2-Sat vector<vector<int>> adj, adj_t; vector<bool> used,assignment; vector<int> order, comp; void dfs1(int v) { used[v] = true; for (int u : adj[v]) { if (!used[u]) dfs1(u); order.push_back(v); void dfs2(int v, int cl) { comp[v] = cl;for (int $u : adj_t[v]$) { if (comp[u] == -1)dfs2(u, c1); bool solve_2SAT() { order.clear(); used.assign(n, false); for (int i = 0; i < n; ++i) { if (!used[i]) comp.assign(n, -1); for (int i = 0, j = 0; i < n; ++i) { int v = order[n - i - 1]; if (comp[v] == -1) dfs2(v, j++); assignment.assign(n / 2, false); for (int i = 0; i < n; i += 2) { if (comp[i] == comp[i + 1]) return false;</pre> assignment[i / 2] = comp[i] > comp[i + 1]; return true: void add_disjunction(int a, bool na, int b, bool nb) { // na and nb signify whether a and b are to be negated a = 2*a ^ na; b = 2*b ^ nb; int_neg_a = a ^ 1; int_neg_b = b ^ 1; adj[neg_a].push_back(b); adj[neg_b].push_back(a); adj_t[b].push_back(neg_a); adj_t[a].push_back(neg_b);

2 Aho

```
const int K = 26;
class Node{
public:
vector<int>nxt;
vector<int>go;
int suf :
int ssuf
bool leaf
int parent;
int ch;
Node(int p=-1,int ch = -1) {
 this->parent = p;
 this->ch = ch;
 nxt.resize(K,-1);
 go.resize(K,-1);
leaf = false:
 suf = -1;
  ssuf = -1;
class AhoCorasick{
public:
vector<Node>nodes;
AhoCorasick() {
 nodes.emplace_back(Node());
int get_num(char ch) {
 int ret = ch-'a';
 return ret;
void add_string(string &s) {
```

```
int v = 0;
for(int i=0; i<s.size(); i++) {
  int now = get_num(s[i]);</pre>
 if(nodes[v].nxt[now] == -1) {
  nodes[v].nxt[now] = nodes.size();
  nodes.emplace_back(v,now);
 v = nodes[v].nxt[now];
nodes[v].leaf = true;
int go(int v, int ch) {
int &ret = nodes[v].go[ch];
if(ret!=-1)
 return ret;
if(nodes[v].nxt[ch]!=-1) {
 return ret = nodes[v].nxt[ch];
if(v==0) {
 return ret = 0:
return ret = go(get_link(v), ch);
int get_link(int v) {
int &ret = nodes[v].suf:
if(ret!=-1)
 return ret;
if(v==0 || nodes[v].parent==0) {
 return ret =0:
return ret = go(get_link(nodes[v].parent) , nodes[v].ch);
int exit link(int v) {
int &ret = nodes[v].ssuf;
if(ret!=-1)
 return ret;
 if(v==0 || nodes[v].parent==0) {
 return ret = 0;
 int s = get_link(v);
if(nodes[s].leaf) {
  return ret = s:
return ret = exit_link(s);
void feed(string &s) {
int v = 0;
for(int i=0;i<s.size();i++) {</pre>
 int ch = get_num(s[i]);
 v = go(v, ch);
 int u =v :
 while(u!=0)
  u = exit_link(u);
```

ArticulationBridge

```
vector<bool>
vector
vector<int> tin, low;
vector<int> tin, low;
int timer;
void dfs(int v, int p = -1) {
    visited[v] = true;
    tin[v] = low[v] = timer++;
    for (int to : adj[v]) {
    if (to == p) continue;
    if (visited[to]) {
    low[v] = min(low[v], tin[to]);
    } else {
    dfs(to, v);
    low[v] = min(low[v], low[to]);
    if (low[to] > tin[v])
    IS_BRIDGE(v, to);
    }
```

```
void find_bridges() {
   timer = 0;
   visited.assign(n, false);
   tin.assign(n, -1);
   low.assign(n, -1);
   for (int i = 0; i < n; ++i) {
      if (!visited[i]) dfs(i);
   }
}</pre>
```

