the sample composition of the tastes of workers through entry–exit decisions when wages change. The cross-section evidence we survey below suggests that the participation response elasticities are substantially higher.

There is a vast literature on static labor supply models that is not directly relevant to the calibration of macro general equilibrium models, although it is often, used this way especially by economists in public finance; see, for example, Auerbach and Kotlikoff (1987) or Fullerton and Rogers (1993). In their simplest form, static models relate hours worked at age t to wages at age t and either asset income or assets at t (A_t). The resulting labor supply equation is not captured by any of the four models of labor supply discussed in Section 3.2. It does not condition on current expenditure; it does not condition on current consumption; it does not condition on the marginal utility of wealth, although A_t may be a partial proxy for λ_t; it also does not condition on e_t but rather on earnings from asset income; and it typically does not allow for corner solutions.

To see what is estimated in this literature, consider a simple model in which consumption and leisure are contemporaneously additively separable. Suppose preferences for leisure are given by a simple iso-elastic specification. We initially assume perfect certainty with a constant real interest rate r and we restrict this rate to satisfy $\beta(1 + r) = 1$ where β is the subjective discount factor. Also, for the time being we abstract from cohort effects in wealth accumulation. Then we may write the λ-constant or Frisch function for leisure demand as

$$\ln l_t = \alpha_0 + \alpha_1 \ln w_t + \alpha_1 \ln \lambda_t + \varepsilon_t,$$

where $T - h_t = l_t$ and ε_t is a mean zero measurement error. Instead of running a regression based on this equation, run a misspecified regression of $\ln l_t$ on $\ln w_t$ and A_t:

$$\ln l_t = \alpha_0 + \alpha_1 \ln w_t + \alpha_2 A_t + (\alpha_1 \ln \lambda_t - \alpha_2 A_t + \varepsilon_t),$$

[53] Only Lucas and Rapping (1970) use total person hours as the dependent variable. Implicitly they estimate effects inclusive of entry and exit. See Heckman (1978).

where the expression in braces is the composite error term for the model. Observe that the true value of α_2 is zero, since once we condition on λ_t, wealth does not enter the Frisch labor supply function. We cannot, however, observe λ_t and consequently cannot condition on it. Instead, we include A_t in a misspecified regression equation.

By a standard specification-error analysis, least squares estimates of α_1, α_2, denoted by $\hat{\alpha}_1$ and $\hat{\alpha}_2$, and obtained from a cross section, converge under the usual regularity conditions to

$$\operatorname{plim} \hat{\alpha}_1 = \alpha_1 \left(1 + \frac{\sigma_{AA}\sigma_{w\lambda} - \sigma_{wA}\sigma_{A\lambda}}{|D|} \right),$$

where the covariances are formed over age groups in the cross section: $\sigma_{ww} = \operatorname{Var}(\ln w_t)$; $\sigma_{w\lambda} = \operatorname{Cov}(\ln w_t, \ln \lambda_t)$; $\sigma_{wA} = \operatorname{Cov}(\ln w_t, A_t)$ and $\sigma_{A\lambda} = \operatorname{Cov}(A_t, \ln \lambda_t)$; $\sigma_{AA} = \operatorname{Cov}(A_t, A_t)$ and $D = \begin{vmatrix} \sigma_{ww} & \sigma_{wA} \\ \sigma_{wA} & \sigma_{AA} \end{vmatrix} > 0$. Moreover,

$$\operatorname{plim} \hat{\alpha}_2 = \alpha_1 \left(\frac{\sigma_{ww}\sigma_{A\lambda} - \sigma_{w\lambda}\sigma_{wA}}{|D|} \right).$$

Some of these covariances can be signed. From concavity of preferences, $\sigma_{A\lambda} < 0$ (diminishing marginal utility of wealth); assuming that goods and leisure are normal in all periods, $\sigma_{w\lambda} \leqslant 0$ and in a cross section, it usually happens that $\sigma_{wA} > 0$ (higher wage people have higher wealth). Thus, in general, the bias of $\hat{\alpha}_1$ for α_1 is ambiguous. Observe, however, that there is no bias if income effects are small. If they are negligible, $\sigma_{w\lambda} = \sigma_{A\lambda} = 0$ and the cross-section wage elasticity in a leisure demand equation recovers the Frisch labor supply parameter. Moreover, in an environment of perfect certainty, the weaker the correlation over time in wages, the weaker is $\sigma_{w\lambda}$, the more likely is the bias of $\hat{\alpha}_1$ for α_1 to be downward and the more likely is $\hat{\alpha}_2$ to be negative.

