Human Capital 34300 by Professor Gary S. Becker

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Initial Remarks

- Human capital, because it induces productivity, is very important in modern economies.
- Most of knowledge comes from investment.
  - Knowledge is an investment process.
- 75% of all human capital determines the income of any person.
  - However, human capital is not accounted in "National Income accounts."

Human capital is the link between parent and child: it is hard to get more micro than that. Forms of human capital are deeply complimentary.

- High educated
- High health
- High training
- Better consumers:
  - More contraception
  - Better diet
  - Less marriage breaks
  - Longer marriages

- In general, adapt faster to better technologies

- **These statements are true for developing countries**

  – Better educated people report they are happier.

  – Less education people report more stress (¿broke marriages?)

Interesting exception: more educated women, less marriage.

- Some differences between physical and human capital:

  1. Endowments and genetic structures are part of human capital
  2. Human capital cannot be bought or sold (slavery is forbidden)
  3. There are no markets of stocks for human capital
  4. It is very difficult to use human capital as collateral

- Richer families' guys have an advantage

  – Since human capital builds on itself (recursive process), public policymakers should get guys young:

  – $H_t + z = \psi(\cdot, H_t)$

    * That human capital is *intensive* in human capital is not only a technical point

- 1980s $\rightarrow$ benefits of going to school increased a lot.

- Relative costs of education went up: tuition went up.

Investment by parents on their own children

Early and late human capital are compliments: children build on their own investment

- Heckman
  - Cognitive skills
  - Noncognitive skills
    - Early
  - Noncognitive skills

Unisex adult model

Assumptions

- One adult
- One child
- Two periods: childhood and adulthood

One overlapping period:

Time

t adult parent/child (overlapping period)

$t+1$ child becomes adult/parent $= 0$ (dies)

One decision: how many to consume/how much to invest in children.

$$\underbrace{C_p}_{\substack{\text{parental}\\\text{consumption}}} + \underbrace{Y_c}_{\substack{\text{children}\\\text{goods}}} = \underbrace{W_p}_{\substack{\text{parental}\\\text{income}}} \qquad \text{parental budget constraint}$$

- There is no way for children to repay their parents
  - Selfish parents: $Y_c^* = 0$
  - If all parents are selfish, we can go home

- We are interested in modeling altruistic parents, whose utility function is:

$$U_p(W_p) = U(C_p) + aV(W_c), \frac{dV}{dW_c} > 0 \quad \frac{d^2V}{dW_c^2} \leq 0$$

where W_c is the earnings of the child when he becomes an adult.

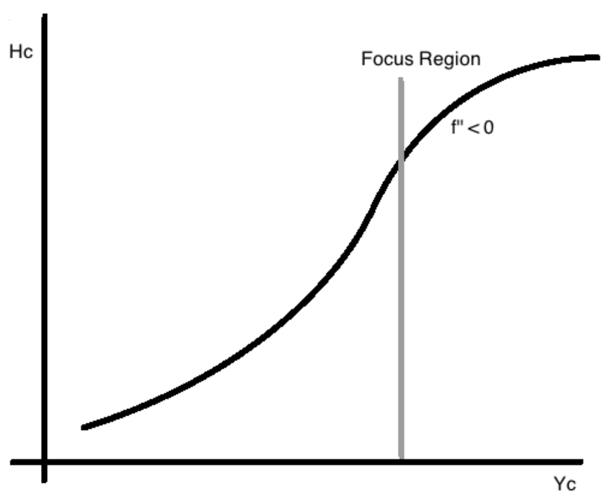
a, degree of altruism, i.e., $a = 0$ implies that the parents are selfish

Human capital of kids

$H_c = f(Y_c)$, household production function for human capital $\frac{dH_c}{dY_c} \neq 0, \frac{d^2H_c}{dY_c^2} \neq 0$, household production function for human capital (the possibility is equal to 0)

The following situation may be possible

Figure 1



You can think of it, by now, in the following way:

$H_c = f(Y_c, \overline{B})$, where $\overline{B}$ is the "fixed brain of the kid." And, therefore, it is more intuitive that:

$$\frac{dF(Y_c)}{dy_c} \geq 0 \quad \text{and} \quad \frac{d^2f(Y_c)}{dy_c^2} \leq 0. \quad \text{(diminishing marginal returns)}$$

Assumption: All families have same production function.

- What determines the earnings of the kids?

  $W_c = RH_c = Rf(Y_c)$; R converts human capital into earnings.

  $R = \psi(\sum_{t \in T} H_t, \text{ technology}, \text{ physical capital})$

Assumptions:

R is constant for each household.

R is "competitive," i.e., a household alone can't change it.

R is taken by parents as given by the market environment.

- In this model, earnings only differ because of human capital, i.e., by goods spent on children. Gaps may differ by economic sector, but this always holds.

Economic Problem

$$\underset{y_c, C_p}{\text{Max}} \;\; \{U_p(W_p) = U(C_p) + a \cdot v(W_c)\}$$

$$\text{s.t} \;\; C_p + y_c = W_p$$

Solution:

$$\mathcal{L} = U_p(\cdot) - \lambda \left[C_p + y_c - W_p\right]$$

F.O.C.'S:

$$U'(C_p) = \lambda, \quad a\frac{dV(W_c)}{dy_c} \leq \lambda$$

- Notice $W_c = 2H_c = 2f(y_c)$

If < 0, $y_C^* = 0$ when?

- if $a = 0$

- if income of parents is too low

Rewriting:

$$a \cdot \frac{dV(W_c)}{dW_c} \cdot \frac{dW_c}{dH_c} \cdot \frac{dH_c}{dy_c} \leq 0 \implies aV' \cdot \underbrace{Rf_y}_{\substack{\text{intuition} \\ \text{quetanto un peso} \\ \text{mas de inversion} \\ \text{be ``retorna''}}} \leq \lambda$$

$$\implies \frac{dW_c}{dy_c} = lti = R_y$$

$$; i \quad \text{marginal rate of return of investment in kids}$$

$$aV'c \underbrace{R_y}_{\substack{\text{marginal} \\ \text{rate of} \\ \text{return}}} \leq \lambda = U'(C_p) \implies \underbrace{\frac{U'(C_p)}{aV'c}}_{\substack{\text{HRS of} \\ \text{parents}}} \geq R_y$$

If $a > 0$, there is an interior solution: let's assume that parents provide at least food and shelter to kids.

Figure 2

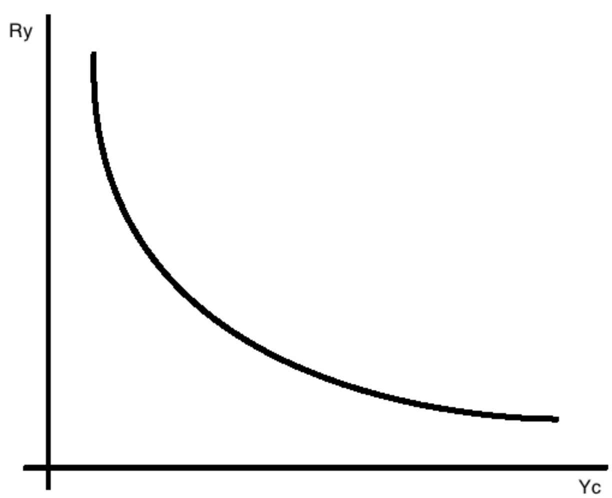
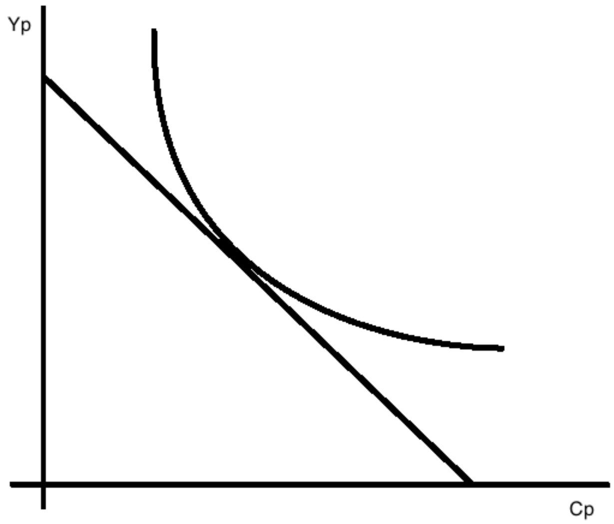


Figure 3



Two basic equations:

$$\begin{aligned} arf_yV_c' - U'(C_p) = 0 \quad \ldots \quad & g_1(y_c, C_p; a, r, W_p) \\ C_p + y_c - W_p = 0 \quad \ldots \quad & g_2(y_c, C_p; a, r, W_p) \end{aligned}$$

- What happens when W_p increases to y_p and to C_p?
  - What happens to endogenous "choice variables" when parameters change, in this case W_p?

