

Hence, there are enough local symmetries to ensure only physical states in the spectrum. Regarding these aspects, the Lagrangian for a massless ( $M_0 = 0$ ) particle with spin  $N/2$  is given by [19].

$$L = \frac{1}{2}E^{-1}\eta_{\mu\nu}\dot{x}^\mu\dot{x}^\nu - \frac{i}{2}\eta_{\mu\nu}\psi_k^\mu\dot{\psi}_k^\nu - \frac{i}{2}E^{-1}\eta_{\mu\nu}\lambda_k\psi_k^\mu\dot{x}^\nu - \frac{1}{8}E^{-1}\eta_{\mu\nu}(\lambda_k\psi_k^\mu)(\lambda_k\psi_k^\nu) + i\eta_{\mu\nu}\psi_k^\mu\psi_l^\nu V_{kl}, \quad (2.1)$$

where repeated Latin indices indicate sums. The action related to the Lagrangian above is invariant under reparameterization of the parameter  $\tau \rightarrow \tau + s(\tau)$ , under supersymmetry and local  $O(N)$  transformation [18, 19] as shown in the table below

Variables	Reparametrization	Supersymmetry	Local $O(N)$
$\delta x^\mu$	$s\dot{x}^\mu$	$i\alpha_k\dot{\psi}_k^\mu$	0
$\delta\psi_k^\mu$	$s\dot{\psi}_k^\mu$	$\alpha_k\left[E^{-1}\dot{x}^\mu - \frac{i}{2}E^{-1}\lambda_l\psi_l^\mu\right]$	0
$\delta E$	$s\dot{E} + \dot{s}E$	$i\alpha_k\lambda_k$	0
$\delta\lambda_k$	$s\dot{\lambda}_k + \dot{s}\lambda_k$	$2\dot{\alpha}_k + 4\alpha_lV_{kl}$	$t_{kl}$
$\delta V_{kl}$	$s\dot{V}_{kl} + \dot{s}V_{kl}$	0	$\frac{1}{2}\dot{t}_{kl} + t_{kn}V_{nl} - t_{ln}V_{nk}$

**Table 1.** Symmetry transformations of the spinning particle model [19].

where  $\alpha_k$  is an odd element of a Grassmann algebra which is an arbitrary function of  $\tau$ , and  $t_{kl}$  are the generators of the group  $O(N)$ . The Euler-Lagrange equations obtained by varying the action related to the Lagrangian, equation (2.1), give us the equations of motion

$$p_\mu = E^{-1}\left(\eta_{\mu\nu}\dot{x}^\nu - \frac{i}{2}\lambda_k\eta_{\mu\nu}\psi_k^\nu\right) \quad (2.2)$$

$$\dot{p}_\mu = 0 \quad (2.3)$$

$$\dot{\psi}_k^\mu = \frac{\lambda_k}{2}p^\mu + 2\psi_l^\mu V_{kl}, \quad (2.4)$$

and the constraints of first kind

$$\eta_{\mu\nu}p^\mu p^\nu \approx 0 \quad (2.5)$$

$$\eta_{\mu\nu}\psi_k^\mu p^\nu \approx 0 \quad (2.6)$$

$$\eta_{\mu\nu}\psi_k^\mu\psi_l^\nu \approx 0 \quad (2.7)$$

$$\pi_k^\mu - \frac{i}{2}\psi_k^\mu \approx 0, \quad (2.8)$$

where  $\pi_k^\mu = \frac{\partial L}{\partial \dot{\psi}_k^\mu}$ . These equations describe the motion of a free massless spinning particle with spin  $N/2$  after quantization.

For a massive spinning particle, it was proposed in [18] the introduction of an additional (Minkowsky scalar) Grassmann variable  $\psi^5$  which goes over to  $\gamma^5$  in the quantization procedure, and this field could carry a mass in the constraint. We further introduced a