If A_t is omitted from the model, as was the common practice in many of the cross-section studies surveyed in Table 3.4, the OLS estimate of α_1 when λ_t is omitted from the regression ($\tilde{\alpha}_1$) has:

$$\operatorname{plim} \tilde{\alpha}_1 = \alpha_1 \left(1 + \frac{\sigma_{w\lambda}}{\sigma_{ww}} \right), \tag{3.27}$$

which is definitely downward biased since $\sigma_{w\lambda} < 0$. From $\tilde{\alpha}_1$, we can produce a lower bound on α_1 [54].

The preceding arguments remains valid if there are cohort effects or if we introduce age effects by relaxing the restriction that $\beta(1 + r) = 1$ back into the model, provided that the assumed properties of the covariances also apply to age-adjusted versions of

[54] A version of this bounding argument was first presented in Heckman (1971). The argument is more general and does not require contemporaneous additive separability.

Table 3.4
Cross section labor supply estimates[a]

Study	Group (dimension)	e	MPE	η
1. Ashenfelter and Heckman (1973)	Males (annual hours)	−0.16	−0.27	0.12
2. Boskin (1983)	Males (annual hours)	−0.29	−0.41	0.12
3. DaVanzo, DeTray and Greenberg (1973)	Males (annual hours)	−0.15	−0.004	−0.14
4. Dickinson (1974)	Males (annual hours)	−0.11	0.08	−0.19
5. Kosters (1966)	Males (annual hours)	−0.09	−0.14	0.04
6. Burtless and Greenberg (1983) (Survey NIT, males)	Males (hours/week)	0.0043	−0.0757	0.0795
7. Moffitt and Kehrer (1981)	Males (hours/week)	[0.09, 0.16]	[−.09, 0]	[0, 0.16]
8. Burtless and Greenberg (1983) (Survey NIT, females)	Wives (inc. N.J.) (excl. N.J.)	−0.0420 0.0957	−0.1515 −0.0957	0.1105 0.1907
Experiments (hours/week)	Females (hours)	−0.0373	−.1709	0.1346
9. Moffitt and Kehrer (1981)	Females (hours)		[-0.34, 0]	[0.08, 0.42]
10. Nakamura and Nakamura (1981)	Married females (generalized tobit)	0.27 (30–34) −0.17 (35–39)[b] −0.05 (40–44)	−0.50 −0.05 −0.19	0.23 −0.12 0.14
11. Mroz (1984)	Females (annual hours)	[0.02, 0.09]	[0.013, 0.016]	[0.03, 0.074]

continued on next page

Table 3.4, *continued*

Study	Group (dimension)	e	MPE	η
12. Juhn, Murphy and Topel (1991), CPS-67–87	Males (weeks worked) (CPS)	0.299 (1–10)[c] 0.232 (11–20) 0.186 (21–40) 0.139 (41–60) 0.062 (61–100)	– – – – –	– – – – –
13. Bowen and Finegan (1969)	Males (labor force participation rate) (1960)	0.06	–0.20	0.26
14. MaCurdy, Green and Paarsch (1990) Tax & labor supply; "New" and "Old"	Male labor supply, tax-adjusted (annual hours)[d]	$[-0.31, -0.267]$ 0	$[0.09, 0.14]$ 0	$[-0.41, -0.41]$ 0
15. McElroy (1981) Survey	Household (survey hours)	$e_M \in [-0.261, -0.130]$ $e_F \in [-.0075, 0.067]$ $e_{FM} \in [-.0147, -0.091]$ $e_{MF} \in [-.0013, 0.031]$ $S_{MF} > 0 > S_{FM}$ NSD: rejected		

[a] Equations: $\frac{\partial H}{\partial W} = S + H\frac{\partial H}{\partial Y}$ (= Hicks-uncompensated); S = Hicks-compensated effect.
$e = \left(\frac{W}{H}\frac{\partial H}{\partial W}\right) = \eta + W\frac{\partial H}{\partial Y}$, in elasticity form, where $\eta = \left(S\frac{W}{H}\right)$; $MPE = W\frac{\partial H}{\partial Y}$.

[b] Age group in parentheses.

[c] Wage quantile in parentheses.

[d] First line: unconstrained; second line: Slutsky constrained.

them. In the case of secular exponential growth in wealth across cohorts, the coefficient on age includes a growth rate of wealth term. Provided that $\sigma_{\lambda w} < 0$, the argument also extends to an uncertain environment with less than full insurance.

In Table 3.4 we present cross-section estimates from 15 studies. The estimates correspond to $\hat{\alpha}_1$ or $\tilde{\alpha}_1$ above. The first nine studies in the table produce estimates of $\frac{\partial \ln h}{\partial \ln w} = -\left(\frac{T-h}{h}\right)\frac{\partial \ln l}{\partial \ln w}$, where $T - h = l$. Thus, the upper bounds for Frisch leisure demands are lower bounds for Frisch labor supply elasticities The first nine studies are the traditional least squares estimates which are plagued by bad data on asset income. All but one of the numbers reported in the first nine rows of this table are below the estimates of the Frisch labor supply elasticity reported in Table 3.3 vindicating our analysis and suggesting that the cross-section labor supply estimates constitute lower bounds for the Frisch labor supply parameter.

Studies (10) and (11) of Table 3.4 report that cross-sectional annual-hours-of-work labor supply elasticities for married women are as negative as they are for married men. Studies (12) and (13) reveal that labor supply elasticities are more positive when participation is the dependent variable. (But recall from our earlier discussion that these elasticities are not the *eis* or the Hicks–Slutsky elasticity.) This point is emphasized in the studies of Heckman (1978, 1993) and Blundell and MaCurdy (1999). Study (12) reveals that the curvature of labor supply is more elastic the lower the wage. Study (14) attempts to replicate the influential cross-sectional estimates of Hausman (1981), which are sometimes used by macroeconomists as measures of the Frisch parameter. His estimates are much higher than the other estimates reported in the table. In a careful analysis, the authors of study (14) are unable to replicate Hausman's reported estimates using his own data. Instead their unrestricted estimates of labor supply elasticities are negative and inconsistent with utility maximization. When the restrictions of utility maximization are imposed, the model exhibits a zero labor supply elasticity for males. It is the estimates in row (14), and not Hausman's (1981), that fall in line with the other estimates reported in the tables and should be used for evidence on uncompensated static labor supply.

Study (15) summarizes the literature that establishes that the traditional labor supply model does not satisfy Slutsky symmetry or integrability conditions. However, Heckman (1971) establishes that when the cross-section model is embedded in a life-cycle setting, the evidence is much stronger for the traditional household model of labor supply.

3.5. *Heterogeneity in the marginal rate of substitution between goods and leisure*

Beginning with the work of Heckman (1974), and continuing on in the work of Burtless and Hausman (1978), Hausman (1981), MaCurdy (1983) and MaCurdy, Green and Paarsch (1990), labor economists and econometricians have estimated the extent of variability in the marginal rate of substitution between goods and leisure.

Heckman (1974) builds a life-cycle model of labor supply, asset accumulation, and the demand for child care for an environment of perfect certainty. In his model the marginal rate of substitution between goods and leisure is explicitly parameterized and allowed to depend on both observed and unobserved factors. In later work, MaCurdy (1983) applies and extends this framework to an environment of uncertainty using Euler equation methods. Other researchers have used static one-period models of labor supply to document heterogeneity in the preference for leisure. For brevity we only summarize the static labor supply evidence reported by Heckman (1974), who presents the clearest evidence on preference heterogeneity.

The marginal rate of substitution function (or slope of the indifference curve) at a given level of prework income Y is

$$m = m(Y, h), \tag{3.28}$$

where we ignore variations in the prices of other goods and where h is hours of work[55]. A consumer possesses a family of indifference curves indexed by level indicator Y, the no-work level of income or consumption We know that $\partial m/\partial Y > 0$ if leisure is a normal good. From diminishing marginal rate of substitution between goods and time, we know that $\partial m/\partial h > 0$.

If a consumer faces a parametric wage w, at initial income position Y, she works if $w > m(Y, 0)$. If this inequality applies, the consumer's decision of how much to work is characterized by

$$w = m(Y^*, h^*), \tag{3.29}$$

where Y^* is a level index appropriate to the indifference curve and is the amount of income (or consumption) that would make the consumer indifferent to a choice between working h^* hours at wage rate w to gain total resources $wh^* + Y$, or not working and receiving income Y^*. Without knowledge of Y^*, one cannot deduce the relationship between w and h^* predicted by consumer optimization. However, given Y^*, we know that optimality also requires that h^* satisfy

$$wh^* + Y = \int_0^{h^*} m(Y^*, h)\,\mathrm{d}h + Y^*. \tag{3.30}$$

From Equation (3.29), if m is monotonic in Y^* we may solve for Y^* as a function of w and h: $Y^* = g(w, h)$. Using this value in Equation (3.30), we implicitly define the labor supply function by

$$wh + Y = \int_0^{h} m\,[g(w, s), s]\,\mathrm{d}s + g(w, h).$$

[55] In the life-cycle version of the model, Y is determined by a two-stage budgeting argument.

Table 3.5
Parameter estimates of marginal rate of substitution between goods and leisure for blacks and whites including and excluding wife's education

Parameter	Estimate							
	Whites				Blacks			
	Excluded (1)		Included (2)		Excluded (3)		Included (4)	
Intercept, α_0	−0.046	(−2.83)	−1.2453	(3.37)	0.068	(1.19)	−0.233	(−1.56)
Price effect of child care	0.143	(2.66)	0.110	(2.05)	2.97	(8.11)	0.237	(9.6)
Income term, α_1	0.22×10^{-4}	(2.62)	0.198×10^{-3}	(2.43)	0.13×10^{-4}	(1.43)	0.12×10^{-4}	(14.9)
Hours effect, α_2	0.51×10^{-3}	(4.88)	0.487×10^{-3}	(3.83)	0.43×10^{-3}	(4.2)	0.314×10^{-3}	(5.98)
No. of children aged 0–3, α_{31}	0.243	(3.32)	0.234	(2.95)	0.124	(5.26)	0.076	(5.84)
No. of children aged 4–6, α_{32}	0.084	(2.11)	0.081	(2.59)	0.040	(1.82)	0.022	(1.37)
No. of children aged 7–10, α_{33}	0.020	(0.66)	0.032	(2.09)	−0.036	(−2.1)	−0.031	(1.37)
Wife's education, α_{84}	–	–	0.100	(14.5)	–	–	0.054	(2.14)
Flow coefficients on assets, S	0.038	(1.29)	0.024	(0.88)	0.159	(0.05)	0.129	(0.89)
Standard deviation, σ_u	0.668	(6.73)	0.598	(5.1)	0.607	(8.4)	0.492	(14.6)

[a] Source: Heckman (1974).
Group is married, spouse present, with at least one child under 10. t-statistics given in parentheses are asymptotic normal statistics estimated from the information matrix.

A wide variety of functional forms may be used to specify m. Heckman's (1974) preferred specification is

$$\ln m = \alpha_0 + \alpha_1 Y + \alpha_2 h + \alpha_3 Z + u, \tag{3.31}$$

where Y is the prework level of income, h is hours of work, and Z is a vector of variables to be discussed more fully below. A random variable designated "u" with zero mean and variance σ_u^2, reflects variation in preferences for work among individuals. The previous analysis leads us to the prediction that $\alpha_1 > 0$ (normality of leisure) and $\alpha_2 > 0$ (diminishing marginal rate of substitution between goods and leisure). The resulting static labor supply function is implicitly defined by

$$u = \ln w - \alpha_0 - \alpha_2 h - \alpha_3 Z - \alpha_1 \left[wh + Y - \frac{w}{\alpha_2}(1 - \mathrm{e}^{-\alpha_2 h}) \right]. \tag{3.32}$$

This labor supply curve can become backward bending beyond a certain value of hours worked.

Estimates for this specification are presented in Table 3.5 where the price of child care is also introduced as a determinant of the marginal rate of substitution. This table reveals that the estimated marginal rate of substitution function for married women has considerable heterogeneity depending both on observed variables (education, the number of children and the wealth level of the family) and on unobservables. Children, education, and asset income all raise the value of leisure. The standard deviation in the unobservables is also large. This heterogeneity arises from variation over people and not sampling variation. Figure 3.1 plots the median, third quartile and first quartile standard deviation in the population marginal rate of substitution for a group with median non-labor income and other characteristics, excluding education from preferences. Population variation in tastes is a central empirical regularity in the micro literature. Moreover, as documented in Heckman and Willis (1977) and Heckman (1982), these tastes are stable over time giving rise to persistence in employment status over time.