4 ArticulationPoint

```
vector<bool> visited;
vector<int> tin, low;
int timer;
void dfs(int v, int p = -1) {
visited[v] = true;
tin[v] = low[v] = timer++;
int children=0;
for (int to : adj[v]) {
  if (to == p) continue;
 if (visited[to]) {
  low[v] = min(low[v], tin[to]);
 } else {
  dfs(to, v);
low[v] = min(low[v], low[to]);
if (low[to] >= tin[v] && p!=-1)
   IS_CUTPOINT(v);
   ++children;
if(p == -1 \&\& \text{ children} > 1)
 IS_CUTPOINT(v);
void find_cutpoints() {
timer = 0;
visited.assign(n, false);
tin.assign(n, -1);
low.assign(n, -1);
for (int i = 0; i < n; ++i) {
  if (!visited[i])</pre>
  dfs (i);
```

$_{ m 5}$ BIT2D

```
struct FenwickTree2D {
  vector<vector<int>> bit;
  int n, m;
  // init(...) { ... }

  int sum(int x, int y) {
    int ret = 0;
    for (int i = x; i >= 0; i = (i & (i + 1)) - 1)
        for (int j = y; j >= 0; j = (j & (j + 1)) - 1)
        ret += bit[i][j];
    return ret;
  }

void add(int x, int y, int delta) {
    for (int i = x; i < n; i = i | (i + 1))
        for (int j = y; j < m; j = j | (j + 1))
        bit[i][j] += delta;
  }
};</pre>
```

6 Berlekamp

```
#define SZ 233333
const int MOD=1e9+7;
ll qp(ll a,ll b){
    ll x=1; a%=MOD;
    while(b) {
        if(b&1) x=x*a%MOD;
        a=a*a%MOD; b>>=1;
    }
    return x;
```

```
namespace linear_seq{
inline vector<int> BM(vector<int> x){
 vector<int> ls.cur:
 for(int i=0; i<int(x.size()); ++i) {</pre>
   11 t=0;
  for(int j=0; j<int(cur.size()); ++j)
  t=(t+x[i-j-1]*(l1)cur[j])%MOD;</pre>
   if((t-x[i])%MOD==0) continue;
   if(!cur.size()) {
    cur.resize(i+1): lf=i: ld=(t-x[i])%MOD:
    continue:
   11 k=-(x[i]-t)*qp(1d,MOD-2)%MOD;
   vector<int> c(i-lf-1):
   c.push_back(k);
  c.pusn_back(x),
for(int j=0; j<int(ls.size()); ++j)
    c.pusn_back(-ls[j]*k%MOD);
if(c.size()<cur.size()) c.resize(cur.size());</pre>
  for(int j=0; j<int(cur.size()); ++j) c[j]=(c[j]+cur[j])%MOD; if(i-lf+(int)ls.size()>=(int)cur.size()) ls=cur,lf=i,ld=(t-x[i])%MOD; cur=c;
 for(int i=0; i<int(cur.size()); ++i)
    cur[i]=(cur[i]%MOD+MOD)%MOD;</pre>
110 m,
11 a[SZ],h[SZ],t_[SZ],s[SZ],t[SZ];
11 inline void mull(ll*p,ll*q){
 for(int i=0; i<m+m; ++i) t_[i]=0;
for(int i=0; i<m; ++i) if(p[i])
for(int j=0, j<m; ++j)

for(int j=0; j<m; ++j)

t_[i+j]=(t_[i+j]+p[i]*q[j])%MOD;

for(int i=m+m-1; i)=m; --i) if(t_[i])

for(int j=m-1; j; --j)

t_[i-j-1]=(t_[i-j-1]+t_[i]*h[j])%MOD;
 for(int i=0; i<m; ++i) p[i]=t_[i];
inline ll calc(ll K){
  for(int i=m: ~i: --i) s[i]=t[i]=0:
 if(m!=1) t[1]=1; else t[0]=h[0];
while(K){if(K&1)mull(s,t);mull(t,t);K>>=1;}
 11 su=0;
 for(int i=0; i<m; ++i) su=(su+s[i]*a[i])%MOD; return (su%MOD+MOD)%MOD;
inline int work(vector<int> x,ll n){
  if(n<int(x.size())) return x[n];</pre>
 vector<int> v=BM(x); m=v.size();
 if(!m) return 0:
 for(int i=0; i<m; ++i) h[i]=v[i],a[i]=x[i];
 return calc(n);
using linear_seq::work;
```

