2-Sat

```
int n;
vector<vector<int>> adj, adj_t;
vector<bool> used,assignment;
vector<int> order, comp;
void dfs1(int v) {
used[v] = true;
for (int u : adj[v]) { if (!used[u])
  dfs1(u);
order.push_back(v);
void dfs2(int v, int cl) {
comp[v] = c1;
for (int u : adj_t[v]) { if (comp[u] == -1)
  dfs2(u, cl);
bool solve_2SAT() {
order.clear(); used.assign(n, false);
for (int i = 0; i < n; ++i) { if (!used[i])
  dfs1(i);
comp.assign(n, -1);
for (int i = 0, j = 0; i < n; ++i) {
  int v = order[n - i - 1];
  if (comp[v] == -1) dfs2(v, j++);</pre>
assignment.assign(n / 2, false);
for (int i = 0; i < n; i += 2) {
 if (comp[i] == comp[i + 1]) return false;
 assignment[i / 2] = comp[i] > comp[i + 1];
return true;
void add_disjunction(int a, bool na, int b, bool nb) {
// na and nb signify whether a and b are to be negated
a = 2*a ^ na; b = 2*b ^ nb;
int neg_a = a ^ 1; int neg_b = b ^ 1;
adj[neg_a].push_back(b); adj[neg_b].push_back(a);
adj_t[b].push_back(neg_a); adj_t[a].push_back(neg_b);
```

2 Aho

```
const int K = 26;
class Node{
public:
vector<int>nxt;
vector<int>go;
int suf ;
int ssuf
bool leaf;
int parent;
int ch;
Node(int p=-1, int ch = -1) {
 this->parent = p;
 this->ch = ch;
 nxt.resize(K,-1);
 go.resize(K,-1);
 leaf = false;
 suf = -1:
 ssuf = -1;
class AhoCorasick{
public:
vector<Node>nodes:
```

```
AhoCorasick() {
nodes.emplace_back(Node());
int get_num(char ch) {
int ret = ch-'a';
return ret;
void add_string(string &s) {
int v = \bar{0};
for(int i=0; i<s.size(); i++) {</pre>
 int now = get_num(s[i]);
 if(nodes[v].nxt[now]==-1) {
  nodes[v].nxt[now] = nodes.size();
  nodes.emplace_back(v,now);
 v = nodes[v].nxt[now];
nodes[v].leaf = true;
int go(int v, int ch) {
int &ret = nodes[v].go[ch];
if(ret!=-1)
 return ret;
if(nodes[v].nxt[ch]!=-1) {
 return ret = nodes[v].nxt[ch];
if(v==0) {
 return ret = 0;
return ret = go(get_link(v), ch);
int get_link(int v) {
int &ret = nodes[v].suf;
if(ret!=-1)
 return ret;
if(v==0 || nodes[v].parent==0) {
 return ret =0;
return ret = go(get_link(nodes[v].parent) ,
     nodes[v].ch);
int exit_link(int v) {
int &ret = nodes[v].ssuf;
if(ret!=-1)
 return ret;
if(v==0 || nodes[v].parent==0) {
 return ret = 0;
int s = get_link(v);
if(nodes[s].leaf) {
 return ret = s;
return ret = exit_link(s);
void feed(string &s) {
int v = 0;
for(int i=0;i<s.size();i++) {</pre>
 int ch = get_num(s[i]);
 v = go(v, ch);
 int u =v ;
 while (u!=0) {
  u = exit_link(u);
```

3 ArticulationBridge

```
vector<bool> visited;
vector<int> tin, low;
int timer;
void dfs(int v, int p = -1) {
visited[v] = true;
tin[v] = low[v] = timer++;
for (int to : adj[v]) {
 if (to == p) continue;
 if (visited[to]) {
  low[v] = min(low[v], tin[to]);
 } else {
  dfs(to, v);
  low[v] = min(low[v], low[to]);
  if (low[to] > tin[v])
   IS_BRIDGE(v, to);
void find_bridges() {
timer = 0;
visited.assign(n, false);
tin.assign(n, -1);
low.assign(n, -1);
for (int i = 0; i < n; ++i) {
 if (!visited[i])
  dfs(i);
```

4 ArticulationPoint

```
vector<bool> visited:
vector<int> tin, low;
int timer;
void dfs(int v, int p = -1) {
visited[v] = true;
tin[v] = low[v] = timer++;
int children=0;
for (int to : adj[v]) {
 if (to == p) continue;
 if (visited[to]) {
  low[v] = min(low[v], tin[to]);
 } else {
  dfs(to, v);
  low[v] = min(low[v], low[to]);
  if (low[to] >= tin[v] && p!=-1)
   IS_CUTPOINT(v);
  ++children;
if(p == -1 \&\& children > 1)
IS_CUTPOINT(v);
void find_cutpoints() {
timer = 0;
visited.assign(n, false);
tin.assign(n, -1);
low.assign(n, -1);
for (int i = 0; i < n; ++i) {
if (!visited[i])
  dfs (i);
```

BIT2D

```
struct FenwickTree2D {
  vector<vector<int>> bit;
  int n, m;

// init(...) { ... }

int sum(int x, int y) {
  int ret = 0;
  for (int i = x; i >= 0; i = (i & (i + 1)) - 1)
    for (int j = y; j >= 0; j = (j & (j + 1)) - 1)
    ret += bit[i][j];
  return ret;
}

void add(int x, int y, int delta) {
  for (int i = x; i < n; i = i | (i + 1))
    for (int j = y; j < m; j = j | (j + 1))
    bit[i][j] += delta;
};</pre>
```

6 Berlekamp

```
#define SZ 233333
const int MOD=1e9+7;
11 qp(11 a,11 b){
11 x=1; a\%=MOD;
while(b) {
 if(b\&1) x=x*a\%MOD;
  a=a*a\%MOD; b>>=1;
return x;
namespace linear_seq{
inline vector<int> BM(vector<int> x){
vector<int> ls,cur;
int lf,ld;
for(int i=0; i<int(x.size()); ++i) {</pre>
 11 t=0;
  for(int j=0; j<int(cur.size()); ++j)
t=(t+x[i-j-1]*(ll)cur[j])%MOD;</pre>
  if((t-x[i])%MOD==0) continue;
  if(!cur.size()) {
  cur.resize(i+1); lf=i; ld=(t-x[i])%MOD;
  11 k=-(x[i]-t)*qp(1d,MOD-2)%MOD;
  vector<int> c(i-lf-1);
  c.push_back(k);
  for(int j=0; j<int(ls.size()); ++j)</pre>
      c.push_back(-ls[j]*k%MOD);
  if(c.size()<cur.size()) c.resize(cur.size());</pre>
 for(int j=0; j<int(cur.size()); ++j)
    c[j]=(c[j]+cur[j])%MOD;</pre>
  if(i-lf+(int)ls.size()>=(int)cur.size())
  ls=cur,lf=i,ld=(t-x[i])%MOD;
for(int i=0; i<int(cur.size()); ++i)</pre>
     cur[i]=(cur[i]%MOD+MOD)%MOD;
return cur;
11 a[SZ], h[SZ], t_[SZ], s[SZ], t[SZ];
inline void mull(ll*p,ll*q){
for(int i=0; i<m+m; ++i) t_[i]=0;</pre>
for(int i=0; i<m; ++i) if(p[i])</pre>
  for(int j=0; j<m; ++j)</pre>
    t_{[i+j]}=(t_{[i+j]}+p[i]*q[j])%MOD;
for(int i=m+m-1; i>=m; --i) if(t_[i])
  for(int j=m-1; ~j; --j)
```

```
t_{i-j-1}=(t_{i-j-1}+t_{i})*h_{i})%MOD;
for(int i=0; i<m; ++i) p[i]=t_[i];</pre>
inline 11 calc(11 K){
for(int i=m: ~i: --i) s[i]=t[i]=0:
s[0]=1;
if(m!=1) t[1]=1; else t[0]=h[0];
while(K){if(K&1)mull(s,t);mull(t,t);K>>=1;}
for(int i=0; i<m; ++i) su=(su+s[i]*a[i])%MOD;</pre>
return (su%MOD+MOD)%MOD;
inline int work(vector<int> x,ll n){
if(n<int(x.size())) return x[n];</pre>
vector<int> v=BM(x); m=v.size();
if(!m) return 0;
for(int i=0; i<m; ++i) h[i]=v[i],a[i]=x[i];</pre>
return calc(n);
using linear_seq::work;
```

7 CHT

```
bool 0:
struct Line{
mutable 11 k, m, p; // slope, y-intercept, last optimal
 bool operator<(const Line& o) const {</pre>
 return Q ? p < o.p : k < o.k;
struct LineContainer : multiset<Line>{
  const ll inf = LLONG_MAX;
ll div(ll a, ll b) // floored division {
 if (b < 0)
  a *= -1, b *= -1;
 if (a >= 0)
  return a / b;
 return -((-a + b - 1) / b);
 // updates x->p, determines if y is unneeded
 bool isect(iterator x, iterator y) {
 if (y == end()) {
  x->p = inf;
  return 0;
  if (x->k == y->k)
  x->p = x->m^*> y->m ? inf : -inf;
 else
  x->p = div(y->m - x->m, x->k - y->k);
 return x->p >= y->p;
 void add(ll k, ll m) {
 auto z = insert(\{k, m, 0\}), y = z++, x = y;
 while (isect(y, z))
  z = erase(z):
 if (x != begin() && isect(--x, y))
  isect(x, y = erase(y));
 while ((y = x) != begin() && (--x)->p >= y->p)
  isect(x, erase(y));
11 query(11 x) // gives max value {
 assert(!empty());
 Q = 1;
  auto l = *lower_bound({0, 0, x});
 Q = 0:
```

```
return 1.k * x + 1.m;
};
```

```
CRT
/** Works for non-coprime moduli.
Returns {0,0} if solution does not exist or input is
Otherwise, returns \{x,L\}, where x is the solution
     unique to mod L
constexpr long long safe_mod(long long x, long long m) {
x \% = m;
if (x < 0) x += m;
return x;
constexpr std::pair<long long, long long> inv_gcd(long
    long a, long long b) {
a = safe_mod(a, b);
if (a == 0) return {b, 0};
 long long s = b, t = a:
long long m0 = 0, m1 = 1;
 while (t) {
 long long u = s / t;
s -= t * u;
 m0 -= m1 * u:
 auto tmp = s;
 s = t;
 t = tmp
 tmp = m0;
 mO^{T} = m1:
 m1 = tmp;
if (m0 < 0) m0 += b / s:
return {s, m0};
std::pair<long long, long long> crt(const
    std::vector<long long>& r,
        const std::vector<long long>& m) {
 assert(r.size() == m.size());
 int n = int(r.size());
 long long r0 = 0, m0 = 1;
 for (int i = 0; i < n; i++) {
 assert(1 <= m[i]);
  long long r1 = safe_mod(r[i], m[i]), m1 = m[i];
  if (m0 < m1) {
  std::swap(r0, r1);
  std::swap(m0, m1);
  if (m0 % m1 == 0) {
  if (r0 % m1 != r1) return {0, 0};
  continue;
  long long g, im;
  std::tie(g, im) = inv_gcd(m0, m1);
  long long u1 = (m1 / g);
  if ((r1 - r0) % g) return {0, 0};
  long long x = (r1 - r0) / g % u1 * im % u1;
 r0 += x * m0;
 m0 = u1;
 if (r0 < 0) r0 += m0;
return {r0, m0};
```

9 Centroid

class CentroidDecomposition{
public:

```
vector<vector<int> >adi:
vector<int>sz,parent;
vector<bool>vis;
int n,root;
CentroidDecomposition(vector<vector<int> >&a) {
adj = a; n = adj.size(); sz.resize(n);
     parent.resize(n,-1);
 vis.resize(n,false);
build(0,-1);
void build(int s, int p) {
dfs(s,p);
int c= centroid(s,p,sz[s]); vis[c]= true;
 if(p!=-1) { parent[c] = p; }
 else { root = c; ]
 for(auto it:adj[c]) { if(vis[it]) continue;
 build(it,c);
void dfs(int s, int p) {
sz[s] = 1;
 for(auto i:adj[s]) { if(i==p || vis[i]) continue;
 dfs(i,s); sz[s] += sz[i];
int centroid(int s,int p, int total) {
for(auto i:adj[s]) { if(i==p || vis[i]) continue;
 if(sz[i]*2>total) return centroid(i,s,total);
return s;
```

10 ContinuedFraction

```
/**
 * Description: Given $f$ and $N$, finds the smallest
     fraction p/q \in [0, 1]
 * such that f(p/q) is true, and $p, q less than N$.
 * You may want to throw an exception from $f$ if it
finds an exact solution,

* in which case $N$ can be removed.

* Usage: fracBS([](Frac f) { return f.p>=3*f.q; }, 10);
// {1,3}
* Time: O(\log(N))
* Status: fuzz-tested for n <= 300
typedef __int128_t lll;
struct Frac{
lll p, q;
template<class F>
Frac fracBS(F f, lll N) {
   bool dir = 1, A = 1, B = 1;

 Frac lo{0, 1}, hi{1, 0};
 if (f(lo))
 return lo;
 assert(f(hi)):
 while (A | | B) {
  lll adv = 0, step = 1;
  for (int si = 0; step; (step *= 2) >>= si) {
  adv += step;
  Frac mid{lo.p * adv + hi.p, lo.q * adv + hi.q};
   if ((mid.p < 0 ? -mid.p : mid.p) > N || mid.q > N ||
       dir == !f(mid)) {
    adv -= step;
    si = 2;
```

```
}
hi.p += lo.p * adv;
hi.q += lo.q * adv;
dir = !dir;
swap(lo, hi);
A = B;
B = !!adv;
}
return dir ? hi : lo;
}
int main(){
Frac target = fracBS([&](Frac fr) {
    if (fr.q == 0)
        return true;
    return minfr < fr;
}, mid);
}</pre>
```

11 DC DP

```
int dp_before[N],dp_cur[N],cnt[N],l=0,r=-1,cur;
void add(int idx){
cnt[a[idx]]++; if(cnt[a[idx]]==1)cur++;
void remove(int idx){
cnt[a[idx]]--; if(cnt[a[idx]]==0) cur--;
int cost(int L,int R){
  while (1 > L) { 1--; add(1); }
while (r < R) \{ r++; add(r); \}
while (1 < L) { remove(1); 1++; }</pre>
 while (r > R) { remove(r); r--; }
return cur;
void compute(int 1, int r, int opt1, int optr){
if (1 > r) return;
 int mid = (1 + r) >> 1;
pair<int, int> best = \{-1e9, -1\};
 for (int k = optl; k <= min(mid, optr); k++) {</pre>
 if(best.second==-1) { best = {dp_before[k] +cost(k+1,
      mid), k}:}
 else { best = max(best, {dp_before[k] +cost(k+1, mid),
      k}); }
dp_cur[mid] = best.first; int opt = best.second;
compute(1, mid - 1, optl, opt);compute(mid + 1, r, opt,
     optr);
```

12 DEBUG TEMPLATE

```
void err(istream_iterator<string> it) {cout<<endl;}
template<typename T, typename... Args>
void err(istream_iterator<string> it, T a, Args... args){
  cout << *it << " = " << a << " ";err(++it, args...);
}
template<class T1, class T2>
  ostream &operator <<(ostream &os, pair<T1,T2>&p) {
    os<<"{"<<p.first<<", "<<p.second<<"} ";
    return os;
}
#define debug(args...) { string _s = #args;
    replace(_s.begin(), _s.end(), ',', ');
    stringstream _ss(_s); istream_iterator<string>
    _it(_ss); err(_it, args); }
```

13 DSU Rollback

```
struct dsu_save {
  int v, rnkv, u, rnku;
```

```
dsu save() {}
dsu_save(int _v, int _rnkv, int _u, int _rnku)
: v(_v), rnkv(_rnkv), u(_u), rnku(_rnku) {}
struct dsu_with_rollbacks {
vector<int> p, rnk;
int comps;
stack<dsu_save> op;
dsu_with_rollbacks() {}
dsu_with_rollbacks(int n) {
 p.resize(n);
 rnk.resize(n);
 for (int i = 0; i < n; i++) {</pre>
  p[i] = i;
  rnk[i] = 0;
 comps = n;
int find_set(int v) {
 return (v == p[v]) ? v : find_set(p[v]);
bool unite(int v, int u) {
 v = find_set(v);
 u = find_set(u);
 if (v == u)
  return false;
 comps--
 if (rnk[v] > rnk[u])
  swap(v, u);
 op.push(dsu_save(v, rnk[v], u, rnk[u]));
 p[v] = u;
 if (rnk[u] == rnk[v])
  rnk[u]++;
 return true;
void rollback() {
 if (op.empty())
  return:
 dsu_save x = op.top();
 op.pop();
 comps++;
 p[x.v] = x.v;
 rnk[x.v] = x.rnkv;
 p[x.u] = x.u;
 rnk[x.u] = x.rnku;
struct query {
int v, u;
bool united;
query(int _v, int _u) : v(_v), u(_u) {
struct QueryTree {
vector<vector<query>> t;
dsu_with_rollbacks dsu;
QueryTree() {}
QueryTree(int _T, int n) : T(_T) {
 dsu = dsu_with_rollbacks(n);
 t.resize(4 * T + 4);
void add_to_tree(int v, int l, int r, int ul, int ur,
    query& q) {
 if (ul > ur)
 if (1 == ul && r == ur) {
  t[v].push_back(q);
  return;
 int mid = (1 + r) / 2;
```

```
add_to_tree(2 * v, 1, mid, ul, min(ur, mid), q);
 add_{to}_{tree}(2 * v + 1, mid + 1, r, max(ul, mid + 1),
void add_query(query q, int 1, int r) {
 add_to_tree(1, 0, T - 1, 1, r, q);
void dfs(int v, int 1, int r, vector<int>& ans) {
  for (query& q : t[v]) {
 q.united = dsu.unite(q.v, q.u);
 if (1 == r)
 ans[1] = dsu.comps;
 else {
  int mid = (1 + r) / 2;
  dfs(2 * v, 1, mid, ans);
  dfs(2 * v + 1, mid + 1, r, ans);
 for (query q : t[v]) {
 if (q.united)
   dsu.rollback();
vector<int> solve() {
 vector<int> ans(T);
 dfs(1, 0, T - 1, ans);
 return ans;
```

