- Hall's Marriage Theorem In a bipartite graph G = A ∪ B, a matching saturating A exists iff $|N(S)| >= |S|$ for all S ⊂ A
- - $c'((s', v)) = \sum_{u \in V} d((u, v))$ for each edge (s', v).
  - $c'((v, t')) = \sum_{w \in V} d((v, w))$ for each edge (v, t').
  - $c'((u, v)) = c((u, v)) - d((u, v))$ for each edge (u, v) in the old network.
  - $c'((t, s)) = \infty$
- Every positive integer has a unique representation as a sum of Fibonacci numbers such that no two numbers are equal or consecutive Fibonacci numbers. Pythagorean triples $(n^2 - m^2, 2nm, n^2 + m^2)$ where $0 < m < n$, n and m are coprime and at least one of n and m is even. Each box may contain at most one ball, and in addition, no two adjacent boxes may both contain a ball.$\binom{n-k+1}{n-2k+1}$
- The Catalan number C_n equals the number of valid parenthesis expressions that consist of n left parentheses and n right parentheses. $1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796$
  $C_n = \frac{1}{n+1}\binom{2n}{n}$ there are C_n binary trees of n nodes, there are C_{n-1} rooted trees of n nodes
- The number of derangements of elements $\{1, 2, ..., n\}$, i.e., permutations where no element remains in its original place.
  $f(n) = (n-1)(f(n-2) + f(n-1)), n > 2$ There are $n-1$ ways to choose an element x that replaces the element 1. Option 1: We also replace the element x with the element 1. After this, the remaining task is to construct a derangement of $n-2$ elements. Option 2: We replace the element x with some other element than 1. Now we have to construct a derangement of $n-1$ element, because we cannot replace the element x with the element 1, and all other elements must be changed.
  Another formula $F(n) = n! - \binom{n}{1} \times (n-1)! + \binom{n}{2} \times (n-2)! - \ldots.. + (-1)^k \times \binom{n}{k} \times (n-k)! + \ldots. + (-1)^n$
- $\sum_{m=0}^{n} \binom{m}{k} = \binom{n+1}{k+1}$ $\quad \sum_{k=0}^{m} \binom{n+k}{k} = \binom{n+m+1}{m}$ $\quad \binom{n}{0}^2 + \binom{n}{1}^2 + \cdots + \binom{n}{n}^2 = \binom{2n}{n}$
- $1\binom{n}{1} + 2\binom{n}{2} + \cdots + n\binom{n}{n} = n2^{n-1}$ $\quad \binom{n}{0} + \binom{n-1}{1} + \cdots + \binom{n-k}{k} + \cdots + \binom{0}{n} = F_{n+1}$
- The number of necklaces of n pearls, where each pearl has m possible colors $\sum_{i=0}^{n-1} \frac{m^{gcd(i,n)}}{n}$
- Number of bracket sequence with n open and m close brackets starts with value k C(m+n,m)- C(m+n,m-k-1)
- The area of the polygon is $a + b/2 - 1$ where a is the number of integer points inside the polygon and b is the number of integer points on the boundary of the polygon.
- Number of lattice point between 2 points $gcd(abs(p1.x - p2.x), abs(p1.y - p2.y)) - 1$
- A useful technique related to Manhattan distances is to rotate all coordinates 45 degrees so that a point (x, y) becomes $(x + y, y - x)$. $|x1 - x2| + |y1 - y2| = \max(|x1' - x2'|, |y1' - y2'|)$
- Size of maximum matching in a bipartite graph is equal to the size of its minimum vertex cover, and the minimum vertex cover can be reconstructed after finding the maximum matching. If we remove a vertex from the minimum vertex cover, the size of the minimum vertex cover of the remaining graph is reduced by 1, so the size of the maximum matching is reduced by 1 as well. It means that we can always choose to remove a vertex from the minimum vertex cover we found. By the way, it also proves that it's always possible to remove a vertex from a bipartite graph so the size of the maximum matching decreases by 1 (obviously, if it's not 0 already).
- Zigzag numbers are as follows $1, 1, 1, 2, 5, 16, 61, 272, 1385, 7936, 50521 \ldots\ldots$
  $a(n+1) = (\sum_{k=0}^{n} \binom{n}{k} * a(k) * a(n-k))/2$
  $E(n, k) = E(n, k-1) + E(n-1, n-k)$ if $k \geq n$ or $k < 1$ $E(n, k) = 0$
- The first few values of the partition function, starting with $p(0) = 1$, are:
  $1, 1, 2, 3, 5, 7, 11, 15, 22, 30, 42, 56, 77, 101, 135, 176, 231, 297, 385, 490, 627, 792, 1002, 1255, 1575, 1958, ...$
- St kind 2 : $A(n, k) = A(n-1, k-1) + k * A(n-1, k)$ $\quad$ St kind 1 : $A(n, k) = A(n-1, k-1) + (n-1) * A(n-1, k)$
- Eulerian number with k ascents : $A(n, k) = (k+1) * A(n-1, k) + (n-k) * A(n-1, k-1)$
- Mo on tree : 1: P = u then [ST(u),ST(v)] , 2:[EN(u),ST(v)] + [ST(P),ST(P)]

St kind 2: 1	St kind 1: 1	Euler number: 1
0,1	0,1	1,0
0,1,1	0,1,1	1,1,0
0,1,3,1	0,2,3,1	1,4,1,0
0,1,7,6,1	0,6,11,6,1	1,11,11,1,0
0,1,15,25,10,1	0,24.50,35,10,1	1,26,66,26,1,0
0,1,31,90,65,15,1	0,120,274,225,85,15,1	1,57,302,302,57,1,0
0,1,63,301,350,140,21,1	0,720,1764,1624,735,175,21,1	