Human capital 34300

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• Human capital, because it induces productivity, is very important in modern economies.

• Most of knowledge come from investment

→ Knowledge is an investment process.

• 75% of human capital determines the income of any person.

→ However, Human Capital is not accounted in "National Income accounts".

Human capital is the link between parents and child: is hard to get more Micro thant that.

Forms of human capital are deeply complimentary.

→ High educated → High health → High training → better consumers:
→ adapt faster to better technologies.
+ contraception
+ diet
− marry brakes
+ stay married

• This statements are true for developing countries.•

→ Better educated people report they are happier.
→ less educated people report more stress (¿broke marriages?)

Interesting exception: more educated women, less mariage.

• Some differences between physical and human capital:

1. Endowments and genetic structures are part of human capital.

2. Human capital cannot be bought or sold (slavery is forbidden)

3. There are no markets of stocks for human capital

4. It is very difficult to use human capital as collateral.

★ Richer families' guys have an advantage.

→ Since human capital builds on itself (recursive process), public policy makers should get guys young:

$H_{t+1} = \psi(\cdot, H_t)$

↳ that human capital is <u>intensive</u> in human capital is not only a technical point.

• 1980's → benefits of going to school increased a lot.

• Relative costs of education went up: tuition went up.

• Investment by parents on their own children •

Early and late human capital are compliments: childrens build on their own investment.

Heckman → cognitive skills → early
Heckman → non-cognitive skills

• Unisex adult model •

Assumptions:

- one adult - one child
- two periods: childhood / adulthood.

One overlapping period:

time

t adult parent / child overlapping period

t+1 child becomes adult / parent = 0 (dies)

One decision: how many to consume / how much to invest in children.

$$\underbrace{C_p}_{\text{parental consumption}} + \underbrace{Y_c}_{\text{children goods}} = \underbrace{W_p}_{\text{parental income}} \quad \Big\} \text{ parental budget constraint}$$

• There is no way for children to repay their parents.

→ selfish parents : $Y_c^* = 0$

→ if all parents are selfish, we can go home

• We are interested in modeling altruistic parents, which utility function is:

$$U_p(w_p) = U(c_p) + a\,V(w_c) \quad , \quad \frac{dV}{dw_c} > 0 \quad \frac{d^2V}{dw_c^2} \leq 0$$

where w_c is the earning of the child when he becomes an adult.

a, degree of altruism, i.e. $a=0$ implies that the parents are selfish

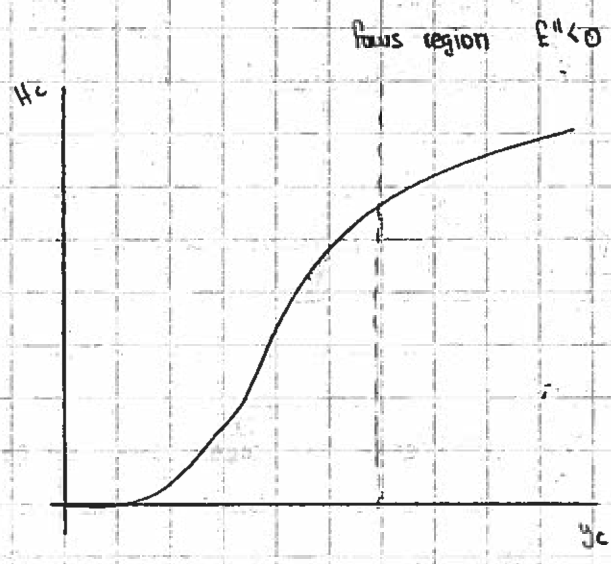
• **Human capital of kids**

$H_c = f(y_c)$, household production function for human capital

$$\frac{dH_c}{dy_c} \geq 0 \quad , \quad \frac{d^2H_c}{dy_c^2} \leq 0$$

(ojo con intuición de la posibilidad de que sea igual a cero).

Following situation may be possible:



You can think it, by now, in the following way:

$H_c = f(y_c, \bar{B})$, where $\bar{B}$ is the "fixed brain of the kid". And, therefore, it is more intuitive that:

$$\frac{\partial f(y_c)}{\partial y_c} \geq 0 \quad \text{and} \quad \frac{\partial^2 f(y_c)}{\partial y_c^2} \leq 0. \quad \text{(diminishing marginal returns)}$$

<u>Assumption</u>: all the families have same production function.

• What determines the earnings of the kids?

$W_c = R\, H_c = R\, f(y_c)$; R converts human capital into earnings.

$R = \psi\left(\sum_{t \in T} H_t \,,\ \text{technology},\ \text{physical capital}\right)$

<u>Assumptions</u>:

R is constant for each household
is "competitive" i.e. a household alone can't change it.
is taken by parents as given by the mkt environment.

• In this model earnings only differ because of human capital, i.e. by goods spent in children. Gaps may differ by economic sector, but this always holds.

• Economic problem •

$$\max_{y_c, c_p} \{ U_p(w_p) = U(c_p) + a\,V(w_c) \}$$

$$\text{s.t.} \quad c_p + y_c = w_p$$

Solution:

$$\mathcal{L} = U_p(\cdot) - \lambda [c_p + y_c - w_p]$$

F.O.C.'s:

$$U'(c_p) = \lambda \quad , \quad a\,\frac{dV(w_c)}{dy_c} \leq \lambda$$

If < 0, $y_c^* = 0$
when?
- if $a = 0$
- if income of parents is too low

• notice $w_c = R\,H_c = R\,f(y_c)$

Rewriting:

$$a \cdot \frac{dV(w_c)}{dw_c} \cdot \frac{dw_c}{dH_c} \cdot \frac{dH_c}{dy_c} \leq 0 \Rightarrow a\,V' \cdot \underbrace{R\,f_y}_{\text{intuicion}} \leq \lambda$$

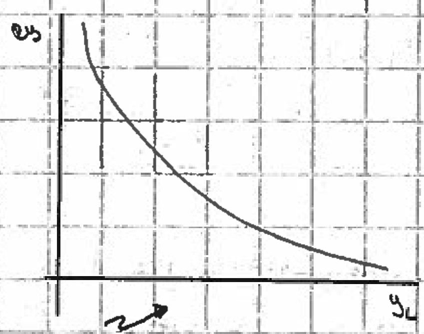
qué tanto un peso más de inversión te "retorna"

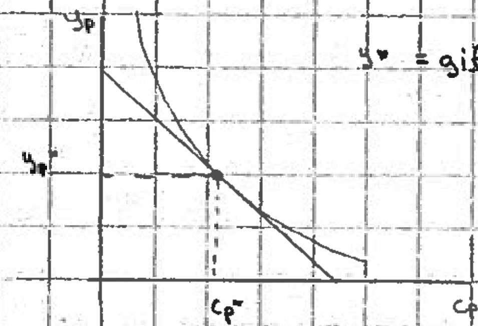
$$\Rightarrow \frac{dw_c}{dy_c} = 1 + i = R_y$$

i marginal rate of return of investment in kids

$$a\,V'_c \underbrace{R_y}_{\substack{\text{marginal}\\\text{rate of}\\\text{return}}} \leq \lambda = U'(c_p) \Rightarrow \underbrace{\frac{U'(c_p)}{aV'_c}}_{\text{MRS of parents}} \geq R_y$$

If $a > 0$, there is an interior solution: let's assume that parents provide at least food and shelter to kids.





$y^* =$ gift

El retorno es decreciente en y_c. $R f_y = R_y = 1+i$ es decreciente en y_c.

→ Recuerda por qué esto es la tasa de retorno: es lo que me da un peso más en "regalo paternal" dado el R en el mercado.

ojo $w_c = r H_c$; $H_c = f(y_c)$.

$\frac{\partial H_c}{\partial y_c} = r f_y$; $f''(y_c) < 0$

• Two basic equations: aquí empieza a cambiar R por r

$$a r f_y V_c' - U'(c_p) = 0 \quad \ldots \quad g_1(y_c, c_p; a, r, w_p)$$
$$c_p + y_c - w_p = 0 \quad \ldots \quad g_2(y_c, c_p; a, r, w_p)$$

• ¿What happens when W_p increases to y_p and to c_p?
→ what happens to endogenous "choice variables" when parameters change, in this case w_p?