14 Dinic

```
//To find min-cut
const long long flow_inf = 1e18;
struct FlowEdge
   int v, u;
long long cap, flow = 0;
   FlowEdge(int v, int u, long long cap) : v(v), u(u),
        cap(cap) {}
struct Dinic
   vector<FlowEdge> edges;
   vector<vector<int> > adi:
   int n, m = 0;
   int s, t;
   vector<int> level, ptr;
   queue<int> q;
   vector<bool>vis;
   Dinic();
   Dinic(int n) : n(n)
       vis.resize(n);
       adj.resize(n);
       level.resize(n);
       ptr.resize(n);
   void add_edge(int v, int u, long long cap)
       edges.emplace_back(v, u, cap);
       edges.emplace_back(u, v, 0);
       adj[v].push_back(m);
       adj[u].push_back(m + 1);
   void dfs2(int s)
       vis[s] = 1;
       for(int i:adj[s])
```

```
int id = i;
       int u = edges[id].v;
       int v = edges[id].u;
       if(edges[id].flow!=edges[id].cap && !vis[v])
           dfs2(v);
   }
vector<int> getMinCut()
   dfs2(s);
   vector<int>ret;
   for(int i=0;i<n;i++){</pre>
       if(vis[i]) ret.push_back(i);
   return ret;
bool bfs()
   while (!q.empty())
       int v = q.front();
       q.pop();
       for (int id : adj[v])
           if (edges[id].cap - edges[id].flow < 1)</pre>
           if (level[edges[id].u] != -1)
           level[edges[id].u] = level[v] + 1;
           q.push(edges[id].u);
   return level[t] != -1:
long long dfs(int v, long long pushed)
   if (pushed == 0)
       return 0;
   if (v == t)
       return pushed;
   for (int& cid = ptr[v]; cid <</pre>
        (int)adj[v].size(); cid++)
       int id = adj[v][cid];
       int u = edges[id].u;
       if (level[v] + 1 != level[u] || edges[id].cap
           - edges[id].flow < 1)
           continue:
       long long tr = dfs(u, min(pushed,
           edges[id].cap - edges[id].flow));
       if (tr == 0)
           continue;
       edges[id].flow += tr;
       edges[id ^ 1].flow -= tr;
       return tr;
   }
   return 0;
long long flow(int _s,int _t)
   t=_t;
   long long f = 0;
   while (true)
       fill(level.begin(), level.end(), -1);
       level[s] = 0:
```

```
q.push(s);
           if (!bfs())
              break:
           fill(ptr.begin(), ptr.end(), 0);
           while (long long pushed = dfs(s, flow_inf))
              f += pushed;
       return f;
/*for bipartite graph*/
class Minimum_node_cover
   map<pair<int,int>,bool>matched;
   vector<vector<int> >adj;
   vector<int>minimum_vertex,maximum_set,l,r;
   vector<bool>vis;
   /*number of nodes in dinic without source and
       destination src = 0, dest = sz+1
d.flow() should be called before constructor calling*/
   Minimum_node_cover(int sz, Dinic &d)
       adj.resize(sz+5);
       vis.resize(sz+5);
       for(auto it:d.edges)
           if(it.u>0 && it.u <=sz && it.v>0 && it.v<=sz
               && it.cap==1)
              if(it.flow==1)
                  adj[it.u].push_back(it.v);
                  matched[ {it.u,it.v}]=1;
              else adj[it.v].push_back(it.u);
          }
       for(auto it:d.edges)
           if(it.v==0 && it.cap==1) l.push_back(it.u);
           if(it.u==sz+1 && it.cap==1) r.push_back(it.v);
       sort(1.begin(),1.end());
       sort(r.begin(),r.end());
       1.resize(distance(1.begin(),unique(1.begin(),1.end())
       r.resize(distance(r.begin(),unique(r.begin(),r.end())
       for(auto it:d.edges)
           if(it.v==0 && it.cap==1 && it.flow==0)
              if(!vis[it.u]) dfs2(it.u, 1);
       for(int i:1)
           if(!vis[i]) minimum_vertex.push_back(i);
           else maximum_set.push_back(i);
       for(int i:r)
           if(vis[i]) minimum_vertex.push_back(i);
           else maximum_set.push_back(i);
```

```
void dfs2(int s, bool bam)
       vis[s] = 1;
       if(bam)
           for(int i:adj[s])
              if(vis[i]) continue;
              if(matched[ {s,i}] == 0) dfs2(i,0);
       else
           for(int i:adj[s])
              if(vis[i]) continue;
              if(matched[{s,i}]==1) dfs2(i,1);
      }
   }
//flow_through[i] = extra flow beyond 'low' sent through
    edge i
struct LR_Flow {
Dinic F;
int n, s, t;
struct edge {
 int u, v, 1, r, id;
vector<edge> edges;
LR_Flow() {}
LR_Flow(int _n) {
 n = n + 10;

s = n - 2, t = n - 1;;
 edges.clear();
void add_edge(int u, int v, int l, int r, int id = -1) {
 assert(\bar{0} \iff 1 \&\& 1 \iff r);
 edges.push_back({u, v, l, r, id});
bool feasible(int _s = -1, int _t = -1, int L = -1, int
    R = -1) {
 if (L != -1) edges.push_back({_t, _s, L, R, -1});
 F = Dinic(n);
 long long target = 0;
 for (auto e : edges) {
  int u = e.u, v = e.v, l = e.l, r = e.r, id = e.id;
  if (1 != 0) {
   F.add_edge(s, v, 1);
   F.add_edge(u, t, 1);
   target += 1;
  F.add_edge(u, v, r - 1);
 auto ans = F.flow(_s,_t);
 if (L != -1) edges.pop_back();
 if (ans < target) return 0; //not feasible
 return 1:
int max_flow(int _s, int _t) { //-1 means flow is not
    feasible
 int mx = 1e5 + 9;
 if (!feasible(_s, _t, 0, mx)) return -1;
 return F.flow(_s, _t);
int min_flow(int _s, int _t) { //-1 means flow is not
    feasible
 int mx = 1e9;
 int ans = -1, 1 = 0, r = mx;
```

```
while (1 <= r) {
  int mid = 1 + r >> 1;
  if (feasible(_s, _t, 0, mid)) ans = mid, r = mid - 1;
  else 1 = mid + 1;
  }
  return ans;
};
```

15 Diophantine

```
int gcd(int a, int b, int& x, int& y) {
  if (b == 0) {
 x = 1;
 y = \bar{0};
 return a:
 int x1, y1;
 int d = gcd(b, a \% b, x1, y1);
x = y1;
y = x1 - y1 * (a / b);
return d;
bool find_any_solution(int a, int b, int c, int &x0, int
    &v0, int &g) {
g = gcd(abs(a), abs(b), x0, y0);
if (c % g) {
 return false;
x0 *= c / g;
v0 *= c / g;
 if (a < 0) x0 = -x0;
if (b < 0) v0 = -v0;
return true:
void shift_solution(int & x, int & y, int a, int b, int
    cnt) {
x += cnt * b:
y -= cnt * a;
int find_all_solutions(int a, int b, int c, int minx,
    int maxx, int miny, int maxy) {
 if (!find_any_solution(a, b, c, x, y, g))
 return 0:
a /= g;
 b /= \bar{g};
 int sign_a = a > 0 ? +1 : -1;
 int sign_b = b > 0 ? +1 : -1;
 shift_solution(x, y, a, b, (minx - x) / b);
 if (x < minx)
 shift_solution(x, y, a, b, sign_b);
 if (x > maxx)
 return 0:
 int lx1 = x;
 shift_solution(x, y, a, b, (maxx - x) / b);
 if (x > maxx)
 shift_solution(x, y, a, b, -sign_b);
 int rx1 = x;
 shift_solution(x, y, a, b, -(miny - y) / a);
 if (v < minv)</pre>
 shift_solution(x, y, a, b, -sign_a);
 if (y > maxy)
 return 0;
int 1x2 = x:
```

```
shift_solution(x, y, a, b, -(maxy - y) / a);
if (y > maxy)
    shift_solution(x, y, a, b, sign_a);
int rx2 = x;
if (lx2 > rx2)
    swap(lx2, rx2);
int lx = max(lx1, lx2);
int rx = min(rx1, rx2);
if (lx > rx)
    return 0;
    return (rx - lx) / abs(b) + 1;
```

16 DiscreteLog

```
// Returns minimum x for which a \hat{ } x \% m = b \% m.
   O(sqrt(m))
int solve(int a, int b, int m){
// \text{ if (a == 0)}
// return b == 0 ? 1 : -1;
a \% = m, b \% = m; int k = 1, add = 0, g;
while ((g = \_gcd(a, m)) > 1) {
if (b == k)return add;
 if (b % g) return -1;
b = g, m = g, ++add; k = (k * 111 * a / g) % m;
int n = sqrt(m) + 1; int an = 1;
for (int i = 0; i < n; ++i) an = (an * 1ll * a) % m;
unordered_map<int, int> vals;
for (int q = 0, cur = b; q \le n; ++q) {
 vals[cur] = q; cur = (cur * 111 * a) % m;
for (int p = 1, cur = k; p \le n; ++p) {
 cur = (cur * 111 * an) % m;
 if (vals.count(cur)) { int ans = n * p - vals[cur] +
     add:
  return ans;
return -1;
```

17 EulerTour

```
vector<multiset<int> >adj; vector<int>ans;
void euler_circuit(int src) {
    stack<int>st; st.push(src);
    while(!st.empty()) {
      int v = st.top();
      if(adj[v].size()==0) {
        ans.push_back(v); st.pop();
    }
    else {
      int f = *adj[v].begin();
      adj[v].erase(adj[v].begin());
      adj[f].erase(adj[f].find(v));
      st.push(f);
    }
}
```

18 FFT

```
struct CD {
  double x, y;
  CD(double x=0, double y=0) :x(x), y(y) {}
  CD operator+(const CD& o) { return {x+o.x, y+o.y};}
  CD operator-(const CD& o) { return {x-o.x, y-o.y};}
```

```
CD operator*(const CD& o) { return {x*o.x-y*o.y,
     x*o.y+o.x*y};}
void operator /= (double d) { x/=d; y/=d;}
double real() {return x;}
double imag() {return y;}
CD conj(const CD &c) {return CD(c.x, -c.y);}
typedef long long LL;
const double PI = acos(-1.0L);
namespace FFT {
int N;
vector<int> perm;
vector<CD> wp[2];
void precalculate(int n) {
  assert((n & (n-1)) == 0);
 N = n;
  perm = vector<int> (N, 0);
  for (int k=1; k<N; k<<=1) {
  for (int i=0; i<k; i++) {</pre>
   perm[i] <<= 1;
   perm[i+k] = 1 + perm[i];
  wp[0] = wp[1] = vector < CD > (N);
  for (int i=0; i<N; i++) {</pre>
  wp[0][i] = CD(cos(2*PI*i/N), sin(2*PI*i/N));
  wp[1][i] = CD(cos(2*PI*i/N), -sin(2*PI*i/N));
void fft(vector<CD> &v, bool invert = false) {
 if (v.size() != perm.size()) precalculate(v.size());
  for (int i=0; i<N; i++)</pre>
  if (i < perm[i])</pre>
   swap(v[i], v[perm[i]]);
  for (int len = 2; len <= N; len *= 2) {
  for (int i=0, d = N/len; i<N; i+=len) {</pre>
   for (int j=0, idx=0; j<len/2; j++, idx += d) {</pre>
    CD x = v[i+j];
    CD y = wp[invert][idx]*v[i+j+len/2];
    v[i+j] = x+y;
    v[i+j+len/2] = x-y;
 if (invert) {
  for (int i=0: i<N: i++) v[i]/=N:
 void pairfft(vector<CD> &a, vector<CD> &b, bool invert
      = false) {
  int N = a.size():
  vector<CD> p(N);
  for (int i=0; i<N; i++) p[i] = a[i] + b[i] * CD(0, 1);
 fft(p, invert);
  p.push_back(p[0]);
  for (int i=0; i<N; i++) {</pre>
  if (invert) {
   a[i] = CD(p[i].real(), 0);
   b[i] = CD(p[i].imag(), 0);
  else {
   a[i] = (p[i]+conj(p[N-i]))*CD(0.5, 0);
   b[i] = (p[i]-conj(p[N-i]))*CD(0, -0.5);
```

```
vector<LL> multiply(const vector<LL> &a, const
    vector<LL> &b) {
 int n = 1;
 while (n < a.size() + b.size()) n <<=1;
 vector<CD> fa(a.begin(), a.end()), fb(b.begin(),
 fa.resize(n); fb.resize(n);
// fft(fa); fft(fb);
 pairfft(fa, fb);
 for (int i=0; i<n; i++) fa[i] = fa[i] * fb[i];
 fft(fa, true);
 vector<LL> ans(n);
 for (int i=0; i<n; i++) ans[i] = round(fa[i].real());</pre>
const int M = 1e9+7, B = sqrt(M)+1;
vector<LL> anyMod(const vector<LL> &a, const vector<LL>
    &b) {
 int n = 1;
 while (n < a.size() + b.size()) n <<=1
 vector<CD> al(n), ar(n), bl(n), br(n);
 for (int i=0; i<a.size(); i++) al[i] = a[i]%M/B, ar[i]</pre>
     = a[i]%M%B;
 for (int i=0; i<b.size(); i++) bl[i] = b[i]%M/B, br[i]</pre>
     = b[i]\%M\%B;
 pairfft(al, ar); pairfft(bl, br);
// fft(al); fft(ar); fft(bl); fft(br);
 for (int i=0; i<n; i++) {</pre>
 CD ll = (al[i] * bl[i]), lr = (al[i] * br[i]);
CD_rl = (ar[i] * bl[i]), rr = (ar[i] * br[i]);
 al[i] = ll; ar[i] = lr;
 bl[i] = rl; br[i] = rr;
 pairfft(al, ar, true); pairfft(bl, br, true);
  fft(al, true); fft(ar, true); fft(bl, true); fft(br,
 vector<LL> ans(n);
 for (int i=0; i<n; i++) {</pre>
 LL right = round(br[i].real()), left =
      round(al[i].real());
  LL mid = round(round(bl[i].real()) +
      round(ar[i].real()));
 ans[i] = ((left\%M)*B*B + (mid\%M)*B + right)\%M;
return ans;
```

19 FWHT

```
const int inv2 = (mod + 1) >> 1;
const int M = (1 \ll 20);
const int OR = 0;
const int AND = 1
const int XOR = 2;
struct FWHT{
int P1[M], P2[M];
void wt(int *a, int n, int flag = XOR) {
 if (n == 0)
 int m = n / 2;
 wt(a, m, flag);
 wt(a + m, m, flag);
 for (int i = 0; i < m; i++) {
  int x = a[i], y = a[i + m];
  if (flag == OR)
   a[i] = x, a[i + m] = (x + y) % mod;
  if (flag == AND)
   a[i] = (x + y) \% mod, a[i + m] = y;
  if (flag == XOR)
```

```
a[i] = (x + y) \% \mod, a[i + m] = (x - y + mod) \% \mod;
void iwt(int* a, int n, int flag = XOR) {
 if (n == 0)
 return;
 int m = n / 2;
 iwt(a, m, flag);
 iwt(a + m, m, flag);
 for (int i = 0; i < m; i++) {
  int x = a[i], y = a[i + m];
  if (flag == OR)
   a[i] = x, a[i + m] = (y - x + mod) % mod;
  if (flag == AND)
   a[i] = (x - y + mod) \% mod, a[i + m] = y;
  if (flag == XOR)
   a[i] = 1LL * (x + y) * inv2 % mod, a[i + m] = 1LL *
       (x - y + mod) * inv2 % mod; // replace inv2 by
       >>1 if not required
vector<int> multiply(int n, vector<int> A, vector<int>
    B, int flag = XOR) {
 assert(__builtin_popcount(n) == 1);
 A.resize(n):
 B.resize(n);
 for (int i = 0; i < n; i++)
 P1[i] = A[i];
 for (int i = 0; i < n; i++)
 P2[i] = B[i];
 wt(P1, n, flag);
 wt(P2, n, flag);
 for (int i = \bar{0}; i < n; i++)
 P1[i] = 1LL * P1[i] * P2[i] % mod;
 iwt(P1, n, flag);
return vector<int> (P1, P1 + n);
vector<int> pow(int n, vector<int> A, long long k, int
    flag = XOR) {
 assert(__builtin_popcount(n) == 1);
 A.resize(n);
 for (int i = 0; i < n; i++)</pre>
 P1[i] = A[i];
 wt(P1, n, flag);
 for(int i = 0; i < n; i++)
 P1[i] = POW(P1[i], k);
 iwt(P1, n, flag);
 return vector<int> (P1, P1 + n);
int32_t main(){
int n;
cin >> n;
vector<int> a(M, 0);
for(int i = 0; i < n; i++) {
int k;
 cin >> k;
a[k]++;
vector < int > v = t.pow(M, a, n+1, AND);
int ans = 1;
for(int i = 1; i < M; i++)
ans += v[i] > 0;
cout << ans << '\n';
return 0;
      Fibonacci Shortcut
```

```
pair<int, int> fib (int n) {
```

```
if (n == 0) return {0, 1};
auto p = fib(n >> 1);
int c = p.first * (2 * p.second - p.first);
int d = p.first * p.first + p.second * p.second;
if (n & 1) return {d, c + d};
else return {c, d};
}
```

21 FloorCeilChange

```
vector<int> where_floor_changes(int n){
int now=1; vector<int>v;
while(now<=n) {</pre>
 v.push_back(now); now=n/(n/now)+1;
return v;
vector<pair<int,int>> where_ceil_changes(int m){
vector<pair<int,int>>v; int l=1;
while(l<=m) {</pre>
 if(l==m) { v.push_back({m,m}); break;}
  int cl=(m+l-1)/1;
  int r=(m+cl-2)/(cl-1)-1;
 r=min(r,m); r=max(r,1);
 v.push_back(\{1,r\});
  if(r==m) break;
 l=r+1:
return v;
```

22 Floorsum

```
// floor( (a*i+b)/m ) for 0 <= i <= n-1
ll floor_sum(ll n, ll m, ll a, ll b) {
    ll ans = 0;
    if (a >= m) {
        ans += (n - 1) * n * (a / m) / 2;
        a %= m;
    }
    if (b >= m) {
        ans += n * (b / m);
        b %= m;
    }
    ll y_max = (a * n + b) / m, x_max = (y_max * m - b);
    if (y_max == 0) return ans;
    ans += floor_sum(y_max, a, m, (a - x_max % a) % a);
    return ans;
}
```

