DSP Lab- Homework6

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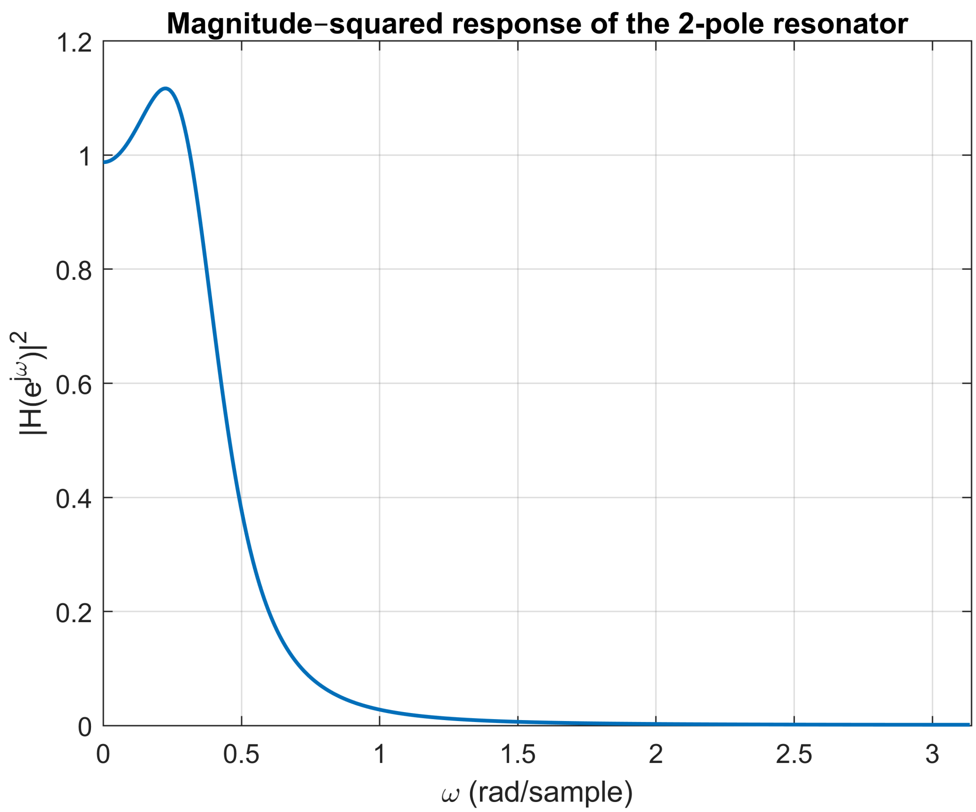
```
clc; clear; close;
```

```
R  = 0.8;
f0 = 100;
fs = 2000;
w0 = 2*pi*f0/fs;

a1 = [ 1 , -2*R*cos(w0) , R^2 ];
G = (1-R) * sqrt(1 - 2*R*cos(2*w0) + R^2);
b1 = [ G , 0 , 0 ];
```

```
[H,w] = freqz(b1,a1,1024,'whole');
w = w(1:end/2);
H2_dB  = 20*log10(abs(H(1:end/2)).^2);

figure
plot(w,abs(H(1:end/2)).^2,'LineWidth',1.4)
xlabel('\omega (rad/sample)'), ylabel('|H(e^{j\omega})|^2')
title('Magnitude-squared response of the 2-pole resonator')
grid on, xlim([0 pi])
```