For notation, let $g = \begin{bmatrix} g_1 \\ g_2 \end{bmatrix}$ and $x = (c_p, y_c)$
$q = (a, r, w_p)$

The general implicit function theorem establishes:

notice, $g_1 = ar\, f_y\, V'_c\, (r\, f(y_c)) - u'(c_p)$

$$\begin{bmatrix} dc_p / dw_p \\ dy_c / dw_p \end{bmatrix} = - \begin{bmatrix} \partial g_1 / \partial c_p & \partial g_1 / \partial y_c \\ \partial g_2 / \partial c_p & \partial g_2 / \partial y_c \end{bmatrix}^{-1} \begin{bmatrix} \partial g_1 / \partial w_p \\ \partial g_2 / \partial w_p \end{bmatrix}$$

$$= - \begin{bmatrix} -u''(c_p) & ar\left[f_{yy} V'_c + r f_y f_y V''_c\right] \\ 1 & 1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 0 \\ -1 \end{bmatrix} =$$

$$= \frac{-1}{\underbrace{-u''(c_p) - ar\left[f_{yy} V'_c + r f_y f_y V''_c\right]}_{D>0}} \cdot \begin{bmatrix} 1 & ar\left[f_{yy} V'_c + r f_y f_y V''_c\right] \\ -1 & -u''(c_p) \end{bmatrix} \cdot \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} -ar\left[f_{yy} V'_c + r f_y f_y V''_c\right] / D \\ -u''(c_p) / D \end{bmatrix} \begin{matrix} >0 \\ >0 \end{matrix}$$

make sense because goods are normal and this change in w_p is an income effect.

- Intergenerational income mobility: studies relation between parents and children income.

It is say that there exists regression to the mean mobility in this matter:

$$\ln W_c = \alpha + h \ln W_p + \varepsilon_c$$

if:
- $h = 1$ no regression to the mean
- $h < 1$ regression to the mean
- $h > 1$ regression away the mean.

• La discusión sobre "intergenerational income mobility es intermitente".

↳ Continuing with the discussion about the previous model...

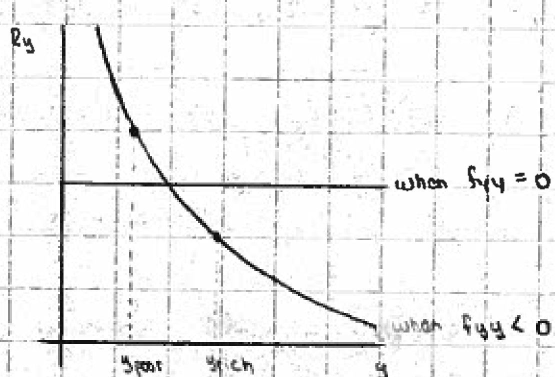
What happens with the specifications of the function $H_c = f(y_c)$
Remember, $w_c = r\, f(y_c)$, and we are supposing r is competitive.

If $f_{yy} = 0$, suppose $H_c = K y_c$; $K \in \mathbb{R}_{++}$.
The situation with the rate of return is the following.
We saw the rate of return is $R_y = r\, f_y$. In this case $f_y = K$, so R_y is the same for all the families, rK.

This means that "capital markets are working perfectly".

↳ Now, imagine we have $f_{yy} < 0$. I.E.g. $K y_c^{\tau}$ with $\tau < 1$.
Then, $R_y = r\, f_y$ and $\frac{dR_y}{dy} = r\, f_{yy} < 0$.

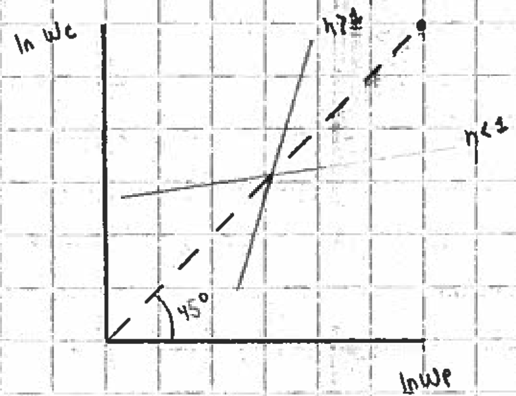
In this case, there is an inefficiency: parents can't lend to other poor kids. There is a "price effect", diminishing returns on investment: "invest more rises the cost of investing". Graphically:



R_y^* (poor) > R_y^* (rich). Rates of return are different and there is no "lending access". Capital markets are not working perfectly.

2. returning to the intergenerational income mobility aspects. Take the regression $\ln w_c = \alpha + h \ln w_p + v_c$. Since we prove that $\frac{dk}{dw_p} > 0$, then it should be clear that $\frac{dw_c}{dw_p} > 0$, so $h > 0$.

But, what happens with regression to the mean?



→ In this context, regression to the mean means that if a parent is on average richer, the kid is also on average richer but less. The literature says that $h \approx .5 - .6$.

What happens with poverty? Take the variance as a measure of poverty. Then $VAR(\ln w_c) = h^2 \, VAR(\ln w_p) + VAR(v)$.

In equilibrium, $VAR(\ln w_c) = VAR(\ln w_p)$ and $VAR(\ln w_c) = \frac{VAR(v)}{1-h^2}$.

If $h = 1$, the poverty process is "not stationary".

- Some people speaks about "underclass". Born poor, stay poor. At least from one generation to another.

- Ejercicios para antes del examen: cambios en α y cambios en c para el modelo más básico.

• It will make a lot of sense to suppose that the human capital of the kids is also depending on the human capital of the parents. That is: $H_c = f(y, H_p)$, where we are going to take H_p as a constant (there is just one overlapping generation).

The "intuitively economic" assumptions of this process are:

$$\underbrace{\frac{\partial H_c}{\partial H_p} > 0 \quad \text{or} \quad \frac{\partial f(y_c, H_p)}{\partial H_p} > 0}_{\text{mayor capital de los padres lleva a mayor capital de los niños}} \quad \text{and} \quad \underbrace{\frac{\partial^2 f(y_c, H_p)}{\partial y_c \, \partial H_p} > 0}_{\text{complementariedad: padres con más capital saben invertir mejor en sus hijos}}$$

Think of this: $H_c = f(y_c, H_p)$, different household technology for each individual

f → same for all families
y_c → choose
H_p → given by g. parents

→ This is a recursive property:

1. As a person increases her human capital she affects her late. (no aprendes de chico, te cuesta más aprender).

2. Economy's production today depends on how much you start with. This is a process that you build in what you have.

→ Recall that diminishing returns to investment in kids, $f_{yy} < 0$. (more information, harder and harder to get it given a brain B).

• one special case: suppose $H_c = \psi(y_c) H_p$; $\frac{\partial \psi(\cdot)}{\partial y_c} >$, $\frac{\partial^2 \psi(\cdot)}{\partial y_c^2} \le 0$.

Let's see an "economic growth" consequence of this :

- $\frac{\partial H_c}{\partial y_c} = \psi_y(\cdot)\, H_p$, $\frac{\partial^2 H_c}{\partial y_c\, \partial H_p}$ $\psi_y > 0$

more investment, more human capital. complementarity

2) $\frac{H_p}{H_c} = 1 + g_H = \psi(y_c)$, so if you hold y_c constant, growth rate will be constant for the countries.

- This is saying that poor economies have it difficult to catch up. (inequality maintains itself constant over time, because growth is constant).

- If we analyse world per capita income, richer countries have not become richer.

