In [1]:
```
using DifferentialEquations, Plots, Polynomials
```

The ODE system

We will work with the following ODE system and analyze its identifiability from th state data for the variables X, T_1, T_2. The values of interest are the values of pa k_{D_1} and k_{D_2}.

In [2]:
```
function ss(du,u,p,t)
    X, T₁, T₂, D₁, D₂, Y = u
    kn₁, kf₁, kn₂, kf₂ = p
    du[1]= -kn₁*T₁*X -kn₂*T₂*X +kf₁*D₁ +kf₂*D₂
    du[2]= -kn₁*T₁*X +kf₁*D₁ -kn₁*T₁*D₂ +kf₁*Y
    du[3]= -kn₂*T₂*X +kf₂*D₂ -kn₂*T₂*D₁ +kf₂*Y
    du[4]= +kn₁*T₁*X -kf₁*D₁ -kn₂*T₂*D₁ +kf₂*Y
    du[5]= +kn₂*T₂*X -kf₂*D₂ -kn₁*T₁*D₂ +kf₁*Y
    du[6]= +kn₁*T₁*D₂ +kn₂*T₂*D₁ -(kf₁+kf₂)*Y
end
p = [1.72,19.66,1.57,0.74]
kD = [p[2] / p[1], p[4] / p[3]] # <----- WANTED
```

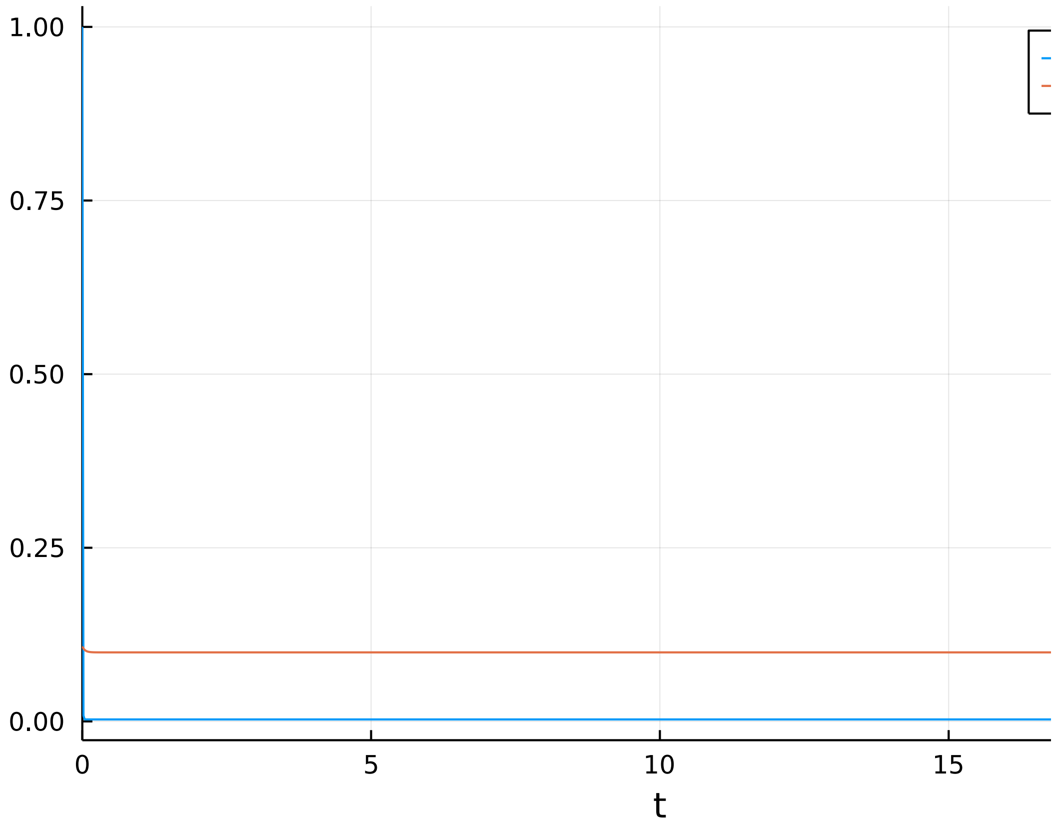
Out[2]:
```
2-element Vector{Float64}:
 11.430232558139535
  0.47133757961783435
```

