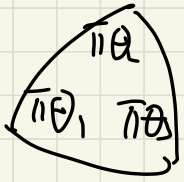
Spherical surf — is a surface glued from sph. triangl

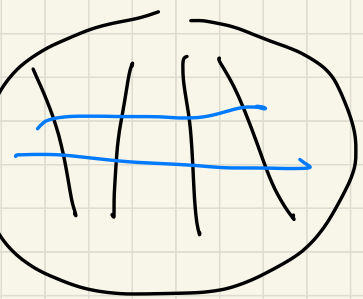
Q1 For which numbers

$0 < \theta_1, \theta_2, \theta_3 < 1$

$\exists$ a spherical triangle with angles



Q2 Let L_1, L_2, L_3, L_4 be 4 lines in $\mathbb{CP}^3$



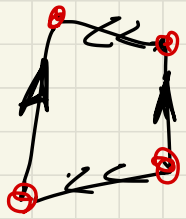
Q. How many lines intersect these 4?

Schubert calculus!

Q. $1+(+)+\dots = -\frac{1}{12} = \zeta(-1) = \sum_{n>0} \frac{1}{n^s}$

How is $-\frac{1}{12}$ related to $M_{1,1}$ - moduli sp. of Elliptic curves!

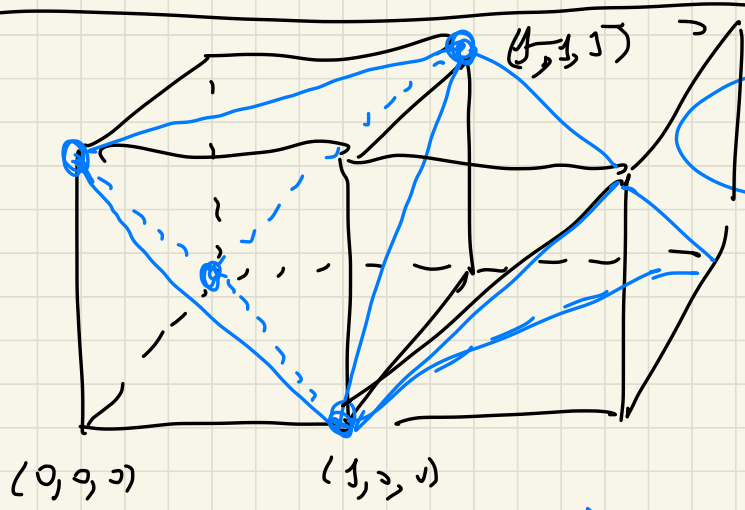
Q. Take a topological polygon with even number of edges that pairwise sides, so that



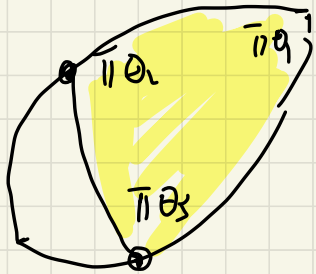
all vertices are given to one.

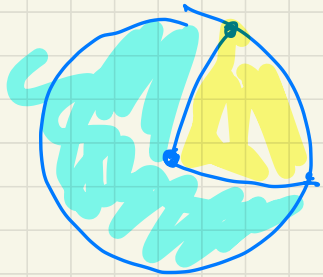
What surfaces can you get this way?

Q 1.



$\theta_1 + \theta_2 + \theta_3 > 1$

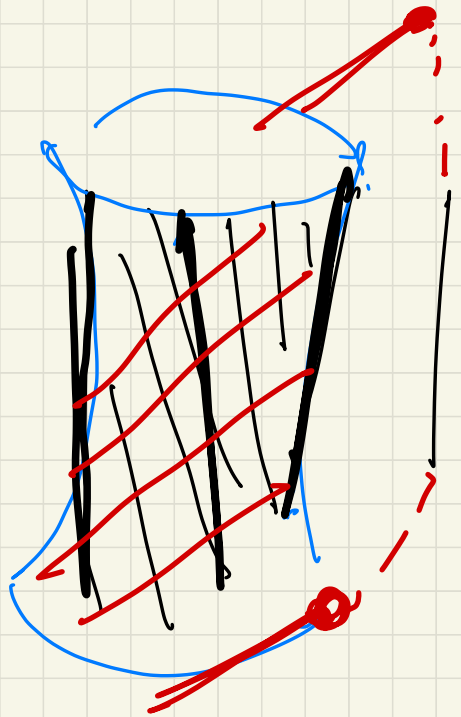




$\theta_1, \theta_2, \theta_3 < 1$

1552 Sir Michael Atiyah

The lines generically !!!



Claim. $\exists$ a Quadric in $\mathbb{CP}^3$ containing 3 lines

$Q \simeq \mathbb{CP}^1 \times \mathbb{CP}^1$

Q3

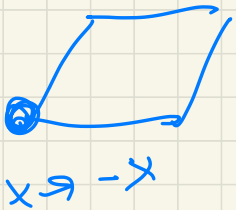
$$\zeta(-1) = 1+2+3+\cdots = -\frac{1}{12}$$

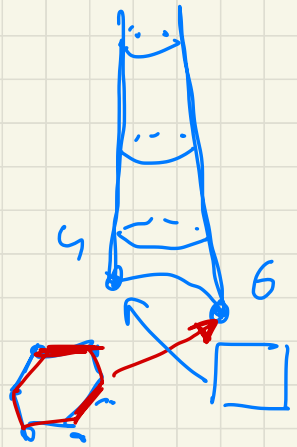
Euler char of $\mathcal{M}_{1,1}$ $\mathcal{M}_{g,n}$ stack

6

Moduli space of Elliptic curves with a marked point.

Elliptic curve $\mathbb{C}$ / lattice





$$\chi(S^2 - 3\text{ point}) = -1$$

$\frac{1}{2}(S^2 - 3\text{ point})$

$+\frac{1}{4} + \frac{1}{8} = -\frac{1}{\pi}$

Th. 4.1 Hauer-Łagiew 1986

$$\chi(M_{5,1}) = 3(1-25)$$

Th (2023) $\theta \in (3, \infty)$ not a odd integer

$\rightarrow$.

$$m = \left\lfloor \frac{\theta+1}{2} \right\rfloor$$

The moduli space of spherical tori with one conical point of angle $2\pi\theta$

$$\chi\left(M_{1,1}(\theta)\right) = -\frac{n^2}{12}$$