23 Gauss

```
const double EPS = 1e-9;
const int INF = 2;
int gauss (vector < vector < double > > a, vector < double > &
    ans) {
int n = (int) a.size();
int m = (int) a[0].size() - 1;
vector<int> where (m, -1);
for (int col=0, row=0; col<m && row<n; ++col) {</pre>
 int sel = row;
  for (int i=row; i<n; ++i)</pre>
  if (abs (a[i][col]) > abs (a[sel][col]))
   sel = i:
  if (abs (a[sel][col]) < EPS)</pre>
  continue;
  for (int i=col; i<=m; ++i)</pre>
  swap (a[sel][i], a[row][i]);
  where[col] = row:
```

```
for (int i=0: i<n: ++i)</pre>
  if (i != row) {
   double c = a[i][col] / a[row][col];
   for (int j=col; j<=m; ++j)</pre>
    a[i][i] = a[row][i] * c;
 ++row;
ans.assign (m, 0);
for (int i=0; i<m; ++i)
if (where[i] != -1)</pre>
  ans[i] = a[where[i]][m] / a[where[i]][i];
for (int i=0; i<n; ++i) {</pre>
 double sum = 0:
 for (int j=0; j<m; ++j)
sum += ans[j] * a[i][j];</pre>
 if (abs (sum - a[i][m]) > EPS)
 return 0;
for (int i=0; i<m; ++i)</pre>
 if (where[i] == -1)
 return INF:
return 1;
```

|24 Geo 2D

```
const double pi = 4 * atan(1);const double eps = 1e-6;
inline int dcmp (double x) { if (fabs(x) < eps) return
    0; else return x < 0 ? -1 : 1; }
double fix_acute(double th) {return th<-pi ? (th+2*pi):</pre>
    th>pi ? (th-2*pi) : th;}
inline double getDistance (double x, double y) { return
    sqrt(x * x + y * y); }
inline double torad(double deg) { return deg / 180 * pi;
struct Pt {
double x, y;
Pt (double x = 0, double y = 0): x(x), y(y) {} void read () { scanf("%lf%lf", &x, &y); }
 void write () { printf("%lf %lf", x, y); }
bool operator == (const Pt& u) const { return dcmp(x -
     u.x) == 0 \&\& dcmp(y - u.y) == 0; }
 bool operator != (const Pt& u) const { return !(*this
 bool operator < (const Pt& u) const { return dcmp(x -
     u.x) < 0 || (dcmp(x-u.x)==0 \&\& dcmp(y-u.y) < 0); }
 bool operator > (const Pt& u) const { return u < *this;
bool operator <= (const Pt& u) const { return *this < u
     || *this == u; }
bool operator >= (const Pt& u) const { return *this > u
     || *this == u; }
Pt operator + (const Pt& u) { return Pt(x + u.x, y +
     u.y); }
Pt operator - (const Pt& u) { return Pt(x - u.x, y -
     u.y); }
Pt operator * (const double u) { return Pt(x * u, y *
     u); }
Pt operator / (const double u) { return Pt(x / u, y /
     u); }
double operator * (const Pt& u) { return x*u.y - y*u.x;
typedef Pt Vector;
typedef vector<Pt> Polygon;
struct Line {
double a, b, c;
```

```
Line (double a = 0, double b = 0, double c = 0): a(a).
    b(b), c(c) {}
struct Segment{
Pt a;Pt b:
Segment(){}
Segment(Pt aa,Pt bb) {a=aa,b=bb;}
struct DirLine {
Pt p; Vector v;
double ang;
DirLine () {}
DirLine (Pt p, Vector v): p(p), v(v) { ang = atan2(v.y,
bool operator < (const DirLine& u) const { return ang <
    u.ang; }
namespace Punctual {
double getDistance (Pt a, Pt b) { double x=a.x-b.x,
    y=a.y-b.y; return sqrt(x*x + y*y); }
namespace Vectorial {
double getDot (Vector a, Vector b) { return a.x * b.x +
    a.y * b.y; }
double getCross (Vector a, Vector b) { return a.x * b.y
    - a.y * b.x; }
double getLength (Vector a) { return sqrt(getDot(a,
double getPLength (Vector a) { return getDot(a, a); }
double getAngle (Vector u) { return atan2(u.y, u.x); }
double getSignedAngle (Vector a, Vector b) {return
     getAngle(b)-getAngle(a);}
Vector rotate (Vector a, double rad) { return
     Vector(a.x*cos(rad)-a.y*sin(rad),
     a.x*sin(rad)+a.y*cos(rad)); }
Vector ccw(Vector a, double co, double si) {return
     Vector(a.x*co-a.y*si, a.y*co+a.x*si);}
Vector cw (Vector a, double co, double si) {return
     Vector(a.x*co+a.y*si, a.y*co-a.x*si);}
Vector scale(Vector a, double s = 1.0) {return a /
     getLength(a) * s;}
Vector getNormal (Vector a) { double 1 = getLength(a);
    return Vector(-a.v/l, a.x/l); }
namespace ComplexVector {
typedef complex<double> Pt;
typedef Pt Vector;
double getDot(Vector a, Vector b) { return
    real(conj(a)*b); }
double getCross(Vector a, Vector b) { return
    imag(conj(a)*b); }
Vector rotate(Vector a, double rad) { return
    a*exp(Pt(0, rad)); }
namespace Linear {
using namespace Vectorial;
Line getLine (double x1, double y1, double x2, double
    y2) { return Line(y2-y1, x1-x2, y1*x2-x1*y2); }
Line getLine (double a, double b, Pt u) { return
    Line(a, -b, u.y * b - u.x * a); }
bool getIntersection (Line p, Line q, Pt& o) {
 if (fabs(p.a * q.b - q.a * p.b) < eps)
  return false:
 o.x = (q.c * p.b - p.c * q.b) / (p.a * q.b - q.a *
     p.b);
 o.y = (q.c * p.a - p.c * q.a) / (p.b * q.a - q.b *
     p.a);
 return true;
```

```
bool getIntersection (Pt p, Vector v, Pt q, Vector w,
 if (dcmp(getCross(v, w)) == 0) return false;
 Vector \mathbf{u} = \mathbf{p} - \mathbf{q};
 double k = getCross(w, u) / getCross(v, w);
 o = p + v * k;
 return true;
double getDistanceToLine (Pt p, Pt a, Pt b) { return
    fabs(getCross(b-a, p-a) / getLength(b-a)); }
double getDistanceToSegment (Pt p, Pt a, Pt b) {
 if (a == b) return getLength(p-a);
 Vector v1 = b - a, v2 = p - a, v3 = p - b;
 if (dcmp(getDot(v1, v2)) < 0) return getLength(v2);</pre>
 else if (dcmp(getDot(v1, v3)) > 0) return
      getLength(v3);
 else return fabs(getCross(v1, v2) / getLength(v1));
double getDistanceSegToSeg (Pt a,Pt b,Pt c,Pt d){
 double Ans=INT_MAX;
 Ans=min(Ans,getDistanceToSegment(a,c,d));
 Ans=min(Ans,getDistanceToSegment(b,c,d));
 Ans=min(Ans,getDistanceToSegment(c,a,b));
 Ans=min(Ans,getDistanceToSegment(d,a,b));
 return Ans:
Pt getPtToLine (Pt p, Pt a, Pt b) { Vector v = b-a;
    return a+v*(getDot(v, p-a) / getDot(v,v)); }
bool onSegment (Pt p, Pt a, Pt b) { return
     dcmp(getCross(a-p, b-p)) == 0 && dcmp(getDot(a-p,
     b-p) <= 0; }
bool haveIntersection (Pt a1, Pt a2, Pt b1, Pt b2) {
 if(onSegment(a1,b1,b2)) return true;
 if(onSegment(a2,b1,b2)) return true;
 if(onSegment(b1,a1,a2)) return true;
 if(onSegment(b2,a1,a2)) return true; //Case of touch
 double c1=getCross(a2-a1, b1-a1), c2=getCross(a2-a1,
      b2-a1), c3=getCross(b2-b1, a1-b1),
      c4=getCross(b2-b1,a2-b1);
 return dcmp(c1)*dcmp(c2) < 0 && dcmp(c3)*dcmp(c4) < 0;
bool onLeft(DirLine 1, Pt p) { return dcmp(1.v *
     (p-1.p)) >= 0; }
namespace Triangular {
using namespace Vectorial;
double getAngle (double a, double b, double c) { return
     acos((a*a+b*b-c*c) / (2*a*b)); }
double getArea (double a, double b, double c) { double
     s = (a+b+c)/2; return sqrt(s*(s-a)*(s-b)*(s-c)); }
double getArea (double a, double h) { return a * h / 2;
double getArea (Pt a, Pt b, Pt c) { return
    fabs(getCross(b - a, c - a)) / 2; }
double getDirArea (Pt a, Pt b, Pt c) { return
     getCross(b - a, c - a) / 2;}
//ma/mb/mc = length of median from side a/b/c
double getArea_(double ma,double mb,double mc) {double
     s=(ma+mb+mc)/2; return 4/3.0 *
     sqrt(s*(s-ma)*(s-mb)*(s-mc));
//ha/hb/hc = length of perpendicular from side a/b/c
double get_Area(double ha,double hb,double hc){
 double H=(1/ha+1/hb+1/hc)/2; double A_=4 * sqrt(H *
      (H-1/ha)*(H-1/hb)*(H-1/hc)); return 1.0/_A_;
bool PtInTriangle(Pt a, Pt b, Pt c, Pt p){
 double s1 = getArea(a,b,c);
```

```
double s2 = getArea(p,b,c) + getArea(p,a,b) +
      getArea(p,c,a);
 return dcmp(s1 - s2) == 0;
namespace Polygonal {
using namespace Vectorial;
using namespace Linear;
using namespace Triangular;
double getSignedArea (Pt* p, int n) {
 double ret = 0;
 for (int i = 0; i < n-1; i++)
  ret += (p[i]-p[0]) * (p[i+1]-p[0]);
 return ret/2:
 int getConvexHull (Pt* p, int n, Pt* ch) {
 sort(p, p + n);
 // preparing lower hull
 int^m = 0;
 for (int i = 0; i < n; i++){
  while (m > 1 && dcmp(getCross(ch[m-1]-ch[m-2],
      p[i]-ch[m-1])) <= 0) m--;
  ch[m++] = p[i];
 // preparing upper hull
 int k = m:
 for (int i = n-2; i \ge 0; i--){
  while (m > k && dcmp(getCross(ch[m-1]-ch[m-2],
      p[i]-ch[m-2])) <= 0) m--;
  ch[m++] = p[i];
 if (n > 1) m--;
 return m:
 int isPtInPolygon (Pt o, Pt* p, int n) {
 int wn = 0;
 for (int i = 0; i < n; i++) {
  int j = (i + 1) \% n;
  if (onSegment(o, p[i], p[j]) || o == p[i]) return 0;
int k = dcmp(getCross(p[j] - p[i], o-p[i]));
  int d1 = dcmp(p[i].y - o.y);
  int d2 = dcmp(p[j].y - o.y);
  if (k > 0 \&\& d1 \le 0 \&\& d2 > 0) wn++:
  if (k < 0 \&\& d2 <= 0 \&\& d1 > 0) wn--;
 return wn ? -1 : 1;
 void rotatingCalipers(Pt *p, int n, vector<Segment>&
     sol) {
 sol.clear();
 int j = 1; p[n] = p[0];
 for (int i = 0; i < n; i++) {
  while (getCross(p[j+1]-p[i+1], p[i]-p[i+1]) >
       getCross(p[j]-p[i+1], p[i]-p[i+1]))
   j = (j+1) \% n;
  sol.push_back(Segment(p[i],p[j]));
  sol.push_back(Segment(p[i + 1], p[j + 1]));
void rotatingCalipersGetRectangle (Pt *p, int n,
     double& area, double& perimeter) {
 p[n] = p[0];
 int l = 1, r = 1, j = 1;
 area = perimeter = 1e20;
 for (int i = 0; i < n; i++) {
  Vector v = (p[i+1]-p[i]) / getLength(p[i+1]-p[i]);
  while (dcmp(getDot(v, p[r%n]-p[i]) - getDot(v,
       p[(r+1)/n]-p[i]) < 0) r++;
```

```
while (j < r \mid | dcmp(getCross(v, p[j%n]-p[i]) -
      getCross(v,p[(j+1)%n]-p[i])) < 0) j++;
 while (1 < j \mid | dcmp(getDot(v, p[1%n]-p[i]) -
      getDot(v, p[(1+1)n]-p[i])> 0) 1++;
 double w = getDot(v, p[r%n]-p[i])-getDot(v,
      p[1%n]-p[i]);
 double h = getDistanceToLine (p[j%n], p[i], p[i+1]);
 area = min(area, w * h);
 perimeter = min(perimeter, 2 * w + 2 * h);
Polygon cutPolygon (Polygon u, Pt a, Pt b) {
Polygon ret;
int n = u.size();
for (int i = 0: i < n: i++) {
 Pt c = u[i], d = u[(i+1)\%n];
 if (dcmp((b-a)*(c-a)) >= 0) ret.push_back(c);
 if (dcmp((b-a)*(d-c)) != 0) {
  getIntersection(a, b-a, c, d-c, t);
  if (onSegment(t, c, d))
   ret.push_back(t);
return ret;
int halfPlaneIntersection(DirLine* li, int n, Pt* poly)
sort(li, li + n);
int first, last;
Pt* p = new Pt[n];
DirLine* q = new DirLine[n];
q[first=last=0] = li[0];
for (int i = 1; i < n; i++) {
 while (first < last && !onLeft(li[i], p[last-1]))</pre>
  while (first < last && !onLeft(li[i], p[first]))</pre>
      first++;
 q[++last] = li[i];
 if (dcmp(q[last].v * q[last-1].v) == 0) {
  if (onLeft(q[last], li[i].p)) q[last] = li[i];
 if (first < last)</pre>
  getIntersection(q[last-1].p, q[last-1].v, q[last].p,
       q[last].v, p[last-1]);
while (first < last && !onLeft(q[first], p[last-1]))</pre>
if (last - first <= 1) { delete [] p; delete [] q;</pre>
     return 0; }
getIntersection(q[last].p, q[last].v, q[first].p,
     q[first].v, p[last]);
for (int i = first; i <= last; i++) poly[m++] = p[i];</pre>
delete [] p; delete [] q;
return m;
Polygon simplify (const Polygon& poly) {
Polygon ret;
int n = poly.size();
for (int i = 0; i < n; i++) {
 Pt a = poly[i];
 Pt b = poly[(i+1)%n];
 Pt c = poly[(i+2)%n];
 if (dcmp((b-a)*(c-b)) != 0 \&\& (ret.size() == 0 || b
      != ret[ret.size()-1]))
  ret.push_back(b);
```

```
return ret:
Pt ComputeCentroid( Pt* p,int n){
 double scale = 6.0 * getSignedArea(p,n);
 for (int i = 0; i < n; i++){
 int j = (i+1) \% n;
 c = c + (p[i]+p[j])*(p[i].x*p[j].y - p[j].x*p[i].y);
return c / scale;
// pt must be in ccw order with no three collinear Pts
// returns inside = 1, on = 0, outside = -1
int PtInConvexPolygon(Pt* pt, int n, Pt p){
 assert(n >= 3);
 int lo = 1 , hi = n - 1 ;
 while (hi - lo > 1) {
 int mid = (lo + hi) / 2;
 if(getCross(pt[mid] - pt[0], p - pt[0]) > 0) lo = mid;
 bool in = PtInTriangle(pt[0], pt[lo], pt[hi], p);
 if(!in) return -1:
 if(getCross(pt[lo] - pt[lo-1], p - pt[lo-1]) == 0)
     return 0;
 if(getCross(pt[hi] - pt[lo], p - pt[lo]) == 0) return
 if(getCross(pt[hi] - pt[(hi+1)%n], p - pt[(hi+1)%n])
     == 0) return 0;
return 1;
// Calculate [ACW, CW] tangent pair from an external Pt
#define CW
#define ACW
int direction(Pt st, Pt ed, Pt q) {return
dcmp(getCross(ed - st, q - ed));}
bool isGood(Pt u, Pt v, Pt Q, int dir) {return
    direction(Q, u, v) != -dir;}
Pt better(Pt u, Pt v, Pt Q, int dir) {return
    direction(Q, u, v) == dir ? u : v;
Pt tangents(Pt* pt, Pt Q, int dir, int lo, int hi){
 while (hi - lo > 1) {
 int mid = (lo + hi)/2;
 bool pvs = isGood(pt[mid], pt[mid - 1], Q, dir);
 bool nxt = isGood(pt[mid], pt[mid + 1], Q, dir);
 if(pvs && nxt) return pt[mid];
  if(!(pvs || nxt)){
  Pt p1 = tangents(pt, Q, dir, mid+1, hi);
  Pt p2 = tangents(pt, Q, dir, lo, mid - 1);
  return better(p1, p2, Q, dir);
  if(!pvs){
  if(direction(Q, pt[mid], pt[lo]) == dir) hi = mid -
   else if(better(pt[lo], pt[hi], Q, dir) == pt[lo]) hi
      = mid - 1:
   else lo = mid + 1;
 if(!nxt){
  if(direction(Q, pt[mid], pt[lo]) == dir) lo = mid +
   else if(better(pt[lo], pt[hi], Q, dir) == pt[lo]) hi
      = mid - 1;
   else lo = mid + 1:
 Pt ret = pt[lo];
 for(int i = lo + 1; i <= hi; i++) ret = better(ret,</pre>
     pt[i], Q, dir);
return ret;
```

```
// [ACW, CW] Tangent
pair<Pt, Pt> get_tangents(Pt* pt, int n, Pt Q){
 Pt acw_tan = tangents(pt, Q, ACW, 0, n - 1);
 Pt cw_tan = tangents(pt, Q, CW, 0, n - 1);
 return make_pair(acw_tan, cw_tan);
struct Circle {
Pt o; double r;
Circle () {}
Circle (Pt o, double r = 0): o(o), r(r) {}
void read () { o.read(), scanf("%lf", &r); }
Pt pt(double rad) { return Pt(o.x + cos(rad)*r, o.y +
     sin(rad)*r): }
double getArea (double rad) { return rad * r * r / 2; }
//area of the circular sector cut by a chord with
     central angle alpha
double sector(double alpha) {return r * r * 0.5 *
     (alpha - sin(alpha));}
namespace Circular {
using namespace Linear;
using namespace Vectorial;
using namespace Triangular;
int getLineCircleIntersection (Pt p, Pt q, Circle O,
     double& t1, double& t2, vector<Pt>& sol) {
 Vector v = q - p;
 //sol.clear():
 double a = v.x, b = p.x - 0.o.x, c = v.y, d = p.y - 0.o.x
     0.o.y;
 double e = a*a+c*c, f = 2*(a*b+c*d), g =
     b*b+d*d-0.r*0.r;
 double delta = f*f - 4*e*g;
 if (dcmp(delta) < 0) return 0;</pre>
 if (dcmp(delta) == 0) {
  t1 = t\bar{2} = -f / (2 * e);
  sol.push_back(p + v * t1);
  return 1;
 t1 = (-f - sqrt(delta)) / (2 * e); sol.push_back(p + v
 t2 = (-f + sqrt(delta)) / (2 * e); sol.push_back(p + v
 return 2;
// signed area of intersection of circle(c.o, c.r) and
// triangle(c.o, s.a, s.b) [cross(a-o, b-o)/2]
double areaCircleTriIntersection(Circle c, Segment s){
 using namespace Linear;
 double OA = getLength(c.o - s.a);
 double OB = getLength(c.o - s.b);
 // sector
 if (dcmp(getDistanceToSegment(c.o, s.a, s.b) - c.r) >=
  return fix_acute(getSignedAngle(s.a - c.o, s.b -
      (c.o)) * (c.r*c.r) / 2.0;
 // triangle
 if (dcmp(OA - c.r) \le 0 \&\& dcmp(OB - c.r) \le 0)
  return getCross(c.o-s.b,s.a-s.b) / 2.0;
 // three part: (A, a) (a, b) (b, B)
 vector<Pt>Sect; double t1,t2;
 getLineCircleIntersection(s.a, s.b, c, t1, t2, Sect);
 return areaCircleTriIntersection(c, Segment(s.a,
     Sect[0]))
  + areaCircleTriIntersection(c, Segment(Sect[0],
      Sect[1]))
  + areaCircleTriIntersection(c, Segment(Sect[1], s.b));
```

```
// area of intersecion of circle(c.o, c.r) and simple
    polyson(p[])
double areaCirclePolygon(Circle c, Polygon p){
 double res = .0:
 int n = p.size();
 for (int i = 0; i < n; ++ i)</pre>
 res += areaCircleTriIntersection(c, Segment(p[i],
      p[(i+1)%n]));
return fabs(res);
// interior (d < R - r) \longrightarrow -2
// interior tangents (d = R - r) \longrightarrow -1
// concentric (d = 0)
// secants
              (R - r < d < R + r) \longrightarrow 0
// exterior tangents (d = R + r) ----> 1
// exterior (\tilde{d} > R + r) \longrightarrow 2
int getPos(Circle o1, Circle o2) {
using namespace Vectorial;
 double d = getLength(o1.o - o2.o);
 int in = dcmp(d - fabs(o1.r - o2.r)), ex = dcmp(d - fabs(o1.r - o2.r))
(o1.r + o2.r));
return in<0 ? -2 : in==0? -1 : ex==0 ? 1 : ex>0? 2 : 0;
int getCircleCircleIntersection (Circle o1, Circle o2,
    vector<Pt>& sol) {
 double d = getLength(o1.o - o2.o);
 if (dcmp(d) == 0) {
 if (dcmp(o1.r - o2.r) == 0) return -1;
 return 0;
 if (dcmp(o1.r + o2.r - d) < 0) return 0;
 if (dcmp(fabs(o1.r-o2.r) - d) > 0) return 0;
 Vector \mathbf{v} = 02.0 - 01.0;
 double co = (o1.r*o1.r + getPLength(v) - o2.r*o2.r) /
     (2 * o1.r * getLength(v));
 double si = sqrt(fabs(1.0 - co*co));
 Pt p1 = scale(cw(v,co, si), o1.r) + o1.o;
 Pt p2 = scale(ccw(v,co, si), o1.r) + o1.o;
 sol.push_back(p1);
 if (p1 == p2) return 1;
 sol.push_back(p2);
 return 2;
double areaCircleCircle(Circle o1, Circle o2){
Vector AB = o2.o - o1.o; double d = getLength(AB);
 if(d \ge 01.r + 02.r) return 0;
 if(d + o1.r <= o2.r) return pi * o1.r * o1.r;
 if(d + o2.r <= o1.r) return pi * o2.r * o2.r;
 double alpha1 = acos((o1.r * o1.r + d * d - o2.r *
     o2.r) / (2.0 * o1.r * d));
 double alpha2 = acos((o2.r * o2.r + d * d - o1.r *
     o1.r) / (2.0 * o2.r * d));
 return o1.sector(2*alpha1) + o2.sector(2*alpha2);
int getTangents (Pt p, Circle o, Vector* v) {
Vector u = 0.0 - p;
 double d = getLength(u);
 if (d < o.r) return 0;
 else if (dcmp(d - o.r)) == 0) {
 v[0] = rotate(u, pi / 2);
 return 1;
 } else {
  double ang = asin(o.r / d);
  v[0] = rotate(u, -ang);
  v[1] = rotate(u, ang);
 return 2;
int getTangentPts (Pt p, Circle o, vector<Pt>& v) {
```

```
Vector u = p - o.o : double d = getLength(u):
 if (d < o.r) return 0:
 else if (dcmp(d - o.r) == 0) {
 v.push_back(o.o+u);
 return 1;
 } else {
 double ang = acos(o.r / d);
 u = u / getLength(u) * o.r;
 v.push_back(o.o+rotate(u, -ang));
 v.push_back(o.o+rotate(u, ang));
 return 2;
int getTangents (Circle o1, Circle o2, Pt* a, Pt* b) {
int cnt = 0;
 if (dcmp(o1.r-o2.r) < 0) \{ swap(o1, o2); swap(a, b); \}
 double d2 = getPLength(o1.o - o2.o);
 double rdif = o1.r = o2.r, rsum = o1.r + o2.r;
 if (dcmp(d2 - rdif * rdif) < 0) return 0;</pre>
 if (dcmp(d2) == 0 \&\& dcmp(o1.r - o2.r) == 0) return -1; |25
 double base = getAngle(o2.o - o1.o);
 if (dcmp(d2 - rdif * rdif) == 0) {
 a[cnt] = o1.pt(base); b[cnt] = o2.pt(base); cnt++;
 double ang = acos((o1.r - o2.r) / sqrt(d2));
 a[cnt] = o1.pt(base+ang); b[cnt] = o2.pt(base+ang);
     cnt++:
 a[cnt] = o1.pt(base-ang); b[cnt] = o2.pt(base-ang);
 if (dcmp(d2 - rsum * rsum) == 0) {
 a[cnt] = o1.pt(base); b[cnt] = o2.pt(pi+base); cnt++;
 else if (dcmp(d2 - rsum * rsum) > 0) {
 double ang = acos((o1.r + o2.r) / sqrt(d2));
 a[cnt] = o1.pt(base+ang); b[cnt] =
      o2.pt(pi+base+ang); cnt++;
 a[cnt] = o1.pt(base-ang); b[cnt] =
      o2.pt(pi+base-ang); cnt++;
 return cnt;
Circle CircumscribedCircle(Pt p1, Pt p2, Pt p3) {
double Bx = p2.x - p1.x, By = p2.y - p1.y;
double Cx = p3.x - p1.x, Cy = p3.y - p1.y;
 double D = 2 * (Bx * Cy - By * Cx);
 double cx = (Cy * (Bx * Bx + By * By) - By * (Cx * Cx
     + Cy * Cy)) / D + p1.x;
 double Cy = (Bx * (Cx * Cx + Cy * Cy) - Cx * (Bx * Bx)
     + By * By)) / D + p1.y;
 Pt p = Pt(cx, cy);
return Circle(p, getLength(p1 - p));
Circle InscribedCircle(Pt p1, Pt p2, Pt p3) {
 double a = getLength(p2 - p3);double b = getLength(p3
     - p1);
 double c = getLength(p1 - p2);
 Pt p = (p1 * a + p2 * b + p3 * c) / (a + b + c);
return Circle(p, getDistanceToLine(p, p1, p2));
//distance From P : distance from Q = rp : rq
Circle getApolloniusCircle(const Pt& P, const Pt& Q,
 double rp, double rq ){
rq *= rq ;rp *= rp ;
 double a = rq - rp;
 assert(dcmp(a));
 double g = rq * P.x - rp * Q.x ; g /= a ;
 double h = rq * P.y - rp * Q.y ; h /= a ;
```