Identifiability from two experiments

We start with computing the steady states by taking a sufficiently long timespan

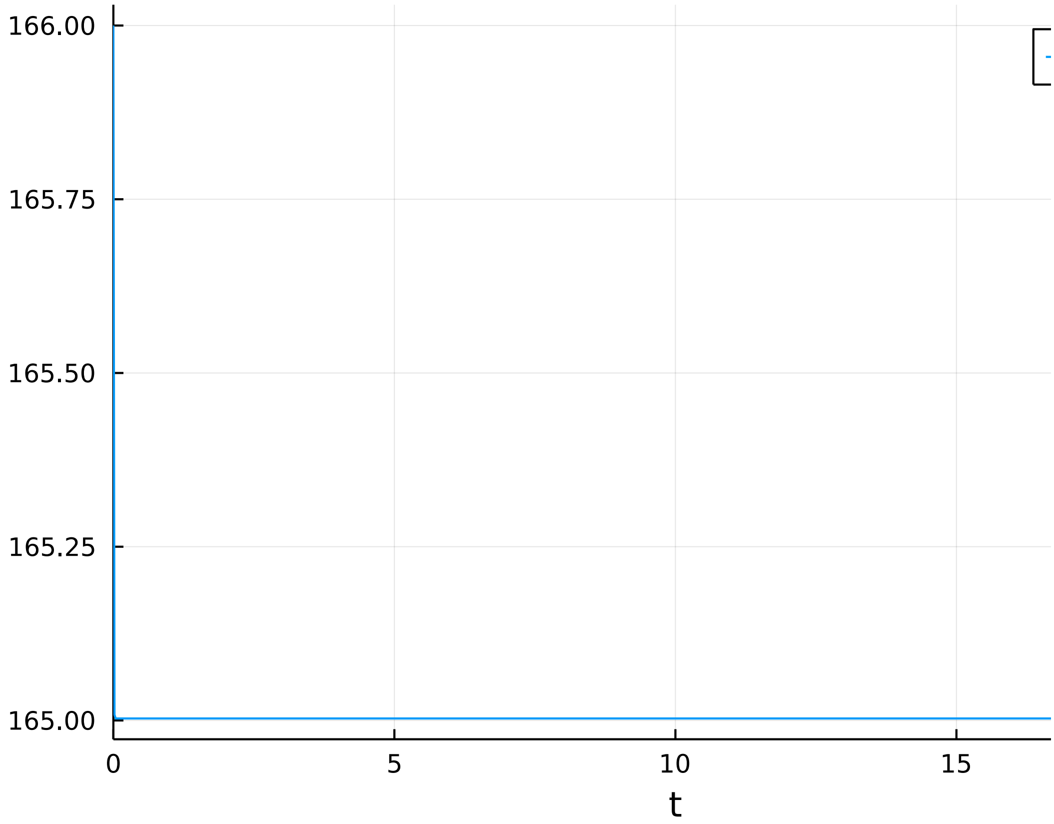
In [3]:
```
timespan = (0.0, 20.0)
initconc = [1e0, 1.08e-1, 166, 0.0, 0.0, 0.0]
dproblem = ODEProblem(ss, initconc, timespan, p)
sol = solve(dproblem)
plot(sol, vars=(0, 1), label="X")
plot!(sol, vars=(0, 2), label="T_1")
```

Out[3]:



In [4]:
```
plot(sol, vars=(0, 3), label="T_2")
```

Out[4]:



In [5]:
```
# steady state for X, T1, T2
sol[1, end], sol[2, end], sol[3, end]
```

Out[5]:
```
(0.002823853430159511, 0.09938042011368256, 165.00284840548
```

Now we perform experiment with perturbed X. For each of these new experime the formulas from the paper to compute k_{D_1} and k_{D_2} from the original steady s new one.