For notation, let

$$g = \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} \quad \text{and} \quad x = (C_p, y_c)$$

$$g = (a, r, W_p)$$

The general implicit function theorem establishes: notice, $\quad g_1 = arf_y V_c'(rf(y_c)) - U'(C_p)$

$$\begin{bmatrix} \frac{dC_p}{dW_p} \\ \frac{dy_c}{dW_p} \end{bmatrix} = - \begin{bmatrix} \frac{dg_1}{dc_p} & \frac{dg_1}{dy_c} \\ \frac{dg_2}{dc_p} & \frac{dg_2}{dy_c} \end{bmatrix}^{-1} \begin{bmatrix} \frac{dg_1}{dW_p} \\ \frac{dg_2}{dW_p} \end{bmatrix}$$

$$= - \begin{bmatrix} -U''(c_p) & ar[f_{yy}V_c' + rf_y f_y V_c''] \\ 1 & 1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

$$= \frac{-1}{-U''(c_p) - ar[f_{yy}V_c' + rf_y f_y V_c'']}(ar = 0 > 0) \cdot \begin{bmatrix} 1 & ar[f_{yy}V_c' + rf_y f_y V_c''] \\ -1 & -U''(c_p) \end{bmatrix} \cdot \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-ar[f_{yy}V_c' + rf_y f_y V_c'']}{0} \\ \frac{-U''(c_p)}{0} \end{bmatrix} > 0$$

Make sense because goods are normal and this change in W_p is an income effect.

- Intergenerational income mobility: studies relation between parents and children income. It is said that there exists regression to the mean mobility in this matter:

$$ln \;\; W_c = \alpha + hln \;\; W_p + \epsilon_c$$

if : $\quad h = 1$ no regression to the mean

$h < 1$ regression to the mean

$h > 1$ regression away from the mean.

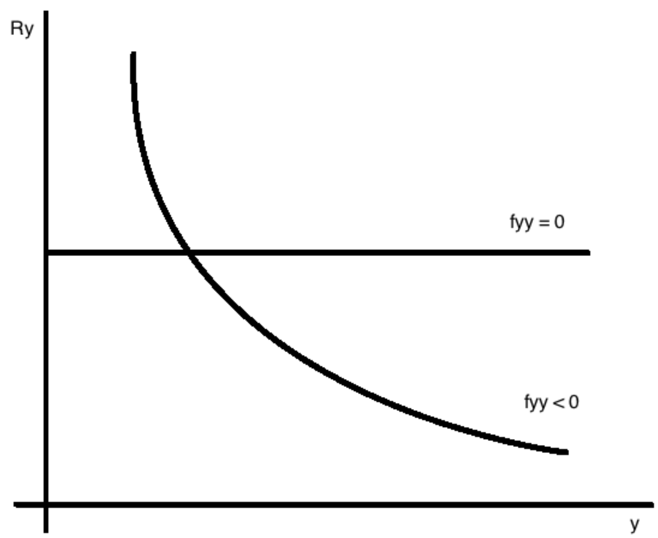
  - Continuing with the discussion about the previous model ...

What happens with the specifications of the function $H_c = f(y_c)$. Remember, $W_c = rf(y_c)$, and we are supposing r is competitive. If $f_{yy} = 0$, suppose $H_c = Ky_c; K \in R_{++}$. The situation with the rate of return is the following. We saw the rate of return is $R_y = rf_y$. In this case $f_y = K$, so R_y is the same for all the families, rK. This means that "capital markets are working perfectly".

  - Now, imagine we have $f_{yy} < 0$. E.g. ky_c with $r < 1$. Then, $R_y = rf_y$ and $\frac{dR_y}{dy} = rf_{yy} < 0$.

In this case, there is an inefficiency: parents can't lend to other poor kids. There is a "price effect", diminishing returns on investment: "invest more rises the cost of investing". Graphically :

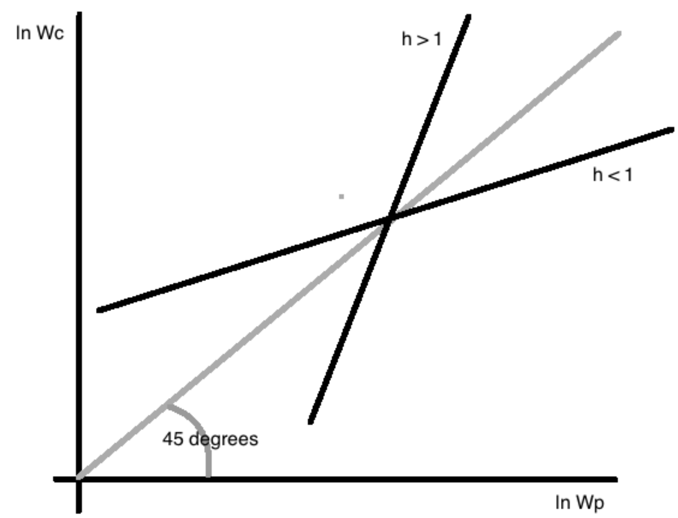
Figure 4



R_y^*(poor) $>$ R_y^*(rich). Rates of return are different and there is no "lending access". Capital markets are not working perfectly.

– returning to the intergenerational income mobility aspects. Take the regression $lnW_c = \alpha + hlnW_p + 2c$. Since we prove that $\frac{dy_c}{dW_p} > 0$, then it should be clear that $\frac{dW_c}{dW_p} > 0$, so $h > 0$. But, what happens with regression to the mean:

Figure 5



- In this context, regression to the mean means that if a parent is an average richer, the kid is also an average reacher but less. The literature says that $h \approx .5 - .b$.

What happens with poverty? Take the variance as a measure of poverty. Then $\text{VAR}(lnW_c) = h^2\text{VAR}(lnW_p) + \text{VAR}(2)$.

In equilibrium, $\text{VAR}(lnW_c) = \text{VAR}(lnW_p)$ and $\text{VAR}(lnW_c) = \frac{\text{VAR}(V)}{1-h^2}$. If $h = 1$, the poverty process is "not stationary".

- Some people speak about "underclass". Born poor, stay poor. At least from one generation to another.

- It will make a lot of sense to suppose that the human capital of the kids is also depending on the human capital of the parents. That is: $H_c = f(y, H_p)$, where we are going to take H_p as a constant (there is just one overlapping generation). The "intuitively economic" assumptions of this process are:

$$\frac{dH_c}{dH_p} > 0 \quad \text{or} \quad \frac{df(y_c, H_p)}{dH_p} > 0 \quad \text{and} \quad \frac{d^2 f(y_c, H_p)}{dy_c dH_p} > 0$$

Think of this: $H_c = \overbrace{f}^{\text{same for all families}}(\underbrace{y_c}_{\text{choose}}, \underbrace{H_p}_{\text{given by g.parents}})$, different household technology for each individual.

- This is a recursive property:
  1. As a person increases her human capital she affects her late.
  2. Economy's production today depends on how much you start with. This is a process that you build in what you have.
- Recall that diminishing returns to investment in kids, $f_{yy} \leq 0$. (more information, harder and harder to get it given a brain $\overline{B}$).
  * **one special case**: Suppose $H_c = \psi(y_c)H_p$; $\frac{d\psi(\cdot)}{dy_c} >, \frac{d^2\psi(\cdot)}{dy_c{}^2} \leq 0$.

    Let's see an "economic growth" consequence of this:
  * $\underbrace{\frac{dH_c}{dy_c} = \psi_y(\cdot)H_p}_{\text{more investment, more human capital.}}, \underbrace{\frac{d^2H_c}{dy_c dH_p}\psi_y > 0}_{\text{complementarity}}$
    - $\frac{H_p}{H_c} = 1 + g_H = \psi(y_c)$, so if you hold y_c constant, growth rate will be constant for the countries.

- This is saying that poor economies have it difficult to catch up. (inequality maintains itself constant over time, because growth is constant).
- If we analyse world per capita income, richer countries have not become richer.

  Now, let's analyse the parental problem with this modification:

$$\underset{c_p, y_c}{\text{Max}} \quad U(c_p) + aV_c(rf(y_c, H_p))$$

$$[cp] : U'(c_p) = \lambda \quad [y_c] = a \cdot V_c' \cdot r \cdot f_y = \lambda$$

The first order conditions are the same, but in this case $g = (a, r, W_p, H_p)$, $\quad x = (c_p, y_c)$.

It might be interesting to know what happens when H_p changes. This is:

$$\begin{bmatrix} \frac{dc_p}{dH_p} \\ \frac{dy_c}{dH_p} \end{bmatrix} = - \begin{bmatrix} \frac{dg_1}{dc_p} & \frac{dg_1}{dy_c} \\ \frac{dg_2}{dc_p} & \frac{dg_2}{dy_c} \end{bmatrix}^{-1} \begin{bmatrix} \frac{dg_1}{dH_p} \\ \frac{dg_2}{dH_p} \end{bmatrix}$$

Notice,

$$ar f_y(y_c, H_p) V_c'(r f(y_c, H_p)) - U'(c_p) = 0 \quad \ldots \quad g_1$$

$$c_p + y_c - W_p = 0 \quad \ldots \quad g_2$$

$$\begin{bmatrix} \frac{dg_1}{dH_p} \\ \frac{dg_2}{dH_p} \end{bmatrix} = \begin{bmatrix} ar[f_{yH} V_c' + f_y \cdot V_c'' \cdot r f_H] \\ 0 \end{bmatrix}$$

Set $\frac{dW_p}{dH_p} = r = 0$ by taxing. ; $W_p = rH_p = rf(y_p)$

$$\begin{bmatrix} \frac{dc_p}{dH_p} \\ \frac{dy_c}{dH_p} \end{bmatrix} = \frac{-1}{0} \begin{bmatrix} 1 & -ar(f_{yy} V_c' + r f_y f_y V_c'') \\ -1 & -V''(c_p) \end{bmatrix} \cdot \begin{bmatrix} ar[f_{yH} V_c' + f_y V_c'' r f_H] \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{\overbrace{-ar f_{yH} V_c'}^{(4)<0} - \overbrace{ar f_y V_c'' r f_H}^{>0(3)}}{0} \\ \overbrace{+\underbrace{(ar f_{yH} V_c')}_{>}}^{>0(2)} \overbrace{+\underbrace{(ar f_y V_c'' r f_H)}_{<}}^{<0(1)} \end{bmatrix} \ldots \text{can't sign}$$

$\frac{dy_c}{dH_p}$: (1) “Income effect” (feels reacher because H_p increases), consume more for himself. Since the budget constraint is fixed, “eat more for me, as parent”.

$\frac{dy_c}{dH_p}$: (2) "Substitution effect" (changing the prices of investing) pushing you to invest more on kid capital

$\frac{dc_p}{dH_p}$: reversed.

- This is why the question, do more educated parents spend more time (y_c) with their kids?, remains unclear theoretically.
- Empirically, measuring the magnitude of the "income effect" and the "substitution effect", more educated parents spend more time (y_p) with their kids.
- Two properties of "kids markets" are the following:
  (recall $f(y, H_p)$ and $f_y > 0, f_{yy} < 0$).
  - you **do affect** them when you give them things (towards negating competition)
  - every household is having its own production function.

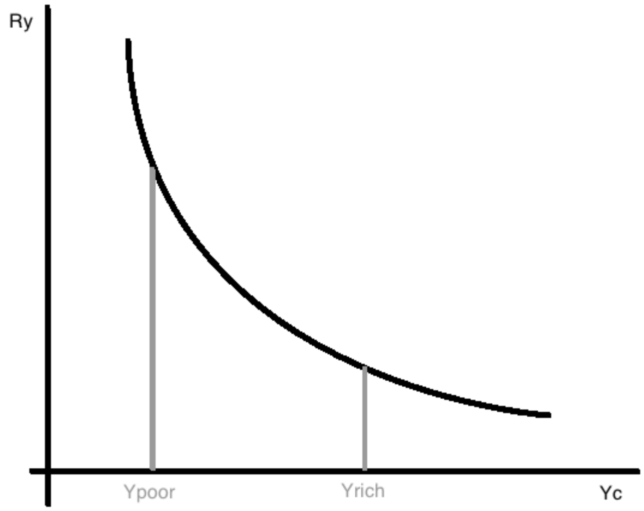
Think: $H_c = f(y_p, y_c, H_{gp})$, and so on ...

Equity v.s. efficiency:

If $H_c = f(y_c)$:

There is no trade-off between efficiency and equity: go ahead and give to the poor because they have more R_y.

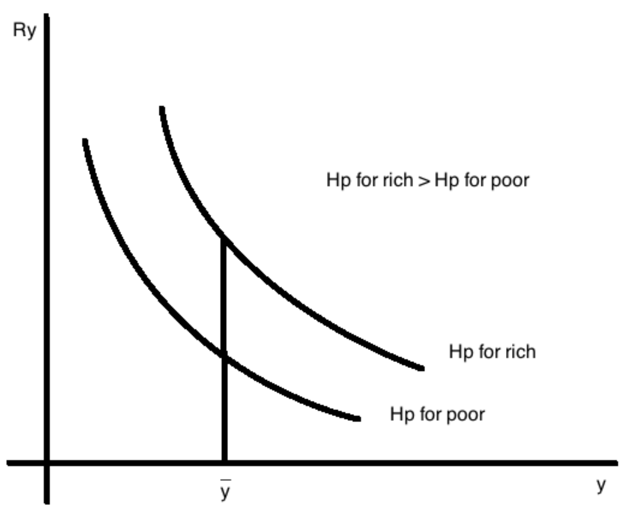
Figure 6



If $H_c = f(y_c, H_p)$:

- For g, it is more efficient y_c spending for the rich, but this varies across different $y's$.

Figure 7



Ability (may be important in regression to the mean)

- How to introduce ability:

$$A_c = (1-h)\overline{A} + hA_p + V_c \quad E[V_c] = 0 \quad E[A_c] = E[A_p] = \overline{A}$$

$h \to$ degree of inheritability: $\quad 0 \leq h \leq 1.$

In our analysis we assume there is a single dimension abilities and is early determined.

- Why is ability important?
  - determines inequality (e.g. earnings in cohort)
  - intergenerational mobility

Bring in abilities: $\quad H_c = f(y_c, H_p, A_c); \quad \frac{df}{dA_c} = f_A > 0$

- If you are more able investment is better for you: $f_{yA} \geq 0$
- df_{HA}? not going to work much.
  - What happens if A_c raises?

$$\frac{dW_c}{dA_c} > 0 \quad \frac{dH_c}{dA_c} > 0, \quad \text{but} \qquad \frac{dy_c}{dA_c} < 0 \quad \text{income effect}$$

$$R_y = rf_y \quad \text{raises} \qquad \frac{dy_c}{dA_c} > 0 \quad \text{substitution effect}$$

- You can't sign neither $\frac{dc_p}{dA_c}$.

Intergenerational perspective

Parental ability: $\quad \frac{dW_p}{dA_p} > 0, \quad \frac{dH_p}{dA_p} > 0.$

- Ability increases means more earning and more human capital.
- Higher ability of parents will increase H_c of child through effect on income on H_p of parents (indirect channel). There may also be a direct channel. Assume a transmission

mechanism:

$$A_c = \underbrace{(1-h)\overline{A}}_{\substack{\text{average ability}\\ \text{of society}}} + hA_p + V_c$$

h, degree of inheritability of adult to child

V_c, random determinant of ability of child. (independent across generations and individuals).

If $0 < h < 1$, regression to the mean.

If $h = 0$, ability is just random.

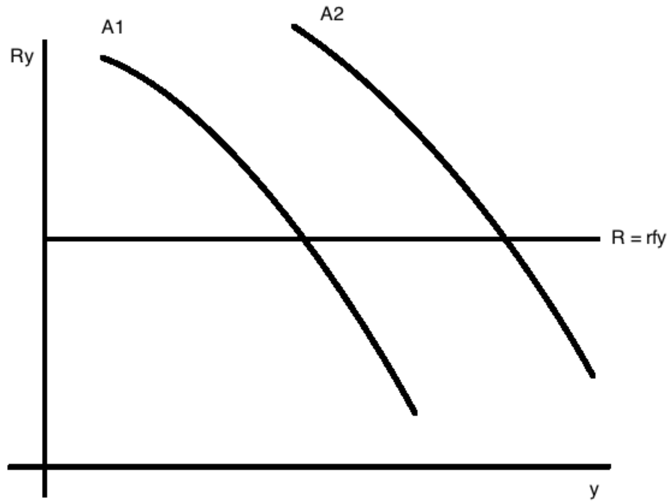
In equilibrium, $\text{VAR}(A_c) = r^2 A_c = r^2 A_p = r^2 A = \text{VAR}(A_0)$, then

$\text{VAR}(A_c) = h^2 \text{VAR}(A_p) + \text{VAR}(V_c) \implies \text{VAR}(A^*) = \frac{\text{VAR}(V_c)}{1-h^2}$. Note $\frac{1}{1-h^2}$ multiplier.

- We want to link this to earnings in two situations:
  - perfect capital markets
  - imperfect capital markets
- Recall $H_c = f(y_c, A_c)$ and assume perfect capital markets.

  This means ${R_y}^* = R$ for all families:

Figure 8



everyone will have the same rf_y around this point.

Let's say two persons have A_1, but they have different incomes, say W_{pLow}, W_{pHigh}. This will cause borrowing-repayment situations.

Perfect capital model

If capital market are perfect, the income of the child will only depend on his ability:

$$W_c = a + bA_c + V_c \tag{1}$$

$$W_p = a \quad bA_p + V_p \tag{2}$$

$$\Leftrightarrow bA_p = W_p - a - V_p$$

And we know that the ability process is:

$$A_c = (1-h)\overline{A} + hA_p + \epsilon_c$$

Then

$$W_c = a + b[(1-h)\overline{A} + hA_p + \epsilon_c] = a + b(1-h)\overline{A} + bhA_p + b\epsilon_c$$

So

$$W_c = a + b(1-h)\overline{A} + h[W_p - a - V_p] + b\epsilon_c$$
$$W_c = a + b(1-h)\overline{A} + hW_p - ah - hV_p + b\epsilon_c$$
$$W_c = \underbrace{a + b(1-h)\overline{A} - ah}_{c} + hW_p - \underbrace{hV_p + b\epsilon_c}_{\theta c}$$

Then $W_c = c + hW_p + \theta_c$, which depicts that W_p is a "perfect signal" of A_c when capital markets are perfect.