Now, let's analyse the parental problem with this modification :

$$\max_{c_p, y_c} \; U(c_p) + a\, V_c\,(r\, f(y_c, H_p))$$

$$[c_p] : \quad U'(c_p) = \lambda \qquad [y_c] = \; a \cdot V'_c \cdot r \cdot f_y = \lambda$$

The first order conditions are the same, but in this case $q = (a, r, w_p, H_p)$, $x = (c_p, y_c)$. It might be interesting to know what happens when H_p changes ? This is :

$$\begin{bmatrix} \frac{\partial c_p}{\partial H_p} \\ \frac{\partial y_c}{\partial H_p} \end{bmatrix} = - \begin{bmatrix} \partial g_1/\partial c_p & \partial g_1/\partial y_c \\ \partial g_2/\partial c_p & \partial g_2/\partial y_c \end{bmatrix}^{-1} \begin{bmatrix} \partial g_1/\partial H_p \\ \partial g_2/\partial H_p \end{bmatrix}$$

Notice, $ar f_y(y_c, H_p) V_c'(r f(y_c, H_p)) - U'(c_p) = 0$ ... g_1

$c_p + y_c - w_p = 0$ ... g_2

$$\begin{bmatrix} \partial g_1 / \partial H_p \\ \partial g_2 / \partial H_p \end{bmatrix} = \begin{bmatrix} ar\,[f_{yH} V_c' + f_y \cdot V''_c \cdot r f_H] \\ 0 \end{bmatrix}$$

Set $\frac{\partial W_p}{\partial H_p} = r = 0$ } by taxing. ; $w_p = r H_p = r f(y_p)$

$$\begin{bmatrix} \partial c_p / \partial H_p \\ \partial y_c / \partial H_p \end{bmatrix} = \frac{-1}{D} \begin{bmatrix} 1 & -ar\,(f_{yy} V_c' + r f_y f_y V''_c) \\ -1 & -V''(c_p) \end{bmatrix}$$

$$\cdot \begin{bmatrix} ar\,[f_{yH} V_c' + f_y V''_c\, r f_H] \\ 0 \end{bmatrix} = \begin{bmatrix} \dfrac{\overbrace{-ar f_{yH} V_c'}^{① <0} - \overbrace{ar f_y V''_c\, r f_H}^{>0 ②}}{D} \cdots \text{can't sign} \\ \underbrace{\overbrace{+(ar f_{yH} V_c')}^{>0 ②}}_{>} + \underbrace{\overbrace{(ar f_y V''_c\, r f_H)}^{<0 ①}}_{<} \cdots \text{can't sign} \end{bmatrix}$$

$\frac{dy_c}{\partial H_p}$: ① "Income effect" (feels richer because H_p increases), consume more for himself. Since the budget constraint is fixed, "eat more for me, as parent".

: ② "Substitution effect" (changing the prices of investing) pushing you to invest more on kid capital

$\frac{dc_p}{\partial H_p}$: reversed

• This is why the question, do more educated parents spent more time (y_c) with their kids?, remains unclear theoritically.

• Empirically, measuring the magnitude of the "income effect" and the substitution effect, more educated parents spent more time (y_p) with their kids.

• Two properties of "kids markets" are the following: (recall $f(y, H_p)$ and $f_y > 0$, $f_{yy} < 0$).

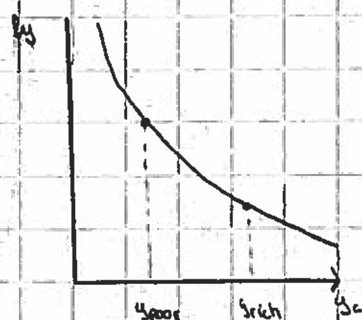
→ you do affect them when you give them things (towards negating competition)

→ every household is having its own production function

Think: $H_c = f(y_p, y_c, H_{gp})$, and so on ...

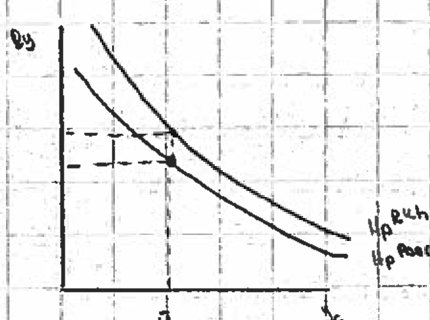
• Equity v.s. efficiency:

If $H_c = f(y_c)$:



There is no trade-off between efficiency and equity: go ahead an give to the poor because they have more f_y.

If $H_c = f(y_c, H_p)$:



• For $\bar{y}$, it is more efficient y_c spending for the rich, but this varies across different $\bar{y}$'s.

$H_p^{Rich} > H_p^{Poor}$

• **Ability:** (may be important in regression to the mean)

I.Q.: cognitive and non-cognitive abilities
=> transmitted automatically from family (cognitive)
=> Interaction between genetics and environment (non-cognitive)

→ How to introduce ability:

$A_c = (1-h)\bar{A} + hA_p + v_c \qquad E[v_c]=0 \qquad E[A_c]=E[A_p]=\bar{A}$

h ⇒ degree of inheritability ; $0 \leq h \leq 1$.

In our analysis we assume there is a single dimension abilities and is early determined.

• Why is ability relevant?
- determines inequality (e.g. earnings in cohort)
- intergenerational mobility

Bring in abilities: $H_c = f(y_c, H_p, A_c)$; $\frac{\partial f}{\partial A_c} = f_A > 0$

• if you are more able investment is better for you: $f_{yA} > 0$
• f_{HA}? not going to work much.

• What happens if A_c raises?

$\frac{dw_c}{dA_c} > 0 \qquad \frac{dH_c}{dA_c} > 0$, but $\frac{dy_c}{dA_c} < 0$ income effect

$R_y = r f_y$ raises. $\qquad \frac{dy_c}{dA_c} > 0$ substitution effect

• You can't sign neither $\frac{dC_p}{dA_c}$.

• Intergenerational perspective •

Parental ability : $\frac{\partial w_p}{\partial A_p} > 0$, $\frac{\partial H_p}{\partial A_p} > 0$.

• Ability increases means more earning and more human capital.
• Higher ability of parents will increase H_c of child through effect on income on H_p of parents (indirect channel).

There may also be a direct channel. Assume a transmission mechanism : $A_c = \underbrace{(1-h)\bar{A}}_{\text{average ability of society}} + h A_p + v_c$

h, degree of inherability of adult to child
v_c, random determinant of ability of child (independent across generations and individuals).

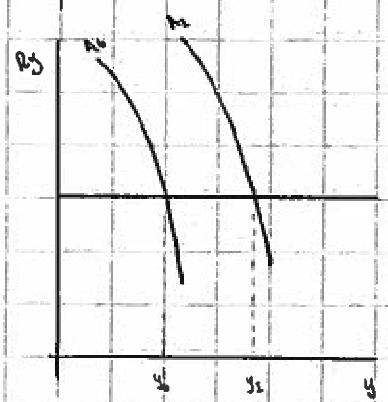
If $0 < h < 1$, regression to the mean.
If $h = 0$, ability is just random.

In equilibrium, $VAR(A_c) = \sigma^2 A_c = \sigma^2 A_p = \sigma^2_A = VAR(A_p)$, then
$VAR(A_c) = h^2 \, VAR(A_p) + VAR(v_c)$ $\Rightarrow$ $VAR(A^*) = \frac{VAR(v_c)}{1-h^2}$ Note $\frac{1}{1-h^2}$ multiplier.

• We want to link this to earnings in two situations :
  • perfect capital markets
  • imperfect capital markets.

• Recall $H_c = f(y_c, A_c)$ and assume perfect capital markets.

This means $R_y^* = R$ for all families :



$R = rfy$: everyone will have the same rfy around this point.

let's say two persons have A_1, but they have different incomes, say w_p low, w_p high. This will cause borrowing-repayment situations.

- Perfect capital market :

If capital market are perfect, the income of the child will only depend on his ability :

$$w_c = a + bA_c + \nu_c \quad \ldots \quad (1)$$
$$w_p = a \quad bA_p \quad + \nu_p \quad \ldots \quad 2$$
$$\Leftrightarrow \quad bA_p = w_p - a - \nu_p$$

And we know that the ability process is :

$$A_c = (1-h)\bar{A} + hA_p + \varepsilon_c$$

Then $w_c = a + b\left[(1-h)\bar{A} + hA_p + \varepsilon_c\right] = a + b(1-h)\bar{A} + bhA_p + b\varepsilon_c$

so $w_c = a + b(1-h)\bar{A} + h\left[w_p - a - \nu_p\right] + b\varepsilon_c$

$w_c = a + b(1-h)\bar{A} + hw_p - ah - h\nu_p + b\varepsilon_c$

$w_c = \underbrace{a + b(1-h)\bar{A} - ah}_{c} + hw_p \underbrace{- h\nu_p + b\varepsilon_c}_{\theta_c}$

Then $w_c = c + hw_p + \theta_c$, which depicts that w_p is a "perfect signal" of A_c when capital markets are perfect.

