In this example we consider two asset price $(S_t)_{0\leq t\leq T}$ and $(\tilde{S}_t)_{0\leq t\leq T}$ defined by :

$$\tilde{S}_t = S_0 e^{(r-\frac{\sigma^2}{2})t+\sigma\tilde{W}_t}$$

where $(\tilde{W}_t)_{t\geq 0}$ is a standard Brownian motion independent of $(W_t)_{t\geq 0}$. In what follows, we set

$$W_T = \sqrt{T}G \text{ and } \tilde{W}_T = \sqrt{T}\tilde{G}$$

- Using Monte Carlo we compute the price of the option given :

$$\Pi = \mathbb{E}[\phi(G,\tilde{G})]$$

Where

$$\phi(G,\bar{G}) = \left(\frac{s_0 e^{(r-\frac{\sigma^2}{2}+\sigma\sqrt{T}G)} + s_0 e^{(r-\frac{\sigma^2}{2}+\sigma\sqrt{T}\bar{G})}}{2} - k\right)_+$$

- We use the the formula below to compute the price by adding λ, $\tilde{\lambda} \in \mathbb{R}$ to reduce the variance :

  $$\Pi = \tilde{\Pi} = e^{-rT}\mathbb{E}(\phi(G+\lambda, \bar{G}+\bar{\lambda})e^{-\lambda G-\frac{-\lambda^2}{2}}e^{-\bar{\lambda}\bar{G}-\frac{-\bar{\lambda}^2}{2}})$$

- We plot in a three dimensional graph the evolution of the variance of the Monte Carlo method associated to the computation of $\bar{\Pi}$ as function of $(\lambda, \bar{\lambda})$ and find numerically the optimal $(\lambda^*, \bar{\lambda}^*)$

- We implement a Gradient Descent algorithm to find automatically the couple $(\lambda^*, \bar{\lambda}^*)$ that minimize the empirical variance.