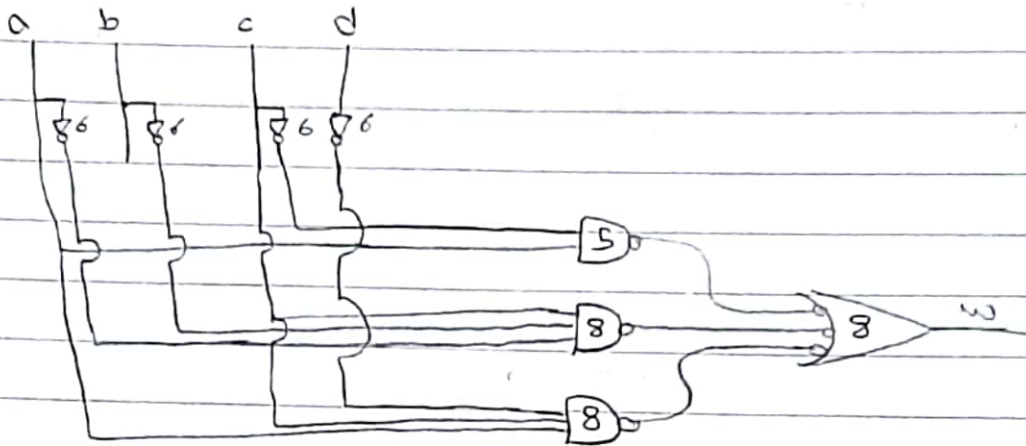1) $f(a,b,c,d) = \sum_m (2,3,4,6,8,9,12,13)$

$f(a,b,c,d) = a\bar{c} + \bar{a}bc + \bar{a}b\bar{d}$

ab cd	00	01	11	10
00		1	1	1
01			1	1
11	1			
10	1	1		

b) worst_case $d: 0 \rightarrow 1 \rightarrow 6 + 8 + 8 = 22ns$

c) assign #22 W = (a & ~c) | (~a & ~b & c) | (~a & b & ~d)

d) C: $1 \rightarrow 0: 6+5+8 = 19ns$
a: $0 \rightarrow 1: 8+5 = 13ns$
b: $0 \rightarrow 1: 8+8 = 16ns$

2) $f(a,b,c,d)$

cd\ab	00	01	11	10
00	1 (A)	1	1	
01			1	1
11	1			
10	1 (B)			

$f(a,b,c,d) = a\bar{c} + \bar{a}b\bar{d} + \bar{a}bc$

(A)(B) → Potential Hazard
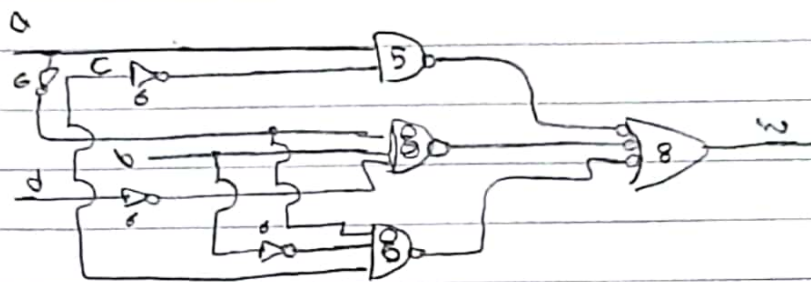
A: $1100 \leftrightarrow 0100$

logical Hazard → short path: 13 ns / long path: 22 ns } → duration: 9 ns / → time: After 13 ns; $a: 1 \to 0$ } → 9 > 8 → Electrical Hazard

B: $0110 \leftrightarrow 0010$

logical Hazard → short path: 16 ns / long path: 22 ns } → duration: 6 ns / → time: After 16 ns; $b: 1 \to 0$ } → 6 < 8 → No Electrical Hazard

D:

با اضافه شدن دو نات به لبه فرودی دلی آن بیشتر از 8 و مقداری نزدیک به 16 یا بیشتر انتخاب می کند
که در آن زمان هیچ Electrical Hazard نخواهیم داشت.