• **Imperfect capital markets model** •

$\ln A_c = a + h \ln A_p + \nu_c$; $\overline{\ln A_c} = \overline{\ln A_p} = \overline{\ln A}$,

$\ln A_c = (1-h)\,\overline{\ln A} + h \ln A_p + \nu_c$

$\ln w_c = b + c \ln w_p + \varepsilon$

$\to \ln w_c = \alpha + \beta \ln A_c + \delta \ln w_p$

• Substituting $\ln A_c$:

$\to \ln w_c = \alpha + \beta (1-h)\,\overline{\ln A} + \beta h \ln A_p + \delta \ln w_p$

but we know : $\ln w_p = \alpha + \beta \ln A_p + \delta' \ln w_{gp}$

$\iff$ $\beta \ln A_p = \ln w_p - \alpha - \delta' w_{gp}$

Then $\ln w_c = \alpha + \beta(1-h)\,\overline{\ln A} + h \ln w_p - h\alpha - h\delta' \ln w_{gp} + \beta \nu_c + \delta \ln w_p$

$= (1-h)(\alpha + \beta \overline{\ln A}) + (h+\delta) \ln w_p \underbrace{- h\delta' \ln w_{gp}}_{\text{why ?}} + \beta \nu_c$

• why impact of w_{gp} is negative ?

Recall $-h\delta'$ is a coefficient that holds everything else constant.

Suppose there are two persons , 1 and 2 :

1	2
w_0	w_0
$\uparrow A_p$	$A_p \downarrow$
$\downarrow y \Rightarrow w_{gp} \downarrow$	$\uparrow y \Rightarrow w_{gp} \uparrow$

$\ln w_p = \alpha + \beta \ln A_p + \delta \ln w_{gp}$

if w_p is fixed, when someone has more w_p she has less A_p.

• Now, we can model this including the human capital of the parents:

$$\ln A_c = (1-h)\ln \bar{A} + h \ln A_p + \nu_c$$

$$\ln w_c = \alpha + \beta \ln A_c + \gamma \ln w_p + \delta \ln H_p \quad ; \quad r H_p = w_p \Leftrightarrow \ln r + \ln H_p = \ln w_p$$

$$\ln w_c = \alpha + \beta \ln A_c + \gamma \ln w_p - \delta \ln r + \delta \ln w_p$$

$$\ln w_c = \alpha + \beta \ln A_c + (\gamma + \delta) \ln w_p - \delta \ln r \quad \Leftrightarrow \quad \ln w_p = \alpha + \beta \ln A_p + (\gamma+\delta) \ln G_p - \delta \ln r$$

$$\Leftrightarrow \quad \beta \ln A_p = \ln w_p - \alpha - (\gamma+\delta)\ln G_p + \delta \ln r$$

$$\ln w_c = \alpha + \beta \left[(1-h)\ln \bar{A} + h \ln A_p + \nu_c\right] + (\gamma+\delta)\ln w_p - \delta \ln r$$

$$= \alpha + \beta(1-h)\ln \bar{A} + \beta h \ln A_p + \beta \nu_c + (\gamma+\delta)\ln w_p - \delta \ln r$$

$$= \alpha + \beta(1-h)\ln \bar{A} + h\left[\ln w_p - \alpha - (\gamma+\delta)\ln w_{Gp} + \delta \ln r\right] + (\gamma+\delta)\ln w_p - \delta \ln r$$

$$= \alpha + \beta(1-h)\ln \bar{A} + h \ln w_p - h\alpha - h(\gamma+\delta)\ln w_{Gp} + h\delta \ln r + (\gamma+\delta)\ln w_p - \delta \ln r$$

$$\ln w_c = \alpha + \beta(1-h) - h\alpha + (h+\gamma+\delta)\ln w_p - h(\gamma+\delta)\ln w_{Gp} + \delta(h-1)\ln r$$

• Returning to our household production problem:

• Another question that might be interesting is if there is more than one child, what happens?

Notation: consider the abilities of the two childs A_1, A_2 with $A_1 > A_2$.

$H_c = f(y_c, A)$, with $f_{y_c} > 0$, $f_A > 0$, $f_{y_c A} > 0$.

What happens with y_1^* and y_2^*. Clearly, efficiency establishes $y_1^* > y_2^*$

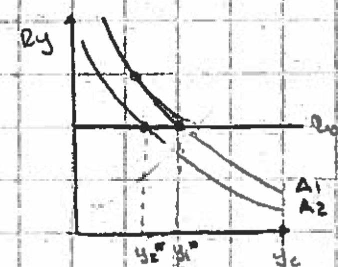
• If $H_1^* = H_2^*$, then $w_1 = w_2 = rH$ only possible if $y_1^* < y_2^*$.

The parent is a social planner inside his family.

• Natural generalization •

$U(w_p) = v(c_p) + a_1 V_1(w_{c1}) + a_2 V_2(w_{c2})$, we can assume both $a_1 = a_2$ and $V_1 = V_2$.

• Perfect capital market which implies $R_1^* = R_2^* = R$ wich implies $y_1^* > y_2^* \Rightarrow H_1^* > H_2^* \Rightarrow Y_1^* > Y_2^*$.



• The problem is:

$$U(w_p) = U(c_p) + a_1 V_1(w_{c1}) + a_2 V_2(w_{c2})$$
$$w_{c1} = rH_{c1} = rf_{c1}$$
$$w_2 = rH_{c2} = rf_{c2}$$

$H = f(y_c)$

budget constraint of parents: $c_p + y_1 + y_2 = w_p$

First order conditions:

$$\underbrace{U'(c_p) = \lambda = a_1 \frac{\partial V_1}{\partial y_1} = a_2 \frac{\partial V_2}{\partial y_2}}$$

$$\Rightarrow \quad a_1 V_1' r f_{y1} = a_2 V_2' r f_{y2} \quad \Leftarrow$$

if $a_1 = a_2 = a$, then $\underbrace{V_1' r f_{y1} = V_2' r f_{y2}}$

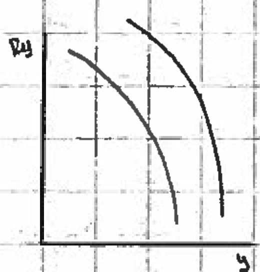
last dollar spent in each kids gives the same utility for parents

• Could it be in equilibriem that $V_1^* < V_2^*$?

If this is true, $V_1' < V_2'$, then $f_{y_1}^* < f_{y_2}^*$. This means $y_1^* > y_2^*$, which implies $H_1^* > H_2^*$ $\Leftrightarrow$ $w_1^* > w_2^*$ $\Leftrightarrow$ $V_1 > V_2$ ∇.

Then, in equilibrium $V_1^* \geq V_2^*$.

Parents take compromises balancing theirs spendings between kids. Degree of concavity of V' matters:



$y_1^* < y_2^*$ possible, but implies $H_1^* > H_2^*$ because $w_1^* > w_2^*$ (ability).

• Since $a_1 f_{y_1} V_1' = a_2 f_{y_2} V_2'$, it is more likely that $y_2 > y_1$ as a_2/a_1 increases.

In general, $w_{k1} \neq w_{k2}$ because rates the return differ by kid.

• Parents do all the time a trade-off between efficiency and equality as government do with inequality and D.W.L.'s that taxes generate.

• Now, suppose the parents don't know the abilities of the child. Then, ability can be A_1 or A_2.

• Suppose $Prob(A_1) = .5$ and $Prob(A_2) = .5$, this means that the problem of the parents is:

The problem that parents face now is:

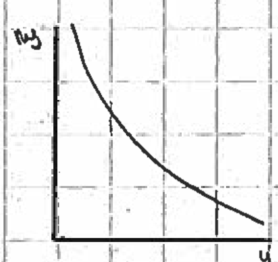
$$U(wp) = U(cp) + .5\, a\, V_1(A=A_2) + .5\, a\, V_1(A=A_1) + .5\, a V_2(A=A_2) + .5\, a\, V_2(A=A_1)$$

and they have to choose cp, y_1, y_2. In equilibrium $y_1^* = y_2$: they will have same expected utility of their investments. (notice that variance may matter as well).

• Other capital than Human •

Define other form of capital: there is a return from capital that is the same for everyone: R_k. What are the consequences for our household production model.

Simplicity: return to the single child household production. We model the situation in which very little y will give very high R_y:



• very little y gives very high R_y. Think: $R_y \to \infty$ as $y \to 0$, so $y > 0$.

