Project

Course: EE 867.3 Economic System Operation

Submitted By:

Mahdi Md. Mostafa

Student ID: 11270121

NSID: mmm874

Submitted to:

Associate Professor Dr. N. Chowdhury

Department of Electrical and Computer Engineering

University of Saskatchewan

Department of Electrical & Computer Engineering

QUESTION 1

The given problem provides us information about the two thermal units committed to satisfy different loads for a period of 24 hour. It gives details about the cost functions, maximum operating limit and minimum operating limit of all 2 units. To satisfy the load over 24-hour period, a hydro unit is available in addition to these 2 thermal units. The hydro unit has a limited Volume of water and should be operated in parallel with thermal units in such a way that the overall operating cost of thermal units is minimized for the given period of 24 hour. The transmission loss is equation is a function of thermal and hydro generation.

Now, we know that, for max cost efficiency it is best to operate hydro units in such hours when the demand is high and the incremental running cost of thermal units are high. But here the volume of water is limited . As a result the optimum dispatch of hydro in combination of thermal units are obtained by assigning the best worth to the value of water.

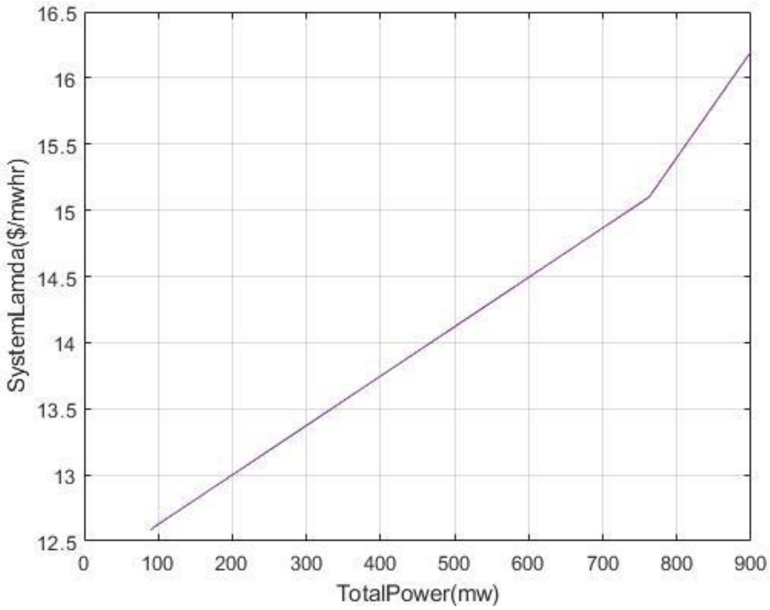
Unlike thermal units, hydro units do not use fuel to operate, so they do not have incremental running cost. Still, we can assign a virtual worth to the hydro plants(water). This is called water worth. If we assign a very low value to water, then we end up using more water than the available volume and if we assign a very high value to the water, that results in some unused water. So, the optimum value of water worth should be calculated to make sure that we use all the available water and simultaneously we minimize the cost of thermal units.

So, to obtain the hydro dispatch to maximize savings over 24-hour period, following steps are followed:

Step-1

- First, the given 2 thermal units are represented by an equivalent unit for which the minimum output=50+40=90MW and the maximum output=500+400= 900MW. The variation of incremental running cost(λ) and Power output is shown by a *Lambda(λ)-Total Power* curve shown below

power	*Lambda(λ)*
90	12.58
92.8	12.60
762.5	15.10
900	16.20



Step-2

- Now, to incorporate the hydro units, we start by assigning one particular virtual water worth value which is called marginal incremental running cost value ($\lambda_{marginal}$). The corresponding marginal power output is $P_{marginal}$ which is obtained from the Lambda-Power curve by interpolation. It means,in any hour if power demand is higher than $P_{marginal}$, the output power greater than $P_{marginal}$ is supplied from Hydro units.
  For this case, we check the total volume of water used over 24-hour period. If more water is used than the allotted volume, it means we assigned a low value to the water, so we increase the value of marginal incremental running cost ($\lambda_{marginal}$).But, if less water is used than the allotted volume ,it means we assigned a very high value to the water, so we decrease $\lambda_{marginal}$ and corresponding $P_{marginal}$.

  For example, We assigned $\lambda_{marginal} = 14.5\$/\mathrm{MWHr}$
  For this lamda, corresponding $P_{marginal} = 600\mathrm{MW}$ (from the Total Power Vs λ curve)
  It means, all the power above 600MW is supplied from thermal units.
  Here,
  ph= Load - $P_{marginal}$,
  And we know ,
  ph+pt=load+Ploss....................(1)
  $P_L = 0.0004P_h^2 + 0.0002P_s^2 + 0.0003P_hP_s - 0.0006P_h - 0.0005P_s$ …………(4)
  Using this loss equation in equation 1 we get a 2nd order quadratic equation which is-

  pt(0.0003ph-1.0005)+0.0002Pt*pt+ load+0.0004ph*ph-1.0006ph=0……(2)

```
a=0.0002;
 c=0.0004*Ph*Ph+Load-(1.0006*Ph);
 b=(0.0003*Ph)-1.0005;
```

```
Pt=(-b-sqrt(abs(b.*b-4*a*c)))/(2*a)
```
...... (3)

We already know the value of ph so from equation 3 we can get the value of thermal generation pt.

Now we know both ph and pt so from equation 4 we can get the value of **Ploss** .

For this case, let us calculate total volume of water used,

From 8AM to 10AM, Load=700MW of which 100MW is supplied by hydro and corresponding Volume used is:

Volume_used=q*t, where q is the discharge

And, $q = 30P_h + 0.02P_h^2$

So, Volume used (1) =30*100+0.02*100*100*(2*60*60) =1443000m^3

Also, from 6AMto 8AM, Load=650MW of which 50MW is supplied by hydro and corresponding Volume used is:

Volume used (2) =30*50+0.02*50*50*(2*60*60) =361500m^3

From 10AM to 12PM, Load=800MW of which 200MW is supplied by hydro and corresponding Volume used is:

Volume used (3) =30*200+0.02*200*200*(2*60*60) =5766000m^3

From 12PM to 2PM, Load=850MW of which 250MW is supplied by hydro and corresponding Volume used is:

Volume used (4) =30*250+0.02*250*250*(2*60*60) =9007500m^3

From 2PM to 4PM, Load=800MW of which 200MW is supplied by hydro and corresponding Volume used is:

Volume used (5) =30*200+0.02*200*200*(2*60*60) =5766000m^3

From 4PM to 6pm, Load=750MW of which 150MW is supplied by hydro and corresponding Volume used is:

Volume used (6) =30*150+0.02*150*150*(2*60*60) =3244500m^3

From 6PM to 8pm, Load=700MW of which 100MW is supplied by hydro and corresponding Volume used is:

Volume used (7) =30*100+0.02*100*100*(2*60*60) =1443000m^3

For all other hours, Load is less than or equal to 600MW and the hydro plant is not operated.