Imperfect capital markets model

$$lnA_c = a + hlnA_p + V_c; \quad ln\overline{A_c} = ln\overline{A_p} = ln\overline{A},$$
$$lnA_c = (1-h)ln\overline{A} + hlnA_p + V_c$$
$$lnW_c = b + clnW_p + \epsilon$$

- $lnW_c = \alpha + \beta lnA_c + \gamma lnW_p$

Substitutions ln A_c:

- $lnW_c = \alpha + \beta(1-h)ln\overline{A} + \beta hlnA_p + \gamma lnW_p$

but we know:

$$lnW_p = \alpha + \beta lnA_p + \gamma lnW_{GP}$$
$$\Leftrightarrow \beta lnA_p = lnW_p - \alpha - \gamma W_{GP}$$

Then

$$
\begin{aligned}
lnW_c &= \alpha + \beta(1-h)ln\overline{A} + hlnW_p - h\alpha - h\gamma lnW_{GP} + \beta V_c + \gamma lnW_p \\
&= (1-h)(\alpha + \beta ln\overline{A}) + (h+\gamma)lnW_p - \underbrace{h\gamma lnW_{GP}}_{\text{Why?}} + \beta V_c
\end{aligned}
$$

- Why impact of W_{GP} is negative?

  Recall $-h\gamma$ is a coefficient that holds everything else constant.

  Suppose there are two persons, 1 and 2:

$$
\begin{array}{lr}
1 & 2 \\
W_0 & W_0 \\
\uparrow A_p & A_p \downarrow \\
\downarrow y \implies W_{GP} \downarrow & \uparrow y \implies W_{GP} \uparrow
\end{array}
$$

$$lnW_p = \alpha + \beta lnA_p + \gamma lnW_{GP}$$

If W_p is fixed, when someone has more W_p she has less A_p.

Now, we can model this including the human capital of the parents:

$$
\begin{aligned}
&lnA_c = (1-h)ln\overline{A} + hlnA_p + V_c \\
&lnW_c = \alpha + \beta lnA_c + \gamma lnW_p + \delta lnH_p; \quad rH_p = W_p \Leftrightarrow lnr + lnH_p = lnW_p \\
&lnW_c = \alpha + \beta lnA_c + \gamma lnW_p - \delta lnr + \delta lnW_p \\
&lnW_c = \alpha + \beta lnA_c + (\gamma + \delta) lnW_p - \delta lnr \\
&\qquad \Leftrightarrow lnW_p = \alpha + \beta lnA_p + (\gamma + \delta) lnG_P - \delta lnr \\
&\qquad \Leftrightarrow \beta lnA_p = lnW_p - \alpha - (\gamma + \delta) lnG_P + \delta lnr
\end{aligned}
$$

$$
\begin{aligned}
lnW_c &= \alpha + \beta\big[(1-h)ln\overline{A} + hlnA_p + V_c\big] + (\gamma+\delta)lnW_p - \delta lnr \\
&= \alpha + \beta(1-h)ln\overline{A} + \beta hlnA_p + \beta V_c + (\gamma+\delta)lnW_p - \delta lnr \\
&= \alpha + \beta(1-h)ln\overline{A} + h\big[lnW_p - \alpha - (\gamma+\delta)lnW_{GP} + \delta lnr\big] + (\gamma+\delta)lnW_p - \delta lnr \\
&= \alpha + \beta(1-h)ln\overline{A} + hlnW_p - h\alpha - h(\gamma+\delta)lnW_{GP} + h\delta lnr + (\gamma+\delta)lnW_p - \delta lnr \\
lnW_c &= \alpha + \beta(1-h) - h\alpha + (h+\gamma+\delta)lnW_p - h(\gamma+\delta)lnW_{GP} + \delta(h-1)lnr.
\end{aligned}
$$

Returning to our household production problem

- Another question that might be interesting is if there is more than one child, what happens?

  Motivation: Consider the abilities of the two childs A_1, A_2 with $A_1 > A_2$.

  $$H_c = f(y_c, A), \quad \text{with} \quad f_{yc} > 0, \quad f_A > 0, \quad f_y \cdot A > 0.$$

  What happens with y_1^* and y_2^*. Clearly, efficiency establishes $y_1^* > y_2^*$.

- If $H_1^* = H_2^*$, then $W_1 = W_2 = rH$ only possible if $y_1^* < y_2^*$. The parent is a social planner inside his family.

Natural generalization

$U(W_p) = U(c_p) + a_1 V_1(Wc_1) + a_2 V_2(Wc_2)$, we can assume both $a_1 = a_2$ and $V_1 = V_2$.

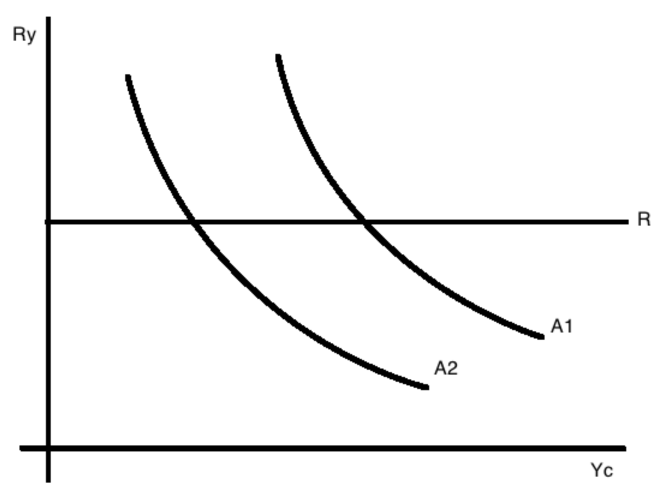
- Perfect capital market which implies $R_1^* = R_2^* = R$ which implies $y_1^* > y_2^* \implies H_1^* > H_2^* \implies V_1^* > V_2^*$.
- The problem is:

$$U(W_p) = U(c_p) + a_1 V_1(Wc_2) + a_2 V_2(Wc_2)$$

$$Wc_1 = rH_{c1} = rf_{c1} \quad H = f(y_c)$$

$$W_2 = rH_{c2} = rf_{c2}$$

Figure 9



budget constraint of parents: $c_p + y_1 + y_2 = W_p$

First order conditions

$$U'(c_p) = \lambda = \underbrace{a_1 \frac{dV_1}{dy_1} = a_2 \frac{dV_2}{dy_2}}_{\substack{\text{last dollar spent in} \\ \text{each kids gives the} \\ \text{same utility for parents}}}$$

$$\Rightarrow a_1 V_1' r f_{yz} = a_2 V_2' r f_y z \Leftarrow$$

$$\text{if} \quad a_1 = a_2 = a, \quad \text{then } \underbrace{V_1' r f_{y_1} = V_2' r f_{y_2}}_{\substack{\text{last dollar spent in} \\ \text{each kids gives the} \\ \text{same utility for parents}}}$$

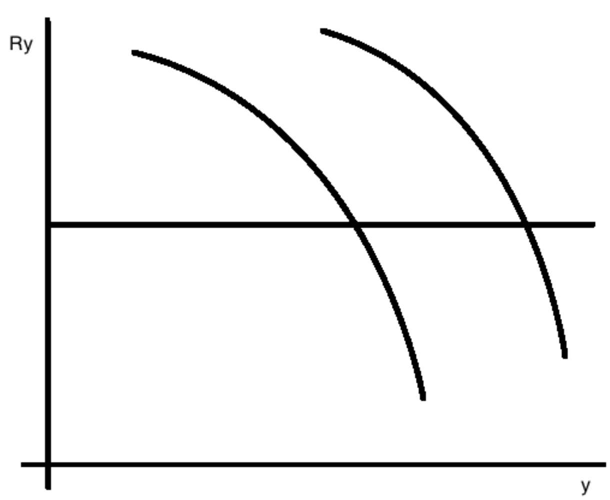
- Could it be in equilibrium that $V_1^* < V_2^*$?

  If this is true, $V_1' < V_2'$, then $f_{y_1}^* < f_{y_2}^*$. This means $y_1^* > y_2^*$, which implies $H_1^* > H_2^* \Leftrightarrow W_1^* > W_2^* \Leftrightarrow V_1 > V_2 \quad \mathbf{!}$.

  Then, in equilibrium $V_1^* \geq V{*}_2$.

  Parents take compromises balancing their spendings between kids. Degree of concavity of V_1' matters:

Figure 10



$y_1^* < y_2^*$ possible, but implies $H_1^* > H_2^*$ because $W_1^* > W_2^*$ (ability).

- Since $a_1 f_{y1} V_1' = a_2 f_{y2} V_2'$, it is more likely that $y_2 > y_1$ as $\frac{a_2}{a_1}$ increases.

  In general, $W_{c_1} \neq W_{c_2}$ because rates the return differ by kid.

- Parents do all the time a trade-off between efficiency and equality as government do with inequality and OWL's that taxes generate.

