$$y = ax + b$$

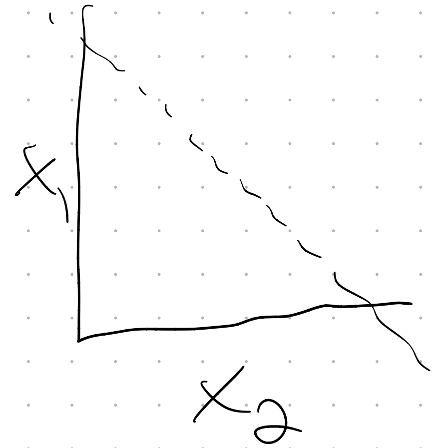
$$y - ax - b = 0$$

$$\boxed{y} + ax + b = 0$$

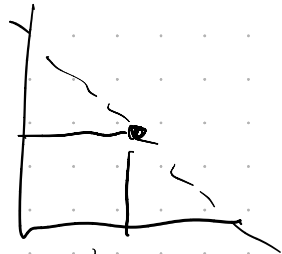
$$x_1 + a x_2 + b = 0$$

$$c x_1 + c a x_2 + c b = 0$$

$$c x_1 + a x_2 + b = 0$$



$w_2 x_2 + w_1 x_1 + b = 0$



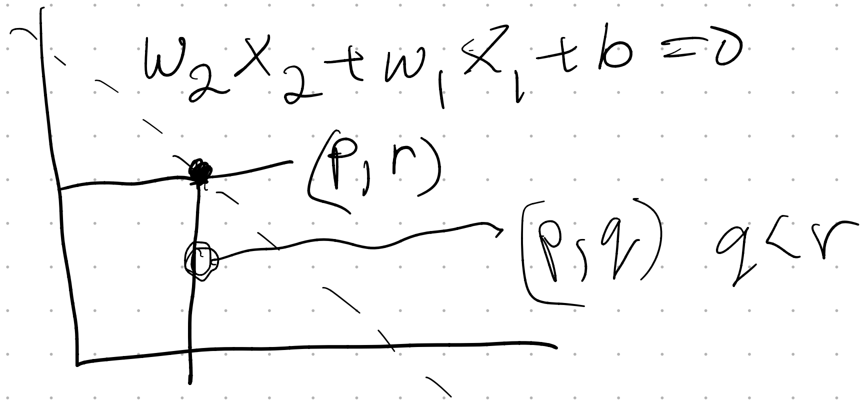
F1: all points x_1 x_2 ~~are~~ lying on decision boundary satisfy the equality

F2: Line creates two half planes

F3: all points (x_1, x_2) on one half of the plane will give an inequality of the same type

F4: Conversely for the other side the other inequality

Fact 3 proof



(p, r) makes the equality hold

$$w_2 X_2 + w_1 X_1 + b = 0$$

$$X_2 = \frac{-w_1}{w_2} x_1 - \frac{b}{w_2}$$

$$r = \frac{-w_1}{w_2} p - \frac{b}{w_2}$$

$$q < \frac{-w_1}{w_2} p - \frac{b}{w_2}$$

$w_2 q + w_1 p + b < 0$ if $w_2 > 0$

$w_2 q + w_1 p + b > 0$ $\quad w_2 < 0$

$X = [x_1 \ldots x_n]$ $w = [w_1 \ldots w_n]$

$h(x, w, b) = \text{sign}(w \cdot x + b)$

$+1 / -1$

depending on the side

learn w, b so that the assignment $+1/-1$ matches the data for mathmatical convenience

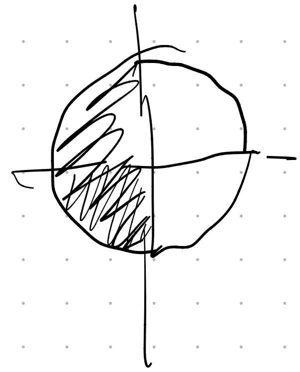
$$x \begin{bmatrix} 1 & x_1 & \cdots & x_n \end{bmatrix} \quad w = \begin{bmatrix} w_0 & w_1 & \cdots & w_n \end{bmatrix}$$

$$w_0 = b$$

now $h(x, w) = \text{sign}(w \cdot x)$

$$= \text{sign}(w^T x)$$

$$a \cdot b = \underbrace{\|a\| \, \|b\|}_{\text{always positive}} \cdot \cos(a, b)$$



negative

$\frac{1}{2}\pi \rightarrow \frac{3}{2}\pi$

negative for obtuse angles

$w \cdot x$ obtuse when $y = -1$

$w \cdot x$ acute when $y = +1$

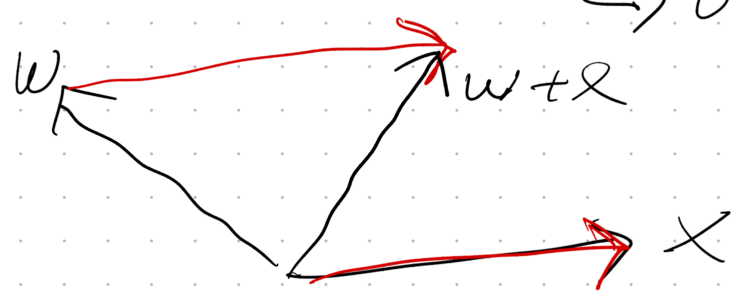
in general we want

$$\underline{\underline{w^T x \, y > 0}}$$

According to our update rule

$$\text{sign}(w^T x_k) \neq y_k$$

1) $y = +1$ but $\underbrace{w^T x < 0}$ → obtuse



$$w = w + yx$$
$$= w + x$$

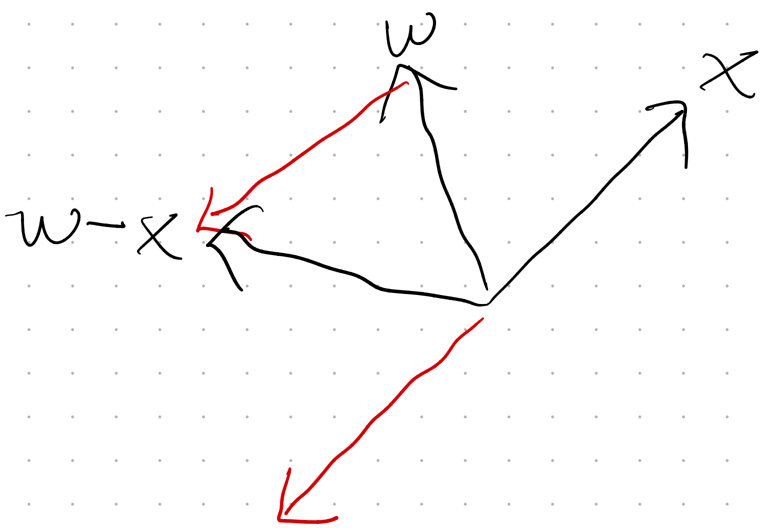
pulls w towards x making ang more acute

hopefully $wx > 0$ next time

2) $y = -1$ but $\underbrace{w^T x}_{\text{acute}} > 0$

$$w = w + \underline{y} x$$

$$= w - x$$



w^* final solution

$$w^{k+1} \cdot w^* = (w^k + \underbrace{y_j}_{1} x_j) w^*$$

$$\downarrow$$

$$f(w^{k-1})$$

$$\vdots$$

$$w^1 \longrightarrow 0$$