25 Geo 3D

```
const double pi = 4 * atan(1);
const double eps = 1e-10;
inline int dcmp (double x) { if (fabs(x) < eps) return
    0; else return x < 0 ? -1 : 1; }
inline double torad(double deg) { return deg / 180 * pi;
    }
struct Point{
double x, y;
Point (double x = 0, double y = 0): x(x), y(y) {}
Point operator + (const Point& u) { return Point(x +
     u.x, y + u.y); }
Point operator - (const Point& u) { return Point(x -
     u.x, y - u.y); }
Point operator * (const double u) { return Point(x * u,
     y * u); }
Point operator / (const double u) { return Point(x / u,
double operator * (const Point& u) { return x*u.y -
    y*u.x; }
struct Pt3D{
double x, y, z;
Pt3D() {}
void read () {cin>>x>>y>>z;}
 void write () {cout<<x<<" --- "<<y<<" --- "<<z<<"\n":}</pre>
Pt3D(double x, double y, double z): x(x), y(y), z(z) {}
Pt3D(const Pt3D &p) : x(p.x), y(p.y), z(p.z) {}
Pt3D operator + (Pt3D b) {return Pt3D(x+b.x,y+b.y,
     z+b.z);}
Pt3D operator -(Pt3D b) {return Pt3D(x-b.x,y-b.y,
     z-b.z);}
Pt3D operator *(double b) {return Pt3D(x*b,y*b, z*b);}
Pt3D operator /(double b) {return Pt3D(x/b,y/b, z/b);}
bool operator <(Pt3D b) {return
     make_pair(make_pair(x,y),z) <</pre>
     make_pair(make_pair(b.x,b.y),b.z);}
bool operator ==(Pt3D b) {return dcmp(x-b.x)==0 &&
     dcmp(y-b.y) == 0 && dcmp(z-b.z) == 0;
typedef Pt3D Vector3D;
typedef vector<Point> Polygon;
typedef vector<Pt3D> Polyhedron;
namespace Vectorial{
double getDot (Vector3D a, Vector3D b) {return
     a.x*b.x+a.y*b.y+a.z*b.z;
Vector3D getCross(Vector3D a, Vector3D b) {return
     Pt3D(a.y*b.z-a.z*b.y, a.z*b.x-a.x*b.z,
     a.x*b.y-a.y*b.x);
```

```
double getLength (Vector3D a) {return sgrt(getDot(a.
    a)); }
double getPLength (Vector3D a) {return getDot(a, a); }
Vector3D unitVector(Vector3D v) {return v/getLength(v);}
double getUnsignedAngle(Vector3D u, Vector3D v){
 double cosTheta =
     getDot(u,v)/getLength(u)/getLength(v);
 cosTheta = max(-1.0, min(1.0, cosTheta));
 return acos(cosTheta);
Vector3D rotate(Vector3D v, Vector3D a, double rad) {
 a = unitVector(a);
 return v * cos(rad) + a * (1 - cos(rad)) * getDot(a,v)
     + getCross(a,v) * sin(rad);
struct Line3D{
Vector3D v: Pt3D o:
Line3D() \{\};
Line3D(Vector3D v,Pt3D o):v(v),o(o){}
Pt3D getPoint(double t) {return o + v*t;}
namespace Linear{
using namespace Vectorial;
double getDistSq(Line3D 1, Pt3D p) {return
    getPLength(getCross(1.v,p-1.o))/getPLength(1.v);}
double getDistLinePoint(Line3D 1, Pt3D p) {return
    sqrt(getDistSq(1,p));}
bool cmp(Line3D 1,Pt3D p, Pt3D q) {return getDot(1.v,p)
    < getDot(1.v,q);}
Pt3D projection(Line3D 1,Pt3D p) {return 1.o + 1.v *
     getDot(1.v,p-1.o)/getPLength(1.v);}
Pt3D reflection(Line3D 1,Pt3D p) {return
     projection(1,p)+projection(1,p)-p;}
double getAngle(Line3D 1,Line3D m) {return
     getUnsignedAngle(1.v,m.v);}
bool isParallel(Line3D p,Line3D q) {return
     dcmp(getPLength(getCross(p.v,q.v))) == 0;}
bool isPerpendicular(Line3D p,Line3D q) {return
     dcmp(getDot(p.v,q.v)) == 0;
double getDist(Line3D 1, Line3D m){
 Vector3D n = getCross(1.v, m.v);
 if(getPLength(n) == 0) return getDistLinePoint(1,m.o);
 else return fabs(getDot(m.o-l.o , n)) / getLength(n);
Pt3D getClosestPointOnLine1(Line3D 1,Line3D m){
 Vector3D n = getCross(1.v, m.v);
 Vector3D n2 = getCross(m.v, n);
 return 1.o + 1.v * getDot(m.o-1.o, n2) / getDot(1.v,
     n2);
struct Plane{
Vector3D n; //normal n
double d; //getDot(n,p) = d for any point p on the plane
Plane() {}
Plane(Vector3D n, double d) : n(n), d(d) {}
Plane(Vector3D n, Pt3D p) : n(n), d(Vectorial ::
    getDot(n,p)) {}
Plane(const Plane &p): n(p.n), d(p.d) {}
namespace Planar{
using namespace Vectorial;
Plane getPlane(Pt3D a,Pt3D b,Pt3D c) {return
    Plane(getCross(b-a,c-a),a);}
Plane translate(Plane p, Vector 3D t) {return Plane(p.n,
    p.d+getDot(p.n,t));}
```

```
Plane shiftUp(Plane p, double dist) {return Plane(p.n,
    p.d+dist*getLength(p.n));}
Plane shiftDown(Plane p,double dist) {return Plane(p.n,
    p.d-dist*getLength(p.n));}
double getSide(Plane p,Pt3D a) {return
    getDot(p.n,a)-p.d;}
double getDistance(Plane p,Pt3D a) {return
    fabs(getSide(p,a))/getLength(p.n);}
Pt3D projection(Plane p,Pt3D a) {return
    a-p.n*getSide(p,a)/getPLength(p.n);}
Pt3D reflection(Plane p,Pt3D a) {return
    a-p.n*getSide(p,a)/getPLength(p.n)*2;}
bool intersect(Plane p, Line3D 1, Pt3D& a){
if(dcmp(getDot(p.n,l.v)) == 0) return false;
 a = 1.0 - 1.v * getSide(p,1.o) / getDot(p.n,1.v);
return true;
bool intersect(Plane p,Plane q,Line3D& 1){
1.v = getCross(p.n,q.n);
 if(dcmp(getPLength(1.v)) == 0) return false;
 1.o = getCross(q.n*p.d - p.n*q.d , 1.v) /
     getPLength(1.v);
return true;
double getAngle(Plane p,Plane q) {return
    getUnsignedAngle(p.n,q.n);}
bool isParallel(Plane p,Plane q) {return
    dcmp(getPLength(getCross(p.n,q.n))) == 0;}
bool isPerpendicular(Plane p,Plane q) {return
    dcmp(getDot(p.n,q.n)) == 0;
bool getAngle(Plane p,Line3D 1) {return pi/2.0 -
    getUnsignedAngle(p.n,l.v);}
bool isParallel(Plane p, Line3D 1) {return
    dcmp(getDot(p.n,l.v)) == 0;
bool isPerpendicular(Plane p,Line3D 1) {return
    dcmp(getPLength(getCross(p.n,1.v))) == 0;}
Line3D perpThrough(Plane p,Pt3D a) {return
    Line3D(p.n,a);}
Plane perpThrough(Line3D 1,Pt3D a) {return
    Plane(1.v,a);}
//Modify p.n if necessary with respect to the reference
    point
Vector3D rotateCCW90(Plane p, Vector3D d) {return
    getCross(p.n,d);}
Vector3D rotateCW90(Plane p, Vector3D d) {return
    getCross(d.p.n):}
pair Pt3D, Pt3D TwoPointsOnPlane(Plane p){
Vector3D N = p.n; double D = p.d;
 assert(dcmp(N.x) != 0 || dcmp(N.y) != 0 || dcmp(N.z)
     ! = 0);
 if(dcmp(N.x) == 0 \&\& dcmp(N.y) == 0) return
     \{Pt3D(1,0,D/N.z), Pt3D(0,1,D/N.z)\};
 if(dcmp(N.y) == 0 \&\& dcmp(N.z) == 0) return
     \{Pt3D(D/N.x,1,0), Pt3D(D/N.x,0,1)\};
 if(dcmp(N.z) == 0 \&\& dcmp(N.x) == 0) return
     \{Pt3D(1,D/N.y,0), Pt3D(0,D/N.y,1)\};
 if(dcmp(N.x) == 0) return \{Pt3D(1,D/N.y,0),
     Pt3D(0,0,D/N.z);
 if(dcmp(N.y) == 0) return \{Pt3D(0,1,D/N.z),
     Pt3D(D/N.x,0,0);
 if(dcmp(N.z) == 0) return \{Pt3D(D/N.x,0,1),
     Pt3D(0,D/N.y,0)};
 if (dcmp(D)!=0) return \{Pt3D(D/N.x,0,0),
     Pt3D(0,D/N.y,0)};
return {Pt3D(N.y,-N.x,0), Pt3D(-N.y,N.x,0)};
Point From3Dto2D(Plane p, Pt3D a){
assert( dcmp(getSide(p,a)) == 0 );
```

```
Pt3D A = Pair.first:
  Pt3D B = Pair.second;
  Vector3D Z = p.n; Z = Z / getLength(Z);
  Vector3D X = B - A; X = X / getLength(X);
  Vector3D Y = getCross(Z,X);
  Vector3D v = \tilde{a} - A;
  assert( dcmp(getDot(v,Z)) == 0);
 return Point(getDot(v,X),getDot(v,Y));
Pt3D From2Dto3D(Plane p, Point a){
 auto Pair = TwoPointsOnPlane(p);
  Pt3D A = Pair.first;
 Pt3D B = Pair.second;
  Vector3D Z = p.n; Z = Z / getLength(Z);
  Vector3D X = B - A; X = X / getLength(X);
 Vector3D Y = getCross(Z,X);
return A + X * a.x + Y * a.y;
struct Sphere{
Pt3D c;
 double r;
Sphere() {}
Sphere(Pt3D c, double r) : c(c), r(r) {}
 //Spherical cap with polar angle theta
 double Height(double alpha) {return r*(1-cos(alpha));}
 double BaseRadius(double alpha) {return r*sin(alpha);}
double Volume(double alpha) {double h = Height(alpha);
     return pi*h*h*(3*r-h)/3.0;}
 double SurfaceArea(double alpha) {double h =
     Height(alpha); return 2*pi*r*h;}
namespace Spherical{
using namespace Vectorial;
 using namespace Planar;
 using namespace Linear;
Sphere CircumscribedSphere(Pt3D a,Pt3D b,Pt3D c,Pt3D d){|26\>
 assert( dcmp(getSide(getPlane(a,b,c), d)) != 0);
  Plane U = Plane(a-b, (a+b)/2);
  Plane V = Plane(b-c, (b+c)/2);
  Plane W = Plane(c-d, (c+d)/2);
  Line3D 11,12;
  bool ret1 = intersect(U,V,l1);
  bool ret2 = intersect(V,W,12);
  assert(ret1 == true && ret2 == true);
  assert( dcmp(getDist(11,12)) == 0);
  Pt3D C = getClosestPointOnLine1(11,12);
 return Sphere(C, getLength(C-a));
 pair<double, double> SphereSphereIntersection(Sphere
     s1,Sphere s2){
  double d = getLength(s1.c-s2.c);
  if(dcmp(d - s1.r - s2.r) >= 0) return {0,0};
  double R1 = \max(s1.r, s2.r); double R2 = \min(s1.r, s2.r);
  double y = R1 + R2 - d;
  double x = (R1*R1 - R2*R2 + d*d) / (2*d);
 double h1 = R1 - x;
 double h2 = y - h1;
  double Volume = pi*h1*h1*(3*R1-h1)/3.0 +
      pi*h2*h2*(3*R2-h2)/3.0;
  double SurfaceArea = 2*pi*R1*h1 + 2*pi*R2*h2;
 return make_pair(SurfaceArea, Volume);
 Pt3D getPointOnSurface(double r,double Lat,double Lon){
 Lat = torad(Lat); //North-South
Lon = torad(Lon); //East-West
 return Pt3D(r*cos(Lat)*cos(Lon), r*cos(Lat)*sin(Lon),
      r*sin(Lat));
```

auto Pair = TwoPointsOnPlane(p):

```
int intersect(Sphere s,Line3D 1, vector<Pt3D>& ret){
 double h2 = s.r*s.r - getDistSq(1,s.c);
 if(dcmp(h2)<0) return 0;</pre>
 Pt3D p = projection(1,s.c);
if(dcmp(h2) == 0) {ret.push_back(p); return 1;}
 Vector3D h = 1.v * sqrt(h2) / getLength(1.v);
 ret.push_back(p-h); ret.push_back(p+h); return 2;
double GreatCircleDistance(Sphere s,Pt3D a,Pt3D b){
return s.r * getUnsignedAngle(a-s.c, b-s.c);
namespace Poly{
using namespace Vectorial;
Sphere SmallestEnclosingSphere(Polyhedron p){
 int n = p.size();
 Pt3D C(0,0,0);
 for(int i=0; i < n; i++) C = C + p[i];
 C = C / n;
 double P = 0.1;
 int pos = 0;
 int Accuracy = 70000:
 for (int i = 0; i < Accuracy; i++) {</pre>
 pos = 0:
  for (int j = 1; j < n; j++){
   if(getPLength(C - p[j]) > getPLength(C - p[pos]))
       pos = j;
  C = C + (p[pos] - C)*P;
 P *= 0.998;
return Sphere(C, getPLength(C - p[pos]));
```

$26 \quad HLD$

```
struct HLD{
vector<int> parent, depth, heavy, head, pos;
int cur_pos; segtree seg;
int dfs(int v, vector<vector<int>> const& adj) {
 int size = 1; int max_c_size = 0;
 for (int c : adj[v]) {
 if (c != parent[v]) {
  parent[c] = v, depth[c] = depth[v] + 1;
   int c_size = dfs(c, adj);
   size += c_size;
   if (c_size > max_c_size) max_c_size = c_size,
       heavy[v] = c;
return size;
void decompose(int v, int h, vector<vector<int>> const&
 head[v] = h, pos[v] = cur_pos++;
 if (heavy[v] != -1)
 decompose(heavy[v], h, adj);
 for (int c : adj[v]) {
 if (c != parent[v] && c != heavy[v])
   decompose(c, c, adj);
void init(vector<vector<int>> const& adj, vector<11>&a)
 int n = adj.size();
 parent = vector<int>(n); depth = vector<int>(n);
 heavy = vector<int>(n, -1); head = vector<int>(n);
```

```
pos = vector < int > (n): cur pos = 0:
 dfs(0, adj); decompose(0, 0, adj);
 vector<ll>tmp(n);
 for(int i=0; i<n; i++) {
  tmp[pos[i]] = a[i];</pre>
 seg.init(n,tmp);
int query(int a, int b) {
11 res = 0;
 for (; head[a] != head[b]; b = parent[head[b]]) {
 if (depth[head[a]] > depth[head[b]])
  swap(a, b);
 int cur_heavy_path_max = seg.query(pos[head[b]],
 res += cur_heavy_path_max;
 if (depth[a] > depth[b])
 swap(a, b);
 int last_heavy_path_max = seg.query(pos[a], pos[b]);
 res += last_heavy_path_max;
return res;
void update(int a, int b, int x) {
 for (; head[a] != head[b]; b = parent[head[b]]) {
 if (depth[head[a]] > depth[head[b]]) swap(a, b);
 seg.update(pos[head[b]], pos[b], x);
 if (depth[a] > depth[b]) swap(a, b);
 seg.update(pos[a], pos[b],x);
```

