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Physics Procedia 24 (2012) 678 – 685

Physics Procedia

2012 International Conference on Applied Physics and Industrial Engineering

Denoising algorithm based on wavelet adaptive threshold

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Abstract

the main idea of denoising algorithm based on wavelet adaptive threshold is that speech signals should be packet transformed to get the wavelet coefficients used in optimal wavelet. Since the signal and the noise have different relevance, there will be different attenuations in wavelet decomposition process. Based on above characteristics, the appropriate threshold can be calculated by a new threshold function and the minimum mean square algorithm, even if the noise coefficients can be removed and the signal coefficients can be saved. Finally, the retained coefficients can be reconstructed to restore the original signal for the purpose of de-noising.

Keywords:denoising algorithm, wavelet, adaptive, threshold.

1.The basic principles of denoising based on wavelet threshold

The model of noisy signal can be expressed as the following [1]:

$$y(t) = f(t) + n(t) \tag{1}$$

In (1), *y(t), f(t) and n(t)* represent the original signal, noisy signal and noise signal respectively. The process is shown in Fig.1.

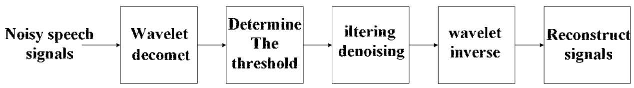


Fig.1 Schematic of denoising using wavelet threshold

doi:10.1016/j.phpro.2012.02.100

2.The main issues of denoising algorithm of wavelet threshold

There are three key issues in denoising using wavelet threshold: selecting wavelet bases, determining the decomposition level and selecting the appropriate threshold and threshold function [2]. This paper is mainly introducing the related methods.

There are two kinds of threshold function commonly used [3]:

2.1.Hard threshold

When the absolute value of wavelet coefficients is less than a given threshold, it is zero; when it is greater than the threshold value, then make it remain the same.

$$w_\lambda = \begin{cases} w & |w| \geq \lambda \\ 0 & |w| < \lambda \end{cases} \tag{2}$$

- *Soft threshold*

$$w_\lambda = \begin{cases} [sign(w)](|w| - \lambda) & |w| \geq \lambda \\ 0 & |w| < \lambda \end{cases} \tag{3}$$

In (2)and (3) w are the values of the wavelet coefficients, w_λ is the value of the wavelet coefficients imposed threshold, λ is a threshold value.

In order to meet the signal processing requirements of time-varying and non-stationary, it is hoped to find a new threshold function to make up for the lack of traditional threshold function [4].

- *New threshold function*

Therefore, a new threshold function is introduced as the followings using the characteristics of the exponential function [5]:

$$\lambda(x,t) = x - t + \frac{2t}{1 + e^{cx}} \tag{4}$$

In (4), c is an unknown constant, when x is equal to positive infinity, λ is approximately equal to x minus t. It shows the trend of (4) and the soft threshold function is coincided. Making x equals to $-x$ it will get the following relations from (4): $\lambda(-x,t) = -\lambda(x,t)$, it can be seen the new threshold function is an odd function.

3.Denoising Algorithm of Adaptive Wavelet Threshold

3.1.Unbiased SURE Estimate

Suppose $X + G(X)$ is the estimate of the mean of X, which $g: R^P \to R^P$ is the weakly differentiable function and satisfies $E\left\langle \sum_i |\nabla_i g_i(X)| \right\rangle < \infty$, for each $i \in \{1,2,...p\}$, the following equations are all tenable:

$$E\left\langle [X_i + g_i(X) - \xi]^2 \right\rangle = \sigma^2 + 2E\left\langle \nabla_i g(X) \right\rangle + E\left\langle \|g_i(X)\|^2 \right\rangle \tag{5}$$

For the whole sequence:

$$E\left\langle [X + g(X) - \xi]^2 \right\rangle = P\sigma^2 + 2E\left\langle \nabla \bullet g(X) \right\rangle + E\left\langle \|g(X)\|^2 \right\rangle \tag{5}$$

In (5) and (6), $P\sigma^2 + 2\nabla \bullet g(X) + \|g(X)\|$ is the unbiased estimate of the mean square error between the mean of *X*, ξ and its estimate, $X + g(X)$, it is known as SURE unbiased.

