> Supplementary code for Proposition 1 and Theorem 1

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> with(Groebner) :
  with(PolynomialIdeals) :
>
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> We consider two experiments with the steady state data for T1, T2, X and the same values of the parameters and c1, c2. We use the formulas from Lemma 1 to generate a system of equations.

```
> # Generates an i-th copy of the formulas in terms of states with c1 and c2 plugged in
  gen_replica := proc(i)
     return [(c1 - Y[i] - T1[i]) * (c2 - Y[i] - T2[i]) - X[i] * Y[i], T1[i] * X[i] - kD1 * (c1
       - Y[i] - T1[i]), T2[i] * X[i] - kD2 * (c2 - Y[i] - T2[i])] :
   end proc:

   n := 2 :

  eqs := [seq(op(gen_replica(i)), i = 1 ..n)] :

  vars1 := [seq(Y[i], i = 1 ..n), c1, c2, seq(X[i], i = 1 ..n), seq(T1[i], i = 1 ..n), seq(T2[i], i = 1
       ..n), kD1, kD2] :

  gbtdeg := Basis(eqs, tdeg(op(vars))) :
  elim := EliminationIdeal(⟨op(eqs)⟩, {seq(X[i], i = 1 ..n), seq(T1[i], i = 1 ..n), seq(T2[i], i = 1
       ..n), kD1, kD2}) :
  gb := Basis(elim, plex(kD2,  kD1, seq(X[i], i = 1 ..n), seq(T1[i], i = 1 ..n), seq(T2[i], i = 1
       ..n)));
```

$$
\begin{aligned}
gb := \Big[& kD1^{2}\, T1_{1}\, T2_{1}\, X_{1}^{2}\, X_{2} - kD1^{2}\, T1_{1}\, T2_{2}\, X_{1}\, X_{2}^{2} - kD1^{2}\, T1_{2}\, T2_{1}\, X_{1}^{2}\, X_{2} + kD1^{2}\, T1_{2}\, T2_{2}\, X_{1}\, X_{2}^{2} \\
& + kD1\, T1_{1}^{2}\, T2_{1}\, X_{1}^{2}\, X_{2} - kD1\, T1_{1}\, T1_{2}\, T2_{1}\, X_{1}^{2}\, X_{2} - kD1\, T1_{1}\, T1_{2}\, T2_{2}\, X_{1}\, X_{2}^{2} - kD1\, T1_{1}\, T2_{1}^{2} \\
& X_{1}^{2}\, X_{2} + kD1\, T1_{1}\, T2_{1}\, T2_{2}\, X_{1}^{2}\, X_{2} + kD1\, T1_{1}\, T2_{1}\, X_{1}^{3}\, X_{2} - kD1\, T1_{1}\, T2_{2}\, X_{1}^{2}\, X_{2}^{2} + kD1 \\
& T1_{2}^{2}\, T2_{2}\, X_{1}\, X_{2}^{2} + kD1\, T1_{2}\, T2_{1}\, T2_{2}\, X_{1}\, X_{2}^{2} - kD1\, T1_{2}\, T2_{1}\, X_{1}^{2}\, X_{2}^{2} - kD1\, T1_{2}\, T2_{2}^{2}\, X_{1}\, X_{2}^{2} \\
& + kD1\, T1_{2}\, T2_{2}\, X_{1}\, X_{2}^{3} + T1_{1}^{2}\, T2_{1}\, X_{1}^{3}\, X_{2} - T1_{1}\, T1_{2}\, T2_{1}\, X_{1}^{2}\, X_{2}^{2} - T1_{1}\, T1_{2}\, T2_{2}\, X_{1}^{2}\, X_{2}^{2} + \\
& T1_{2}^{2}\, T2_{2}\, X_{1}\, X_{2}^{3}, -kD1\, T1_{1}\, T2_{1}\, X_{1}^{2}\, X_{2} + kD1\, T1_{1}\, T2_{2}\, X_{1}\, X_{2}^{2} + kD1\, T1_{2}\, T2_{1}\, X_{1}^{2}\, X_{2} \\
& - kD1\, T1_{2}\, T2_{2}\, X_{1}\, X_{2}^{2} + kD2\, T1_{1}\, T2_{1}\, X_{1}^{2}\, X_{2} - kD2\, T1_{1}\, T2_{2}\, X_{1}^{2}\, X_{2} - kD2\, T1_{2}\, T2_{1}\, X_{1}\, X_{2}^{2} \\
& + kD2\, T1_{2}\, T2_{2}\, X_{1}\, X_{2}^{2} - T1_{1}^{2}\, T2_{1}\, X_{1}^{2}\, X_{2} + T1_{1}\, T1_{2}\, T2_{1}\, X_{1}^{2}\, X_{2} + T1_{1}\, T1_{2}\, T2_{2}\, X_{1}\, X_{2}^{2} + T1_{1} \\
& T2_{1}^{2}\, X_{1}^{2}\, X_{2} - T1_{1}\, T2_{1}\, T2_{2}\, X_{1}^{2}\, X_{2} - T1_{2}^{2}\, T2_{2}\, X_{1}\, X_{2}^{2} - T1_{2}\, T2_{1}\, T2_{2}\, X_{1}\, X_{2}^{2} + T1_{2}\, T2_{2}^{2}\, X_{1}\, X_{2}^{2}, \\
& kD1\, kD2\, T1_{1} - kD1\, kD2\, T1_{2} - kD1\, kD2\, T2_{1} + kD1\, kD2\, T2_{2} - kD1\, T2_{1}\, X_{1} + kD1\, T2_{2}\, X_{2}
\end{aligned} \quad \textbf{(1)}
$$

$+ kD2\, T1_1\, X_1 - kD2\, T1_2\, X_2, kD1\, kD2\, T2_1\, X_1\, X_2 - kD1\, kD2\, T2_2\, X_1\, X_2 + kD1\, T2_1\, X_1^2\, X_2$
$- kD1\, T2_2\, X_1\, X_2^2 + T1_1\, T2_1\, X_1^2\, X_2 - T1_2\, T2_2\, X_1\, X_2^2]$

>

> **We get two relations for kD1 and kD2. Let us investigate their leading terms.**

> $d1 := degree(gb[1], kD1);$

$factor(coeff(gb[1], kD1, d1));$

$$d1 := 2$$

$$X_1\, X_2\, (T2_1\, X_1 - T2_2\, X_2)\,(T1_1 - T1_2) \quad (2)$$

> $d2 := degree(gb[2], kD2);$

$factor(coeff(gb[2], kD2,\ d2));$

$$d2 := 1$$

$$X_1\, X_2\, (T2_1 - T2_2)\,(T1_1\, X_1 - T1_2\, X_2) \quad (3)$$

> **These are exactly the factorizations used in Proposition 2. In Theorem 3, in addition to this, we are interested in the constant term of the first relation. Here it is:**

> $factor(coeff(gb[1], kD1, 0));$

$$X_1\, X_2\, (T1_1\, X_1 - T1_2\, X_2)\,(T1_1\, T2_1\, X_1 - T1_2\, T2_2\, X_2) \quad (4)$$

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