• Every family $y > 0$, as long as $R_y > R_k$, which is a weak assumption.

° As parents become richer, $R_y < R_k$.

→ Now, we are going to consider the income (welfare) of the kid: $I_c = W_c + \underbrace{R_k \cdot k_c}_{i?}$ si, porque crece

→ Now, we have two forms of investment, then the parents budget constraint is: $C_p + y_c + k_c = I_p$.

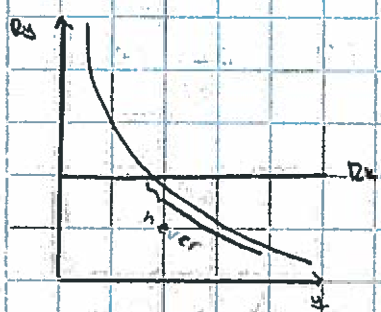
2) The utility function of the parents now is:

$$U(I_p) = u(c_p) + a\, V(\underbrace{w_c + R_K K_c}_{I_c}).$$

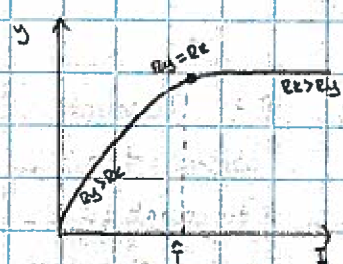
Then, the first order conditions of the problem are:

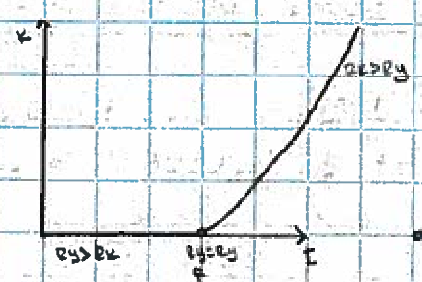
- $U'(c_p) = \lambda$
- $a V'_c \dfrac{\partial I}{\partial y_c} = \lambda$
- $a V'_c \cdot \dfrac{\partial I}{\partial k_c} \leq \lambda$, if $<$, $k_c = 0$.
- $a V'_c R_K \leq a V'_c R_y \iff R_K <$

2) corner solution may be possible in K_c.



- $\dfrac{\partial y^*}{\partial I_p}$ } normal good as long as $R_y > R_y$.





• Household production with an overlapping period when children are adults.

• Initially, parents can't leave debt to their children.

• Parents can also invest in other capital rather than the one of their kids.

• Parents can leave "bequest" to their kids.

• Parents also receive "bequest" from grandparents.

• Ages: middle (m) and old (o).

• Budget Constraint 1: (middle age constraint)

$$C_m + y_c + k_m = E_p = w_p + b_p \quad ; \quad \text{always } b \geq 0.$$

• Budget Constraint 2: (old age constraint)

$$C_o + b_c = R_k \cdot k_m$$

Recall: $w_c = r H_c = r f(y_c)$; $f_y > 0$, $f_{yy} < 0$, $\frac{dR_y}{dy} < 0$.

• Parents' utility function:

$U[E_p] = u(c_p) + \beta\, u(c_o)$; β = discount factor on "old days consumption". (lifecycle degree of discount).

$+ \beta a\, V_c\,(E_c = w_c + b_c)$

a = altruism.

$\underbrace{\quad\quad}_{\beta}$ "discount for future" vs. $\underbrace{\quad\quad}_{a}$ "discount for altruism"

• Recall: $u' > 0$, $u'' < 0$, $V' > 0$, $V'' < 0$.

- The first condition of this problem induce the following problem:

$\to$ $U'(c_m) = \lambda$ marginal utility of middle age consumption
$\to$ $\beta \cdot U'(c_o) = \lambda / R_K$ discounted marginal utility of old age consumption

- Then, $U'(c_m) = R_K \cdot \beta \cdot U'(c_o)$ optimality between consumption

$\to$ $\beta a V'_t R_y = \lambda = U'(c_m)$; $y_c > 0$
$\to$ $\beta a V'_c \leq \lambda / R_K$; $< \Rightarrow b_c = 0$
$= \Rightarrow b_c > 0$

$\to$ $\beta R_K a V'_c = \beta a V'_c R_y \Leftrightarrow R_K \leq R_y$; $< \Rightarrow b_c = 0$
$= \Rightarrow b_c > 0$

If $aV'_c < U'(c_o)$, then $b_c = 0$. The more selfish they are, the less bequest they give. $(R_y > R_K)$.

It will be more efficient if you can do negative b_c. This will imply $R_y^* = R_K^*$.

<u>Public policy problem</u>: there may be very selfish parents:

$aV'_c < U'(c_o)$, when $b_c = 0$.

"Preference transmission" model

- Suppose kids have a utility function $V_c(E_c, G)$; $\frac{\partial V_c}{\partial G} < 0$ where G = guilt.

- Suppose parents have the following utility function:

$$U_p = U(c_m) + \beta U(c_o) + \alpha\beta V_c(E_c, G)$$

- Assume $G(z_c)$, where $\frac{dG(z_c)}{dz_c} > 0$ and z_c is the expenditure made by parents in making children guilty.

Budget constraints of the parents:

$$c_m + y_c + z_c + k_m = E_p + S_p(G) \quad \text{(middle age)}$$

grandparents "guilty gift".

$$c_o + b_c = Rk + \cancel{S_p(G)}$$

; $S(G)$ is the child support of the parents and $\frac{dS(G)}{dG} > 0$.

You lower kids utility in an special way; they fill obligation to come back and give you money in your old days.

- Notice: in this case $b_c = 0$, what for you will give them "bequest" and then spend for them to comeback.

- There are no "net welfare conclusions". You lower their utility but you can spend more if they comeback.

• If $b_c > 0$, then $\beta_c = 0$, because otherwise the spendings will be inefficient.

• If you can avoid buying β_c you will do it: you can sign a contract with your son or daughter:

$$R_z < R_k.$$

• A necessary, but not sufficient condition, for parents spending in z is:

$$R_z > R_k.$$

2. The point of this model is to "endogenize the preferences".

• More traditional Human Capital Problem: Education •

This studies the investment of young adults in their selves. They take as given H^0, the human capital they have until the period they decide wether or not to go into college. H_0, of course, is different for every adult.

__Life__: E_{Hi} ; $i = 1, \ldots, H$ } earning per period in each situation
E_{Ci} ; $i = 1, \ldots, H$

2. College tuition: f. $R = \dfrac{1}{1+r}$; rate of discount.

$V_H = \sum_{i=0}^{H} E_{iH} R^i$ total earnings if stay just with highschool.

$V_C = \sum_{i=0}^{H} E_{iC} R^i - f$ total earnings if going to college

• If just stay with highschool : $E_{0H} = W_{0H}\, T$

$E_{0H} = W_{0H} \left[T - t_c \right]$; t_c = time in college.

• In $H=0$, both get the same payment.

• So the point is to compare what happens when you go to highschool to what happen when you go to college.

→ This comparison may be done by the following equation:

$$\sum_{i=1}^{M} T \; \Delta w_i \; R_i \quad \boxed{?} \quad \underbrace{T_c \; W_0}_{\text{foregone earnings}} + \underbrace{f}_{\text{tuition}}$$

Notice that $\Delta w_i = w_{ci} - w_{Hi}$, is the wage differential of "College to highschool" and the equation asumes that:

$$\Delta w_i = \Delta w_j \quad \forall\, i, j$$

→ There are six relevant parameters in the study of this decision:

• T_c = hours of College.
• W_0 = wage at $t=0$, which is the same for both highschool and college.
• H = life expectancy
• Δw = " benefits of College"
• $w_0 T_c$ = foregone earnings
• f = tuition

• Now, if we consider the complete serie:

$$\sum_{i=1} T\Delta w_i \, \varrho_i = T\Delta w \cdot \left[\frac{1 - \varrho^{H+1}}{1-\varrho} \right] \boxed{?} f + T_c \cdot w_0$$

↳ This analysis can be extended to "add" the probability of dying at age i., which will be different in each period of the agent's life. So if we let m_i be this probability:

$$\sum_{i=1}^{H} T(1-m_i)\, \Delta w_i \, \varrho_i \; \boxed{?} \; f + T_c \, w_0$$

↳ Suppose, for a conceptual excercise, that there is no tuition. Then, a flat tax to earnings will not affect the decision.. (i.e. the term $(1-T)$ will drop out).

↳ If there is tuition, on the other hand, costs are falling by less than τ and the costs are less (i.e. the decision may be affected.

↳ Clearly, ability affects both foregone earnings and wage differentials. We can think of this cancelling out on both sides of the equation.

• ↳ **Why do we have this large increases in tuition?**

↳ Education is intensive in education

↳ I.e.: increase in tuition is due to increase an increase in costs. Large tuition is related to higher returns to College.

→ Then, it should be true that the following inequality holds:

$$\frac{d}{dt}\left(\frac{W_c}{W_H}\right) > 0 .$$

• Think of inequality: "Why is there no more people taking advantage of "College premium"."

• Dropout highschool: face awful situation in any dimension:

→ employment → earnings → wealth
→ marriage (less getting, more divorced)

Note: Accumulation of capital also explains rise in wages.

• Now, let's say there is a set of abilities:

A = abilities (cognitive and non-cognitive)
H_0 = initial Human Capital at high-school graduation.

If we want to "endogenize" time spent in College, what are some reasonable assumptions:

$T_c(A, H_0)$; $\frac{\partial T_c}{\partial A} \leq 0$, $\frac{\partial T_c}{\partial H_0} \leq 0$

Also, we can consider the determinants of "tuition fees":

$f(A, H_0)$; where f are tuition fees.

Then, $\frac{\partial f}{\partial A} < 0$ and $\frac{\partial f}{\partial H_0} < 0$ —

→ the wage differential can be signed here as well:

$\Delta w^k (Q^k, A^k, Ho^k, H^k, T^k)$; k is an individual.

→ $\dfrac{\partial \Delta w^k}{\partial Q} < 0$ $\dfrac{\partial \Delta w^k}{\partial tc} > 0$.

Empirically, education gains are everywhere:

→ less marriage brakes → better health
→ raise productivity in household production
→ better use of contraception method
→ better use of drugs
→ better adaptation to new environment (e.g. technology)

• Model education with utility •

We model the individual's utility as one given by the following function:

$U(x_i, l_i, H, s_i)$; $i = 1, 2$

where x = goods l = leisure s = college m = hours worked
t = tuition fees T = total time = 1
h = hours spent in investing in college
H = high school education

Then: $l_1 + m_1 + h = T$ and $l_2 + m_2 = T$

The "total" utility function is:

$V = U_1(x_1, l_1, H) + p(H, S)\, \beta\, U_2(x_2, l_2, H, S)$, where $p(\cdot)$ is the probability of surviving in period 2 and β is the discount rate.

In this kind of models we implicitly assume that there is a "perfect annuity market": expected consumption = expected earnings (access to full earnings wherever)

Budget constraint:

$$x_1 + \frac{p(\cdot) \cdot x_2}{1+r} + w_1 l_1 + \frac{p(\cdot) \cdot w_2(\cdot) l_2}{1+r} + f + w_1 h$$

$$= \underbrace{w_1 + \frac{p \cdot w_2(\cdot)}{1+r} + \frac{p \cdot H(\cdot)}{1+r}}_{\text{full income}}$$

Think: $s = f(h, H, A_c, A_n)$; $f_j > 0$ with $j = H, A_c, A_n$

f_{jy} for $j \neq y$

F.O.C.

$U_{1x} = \lambda$ and $\beta \cdot U_{2x} \cdot p = \frac{\lambda p}{1+r}$

suppose that, in equilibrium, $\beta = \frac{1}{1+r}$, then: $\boxed{U_{1x} = U_{2x}}$ //

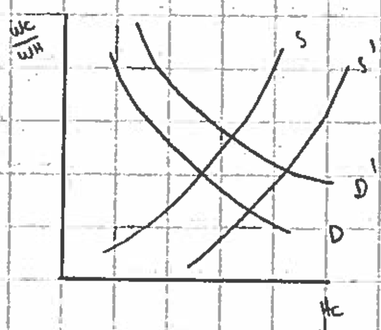
Also: $U_{1l} = \lambda w_1$ and $p \beta U_{2l} = \frac{\lambda p w_2}{1+r}$

then: $\frac{U_{1l}}{\beta U_{2l}} = \frac{w_1}{w_2}(1+r)$. Likewise, if $\beta = \frac{1}{1+r}$, then

$\frac{U_{1l}}{U_{2l}} = \frac{w_1}{w_2}$.

Increase earnings:

Increase in tuition (80's and 90's):



2. Education not only gives you skills, it gives you "tecniques" to know how to "get information": learning how to learn. (schooling is a effective way to do this).

2. Information acquisition aspects

2. Efficient way to get social networking

• Health as Human Capital •

2. we want to treat health as investment per se,

2. mechanical medicine and people's decisions of health (surgery instruments)

2. We want to study "what people do with their health".

In an ordered fashion:
1. Statistical Value of Life (SVL)
2. Optimal investment in health
2.a health as self protection

The "utility maximizing problem" is:

$$\max_{x_1, \ell_1, x_2, \ell_2, h} \; U(x_1, \ell_1) + \beta\, s(h, \text{schooling})\, U(x_2, \ell_2)$$

$$\text{s.t.} \quad x_1 + \frac{s(h)x_2}{1+r} + g(h) = w_1(1-\ell_1) + \frac{s(h, \text{schooling}) \cdot w_2 (1-\ell_2)}{1+r}$$

$\boxed{\text{Total time} = 1}$ $\quad x_i \Rightarrow$ consumption $\quad \beta \Rightarrow$ discount factor

$\ell_i \Rightarrow$ leisure

Basic assumptions: $g' = \frac{\partial g(\cdot)}{\partial h} > 0 \quad g'' > 0$, convex cost function

$s' = \frac{\partial s(\cdot)}{\partial h} > 0$, $s'' < 0$, diminishing marginal returns.

First order conditions:

$[x_1]$: $U_{x_1}(x_1, \ell_1) = \lambda \; \ldots (1)$ $\quad [x_2]$: $\beta s(\cdot)\, U_{x_2}(x_2, \ell_2) = \frac{\lambda\, s(\cdot)}{1+r} \; \ldots (2)$

$[\ell_1]$: $U_{\ell_1}(x_1, \ell_1) = \lambda w_1 \; \ldots (3)$ $\quad [\ell_2]$: $\beta s(\cdot)\, U_{\ell_2}(x_2, \ell_2) = \frac{\lambda \cdot s(\cdot) w_2}{1+r} \; \ldots (4)$

$[h]$: $\beta s'(\cdot)\, U(x_2, \ell_2) = \lambda \left[g'(\cdot) + \frac{s'(\cdot)}{1+r} (x_2 - w_2(1-\ell_2)) \right] \; \ldots (5)$

From (1) and (3): $\frac{U_{\ell_1}(x_1, \ell_1)}{U_{x_1}(x_1, \ell_1)} = w_1$

Assumption: persons discounts in the same way than the market.

then: $\frac{U_{x_2}(x_2, \ell_2)}{U_{x_1}(x_1, \ell_1)} = 1$.

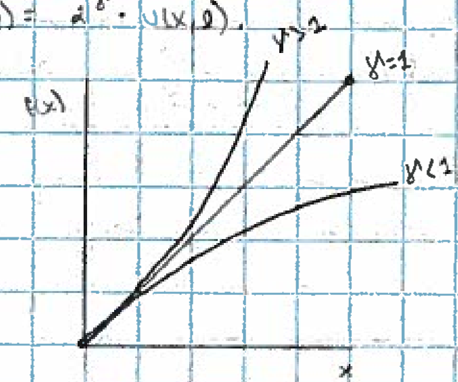
To analyse (5), we can make a "Math Commercial".

Define: an homogeneous function of degree k is:

$$f(\alpha x) = \alpha^k \cdot f(x)$$

If U is h.o.d. γ, then $U(\alpha x, \alpha \ell) = \alpha^\gamma \cdot U(x,\ell)$.

If $\gamma = 1$ the function is "linear"
$\gamma < 1$ " " "concave"
$\gamma > 1$ " " "convex".



Euler's theorem:

Take x, ℓ and define $x' = tx$ and $\ell' = t\ell$, then $U(x', \ell') = t^\gamma \cdot U(x,\ell)$ because U is h.o.d. of degree γ.