Then ploss will be-

```
ploss= 0.0002*(Pt*Pt)-0.0005*Pt
```

and when there is no hydro power available the thermal power is-

```
Pt(hour)=Load(hour)+ ploss(hour)
```

Therefore, Total Volume used=27031500m^3

But, Allotted Volume =$5.7*10^6 m^3$

So, we assigned a low value for the water worth, as a result we used more water than available. So, in the next iteration we increase the value of $\lambda_{marginal}$, by a small amount(0.0001) ,so that the marginal Power above which the hydro operates is increased and corresponding Volume of water used is decreased.

In every iteration Value of $\lambda_{marginal}$ and corresponding $P_{marginal}$ is changed until Hydro units are operated in such a way that the Volume of water used is equal to available volume of water.

Step-3

- When the convergence is reached, all the available volume of water is consumed in such a way that the hydro unit supplies power during the durations where load demand is high and incremental cost of thermal units is high. So, after the convergence the obtained results are:

Virtual water worth (Marginal Incremental running cost that uses all the water):

$\boldsymbol{\lambda_{marginal}}$ ***= 14.9458$/Hr***

Corresponding Marginal Power above which Hydro plant operates:

$\boldsymbol{P_{marginal}}$**= 607.67MW**

Step-4

- After deciding on when to operate Hydro units to maximize the savings over 24-hour period, the output of thermal units are calculated. It should be noted that the load above $P_{marginal}$ is always supplied by Hydro units and thermal units are only responsible to supply load below $P_{marginal}$. .To calculate the output of thermal units, new modified daily load curve is obtained by subtracting the Hydro outputs and adding Ploss from the.

```
modified_load=Load-Ph+ploss
```

Then, for the corresponding Modified Load, System Lambda is calculated using interpolation in Lambda-Capability curve.

From that value of System Lambda output of each units are calculated using following relation:

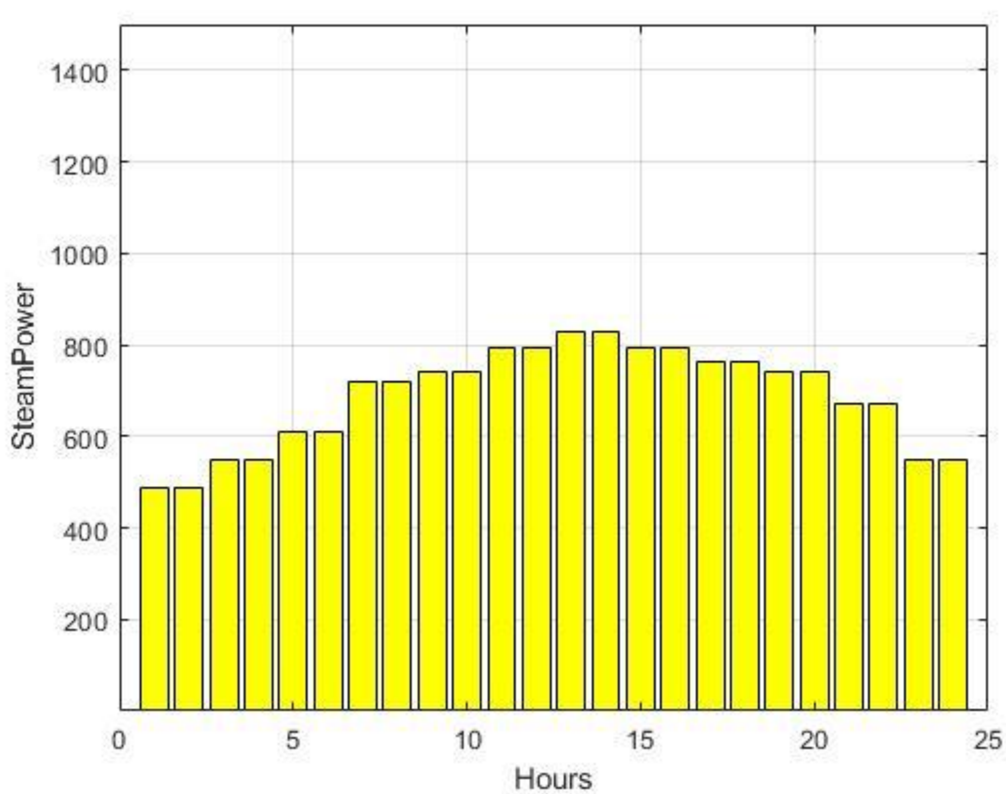
$Pn = (\lambda - bn)/2an$……………………………………………………….*(5)*

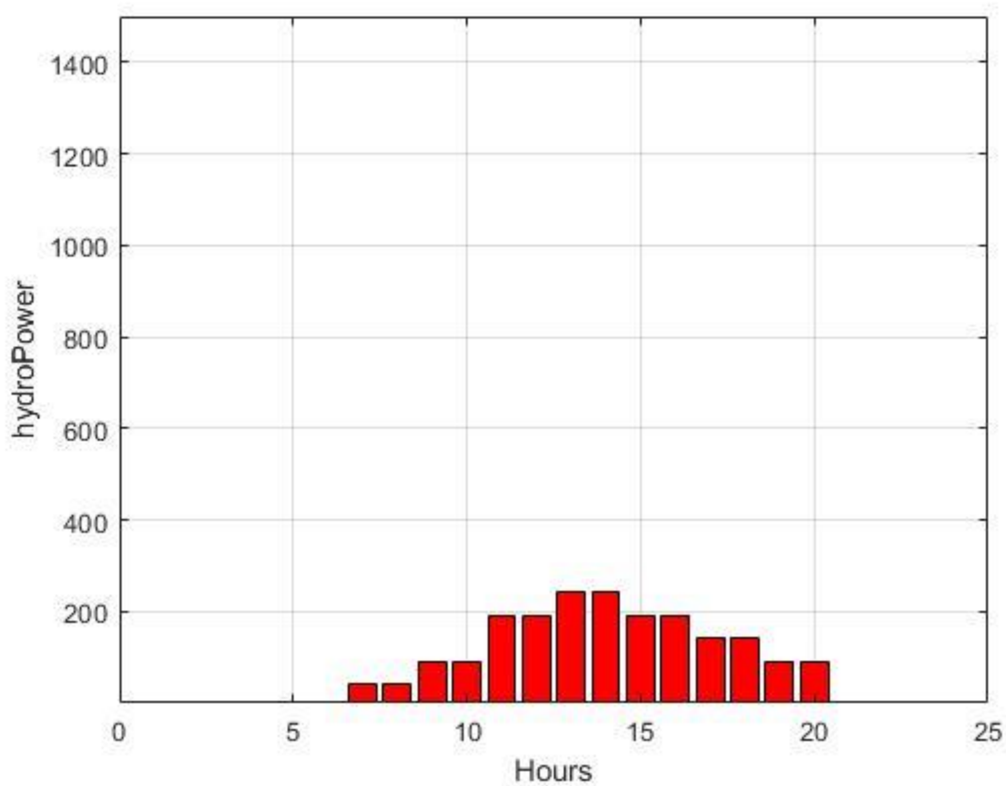
For example,
For 7^{th} hour, Load=650MW
Ph=650-607.67=42.321MW
Modified load(to be supplied by thermal)=607.67
Corresponding System λ=14.9458$/MWHr