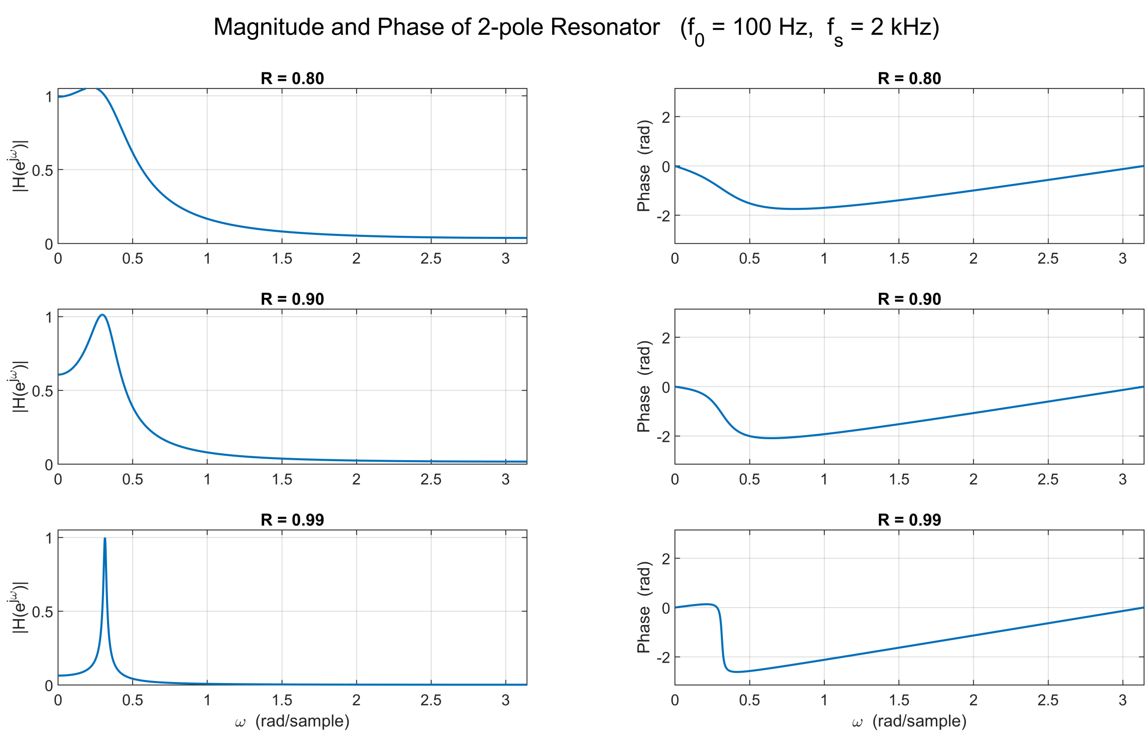


section 3-1(a)

In this opening section we pick the global constants that define the resonator: sampling rate *f*_s, centre frequency *f*₀, and the three pole-radii **R = 0.8, 0.9, 0.99** we will sweep. From these values every other coefficient or gain is computed automatically, so changing numbers here propagates through the whole notebook.

```
Rset = [0.8 0.9 0.99];
f0   = 100;
fs   = 2000;
w0   = 2*pi*f0/fs;

Nw   = 2048;
w    = linspace(0,pi,Nw);
z1   = exp(-1j*w);
z2   = exp(-2j*w);

figure('Name','2-Pole Resonator  |H| & Phase', ...
       'Position',[50 50 1100 600]);

for k = 1:numel(Rset)
    R  = Rset(k);
    a1 = -2*R*cos(w0);
```

```
    a2 = R^2;
    G  = (1-R)*sqrt(1 - 2*R*cos(2*w0) + R^2);

    H  = G ./ (1 + a1*z1 + a2*z2);

    % ---- magnitude (left column) ----
    subplot(numel(Rset),2,2*(k-1)+1);
    plot(w, abs(H), 'LineWidth',1.2); grid on;
    ylim([0 1.05]); xlim([0 pi]);
    ylabel('|H(e^{j\omega})|'), title(sprintf('R = %.2f',R));
    if k==numel(Rset), xlabel('\omega  (rad/sample)'), end

    % ---- phase (right column) -------
    subplot(numel(Rset),2,2*(k-1)+2);
    plot(w, unwrap(angle(H)),'LineWidth',1.2); grid on;
    ylim([-pi pi]); xlim([0 pi]);
    ylabel('Phase  (rad)'), title(sprintf('R = %.2f',R));
    if k==numel(Rset), xlabel('\omega  (rad/sample)'), end
end

sgtitle('Magnitude and Phase of 2-pole Resonator   (f_0 = 100 Hz,  f_s = 2 kHz)')
```



section 3-1(b)