7 CHT

```
bool Q;
struct Line{
   mutable 11 k, m, p; // slope, y-intercept, last optimal x
   bool operator<(const Line& o) const {
     return Q ? p < o.p : k < o.k;
   }
};
struct LineContainer : multiset<Line>{
   const 11 inf = LLONG_MAX;
11 div(11 a, 11 b) // floored division {
   if (b < 0)
        a *= -1, b *= -1;
   if (a >= 0)
        return a / b;
   return -((-a + b - 1) / b);
}
```

```
// updates x->p, determines if y is unneeded
bool isect(iterator x, iterator y) {
 if (y == end()) {
 x->p = inf:
 return 0;
 if (x->k == y->k)
 x->p = x->m'> y->m ? inf : -inf;
 x->p = div(y->m - x->m, x->k - y->k);
 return x->p >= y->p;
void add(11 k, 11 m) {
  auto z = insert({k, m, 0}), y = z++, x = y;
 while (isect(y, z))
 z = erase(z):
 if (x != begin() && isect(--x, y))
 isect(x, y = erase(y));
while ((y = x) != begin() && (--x)->p >= y->p)
 isect(x, erase(y));
11 query(11 x) // gives max value {
  assert(!empty());
 auto 1 = *lower_bound(\{0, 0, x\});
 return 1.k * x + 1.m:
```

8 CRT

```
/** Works for non-coprime moduli.
Returns {0.0} if solution does not exist or input is invalid.
Otherwise, returns \{x,L\}, where x is the solution unique to
constexpr long long safe mod(long long x, long long m) {
x \% = m;
if (x < 0) x += m;
return x;
constexpr std::pair<long long, long long> inv gcd(long long a.
    long long b) {
 a = safe_mod(a, b);
 if (a == 0) return {b, 0};
long long s = b, t = a;
long long m0 = 0, m1 = 1;
 while (t) {
 long long u = s / t;
s -= t * u;
m0 -= m1 * u;
 auto tmp = s;
 s = t;
t = tmp;
 tmp = mO;
 mO' = m1;
 m1 = tmp;
if (m0 < 0) m0 += b / s;
return {s. m0}:
std::pair<long long, long long> crt(const std::vector<long
     fong>& r,
         const std::vector<long long>& m) {
 assert(r.size() == m.size());
int n = int(r.size());
long long r0 = 0, m0 = 1;
for (int i = 0; i < n; i++) {
 assert(1 <= m[i]);
  long long r1 = safe_mod(r[i], m[i]), m1 = m[i];
  if (m0 < m1) {
  std::swap(r0, r1):
  std::swap(m0, m1);
  if (m0 \% m1 == 0) {
  if (r0 % m1 != r1) return {0, 0};
  continue:
```

```
} long long g, im;
std::tie(g, im) = inv_gcd(m0, m1);
long long u1 = (m1 / g);
if ((r1 - r0) % g) return {0, 0};
long long x = (r1 - r0) / g % u1 * im % u1;
r0 += x * m0;
m0 *= u1;
if (r0 < 0) r0 += m0;
}
return {r0, m0};
}</pre>
```

9 Centroid

```
class CentroidDecomposition{
public:
vector<vector<int> >adj;
vector<int>sz,parent;
vector<bool>vis:
int n,root;
CentroidDecomposition(vector<vector<int> >&a) {
 adj = a; n = adj.size(); sz.resize(n); parent.resize(n,-1);
 vis.resize(n.false):
 build(0,-1);
void build(int s, int p) {
 dfs(s,p);
 int c= centroid(s,p,sz[s]); vis[c]= true;
if(p!=-1) { parent[c] = p; }
 else { root = c; }
for(auto it:adj[c]) { if(vis[it]) continue;
  build(it,c);
void dfs(int s. int p) {
 sz[s] = 1;
for(auto i:adj[s]) { if(i==p || vis[i]) continue;
  dfs(i,s); sz[s] += sz[i];
int centroid(int s,int p, int total) {
  for(auto i:adj[s]) { if(i==p || vis[i]) continue;
  if(sz[i]*2>total) return centroid(i,s,total);
 return s;
```

