

We consider, the payoff of the following Down-Out Call option

$$e^{-rT}(S_T - K)_+ \mathbf{1}_{\inf_{u \in [0, T]} S_u \geq L}$$

- We compute the price of the option using Monte Carlo and then we plot the evolution of the price as a function of number of steps.
- then we implement another method to evaluate the price of the same option. to do so we rely on the following formula :

$$e^{-rT} \mathbb{E}[(S_T - K)_+ \mathbf{1}_{\inf_{u \in [0, T]} S_u \geq L}] = e^{-rT} \mathbb{E}[(S_T - K)_+ \prod_{i=1}^n P(\inf_{u \in [t_{i-1}, t_i]} S_u \geq L | S_{t_{i-1}}, S_{t_i})]$$

where for all $s < t$, $L > 0$ and $L < x$ and $L < y$:

$$P(\inf_{u \in [s, t]} S_u \geq L | S_s = x, S_t = y) = 1 - \exp\left(-\frac{2 \log \frac{L}{x} \log \frac{L}{y}}{\sigma^2(t-s)}\right)$$