Taking the derivative with respect to t of each side:

$$\frac{\partial U}{\partial x'} \cdot \frac{\partial (tx)}{\partial t} + \frac{\partial U}{\partial \ell'} \cdot \frac{\partial (t\ell)}{\partial t} = \gamma t^{\gamma-1} \cdot U(x,\ell)$$

$\Leftrightarrow$

$$U_x(tx, t\ell) \cdot x + U_\ell(tx, t\ell) \cdot \ell = \gamma \cdot t^{\gamma-1} \cdot U(x,\ell)$$

if $t=1$:

$$U_x(x,\ell) \cdot x + U_\ell(x,\ell) \cdot \ell = \gamma \cdot U(x,\ell)$$

Then: $\frac{1}{\gamma}\left[U_x(x,\ell) \cdot x + U_\ell \cdot (x,\ell)\right] = U(x,\ell)$

we can take (5) and substitute $\lambda = U_{x2}(x_2, \ell_2)$:

$$\beta \, s'(\cdot) \cdot U(x_2, \ell_2) = U_{x2}(x_2, \ell_2) \left[g'(\cdot) + \frac{s'(\cdot)}{1+r} (x_2 - w_2(1-\ell_2)) \right]$$

recalling that $\beta = \frac{1}{1+r}$, then:

$$\frac{s'(\cdot)}{1+r} \cdot \frac{U(x_2, \ell_2)}{U_{x2}(x_2, \ell_2)} = \boxed{g'(\cdot) + \frac{s'(\cdot)}{1+r} (x_2 - w_2(1-\ell_2))} \Rightarrow RHS$$

If U is h.o.d. γ, then $\frac{U(x_2, \ell_2)}{U_{x2}(x_2, \ell_2)} = \frac{1}{\gamma} \left(U_x \cdot x_2 + \underbrace{\frac{U_{\ell 2}}{U_{x2}}}_{wL} \cdot \ell_2 \right)$

Then $\frac{s'(\cdot)}{1+r} \cdot \frac{1}{\gamma} \cdot (x_2 - w_2 \ell_2) = RHS$

or:

$$s'(h) \cdot \frac{1}{\gamma} (x_2 - w_2 \ell_2) - \frac{s'(\cdot)}{1+r} (x_2 - \ell_2 w_2) = g'(\cdot) + (-w_2) \frac{s'(\cdot)}{1+r}$$

then:

$$\frac{s'(h)}{1+r} \left[\frac{1}{\gamma} - 1 \right] (x_2 + w_2 \ell_2) = g'(\cdot) + \frac{s'(\cdot)}{1+r} (-w_2).$$

• Statistical value of life •

Tipical literature: Money value of MAN.

- Tipical income in US $40,000
- Take 1/8 of that for maintainance and leisure: $72,000
- Anual value $110,000, but missing the value of $1.

let's say it is $1 = 1/2.

Then $W = 2(110,000) = 220,000$.

- If the discount rate is 4%, then:

$$VSL = \frac{220{,}000}{.04} = \$5{,}500{,}000$$ in the U.S.

Exercise:

- How costly was the AH1N1 flu in Mexico:

GDP per capita in Mexico: $13,500

Number of deaths 119

$$VSL = 5{,}500{,}000 \cdot \frac{13{,}500}{40{,}000} = 1{,}856{,}250$$

Stat. Loss = $119(1{,}856{,}250) = 220{,}893{,}750$

- What about Haiti earthquake.

GDP per capita Haiti: $1,300

number of deaths: 230,000

$$VSL = 5{,}500{,}000 = \frac{1{,}300}{40{,}000} \; 5{,}500{,}000 = 178{,}750$$

Stat loss = $= 4.1125 \times exp^{10}$

$41{,}112{,}500{,}000 = 41{,}112{,}500{,}000.000$

• More on health investment:

Suppose s_1 is the conditional probability of surviving age 1.

s_2 " " " age 2.

⋮

s_n " " " age n.

• Note: $s_1 = S_1$.

Likewise, S_i is the unconditional probability of surviving to age i.

• Individuals' maximization problem will be:

$$V = S_1 U_1(x_1, l_1) + \beta S_2 U(x_2, l_2) + \beta^2 S_3 U(x_3, l_3)$$

g(h) is the expenditure cost function.
in health

For simplicity, we assume that the individual "spend" in health on age 1.

Assume $s_2(h)$ and diminishing marginal returns:

$$\frac{d s_2(h)}{dh} > 0, \quad \frac{d s_2(h)}{dh} \leq 0.$$

In this case, the budget constraint is:

$$s_1 x_1 + \frac{s_2 x_2}{1+r} + \frac{s_3 x_3}{(1+r)^2} = s_1 w_1 (1-l_1) + \frac{s_2 w_2 (1-l_2)}{(1+r)^2} + \frac{s_3 w_3 (1-l_3)}{(1+r)^3}$$

The relevant first condition in this problem is:

For x_1, ℓ_1, x_2, ℓ_2 s dropsout. Simply, MRS in each good should be the same.

$[h]$:

$$\beta \frac{ds_2}{dh} + \beta^2 \frac{d s_3 V_3}{dh} U_3 + \lambda \left[g'(h) + \frac{ds_2}{dh} \frac{1}{1+r} \left[x_2 - w_2(1-\ell_2) \right] + \frac{ds_3}{dh} \cdot \frac{1}{(1+r)^2} \left[x_3 - w_3(1-\ell_3) \right] \right]$$

Notice that, since $S_2 = s_1 s_2$, $S_3 = s_1 s_2 s_3$ and so on, even when only $s_2(h)$, investing in h impacts (Becker called it weight) the unconditional probability of surviving to period $S_1 s_2$.

"Investing in h helps the unconditional probability of surviving to the posterior to 2 periods".

Notice, other things the same, it is rational to "spend in the probability of surviving" in earlier periods. (the product $S_n = s_1 \cdot \cdots \cdot s_n$ is smaller each time).

The marginal benefit of "the general expenditure" in health is:

$$\boxed{HB = \sum_{i=1}^{\infty} \beta^i \frac{d s_2}{dh} s_3 \cdot \cdots \cdot s_i}$$

- Expenditure in different health goods (two periods)

$S_1 = 1$

$S_2 = d_1(h_1) \cdot d_2(h_2)$ } unconditional probabilities

h_i = expenditure on health, 1 = cardiovascular 2 = cancer

For example: there may be sorting by rank. In this case it will be perfect. Smarter girl with smarter boy and so on.

Example of complementarity: investing in child.

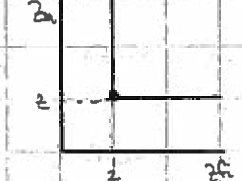
- Rank determines the sorting level. Then, you have incentives to educate yourself because that is the way to jump in the ranking.

Sociologist claim that this is a "zero-sum game". That is false because the "ranking jumps" increase the "social pie". jump and other decreases in rankins

- What happens when utility is not transferable?

In that case, $z_{im} = z = z_{jf}$, where z is the total output and both, the male and the female, receive exactly the same of it.

In that case, there is a extrictly positive sorting: you want z to be as big as possible and the way to have that and "don't waste" your resources is to positively sort.



- Is there any case when there are no gains from marriage?

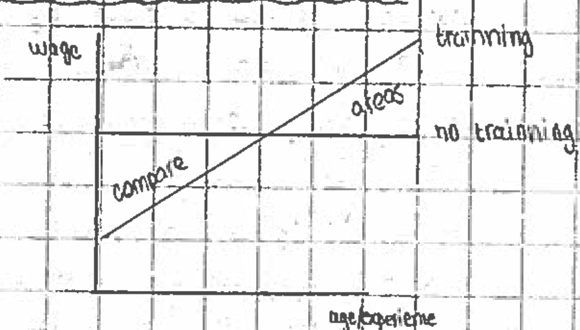
Yes, when $z_{ms} \leq 0$, complementarity doesn't (negatively) affect the household production good utility.

• **On the job investment:**

Two ways of "job investment": learning by doing and explicit investment.

There is a more basic "data problem" in this context than selection bias, namely, firms do not gather data on their "job investment spending".