Using equation 5,
P1=(14.9458-12.2)/(2*0.004)=343.22MW
P2=(14.9458-12.3)/(2*0.0035)=377.97MW
Ploss=113.51
Ph=42.321
So,
ph+pt= load +Ploss
The corresponding operating cost=F(1) +F(2)
Where $F(n)=an^2Pn+bnPn+cn$.
The coefficents an,bn and cn for all the units are provided . therefore,
Total operating cost=10307$

So, the **marginal Cost of thermal units at which hydro dispatch begins= 10307$**

The algorithm was implemented by writing a code in MATLABThe distribution of Power among thermal and hydro units for each hourly Load can be shown in the figure below:





The Hourly Load, output of each thermal Units, output of hydro unit, corresponding discharge in (m^3/hour) and hourly running cost for the 24 hour is shown in the table below:

hour	Load	Ph	P1	P2	ploss	Q	Total_cost
1	450	0	235.46	254.81	40.275	0	6955.9
2	450	0	235.46	254.81	40.275	0	6955.9
3	500	0	263.22	286.53	49.75	0	7800.1
4	500	0	263.22	286.53	49.75	0	7800.1
5	550	0	291.44	318.79	60.225	0	8672.1
6	550	0	291.44	318.79	60.225	0	8672.1
7	650	42.321	343.22	377.97	113.51	4.6997e+06	10307

8	650	42.321	343.22	377.97	113.51	4.6997e+06	10307
9	700	92.321	352.47	388.53	133.32	1.0584e+07	10604
10	700	92.321	352.47	388.53	133.32	1.0584e+07	10604
11	800	192.32	393.78	435.74	186.1	2.3434e+07	11948
12	800	192.32	393.78	435.74	186.1	2.3434e+07	11948
13	850	242.32	427.86	474.7	220.18	3.0399e+07	13080
14	850	242.32	427.86	474.7	220.18	3.0399e+07	13080
15	800	192.32	393.78	435.74	186.1	2.3434e+07	11948
16	800	192.32	393.78	435.74	186.1	2.3434e+07	11948
17	750	142.32	365.03	402.89	157.35	1.6829e+07	11010
18	750	142.32	365.03	402.89	157.35	1.6829e+07	11010
19	700	92.321	352.47	388.53	133.32	1.0584e+07	10604
20	700	92.321	352.47	388.53	133.32	1.0584e+07	10604
21	600	0	320.13	351.57	71.7	0	9572.4
22	600	0	320.13	351.57	71.7	0	9572.4
23	500	0	263.22	286.53	49.75	0	7800.1
24	500	0	263.22	286.53	49.75	0	7800.1

Discussion: It is found that decreasing the error value increases the number of used loop which simply increase the computation time as the memory required to store the values is comparatively small. And if the error value is less than 100 then the program goes to debugging mode as it needs more lamda values for checking purpose.

MATLAB CODE FOR QUESTION NO 1

```
clc;
clear;
%%
%given LOAD
Load=[450 450 500 500 550 550 650 650 700 700 800 800 850 850 800 800 750 750
700 700 600 600 500 500  ];

gen_data=[ 0.004 12.2 300 50 500
           0.0035 12.3 200 40 400
          ];
       Volume_allot=2.4*10^8;
%%
%represents the given thermal unit by an equivalent unit
% values of lambda(2 lambda for each units)
     count=2*2; %2 unit * 2 lamda
     lamda=zeros(count,1);
     power_fordifflamdas=zeros(count,1);
     j=1;
     a=1;
     while(j<=count) % this loop calculates lamda
         lamda(j)=2*gen_data(a,1)*gen_data(a,4)+gen_data(a,2);
         lamda(j+1)=2*gen_data(a,1)*gen_data(a,5)+gen_data(a,2);
         a=a+1;
         j=j+2;
     end
     a=1;
     while(a<=count) % this loop calculates power fo lemda
     for j=1:1:2
         powerofunit(j)=(lamda(a)-gen_data(j,2))/(2*gen_data(j,1));
         if ( powerofunit(j)>gen_data(j,5))
              powerofunit(j)=gen_data(j,5);
         end
             if ( powerofunit(j)<gen_data(j,4))
              powerofunit(j)=gen_data(j,4);
             end
             power_fordifflamdas(a)=power_fordifflamdas(a)+powerofunit(j);
     end
         a=a+1;
     end
     %plot of lemda
     lamda=sort(lamda);
     power_fordifflamdas=sort(power_fordifflamdas);
```

```
     t=table(power_fordifflamdas,lamda);
     figure(1);
     plot(power_fordifflamdas,lamda);
     xlabel('TotalPower(mw)');
     ylabel('SystemLamda($/mwhr)');
     grid on;
    for hour=1:1:24 % linear interpolation sys lamda according to load value
from powerf_fordifflamdas vs lamda)
    lamda_thermal(hour)=interp1(power_fordifflamdas,lamda,Load(hour));
    end
    %%
    % hydro part starts
  lamda_marginal=max(lamda_thermal); % assuming the max val of lamdasys from
interpolation as marginal val
  error=1;
  loop_n=1;
  while(1)% continues loos until there is a break
  Volume_used=0;
  Ph=zeros(24,1);
  ploss=zeros(24,1);
  r=zeros(1,24); %using r mat as the ranspose of ph
  for hour=1:1:24
      Pt(hour)=Load(hour);
      if lamda_thermal(hour)>lamda_marginal %dispatch of hydro occurs satisfing
this condition only
            Power_marginal=interp1(lamda,power_fordifflamdas,lamda_marginal);
%finding the marginal power according to marginal lamda, any power above this
will be served by hydro
             Ph(hour)=Load(hour)-Power_marginal;
             r=Ph.';
              %used loss equation and basic power consumtion equation to find
             %the 2nd order eqn and using its root deriving formula to find
             %pt
              a=0.0002;
             c=0.0004*Ph(hour)*Ph(hour)+Load(hour)-(1.0006*Ph(hour));
             b=(0.0003*Ph(hour))-1.0005;
            Pt(hour)=(-b-sqrt(abs(b.*b-4*a*c)))/(2*a);
            ploss(hour)=
0.0002*(Pt(hour).*Pt(hour))+0.0004*(r(hour).*r(hour))+0.0003*(r(hour).*Pt(hou
r))-0.0006*r(hour)-0.0005*Pt(hour);%loss wqn


Volume_used=Volume_used+(30*Ph(hour)+0.02*Ph(hour).*Ph(hour))*60*60;

      else

          ploss(hour)= 0.0002*(Pt(hour).*Pt(hour))-0.0005*Pt(hour);% loss with
out hydro as this part involves no hydro
          Pt(hour)=Load(hour)+ ploss(hour);%no hydro
      end
  end
   error=abs(Volume_used-Volume_allot); %missmatch
   if error<=100000
       break; % breaks when converges and goes to result
   end
  lamda_marginal= lamda_marginal-0.0001;% check for new marginal lamda
```

```
   loop_n=loop_n+1;% counts needed loops
   end
  %%
%calculating power_outputs
Load=transpose(Load);
modified_load=Load-Ph+ploss;
for hour=1:1:24

new_lamda(hour)=interp1(power_fordifflamdas,lamda,modified_load(hour));%findi
ng newlamda for modified load
    P1(hour,1)=(new_lamda(hour)-gen_data(1,2))/(2*gen_data(1,1));%p1
    P2(hour,1)=(new_lamda(hour)-gen_data(2,2))/(2*gen_data(2,1));%p2

cost1(hour,1)=gen_data(1,1)*P1(hour)*P1(hour)+gen_data(1,2)*P1(hour)+gen_data
(1,3);

cost2(hour,1)=gen_data(2,1)*P2(hour)*P2(hour)+gen_data(2,2)*P2(hour)+gen_data
(2,3);

    Total_cost(hour,1)=cost1(hour)+cost2(hour);
end
hour=transpose(1:24);
%%%calculate hourly dischare in m3/hr
Q=(30*Ph(hour)+0.02*Ph(hour).*Ph(hour))*60*60;%vol per hr
final_table=table(hour,Load,Ph,P1,P2,ploss,Q,Total_cost);
disp(final_table);
figure(2);
bar(hour,Pt,'y');
grid on;
axis([0 25 1 1500]);
xlabel('Hours');
ylabel('SteamPower');
figure(3);
bar(hour,Ph,'r');
grid on;
axis([0 25 1 1500]);
xlabel('Hours');
ylabel('hydroPower');
```

QUESTION NO 2

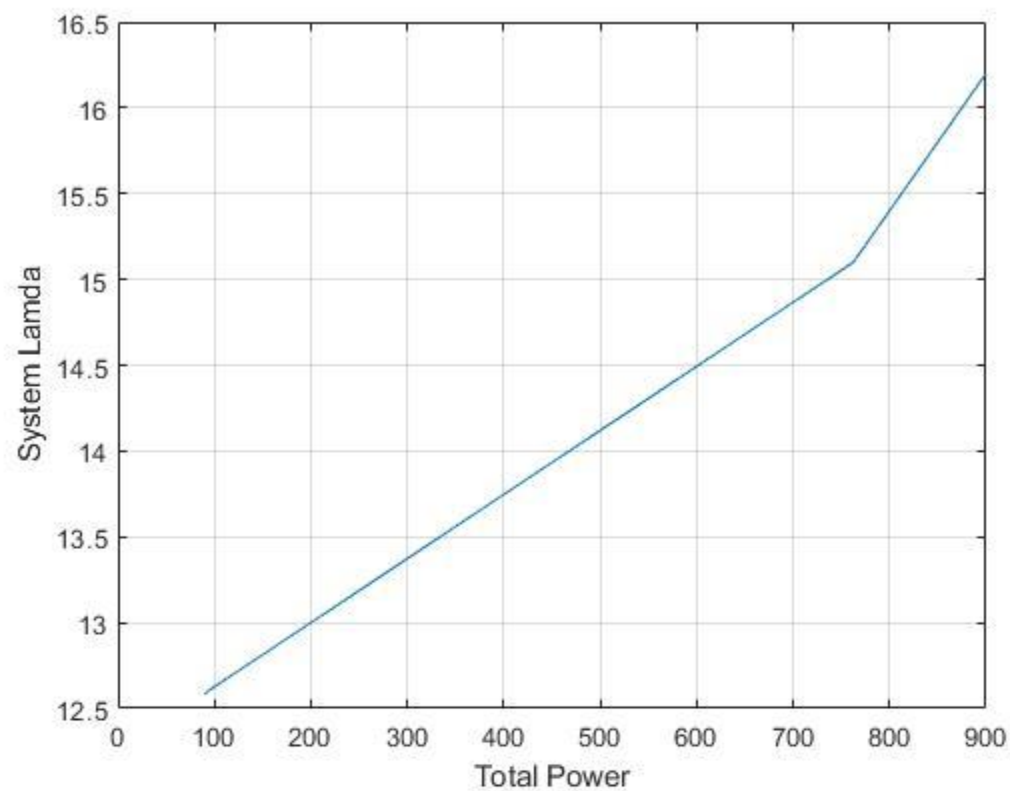
The given problem provides us information about the two thermal units committed to satisfy different loads for a period of 24 hour. It gives details about the cost functions, maximum operating limit and minimum operating limit of all 2 units. To satisfy the load over 24-hour period, a hydro unit is available in addition to these 2 thermal units. The hydro unit has a limited Volume of water and should be operated in parallel with thermal units in such a way that the overall operating cost of thermal units is minimized for the given period of 24 hour. The transmission loss is equation is a function of thermal and hydro generation.

We need to obtain the optimal load dispatch for the hydro thermal system by utilizing incremental dynamic programming approach and then calculate the output of the thermal and hydro units, hourly discharge of hydro plant, Volume states of hydro reservoir, corresponding running cost of thermal plants, transmission loss and Penalty factors (hydro and thermal) for a period of 24 hours.