Summary and conclusion

This chapter has documented the empirical evidence supporting heterogeneity in preferences, constraints and skills and some of their consequences for modern macroeconomics. It has also examined the problems of measuring uncertainty from time series processes of earnings and wage functions. Finally, it has discussed the problem of extracting the parameter values required in dynamic general equilibrium theory from the "large shelf" of micro estimates. The gulfs between the theoretical environments presumed by the dynamic economic models and estimation environments used by empirical microeconomists leaves the shelf of directly usable numbers virtually empty. What is there requires careful interpretation and qualification.

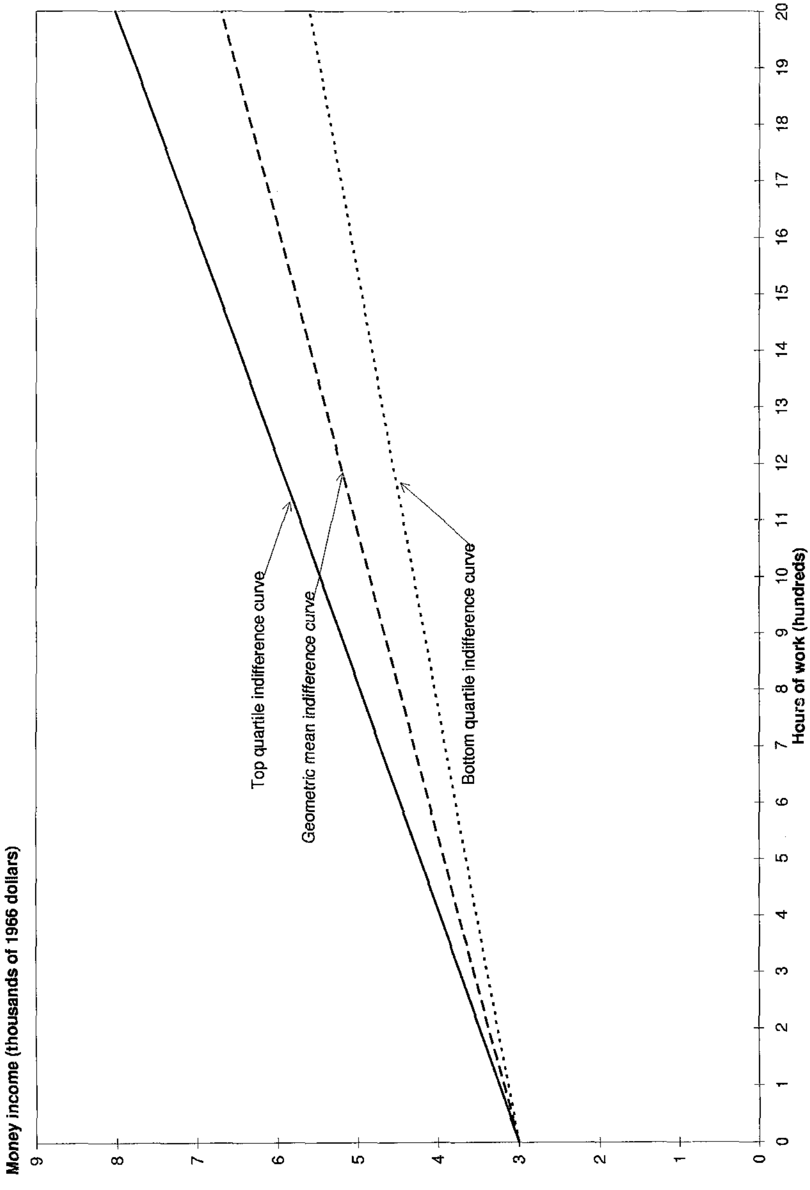

Fig. 3.1. Estimated indifference curves between goods and leisure. Data from Heckman (1974), Fig. 4.

While dynamic general equilibrium models may suggest new directions for empirical macroeconomic research, it is essential to build the dynamic economic models so that the formal incorporation of microeconomic evidence is more than an afterthought. Macroeconomic theory will be enriched by learning from many of the lessons from modern empirical research in microeconomics. At the same time, microeconomics will be enriched by conducting research within the paradigm of modern dynamic general equilibrium theory, which provides a framework for interpretation and synthesis of the micro evidence across studies.

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