10 ContinuedFraction

```
* Description: Given $f$ and $N$, finds the smallest fraction
       p/q \in [0, 1]
 * such that $f(p/q)$ is true, and $p, q less than N$.

* You may want to throw an exception from $f$ if it finds an
       exact solution.
* in which case $N$ can be removed.

* Usage: fracBS([](Frac f) { return f.p>=3*f.q; }, 10); //
{1,3}
 * Time: O(\log(N))
 * Status: fuzz-tested for n <= 300
typedef __int128_t lll;
struct Frac{ lll p, q;
template<class F>
Frac fracBS(F f, 111 N) {
bool dir = 1, A = 1, B = 1;
Frac lo{0, 1}, hi{1, 0}; if (f(lo))
 return lo;
assert(f(hi))
while (A | B) {
 lll adv = 0, step = 1;
  for (int si = 0; step; (step *= 2) >>= si) {
  adv += step:
```

```
Frac mid{lo.p * adv + hi.p, lo.q * adv + hi.q};
  if ((mid.p < 0 ? -mid.p : mid.p) > N || mid.q > N || dir ==
       !f(mid)) {
   adv -= step;
   si = 2;
 hi.p += lo.p * adv;
 hi.q += lo.q * adv;
 dir = !dir:
  swap(lo, hi);
 A = B;
B = !!adv;
return dir ? hi : lo;
int main(){
Frac target = fracBS([&](Frac fr) {
 if (fr.q == 0) return true:
 return minfr' < fr;
}, mid);
```

11 DC DP

```
int dp_before[N],dp_cur[N],cnt[N],l=0,r=-1,cur;
void add(int idx){
 cnt[a[idx]]++; if(cnt[a[idx]]==1)cur++;
void remove(int idx){
 cnt[a[idx]]--; if(cnt[a[idx]]==0) cur--;
int cost(int L,int R){
  while (1 > L) { 1--; add(1); }
  while (r < R) { r++; add(r); }</pre>
 while (1 < L) { remove(1); 1++; } while (r > R) { remove(r); r--;}
 return cur:
void compute(int 1, int r, int optl, int optr){
 if (1 > r) return;
 int mid = (1 + r) >> 1;
 pair<int, int> best = {-1e9, -1};
 for (int k = optl; k <= min(mid, optr); k++) {
  if(best.second==-1) { best = {dp_before[k] +cost(k+1, mid),
  else { best = max(best, {dp_before[k] +cost(k+1, mid), k}); }
 dp_cur[mid] = best.first; int opt = best.second;
 compute(1, mid - 1, optl, opt);compute(mid + 1, r, opt, optr);
```

DEBUG TEMPLATE

```
void err(istream_iterator<string> it) {cout<<endl;}</pre>
template<typename T, typename... Args>
void err(istream_iterator<string> it, T a, Args... args){
cout << *it << " = " << a << " ":err(++it, args...);
template < class T1, class T2>
ostream &operator <<(ostream &os, pair<T1,T2>&p) {
os<<"{"<<p.first<<", "<<p.second<<"} ";
return os:
#define debug(args...) { string _s = #args
    replace(_s.begin(), _s.end(), ',', '); stringstream
    _ss(_s); istream_iterator<string> _it(_ss); err(_it,
    args); }
```