3.2.The steepest descent algorithm

The basic idea of steepest descent algorithm is to adjust the weight vector w continuous along the direction of steepest descent, that is:

$$w(n+1) = w(n) - \frac{1}{2}\mu g(n) \tag{7}$$

In (7), n means iterative process, $g = \nabla J(w) = \frac{\partial J(w)}{\partial w}$ is the gradient vector, μ represents the step and usually is a normal number.

3.3.LMS Adaptive Algorithm

The algorithm LMS (Least mean square, LMS) is a linear adaptive filter algorithm, which makes the error squared e^2 of time i as the estimate of instantaneous mean square error, ∇ is the gradient of e^2 to w, so the LMS algorithm updated by the weight vector is expressed as the following:

$$w(n+1) = w(n) - \frac{1}{2}\mu\, \hat{g}(n) = w(n) + \mu e(n)u(n) \tag{8}$$

3.4.Denoising Algorithm Adaptive Wavelet Threshold

According to the (1), the estimated value $\hat{f}(Y)$ of signal

$$\Delta T_2 = \frac{8\hat{d}_{m,n} e^{2\hat{d}_{m,n}/t_m(k)}}{t^2{}_m(k)\left(1 + e^{2\hat{d}_{m,n}/t_m(k)}\right)^2} - \frac{16\hat{d}_{m,n} e^{4\hat{d}_{m,n}/t_m(k)}}{t^2{}_m(k)\left(1 + e^{2\hat{d}_{m,n}/t_m(k)}\right)^3}$$

is desired, since $\hat{f}(Y)$ is noisy, in order to reduce the noise amount, it usually makes $\hat{f}(Y)$ and the mean square error $R_m = N\sigma^2{}_m + 2E\left\langle \nabla \cdot g(\hat{d}_{m,n})\right\rangle + E\left\langle \left\|g(\hat{d}_{m,n})\right\|^2\right\rangle$ of *f* minimum and replaces expectation with the mean value, it will get the following equation:

$$R(f, \hat{f}) = \frac{1}{N}\left\|\hat{f} - f\right\| = \frac{1}{N}\sum_{i=0}^{N-1}(\hat{f} - f)^2 \tag{9}$$

Together with (9), making the threshold $t > 0$, then the estimate $\hat{f}(Y)$ can be expressed as following:

$$\hat{f}(Y) = \sum_n \hat{c}_{m_0,n}\phi_{m_0,n}(t) + \sum_{m=1}^{m_0}\sum_n d^*{}_{m,n}\psi_{m,n}(t) \tag{10}$$

In (15), $\hat{c}_{m_0,n}$ and $\hat{d}_{m,n}$ are scale coefficients and wavelet coefficients respectively, $d^*{}_{m,n}$ is a required estimate of wavelet coefficients of new threshold function.

If using the orthogonal wavelet bases and the decomposition scale is large enough, the mean square error can be written as the following:

$$R(\hat{f}, f) = \frac{1}{N} \sum_{m=1}^{m_0} \sum_{n} (\lambda(\hat{d}_{m_0,n}, t_m) - d_{m,n})^2 \tag{11}$$

By the above equation, it must require the mean square errors of each scale of wavelet coefficients to take the minimum to make the total mean square error minimum. In order to compute conveniently, suppose R_m is the mean square error of the wavelet coefficients in scale of m, thus:

$$R_m = \sum_{n} (\lambda(\hat{d}_{m,n}, t_m) - d_{m,n})^2 \tag{12}$$

3.5.Threshold Selection

The threshold t_m is selected by LMS algorithm to make R_m the minimum value. The basic idea of LMS algorithm is to ensure that the threshold $t_m(k+1)$ of the next time is equal to the current threshold $t_m(k)$ adding the gradient value $\Delta t_m(k)$ with a negative proportional to R_m, that is:

$t_m(k+1) = t_m(k) - \rho \cdot \Delta t_m(k)$, $\Delta t_m(k) = \dfrac{\partial R_m(k)}{\partial t_m(k)}$ Where ρ is the step, the key of this algorithm is to find $\Delta t_m(k)$ [6].