The steps followed are:

- First, the given 2 thermal units are represented by an equivalent unit for which the minimum output=50+40=90MW and the maximum output=500+400= 900MW. The variation of incremental running cost(λ) and Power output is shown by a *Lambda(λ)-Total Power* curve shown below

power	***Lambda(λ)***
90	12.58
92.8	12.60
762.5	15.10
900	16.20



- Since we need to dispatch hydro units over 24 hour periods in such a way that there will be maximum savings in fuel cost of thermal units, we start by assuming two limits of hydro operation. They are:

    1. The hydro plant does not operate from the first hour. It starts operating only when there is enough time left to use all the allotted water. This gives the Upper Limit of operation.
    2. The plant is operated from the first hour with maximum discharge until all the allotted water is used. After this, the hydro units are off. This gives Lower limit of operation

- At the beginning of first iteration, we assume that every hour uses the same amount of discharge. It means for every hour we generate same amount of power from hydro
  But it should be noted that the discharge at any hour cannot be greater than Qmax given by, phmax=700 here

$$Qmax=0.02x700^2+30x700=30800m^3/s$$

$$\begin{aligned}So,Q\_initial &=(Volume\_alloted)/24\\ &=(2.4*10^8/24)\\ &=2.583*10^7 m^3/hr\\ &=2777.77 m^3/second\end{aligned}$$

At the beginning in each hour, power from hydro can be calculated using following relation:

$$Q\ initial\ =0.02Phydro^2+30Phydro \quad \text{for Phydro from 0 to 700MW} \ldots\ldots\ldots\ldots(1)$$

```
A=0.02;
B=30;
C=-Q_initial
```

So ph(initial)= $1.58*10^3$

For this corresponding Value of Phydro, Psteam and Ploss are calculated for 24 hours using following relations:

$$Phydro+Psteam=Pload+Ploss, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

where Pload is constant for particular hour.

$$P_L = 0.0004P_h^2 + 0.0002P_s^2 + 0.0003P_hP_s - 0.0006P_h - 0.0005P_s \quad \ldots\ldots\ldots\ldots\ldots(3)$$

So Psteam is calculated for 24 hour and then Ploss is calculated using equation (3) and equation (2).

Our objective is to maximize the savings during each interval. The savings is dependent on the value of G which is given by:

$$\mathrm{G} = \frac{dFs}{dPs} * \frac{PFsteam}{PFhydro} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots.(4)$$

Where, $\frac{dFs}{dPs}$ is the incremental running cost of equivalent thermal unit obtained from interpolation of Lambda-total power curve for the particular value of Psteam.

And $PFsteam$, $PFhydro$ are the penalty factors of thermal and hydro units respectively which can be calculated after we know the values of Phydro and Psteam.

- To start the dynamic programming, from the above step, we know an initial volume state for each hour which is represented by V_{hour}.
  We take a small volume step given by DelV=5000m^3 and assume one volume above and one volume below for every hour. So for each hour we have:
  V_{hour}
  $Vabove_{hour} = V_{hour} + DelV$
  $Vbelow_{hour} = V_{hour} - DelV$
  These volume states should be within the upper limit and lower limit of hydro operation for every hour.

- Now we start from the beginning of 24th hour. At the beginning of 24th hour there are 3 volume states. At the end of 24th hour, we use all the allotted water, which means the Volume state is zero. So we first calculate corresponding discharges for three volume states and then corresponding Phydro, Psteam, Ploss, PFhydro, PFsteam are calculated. The discharges that violates the maximum permissible discharges(Qmax) are neglected. We know the initial value of G(24). So the maximum weighted output for each discharges can calculated and therefore optimal mode of operation during hour 24 is calculated.

  For different Volume states at the beginning of 24th hour, the discharges are:
  $Q1(24) = (Vabove_{24} - 0)/3600 = (V24_{initial} + DelV - 0)/3600$
  $Q2(24) = (V_{24} - 0)/3600 = (V24_{initial} - 0)/3600$
  $Q3(24) = (Vbelow_{24} - 0)/3600 = (V24_{initial} - DelV - 0)/3600$

  Then, Phydro, Psteam and Ploss for these three different discharges are calculated using equation 2 and 3 respectively:

  Phydro1(24) Psteam1(24) Ploss1(24)
  Phydro2(24) Psteam2(24) Ploss2(24)
  Phydro3(24) Psteam3(24) Ploss3(24)

  The corresponding **maximum weighted output** are calculated which is given by **R24**

  R24(state)=G(24)*Phydro(state).
  R24(1)
  R24(2)
  R24(3)

- Using the results of the above step, now we go to one previous hour, i.e 23^{rd} hour and the optimum mode of operation during the last two hours of operations is calculated.
  At the beginning of 23^{rd} hour there are 3 volume states. And at the end of 23^{rd} hour or the beginning of 24^{th} hour, there are other 3 volume states. So corresponding discharges are calculated and corresponding Phydro is calculated using equation 1 and Psteam is calculated using equation 2 and 3.

  <u>For $Vabove_{23}$ state</u>
  $Q1(23)=(Vabove_{23} - Vabove_{24})/3600$
  $Q2(23)=(Vabove_{23} - V_{24})/3600$
  $Q3(23)=(Vabove_{23} \text{-} Vbelow_{24})/3600$

Phydro1(23)	Psteam1(23)
Phydro2(23)	Psteam2(23)
Phydro3(23)	Psteam3(23)

  Among these three discharges, the discharge that corresponds to the maximum weighted output and the corresponding path afterwards should be noted.
  R23(state)=G(23)*Phydro(state)+R24(corresponding to max weighted output)

  Following the above procces we need to find R23 value of other 2 states.