- Now, suppose the parents don't know the abilities of the child. Then, ability can be A_1 or A_2.

- Suppose Prob(A_1) = .5 and Prob(A_2) = .5, this means that the problem of the parents is:

  The problem that parents face now is:

$$U(W_p) = U(c_p) + .5aV_1(A = A_2) + .5aV_1(A = A_1) + .5aV_2(A = A_2) + .5aV_2(A = A_1)$$

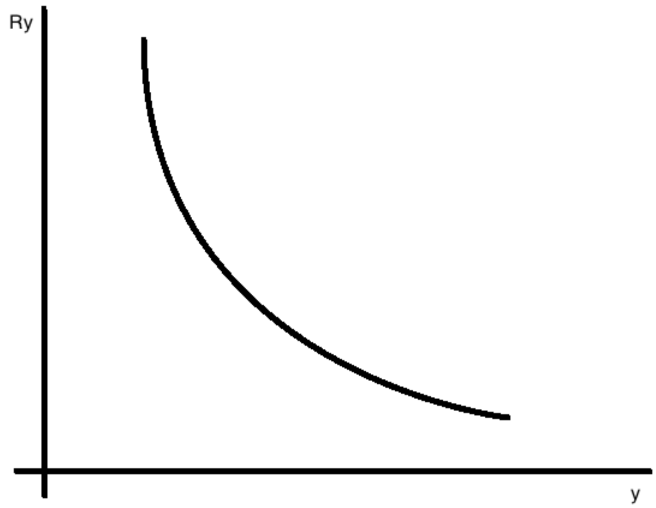
  and they have to choose c_p, y_1, y_2. In equilibrium, $y_1^* = y_2^*$: they will have same expected utility of their investments. (notice that variance may matter as well).

Other Capital than Human

Define other form of capital: there is a return from capital that is the same for everyone: R_k. What are the consequences for our household production model.

Simplicity: return to the single child household production. We model the situation in which very little y will give very high R_y:

Figure 11



- Very little y gives very high R_y. Think: $R_y \to \infty$ as $y \to 0$, so $y > 0$.
- Every family $y > 0$, as long as $R_y > R_k$, which is a weak assumption.
- As parents become richer, $R_y < R_k$.
  - Now, we are going to consider the income (welfare) of the kid: $I_c = W_c + R_k \cdot Ka$
  - Now, we have two forms of investment, then the parents budget constraint is: $c_p + y_c + k_c = I_p$.
  - The utility function of the parents now is:

$$U(T_p) = U(c_p) + aV\underbrace{(W_c + R_k K_c)}_{I_c}.$$

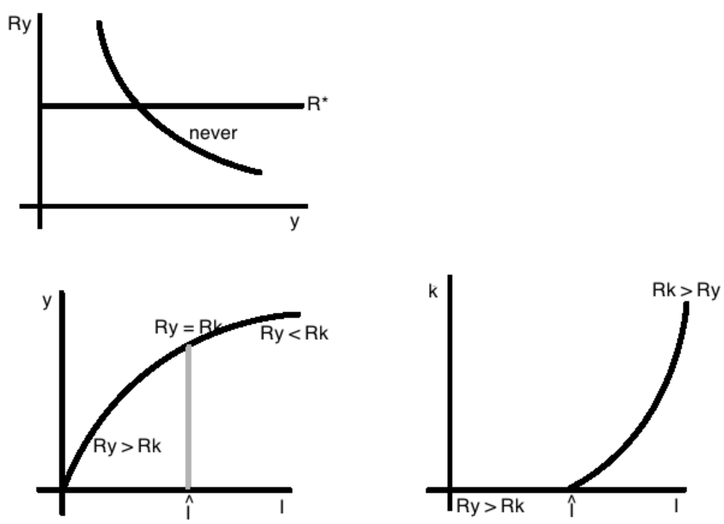
Then, the first order conditions of the problem are:

- $U'(c_p) = \lambda$

- $aV_c' \frac{dI}{dy_c} = \lambda$
- $aV_c' \cdot \frac{dI}{dK_c} \leq \lambda$, if $<, K_c = 0$.
- $aV_c' R_k \leq aV_c' R_y \Leftrightarrow R_k \leq$

- Corner solution may be possible in K_c.
  - $\frac{dy*}{dI_p} \Big\} \begin{matrix}\text{normal}\\ \text{good}\end{matrix}$ as long as $R_y > R_y$.

Figure 12



Household production with an overlapping period when children are adults

- Initially, parents can't leave debt to their children.
- Parents can also invest in other capital rather than the one of their kids.
- Parents can leave "bequest" to their kids.
- Parents also receive "bequest" from grandparents.
- Ages: middle (m) and old (o).

Budget Constraint 1: (middle age constraint)

$$c_m + y_c + k_m = E_p = W_p + b_p; \quad \text{always} \quad b \geq 0.$$

Budget Constraint 2: (old age constraint)

$$c_0 + b_c = R_k \cdot k_m$$

$$\textbf{Recall}: \quad W_c = rH_c = rf(y_c); \quad f_y > 0, f_{yy} < 0, \frac{dR_y}{dy} < 0.$$

Parents' Utility Function:

$U[E_p] = U(c_p) + \beta U(c_0) + \beta a V_c(E_c = W_c + bc);$ β = discount factor on "old days consumption".

(lifecycle degree of discount).

a = altruism.

$$\underbrace{\beta}_{\text{"discount for future"}} \quad \text{vs.} \quad \underbrace{a}_{\text{"discount for altruism"}}$$

- **Recall**: $U' > 0, U'' < 0, V' > 0, V'' < 0$.
- The first condition of this problem induce the following problem:
  - $U'(c_m) = \lambda$ marginal utility of middle age consumption
  - $\beta \cdot U'(c_0) = \frac{\lambda}{R_k}$ discounted marginal utility of old age consumption
- Then, $U'(c_m) = R_k \cdot \beta \cdot U'(c_0)$ optimality between consumption
  - $\beta a V_c' R_y = \lambda = U'(c_m); \quad y_c > 0$

– $\beta a V_c' \leq \frac{\lambda}{R_k}; \quad <\Longrightarrow\ bc = 0, =\Longrightarrow\ bc > 0$

– $\beta R_k a V_c' = \beta a V_c' R_y \Leftrightarrow R_k \leq R_y; \quad <\Longrightarrow\ bc = 0, =\Longrightarrow\ bc > 0$

If $aV_c' < U'(c_0)$, then $bc = 0$. The more selfish they are, the less bequest they give. $(R_y > R_k)$.

It will be more efficient if you can do negative b_c. This will imply ${R_y}^* = {R_k}^*$.

Public policy problem: there may be very selfish parents:

$$aV_c' < U'(c_0), \text{when} \quad b_c = 0.$$

"Preference transmission" model

- Suppose kids have a utility function $V_c(E_c, G)$; $\frac{dV_c}{dG} < 0$ where $G =$ guilt.
- Suppose parents have the following utility function:

$$U_p = U(c_m) + \beta U(c_0) + a\beta V_c(E_c \cdot G)$$

- Assume $G(z_c)$, where $\frac{dG(z_c)}{dz_c} > 0$ and z_c is the expenditure made by parents in making children guilty.

Budget constraints of the parents:

$$c_m + y_c + z_c + k_m = E_p + \overbrace{S_p(G)}^{\text{grandparents "guilty gift".}} \qquad \text{(middle age)}$$

$$c_o + b_c = R_k K + s_c(G); \quad s(G)\text{is the child support of the parents and} \quad \frac{ds(G)}{dG} > 0.$$

You lower kids utility in an special way: they fill obligation to come back and give you money in your old days.

- **Notice**: in this case $b_c = 0$, what for you will give them "bequest" and then spend for them to come back.

- There are no "net welfare conclusions". You lower their utility but you can spend more if they comeback.
- If $b_c > 0$, then $z_c = 0$, because otherwise the spendings will be inefficient.
- If you can avoid buying z_c you will do it: you can sign a contract with your son or daughter:

$$R_z > R_k.$$

- A necessary, but not sufficient condition, for parents spending in z is:

$$R_z > R_k.$$

  - The point of this model is to "endogenize the preferences".

More Traditional Human Capital Problem: Education

This studies the investment of young adults in theirselves. They take as given H°, the Human capital they have until the period they decide whether or not to go into college. H_0, of course is different for every adult.

$$\left.\begin{aligned} \textbf{Life}: \quad & E_{Hi}; \quad i = 1, \quad \ldots \quad , M \\ & E_{ci}; \quad i = 1, \quad \ldots \quad , M \end{aligned}\right\} \quad \text{earning per period in each situation}$$

- College tuition : $f \cdot \quad R = \frac{1}{1+r}$; rate of discount.

$$V_H = \sum_{i=0}^{H} E_{iH} R^i \quad \text{total earnings if stay just with high school.}$$

$$V_c = \sum_{i=0}^{H} E_{ic} R^i - f \quad \text{total earnings if going to college}$$

- If just stay with high school:

$$E_{OH} = W_{OH}T$$

$$E_{OH} = W_{OH}\left[T - t_c\right]; t_c = \text{time in college}$$

- In $H = 0$, both get the same payment.
- So the point is to compare what happens when you go to high school to what happens when you go to college.

