Supplementary code for the proof of Lemma 1

Consider a system of differential equations with the following right-hand sides

```
> eqs := [
    -kn1 * X * T1 - kn2 * X * T2 + kf1 * D1 + kf2 * D2,
    -kn1 * X * T1 + kf1 * D1 - kn1 * T1 * D2 + kf1 * Y,
    -kn2 * X * T2 + kf2 * D2 - kn2 * T2 * D1 + kf2 * Y,
    kn1 * X * T1 - kf1 * D1 - kn2 * T2 * D1 + kf2 * Y,
    kn2 * X * T2 - kf2 * D2 - kn1 * T1 * D2 + kf1 * Y,
    kn1 * T1 * D2 + kn2 * T2 * D1 - (kf1 + kf2) * Y
  ];
```

$$eqs := [-kn1\,X\,T1 - kn2\,X\,T2 + D1\,kf1 + D2\,kf2, -kn1\,T1\,D2 - kn1\,X\,T1 + D1\,kf1 + kf1\,Y, -kn2\,T2\,D1 - kn2\,X\,T2 + D2\,kf2 + kf2\,Y, -kn2\,T2\,D1 + kn1\,X\,T1 - D1\,kf1 + kf2\,Y, -kn1\,T1\,D2 + kn2\,X\,T2 - D2\,kf2 + kf1\,Y, kn1\,T1\,D2 + kn2\,T2\,D1 - (kf1 + kf2)\,Y] \quad (1)$$

where X, T1, T2, D1, D2, Y are the state variables and kn1, kn2, kf1, kf2 are scalar parameters. The goal is to obtain a couple of useful relations between the coordinates of the steady states and the parameters.

```
>
  with(Groebner) :

  gb := Basis(eqs, plex(T1, T2, X, Y, D1, D2, kn1, kn2, kf1, kf2)) :

  factor(gb[1]);
```

$$-(D1\,kf1 + D2\,kf2 + X\,kf1 + X\,kf2)\,(D1\,D2 - X\,Y) \quad (2)$$

If the left bracket vanishes, then, by the positivity of the parameters and nonegativity of the steady state, D1 = D2 = X = 0, so the right bracket vanishes as well. Therefore, the right bracket vanishes in any case, and we can adjoin it to the list of equations.

Error, missing operator or `;`

```
>
  eqs := [op(eqs), X * Y - D1 * D2] :

  gb := Basis(eqs, plex(Y, T1, T2, X, D1, D2, kn1, kn2, kf1, kf2)) :

  factor(gb[1]);
```

$$-(-kn2\,X\,T2 + D2\,kf2)\,(D1 - D2) \quad (3)$$

If D1 != D2, we get an desired relation between T2, X, D2, and kn2/kf2. Otherwise, we add D1 = D2 to the system:

Error, missing operator or `;`

```
> eqs_ext := [op(eqs), D1 - D2] :
  gb := Basis(eqs_ext, plex(Y, T1, D1, T2, X, D2, kn1, kn2, kf1, kf2));
  factor(gb[1]);
```

$$gb := [D2\,T2\,X\,kn2 + T2\,X^2\,kn2 - D2^2\,kf2 - D2\,X\,kf2, D1 - D2, D2\,T1\,kf2\,kn1 - D2\,T2\,kf1\,kn2 - T2\,X\,kf1\,kn2 - T2\,X\,kf2\,kn2 + D2\,kf1\,kf2 + D2\,kf2^2, kn1\,X\,T1 + kn2\,X\,T2 - D2\,kf1 - D2\,kf2, -D2\,T2\,kn2 - kn2\,X\,T2 + D2\,kf2 + kf2\,Y, -kn1\,T1\,D2 + kn2\,X\,T2 - D2\,kf2 + kf1\,Y, -D2^2 + X\,Y]$$

$$-(D2 + X)\,(-kn2\,X\,T2 + D2\,kf2) \tag{4}$$

```
> If the left bracket vanishes, D2 = X = 0,
    so the right bracket vanishes as well
   . Thus the right bracket vanishes in any case.
  Now we can do the same for T1:
```

```
>
gb := Basis(eqs, plex(Y, T2, T1, X, D1, D2, kn1, kn2, kf1, kf2)):

factor(gb[1]);
```

$$-(-kn1\,X\,T1 + D1\,kf1)\,(D1 - D2) \tag{5}$$

```
> gb := Basis(eqs_ext, plex(Y, T2, D2, T1, X, D1, kn1, kn2, kf1, kf2)):
factor(gb[1]);
```

$$-(D1 + X)\,(-kn1\,X\,T1 + D1\,kf1) \tag{6}$$

```
>
```