  Similarly for 22^{nd} hour, we need to determne the the maximum weighted output

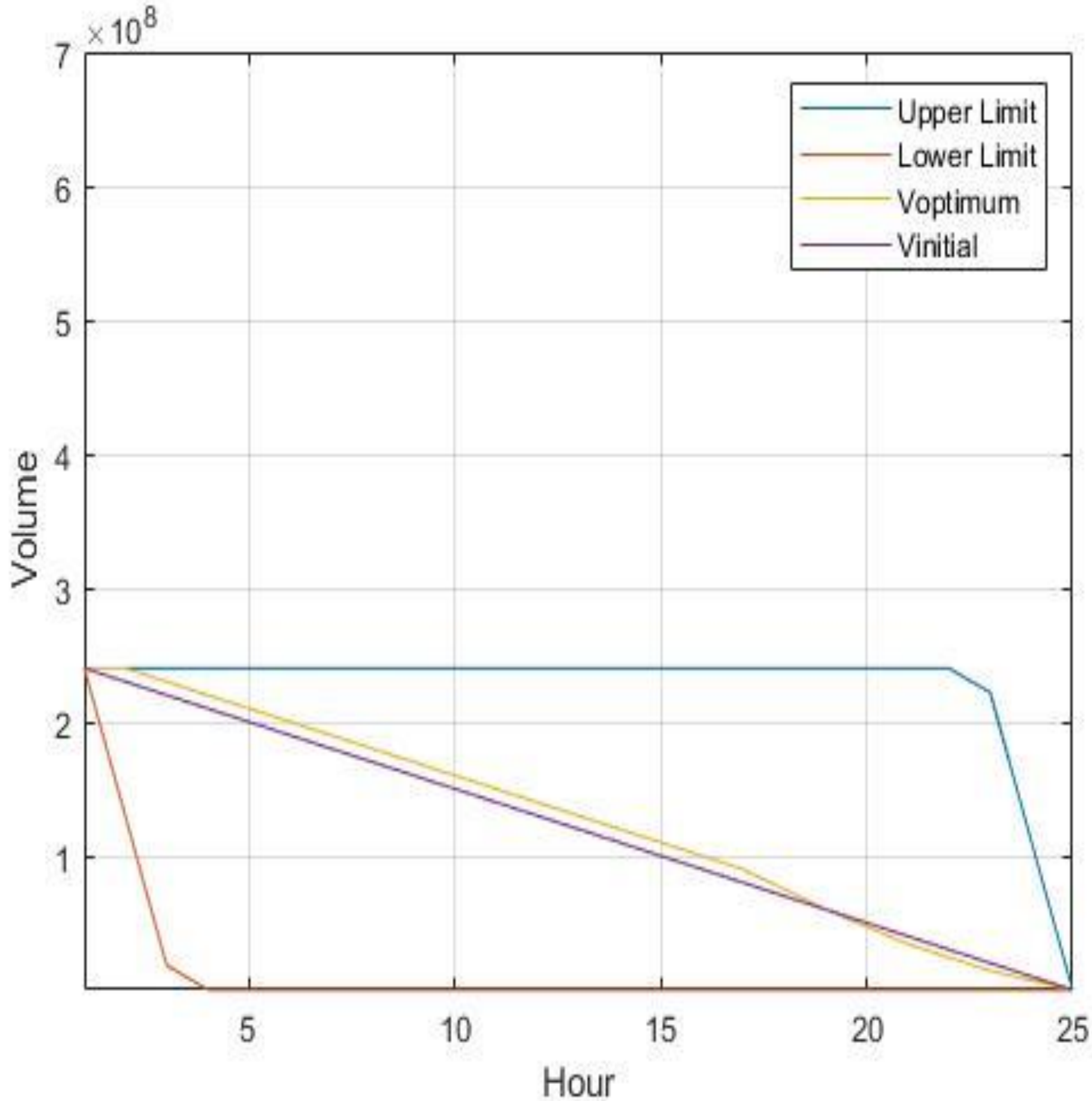
- The similar process is repeated for every hour and calculation is made upto the beginning of first hour. By the end of one iteration,we note the most optimum state at hour i and its corresponding optimum path from upto 24^{th} hour.By the end of one iteration, the Volume state of each hour is changed and corresponding values of Phydro and Psteam are also changed.
- The new volume states are calculated for each hour and the new value of G is calculated using:

$$G = \frac{dFs}{dPs} * \frac{PFsteam}{PFhydro}$$

  Where, $\frac{dFs}{dPs}$is calulated from the lambda-total power curve initially plotted for the corresponding value of Psteam.

- The iterative process is repeated until the optimum solution is obtained.
- After the optimum solution is obtained, the total power of thermal units i.e Psteam is known which is distributed among three thermal units again using the interpolation on Lambda-total power curve to find out the output of individual units (P1,P2) for every hour.

The optimum volume states ,after the iterative process is complete, can be plotted as shown in the figure below:



The final results are obtained using MATLAB code which are shown in the table below:

Load	ph	ps	ploss	PF Hydro	PF Steam	Cost	hour
450	0	499.68	49.9375634148247	1.177	1.2505863	7088.59078215288	1
450	87.48	409.62	47.3191312749025	1.239	1.2354781	5832.14690498752	2
500	87.48	472.29	60.0173259005220	1.269	1.27496	6703.18976504478	3
500	87.48	472.29	60.0173259005220	1.948	1.27496299	6703.18976504478	4
550	87.48	537.02	74.7838585166760	1.3052	1.318490	7618.36205260700	5
550	87.48	537.02	74.7838585166760	1.30952	1.31849	7618.36205260700	6
650	87.48	673.61	111.440833517006	1.343023	1.4208433	9600.70019024321	7
650	87.48	673.61	111.440833517006	1.31743023	1.4208	9600.70019024321	8
700	87.48	746.04	133.907084643858	1.4577944	1.48184296	10680.1513197459	9
700	87.48	746.04	133.907084643858	1.4577944	1.481842	10680.1513197459	10
800	87.48	901.12	189.066934323699	1.517158	1.63184735	NaN	11
800	87.48	901.12	189.066934323699	1.5174158	1.6318473	NaN	12
850	87.48	984.87	222.853396058420	1.571962	1.7262076	NaN	13
850	87.48	984.87	222.853396058420	1.541962	1.72620765	NaN	14
800	87.48	901.12	189.066934323699	1.5174158	1.63184735	NaN	15
800	87.48	901.12	189.066934323699	1.5174158	1.631847	NaN	16
750	127.5	780.00	157.966767714242	1.507502861	1.5402936	11497.0389488104	17
750	127.5	780.00	157.966767714242	1.50702861	1.540293657	11497.0389488104	18
700	112.0	720.63	133.040436706718	1.44377045	1.475725	10299.2073357621	19
700	112.0	720.59	133.039458889669	1.44315724	1.47571621	10298.5911346401	20
600	86.78	604.76	91.8547174236965	1.335093076	1.36694651	8592.81060673620	21
600	86.78	604.76	91.8547174236965	1.3093076	1.366946	8592.81060673620	22
500	63.30	496.81	60.3645549538512	1.25044513	1.279126	7048.05040389493	23
500	63.34	496.77	60.3635449760998	1.250475451	1.2791186	7047.43416551948	24

Discussion: In this program it is noticed that the optimal curve goes out of upperlimit, so the boundary checking is not correctly done here. Also There are some NAN value in the cost which is also a incorrect output.

