PDF 13 to 16 Integral Formulas

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January 3, 2023

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Chapter 1

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1.1 Line Integral

$$\int\limits_C f \,\mathrm{d}s = \int_a^b f\big(\vec{r}(t)\big)\,|r'(t)|\,\mathrm{d}t \tag{1.1}$$

1.2 Physical Aspect

$$m = \int\limits_C \delta(x,y)\,\mathrm{d}s \tag{1.2}$$

$$(\bar{x},\bar{y}) = \begin{cases} \bar{x} = \frac{1}{m}\int\limits_C x\,\delta(x,y)\ \mathrm{d}s \\ \bar{y} = \frac{1}{m}\int\limits_C y\,\delta(x,y)\ \mathrm{d}s \end{cases} \tag{1.3}$$

1.3 Vector Field

$$\int\limits_C F\cdot \mathrm{d}r = \int_a^b F\big(\vec{r}(t)\big)\cdot r'(t)\,\mathrm{d}t \tag{1.4}$$

$$\int\limits_C F\cdot \mathrm{d}r = \int\limits_C \mathtt{p}\,\mathrm{d}x + \mathtt{q}\,\mathrm{d}y + \mathtt{r}\,\mathrm{d}z \tag{1.5}$$

Chapter 2

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2.1 Gradient Vector

$$F = \nabla f \Rightarrow \begin{cases} p = \frac{\partial f}{\partial x} \\ q = \frac{\partial f}{\partial y} \\ r = \frac{\partial f}{\partial z} \end{cases} \tag{2.1}$$

$$\int\limits_C F \cdot \mathrm{d}r = \int_a^b \nabla f \cdot \mathrm{d}r = f\big(\vec{r}(b)\big) - f\big(\vec{r}(a)\big) \tag{2.2}$$

$$\frac{\partial p}{\partial y} = \frac{\partial q}{\partial x} \qquad \frac{\partial p}{\partial z} = \frac{\partial r}{\partial x} \qquad \frac{\partial q}{\partial z} = \frac{\partial r}{\partial y} \tag{2.3}$$

2.2 Green Theorem

$$\int\limits_C P\,\mathrm{d}x + Q\,\mathrm{d}y = \iint\limits_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) \mathrm{d}A \tag{2.4}$$

2.3 Area Calculations

$$A = \int\limits_C x\,\mathrm{d}y = -\int\limits_C y\,\mathrm{d}x = \frac{1}{2}\int\limits_C x\,\mathrm{d}y - y\,\mathrm{d}x \tag{2.5}$$

2.4 Green Theorem 2

$$\int_C p\,\mathrm{d}x + q\,\mathrm{d}y = \int_{C_1} p\,\mathrm{d}x + q\,\mathrm{d}y + \int_{C_2} p\,\mathrm{d}x + q\,\mathrm{d}y = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) \mathrm{d}A \quad (2.6)$$

Chapter 3

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3.1 Smooth Parametrized Curve

$$\vec{r}(u,v) = \big(x(u,v),\, y(u,v),\, z(u,v)\big), \quad (u,v) \in R \tag{3.1}$$

$$\vec{r}_u = (\frac{\partial x}{\partial u}, \frac{\partial y}{\partial u}, \frac{\partial z}{\partial u}) \qquad \vec{r}_v = (\frac{\partial x}{\partial v}, \frac{\partial y}{\partial v}, \frac{\partial z}{\partial v}) \tag{3.2}$$

3.1.1 Sample

Let $z = f(x,y)$, the smooth parametrized curve will be:

$$\vec{r}(u,v) = (u, v, f(u,v)) \tag{3.3}$$

Partial Derivatives will be:

$$\begin{aligned} \vec{r}_u &= (\frac{\partial u}{\partial u}, \frac{\partial v}{\partial u}, \frac{\partial f(u,v)}{\partial u}) \\ &= (1, 0, \frac{\partial f}{\partial u}) = \vec{i} + \frac{\partial f}{\partial u}\vec{k} \end{aligned} \tag{3.4}$$

$$\begin{aligned} \vec{r}_v &= (\frac{\partial u}{\partial v}, \frac{\partial v}{\partial v}, \frac{\partial f(u,v)}{\partial v}) \\ &= (0, 1, \frac{\partial f}{\partial v}) = \vec{j} + \frac{\partial f}{\partial v}\vec{k} \end{aligned} \tag{3.5}$$

Cross Product of the partial derivatives:

$$
\begin{aligned}
\vec{r}_u \times \vec{r}_v &= (\frac{-\partial f}{\partial u}, \frac{-\partial f}{\partial v}, 1) \\
&= \frac{-\partial f}{\partial u}\vec{i} + (\frac{-\partial f}{\partial v}\vec{j}) + \vec{k} \neq 0
\end{aligned}
\tag{3.6}
$$

3.2 Area

$$
A(\mathtt{S}) = \iint\limits_R |\vec{r}_u \times \vec{r}_v| \; \mathrm{d}A \tag{3.7}
$$

3.2.1 Theorem

Let $z = f(x, y)$, then:

$$
A(\mathtt{S}) = \iint\limits_R \sqrt{(\frac{\partial f}{\partial x})^2 + (\frac{\partial f}{\partial y})^2 + 1} \; \mathrm{d}A \tag{3.8}
$$

Chapter 4

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