Logistic regression

Sept 27

$$P(y|X) = \begin{cases} f(x) & y=0 \\ 1-f(x) & y=1 \end{cases}$$

$$S(z) = \frac{1}{1+e^{-z}} \in (0,1)$$

$$z = w \cdot x$$

$$w = [w_0 \; w_1 \; \cdots \; w_m]$$

$$x = [1 \; x_1 \; \cdots \; x_m]$$

$S(w \cdot x)$ as $f(x)$

$$p(y=0 \mid x) = 1/1+e^{wx}$$

$$p(y=1 \mid x) = 1 - \frac{1}{1+e^{wx}}$$

$$= \frac{1+e^{wx}-1}{1+e^{wx}}$$

$$= \frac{e^{wx}}{1+e^{wx}}$$

$$D = [(x_1, y_1), (x_2, y_2) \ldots (x_n y_n)]$$

$$\prod_{i=1}^{n} p(y \mid X w) = \mathcal{L}(w)$$

$$w_{MCLE} = \underset{w}{\operatorname{argmax}} \prod_{i=1}^{n} P(y \mid x, w)$$

$$\log \mathcal{L}(w) = \sum_{i=1}^{n} \log P(y \mid x, w)$$

$$y=0 \quad \frac{1}{1+e^{wx}} \qquad y=1 \quad \frac{e^{wx}}{1+e^{wx}}$$

$$\sum_{i \mid y_i = 0} \quad + \quad \sum_{i \mid y_i = 1}$$

$$\log \mathcal{L}(w) = \sum_{i=1}^{n} y_i \ln P(y=1|x,w) +$$

$$\sum_{i=1}^{n} (1-y_i) \ln P(y=0|x,w)$$

$$= \sum_{i=1}^{n} y_i \left[\ln P(y=1|x,w) - \ln P(y=0|x,w) \right]$$

$$+ \sum_{i=1}^{n} \ln P(y=0|x,w)$$

$$= \sum_{i=1}^{n} y_i \ln \left[\frac{P(y=1|x,w)}{P(y=0|x,w)} \right] +$$

$$\frac{e^{wx}/1+e^{wx}}{1/1+e^{wx}} = \boxed{e^{wx}} \qquad \ln\left(1/1+e^{wx}\right)$$

$$\boxed{-\ln\left(1+e^{wx}\right)}$$

$$= \sum_{i=1}^{n} y_i \underbrace{\ln(e^{wx})} - \sum_{i=1}^{n} \ln(1+e^{wx})$$

$$= \sum_{i=1}^{n} y_i \, wx - \sum_{i=1}^{n} \ln(1+e^{wx})$$

$$\frac{d}{dw} \log \mathcal{L}(w) = \frac{d}{dw} \left[\sum_{i=1}^{n} y_i wx_i - \sum_{i=1}^{n} \ln(1+e^{wx}) \right]$$

$$= \sum_{i=1}^{n} y_i x_i - \sum_{i=1}^{n} \frac{x_i e^{wx_i}}{1+e^{wx_i}}$$

$$= \sum_{i=1}^{n} x_i \left(y_i - \underbrace{\frac{e^{wx_i}}{1+e^{wx_i}}}_{p(y=1 \mid x, w)} \right)$$

$$w^{k+1} = w^k + \kappa \left(\sum_{i=1}^{n} x_i \left(y_i - P(y_i = 1 \mid x_i, w^k) \right) \right)$$

Softmax

$e^{-wx} \rightarrow$ predict 1 if <1

$e^{wx} \rightarrow$ predict 1 if >1

$y = \{1, 2, 3\}$

$$\left.\begin{array}{l} e^{w_1 x} \\ e^{w_2 x} \\ e^{w_3 x} \end{array}\right\} \text{normalize}$$

$$P(y_i = 1 | w\, x) = \frac{e^{w_1 x}}{\sum_{j=}^{3} e^{w_j x}}$$