INTRODUCTION

Lifestyle [1] and pharmacotherapy interventions [2,3] have been shown in randomised trials to be effective in achieving weight loss among individuals with overweight or obesity with high cardiovascular (CVD) risk because of type 2 diabetes, cardiovascular diseases or additional risk factors [3,4] In such high risk patients, there is evidence that weight loss is also beneficial for cardiovascular risk factors [4]. However, there are mixed findings on the effect of weight loss on CVD, as some studies found no effect of weight loss on fatal and non-fatal CVD [1,5,6], while a recent meta-analysis of trials reported moderate lower risk of CVD following weight loss [4]. In any case, there are no randomised controlled trials (RCTs) of any weight loss intervention assessing the effectiveness for primary prevention of cardiovascular diseases (CVDs) in otherwise healthy individuals with overweight or obesity. The prospect of performing such trials is low because of the very high number of participants required for reliable estimation of a primary preventive effect on CVD events. Therefore, it is not known whether weight loss reduces the risk of incident cardiovascular disease among people in the general population with overweight or obesity. This is particularly important as higher body mass index is associated with the onset of cardiovascular diseases [7] and the prevalence of obesity, already high, is predicted to risk further [8].

In such settings emulating target pragmatic trials using causal inference methods in large scale observational data may play a role in distinguishing effects of weight change in people with normal weight, overweight and obesity [9-12]. Observational studies of weight (or BMI) change are conflicting, some reporting increased risk of CVD [13-16], no association [17-20] or lower risk, especially after bariatric surgery in people with severe obesity [21]; weight gain has been associated with increased CVD risk in some studies[13-16], but not others[21]. Emulation of weight loss trials has been carried out in a consented cohort, the Nurses' Health Study, which found no relationship between weight loss and CHD [17,18].

nonempty open subinterval J of I such that $A = \{p : i_p = i \text{ and } n_p = n\}$ contains the closure of J.

Again by Lemma 1.2 we have

$$\bigcup_{p \in A \cap J} U'(p,n) = \bigcup_{q \in \mathbb{Q} \cap J} U'(q,n)$$

This now implies that $f[U'(q,n)] \subseteq K_i$, and hence that $i \cdot \varepsilon \leqslant f(q,0)$, whenever $q \in \mathbb{Q} \cap J$. But $\inf\{f(q,0) : q \in \mathbb{Q} \cap J\} = 0$ and hence $i = -1$ or $i = 0$.

We find that $f[U(x,n)] \subseteq [0, 2\varepsilon]$, and hence $f(x,0) \leqslant 2\varepsilon < 2^{-k}$, for all $x \in J$. $\square$

2. A Book

We apply Jones' method from [3] to N to obtain a regular, non-completely regular space, as follows.

We start with the product $N \times \mathbb{N}$, where $\mathbb{N}$ carries its discrete topology.

For every even $n \in \mathbb{N}$ and $q \in \mathbb{Q}$ we identify the points $\langle\langle q,0\rangle, n\rangle$ and $\langle\langle q,0\rangle, n+1\rangle$, so that $Q \times \{n\}$ and $Q \times \{n+1\}$ become one copy of Q, which we denote Q_n.

Likewise for every odd $n \in \mathbb{N}$ and $p \in \mathbb{P}$ we identify $\langle\langle p,0\rangle, n\rangle$ and $\langle\langle p,0\rangle, n+1\rangle$, thus creating out of $P \times \{n\}$ and $P \times \{n+1\}$ one copy of P that we denote P_n.

The resulting space we call B and we let $\pi : N \times \mathbb{N} \to B$ denote the quotient map.

Remark 2.1. In [5, Examples 90 and 91] Tychonoff's example, mentioned in the introduction, is represented pictorially as a corkscrew.

Our space B looks more like a book made from infinitely many sheets of paper sewn together alternately along the rational and irrational points on the x-axis, hence the title of the present section. We shall call B a *book* from now on.

Lemmas 1.3 and 1.4 above imply that if $f : B \to [0,1]$ is continuous and equal to 0 on the set Q_0 then, by induction, the following holds for every n, where we let $F : N \times \mathbb{N} \to [0,1]$ be the composition $f \circ \pi$.

- if n is even then $\inf\{F(x,0,n) : x \in O \cap \mathbb{Q}\} = 0$ whenever O is an open interval in $\mathbb{R}$, and
- if n is odd then there is a dense G_δ-set G_n, such that $F(x,0,n+1) = F(x,0,n) = 0$ whenever $x \in G_n \cap P_n$.