4) $f(a,b,c,d) = \sum_m (2,3,4,6,8,9,11,12,13) , d(7,15)$

$$f(a,b,c,d) = a\bar{c} + cd + \bar{a}c + \begin{cases} \bar{a}b\bar{d} \\ b\bar{c}\bar{d} \end{cases}$$

$$= a\bar{c} + ad + \bar{a}c + \begin{cases} b\bar{c}d \\ \bar{a}b\bar{d} \end{cases}$$

5) $f(a,b,c,d) = \sum_m (2,3,4,6,8,9,11,12,13), \; d(7,15)$

cd \ ab	00	01	11	10
00	0	1	1	1
01	0	0	1	1
11	1	-	-	1
10	1	1	0	0

$f(a,b,c,d) = (a+b+c) \cdot (a+c+d) \cdot (\bar{a}+\bar{c}+d)$

$= (a+b+c) \cdot (a+\bar{b}+\bar{d}) + (\bar{a}+\bar{c}+d)$

6] $f(a,b,c,d) = \sum_{m}(2,3,4,6,8,9,11,12,13), d(7,15)$

a b c d	w
0 0 0 0 0	
1 0 0 0 1	
2 0 0 1 0	1
3 0 0 1 1	1
4 0 1 0 0	1
5 0 1 0 1	
6 0 1 1 0	1
7 0 1 1 1	1 (d)
8 1 0 0 0	1
9 1 0 0 1	1
10 1 0 1 0	
11 1 0 1 1	1
12 1 1 0 0	1
13 1 1 0 1	1
14 1 1 1 0	
15 1 1 1 1	1 (d)

0-cubes	1-cubes	2-cubes
0010 ✓	001x ✓	0x1x
0100 ✓	(01x0)	1x0x
1000 ✓	(x100)	
0011 ✓	100x ✓	x̶x̶1̶1̶
0110 ✓	0x11 ✓	xx11
1001 ✓	x011 ✓	1̶x̶x̶1̶
1100 ✓	011x ✓	1xx1
0111 ✓	10x1 ✓	
1011 ✓	1x01 ✓	
1101 ✓	110x ✓	
1111 ✓	x111 ✓	
	1x11 ✓	
	11x1 ✓	

	0010	0011	0100	0110	1000	1001	1011	1100	1101
01x0			✓						
x100			✓					✓	
(0x1x)	✓			✓					
(1x0x)					✓	✓		✓	
(xx11)		✓					✓		
1xx1						✓	✓		✓
f	✓	✓	✓	✓	✓	✓	✓	✓	✓

$$f(a,b,c,d) = 0x1x + 1x0x + xx11 + \begin{cases} 01x0 \\ x100 \end{cases} \longrightarrow f(a,b,c,d) = \bar{a}c + a\bar{c} + cd + \begin{cases} \bar{a}b\bar{d} \\ b\bar{c}\bar{d} \end{cases}$$

7) $f_{(a,b,c,d)} = \prod_M (2,3,4,6,8,9,11,12,13), d(7,15)$

$f_{(a,b,c,d)} = (\bar{a}+c)\cdot(a+\bar{c})\cdot(\bar{c}+\bar{d})\cdot\begin{cases}(\bar{b}+c+d) \\ (a+\bar{b}+d)\end{cases}$

$= (\bar{a}+c)\cdot(a+\bar{c})\cdot(\bar{a}+\bar{d})\cdot\begin{cases}(\bar{b}+c+d) \\ (a+\bar{b}+d)\end{cases}$

8) $f(a,b,c,d) = \prod_m (2,3,4,6,8,9,11,12,13),\ d(7,15)$

cd \ ab	00	01	11	10
00	1			
01	1	1		
11		1	–	
10			1	1

$$f(a,b,c,d) = \bar{a}\,\bar{b}\,\bar{c} + a\,c\,\bar{d} + \begin{cases} \bar{a}\,\bar{c}\,d \\ \bar{a}\,b\,d \end{cases}$$