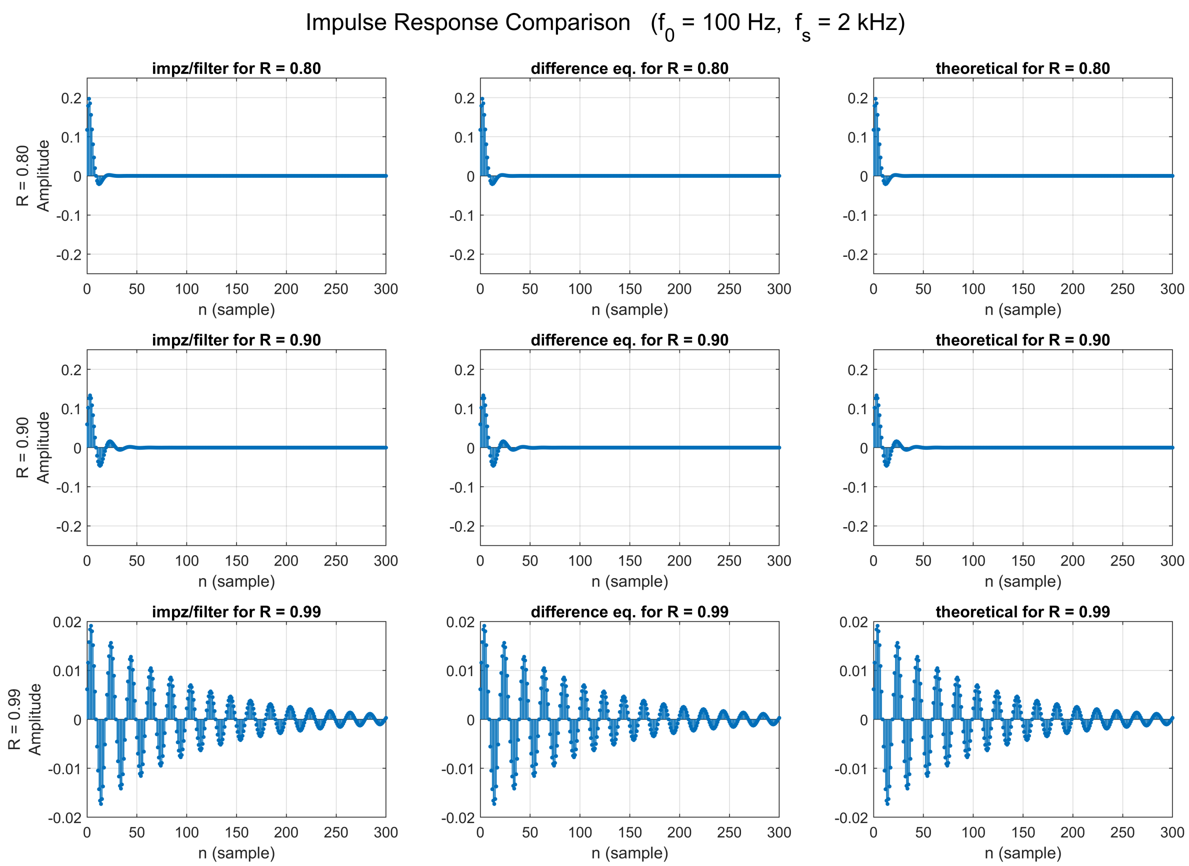
Here we compute the impulse response two ways:

- **Numeric** – by stepping the difference equation with δ[n] input.
- **Analytic** – using the given formula