MATLAB CODE FOR QUESTION 2

```
clc;
clear;
step=5000;
Volume_allot=2.4*10^8;
```

```
Load=[450 450 500 500 550 550 650 650 700 700 800 800 850 850 800 800 750 750
700 700 600 600 500 500  ];

Ph_max=700;
Q_max=(30*Ph_max+0.02*Ph_max*Ph_max);
Volume(25)=0;
Vmax(25)=0;
Vmin(25)=0;
for hour=24:-1:1 %%upper limit detection
    Vmax(hour)=Vmax(hour+1)+Q_max*60*60;
    if Vmax(hour)>Volume_allot
        Vmax(hour)=Volume_allot;
    end
end
Vmin(1)=Volume_allot;
for hour=2:1:24%%lower limit detect
    Vmin(hour)=Vmin(hour-1)-Q_max*60*60;
    if Vmin(hour)<0
        Vmin(hour)=0;
    end
end
Q_initial=(Volume_allot/24);
Volume(25)=0;
Volume_above(25)=0;
Volume_below(25)=0;
for hour=24:-1:1  %%avg/mid line detection
    Volume(hour)=Volume(hour+1)+Q_initial;
    Volume_below(hour)=Volume(hour)-step;
    Volume_above(hour)=Volume(hour)+step;
end
Volume_initial=Volume;
 hour=1:25;
 plot(hour,Vmax,'b',hour,Vmin,'r',hour,Volume,'g');
axis([1 25 1 600000000]);
 grid on;
%%
%%%combine thermal units into 1;
gen_data=[ 0.004 12.2 300 50 500
           0.0035 12.3 200 40 400
          ];
%%represent the given thermal unit by an equivalent unit
%Calculating values of lambda,there will be 2 lambda for each units%
     count=2*2;
     lamda=zeros(count,1);
     power_fordifflamdas=zeros(count,1);
     j=1;
     a=1;
     while(j<=count)
         lamda(j)=2*gen_data(a,1)*gen_data(a,4)+gen_data(a,2);
         lamda(j+1)=2*gen_data(a,1)*gen_data(a,5)+gen_data(a,2);
         a=a+1;
         j=j+2;
     end
     a=1;
     while(a<=count)
     for j=1:1:2
         powerofunit(j)=(lamda(a)-gen_data(j,2))/(2*gen_data(j,1));
```

```
            if  powerofunit(j)>gen_data(j,5)
                 powerofunit(j)=gen_data(j,5);
            end
                if  powerofunit(j)<gen_data(j,4)
                 powerofunit(j)=gen_data(j,4);
                end
                power_fordifflamdas(a)=power_fordifflamdas(a)+powerofunit(j);
        end
            a=a+1;
        end
        lamda=sort(lamda);
        power_fordifflamdas=sort(power_fordifflamdas);
        t=table(power_fordifflamdas,lamda);
        figure(1);
        plot(power_fordifflamdas,lamda);
        xlabel('Total Power');
        ylabel('System Lamda');
        grid on;
        %%
        %%%find initial G
       % G(1:24)=20;
for hour=1:1:24
    A=0.02; %%from given Q equation
    B=30;
    C=-Q_initial;
init_ph= (-B-sqrt(abs(B*B-4*A*C)))/(2*A);
        a=0.0002;
            c=0.0004*init_ph*init_ph+Load(hour)-1.0006*init_ph;
            %used loss equation and basic power consumtion equation to find
            %the 2nd order eqn and using its root deriving formula to find
            %pt
            b=0.0003*init_ph-1.0005;%chg%chg
            ini_Ps(hour)=(-b-sqrt(abs(b*b-4*a*c)))/(2*a);
             PF_hydro__ini(hour)=1/(1-2*0.0004*init_ph+0.0003*ini_Ps(hour)-
0.0006);
            PF_steam_ini(hour)=1/(1-2*0.0002*ini_Ps(hour)+0.0003*init_ph-
0.0005);

G(hour)=interp1(power_fordifflamdas,lamda,ini_Ps(hour))*(PF_steam_ini(hour)/P
F_hydro__ini(hour));
end
Power_hydro_old(1:24)=init_ph;
  %%
Q=zeros(3,3,24);
Ph=zeros(3,3,24);
Ps=zeros(3,3,24);
cost=zeros(3,3,24);
Ploss=zeros(3,3,24);
iteration=2000;
for loop_no=1:1:iteration
  for hour=24:-1:1
  if hour==24
      for state_ini=1:1:3
            if state_ini==1
                Volume_first=Volume_above;
            end
            if state_ini==2
```

```
                Volume_first=Volume;
            end
            if state_ini==3
                Volume_first=Volume_below;
            end
           state_fin=2;
            if state_fin==1
                Volume_second=Volume_above;
            end
            if state_fin==2
                Volume_second=Volume;
            end
            if state_fin==3
                Volume_second=Volume_below;
            end
            Q(state_ini,state_fin,hour)=(Volume_first(hour)-
Volume_second(hour+1))/3600;
             %%%from Q equation


            C=-Q(state_ini,state_fin,hour);
            A=0.02;B=30;
            Ph1=(-B+sqrt(abs(B*B-4*A*C)))/(2*A);
            Ph(state_ini,state_fin,hour)=Ph1; %got ph now need ps
             %used loss equation and basic power consumtion equation to find
            %the 2nd order eqn and using its root deriving formula to find
            %pt
            a=0.0002;

c=0.0004*Ph(state_ini,state_fin,hour)*Ph(state_ini,state_fin,hour)+Load(hour)
-1.0006*Ph(state_ini,state_fin,hour);%chang
            b=0.0003*Ph(state_ini,state_fin,hour)-1.0005;%chang
            Ps1=(-b-sqrt(abs(b*b-4*a*c)))/(2*a);
            Ps(state_ini,state_fin,hour)=Ps1;
            % ploss from quesstion

Ploss(state_ini,state_fin,hour)=0.0004*Ph(state_ini,state_fin,hour)*Ph(state_
ini,state_fin,hour)+0.0002*Ps(state_ini,state_fin,hour)*Ps(state_ini,state_fi
n,hour)+0.0003*Ph(state_ini,state_fin,hour)*Ps(state_ini,state_fin,hour)-
0.0006*Ph(state_ini,state_fin,hour);

Max_cost(state_ini,hour)=G(hour)*Ph(state_ini,state_fin,hour);%chang  3 stage
so 3 cost each hr
      end
  end
  if hour>=2&&hour<=23
      for state_ini=1:1:3
            if state_ini==1
                Volume_first=Volume_above;
            end
            if state_ini==2
                Volume_first=Volume;
            end
            if state_ini==3
                Volume_first=Volume_below;
            end
```

```
            for state_fin=1:1:3