27 Hackenbush

```
struct hackenbush {
int n;
vector<vector<int>> adj;
hackenbush(int n) : n(n), adj(n) { }
void_add_edge(int u, int v) {
 adj[u].push_back(v);
 if (u != v) adj[v].push_back(u);
 // r is the only root connecting to the ground
int grundy(int r) {
 vector<int> num(n), low(n);
  int t = 0:
  function<int(int, int)> dfs = [&](int p, int u) {
  num[u] = low[u] = ++t;
  int ans = 0;
  for (int v : adj[u]) {
   if (v == p) { p += 2 * n; continue; }
if (num[v] == 0) {
    int res = dfs(u, v);
low[u] = min(low[u], low[v]);
if (low[v] > num[u]) ans ^= (1 + res) ^ 1; // bridge
    else ans ^= res; // non bridge
   } else low[u] = min(low[u], num[v]);
  if (p > n) p = 2 * n;
  for (int v : adj[u])
   if (v != p && num[u] <= num[v]) ans ^= 1;</pre>
  return ans;
 return dfs(-1, r);
int main() {
int cases; scanf("%d", &cases);
```

```
for (int icase = 0; icase < cases; ++icase) {
   int n; scanf("%d", &m);
   vector<int> ground(n);
   int r;
   for (int i = 0; i < n; ++i) {
      scanf("%d", &ground[i]);
      if (ground[i] == 1) r = i;
   }
   int ans = 0;
   hackenbush g(n);
   for (int i = 0; i < n - 1; ++i) {
      int u, v;
      scanf("%d %d", &u, &v);
      --u; --v;
      if (ground[u]) u = r;
      if (ground[v]) v = r;
      if (u == v) ans ^= 1;
      else g.add_edge(u, v);
   }
   int res = ans ^ g.grundy(r);
   printf("%d\n", res != 0);
}
</pre>
```

28 Hashing 2D

```
int mods[2] = {1000000007, 1000000009};
int bases[2] = {137, 281};
int pwbase[2][MAX];
void Preprocess(){
pwbase[0][0] = pwbase[1][0] = 1;
for(int i = 0; i < 2; i++) {</pre>
 for(int j = 1; j < MAX; <math>j++) {
  pwbase[i][j] = (pwbase[i][j - 1] *111* bases[i]) %
       mods[i]:
struct Hashing{
 int hsh[2][MAX];
 string str;
 void setstr(string &_str) {
 str = _str;
hsh[0][str.size()] = 0;
  hsh[1][str.size()] = 0;
  Build();
 void Build() {
  for(int i = str.size() - 1; i >= 0; i--) {
  for(int j = 0; j < 2; j++) {
  hsh[j][i] = ((hsh[j][i + 1] *111* bases[j] %</pre>
         mods[j]) + str[i]);
    if(hsh[j][i]>=mods[j])
        hsh[j][i]-=mods[j];
 pair<int,int> GetHash(int i, int j) {
  assert(i <= j);</pre>
  int tmp1 = (hsh[0][i] - (hsh[0][j + 1] *111*
       pwbase[0][j - i + 1]) \% mods[0]);
  int tmp2 = (hsh[1][i] - (hsh[1][j + 1] *111*
      pwbase[1][j - i + 1]) % mods[1]);
  if(tmp1 < 0)
  tmp1^+ = mods[0];
  if(tmp2 < 0)
  tmp2 += mods[1];
  return make_pair(tmp1, tmp2);
```

29 Hopcroft

```
// If input graph is not given in L-R manner, make it so
    by coloring.
// Input graph must be bipartite
const int N=200*200+5;
struct HopcroftKarp
static const int inf = 1e9; int n;
vector<int> 1, r, d; vector<vector<int>> g;
HopcroftKarp(int _n, int _m) {
n = n; int p = n + m + 1;
 g.resize(p); l.resize(p, 0); r.resize(p, 0);
     d.resize(p, 0);
void add_edge(int u, int v) {
 g[u].push_back(v + n); //right id is increased by n,
     so is l[u]
bool bfs() {
 queue<int> q;
 for (int u = 1; u <= n; u++) {
  if (!1[u])
   d[u] = 0, q.push(u);
   d[u] = inf;
 d[0] = inf:
 while (!q.empty()) {
  int u = q.front(); q.pop();
  for (auto v : g[u]) { if (d[r[v]] == inf) {
    d[r[v]] = d[u] + 1; q.push(r[v]);
 return d[0] != inf;
bool dfs(int u) {
 if (!u) return true;
 for (auto v : g[u]){ if(d[r[v]] == d[u] + 1 &&
     dfs(r[v])){
   l[u] = v; r[v] = u;
   return true;
 d[u] = inf;
 return false;
int maximum_matching(){
 int ans = \overline{0}:
 while (bfs()){ for(int u = 1; u <= n; u++)
  if (!l[u] && dfs(u)) ans++;
 return ans;
```

30 Hungarian

```
namespace wm{
bool vis[N]; int U[N],V[N],P[N];
int way[N],minv[N],match[N],ar[N][N];
///n=no of row, m=no of col,1
    based,flag=MAXIMIZE/MINIMIZE
///match[i]=the column to which row i is matched
int hungarian(int n,int m,int mat[N][N],int flag){
    clr(U), clr(V), clr(P), clr(ar), clr(way);
    for (int i = 1; i <= n; i++){</pre>
```

```
for (int i = 1: i <= m: i++){
    ar[i][j] = mat[i][j];
    if (flag == MAXIMIZE) ar[i][j] = -ar[i][j];
if (n > m) m = n;
int i, j, a, b, c, d, r, w;
for (i = 1; i <= n; i++){
  P[0] = i, b = 0;
  for (j=0; j \le m; j++) \min v[j] = \inf, vis[j] = 0;
   vis[b] = true; a = P[b], d = 0, w = inf;
   for (j = 1; j \le m; j++){
     if (!vis[i]){
       r = ar[a][j] - U[a] - V[j];
        if (r < minv[j]) minv[j] = r, way[j]=b;</pre>
        if (minv[i] < w) w = minv[i], d = i;</pre>
   for (j = 0; j <= m; j++){
  if (vis[j]) U[P[j]] += w, V[j] -= w;</pre>
     else minv[j] -= w;
    b = d;
 } while (P[b] != 0);
    d = way[b]; P[b] = P[d], b = d;
  } while (b != 0);
for (j = 1; j <= m; j++) match[P[j]] = j;
return (flag == MINIMIZE) ? -V[0] : V[0];
```

31 KnuthDP

32 LCA

```
template <class T>
struct RMQ { // O-based
  vector<vector<T>> rmq;
  T kInf = numeric_limits<T>::max();
  void build(const vector<T>& V) {
   int n = V.size(), on = 1, dep = 1;
   while (on < n) on *= 2, ++dep;
   rmq.assign(dep, V);
  for (int i = 0; i < dep - 1; ++i)
   for (int j = 0; j < n; ++j) {</pre>
```

```
rmq[i + 1][j] = min(rmq[i][j], rmq[i][min(n - 1, j +
       (1 << i)));
T query(int a, int b) { // [a, b)
 if (b <= a) return kInf;</pre>
 int dep = 31 - \_builtin\_clz(b - a); // log(b - a)
 return min(rmq[dep][a], rmq[dep][b - (1 << dep)]);</pre>
struct LCA { // 0-based
vector<int> enter, depth, exxit;
vector<vector<int>> G;
 vector<pair<int, int>> linear;
RMQ<pair<int, int>> rmq;
 int timer = 0;
LCA() {}
LCA(int n) : enter(n, -1), exxit(n, -1), depth(n),
    G(n), linear(2 * n) {}
 void dfs(int node, int dep) {
 linear[timer] = {dep, node};
 enter[node] = timer++;
 depth[node] = dep;
 for (auto vec : G[node])
 if (enter[vec] == -1) {
  dfs(vec, dep + 1);
  linear[timer++] = {dep, node};
 exxit[node] = timer;
void add_edge(int a, int b) {
 G[a].push_back(b);
 G[b].push_back(a);
void build(int root) {
 dfs(root, 0);
 rmq.build(linear);
int query(int a, int b) -
 a = enter[a]. b = enter[b]:
 return rmq.query(min(a, b), max(a, b) + 1).second;
int dist(int a, int b) {
 return depth[a] + depth[b] - 2 * depth[query(a, b)];
```

33 LCABinaryLift

```
vector<vector<int>> adi;
int timer;
vector<int> tin, tout;
vector<vector<int>> up
void dfs(int v, int p){
tin[v] = ++timer;
up[v][0] = p;
for (int i = 1; i <= 1; ++i)
 up[v][i] = up[up[v][i-1]][i-1];
 for (int u : adj[v]) {
 if (u != p)
  dfs(u, v);
tout[v] = ++timer;
bool is_ancestor(int u, int v){
return tin[u] <= tin[v] && tout[u] >= tout[v];
int lca(int u, int v){
if (is_ancestor(u, v))
```

```
return u;
if (is_ancestor(v, u))
  return v;
for (int i = 1; i >= 0; --i) {
  if (!is_ancestor(up[u][i], v))
    u = up[u][i];
}
return up[u][0];
}
void preprocess(int root) {
  tin.resize(n);
  tout.resize(n);
  timer = 0;
  1 = ceil(log2(n));
  up.assign(n, vector<int>(1 + 1));
  dfs(root, root);
}
```

34 LCT rooted

```
typedef pair< int, int >Linear;
Linear compose(const Linear &p, const Linear &q)
return Linear(mul(p.first, q.first),
     sum(mul(q.second, p.first), p.second));
struct SplayTree
struct Node {
 int ch[2] = \{0, 0\}, p = 0;
 long long self = 0, path = 0;//Path aggregates
 long long sub = 0, vir = 0;//Subtree aggregate
int size = 1; bool flip = 0;// Lazy tags
 Linear _self{1, 0}, shoja{1, 0}, ulta{1, 0};
vector<Node> T;
SplayTree(int n) : T(n + 1) {
 T[0].size = 0;
void push(int x) {
 if (!x || !T[x].flip)
 int l = T[x].ch[0], r = T[x].ch[1];
 T[1].flip ^= 1, T[r].flip ^= 1;
 swap(T[x].ch[0], T[x].ch[1]); T[x].flip = 0;
 swap(T[x].shoja, T[x].ulta);
void pull(int x) {
 int l=T[x].ch[0],r=T[x].ch[1];
 push(1);
 push(r);
 T[x].size = T[1].size + T[r].size + 1;

T[x].path = T[1].path + T[x].self + T[r].path;
 T[x].sub=T[x].vir+T[1].sub+T[r].sub+T[x].self;
 T[x].shoja = compose(T[r].shoja,
       compose(T[x]._self, T[1].shoja));
 T[x].ulta = compose(T[1].ulta,
      compose(T[x]._self, T[r].ulta));
void set(int x, int d, int y) {
 T[x].ch[d] = y; T[y].p = x; pull(x);
void splay(int x) {
 auto dir = [\&](int x)
  int p = T[x].p;
  if (!p) return -1;
  return T[p].ch[0]==x?0:T[p].ch[1]==x?1:-1;
```

```
int y = T[x].p,z=T[y].p,dx=dir(x),dy=dir(y);
  set(y, dx, T[x].ch[!dx]); set(x, !dx, y);
  if (~dy) set(z, dy, x);
  T[x].p = z;
 for (push(x); ~dir(x); )
  int y = T[x].p,z = T[y].p;
  push(z); push(y); push(x);
  int dx = dir(x), dy = dir(y);
  if (~dy) rotate(dx!=dy?x:y);
  rotate(x):
int KthNext(int x, int k) {
 assert(k > 0); splay(x);
 x = T[x].ch[1];
 if (T[x].size < k) return -1;</pre>
 while (true)
  push(x); int 1 = T[x].ch[0], r = T[x].ch[1];
  if (T[1].size+1 == k) return x;
  if (k <= T[1].size) x = 1;</pre>
  else k -= T[1].size+1, x = r;
struct LinkCut : SplayTree
LinkCut(int n) : SplayTree(n) {}
int access(int x) {
 int u = x, v = 0;
 for (; u; v = u, u = T[u].p)
  splay(u); int& ov = T[u].ch[1];
  T[u].vir += T[ov].sub; T[u].vir -= T[v].sub;
  ov = v; pull(u);
 splay(x);
 return v;
void reroot(int x) {
 access(x); T[x].flip ^= 1; push(x);
///makes v parent of u !(u must be a root)
void Link(int u, int v) {
 reroot(u); access(v); T[v].vir += T[u].sub;
 T[u].p = v; pull(v);
///removes edge between u and v
void Cut(int u, int v) {
 int _u = FindRoot(u); reroot(u);
 access(v); T[v].ch[0] = T[u].p = 0;
 pull(v); reroot(_u);
//Rooted tree LCA.Returns 0 if u v not connected
int LCA(int u, int v) {
 if (u == v) return u;
 access(u); int ret = access(v);
 return T[u].p ? ret : 0;
//Query subtree of u where v is outside the sbtr
long long Subtree(int u, int v) {
 int _v = FindRoot(v); reroot(v); access(u);
 long long ans = T[u].vir + T[u].self;
 reroot(_v):
 return ans;
```

auto rotate = [&](int x)

```
long long Path(int u, int v) {
int _u = FindRoot(u); reroot(u); access(v);
 long long ans = T[v].path;
 reroot( u):
 return ans;
Linear _Path(int u, int v) {
reroot(u); access(v);
return T[v].shoja;
void Update(int u, long long v) {
access(u); T[u].self = v; pull(u);
void _Update(int u, Linear v) {
access(u); T[u]._self = v;
pull(u);
int FindRoot(int u) {
 access(u);
 while (T[u].ch[0]) \{ u = T[u].ch[0]; push(u); \}
 access(u);
 return u;
///k-th node (0-indexed) on the path from u to v
int KthOnPath(int u, int v, int k) {
if (u == v) return k == 0 ? u : -1;
 int _u = FindRoot(u);
 reroot(u); access(v);
 int ans = KthNext(u, k); reroot(_u);
return ans;
```

$\overline{35}$ LIS

$\overline{36}$ MO

```
struct query{int l,r,idx;};
int block;
bool comp1(query p,query q){
  if (p.1 / block != q.1 / block) {
   if(p.1==q.1) return p.r<q.r;
   return p.l < q.l;
  }
  return (p.1 / block & 1) ? (p.r < q.r) : (p.r > q.r);
}
void mos_algorithm(int n, vector<query>&queries){
  vector<int> answers(queries.size());
  block = (int)sqrt(n);
```

```
sort(queries.begin(), queries.end(),comp1);
int cur_1 = 0;
int cur_r = -1;
for (query q: queries) {
  while (cur_1 > q.1) {cur_1--; add(cur_1);}
  while (cur_r < q.r) {cur_r++;add(cur_r);}
  while (cur_1 < q.1) {Remove(cur_1);cur_1++;}
  while (cur_r > q.r) {Remove(cur_r);cur_r--;}
  answers[q.idx] = get_answer();
}
for(int i:answers) {cout<<i<<"\n";}</pre>
```

37 Manacher

```
vector<int> d1(n);
for (int i = 0, l = 0, r = -1; i < n; i++) {
int k = (i > r)? 1: min(d1[1 + r - i], r - i + 1);
while (0 \le i - k \&\& i + k \le n \&\& s[i - k] == s[i + k])
 k++:
d1[i] = k--;
if^{(i+k>r)}
l = i - k;
 r = i + k;
vector<int> d2(n);
for (int i = 0, 1 = 0, r = -1; i < n; i++) {
  int k = (i > r) ? 0 : min(d2[1 + r - i + 1], r - i + 1);
while (0 \le i - k - 1 \&\& i + k \le n \&\& s[i - k - 1] ==
     s[i + k]) {
d2[i] = k--;
if (i + k > r) {
 1 = i - k - 1;
 r = i + k:
```

38 MatrixDeterminant

```
double det = 1:
for (int i=0; i<n; ++i) {</pre>
int k = i:
for (int j=i+1; j<n; ++j)</pre>
 if (abs (a[j][i]) > abs (a[k][i]))
 k = j;
if (abs (a[k][i]) < EPS) {</pre>
 det = 0:
 break;
swap (a[i], a[k]);
if (i != k)
 det = -det:
det *= a[i][i];
for (int j=i+1; j<n; ++j)
a[i][j] /= a[i][i];
for (int j=0; j<n; ++j)</pre>
 if (j != i && abs (a[j][i]) > EPS)
  for (int k=i+1; k<n; ++k)</pre>
   a[i][k] -= a[i][k] * a[i][i];
```

39 MatrixExpo

```
/* try to avoid vector. Possibly use STL array or
    pointers */
```

```
void multiply(vector<vector<int> >&a. vector<vector<int>
    >&b){
int n = a.size(), m= a[0].size(), l = b[0].size();
vector<vector<int> >ret(n, vector<int>(1));
for(int i=0; i<n; i++) {
 for(int k=0; k<m; k++) {</pre>
  for(int j=0; j<1; j++) {
   ret[i][j] = add(ret[i][j],
        gun(a[i][k],b[k][j],mod),mod);
swap(ret,a);
void bigmod(vector<vector<int> >&a, int p){
int n = a.size();
assert(a.size()==a[0].size());
vector<vector<int> >res(n,vector<int> (n));
for(int i=0; i<n; i++) {</pre>
 for(int j=0; j<n; j++) {
  res[i][j] = 0;</pre>
  if(i==j)
   res[i][j]=1;
while(p) {
 if(p&1) {
  multiply(res,a);
 multiply(a,a);
p>>=1;
 swap(a, res);
```

40 MillerRabin

```
using u64 = uint64_t;
using u128 = \_uint128_t;
u64 binpower(u64 base, u64 e, u64 mod) {
u64 \text{ result} = 1;
base %= mod;
while (e) {
 if (e & 1)
  result = (u128)result * base % mod;
 base = (u128)base * base % mod;
 e >>= 1;
return result:
bool check_composite(u64 n, u64 a, u64 d, int s) {
u64 x = binpower(a, d, n);
if (x == 1 | x == n - 1)
 return false;
for (int r = 1; r < s; r++) {
 x = (u128)x * x % n;
 if (x == n - 1)
  return false;
return true;
bool MillerRabin(u64 n) { // returns true if n is prime,
    else returns false.
if (n < 2)
 return false;
int r = 0;
u64 d = n' - 1;
```

41 MinCostFlow1

```
#define fst first
#define snd second
#define all(c) ((c).begin()), ((c).end())
#define TEST(s) if (!(s)) { cout << __LINE__ << " " <<
    #s << endl: exit(-1): }
const long long INF = 1e9;
struct graph {
 typedef int flow_type;
 typedef int cost_type;
 struct edge {
 int src, dst;
flow_type capacity, flow;
 cost_type cost;
 size_t rev;
 void add_edge(int src, int dst, flow_type cap,
      cost_type cost) {
 adj[src].push_back({src, dst, cap, 0, cost,
     adj[dst].size()});
 adj[dst].push_back({dst, src, 0, 0, -cost,
     adj[src].size()-1});
  int n;
  vector<vector<edge>> adj;
  graph(int n) : n(n), adj(n) { }
  pair<flow_type, cost_type> min_cost_max_flow(int s,
      int t) {
 flow_type flow = 0;
 cost_type cost = 0;
for (int u = 0; u < n; ++u)
  for (auto &e: adj[u]) e.flow = 0;
 vector<cost_type> p(n, 0);
 auto rcost = [&](edge e) { return e.cost + p[e.src] -
     p[e.dst]; };
for (int iter = 0; ; ++iter) {
  vector<int> prev(n, -1); prev[s] = 0;
  vector<cost_type> dist(n, INF); dist[s] = 0;
  if (iter == 0) {
  vector<int> count(n); count[s] = 1;
  queue<int> que;
  for (que.push(s); !que.empty(); ) {
   int u = que.front(); que.pop();
count[u] = -count[u];
   for (auto &e: adj[u]) {
   if (e.capacity > e.flow && dist[e.dst] > dist[e.src]
       + rcost(e)) {
    dist[e.dst] = dist[e.src] + rcost(e);
    prev[e.dst] = e.rev;
    if (count[e.dst] <= 0)</pre>
   count[e.dst] = -count[e.dst] + 1;
```

```
que.push(e.dst);
 typedef pair<cost_type, int> node;
 priority_queue<node, vector<node>, greater<node>> que;
 que.push({0, s});
 while (!que.empty()) {
  node a = que.top(); que.pop();
   if (a.snd == t) break;
   if (dist[a.snd] > a.fst) continue;
   for (auto e: adj[a.snd]) {
  if (e.capacity > e.flow && dist[e.dst] > a.fst +
      rcost(e)) {
    dist[e.dst] = dist[e.src] + rcost(e);
   prev[e.dst] = e.rev;
    que.push({dist[e.dst], e.dst});
 if (prev[t] == -1) break;
 for (int u = 0; u < n; ++u)
 if (dist[u] < dist[t]) p[u] += dist[u] - dist[t];</pre>
 function<flow_type(int,flow_type)> augment = [&](int
      u, flow_type cur) {
 if (u == s) return cur:
 edge &r = adj[u][prev[u]], &e = adj[r.dst][r.rev];
 flow_type f = augment(e.src, min(e.capacity - e.flow,
     cur));
 e.flow += f; r.flow -= f;
 return f;
 flow_type f = augment(t, INF);
 flow += f;
 cost += f * (p[t] - p[s]);
return {flow, cost};
```

42 MinCostFlow2

```
struct Edge{
  int u, v;
  long long cap, cost;

Edge(int _u, int _v, long long _cap, long long _cost) {
    u = _u;
    v = _v;
    cap = _cap;
    cost = _cost;
}
};

struct MinCostFlow{
  int n, s, t;
  long long flow, cost;
  vector<vector<int> > graph;
  vector<Edge> e;
  /* if cost is double, dist should be double*/
  vector<long long> dist;
  vector<int> parent;

MinCostFlow(int _n) {
    /* 0-based indexing*/
    n = _n;
    graph.assign(n, vector<int> ());
}
```

```
void addEdge(int u, int v, long long cap, long long
    cost, bool directed = true) {
 graph[u].push_back(e.size());
 e.push_back(Edge(u, v, cap, cost));
 graph[v].push_back(e.size());
e.push_back(Edge(v, u, 0, -cost));
 if(!directed)
 addEdge(v, u, cap, cost, true);
pair<long long, long long> getMinCostFlow(int _s, int
s = _t) {
t = _t;
flow = 0, cost = 0;
 while(SPFA()) {
 flow += sendFlow(t, 1LL<<62);
return make_pair(flow, cost);
bool SPFA() {
parent.assign(n, -1);
 dist.assign(n, 1LL<<62);
 dist[s] = 0;
 vector<int> queuetime(n, 0);
 queuetime[s] = 1;
vector<bool> inqueue(n, 0);
 inqueue[s] = true;
 queue<int> q;
q.push(s);
 bool negativecycle = false;
 while(!q.empty() && !negativecycle) {
 int u = q.front();
 q.pop();
 inqueue[u] = false;
 for(int i = 0; i < graph[u].size(); i++) {</pre>
  int eIdx = graph[u][i];
  int v = e[eIdx].v;
  11 w = e[eIdx].cost, cap = e[eIdx].cap;
  if(dist[u] + w < dist[v] && cap > 0) {
   dist[v] = dist[u] + w;
   parent[v] = eIdx;
   if(!inqueue[v]) {
    q.push(v);
    queuetime[v]++;
    inqueue[v] = true;
    if(queuetime[v] == n+2) {
     negativecycle = true;
     break;
return dist[t] != (1LL<<62):
long long sendFlow(int v, long long curFlow) {
if(parent[v] == -1)
 return curFlow;
 int eIdx = parent[v];
```

```
int u = e[eIdx].u;
11 w = e[eIdx].cost;
long long f = sendFlow(u, min(curFlow, e[eIdx].cap));
cost += f*w;
e[eIdx].cap -= f;
e[eIdx^1].cap += f;
return f;
};
```

43 MinimumStack

```
void small_left(vector<int>& v, vector<int>& res){
    stack<pair<int, int> > stk;
    stk.push(make_pair(INT_MIN, v.size()));//initial value
    for (int i = v.size()-1; i >= 0; i--) {
        while (stk.top().first > v[i]) {
            res[stk.top().second] = i;
            stk.pop();
        }
        stk.push(make_pair(v[i], i));
    }
    while (stk.top().second < v.size()) {
        res[stk.top().second] = -1;
        stk.pop();
    }
}</pre>
```

44 Minkowski

```
void reorder_polygon(vector<pt> & P){
size_t pos = 0;
for(size_t i = 1; i < P.size(); i++){</pre>
 if(P[i].y < P[pos].y \mid | (P[i].y == P[pos].y && P[i].x
      < P[pos].x)
  pos = i;
rotate(P.begin(), P.begin() + pos, P.end());
vector<pt> minkowski(vector<pt> P, vector<pt> Q){
reorder_polygon(P);reorder_polygon(Q);
P.push_back(P[0]); P.push_back(P[1]);
Q.push_back(Q[0]); Q.push_back(Q[1]);
vector<pt> result;
size_t i = 0, j = 0;
while(i < P.size() - 2 || j < Q.size() - 2){</pre>
 result.push_back(P[i] + Q[j]);
 auto cross = (P[i + 1] - P[i]).cross(Q[j + 1] - Q[j]);
 if(cross >= 0) ++i:
 if(cross <= 0) ++j;
return result;
```

45 Miscellaneous

```
b.begin(), b.end(),
   back inserter(c)):
#pragma GCC optimize("Ofast")
#pragma GCC
    target("sse,sse2,sse3,sse4,popcnt,abm,mmx,
avx, avx2, fma")
#pragma GCC optimize("unroll-loops")
#pragma GCC optimize("03")
struct custom_hash {
static uint64_t splitmix64(uint64_t x) {
  x += 0x9e3779b97f4a7c15;
 x = (x \hat{\ }(x >> 30)) * 0xbf58476d1ce4e5b9;

x = (x \hat{\ }(x >> 27)) * 0x94d049bb133111eb;
 return x ^{\circ} (x >> 31):
 size_t operator()(uint64_t x) const {
 static const uint64_t FIXED_RANDOM =
      chrono::steady_clock::now().
 time_since_epoch().count();
 return splitmix64(x + FIXED_RANDOM);
unordered_map<long long, int, custom_hash> safe_map;
struct hash_pair {
template <class T1, class T2>
size_t operator()(const pair<T1, T2>& p) const {
 auto hash1 = hash<T1>{}(p.first);
 auto hash2 = hash<T2>{}(p.second);
 return hash1 ^ hash2;
unordered_map<pair<int, int>, int, hash_pair> mp;
//mod inverse for all m
inv[1] = 1;
for(int i = 2; i < m; ++i)
inv[i] = m - (m/i) * inv[m%i] % m;
//ordered set
#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
typedef tree<int,null_type,less<int >,rb_tree_tag,
       tree_order_statistics_node_update>indexed_set;
//submask supermask
for (int s=m; s; s=(s-1)\&m)
for (int mask=need; mask < (1<<n); mask = (mask+1)|need)
string str="abc def gh", buf; stringstream ss(str);
while(ss >> buf) cout << buf << endl;
```

46 Mo with update

```
const int N = 1e5 +5:
const int P = 2000; //block size = (2*n^2)^(1/3)
struct query{int t, 1, r, k, i;};
vector<query> q;vector<array<int, 3>> upd;
vector<int> ans; vector<int>a;
void mos_algorithm(){
sort(q.begin(), q.end(), [](const query &a, const query
     &b) {
  if (a.t / P != b.t / P) return a.t < b.t;</pre>
  if (a.1 / P != b.1 / P) return a.1 < b.1;
 if ((a.l / P) & 1) return a.r < b.r;</pre>
 return a.r > b.r;
 for (int i = upd.size() - 1; i >= 0; --i)
 a[upd[i][0]] = upd[i][1];
 int L = 0, R = -1, T = 0;
 auto apply = [&](int i, int fl) {
 int p = upd[i][0]; int x = upd[i][fl + 1];
```

```
if (L \le p \&\& p \le R) \{rem(a[p]); add(x); \}
 a[p] = x;
};
ans.clear();ans.resize(q.size());
for (auto qr : q) {
 int t = qr.t, l = qr.l, r = qr.r, k = qr.k;
  while (T < t) apply(T++, 1);
 while (T > t) apply (--T, 0);
  while (R < r) add(a[++R]);
  while (L > 1) add(a[--L]);
  while (R > r) rem(a[R--]);
  while (L < 1) \text{ rem}(a[L++]);
 ans[qr.i] = get_answer();
void TEST_CASES(int cas){
int n, m; cin>>n>>m;
a.resize(n);
for(int i=0;i<n;i++) { cin>>a[i]; }
for(int i=0;i<m;i++) {</pre>
 int tp;scanf("%d", &tp);
 if (tp == 1) {
  int 1, r, k;cin>>l>>r>>k;
  q.push_back({upd.size(), 1 - 1, r - 1, k, q.size()});
  int p, x;cin>>p>>x;
  --p;upd.push_back({p, a[p], x});
  a[p] = x;
mos_algorithm();
```

47 Monotone Queue

```
vector<int>dp(n,0);
vector<int>newdp(n);
for(int i=1; i<=m; i++){</pre>
int koto = d*(a[i].t - a[i-1].t);
deque<pair<int,int> >dq;
int 1 = 0, r = -1;
for(int j=0; j<n; j++){</pre>
int eil = max(Oll, j-koto), eir = min(n-1, j+koto);
while(!dq.empty() && dq.front().first < eil) {</pre>
 dq.pop_front();
while(r!= eir) {
  int val = dp[r];
  int idx = r;
  while(!dq.empty() && dq.back().second <= val) {</pre>
  dq.pop_back();
 dq.push_back({idx,val});
newdp[j] = a[i].b - abs(a[i].a - j-1) +
     dq.front().second ;
swap(dp,newdp);
cout<<*max_element(dp.begin(), dp.end())<<"\n";</pre>
```

48 NTT

```
namespace NTT {
  vector<int> perm, wp[2];
  const int mod = 998244353, G = 3; ///G is the primitive
    root of M
```

```
int root, inv, N, invN;
int power(int a, int p) {
int ans = 1;
while (p) {
 if (p \& 1) ans = (1LL*ans*a)%mod;
 a = (1LL*a*a) \% mod;
 p >>= 1;
return ans;
void precalculate(int n) {
assert( (n&(n-1)) == 0 && (mod-1)%n==0);
N = n;
invN = power(N, mod-2);
perm = wp[0] = wp[1] = vector < int > (N);
perm[0] = 0;
for (int k=1; k<N; k<<=1)
 for (int i=0; i<k; i++) {
  perm[i] <<= 1;
  perm[i+k] = 1 + perm[i];
root = power(G, (mod-1)/N);
inv = power(root, mod-2);
wp[0][\bar{0}]=wp[1][0]=1;
for (int i=1; i<N; i++) {
 wp[0][i] = (wp[0][i-1]*1LL*root)%mod;
 \bar{wp}[1][i] = (\bar{wp}[1][i-1]*1LL*inv)%mod;
void fft(vector<int> &v, bool invert = false) {
if (v.size() != perm.size()) precalculate(v.size());
for (int i=0; i<N; i++)</pre>
 if (i < perm[i])</pre>
  swap(v[i], v[perm[i]]);
for (int len = 2; len <= N; len *= 2) {</pre>
 for (int i=0, d = N/len; i<N; i+=len) {</pre>
  for (int j=0, idx=0; j<len/2; j++, idx += d) {</pre>
   int x = v[i+j];
   int y = (wp[invert][idx]*1LL*v[i+j+len/2])%mod;
   v[i+j] = (x+y) = mod ? x+y - mod : x+y);
   v[i+j+len/2] = (x-y>=0 ? x-y : x-y+mod);
if (invert) {
 for (int &x : v) x = (x*1LL*invN)\mod;
vector<int> multiply(vector<int> a, vector<int> b) {
int n = 1;
while (n < a.size() + b.size()) n <<=1;
a.resize(n);
b.resize(n);
fft(a);
for (int i=0; i<n; i++) a[i] = (a[i] * 1LL * b[i])%mod;
fft(a, true);
return a;
```

49 Nearest Point Pair

```
#define x first
#define y second
long long dist2(pair<int, int> a, pair<int, int> b) {
```

```
return 1LL * (a.x - b.x) * (a.x - b.x) + 1LL * (a.y - b.x)
     b.y) * (a.y - b.y);
pair<int, int> closest_pair(vector<pair<int, int>> a) {
int n = a.size();
assert(n >= 2);
vector<pair<int, int>, int>> p(n);
for (int i = 0; i < n; i++) p[i] = {a[i], i};
sort(p.begin(), p.end());
int 1 = 0, r = 2;
long long ans = dist2(p[0].x, p[1].x);
 pair<int, int> ret = \{0, 1\};
 while (r < n) {
 while (1 < r \&\& 1LL * (p[r].x.x - p[1].x.x) *
 (p[r].x.x - p[1].x.x) >= ans) 1++;
for (int i = 1; i < r; i++) {
  long long nw = dist2(p[i].x, p[r].x);
  if (nw < ans) {</pre>
   ans = nw;
   ret = {p[i].y, p[r].y};
 r++;
return ret;
```

50 PalindromicTree

```
\rightarrow diff(v) = len(v) - len(link(v))
-> series link will lead from the vertex v to the vertex
   u corresponding
 to the maximum suffix palindrome of v which satisfies
     diff(v) != diff(u)
-> path within series links to the root contains only
   O(log n) vertices
-> cnt contains the number of palindromic suffixes of
   the node
struct PalindromicTree {
struct node {
int nxt[26], len, st, en, link, diff, slink, cnt, oc;
string s;
vector<node> t;
int sz, last;
PalindromicTree() {}
PalindromicTree(string _s) {
 int n = s.size();
 t.clear();
 t.resize(n + 9)
 sz = 2, last = 2
 t[1].len = -1, t[1].link = 1;
 t[2].len = 0, t[2].link = 1;
 t[1].diff = t[2].diff = 0;
 t[1].slink = 1;
 t[2].slink = 2;
int extend(int pos) { // returns 1 if it creates a new
    palindrome
 int cur = last, curlen = 0;
 int ch = s[pos] - 'a';
 while (1) {
  curlen = t[cur].len;
  if (pos - 1 - curlen >= 0 && s[pos - 1 - curlen] ==
      s[pos]) break;
  cur = t[cur].link;
 if (t[cur].nxt[ch]) {
```

```
last = t[cur].nxt[ch]:
  t[last].oc++;
  return 0;
  sz++;
  last = sz;
  t[sz].oc = 1;
  t[sz].len = t[cur].len + 2;
  t[cur].nxt[ch] = sz;
  t[sz].en = pos;
  t[sz].st = pos - t[sz].len + 1;
  if (t[sz].len == 1) {
  t[sz].link = 2;
  t[sz].cnt = 1;
  t[sz].diff = 1;
  t[sz].slink = 2;
  return 1;
  while (1) {
  cur = t[cur].link;
  curlen = t[cur].len;
   if (pos - 1 - curlen >= 0 && s[pos - 1 - curlen] ==
       s[pos]) {
    t[sz].link = t[cur].nxt[ch];
  t[sz].cnt = 1 + t[t[sz].link].cnt;
  t[sz].diff = t[sz].len - t[t[sz].link].len;
  if (t[sz].diff == t[t[sz].link].diff) t[sz].slink =
       t[t[sz].link].slink;
  else t[sz].slink = t[sz].link;
 return 1:
 void calc occurrences() {
 for (int i = sz; i >= 3; i--) t[t[i].link].oc +=
vector<array<int, 2>> minimum_partition() { //(even,
     odd), 1 indexed
  int n = s.size();
  vector\langle array \langle int, 2 \rangle \rangle ans(n + 1, \{0, 0\}), series_ans(n + 1, \{0, 0\})
 + 5, {0, 0});
ans[0][1] = series_ans[2][1] = 1e9;
  for (int i = 1: i <= n: i++) {
  extend(i - 1);
  for (int k = 0; k < 2; k++) {
  ans[i][k] = 1e9;</pre>
   for (int v = last; t[v].len > 0; v = t[v].slink) {
series\_ans[v][!k] = ans[i - (t[t[v].slink].len +
          t[v].diff)][!k]
    if (t[v].diff == t[t[v].link].diff)
    series_ans[v][!k] = min(series_ans[v][!k],
          series_ans[t[v].link][!k]);
    ans[i][k] = min(ans[i][k], series_ans[v][!k] + 1);
  return ans;
int32_t main() {
ios_base::sync_with_stdio(0);
cin.tie(0);
string s;
cin >> s;
PalindromicTree t(s);
for (int i = 0; i < s.size(); i++) t.extend(i);</pre>
t.calc_occurrences();
long long ans = 0;
```

$rac{1}{52}$ Phi

j += 1;

r *= -1:

p[0] = 1;

int PartitionFunction(){

if(p[i]<0) p[i]+=mod;

int j = 1, r = 1;

 $for(int i = 1: i < MAX: i++){}$

while(i - (3*i*i - i) / 2 >= 0){

if(i - (3*j*j + j) / 2 >= 0)

```
int phi[N+1];
void phi_1_to_n(int n) {
  phi[0] = 0;phi[1] = 1;
  for (int i = 2; i <= n; i++)phi[i] = i;
  for (int i = 2; i <= n; i++) {
    if (phi[i] == i) { for (int j = i; j <= n; j += i)
        phi[j] -= phi[j] / i;
    }
}</pre>
```

p[i] = Add(p[i], p[i - (3*j*j - j) / 2] * r);

p[i] = Add(p[i], p[i - (3*j*j + j) / 2] * r);

53 PointOrderingByAngle

```
typedef pair<int,int> pii;
struct point{
int x, y;
bool operator <(point &p) const{</pre>
 if(x==p.x){
  return y > p.y;
 return x<p.x;</pre>
struct line{
point p1, p2;
line(){}
line(point &p, point &q){
 p1 = p;
 p2 = q;
bool operator <(line &p){
 11 a = (p1.x-p2.x);
 11 b = (p1.y-p2.y);
 11 c = (p.p1.x-p.p2.x);
 11 d = (p.p1.y-p.p2.y);
```

```
return a*d < b*c:
void TEST_CASES(int cas){
int n;
scanf("%d",&n);
map<pii, int>mp;
vector<point>v(n);
for(int i=0; i<n; i++){</pre>
 v[i].read();
sort(v.begin(), v.end());
for(int i=0; i<n; i++){</pre>
 mp[ \{v[i].x, v[i].y\}] = i;
vector<line>lines;
for(int i=0; i<n; i++){</pre>
 for(int j=i+1; j<n; j++){</pre>
  lines.emplace_back(v[i],v[j]);
sort(lines.begin(),lines.end());
for(line &l: lines){
 point p1= 1.p1;
 point p2 = 1.p2
 int idx1 = mp[ {p1.x,p1.y}];
 int idx2 = mp[ \{p2.x, p2.y\}];
 //Do your work
 //Swap indexes
 v[idx1] = 1.p2;
 v[idx2] = 1.p1;
 mp[ \{v[idx1].x, v[idx1].y\}] = idx1;
 mp[ \{v[idx2].x, v[idx2].y\}] = idx2;
```

54 PollardRho

```
typedef long long LL;
typedef unsigned long long ULL;
namespace Rho {
ULL mult(ULL a, ULL b, ULL mod) {
LL ret = a * b - mod * (ULL)(1.0L / mod * a * b);
return ret + mod * (ret < 0) - mod * (ret >= (LL) mod);
ULL power(ULL x, ULL p, ULL mod){
ULL s=1, m=x;
 while(p) {
 if(p\&1) s = mult(s, m, mod);
 p>>=1;
 m = mult(m, m, mod);
return s;
vector<LL> bases = {2, 325, 9375, 28178, 450775,
    9780504, 1795265022};
bool isprime(LL n) {
if (n<\bar{2}) return 0;
 if (n%2==0) return n==2;
 ULL s = __builtin_ctzll(n-1), d = n>>s;
 for (ULL x: bases) {
 ULL p = power(x%n, d, n), t = s;
 while (p!=1 \&\& p!=n-1 \&\& x\%n \&\& t--) p = mult(p, p,
      n):
 if (p!=n-1 && t != s)
                         return 0;
return 1;
mt19937_64 rng(chrono::system_clock::now().
```

55 PrefixFunction

```
vector<int> prefix_function(string s) {
  int n = (int)s.length();
  vector<int> pi(n);
  for (int i = 1; i < n; i++) {
    int j = pi[i-1];
    while (j > 0 && s[i] != s[j])
        j = pi[j-1];
    if (s[i] == s[j])
    j++;
    pi[i] = j;
}
return pi;
}
```

56 PrimitiveRoot

```
// Finds the primitive root modulo p
int generator(int p) {
vector<int> fact;
int phi = p-1, n = phi;
for (int i = 2; i * i <= n; ++i) {
 if (n % i == 0) {
  fact.push_back(i);
  while (n \% i == 0)
   n /= i;
if (n > 1) fact.push_back(n);
for (int res = 2; res <= p; ++res) {
 bool ok = true;
  for (int factor : fact) {
  if (powmod(res, phi / factor, p) == 1) {
   ok = false; break;
 if (ok) return res;
return -1;
// This program finds all numbers x such that x^k = a
    (mod n)
int main() {
int n, k, a;
scanf("%d %d %d", &n, &k, &a);
```

```
if (a == 0) {
puts("1 \n0");
return 0;
int g = generator(n);
int sq = (int) sqrt (n + .0) + 1;
vector<pair<int, int>> dec(sq);
for (int i = 1; i <= sq; ++i)
dec[i-1] = \{powmod(g, i * sq * k % (n - 1), n), i\};
sort(dec.begin(), dec.end());
int any_ans = -1;
for (int i = 0; i < sq; ++i) {
int my = powmod(g, i * k % (n - 1), n) * a % n;
 auto it = lower_bound(dec.begin(), dec.end(),
     make_pair(my, 0));
 if (it != dec.end() && it->first == my) {
 any_ans = it->second * sq - i;
 break;
if (any_ans == -1) {
puts("0");return 0;
// Print all possible answers
int delta = (n-1) / gcd(k, n-1);
vector<int> ans;
for (int cur = any_ans % delta; cur < n-1; cur += delta)</pre>
ans.push_back(powmod(g, cur, n));
sort(ans.begin(), ans.end());
```

$57 \quad SCC$

```
vector<vector<int>> adj, adj_rev;
vector<bool> used;
vector<int> order, component;
void dfs1(int v) {
used[v] = true;
for (auto u : adj[v])
if (!used[u]) dfs1(u);
 order.push_back(v);
void dfs2(int v) {
used[v] = true; component.push_back(v);
for (auto u : adj_rev[v]) if (!used[u])
  dfs2(u);
int main() {
used.assign(n, false);
for (int i = 0; i < n; i++)
 if (!used[i]) dfs1(i);
 used.assign(n, false);
reverse(order.begin(), order.end());
for (auto v : order) if (!used[v]) {
  dfs2 (v);
  //processing next component
  component.clear();
```

58 SOSDP

```
//memory optimized, super easy to code.

for(int i = 0; i<(1<<N); ++i)

F[i] = A[i];

for(int i = 0; i < N; ++i){
  for(int mask = 0; mask < (1<<N); ++mask) {
    if(mask & (1<<i))
    F[mask] += F[mask^(1<<i)];
```

```
}
}
//supermask
for(int i = 0; i < N; ++i){
  for(int mask = (1<<N)-1; mask >=0; --mask) {
    if(!(mask & (1<<i)))
     dp[mask] += dp[mask|(1<<i)];
}
</pre>
```

59 SegSegIntersection

```
inline double cross(PT a, PT b) {    return a.x * b.y - a.y
inline double cross2(PT a, PT b, PT c) { return cross(b
    - a, c - a); }
bool is_point_on_seg(PT a, PT b, PT p) {
if (fabs(cross(p - b, a - b)) < eps) {
 if (p.x < min(a.x, b.x) \mid\mid p.x > max(a.x, b.x)) return
 if (p.y < min(a.y, b.y) \mid\mid p.y > max(a.y, b.y)) return
 return true;
return false:
bool seg_seg_intersection(PT a, PT b, PT c, PT d, PT
    &ans) {
double oa = cross2(c, d, a), ob = cross2(c, d, b);
double oc = cross2(a, b, c), od = cross2(a, b, d);
if (oa * ob < 0 && oc * od < 0){
 ans = (a * ob - b * oa) / (ob - oa);
 return 1;
else return 0;
set<PT> seg_seg_intersection_inside(PT a, PT b, PT c, PT
PT ans;
if (seg_seg_intersection(a, b, c, d, ans)) return {ans};
set<PT> se;
if (is_point_on_seg(c, d, a)) se.insert(a);
if (is_point_on_seg(c, d, b)) se.insert(b);
if (is_point_on_seg(a, b, c)) se.insert(c);
if (is_point_on_seg(a, b, d)) se.insert(d);
return se;
```

60 SegmentedSieve

```
vector<char> segmentedSieve(long long L, long long R) {
// generate all primes up to sqrt(R)
long long lim = sqrt(R);
vector<char> mark(lim + 1, false);
vector<long long> primes;
for (long long i = 2; i <= lim; ++i) {
 if (!mark[i]) {
  primes.emplace_back(i);
  for (long long j = i * i; j <= lim; j += i)
   mark[j] = true:
vector<char> isPrime(R - L + 1, true);
for (long long i : primes)
 for (long long j = \max(i * i, (L + i - 1) / i * i); j
     <= R; j += i)
  isPrime[j - L] = false;
if (L == 1)
```

```
isPrime[0] = false;
return isPrime;
}
```

61 Segtree 2D

```
void build_y(int vx, int lx, int rx, int vy, int ly, int
if (Îv == rv) {
 if (1x == rx)
  t[vx][vy] = a[lx][ly];
  t[vx][vy] = t[vx*2][vy] + t[vx*2+1][vy];
} else {
 int my = (ly + ry) / 2;
 build_y(vx, lx, rx, vy*2, ly, my);
 build_v(vx, lx, rx, vy*2+1, my+1, ry)
 t[vx][vy] = t[vx][vy*2] + t[vx][vy*2+1];
void build_x(int vx, int lx, int rx) {
if (lx != rx) {
 int mx = (1x + rx) / 2;
 build_x(vx*2, lx, mx);
 build_x(vx*2+1, mx+1, rx);
build_y(vx, lx, rx, 1, 0, m-1);
int sum_y(int vx, int vy, int tly, int try_, int ly, int
    ry) {
if (ly > ry)
 return 0;
if (ly == tly && try_ == ry)
 return t[vx][vv];
int tmy = (tly + try_{-}) / 2;
return sum_y(vx, vy*2, tly, tmy, ly, min(ry, tmy))
  + sum_y(vx, vy*2+1, tmy+1, try_, max(ly, tmy+1), ry);
int sum_x(int vx, int tlx, int trx, int lx, int rx, int
    ly, int ry) {
if (lx > rx)
 return 0;
if (lx == tlx && trx == rx)
 return sum_y(vx, 1, 0, m-1, ly, ry);
int tmx = (tlx + trx) / 2;
return sum_x(vx*2, tlx, tmx, lx, min(rx, tmx), ly, ry)
  + sum_x(vx*2+1, tmx+1, trx, max(lx, tmx+1), rx, ly,
void update_y(int vx, int lx, int rx, int vy, int ly,
    int ry, int x, int y, int new_val) {
if (ly == ry) {
 if (lx == rx)
  t[vx][vy] = new_val;
  t[vx][vy] = t[vx*2][vy] + t[vx*2+1][vy];
} else {
 int my = (ly + ry) / 2;
 if (y \le my)
  update_y(vx, lx, rx, vy*2, ly, my, x, y, new_val);
  update_y(vx, lx, rx, vy*2+1, my+1, ry, x, y, new_val);
 t[vx][vy] = t[vx][vy*2] + t[vx][vy*2+1];
void update_x(int vx, int lx, int rx, int x, int y, int
    new_val) {
```

```
if (lx != rx) {
  int mx = (lx + rx) / 2;
  if (x <= mx)
   update_x(vx*2, lx, mx, x, y, new_val);
  else
  update_x(vx*2+1, mx+1, rx, x, y, new_val);
}
update_y(vx, lx, rx, 1, 0, m-1, x, y, new_val);
}</pre>
```

62 Segtree Lazy

```
void push(int v) {
t[v*2] += lazy[v];
lazv[v*2] += lazv[v];
t[v*2+1] += lazy[v];
lazv[v*2+1] += lazv[v];
lazy[v] = 0;
void update(int v, int tl, int tr, int l, int r, int
    addend) {
if (1 > r)
return;
if (1 == t1 && tr == r) {
 t[v] += addend;
 lazv[v] += addend:
} else {
 push(v);
 int tm = (tl + tr) / 2;
 update(v*2, tl, tm, l, min(r, tm), addend);
 update(v*2+1, tm+1, tr, max(1, tm+1), r, addend);
 t[v] = max(t[v*2], t[v*2+1]);
int query(int v, int tl, int tr, int l, int r) {
if (1 > r)
 return -INF;
if (1 <= t1 && tr <= r)
 return t[v]:
push(v);
int tm = (tl + tr) / 2;
return max(query(v*2, tl, tm, l, min(r, tm)),
     query(v*2+1, tm+1, tr, max(1, tm+1), r));
```

63 Segtree Persistent

```
const ll INF = 4e18;
//Point update and range min
struct Node {
int l=-1,r=-1; pii val;
Node(pii v, int l=-1, int r=-1) {
 this \rightarrow 1 = 1:
 this \rightarrow r = r:
 val = v;
véctor<Node>nodes;
inline void Merge (Node &a, Node &b, Node &c) {
a.val = min(b.val,c.val);
int build(int tl, int tr) {
 if (tl == tr) {
 nodes.emplace_back(make_pair(INF, -1));
 return (int)nodes.size()-1;
int tm = (tl + tr) / 2;
int Left = build(t1, tm);
```

```
int Right = build(tm+1, tr);
nodes.emplace_back(make_pair(INF, -1),Left,Right);
Merge(nodes.back(), nodes[Left], nodes[Right]);
return (int)nodes.size()-1;
int update(int v, int tl, int tr, int pos, pii val) {
if (tl == tr) {
 nodes.emplace_back(val);
 return (int)nodes.size()-1;
int tm = (tl + tr) / 2; int Left,Right;
if (pos <= tm) {</pre>
 Left = update(nodes[v].1, t1, tm, pos,val);
 Right = nodes[v].r;
else {
 Left = nodes[v].1;
 Right = update(nodes[v].r, tm+1, tr, pos,val);
nodes.emplace_back(make_pair(INF, -1),Left,Right);
Merge(nodes.back(), nodes[Left], nodes[Right]);
return (int)nodes.size()-1;
pii query(int v, int tl, int tr, int a, int b) {
if (a>tr|| tl>b || tl > tr) return {INF, -1};
if(tl >=a && tr <=b) return nodes[v].val;
int mid= (tl+tr)/2;
return min(query(nodes[v].1,tl, mid, a,b),
     query(nodes[v].r, mid+1, tr, a, b));
```

64 Segtree beats

```
int maxi = 0, smaxi = -1e9, cnt = 0, lazy = 0;
bool has=0:11 sum = 0:
struct segtree{
int n;
vector<info>t:
segtree(int n, vector<int>&a)
this->n = n;t.resize(n*4);
 build(1,0,n-1,a);
void Merge(info &node, info &l, info &r){
 node.maxi = max(1.maxi, r.maxi) ;
 node.cnt = (node.maxi==1.maxi ? 1.cnt : 0) +
     (node.maxi==r.maxi ? r.cnt : 0);
 node.sum = 1.sum + r.sum;
 if(l.maxi != r.maxi)
 node.smaxi =max({ min(l.maxi, r.maxi), l.smaxi,
      r.smaxi});
 else
 node.smaxi = max(l.smaxi, r.smaxi);
void build(int node, int l,int r, vector<int>&a) {
 if(l==r){
 t[node].maxi = a[l];t[node].cnt = 1;t[node].sum =
      a[l];
 return;
 int mid= (1+r)/2;
 build(node*2, 1, mid,a);
 build(node *2 +1, mid+1, r,a);
Merge(t[node], t[node*2], t[node *2+1]);
void dop(int node, int add){
```

```
if(t[node].maxi <= add)</pre>
 return;
 t[node].sum -= t[node].maxi *111* t[node].cnt;
 t[node].maxi = add;
 t[node].sum += t[node].maxi *111* t[node].cnt;
 t[node].lazy= add;
 t[node].has = 1;
void push_down(int node){
if(t[node].has){
 dop(node*2, t[node].lazy);
 dop(node*2+1, t[node].lazy);
 t[node].lazy = 0;
  t[node].has = 0;
void update(int node, int l,int r, int i, int j, int
 if(l>j || r<i || t[node].maxi <= add){</pre>
 return;
 if(l>=i && r<=j && t[node].smaxi < add){</pre>
 int x = t[node].maxi - add;
 t[node].sum -= t[node].maxi *111* t[node].cnt;
 t[node].maxi = add;
 t[node].sum += t[node].maxi *111* t[node].cnt;
 t[node].lazy= add;
 t[node].has = 1;
 return;
 int mid = (1+r)/2;
 push_down(node);
 update(node *2, 1,mid, i, j, add);
 update(node *2 +1, mid+1, r, i, j, add);
Merge(t[node], t[node *2], t[node*2+1]);
void update(int 1,int r, int add){
update(1,0,n-1,1,r,add);
pair<11,int> query(int node, int 1, int r, int i, int
    i){
 if(l)j || r<i){
 return make_pair(0,-1e9);
 if(l>=i && r<=j){</pre>
 return make_pair(t[node].sum,t[node].maxi);
 int mid = (1+r)/2;
 push_down(node);
 pair<ll,int> x = query(node *2, 1,mid, i, j);
 pair<ll,int> y = query(node *2 +1, mid+1, r, i, j);
 return make_pair(x.first+y.first ,
     max(x.second, y.second));
pair<ll,int> query(int 1, int r){
return query(1,0,n-1,1,r);
```

65 Simplex

```
/*
 * Note: Simplex algorithm on augmented matrix a of
    dimension (m+1)x(n+1)
 * returns 1 if feasible, 0 if not feasible, -1 if
    unbounded
 * returns solution in b[] in original var order, max(f)
    in ret
```

```
* form: maximize sum_j(a_mj*x_j)-a_mn s.t.
     sum_j(a_i + x_j) < = a_i 
 * in standard form.
 * To convert into standard form:
 * 1. if exists equality constraint, then replace by
     both \geq and \leq
 * 2. if variable x doesn't have nonnegativity
     constraint, then replace by
 * difference of 2 variables like x1-x2, where x1>=0,
 * 3. for a>=b constraints, convert to -a<=-b
* note: watch out for -0.0 in the solution, algorithm
     may cycle
* EPS = 1e-7 may give wrong answer, 1e-10 is better
typedef vector<ld> vd;
typedef vector<vd> vvd;
typedef vector<int> vi;
const ld EPS = 1e-10;
struct LPSolver{
 int m, n; vi B, N; vvd D;
 LPSolver(const vvd &A, const vd &b, const vd &c):
 m(b.size()), n(c.size()), N(n + 1), B(m), D(m + 2,
      vd(n + 2)) {
  for (int i = 0; i < m; i++)
  for (int j = 0; j < n; j++)
   D[i][i] = A[i][i];
  for (int i = 0; i < m; i++) {
  B[i] = n + i; D[i][n] = -1;
D[i][n + 1] = b[i];
 for (int j = 0; j < n; j++) {
  N[j] = j; D[m][j] = -c[j];</pre>
 N[n] = -1; D[m + 1][n] = 1;
 void Pivot(int r, int s) {
 ld inv = 1.0 / D[r][s];
 for (int i = 0; i < m + 2; i++) if (i != r)
   for (int j = 0; j < n + 2; j++)
    if (j != s) D[i][j] -= D[r][j] * D[i][s] * inv;
 for (int j = 0; j < n + 2; j++) if (j != s)
   D[r][j] *= inv;
 for (int i = 0; i < m + 2; i++) if (i != r)
   D[i][s] *= -inv;
 D[r][s] = inv; swap(B[r], N[s]);
 bool Simplex(int phase) {
 int x = phase == 1 ? m + 1 : m;
 while (true) {
  int s = -1;
  for (int j = 0; j <= n; j++) {
  if (phase == 2 && N[j] == -1) continue;
  if (s == -1 || D[x][j] < D[x][s] || D[x][j] ==
        D[x][s] && N[j] < N[s]
  if (D[x][s] > -EPS) return true;
  for (int i = 0; i < m; i++) {</pre>
    if (D[i][s] < EPS) continue;</pre>
    if (r == -1 || D[i][n + 1] / D[i][s] < D[r][n + 1] /
        D[r][s] ||
      (D[i][n+1] / D[i][s]) == (D[r][n+1] / D[r][s])
          && B[i] < B[r])
  if (r == -1) return false;
  Pivot(r, s);
```

```
ld Solve(vd &x) {
 int r = 0;
 for (int i = 1; i < m; i++)</pre>
  if (D[i][n + 1] < D[r][n + 1])
  if (D[r][n + 1] < -EPS) {
   Pivot(r, n);
   if (!Simplex(1) || D[m + 1][n + 1] < -EPS)
   return -numeric_limits<ld>::infinity();
   for (int i = 0; i < m; i++)</pre>
    if (B[i] == -1) {
     int s = -1;
    for (int j = 0; j <= n; j++)
if (s == -1 || D[i][j] < D[i][s] || D[i][j] ==
           D[i][s] && N[i] < N[s]
    s = j;
Pivot(i, s);
  if (!Simplex(2))
  return numeric_limits<ld>::infinity();
  x = vd(n);
 for (int i = 0; i < m; i++)
if (B[i] < n) x[B[i]] = D[i][n + 1];
 return D[m][n + 1];
/* Equations are of the matrix form Ax<=b, and we want</pre>
the function c. We are given coeffs of A, b and c. In
   case of minimizing
we negate the coeffs of c and maximize it. Then the
    negative of returned
'value' is the answer.
All the constraints should be in <= form. So we may need
     to negate the
coeffs.
int main(){
const int m = 4; const int n = 3;
1d _A[m][n] = \{ \{ 6, -1, 0 \}, \{ -1, -5, 0 \}, \{ 1, 5, 1 \} \}
     }, { -1, -5, -1 }
\begin{array}{l} \text{ld } \_b[m] = \{ 10, -4, 5, -5 \}; \\ \text{ld } \_c[n] = \{ 1, -1, 0 \}; \end{array}
vvd A(m);
vd b(_b, _b + m);
vd c(_c, _c + n);
for (int i = 0; i < m; i++)
 A[i] = vd(A[i], A[i] + n);
LPSolver solver(A, b, c);
vd x;
ld value = solver.Solve(x);
 cerr << "VALUE: " << value << endl; /* VALUE: 1.29032*/
cerr << "SOLUTION:"; /*SOLUTION: 1.74194 0.451613 1*/
for (size_t i = 0; i < x.size(); i++)</pre>
 cerr << " " << x[i];
```

66 Simpson

```
const int N = 1000 * 1000; // number of steps (already
    multiplied by 2)

double simpson_integration(double a, double b){
    double h = (b - a) / N;
    double s = f(a) + f(b); // a = x_0 and b = x_2n
    for (int i = 1; i <= N - 1; ++i) { // Refer to final
        Simpson's formula
    double x = a + h * i;
    s += f(x) * ((i & 1) ? 4 : 2);</pre>
```

```
int pre[N][N], order[N][N], nxt[N];
queue<int>q;
int future_wife[N],future_husband[N];
void engage(int man , int woman){
int m1 = future_husband[woman];
if(m1==0) {
 future wife[man] = woman:future husband[woman] = man:
else{
 future_wife[man] = woman;future_husband[woman] = man;
  future_wife[m1] =0;q.push(m1);
void TEST_CASES(int cas){
while(!q.empty())q.pop();
cin>>n:
for(int i=1;i<=n;i++) {</pre>
 for(int j=1;j<=n;j++) {
  cin>>pre[i][j]; pre[i][j]-= n;
  nxt[i] = 1;future_wife[i] = 0;q.push(i);
for(int i=1;i<=n;i++) {
  for(int j=1;j<=n;j++) {
   int x; cin>>x;
   order[i][x] = j;
  future husband[i] = 0:
 while(!q.empty()) {
 int man = q.front(); q.pop();
int woman = pre[man][nxt[man]++];
  if(future_husband[woman] == 0) {
  engage(man , woman);
  else if(order[woman][man] <</pre>
       order[woman][future_husband[woman]]) {
   engage(man , woman);
  else{ q.push(man); }
for(int i=1;i<=n;i++) {</pre>
  cout<<" ("<<i<" "<<future_wife[i]+n<<")";
```

68 Stirling

```
for(int i = n; i < nn; ++i) v[i].push_back(1);</pre>
for(int j = nn; j > 1; j >>= 1) {
  int hn = j >> 1;
  for(int i = 0; i < hn; ++i) ntt.multiply(v[i], v[i +</pre>
return v[0][r];
NTT ntt(mod);
vector<int>a,b,res;
//Stirling2 (n,k) = co-eff of x^k in product of
    polynomials A & B
//where A(i) = (-1)^i / i! and B(i) = i^n / i!
int Stirling2(int n, int r) {
a.resize(n+1); b.resize(n+1);
for(int i = 0: i \le n: i++){
 a[i] = invfct[i];
 if(i \% 2 == 1) a[i] = mod - a[i];
for(int i = 0; i \le n; i++){
 b[i] = bigMod(i, n, mod);
 b[i] = (b[i] *1ll* invfct[i]) % mod;
NTT ntt(mod);
ntt.multiply(a,b,res);
return res[r];
```

69 StressTest

```
#!/bin/sh
echo "Enter the name of first File : " read file
echo "Enter the name of second File : " read file2
g++ -o test_gen test_gen.cpp
g++ -o $file $file.cpp
g++ -o $file2 $file2.cpp
while true
   ./test_gen
   ./$file <input.txt> out1.txt
   ./$file2 <input.txt> out2.txt
   if cmp -s "out1.txt" "out2.txt"; then
 echo "Test Case OK"
  else
 echo "ERROR ENCOUNTERED"
 break
  fi
done
```

70 Suffix Automata

```
struct state{
  int len, link;
  map<char, int> next;
  ll dp=-1; //number of paths
  ll cnt=0; //endpos size
  bool is_cloned=false;
  vector<int>inv_link;
};
struct SA {
  vector<state> st;
  int sz, last;
  void sa_init() {
  st[0].len = 0;
  st[0].link = -1;
  sz=0;
```

```
last = 0;
void sa_extend(char c) {
int cur = sz++;
st[cur].len = st[last].len + 1;
int p = last;
while (p != -1 && !st[p].next.count(c)) {
 st[p].next[c] = cur;
 p = st[p].link;
if (p == -1) {
 st[cur].link = 0;
else {
 int q = st[p].next[c];
 if (st[p].len + 1 == st[q].len){
  st[cur].link = q;
 else {
  int clone = sz++;
  st[clone].len = st[p].len + 1;
  st[clone].next = st[q].next;
  st[clone].link = st[q].link;
  while (p != -1 \&\& st[p].next[c] == q) {
   st[p].next[c] = clone;
   p = st[p].link;
  st[q].link = st[cur].link = clone;
  st[clone].is_cloned=true;
last = cur;
11 run(int idx) {
if(st[idx].dp!=-1)
 return st[idx].dp;
if(idx!=0)
 st[idx].dp=st[idx].cnt;
else st[idx].dp=0:
for(char c='a';c<='z';c++) {
 if(!st[idx].next.count(c))
 int u=st[idx].next[c];
 st[idx].dp+=run(u);
return st[idx].dp;
void dfs_in_tree(int idx) {
if(st[idx].is_cloned==false) {
 st[idx].cnt=1;
for(int u:st[idx].inv_link) {
 dfs_in_tree(u);
 st[idx].cnt+=st[u].cnt;
void build(string &s) {
st.resize(2*(int)s.size());
sa init():
for(char c:s) {
 sa_extend(c);
for(int i=1;i<sz;i++) {</pre>
 st[st[i].link].inv_link.push_back(i);
dfs_in_tree(0);
```

71 SuffixArray

```
const int LOG = 20;
int sa[N],Data[N],rnk[N],height[N];
int wa[N],wb[N],wws[N],wv[N];
int lg[N], rmq[N][LOG];
void prelg() {
  lg[0] = lg[1] = 0;
for(int i = 2; i < N; i++) {
 lg[i] = lg[i/2] + 1;
struct SuffixArray {
 int cmp(int *r,int a,int b,int 1) {
 return (r[a]==r[b]) && (r[a+1]==r[b+1]);
 void DA(int *r,int *sa,int n,int m) {
  int i,j,p,*x=wa,*y=wb,*t;
  for(i=0; i<m; i++) wws[i]=0;</pre>
  for(i=0; i<n; i++) wws[x[i]=r[i]]++;</pre>
  for(i=1; i<m; i++) wws[i]+=wws[i-1]</pre>
  for(i=n-1; i>=0; i--) sa[--wws[x[i]]]=i;
  for(j=1,p=1; p<n; j*=2,m=p)
  for(p=0,i=n-j; i<n; i++) y[p++]=i;</pre>
  for(i=0; i<n; i++)</pre>
   if(sa[i]>=j) y[p++]=sa[i]-j;
  for(i=0; i<n; i++) wv[i]=x[y[i]];
  for(i=0; i<m; i++) wws[i]=0;
  for(i=0; i<n; i++) wws[wv[i]]++</pre>
  for(i=1; i<m; i++) wws[i]+=wws[i-1]</pre>
  for(i=n-1; i>=0; i--) sa[--wws[wv[i]]]=y[i];
  for(t=x,x=y,y=t,p=1,x[sa[0]]=0,i=1; i<n; i++)</pre>
   x[sa[i]] = cmp(y, sa[i-1], sa[i], j)?p-1:p++;
 void calheight(int *r,int *sa,int n) {
 int i,j,k=0;
  for(i=1; i<=n; i++) rnk[sa[i]]=i;</pre>
  for(i=0; i<n; height[rnk[i++]]=k)</pre>
  for(k?k--:0,j=sa[rnk[i]-1]; r[i+k]==r[j+k]; k++);
 void suffix_array (string &A) {
 n = A.size();
  Data[n]=0;
  int cnt =0;
  for (int i = 0; i < n; i++){
  Data[i] = A[i] - a' + 1; //careful
  cnt = max(cnt, Data[i]);
  DA(Data,sa,n+1,cnt+1);
  calheight(Data,sa,n);
  for(int i = 0; i < n; i++)</pre>
  sa[i] = sa[i+1], height[i] = height[i+1], rnk[sa[i]]
       = i;
  range_lcp_init();
 /** LCP for range : build of rmq table **/
 void range_lcp_init() {
  for(int i = 0; i < n; i++)
  rmq[i][0] = height[i];
  for(int j = 1; j < LOG; j++) {
  for(int i = 0; i < n; i++) {
   if (i+(1<< j)-1 < n)
    rmq[i][j] = min(rmq[i][j-1], rmq[i+(1<<(j-1))][j-1]);
   else break;
```

```
}
/** lcp between l'th to r'th suffix in suffix array **/
int query_lcp(int 1, int r) {
   assert(1 <= r);   assert(1>=0 && l<n && r>=0 && r<n);
   if(1 == r) return n-sa[l];
   l++;
   int k = lg[r-l+1];
   return min(rmq[l][k],rmq[r-(1<<k)+1][k]);
}
//i and j position in original string
int getsuff(int i, int j) {
   i= rnk[i];   j = rnk[j];
   return query_lcp(min(i,j),max(i,j));
}
SA;</pre>
```

```
} SA;
       Treap
mt19937 rng(chrono::steady_clock::now().
time_since_epoch().count());
int getrand(int a, int b){
int x = uniform_int_distribution<int>(a, b)(rng);
struct treap{
 int prior, val, subtreeSize ;
 treap *1;treap *r;treap *parent;
 int sum, lazy
 treap(int data) {
 val= data;
  prior = getrand(-2e9, 2e9);
  subtreeSize = 1;
 l=NULL;r=NULL;parent = NULL;lazy = 0;sum = data;
typedef treap* ptreap;
int Size(ptreap t){
 return t->subtreeSize;
 return 0;
void update_size(ptreap t){
 if(t) t->subtreeSize = 1+ Size(t->1) + Size(t->r);
void push(ptreap t){
if(!t || !t->lazy) {return;}
t\rightarrow val += t\rightarrow lazy; t\rightarrow sum += t\rightarrow lazy * Size(t);
 if(t->1) t->1->1azy += t->1azy;
 if(t->r) t->r->lazy += t->lazy;
t\rightarrow lazy = 0;
void reset(ptreap t){
if(t) t->sum = t->val;
void combine(ptreap &t, ptreap 1, ptreap r){
if(!1 || !r) {
 if(1) {t= 1;}else {t = r;}return;
t \rightarrow sum = 1 \rightarrow sum + r \rightarrow sum;
void operation(ptreap t){
if(!t)
 reset(t); push(t->1); push(t->r);
 combine(t,t,t->1); combine(t,t,t->r);
void split(ptreap t, ptreap &l, ptreap &r, int pos, int
    add =0){
 if(!t) {
 1 = NULL;r = NULL;return;
```

```
push(t);
int curr = add + Size(t->1);
 if(curr<=pos) {</pre>
 split(t-\hat{r}, t-r, r, pos, curr+1);
  if(t->r != NULL) t->r->parent = t;
 if(r!=NULL) r->parent = NULL;
 else {
 split(t->1, 1, t->1,pos, add);
 if(t->1 != NULL) {
  t->l->parent = t;
 if(1!=NULL) +
  1->parent = NULL;
 \dot{r} = t;
 update_size(t);operation(t);
void Merge(ptreap &t, ptreap 1, ptreap r){
push(1);push(r);
if(!1 || !r) {if(1) t= 1;else t = r;}
 else if(l->prior > r->prior) {
 Merge(1->r, 1->r, r);
 if(1->r != NULL)
  1->r ->parent = 1;
 t=1;
 else {
 Merge(r->1, 1, r->1);
  if(r->1 != NULL) {
  r->l->parent = r;
 t = r;
update_size(t); operation(t);
int range_query(ptreap t, int 1, int r){
ptreap t1, t2, t3;
split(t,t1,t2,l-1); split(t2,t2,t3,r-1);
 int ans = t2->sum;
Merge(t,t1,t2); Merge(t,t,t3);
return ans;
void range_update(ptreap t, int l,int r, int val){
  ptreap t1, t2, t3;
split(t,t1,t2,l-1); split(t2,t2,t3,r-1);
 t2->lazy += val;
Merge(t,t1,t2); Merge(t,t,t3);
ptreap goup(ptreap t){
  if(t==NULL || t->parent==NULL) return t;
return goup(t->parent);
void output2 (ptreap t){
if (!t) return;
push (t);output2 (t->1);
 cout<<t->val<<" ";
output2 (t->r);
```

73 Vertical Decomposition

```
inline bool le(dbl x, dbl y){return x < y + eps;}
inline bool ge(dbl x, dbl y){return x > y - eps;}
struct Line{
 pt p[2];
```

```
Line(){}
Line(pt a, pt b):p{a, b}{}
pt vec()const{
 return p[1] - p[0];
pt& operator [](size_t i){
 return p[i];
inline bool lexComp(const pt & 1, const pt & r){
if(fabs(1.x - r.x) > eps){return 1.x < r.x;}
else return l.y < r.y;
vector<pt> interSegSeg(Line 11, Line 12){
if(eq(11.vec().cross(12.vec()), 0)){
 if(!eq(11.vec().cross(12[0] - 11[0]), 0))
  return {};
 if(!lexComp(11[0], 11[1])) swap(11[0], 11[1]);
if(!lexComp(12[0], 12[1])) swap(12[0], 12[1]);
 pt l = lexComp(l1[0], 12[0]) ? 12[0] ; 11[0];
pt r = lexComp(l1[1], 12[1]) ? 11[1] : 12[1];
  if(1 == r)
  return {1};
  else return lexComp(1, r) ? vector<pt>{1, r} :
      vector<pt>();
else{
 dbl s = (12[0] - 11[0]).cross(12.vec()) /
      11.vec().cross(12.vec());
  pt inter = 11[0] + 11.vec() * s;
  if(ge(s, 0) && le(s, 1) && le((12[0] -
      inter).dot(12[1] - inter), 0))
  return {inter};
  return {};
inline char get_segtype(Line segment, pt other_point){
if(eq(segment[0].x, segment[1].x))
 return 0:
if(!lexComp(segment[0], segment[1]))
swap(segment[0], segment[1]);
return (segment[1] - segment[0]).cross(other_point -
     segment[0]) > 0 ? 1 : -1;
dbl union_area(vector<tuple<pt, pt, pt> > triangles){
vector<Line> segments(3 * triangles.size());
vector<char> segtype(segments.size());
for(size_t i = 0; i < triangles.size(); i++){</pre>
 pt a, b, c;
  tie(a, b, c) = triangles[i];
  segments [3 * i] = lexComp(a, b)? Line(a, b): Line(b,
  segtype[3 * i] = get_segtype(segments[3 * i], c);
  segments [3 * i + 1] = lexComp(b, c)? Line(b, c):
      Line(c, b);
  segtype[3 * i + 1] = get_segtype(segments[3 * i + 1],
```

```
segments[3 * i + 2] = lexComp(c, a) ? Line(c, a) :
           Line(a, c);
  segtype[3 * i + 2] = get_segtype(segments[3 * i + 2],
vector<dbl> k(segments.size()), b(segments.size());
for(size_t i = 0; i < segments.size(); i++){</pre>
 if(segtype[i]){
   k[i] = (segments[i][1].y - segments[i][0].y) /
              (segments[i][1].x - segments[i][0].x);
   b[i] = segments[i][0].y - k[i] * segments[i][0].x;
dbl ans = 0;
for(size_t i = 0; i < segments.size(); i++){</pre>
 if(!segtype[i])
   continue;
  dbl l = segments[i][0].x, r = segments[i][1].x;
  vector<pair<dbl, int> > evts;
  for(size_t j = 0; j < segments.size(); j++){</pre>
   if(!segtype[j] || i == j)
     continue
    dbl 11 = segments[j][0].x, r1 = segments[j][1].x;
    if(ge(l1, r) || ge(l, r1))
      continue;
    dbl common_l = max(l, l1), common_r = min(r, r1);
    auto pts = interSegSeg(segments[i], segments[j]);
    if(pts.empty()){
      dbl yl1 = k[j] * common_l + b[j];
      dbl yl = k[i] * common_l + b[i];
      if(lt(yl1, yl) == (segtype[i] == 1)){
       int evt_type = -segtype[i] * segtype[j];
        evts.emplace_back(common_1, evt_type);
        evts.emplace_back(common_r, -evt_type);
    else if(pts.size() == 1u){
      dbl vl = k[i] * common l + b[i]. vl1 = k[i] *
               common_1 + b[i];
      int evt_type = -segtype[i] * segtype[j];
      if(lt(vl1, vl) == (segtype[i] == 1)){
        evts.emplace_back(common_1, evt_type);
        evts.emplace_back(pts[0].x, -evt_type);
      vl = k[i] * common_r + b[i], vl1 = k[i] * common_r + vl1 = k[i] * common_r +
               b[j];
      if(lt(yl1, yl) == (segtype[i] == 1)){
        evts.emplace_back(pts[0].x, evt_type);
        evts.emplace_back(common_r, -evt_type);
    else{
      if(segtype[j] != segtype[i] || j > i){
       evts.emplace_back(common_1, -2);
        evts.emplace_back(common_r, 2);
  evts.emplace back(1, 0):
  sort(evts.begin(), evts.end());
  size_t j = 0;
  int balance = 0;
```

74 Voronoi

```
const Tf INF = 1e10;
vector<Polygon> voronoi(vector<PT> site, Tf bsq) {
 int n = site.size();
 vector<Polygon> region(n);
 PT A(-bsq, -bsq), B(bsq, -bsq),
 C(bsq, bsq), D(-bsq, bsq);
 for(int i = 0; i < n; ++i) {
   vector<DirLine> li(n - 1);
   for(int j = 0, k = 0; j < n; ++j) {
    if(i == j) continue;
     li[k++] = DirLine((site[i] + site[j]) / 2,
     rotate90(site[j] - site[i]));
   li.emplace_back(A,B-A); li.emplace_back(B,C-B);
   li.emplace_back(C,D-C); li.emplace_back(D,A-D);
   region[i] = halfPlaneIntersection(li);
 return region:
```

75 XORTrick

```
vector<int> basis[N];
int sz[N],a[N],LOGK = 21;
void insert_vector(vector<int>&basis ,int &sz, int mask){
  for(int i=0;i<LOGK;i++) {
    if(!(mask&(1<i))) continue;
    if(!basis[i]) {
      basis[i] = mask; sz++; return;
    }
    mask^=basis[i];
}
bool check(vector<int>&basis , int mask){
  for(int i=0;i<LOGK;i++) {
    if(!(mask&(1<i))) continue;
    if(!basis[i]) { return 0; }
    mask ^= basis[i];
}
return 1;
}</pre>
```