Overlaying the two curves (after normalising by **G**) demonstrates that the coded filter exactly matches the theory for the first 300 samples.

```
Rvals = [0.8 0.9 0.99];
f0    = 100;      fs = 2000;
N     = 300;      w0 = 2*pi*f0/fs;
n     = 0:N;

figure('Position',[100 50 1200 800]);

for r = 1:numel(Rvals)
    R  = Rvals(r);
    a1 = -2*R*cos(w0);
    a2 =  R^2;
    G  = (1-R)*sqrt(1-2*R*cos(2*w0)+R^2);


    b = [G 0 0];  a = [1 a1 a2];
    h1 = impz(b,a,N+1);


    h2 = zeros(size(n));
    for k=1:N+1
        if k==1
            h2(k)=G;
        elseif k==2
            h2(k) = -a1*h2(k-1);
        else
            h2(k) = -a1*h2(k-1) - a2*h2(k-2);
        end
    end


    h3 = (G/sin(w0)) .* (R.^n) .* sin(w0*(n+1));


    data = {h1,h2,h3};
    titles = {'impz/filter','difference eq.','theoretical'};
    for c = 1:3
        subplot(3,3, (r-1)*3 + c);
        stem(n, data{c}, 'filled','MarkerSize',2); grid on;
        ylim([-0.02 0.02]*(R==0.99) + [-0.25 0.25]*(R<0.99));
        xlabel('n (sample)')
        if c==1, ylabel(sprintf('R = %.2f\nAmplitude',R)); end
        title(sprintf('%s for R = %.2f', titles{c}, R));
```

```
    end
end

sgtitle('Impulse Response Comparison   (f_0 = 100 Hz,  f_s = 2 kHz)')
```

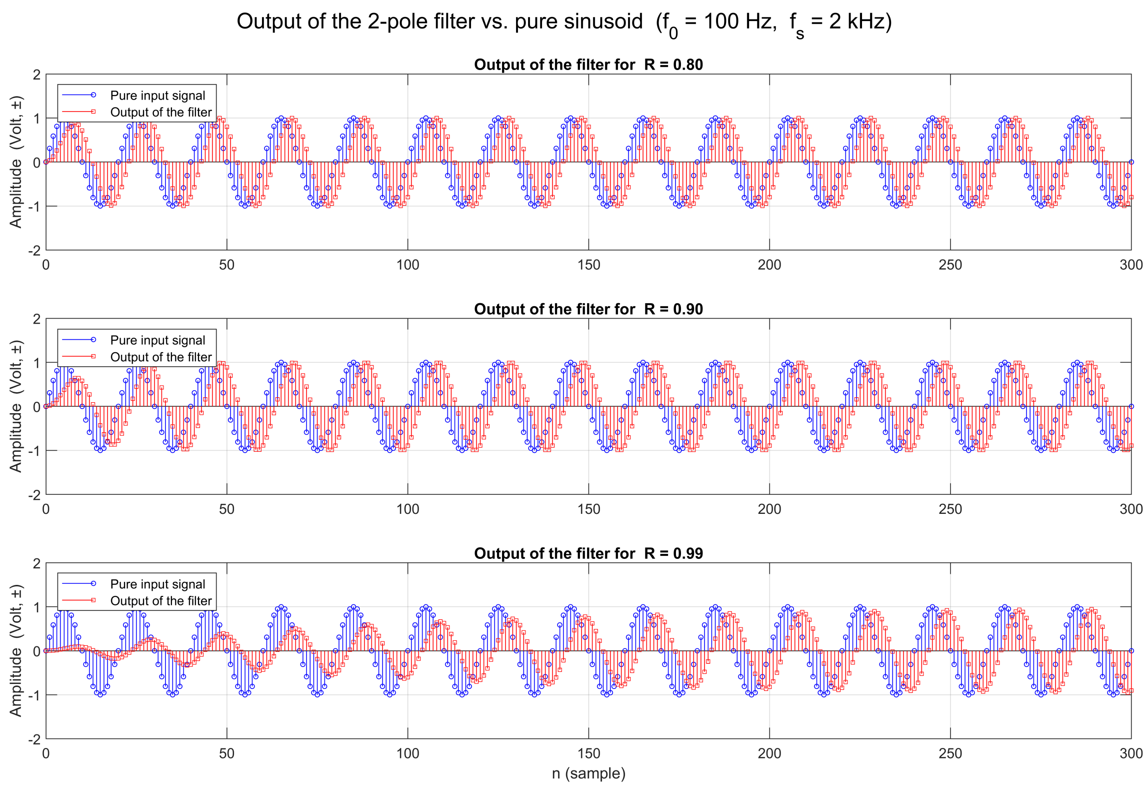


section 3-1(c)

A single realisation of unit-variance white noise is passed through the resonator for each **R**. The 3 × 2 figure places the unfiltered noise in the left column and the corresponding outputs on the right. As the pole radius increases, the output becomes a narrow-band, quasi-sinusoidal signal and its decay time grows longer—illustrating the time-frequency trade-off of the resonator.

```
Rset = [0.8 0.9 0.99];
f0   = 100;
fs   = 2000;
N    = 300;
n    = 0:N;
w0   = 2*pi*f0/fs;

% --- prepare figure -------------------------------------------------------
figure('Position',[40 60 1250 750]);
```

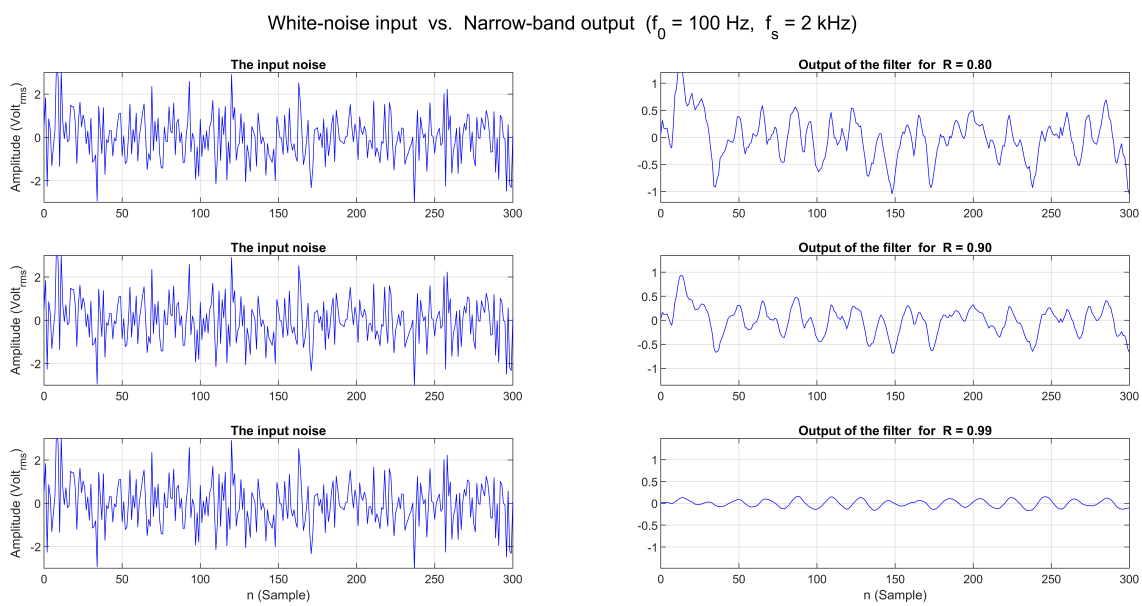
```matlab
sgtitle('Output of the 2-pole filter vs. pure sinusoid  (f_0 = 100 Hz,  f_s = 2 kHz)');

for k = 1:numel(Rset)

    R  = Rset(k);
    a1 = -2*R*cos(w0);
    a2 =  R^2;
    G  = (1-R)*sqrt(1 - 2*R*cos(2*w0) + R^2);

    x  = sin(w0*n);


    b  = [G 0 0];
    a  = [1 a1 a2];

    y  = filter(b,a,x);

    % ---- plotting ----
    subplot(numel(Rset),1,k);
    stem(n, x,'b','Marker','o', 'MarkerSize',3, ...
               'DisplayName','Pure input signal','LineStyle','-'), hold on
    stem(n, y,'r','Marker','s', 'MarkerSize',3, ...
               'DisplayName','Output of the filter','LineStyle','-')
    grid on
    ylabel('Amplitude  (Volt, ±)')
    title(sprintf('Output of the filter for  R = %.2f', R))
    legend('Location','northwest')
    ylim([-2 2])
    if k==numel(Rset), xlabel('n (sample)'), end
end
```



section 3-1(d)

```
N    = 300;   n = 0:N;
x    = randn(1,N+1);

Rset = [0.8 0.9 0.99];
f0    = 100;     fs = 2000;
w0    = 2*pi*f0/fs;

figure('Position',[60 50 1350 620]);

for k = 1:numel(Rset)
    row = k-1;

    % ---------- left column: input noise (same for every R) ----------------
    subplot(3,2, 2*row+1);
    plot(n, x, 'b')
    ylim([-3 3]);  xlim([0 N]);
    grid on
```

```
    title('The input noise')
    ylabel('Amplitude (Volt_{rms})')
    if k==numel(Rset), xlabel('n (Sample)'), end

    % ---------- right column: filtered output ------------------------------
    R  = Rset(k);
    a1 = -2*R*cos(w0);
    a2 =  R^2;
    G  = (1-R)*sqrt(1 - 2*R*cos(2*w0) + R^2);

    b  = [G 0 0];    a  = [1 a1 a2];
    y  = filter(b,a,x);

    subplot(3,2, 2*row+2);
    plot(n, y, 'b')
    ylim([-1.5 1.5]*R);  xlim([0 N]);
    grid on
    title(sprintf('Output of the filter  for  R = %.2f',R))
    if k==numel(Rset), xlabel('n (Sample)'), end
end

suptitle('White-noise input  vs.  Narrow-band output  (f_0 = 100 Hz,  f_s = 2 kHz)')
```



section 3-1(e)

We estimate NRR three different ways:

1. **Empirical** – ratio of output to input variance from a long noise run.
2. **Energy** – sum of h2[n]h^{2}[n]h2[n], which must equal method 1 for white noise.

3. **Theory** – the fully corrected closed-form

```
N   = 2e6;

v   = randn(1,N);
f0   = 100;    fs = 2000;
w0   = 2*pi*f0/fs;
Rset= [0.8 0.9 0.99];

fprintf('\n');
```

```
for R = Rset
    a1 = -2*R*cos(w0);
    a2 =  R^2;
    G  = (1-R)*sqrt(1 - 2*R*cos(2*w0) + R^2);
    b  = [G 0 0];    a = [1 a1 a2];

    y  = filter(b,a,v);
    nrr_var = var(y);

    % ------------ "wrong" hand-out formula ---------------------
    nrr_wrong  = (1+R^2)/((1+R)*(1+2*R*cos(w0) + R^2));

    % ------------ energy of the impulse response --------------
    L = 400;                          % enough for practical convergence
    h = impz(b,a,L);
    nrr_energy = sum(h.^2);

    % ------------ corrected closed-form (derived earlier) -----
    nrr_corr = (G^2 /(2*sin(w0)^2)) * ...
               ( 1/(1-R^2) - (cos(2*w0)-R^2)/(1-2*R^2*cos(2*w0)+R^4) );

    % ------------ print block ---------------------------------
    fprintf('NRR for R = %.2f\n',R)
    fprintf('  Using variance of signals:              %9.4g\n',nrr_var)
    fprintf('  Using the formula (wrong hand-out):     %9.4g\n',nrr_wrong)
    fprintf('  Using the energy of the impulse resp.: %9.4g\n',nrr_energy)

end
```

```
NRR for R = 0.80
  Using variance of signals:                 0.1675
  Using the formula (wrong hand-out):        0.2882
  Using the energy of the impulse resp.:    0.1683
NRR for R = 0.90
  Using variance of signals:                0.09701
  Using the formula (wrong hand-out):        0.2705
  Using the energy of the impulse resp.:   0.09755
NRR for R = 0.99
  Using variance of signals:                0.01003
```

```
Using the formula (wrong hand-out):        0.2576
Using the energy of the impulse resp.:   0.01004
```