In [7]:
```
"""
    Function for finding the values of kD's using the formula
    Takes as input pairs of value of X, T1, and T2 (from the
"""
function find_sol_kDs(X, T1, T2)
    # A1 * kD1^2 + A2 * kD1 + A3 = 0
    A3 = (T1[1]*X[1]-T1[2]*X[2])*(T1[1]*T2[1]*X[1]-T1[2]*T2[2
    A2 = (T1[1]^2*T2[1]*X[1]-T1[1]*T1[2]*T2[1]*X[1]-T1[1]*T1[
    A1 = (T2[1]*X[1]-T2[2]*X[2])*(T1[1]-T1[2])

    # B1 * kD2 + B2 * kD1 + B3 = 0
    B1 = (T2[1]-T2[2])*(T1[1]*X[1]-T1[2]*X[2])
    B2 = -(T2[1]*X[1]-T2[2]*X[2])*(T1[1]-T1[2])
    B3 = -(T1[1]-T1[2]-T2[1]+T2[2])*(T1[1]*T2[1]*X[1]-T1[2]*T
    p = Polynomial([A3, A2, A1])
    println("Roots for kD1: $(roots(p))")
    for r in roots(p)
        kD2 = (-B3 - B2 * r) / B1
        println("Solution: kD1 = $r and kD2 = $kD2")
    end
end

println("True values kD1 = $(kD[1]) and kD2 = $(kD[2])")

for delta_x in [0.1, -0.1]
    initconc2 = [1e0 + delta_x,1.08e-1,166,0.0,0.0,0.0]
    dproblem2 = ODEProblem(ss, initconc2, timespan, p)
    sol2 = solve(dproblem2)
    X = [sol[1, end], sol2[1, end]]
    T1 = [sol[2, end], sol2[2, end]]
    T2 = [sol[3, end], sol2[3, end]]
    println("==================")
    println("For second experiment with perturbing X by $delt
    find_sol_kDs(X, T1, T2)
end
```

```
True values kD1 = 11.430232558139535 and kD2 = 0.47133757961
==================
For second experiment with perturbing X by 0.1
Roots for kD1: [-0.0002602801047430816, 11.430232555196689]
Solution: kD1 = -0.0002602801047430816 and kD2 = -162.5294
6
Solution: kD1 = 11.430232555196689 and kD2 = 0.4713375792512
==================
For second experiment with perturbing X by -0.1
Roots for kD1: [-0.0002641253990880462, 11.430232555652918]
Solution: kD1 = -0.0002641253990880462 and kD2 = -162.7221
7
Solution: kD1 = 11.430232555652918 and kD2 = 0.4713375787464
```

We see that the first solutions coincide with high precision (8 digits) and yield the
parameter values.

For these numerical values, the second solution can be discarded since it is neg
However, if it was positive, the second experiment can be used to discard the in
value thus three experiments would suffice.

Is one experiment sufficient?

In order to show that a single experiment was not sufficient, we will now cook up
initial conditions and parameter values yielding the same steady state data for X
T_2.

In [8]:
```
c = 1.1
D1_ = sol[4, end] * c
D2_ = sol[5, end] * c
Y_ = sol[6, end] * c^2
kD_ = kD ./ c
```

Out[8]:
```
2-element Vector{Float64}:
 10.391120507399576
  0.4284887087434857
```

In [9]:
```
initconc_ = [
    sol[1, end] + Y_ + D1_ + D2_,
    sol[2, end] + Y_ + D1_,
    sol[3, end] + Y_ + D2_,
    0.0, 0.0, 0.0
]
p_ = [1., kD_[1], 1., kD_[2]]
dproblem_ = ODEProblem(ss, initconc_, timespan, p_)
sol_ = solve(dproblem_)
sol_[1, end], sol_[2, end], sol_[3, end]
```

Out[9]:
```
(0.002823853430151104З, 0.09938042011043911, 165.0028484054
```

Here are the original steady state for comparison. We observe that they coincide
order

