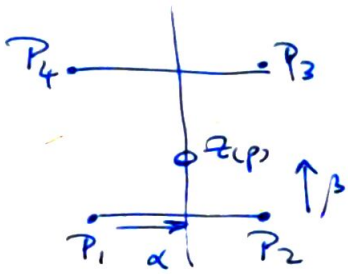
Problem 2

Bilinear interpolation "like method"

$P_i = \begin{pmatrix} x_i \\ y_i \end{pmatrix}$: $\alpha = \frac{x - x_1}{x_2 - x_1}$, $\beta = \frac{y - y_1}{y_4 - y_1}$



$$\Rightarrow \begin{cases} z_a = (1-\alpha) z_1 + \alpha z_2 \\ z_b = (1-\alpha) z_4 + \alpha z_3 \end{cases}$$

$z_{(x,y)} = A + Bx + Cy + Dxy$

$$z_{(p)} = (1-\beta)(1-\alpha) z_1 + (1-\beta)\alpha z_2 + \beta(1-\alpha) z_4 + \beta\alpha z_3$$

Matrix form:

$$\begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix} = \begin{pmatrix} 1 & x_1 & y_1 & x_1 y_1 \\ 1 & x_2 & y_2 & x_2 y_2 \\ 1 & x_3 & y_3 & x_3 y_3 \\ 1 & x_4 & y_4 & x_4 y_4 \end{pmatrix} \begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix}$$

$$\Rightarrow z_{(p)} = \left(\frac{y_4}{y_4 - y_1} - \frac{y}{y_4 - y_1}\right)\left(\frac{x_2}{x_2 - x_1} - \frac{x}{x_2 - x_1}\right) z_1 + \left(\frac{y_4}{y_4 - y_1} - \frac{y}{y_4 - y_1}\right)\left(\frac{x}{x_2 - x_1} - \frac{x_1}{x_2 - x_1}\right) z_2 + \left(\frac{y}{y_4 - y_1} - \frac{y_1}{y_4 - y_1}\right)\left(\frac{x_2}{x_2 - x_1} - \frac{x}{x_2 - x_1}\right) z_4 + \left(\frac{y}{y_4 - y_1} - \frac{y_1}{y_4 - y_1}\right)\left(\frac{x}{x_2 - x_1} - \frac{x_1}{x_2 - x_1}\right) z_3$$

$$\Rightarrow \begin{cases} A = \left(\frac{y_4}{y_4 - y_1}\right)\left(\frac{x_2}{x_2 - x_1}\right) z_1 + \left(\frac{y_4}{y_4 - y_1}\right)\left(\frac{-x_1}{x_2 - x_1}\right) z_2 + \cdots \\ B = \frac{y_4}{(y_4 - y_1)}\left(\frac{-1}{x_2 - x_1}\right) z_1 + \left(\frac{y_4}{y_4 - y_1}\right)\left(\frac{1}{x_2 - x_1}\right) z_2 + \cdots \\ C = \left(\frac{-1}{y_4 - y_1}\right)\left(\frac{x_2}{x_2 - x_1}\right) z_1 + \left(\frac{-1}{y_4 - y_1}\right)\left(\frac{-x_1}{x_2 - x_1}\right) z_2 + \cdots \\ D = \left(\frac{-1}{y_4 - y_1}\right)\left(\frac{-1}{x_2 - x_1}\right) z_1 + \left(\frac{-1}{y_4 - y_1}\right)\left(\frac{1}{x_2 - x_1}\right) z_2 + \cdots \end{cases}$$