  - This comparison may be done by the following equation:

$$\sum_{i=1}^{M} T\Delta W_i R_i \quad \boxed{?} \quad \underbrace{T_c W_o}_{\text{forgone earnings}} + \underbrace{f}_{\text{tuition}}$$

    Notice that $\Delta W_i = W_{ci} - W_{Hi}$, is the wage differential of "college to high school" and the equation assumes that:

$$\Delta W_i = \Delta W_j \quad \forall \;\; i, j$$

- There are six relevant parameters in the study of this decision:
  - T_c = hours of college.
  - W_0 = wage at $t = 0$, which is the same for both high school and college.
  - H = life expectancy
  - ΔW = "benefits of college"
  - $W_0 T_c$ = forgone earnings
  - f = tuition

- Now, if we consider the complete serie:

$$\sum_{i=1} T\Delta W_i R_i = T\Delta W \cdot [\frac{1-R^{M+1}}{1-R}] \quad \boxed{?} \quad f + T_c \cdot W_o$$

  - This analysis can be extended to "add" the probability of dying at age i., which will be different in each period of the agent's life. So if we let m_i be this probability:

$$\sum_{i=1}^{M} T(1-m_i)\Delta W_i R_i \quad \boxed{?} \quad f + T_c \cdot W_o$$

  - Suppose, for a conceptual exercise, that there is no tuition. Then, a flat tax to earnings will not affect the decision. (i.e. the term $(1-\tau)$ will drop out).
  - If there is tuition, on the other hand, costs are falling by less than τ and the costs are less (i.e. the decision may be affected).
  - Clearly, ability affects both: forgone earnings and wage differentials. We can think of this cancelling out on both sides of the equation.

Why do we have this large increases in tuition?

- Education is intensive in education
- I.e.: increase in tuition is due to increase an increase in costs. Large tuition is related to higher returns to college.
- Then, it should be true that the following inequality holds:

$$\frac{d(\frac{W_c}{W_H})}{dt} > 0.$$

Think of inequality: "Why is there no more people taking advantage of "college premium".

- Dropout high school: face awful situation in any dimension:
  - employment

- earnings
- wealth
- marriage (less getting, more divorced).

Note: Accumulation of capital also explains rise in wages

- Now, let's say there is a set of abilities:

  $A =$ abilities (cognitive and non-cognitive)

  $H_o =$ initial Human Capital at high-school graduation.

  If we want to "endogenize" time spent in college, what are some reasonable assumptions:

$$T_c(A, H_o); \quad \frac{dT_c}{dA} \leq 0, \quad \frac{dT_c}{dH_o} \leq 0$$

Also, we can consider the determinants of "tuition fees":

$$f(A, H_o); \quad \text{where } f \text{are tuition fees.}$$

$$\text{Then,} \quad \frac{df}{dA} < 0 \quad \text{and} \quad \frac{df}{dH_o} < 0.$$

- The wage differential can be signed here as well:

$$\Delta W^k(R^k, A^k, H_o^k, M^k, T^k); \quad \text{k is an individual.}$$

- $\frac{d\Delta W^k}{dR} < 0 \quad \frac{dW^k}{dt_c} > 0.$

Empirically, education gains are everywhere:

- less marriage brokes
- better health
- raise productivity in household production
- better use of contraception methods

– better use of drugs

– better adaptation of new environments (e.g. technology)

Model education with Utility:

We model the individual's utility as one given by the following function:

$$U(x_i, l_i, H, s_i); \quad i = 1, 2$$

where x = goods $\quad l$ = leisure $\quad s$ = college $\quad m$ = hours worked

t = tuition fees $\quad T$ = total time = 1

h = hours spent in investing in college

H = high school education

Then:

$$l_1 + m_1 + h = T \quad \text{and} \quad l_2 + m_2 = T.$$

The "total" utility function is:

$V = U_1(x_1, l_1, H) + p(H, s)\beta U_2(x_2, l_2, H, s)$, where $p(\cdot)$ is the probability of surviving in period 2 and p is the discount rate.

In this kind of models we implicitly assume that there is a "perfect annuity market": expected consumption = expected earnings (access to full earnings wherever)

Budget Constraint:

$$x_1 + \frac{p(\cdot) \cdot x_2}{1+r} + w_1 l_1 + \frac{p(\cdot) \cdot W_2(\cdot) l_2}{1+r} + f + W_1 h \qquad = \underbrace{W_1 + \frac{p \cdot W_2(\cdot)}{1+r} + \frac{p \cdot (s)}{1+r}}_{\text{full income}}$$

Think: $\quad s = f(h, H, A_c, A_n); f_j > 0$ with $j = H, A_c, A_n f_{jy}$ for $j \neq y$

F.O.C.

$$U_1x = \lambda \quad \text{and} \quad \beta U_2xp = \frac{\lambda_p}{1+r}$$

$$\text{Suppose that, in equilibrium,} \quad \beta = \frac{1}{1+r}, \quad \text{then :} \quad U_{1x} = U_{2x}$$

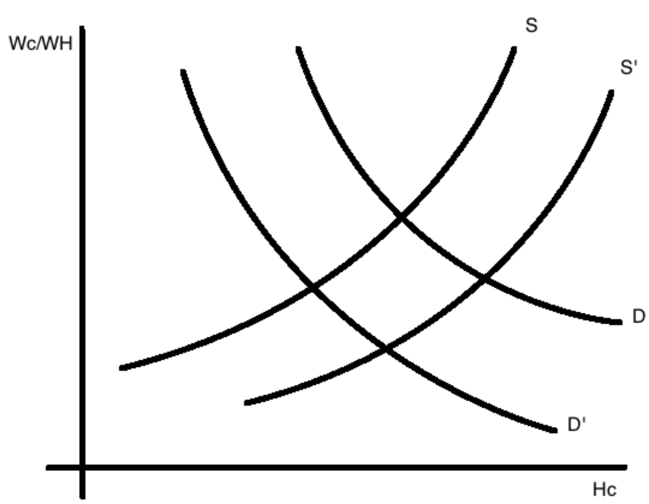
$$\text{Also :} \quad U_{1l} = \lambda W_1 \quad \text{and} \quad p\beta U_{2l} = \frac{\lambda_l W_2}{1+r}$$

$$\text{then :} \ \frac{U_{1l}}{\beta U_{2l}} = \frac{W_1}{W_2}(1+r). \quad \text{Likewise, if } \beta = \frac{1}{1+r}, \quad \text{then} \quad \frac{U_{1l}}{U_{2l}} = \frac{W_1}{W_2}.$$

Increase earnings:

Increase in tuition (80's and 90's):

Figure 13



- Education not only gives you skills, it gives you "techniques" to know how to "get information": learning how to learn. (Schooling is an effective way to do this).
- Information acquisition aspects
- Efficient way to get social networking

Health as Human Capital

- We want to treat health as investment per se.
- Mechanical medicine (surgery instruments) and people's decisions of health
- We want to study "what people do with their health".

In an ordered fashion:

1. Statistical value of life (SVL)
2. Optimal investment in health

   a. health as self protection

   The "utility maximizing problem" is:

$$\begin{aligned} &\underset{x_1,l_1,x_2,p_2,h}{MAX} \quad U(x_1,l_1)+\beta s(h,\text{schooling}) \;\; U(x_2,l_2) \\ &\text{s.t.} \quad x_1+\frac{s(h)x_2}{1+r}+g(h)=W_1(1-l_1)+\frac{s(h,\text{schooling})\cdot W_2(1-l_2)}{1+r} \end{aligned}$$

$\boxed{\text{Total time} = 1}$ $x_i \to$ consumption $p \to$ discount factor $l_i \to$ leisure

Basic assumptions:

$$g' = \frac{dg()}{dh} > 0 \quad g'' \geq 0, \quad \text{convex cost function}$$

$$s' = \frac{ds(\cdot)}{dh} > 0 \quad s'' \leq 0, \quad \text{diminishing marginal returns}$$

First order conditions:

$$[x_1] : U_{x1}(x_1,l_1) = \lambda \tag{3}$$

$$[x_2] : \beta s() U_{x2}(x_2,l_2) = \frac{\lambda s(\cdot)}{1+r} \tag{4}$$

$$[l_1] : U_{l1}(x_1,l_1) = \lambda W_1 \tag{5}$$

$$[l_2] : \beta s(\cdot) U_{l2}(x_2,l_2) = \frac{\lambda \cdot s(\cdot) W_2}{1+r} \tag{6}$$

$$[h] : \beta s'(\cdot) U(x_2,l_2) = \lambda[g'(\cdot) + \frac{s'(\cdot)}{1+r}(x_2 - W_2(1-l_2))] \tag{7}$$

From 3 and 5: $\frac{U_{l1}(x_1,l_1)}{U_{x1}(x_1,l_1)} = W_1$

Assumption: persons discounts in the same way than the market.

Then: $\frac{U_{x2}(x_2,l_2)}{U_{x1}(x_1,l_1)} = 1$

To analyze 7, we can make a "Math Commercial".

