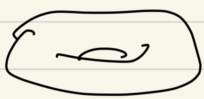
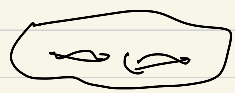
M.M.P.

$X = \{f_1 = \dots = f_k = 0\} \subset \mathbb{P}^N_{\mathbb{C}}$

loc $\simeq \mathbb{C}^n$ $\qquad n = \dim X$

$n = 1$ A.C. $=$ R.S.

3 families

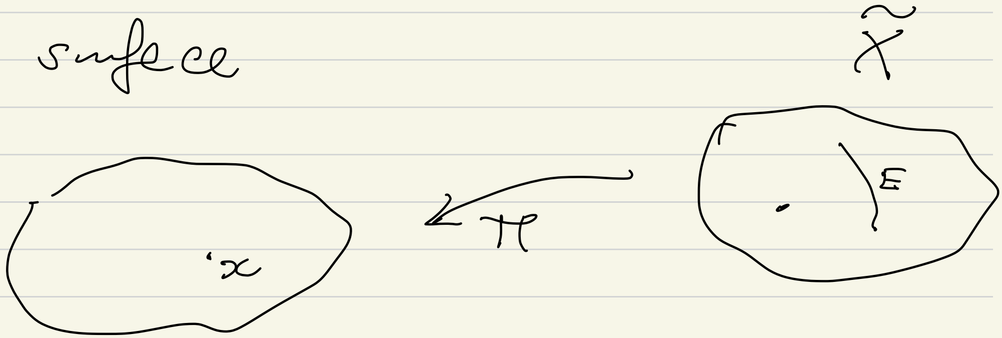
  

	Sphere	Ell. curves	hyp curves.
	>0 curvature	$=0$ curvature	<0 curvature
#Rat sol	Huge	fin gen.	finitely many
π_1	$=0$	something middle	Huge
Aut	Huge	//	Finitely many

$n = 2$ Alg. smooth surfaces

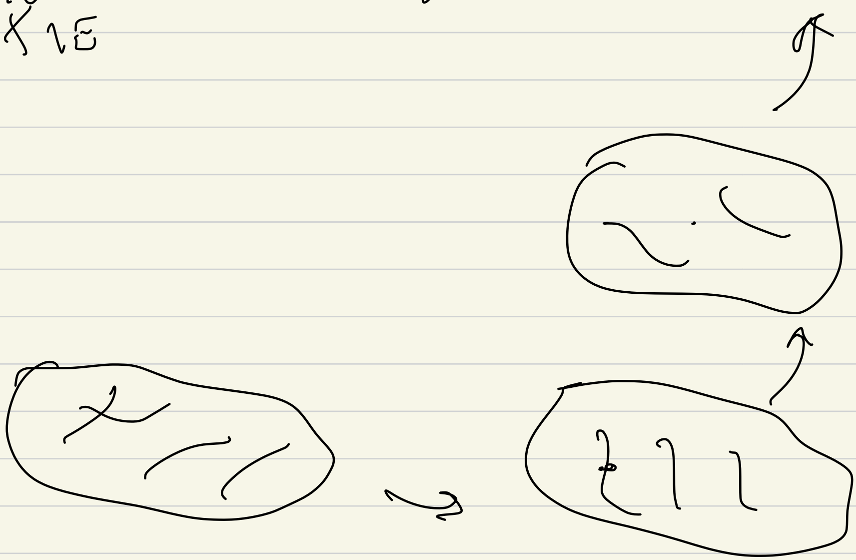
Blow-up of a pt.

X surface



$E \simeq \mathbb{P}^1_{\mathbb{C}}$

$$\pi|_{\tilde{X} \setminus E} : \tilde{X} \setminus E \xrightarrow{\sim} X \setminus \{x\}$$

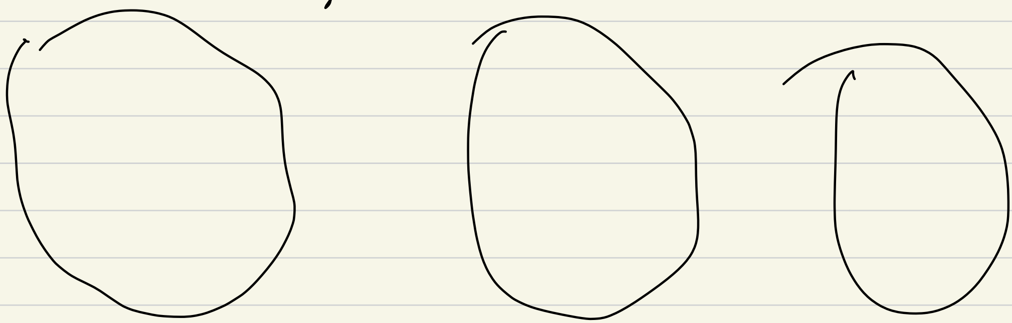


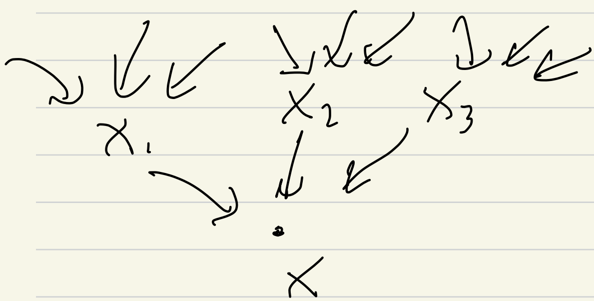
Directed Graph:

{smooth surface} = Vertices

$$Y \cdot \longrightarrow \cdot X \iff Y = \text{blow-up of } X \text{ at a point of } X.$$

connected components





Theorem X_1 and X_2 are contained in the same connected component iff they are birational:

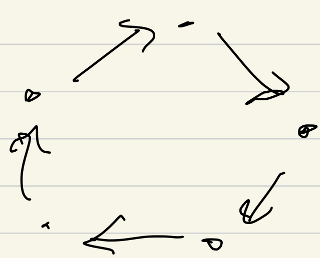
i.e. $\exists\, Y_i \subsetneq X_i$ subvariety

s.t. $X_1 \setminus Y_1 \underset{\text{isom}}{\cong} X_2 \setminus Y_2$

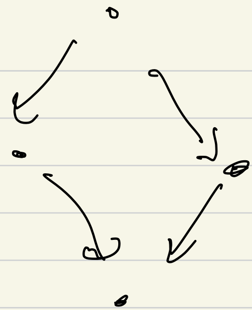
For each connected component we want to find a good representative

Properties: (Topology)

- $\nexists$ cycle



NO



OK

- Given any vertex, $\nexists$ infinite sequence of edges starting from this vertex

$$X \longrightarrow X_1 \longrightarrow X_2 \longrightarrow \cdots$$

$\Rightarrow$ $\exists$ end point. i.e. vertex that does not admit any edges starting from it.

endpoint

The corresponding surface is called minimal model.

$\mathbb{P}^2 \in \{$ rational surface $\}$
connected component.

endpoint

How many endpoints?

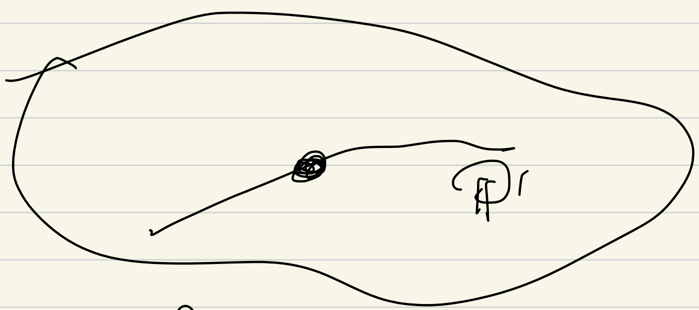
Infinitely many: either $\mathbb{P}^2$

or $\mathbb{P}^1$-bundles over $\mathbb{P}^1$.

X is <u>uniruled</u> if

$\forall x \in X \quad \exists\, \mathbb{P}^1 \xrightarrow{\eta} X$ not constant

s.t. $\eta(\mathbb{P}^1) \ni x$



If uniruled surface

$\exists$ so many endpoint.

If X is <u>not</u> uniruled

then $\exists!$ endpoint.

eg. K3, Abelian surface

hypersurface of $\mathbb{P}^3$ of degree ≥ 5

<u>What about $n \geq 3$</u> ?

- We need to allow some singularities.
- Edges are not just blow-ups,

Theorem Mori '80 .....

$\exists$ graph of such varieties s.t. each connected component corresponds to a birational equivalence class.

($\exists$ of flips)

Properties:

- $\nexists$ cycles.
- $n=3$ then $\nexists$ infinite step starting from a given vertex.

$n \geq 4$ (??)

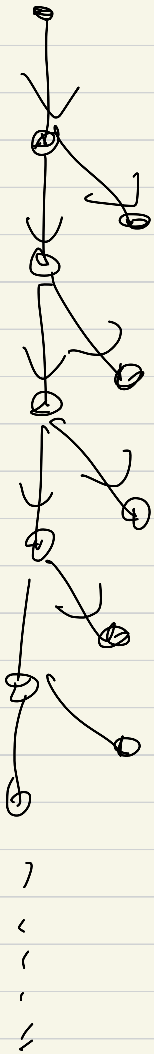
X = general type if

$X \subset \mathbb{P}^N$ is defined by polynomial of large degree

99.9% of varieties of general type.

For these varieties

$\exists$ endpoint (Min Model)



If X is winded,
then $\exists$ endpoint, ?

$$Y \xrightarrow{\quad g \quad} Z \quad \dim Z < n$$

fibre are Fano.

$\exists$ Fano which do not admit a K-E with positive curvature.

<u>Conj</u> We can find such a $g : Y \to Z$ s.t. the fibre admits a metric with positive curvature

$\rightsquigarrow$ We can decompose upto a birational equivalence any variety into objectives that admits a metric with constant curvature.