

Isfahan University of Technology
Algorithm Design Homework 2
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- 1) Prove that there is no comparison based sorting problem running better than $O(n \log n)$.
- 2)
 - a) Find the best, worst and the average cases time complexity of quick sort algorithm when the pivot element is the last element of the array.
 - b) Suggest a better way of choosing pivot which you think can reduce the time of Quick Sort algorithm.
- 3) Imagine a squirrel who lives on a tall tree consisting of n branches. The squirrel wants to find the lowest branch i which an acorn will break open if dropped from that branch. If the acorn breaks open when dropped from the branch i , then it will break open for any branch j if $j > i$. The important point is when an acorn breaks open, the squirrel eats it and can no longer be used for measuring. Answer the following questions (branch 0 has the lowest height):
 - a) Suppose that the squirrel has $\lfloor \log(n) \rfloor + 1$ acorns. Give a procedure that helps him identify the correct branch using $O(\log(n))$ drops.
 - b) Now suppose that he has only one acorn. Help him identify the desired branch using $O(n)$ drops. Explain why the $O(\log(n))$ drops solution won't work.
 - c) Help the squirrel identify the correct branch using $O(\sqrt{n})$ drops when he has two acorns.
 - d) Suppose that he has $a = O(1)$ acorns. Help him find the desired branch using $O(n^{1/a})$ drops.
 - e) What happens to your solution for the previous part if $a = \lfloor \log(n) \rfloor + 1$? Is it like the previous part, or like part a?
- 4) In a stock market in each day you have the price of a stock in future n days and you are supposed to buy a unit of stock in day A and sell it in day B . (Obviously $A \leq B$). Given the price in future n days how much benefit you may get at most? Find the solution and write down the pseudocode for both brute-force algorithm and an algorithm using Divide and Conquer paradigm.