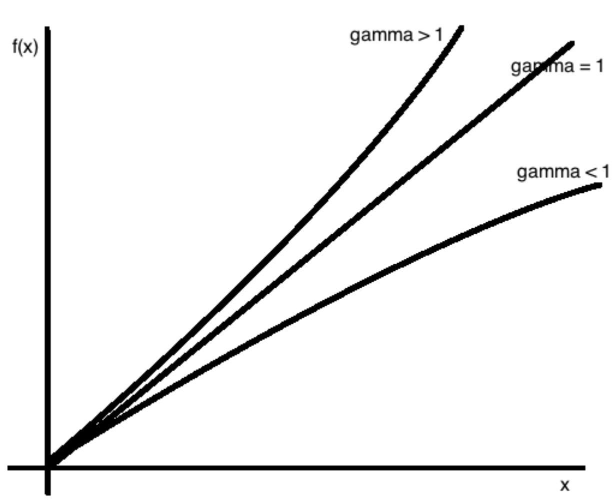
Define: an homogeneous function of degree k is:

$$f(\alpha x) = \alpha^k \cdot f(x).$$

If U is b.o.d γ, then $U(\alpha x, \alpha l) = \alpha^\gamma \cdot U(x,l)$.

If $\gamma = 1$, the function is "linear" $\gamma < 1$, "concave" $\gamma > 1$, "convex"

Figure 14



Euler's theorem:

Take x, l and define $x' = tx$ and $l' = tl$, then $U(x', l') = t^\gamma \cdot U(x,l)$ because U is b.o.d of degree γ.

Taking the derivative with respect to t of each side:

$$\frac{dU}{dx'} \cdot \frac{d(t_x)}{dt} + \frac{dU}{dl'} \cdot \frac{d(tl)}{dt} = \gamma t^{\gamma-1} \cdot U(x,l)$$
$$\Leftrightarrow U_x(tx,tl) \cdot x + U_o(tx,tl) \cdot l = \gamma \cdot t^{\gamma-1} \cdot U(x,l)$$

If t = 1:

$$U_x(x,l) \cdot x + U_l \cdot (x,l) \cdot l = \gamma \cdot U(x,l)$$

Then:

$$\frac{1}{\gamma}[U_x(x,l) \cdot x + U_l \cdot (x,l)] = U(x,l).$$

We can take 7 and substitute $\lambda = U_{x2}(x_2, l_2)$:

$$\beta \cdot s'() \cdot U(x_2,l_2) = U_{x2}(x_2,l_2)[g'(\cdot) + \frac{s'(\cdot)}{1+r}(x_2 - W_2(1-l_2))]$$

Recalling that $\beta = \frac{1}{1+r}$, then:

$$\frac{s'(\cdot)}{1+r} \cdot \frac{U(x_2,l_2)}{U_{x2}(x_2,l_2)} = \boxed{g'(\cdot) + \tfrac{s'(\cdot)}{1+r}(x_2 - W_2(1-l_2))} \rightarrow \quad \text{RHS}$$

$$\boxed{\text{If } U \text{ is b.o.d } \gamma\text{, then } \tfrac{U(x_2,l_2)}{U_{x2}(x_2,l_2)} = \tfrac{1}{\gamma}(U_x \cdot x_2 + \underbrace{\frac{Ul^2}{Ux^2}}_{WL} \cdot l_2)}$$

Then

$$\frac{s'(\cdot)}{1+r} \cdot \frac{1}{\gamma} \cdot (x_2 - w_2 l_2) = RHS$$

or:

$$s'(h) \cdot \frac{1}{\gamma}(x_2 - W_2 l_2) - \frac{s'(\cdot)}{1+r}(x_2 - l_2 w_2) = g'(\cdot) + (-W_2) \cdot \frac{s'(\cdot)}{1+r}$$

Then:

$$\frac{s'(h)}{1+r}[\frac{1}{\gamma}-1](x_2+W_2l_2)=g'(\cdot)+\frac{s'(\cdot)}{1+r}(-W_2).$$

Statistical Value of Life

Tipical Literature: Money value of MAN.

- Tipical income in US $\$40,000$
- Take 1.8 of that for maintenance and leisure: $\$72,000$
- Annual value $\$110,000$, but missing the value of γ. Let's say it is $\gamma = y_2$.
- Then $W = 2(110,000) = 220,000$.
- If the discount rate is 4%, then:

$$VSL = \frac{220,000}{.04} = \$5,500,000 \quad \text{in the U.S.}$$

Exercise

- How costly was the AH_1N_1 flu in Mexico:

  GDP per capita in Mexico: $\$13,500$

  Number of deaths 119

  VSL $= 5,500,000 \cdot \frac{13,500}{40,000} = 1,856,250$

  Stat.loss $= 119(1,856,250) = 220,893,750$

- What about Haiti earthquake

  GDP per capita Haiti: $\$1,300$

  Number of deaths: $230,000$

  VSL $= \$5,500,000 = \frac{1,300}{40,00} \cdot 5,500,000 = 178,750$

  Stat.loss $= 41,112,500,000 = 41,112,500,000.000$

More on health investment

Suppose s_1 is the conditional probability of surviving age 1.

Suppose s_2 is the conditional probability of surviving age 2.

$\vdots$

Suppose s_n is the conditional probability of surviving age n.

- Note: $s_1 = S_1$. Likewise, S_i is the unconditional probability of surviving to age i.
- Individual's maximization problem will be:

$$V = S_1 U_1(x_1, l_1) + \beta S_2 U(x_2, l_2) + \beta^2 S_3 U(x_3, l_3)$$

$g(h)$ is the expenditure in health cost function.

For simplicity, we assume that the individual "spend" in health on age 1.

Assume $S_2(h)$ and diminishing marginal returns:

$$\frac{dS_2(h)}{dh} > 0, \quad \frac{dS_2(h)}{dh} \leq 0.$$

In this case, the budget constraint is:

$$S_1 x_1 + \frac{S_2 x_2}{1+r} + \frac{S_3 x_3}{(1+r)^2} = S_1 W_1(1-l_1) + \frac{S_2 W_2(1-l_2)}{(1+r)^2} + \frac{S_3 W_3(1-l_3)}{(1+r)^3}$$

The relevant first condition in this problem is:

For x_1, l_1, x_2, l_2 $\quad s$ drop out. Simply, MRS in each good should be the same.

$[h]$:

$$\beta \frac{dS_2}{dh} + \beta^2 \frac{dS_3 V_3}{dh} U_3 + \lambda[g'(h) + \frac{dS_2}{dh} \cdot \frac{1}{1+r}[x_2 - W_2(1-l_2)] + \frac{dS_3}{dh} \cdot \frac{1}{(1+r)^2}[x_3 - W_3(1-l_3)]]$$

Notice that, since $S_2 = s_1 s_2$, $S_3 = s_1 s_2 s_3$ and so on, even when **only** $S_2(h)$, investing in

h impacts (Becker called it weight) the unconditional probability of surviving to period $S_{i>2}$.

"Investing in h helps the unconditional probability of surviving to the posterior to 2 periods".

Notice, other things the same, it is rational to "spend in the probability of surviving" in earlier periods. (the product $S_n = s_1 \dots s_n$ is smaller each time).

The marginal benefit of "the general expenditure" in health is:

$$\boxed{MB = \sum_{i=1}^{\infty} \beta^i \frac{dS_2}{dh} s_3 \dots s_i}$$

- Expenditure in different health goods (two periods)

$$\left\{ \begin{array}{l} s_1 = 1 \\ \\ s_2 = d_1(h_1) \cdot d_2(h_2) \end{array} \right\} \text{unconditional probabilities}$$

$$h_i = \text{expenditure on health} \quad 1 = \text{cardiovascular} \quad 2 = \text{cancer}$$

For example: there may be sorting by rank. In this case it will be perfect. Smarter girl with smarter boy and so on.

Example of complementarity: investing in child.

- Rank determines the sorting level. Then, you have incentives to educate yourself because that is the way to jump in the ranking. Sociologist claim that this is a "zero-sum game". *implies* jump and other decreases in ranking. That is false because the "ranking jumps" increase the "social pie".

- What happens when utility is not transferable? In that case, $z_{im} = z = z_{jf}$, where z is the total output and both, the male and the female, receive exactly the same of it.

  In that case, there is a exactly positive sorting: you want z to be as big as possible and the way to have that and "don't waste" your resources is to positively sort.

- Is there any case when there are no gains from marriage? Yes, when $z_{ms} \leq 0$, complementarity doesn't (negatively) affect the household production good utility.

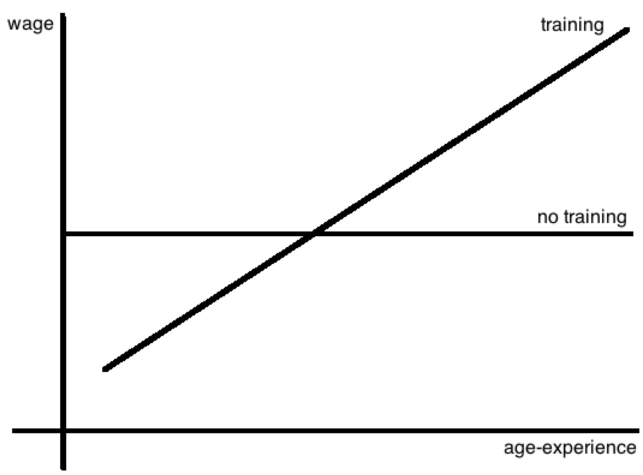
On the job investment

Two ways of "job investment": learning by doing and explicit investment.