             if state_fin==1
                 Volume_second=Volume_above;
             end
             if state_fin==2
                 Volume_second=Volume;
             end
             if state_fin==3
                 Volume_second=Volume_below;
             end
             Q(state_ini,state_fin,hour)=(Volume_first(hour)-
Volume_second(hour+1))/3600;


                  C=-Q(state_ini,state_fin,hour);
             A=0.02;B=30;

             Ph1=(-B+sqrt(abs(B*B-4*A*C)))/(2*A);
             Ph(state_ini,state_fin,hour)=Ph1;

            %used loss equation and basic power consumtion equation to find
             %the 2nd order eqn and using its root deriving formula to find
             %pt
             a=0.0002;%chang

c=0.0004*Ph(state_ini,state_fin,hour)*Ph(state_ini,state_fin,hour)+Load(hour)
-1.0006*Ph(state_ini,state_fin,hour);%chang
             b=0.0003*Ph(state_ini,state_fin,hour)-1.0005;%

             Ps1=(-b-sqrt(abs(b*b-4*a*c)))/(2*a);
             Ps(state_ini,state_fin,hour)=Ps1;

Ploss(state_ini,state_fin,hour)=0.0004*Ph(state_ini,state_fin,hour)*Ph(state_
ini,state_fin,hour)+0.0002*Ps(state_ini,state_fin,hour)*Ps(state_ini,state_fi
n,hour)+0.0003*Ph(state_ini,state_fin,hour)*Ps(state_ini,state_fin,hour)-
0.0006*Ph(state_ini,state_fin,hour);


cost(state_ini,state_fin,hour)=G(hour)*Ph(state_ini,state_fin,hour)+Max_cost(
state_fin,hour+1);
            end
            [Max_cost(state_ini,hour)
Next_state(state_ini,hour)]=max(cost(state_ini,:,hour));
       end
   end
   if hour==1
       for state_fin=1:1:3
            if state_fin==1
                 Volume_second=Volume_above;
             end
             if state_fin==2
                 Volume_second=Volume;
             end
             if state_fin==3
                 Volume_second=Volume_below;
             end
```

```
            Volume_first=Volume;
            Qist(hour,state_fin)=(Volume_first(hour)-
Volume_second(hour+1))/3600;


            Cist=-Q(hour,state_fin); %chng
            Aist=0.02;Bist=30;%chng A=0.02;B=30;
            Ph1ist=(-Bist+sqrt(abs(Bist*Bist-4*Aist*Cist)))/(2*Aist);
            Phist(hour,state_fin)=Ph1ist;
            %used loss equation and basic power consumtion equation to find
            %the 2nd order eqn and using its root deriving formula to find
            %pt
            aist=0.0002;

cist=0.0004*Phist(hour,state_fin)*Phist(hour,state_fin)+Load(hour)-
1.0006*Phist(hour,state_fin);
            bist=0.0003*Phist(hour,state_fin)-1.0005;
            Ps1ist=(-bist-sqrt(abs(bist*bist-4*aist*cist)))/(2*aist);
            Psist(hour,state_fin)=Ps1ist;

Plossist(hour,state_fin)=0.0004*Phist(hour,state_fin)*Phist(hour,state_fin)+0
.0002*Psist(hour,state_fin)*Psist(hour,state_fin)+0.0003*Phist(hour,state_fin
)*Psist(hour,state_fin)-0.0006*Phist(hour,state_fin);

costist(hour,state_fin)=G(hour)*Phist(hour,state_fin)+Max_cost(state_fin,hour
+1);
      end
  end
  end
[Value position]=max(costist);
new_state(1)=2;
new_state(2)=position;
new_state(25)=2;
for i=3:1:24
    new_state(i)=Next_state(new_state(i-1),i-1);
end
Power_hydro(1)=Phist(new_state(2));
Power_steam(1)=Psist(new_state(2));
Power_loss(1)=Plossist(new_state(2));
for i=2:1:23
    Power_hydro(i)=Ph(new_state(i),new_state(i+1),i);
    Power_steam(i)=Ps(new_state(i),new_state(i+1),i);
    Power_loss(i)=Ploss(new_state(i),new_state(i+1),i);
end
Power_hydro(24)=Ph(new_state(i),2,24);
Power_steam(24)=Ps(new_state(i),2,24);
Power_loss(24)=Ploss(new_state(i),2,24);
for hour=1:1:24

    PF_hydro(hour)=1/(1-2*0.0004*Power_hydro(hour)-0.0003*Power_steam(hour)-
0.0006); % usinf d/dph *ploss
    PF_steam(hour)=1/(1-2*0.0002*Power_steam(hour)-0.0003*Power_hydro(hour)-
0.0005);% usinf d/dps *ploss
    G(hour)=interp1(power_fordifflamdas,lamda,Power_steam(hour))*(
PF_steam(hour)/PF_hydro(hour));
end
```

```
Volume(1)=Volume(1);
for hour=2:1:24
    if new_state(hour)==1
        Volume(hour)=Volume(hour)+step;
    end
    if new_state(hour)==2
        Volume(hour)=Volume(hour);
    end
    if new_state(hour)==3
        Volume(hour)=Volume(hour)-step;
    end
    Volume_above(hour)=Volume(hour)+step;
    Volume_below(hour)=Volume(hour)-step;
end
Volume(25)=0;
Volume_above(25)=0;
Volume_below(25)=0;
error=max(abs((Power_hydro-Power_hydro_old)));
end
hour=1:25;
figure(2);
plot(hour,Vmax,hour,Vmin,hour,Volume,hour,Volume_initial);
axis([1 25 1 700000000]);
xlabel('Hour');
ylabel('Volume');
legend('Upper Limit','Lower Limit','Voptimum','Vinitial');
grid on;
%%
%%%find indivudal power outputs
for hour=1:1:24
lamda_sys(hour)=interp1(power_fordifflamdas,lamda,Power_steam(hour));
P1(hour)=(lamda_sys(hour)-gen_data(1,2))/(2*gen_data(1,1));
P2(hour)=(lamda_sys(hour)-gen_data(2,2))/(2*gen_data(2,1));
 PF_hydro(hour)=1/(1-2*0.0004*Power_hydro(hour)-0.0003*Power_steam(hour)-
0.0006);% usinf d/dph *ploss
    PF_steam(hour)=1/(1-2*0.0002*Power_steam(hour)-0.0003*Power_hydro(hour)-
0.0005);% usinf d/dps *ploss


cost1(hour)=gen_data(1,1)*P1(hour)*P1(hour)+gen_data(1,2)*P1(hour)+gen_data(1
,3);
cost2(hour)=gen_data(2,1)*P2(hour)*P2(hour)+gen_data(2,2)*P2(hour)+gen_data(2
,3);

Total_cost(hour)=cost1(hour)+cost2(hour);


Discharge(hour)=0.02*(Power_hydro(hour))*(Power_hydro(hour))+30*(Power_hydro(
hour))*60*60;

if hour==1
    Volume_state(hour)=Volume_allot;
else
Volume_state(hour)=Volume_state(hour-1)-Discharge(hour-1);
end
end
```