Suppose $g(\hat{d}_{m,n})$ is the function of estimate of wavelet coefficients $\hat{d}_{m,n}$ in the scale of m, that is:

$$g(\hat{d}_{m,n}) = \lambda(\hat{d}_{m,n}, t) - \hat{d}_{m,n} \tag{13}$$

Where λ is the new threshold function, put it into the (12), it can get:

$$R_m = E\left\langle \left\| \hat{d}_{m,n} + g(\hat{d}_{m,n}) - d_{m,n} \right\|^2 \right\rangle \tag{14}$$

It is known from (13), $g(\hat{d}_{m,n})$ meets the conditions for unbiased estimation of SURE, that is:

$$R_m = N\sigma^2{}_m + 2E\left\langle \nabla \cdot g(\hat{d}_{m,n}) \right\rangle + E\left\langle \left\| g(\hat{d}_{m,n}) \right\|^2 \right\rangle \tag{15}$$

It can obtain the following equation by simplifying (15): $R_m = N\sigma_m^2 + \left\| g(\hat{d}_{m,n}) \right\|^2 + 2\nabla \cdot g(\hat{d}_{m,n})$

The gradient of mean square error is:

$$\Delta t_m(k) = 2\sum_{n} \left(g_n \cdot \frac{\partial g_n}{\partial t_m(k)} \right) + 2\sum_{n} \frac{\partial^2 g_n}{\partial \hat{d}_{m,n} \partial t_m(k)} \tag{16}$$

Equation (13) was substituted into the following equation:

$$\Delta t_m(k) = 2\sum_{n} \left(g_n \cdot \frac{\partial(\lambda(\hat{d}_{m,n}, t_m(k)))}{\partial t_m(k)} \right) + 2\sum_{n} \frac{\partial^2 (\lambda(\hat{d}_{m,n}, t_m(k)))}{\partial \hat{d}_{m,n} \partial t_m(k)} \tag{17}$$

Suppose $\Delta T_1 = \dfrac{\partial(\lambda(\hat{d}_{m,n}, t_m(k)))}{\partial t_m(k)}$ and $\Delta T_2 = \dfrac{\partial^2 (\lambda(\hat{d}_{m,n}, t_m(k)))}{\partial \hat{d}_{m,n} \partial t_m(k)}$, Equation (17) can be changed to(18):

$$\Delta t_m(k) = 2\sum_n g_n \Delta T_1 + 2\sum_n \Delta T_2 \tag{18}$$

$\lambda(x,t) = x - t + \dfrac{2t}{1+e^{2x/t}}$ can be substituted into above equation, it will get the following equation:

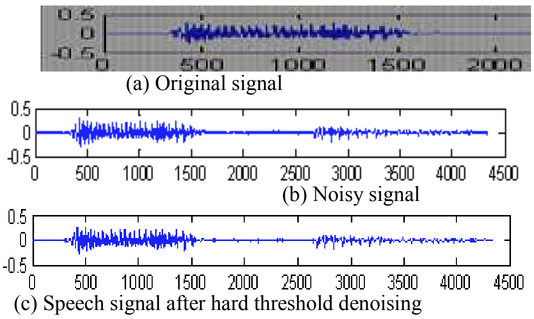
$$\Delta T_1 = -1 + \frac{2}{1+e^{2d_{m,n}/t_m(k)}} + \frac{4\hat{d}_{m,n} e^{2\hat{d}_{m,n}/t_m(k)}}{t_m(k)\left(1+e^{2\hat{d}_{m,n}/t_m(k)}\right)^2} \tag{19}$$

$$\Delta T_2 = \frac{8\hat{d}_{m,n} e^{2\hat{d}_{m,n}/t_m(k)}}{t^2{}_m(k)\left(1+e^{2\hat{d}_{m,n}/t_m(k)}\right)^2} - \frac{16\hat{d}_{m,n} e^{4\hat{d}_{m,n}/t_m(k)}}{t^2{}_m(k)\left(1+e^{2\hat{d}_{m,n}/t_m(k)}\right)^3} \tag{20}$$

Making (13), (19) and (20) substitute into (18) respectively, Δt_m can be obtained. Then substitute Δt_m into the equation
$t_m(k+1) = t_m(k) - \mu \cdot \Delta t_m(k)$, the optimal threshold value t_m of each scale can be obtained, and then the best estimate $\hat{d}^*_{m,n} = \lambda(\hat{d}_{m,n}, t)$ of wavelet coefficients with each scale can be obtained, finally the estimate $\hat{f}$ of signal f can be reconstructed.