There is a more basic "data problem" in this context than selection bias; namely, firms do not gather data on their "job investment spending".

Think of the problem graphically:

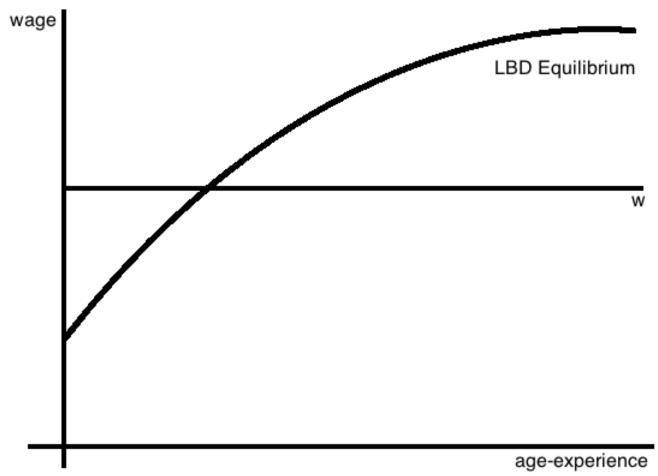
Figure 15



An area comparison ratio will give you the return of the training investment.

- Why does training happens when you are young?
  - If you want to acquire skills you may want to do it know in order to maximize the gain from that acquisition.
- How does learning by doing looks like (happen)?

Figure 16



Who pays for the training?

Clearly, this depends on the market structure and on the kind of training people receive. There are 4 types of training:

- firm specific
- industry specific
- occupational specific
- general training
- If the market is competitive general training raises productivity in all markets. Then, no firm will be willing to pay for that employee's training.
- If there is a competitive industry, the worker will face a higher uncertainty than in the general case and it is uncertain who is paying for the training.
- firm specific: firm pays.

Why is there job specialization

"Division of labor is limited by the extension of the economy". Adam Smith

Is development leading to more specialization or is specialization leading to more development? You may argue in both senses.

Two fundamental approaches for trying to understand why people differ.

1. Roy model of specialization and comparative advantage.

   - take differences as given

Is this arguable?

1. Yes, people become different too. How? Training, learning by doing, human capital investment, etc.

2. Becker model: attack a problem in which individuals are identical ex-ante.

Assumptions:

- identical individuals ex-ante
- specialization in different skills or tasks
- cooperation among specialists to produce widgets
- cooperation is in teams (not only a firm, within a firm)
- there are m tasks
- investment in task specific skills (no dynamics)
- T hours available per individual
- h time spent investing in this task
- $H = g(h)$ investment function, where $g'(\cdot) > 0$ and $g'' \leq 0$.

Think: it pays to develop different kind of skills. Why? increasing returns on doing it.

y **is the output produced**: It is proportional to the effective labor input (works spent working times productivity).

$$Y = \overbrace{\alpha}^{\text{productivity of economy}} \cdot \underbrace{l \cdot H}_{\text{effective labor}}$$

$g(n, A_c, A_{N_c}) \implies$ complementarity with other individuals, etc. is brought by the shape of g.

Time can be allocated working or spending in human capital:

$$\boxed{l + h = T}$$

The problem that any agent faces is:

$$\underset{l,h}{\text{MAX}} \quad \{Y = \alpha \cdot l \cdot g(h)\}$$

F.O.C.s

$$[l] : \alpha g(h) = \lambda \quad [h] : \alpha \cdot l \cdot g'(h) = \lambda \Leftrightarrow \boxed{l^* = \frac{g(h)}{g'(h)}} \boxed{h^* = T - \frac{g(h)}{g'(h)}}$$

Suppose that $g(h) = ch^\theta \quad 0 \leq \theta \leq 1$, where $c(A_{N_c}, A_c)$, etc.

Then:

$$\boxed{l^* = \frac{h^*}{\theta}}$$

and

$$\boxed{l^* + h^* = T}$$

$$\boxed{l^* = \frac{1}{1+\theta} \cdot T} \boxed{h^* = \frac{\theta}{1+\theta} \cdot T}$$

In equilibrium, what is the output?

$$Y^* = \alpha \cdot l^* \cdot g(h^*) = \frac{\alpha \cdot g(h^*)}{g'(h^*)} \cdot g(h^*) = \boxed{\frac{\alpha \cdot g^2(h^*)}{g'(h^*)} = Y^*}$$

Notice that there are increasing returns in investing in specialization. If $g(h) = ch^\theta$, then:

$$Y^* = \frac{\alpha c}{(1+\theta)^{\theta+1}} \rightarrow \quad \text{better to be more able}$$

If $\theta = 1$,

$$\boxed{Y^* = \frac{\alpha c}{4} T^2}$$

- Increasing in T; double your time, more than double your output.
- This is why you don't want to diversify, you want to concentrate.

To show more clearly the benefits of specialization, let's deal with a double task (independent) case.

- Tasks: y_1, y_2
- investment functions: $g(h_1), g(h_2)$
- Restrictions:

$$h_1 + l_1 = t_1$$

$$h_2 + l_2 = t_2$$

$$t_1 + t_2 = T$$

"Obviously":

$$Y_1^* = \frac{\alpha_1 c_1}{\theta_1} \cdot [\frac{\theta_1}{1+\theta_1}]^{1+\theta_1} \cdot t^{1+\theta_1} \quad Y_2^* = \frac{\alpha_2 c_2}{\theta_2} [\frac{\theta_2}{1+\theta_2}]^{1+\theta_2} \cdot t_2^{1+\theta_2}$$

Now, suppose $t_i = \frac{T}{2}$, for $i = 1, 2$

What is the effect of dividing time?

$Y^* = [\quad] \frac{T^{1+\theta}}{2^{1+\theta}}$; output decreases more than half.

Suppose that the joint production function, $\phi = f(Y_1, Y_2)$ is leontief (la favorita del skoto) $\phi(Y_1, Y_2) = min(Y_1, Y_2)$, then $Y_1^* = Y_2^*$.

This will cause people to put a lot of time to the thing they are not good at.

If $\alpha_1 = \alpha_2, c_1 = c_2, \theta_1 = \theta_2$, is same for everyone, then $t_1^* = t_2^*$ obviously $Y_1^* = Y_2^*$ (leontief)

Then:

$$Y^* = Y_1^* = Y_2^* = \frac{\alpha c}{4}(\frac{T}{2})^2 = \boxed{\frac{\alpha c T^2}{16} = Y^*}$$

- If somebody needs to produce in fixed properties, they suffer a lot when they specialize.

Suppose $\alpha_1 = \alpha_2 = \alpha, \theta_1 = \theta_2 = \theta$ and $c_1 \neq c_2$ s.t. $c_1 > c_2$, then $t_1 < t_2$.

More productive in task 1, you allocate less time in it. If $\theta = 1$ and $c_1 = 2c_2 \implies t_1 = \frac{t_2}{\sqrt{2}}$.

Finally, if there are m tasks and $\phi = \min\{y_1, \ldots, y_m\}$:

$$\boxed{Y_i^* = [\cdot]\frac{t^2}{m^2}}$$

- $\boxed{\text{What have we learned today? (Nothing), there are huge gains from specialization}}$

- Entrepreneur coordinates people

Coordination Costs

- communication

- complementarity

- cost function of coordination is increasing in the number of members the team has

- **Model**

$n = \#$ of members $= \frac{m}{s}$, where s is # of independent tasks

cost function: $c(n), c'(n) > 0, c''(n) > 0$

$n(n-1)$ possible interactions

$I =$ per capita income $= y(\frac{m}{n}) - c(n)$

$$\frac{dI}{dn} = \frac{dy}{dn} - c'(n) > 0, \quad \forall n \leq m \quad N < m$$

$N =$ numbers of individuals available (number of the market)

$s = \frac{m}{N} > 1 \rightarrow$ in this case division of labor is limited by extent of the market.

1. ✓ prior

2. $y'(n) < c'(n)$ all $n > 1$ (coordination costs are too big) (inefficient)

3. Division of labor limited by extent of the economy

$$y'(n) > c'(n) \qquad \text{at the beginning}$$

$$c'(n) > y'(n) \qquad \text{at the end } (c'(n) \text{ convex})$$

In this case there is an interior solution.

$$\underset{h}{\text{MAX}} \quad I = y(n) - c(n) \quad 1 < n^* < m$$

$$\text{FOC} \quad y'(n) = c'(n) \quad \# \text{ teams} = m$$

$$\text{SOC} \quad c''(n) = y''(n) > 0$$

- Household

2 members Household markets, $m = 2$.

How do you explain that women are always in the market place

1. Roy-differences

2. Discrimination