Remark 2.2. Note that by the second item there is in fact a single dense G_δ-set G, to wit $\bigcap\{G_n : n \text{ is odd}\}$, such that $F(x,0,n+1) = F(x,0,n) = 0$ whenever $x \in G \cap P_n$ and n is odd.

This implies that if we were to add a 'point at infinity' ∞ to $N \times \mathbb{N}$ with basic neighbourhoods $U_m = \{\infty\} \cup (N \times \{n \in \mathbb{N} : n \geqslant m\})$ and apply the quotient operation above to the new space $(N \times \mathbb{N}) \cup \{\infty\}$, then the resulting quotient space $B \cup \{\infty\}$ is not completely regular at ∞.

Lemma 2.3. *The map* $\pi : (N \times \mathbb{N}) \cup \{\infty\} \to B \cup \{\infty\}$ *is closed.*

Proof. If F is closed in $(N \times \mathbb{N}) \cup \{\infty\}$ then $\pi^{\leftarrow}\big[\pi[F]\big]$ is closed as well. If $n \in \mathbb{N}$ then its intersection with the sheet $N \times \{n\}$ consists of three parts (two if $n = 0$):

- $F \cap (N \times \{n\})$,
- the set $\{\langle\langle x,0\rangle, n\rangle : \langle\langle x,0\rangle, n+1\rangle \in F\}$, and
- the set $\{\langle\langle x,0\rangle, n\rangle : \langle\langle x,0\rangle, n-1\rangle \in F\}$ if also $n > 0$.

The union of these intersections is closed in $N \times \mathbb{N}$.

If $\infty \notin F$ then F intersects only finitely many sheets and so ∞ is not in the closure of $\pi^{\leftarrow}\big[\pi[F]\big]$.

If $\infty \in F$ then all is well. $\square$

Hence, there are enough local symmetries to ensure only physical states in the spectrum. Regarding these aspects, the Lagrangian for a massless ($M_0 = 0$) particle with spin $N/2$ is given by [19].

$$
\begin{aligned}
L = & \frac{1}{2}E^{-1}\eta_{\mu\nu}\dot{x}^\mu\dot{x}^\mu - \frac{i}{2}\eta_{\mu\nu}\psi_k^\mu\dot{\psi}_k^\nu - \frac{i}{2}E^{-1}\eta_{\mu\nu}\lambda_k\psi_k^\mu\dot{x}^\nu \\
& -\frac{1}{8}E^{-1}\eta_{\mu\nu}(\lambda_k\psi_k^\mu)(\lambda_k\psi_k^\nu) + i\eta_{\mu\nu}\psi_k^\mu\psi_l^\nu V_{kl},
\end{aligned}
\tag{2.1}
$$

where repeated Latin indices indicate sums. The action related to the Lagrangian above is invariant under reparameterization of the parameter $\tau \to \tau + s(\tau)$, under supersymmetry and local $O(N)$ transformation [18, 19] as shown in the table below

Variables	Reparametrization	Supersymmetry	Local O(N)
δx^μ	$s\dot{x}^\mu$	$i\alpha_k\dot{\psi}_k^\mu$	0
$\delta\psi_k^\mu$	$s\dot{\psi}_k^\mu$	$\alpha_k\left[E^{-1}\dot{x}^\mu - \frac{i}{2}E^{-1}\lambda_l\psi_l^\mu\right]$	0
δE	$s\dot{E} + \dot{s}E$	$i\alpha_k\lambda_k$	0
$\delta\lambda_k$	$s\dot{\lambda}_k + \dot{s}\lambda_k$	$2\dot{\alpha}_k + 4\alpha_l V_{kl}$	t_{kl}
δV_{kl}	$s\dot{V}_{kl} + \dot{s}V_{kl}$	0	$\frac{1}{2}\dot{t}_{kl} + t_{kn}V_{nl} - t_{ln}V_{nk}$

Table 1. Symmetry transformations of the spinning particle model [19].