4.Simulation

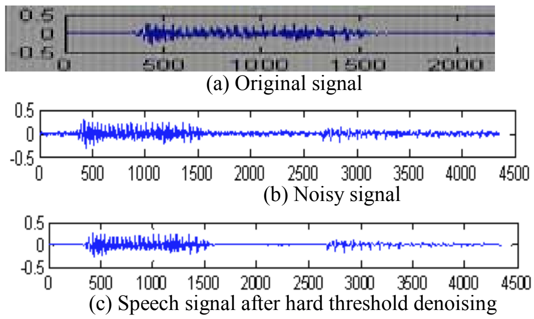
To illustrate the validity of the new threshold function and adaptive wavelet threshold algorithm, the following denoising experiments are carried out: in a quiet environment, gather a voice "works One" as the original speech signal with the sampling frequency of 8KHz, 8bit quantization, Mono and the noise is additive white Gaussian noise, adopt coif5 wavelet, the decomposition of 4 layers, use a hard threshold function, a soft threshold function and the new threshold function respectively to carry on the adaptive wavelet threshold denoising experiment, the results are shown in Fig.2~4, where the abscissa is time, vertical axis is amplitude.



(a) Original signal

(b) Noisy signal

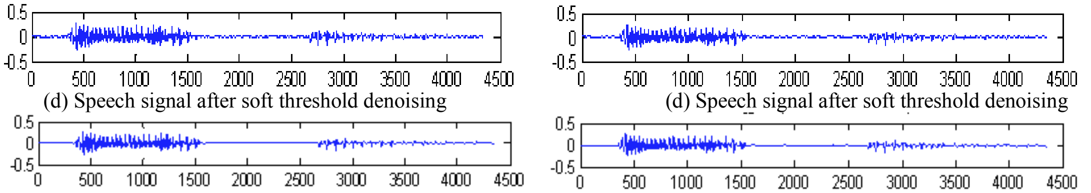
(c) Speech signal after hard threshold denoising



(a) Original signal

(b) Noisy signal

(c) Speech signal after hard threshold denoising

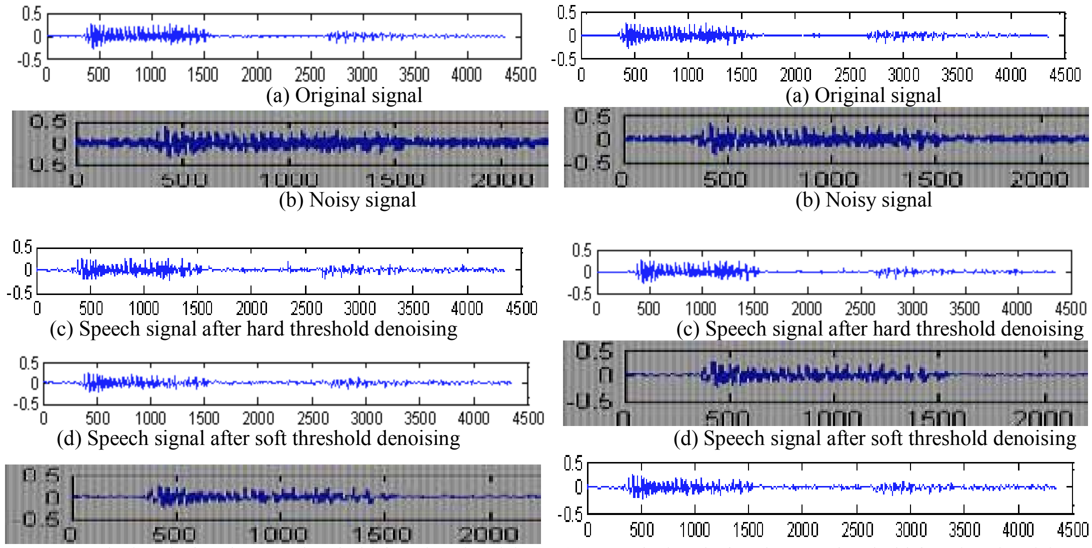


(d) Speech signal after soft threshold denoising (d) Speech signal after soft threshold denoising