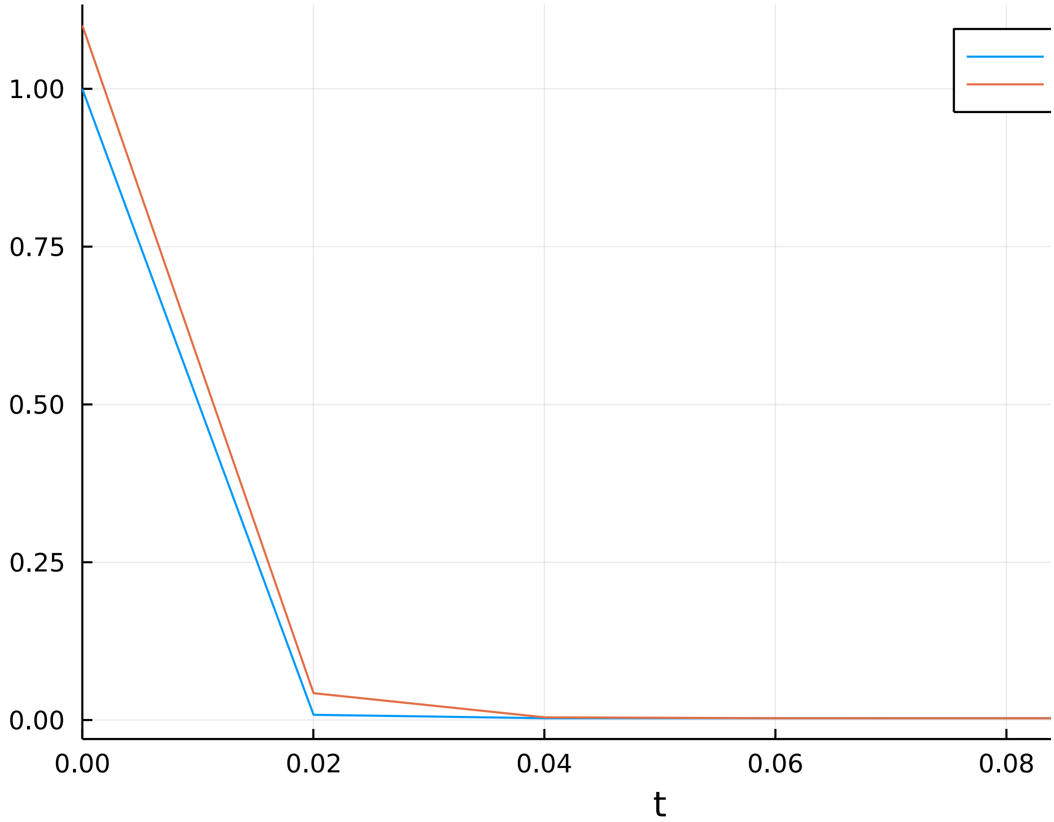
In [10]:
```
sol[1, end], sol[2, end], sol[3, end]
```

Out[10]:
```
(0.002823853430159511, 0.09938042011368256, 165.0028484054
```

However, as the following plots show, the overall dynamics was different

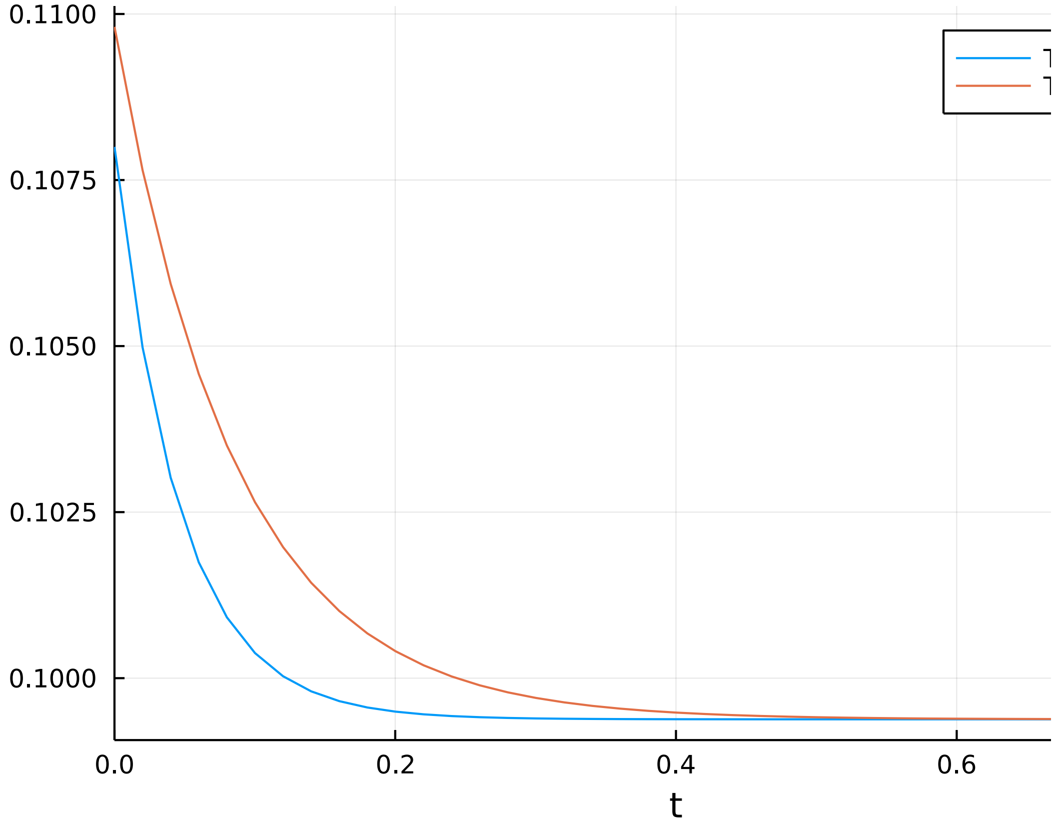
In [11]:
```
plot(sol, vars=(0, 1), label="X original", xlim=(0., 0.1))
plot!(sol_, vars=(0, 1), label="X new", xlim=(0., 0.1))
```