where α_k is an odd element of a Grassmann algebra which is an arbitrary function of τ, and t_{kl} are the generators of the group $O(N)$. The Euler-Lagrange equations obtained by varying the action related to the Lagrangian, equation (2.1), give us the equations of motion

$$
p_\mu = E^{-1}\left(\eta_{\mu\nu}\dot{x}^\nu - \frac{i}{2}\lambda_k\eta_{\mu\nu}\psi_k^\nu\right) \tag{2.2}
$$

$$
\dot{p}_\mu = 0 \tag{2.3}
$$

$$
\dot{\psi}_k^\mu = \frac{\lambda_k}{2}p^\mu + 2\psi_l^\mu V_{kl}, \tag{2.4}
$$

and the constrains of first kind

$$
\eta_{\mu\nu}p^\mu p^\nu \approx 0 \tag{2.5}
$$

$$
\eta_{\mu\nu}\psi_k^\mu p^\nu \approx 0 \tag{2.6}
$$

$$
\eta_{\mu\nu}\psi_k^\mu\psi_l^\nu \approx 0 \tag{2.7}
$$

$$
\pi_k^\mu - \frac{i}{2}\psi_k^\mu \approx 0, \tag{2.8}
$$

where $\pi_k^\mu = \frac{\partial L}{\partial\dot{\psi}_k^\mu}$. These equations describe the motion of a free massless spinning particle with spin $N/2$ after quantization.

For a massive spinning particle, it was proposed in [18] the introduction of an additional (Minkowsky scalar) Grassmann variable ψ^5 which goes over to γ^5 in the quantization procedure, and this field could carry a mass in the constraint. We further introduced a

The RHS of the previous equation is clearly nonpositive, so α' must be nonpositive as well. We can also bound the size of α' with

$$\begin{aligned}
-\varepsilon\alpha' &= \left\lceil \frac{\alpha}{1+\varepsilon} \right\rceil - \alpha \\
1-\varepsilon\alpha' &\geq 1 + \frac{\alpha}{1+\varepsilon} - \alpha \\
-\varepsilon\alpha' &\geq \frac{-\varepsilon\alpha}{1+\varepsilon} \\
(1+\varepsilon)\alpha' &\leq \alpha \\
\alpha' &< \alpha.
\end{aligned}$$

By (1), if we take the lower bound for the change in $\sum E_k^2$ using α', the value of $\sum E_{k-1}^2$ is equal to or smaller than if we had used α instead. By the same logic that was used to prove Lemma 13, it is always helpful to have a lower $\sum E^2$. $\square$

With Lemma 13 and Lemma 14, we now have a strictly stronger set of actions on $\sum E$ and $\sum E^2$ that we can perform with each move.

Definition 15. *A* ***strong reversed move*** *that uses α on the pair of values $\sum E_k$ and $\sum E_k^2$ is given by*

$$\begin{aligned}
\sum E_{k-1} &= \sum E_k - \varepsilon\alpha \\
\sum E_{k-1}^2 &= \sum E_k^2 - 2\sum E_k + (1-\varepsilon)^2\alpha^2.
\end{aligned}$$

For integers $1 \leq k \leq m$, let α_k denote the value of α used in the strong reversed move to get from $\left(\sum E_k, \sum E_k^2\right)$ to $\left(\sum E_{k-1}, \sum E_{k-1}^2\right)$. Then,

$$\begin{aligned}
\sum E_m &= \sum E_0 + \varepsilon\alpha_1 + \varepsilon\alpha_2 + \cdots + \varepsilon\alpha_m \\
\sum E_m &= \sum E_0 + \varepsilon\sum_{j=1}^{m}\alpha_j. \qquad (3)
\end{aligned}$$

The same process is a little more complicated for $\sum E_m^2$.

$$\sum E_m^2 = \sum E_0^2 + 2\sum_{j=1}^{m}\left(\sum E_j\right) - (1-\varepsilon)^2\sum_{j=1}^{m}\alpha_j^2$$

We can rewrite the $\sum E_j$ by plugging in (3).

$$\begin{aligned}
\sum E_m^2 &= \sum E_0^2 + 2\sum_{j=1}^{m}\left(\sum E_j\right) - (1-\varepsilon)^2\sum_{j=1}^{m}\alpha_j^2 \\
\sum E_m^2 &= \sum E_0^2 + 2\sum_{j=1}^{m}\left(\sum E_0 + \varepsilon\sum_{i=1}^{j}\alpha_i\right) - (1-\varepsilon)^2\sum_{j=1}^{m}\alpha_j^2 \\
\sum E_m^2 &= \sum E_0^2 + 2m\sum E_0 + 2\varepsilon\sum_{j=1}^{m}(m-j+1)\alpha_j - (1-\varepsilon)^2\sum_{j=1}^{m}\alpha_j^2 \\
0 &= \sum E_0^2 + 2m\sum E_0 - \sum E_m^2 + 2\varepsilon\sum_{j=1}^{m}(m-j+1)\alpha_j - (1-\varepsilon)^2\sum_{j=1}^{m}\alpha_j^2
\end{aligned}$$