(e) speech signal after the new threshold function denoising (e) speech signal after the new threshold function denoising

Figure 1. Denoising simulation diagram when the input SNR is 9.9172dB

Figure 2. Denoising simulation diagram when the input SNR is 4.5091dB



(a) Original signal (a) Original signal

(b) Noisy signal (b) Noisy signal

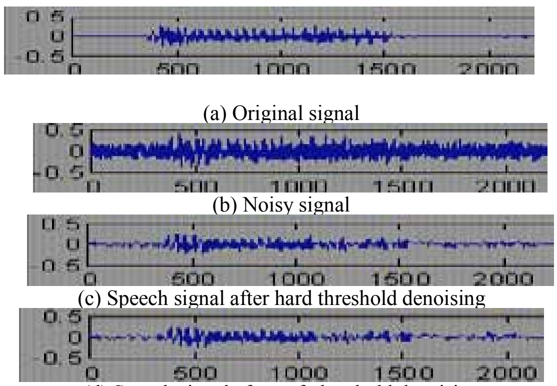
(c) Speech signal after hard threshold denoising (c) Speech signal after hard threshold denoising

(d) Speech signal after soft threshold denoising (d) Speech signal after soft threshold denoising

(e) Speech signal after the new threshold function denoising (e) Speech signal after the new threshold function denoising

Figure 3. Denoising simulation when the input SNR is 0.4604dB

Figure 4. Denoising simulation when the input SNR is -5.0356dB

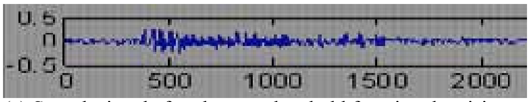


(a) Original signal

(b) Noisy signal

(c) Speech signal after hard threshold denoising

(d) Speech signal after soft threshold denoising



(e) Speech signal after the new threshold function denoising

Figure 5. Denoising simulation when the input SNR is -9.8558dB

The SNR results with three denoising function of wavelet threshold are listed in table 1.

TABLE I. SNR COMPARISON WITH THREE DENOISING FUNCTION OF THRESHOLD

Function type	Input SNR	*Output SNR*
Hard threshold function	-9.8558	*-6.8660*
	-5.0356	*-2.0175*
	0.4604	*3.0235*
	4.5091	*8.8902*
	9.9172	*12.3999*
Soft threshold function	-9.8558	*7.1207*
	-5.0356	*10.5378*
	0.4604	*8.5405*
	4.5091	*9.1641*
	9.9172	*10.4412*
New function	-9.8558	*0.2350*
	-5.0356	*5.0029*
	0.4604	*9.4024*
	4.5091	*11.6146*
	9.9172	*13.1634*

5.Conclusion

It is known from Fig.2 to 6, there are more discontinuous shocking points while using hard threshold function to denoise; using soft threshold function can remove more noise, but it may lose more useful information, the denoising effect is general; the new threshold function is a compromise function between a hard threshold and a soft threshold function, using the new threshold function it will inhibit the shock points to ensure the smooth transition of wavelet coefficients, which is better concluded from the experiment data, and prove the validity of this improved speech enhancement algorithm.

From Table 1, it can be seen that the output SNR after denoising experiments all have been improved with three threshold functions; comparing with the results of the hard threshold and soft threshold function, the denoising effect is more thorough, SNR is increased greatly and the characteristics of speech signal are retained by using the new threshold function with the high input SNR. Under the condition of low input SNR, the denoising effect of the new threshold function is not only better than soft threshold function, but also it is better than the hard threshold function. So denoising effects using wavelet transform is better than others but this kind of algorithms should be further improved with low SNR.

Acknowledgment

This work is supported by science and technology planning of Gansu Province（No1010RJZA071）.

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