Out[11]:



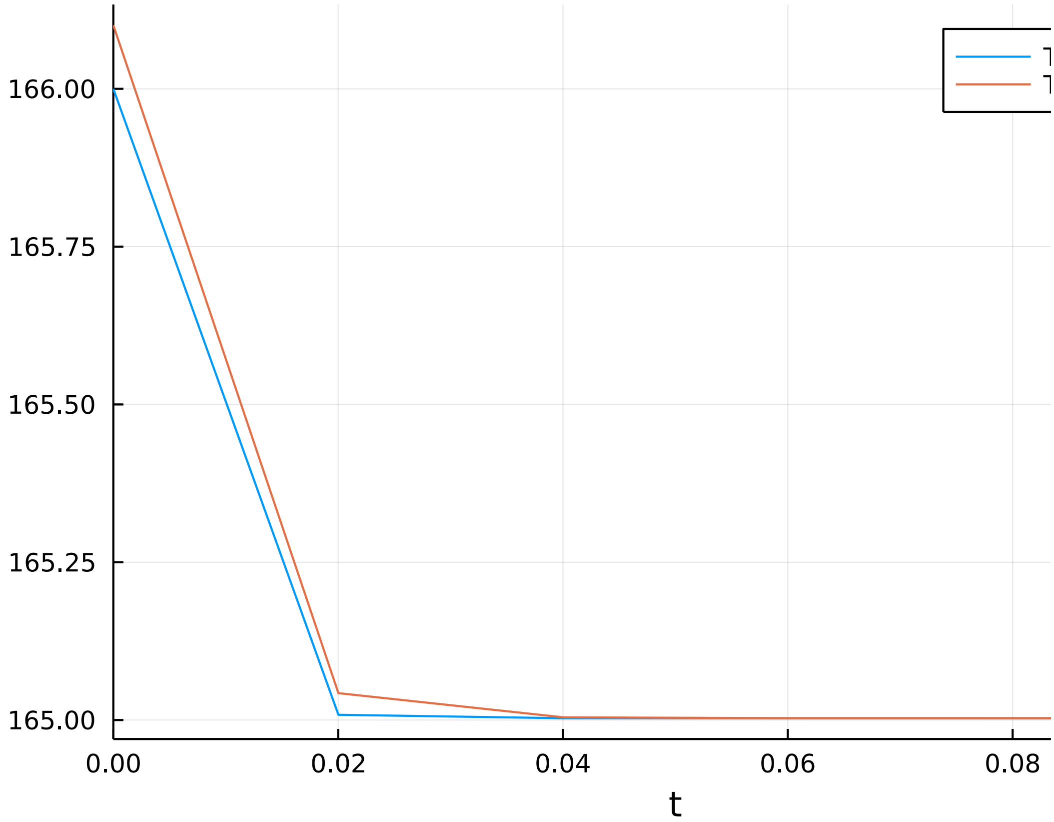
In [12]:
```
plot(sol, vars=(0, 2), label="T1 original", xlim=(0., 0.8))
plot!(sol_, vars=(0, 2), label="T1 new", xlim=(0., 0.8))
```

Out[12]:



In [13]:
```
plot(sol, vars=(0, 3), label="T2 original", xlim=(0., 0.1))
plot!(sol_, vars=(0, 3), label="T2 new", xlim=(0., 0.1))
```

Out[13]:



In [ ]: