PDF 13 to 16 Integral Formulas

Mahdi Haghverdi

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Chapter 1

PDF 13

1.1 Line Integral

$$\int\limits_C f \, \mathrm{d}s = \int_a^b f\big(\vec{r}(t)\big) \, |r'(t)| \, \mathrm{d}t \tag{1.1}$$

1.2 Physical Aspect

$$m = \int\limits_C \delta(x, y) \, \mathrm{d}s \tag{1.2}$$

$$(\bar{x}, \bar{y}) = \begin{cases} \bar{x} = \frac{1}{m} \int\limits_C x \, \delta(x, y) \ \mathrm{d}s \\ \bar{y} = \frac{1}{m} \int\limits_C y \, \delta(x, y) \ \mathrm{d}s \end{cases} \tag{1.3}$$

1.3 Vector Field

$$\int\limits_C F \cdot \mathrm{d}r = \int_a^b F\big(\vec{r}(t)\big) \cdot r'(t) \, \mathrm{d}t \tag{1.4}$$

$$\int\limits_C F \cdot \mathrm{d}r = \int\limits_C \mathtt{p} \, \mathrm{d}x + \mathtt{q} \, \mathrm{d}y + \mathtt{r} \, \mathrm{d}z \tag{1.5}$$

Chapter 2

PDF 14

2.1 Gradient Vector

$$F = \nabla f \Rightarrow \begin{cases} p = \frac{\partial f}{\partial x} \\ q = \frac{\partial f}{\partial y} \\ r = \frac{\partial f}{\partial z} \end{cases} \tag{2.1}$$

$$\int\limits_C F \cdot \mathrm{d}r = \int_a^b \nabla f \cdot \mathrm{d}r = f\big(\vec{r}(b)\big) - f\big(\vec{r}(a)\big) \tag{2.2}$$

$$\frac{\partial p}{\partial y} = \frac{\partial q}{\partial x} \qquad \frac{\partial p}{\partial z} = \frac{\partial r}{\partial x} \qquad \frac{\partial q}{\partial z} = \frac{\partial r}{\partial y} \tag{2.3}$$

2.2 Green Theorem

$$\int\limits_C P\,\mathrm{d}x + Q\,\mathrm{d}y = \iint\limits_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) \mathrm{d}A \tag{2.4}$$

2.3 Area Calculations

$$A = \int\limits_C x\,\mathrm{d}y = -\int\limits_C y\,\mathrm{d}x = \frac{1}{2}\int\limits_C x\,\mathrm{d}y - y\,\mathrm{d}x \tag{2.5}$$

2.4 Green Theorem 2

$$\int\limits_{C} p\,\mathrm{d}x + q\,\mathrm{d}y = \int\limits_{C_1} p\,\mathrm{d}x + q\,\mathrm{d}y + \int\limits_{C_2} p\,\mathrm{d}x + q\,\mathrm{d}y = \iint\limits_{D} \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) \mathrm{d}A \quad (2.6)$$

Chapter 3

PDF 15

3.1 Smooth Parametrized Curve

$$\vec{r}(u,v) = \big(x(u,v),\, y(u,v),\, z(u,v)\big), \quad (u,v) \in R \tag{3.1}$$

$$\vec{r}_u = (\frac{\partial x}{\partial u}, \frac{\partial y}{\partial u}, \frac{\partial z}{\partial u}) \qquad \vec{r}_v = (\frac{\partial x}{\partial v}, \frac{\partial y}{\partial v}, \frac{\partial z}{\partial v}) \tag{3.2}$$

3.1.1 Sample

Let $z = f(x,y)$, the smooth parametrized curve will be:

$$\vec{r}(u,v) = (u, v, f(u,v)) \tag{3.3}$$

Partial Derivatives will be:

$$\begin{aligned} \vec{r}_u &= (\frac{\partial u}{\partial u}, \frac{\partial v}{\partial u}, \frac{\partial f(u,v)}{\partial u}) \\ &= (1, 0, \frac{\partial f}{\partial u}) = \vec{i} + \frac{\partial f}{\partial u}\vec{k} \end{aligned} \tag{3.4}$$

$$\begin{aligned} \vec{r}_v &= (\frac{\partial u}{\partial v}, \frac{\partial v}{\partial v}, \frac{\partial f(u,v)}{\partial v}) \\ &= (0, 1, \frac{\partial f}{\partial v}) = \vec{j} + \frac{\partial f}{\partial v}\vec{k} \end{aligned} \tag{3.5}$$

Cross Product of the partial derivatives:

$$\begin{aligned} \vec{r}_u \times \vec{r}_v &= (-\frac{\partial f}{\partial u}, -\frac{\partial f}{\partial v}, 1) \\ &= -\frac{\partial f}{\partial u}\vec{i} + -\frac{\partial f}{\partial v}\vec{j} + \vec{k} \neq 0 \end{aligned} \tag{3.6}$$

3.2 Area

$$A(\mathtt{S}) = \iint\limits_R |\vec{r}_u \times \vec{r}_v| \, \mathrm{d}A \tag{3.7}$$

3.2.1 Theorem

Let $z = f(x, y)$, then:

$$A(\mathtt{S}) = \iint\limits_R \sqrt{(\frac{\partial f}{\partial x})^2 + (\frac{\partial f}{\partial y})^2 + 1} \, \mathrm{d}A \tag{3.8}$$

Chapter 4

PDF 16