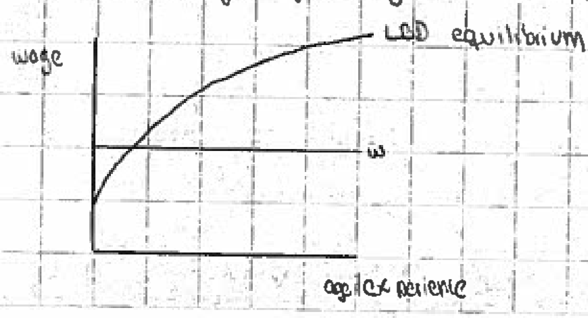
Think of the problem graphically:



1. An area comparison ratio will give you the return of the training investment.

2. Why does trainning happens when you are young?
   • If you want to aquire skills you may want to do it know in order to maximize the gain from that aquisition.

3. How does learning by doing looks like (happen)?



• Who pays for the trainning?

Clearly, this depends on the market structure and on the kind of trainning people receive.

There are 4 types of trainning:

• firm specific • industry specific • occupational specific
• general trainning.

2. If the market is competitive general trainning raises productivity in all markets. Then, no firm will be willing to pay for that employee's trainning.

2. If there is a competitive industry, the worker will face a higher uncertainty than in the general case and it is uncertain who is paying for the training.

2. Firm specific: firm pays.

Why is there job specialization

"Division of labor is limited by the extension of the economy" Adam Smith.

Is development leading to more specialization or is specialization leading to more development? You may argue in both senses.

Two fundamental approaches for trying to understand why people differ.

1. Roy model of specialization and comparative advantage.

• take differences as given

Is this arguable?

Yes, people become different too. How? training, learning by doing, human capital investment, etc.

2. Becker model: attack a problem in which individuals are identical ex-ante.

Assumptions:

→ identical individuals ex-ante
→ specialization in different skills or tasks
→ cooperation among specialists to produce widgets
→ cooperation is in teams (not only a firm, within a firm)
→ there are n tasks
→ investment in task specific skills (no dynamics)
→ T hours available per individual
→ h time spent investing in this task
→ $t = g(h)$ investment function, where $g'(\cdot) > 0$ and $g'' \leq 0$.

Think: it pays to develop different kind of skills. Why? increasing returns in doing it.

Y is the output produced: it is proportional to the effective labor input (works spent working times productivity).

$$Y = \overset{\text{productivity of economy}}{k} \cdot \underbrace{\ell \cdot H}_{\text{effective labor}}$$

[illegible]: $g(h, A_c, A_{nc})$ → complementarity with other individuals, etc. is brought by the shape of g.

Time can be allocated working or spending in human capital:

$$\boxed{\ell + h = T}$$

The problem that one agent faces is:

$$\max_{\ell, h} \{ Y = \alpha \cdot \ell \cdot g(h) \}$$

<u>F.O.C.s</u>

$$[\ell]: \quad \alpha g(h) = \lambda$$
$$[h]: \quad \alpha \cdot \ell \cdot g'(h) = \lambda$$
$$\Leftrightarrow \quad \boxed{\ell^* = \frac{g(h)}{g'(h)}} \quad \boxed{h^* = T - \frac{g(h)}{g'(h)}}$$

Suppose that $g(h) = c h^{\theta}$, $0 \leq \theta \leq 1$, where $c(A_{NC}, A_C)$, etc.

Then: $\boxed{\ell^* = \frac{h^*}{\theta}}$ and $\boxed{\ell^* + h^* = T}$

$$\boxed{\ell^* = \frac{1}{1+\theta} \cdot T}$$
$$\boxed{h^* = \frac{\theta}{1+\theta} \cdot T}$$

In equilibrium, what is the output?

$$Y^* = \alpha \cdot \ell^* \cdot g(h^*) = \frac{\alpha \cdot g(h^*)}{g'(h^*)} \cdot g(h^*) = \boxed{\frac{\alpha \cdot g^2(h^*)}{g'(h^*)} = Y^*}$$

Notice that there are increasing returns in investing in specialization.

If $g(h) = ch^{\theta}$, then: $Y^* = \frac{\alpha c}{(1+\theta)^{\theta+1}} \cdot T^{1+\theta}$ better to be more able.

if $\theta = 1$, $\boxed{Y^* = \frac{\alpha c}{4} T^2}$

↳ Increasing in T; double your time, more than double your output.

↳ This is why you don't want to diversify, you want to concentrate.

To show more clearly the benefits of specialization, let's deal with a double task (independent) data.

- 2 tasks: y_1, y_2
- 2 investment functions: $g(h_1)$, $g(h_2)$

Restrictions: $h_1 + l_1 = t_1$

$h_2 + l_2 = t_2$

$t_1 + t_2 = T$

"Obviously":

$$y_1^* = \frac{\alpha_1 c_1}{\theta_1} \cdot \left[\frac{\theta_1}{1+\theta_1} \right]^{1+\theta_1} \cdot t_1^{1+\theta_1} \qquad y_2^* = \frac{\alpha_2 c_2}{\theta_2} \left[\frac{\theta_2}{1+\theta_2} \right]^{1+\theta_2} \cdot t_2^{1+\theta_2}$$

Now, suppose $t_i = \frac{T}{2}$, for $i = 1, 2$

<u>What is the effect of dividing time?</u>

$$y^* = [\quad] \frac{T^{1+\theta}}{2^{1+\theta}}$$; output decreases more than half.

Suppose that the joint production function, $\phi = f(y_1, y_2)$ is leontief (a favorite del state) $\phi(y_1, y_2) = \min(y_1, y_2)$, then $y_1^* = y_2^*$.

This will cause people to put a lot of time to the thing they are not good at.

If $\alpha_1 = \alpha_2$, $c_1 = c_2$, $\theta_1 = \theta_2$ is same for everyone, then $t_1^* = t_2^*$ obviously $y_1^* = y_2^*$. (leontief)

Then: $$Y^* = y_1^* = y_2^* = \frac{\alpha c}{4} \left(\frac{T}{2} \right)^2 = \boxed{\frac{\alpha c T^2}{16} = Y^*}$$

→ if somebody needs to produce in fixed proportions, they suffer a lot when they specialize.

Suppose $\alpha_1 = \alpha_2 = \alpha$, $\theta_1 = \theta_2 = \theta$ and $c_1 \neq c_2$ s.t. $c_1 > c_2$, then $t_1 < t_2$.

More productive in task 1, you allocate less time in it.

if $\theta = 1$ and $c_1 = 2c_2 \Rightarrow t_1 = \frac{t_2}{\sqrt{2}}$.

Finally, if there are m tasks and $\phi = \min\{y_1, \ldots, y_m\}$,

$$\boxed{y_i^* = [\cdot\ ]\ \frac{t^2}{m^2}}$$

$\boxed{\text{2. what have we learned today? (nothing), there are huge gains from specialization}}$

↳ Entepraneur coordinates people

• Coordination costs •

• comunication.
• complementarity.
• cost function of coordination is increasing in the number of members the team has

↳ Model

$n = \#$ of members $= \frac{m}{s}$, where s is $\#$ of independent tasks

cost function : $c(n)$, $c'(n) > 0$, $c''(n) > 0$

$n(n-1)$ possible interactions

$I =$ per capita Income $= y(m/n) - c(n)$

$$\frac{\partial I}{\partial n} = \frac{\partial y}{\partial n} - c'(n) > 0 \quad , \quad \forall\ n \leq m$$

$$N < m$$

$N =$ numbers of individuals available (number of the market)

$$s = \frac{m}{N} > 1$$

↳ in this case division of labor is limited by extent of the market.

1. ✓ prior

2. $y'(n) < c'(n)$ all $n > 1$ (coordination costs are too big) (inneficient)

3. Division of labor

limited by extent of the economy

$y'(n) > c'(n)$ at the beggining

$c'(n) > y'(n)$ at the end ($c'(n)$ convex)

In this case there is an interior solution.

$$\max_{n} \quad \Pi = y(n) - c(n) \qquad 1 < n^* < m$$

FOC $y'(n) = c'(n)$ # teams $= m$

SOC $c''(n) = y''(n) > 0$

• Household

· 2 members

Household, markets, $m = 2$.

How do you explain that women are always in the market place

1. Roy - differences
2. Discrimination

Fertility

$$V_p = g(c_p) + U(H) + v(n)$$

H = Hc per child

n = # of child