13 DSU Rollback

```
struct dsu save {
int v, rnkv, u, rnku;
dsu_save() {}
dsu_save(int _v, int _rnkv, int _u, int _rnku)
 : v(v), rnkv(rnkv), u(u), rnku(rnku) {}
struct dsu_with_rollbacks {
```

```
vector<int> p, rnk;
int comps;
stack<dsu_save> op;
dsu_with_rollbacks() {}
dsu_with_rollbacks(int n) {
 p.resize(n);
 rnk.resize(n)
 for (int i = 0; i < n; i++) {
  p[i] = i;
  rnk[i] = 0;
 comps = n;
int find_set(int v) {
 return (v == p[v]) ? v : find_set(p[v]);
bool unite(int v, int u) {
 v = find set(v):
 u = find_set(u);
 if (v == u)
  return false;
  if (rnk[v] > rnk[u])
  swap(v, u);
  op.push(dsu_save(v, rnk[v], u, rnk[u]));
 p[v] = u;
  if (rnk[u] == rnk[v])
  rnk[u]++;
 return true;
 void rollback() {
 if (op.empty())
return;
  dsu_save x = op.top();
 op.pop();
  comps++;
 p[x.v] = x.v;
 rnk[x.v] = x.rnkv;
 p[x.u] = x.u;
 rnk[x.u] = x.rnku;
struct query {
int v, u;
bool united:
query(int _v, int _u) : v(_v), u(_u) {
struct QueryTree {
  vector<vector<query>> t;
dsu_with_rollbacks dsu;
int T:
QueryTree() {}
QueryTree(int _T, int n) : T(_T) {
 dsu = dsu_with_rollbacks(n);
t.resize(4 * T + 4);
void add_to_tree(int v, int 1, int r, int ul, int ur, query&
     q) {
    (ul > ur)
  return;
  if (1 == ul && r == ur) {
  t[v].push_back(q);
  return:
  int mid = (1 + r) / 2;
  add_to_tree(2 * v, 1, mid, ul, min(ur, mid), q);
  add_to_tree(2 * v + 1, mid + 1, r, max(ul, mid + 1), ur, q);
void add_query(query q, int 1, int r) {
 add to tree(1, 0, T - 1, 1, r, g):
void dfs(int v, int 1, int r, vector<int>& ans) {
  for (query& q : t[v]) {
  q.united = dsu.unite(q.v, q.u);
 if (1 == r)
  ans[1] = dsu.comps;
  else {
  int mid = (1 + r) / 2;
```

```
dfs(2 * v, 1, mid, ans);
 dfs(2 * v + 1, mid + 1, r, ans):
for (query q : t[v]) {
 if (q.united)
  dsu.rollback();
vector<int> solve() {
vector<int> ans(T);
 dfs(1, 0, T - 1, ans);
return ans;
```

14 Dinic

```
const long long flow_inf = 1e18;
struct FlowEdge {
 int v,u,id; long long cap, flow = 0;
 FlowEdge(int v, int u, long long cap, int id=-1): v(v),
      u(u), cap(cap),id(id) {}
struct Dinic
 vector<FlowEdge> edges; vector<vector<int> > adj;
 int n, m = 0; int s, t;
 vector<int> level, ptr,flow_through; queue<int> q; vector<bool>vis;
 int maxid=0;
 Dinic() {}
 Dinic(int n) : n(n) {
   vis.resize(n); adj.resize(n);
   level.resize(n); ptr.resize(n);
 void add_edge(int v, int u, long long cap,int id=-1) {
   edges.emplace_back(v, u, cap,id);
   edges.emplace_back(u, v, 0);
   adj[v].push_back(m);
   adj[u].push_back(m + 1);
   if(id!=-1)maxid++;
 void dfs2(int s) {
   vis[s] = 1;
   for(int i:adj[s]) {
     int id = i; int u = edges[id].v;
     int v = edges[id].u;
     if(edges[id].flow!=edges[id].cap && !vis[v])
       dfs2(v);
 vector<int> getMinCut() {
   dfs2(s); vector<int>ret;
   for(int i=0; i<n; i++) {
  if(vis[i]) ret.push_back(i);</pre>
   return ret;
 bool bfs() {
   while (!q.empty()) {
     int v = q.front();
     q.pop();
     for (int id : adj[v])
       if (edges[id].cap - edges[id].flow < 1)</pre>
         continue;
       if (level[edges[id].u] != -1)
         continue;
       level[edges[id].u] = level[v] + 1;
       q.push(edges[id].u);
   return level[t] != -1;
 long long dfs(int v, long long pushed) {
   if (pushed == 0) return 0;
```

```
if (v == t) return pushed;
   for (int& cid = ptr[v]; cid < (int)adj[v].size(); cid++){
  int id = adj[v][cid]; int u = edges[id].u;
  if (level[v] + 1 != level[u] || edges[id].cap -</pre>
          edges[id].flow < 1)
       continue;
     long long tr = dfs(u, min(pushed, edges[id].cap -
          edges[id].flow));
     if (tr == 0)
       continue;
     edges[id].flow += tr; edges[id ^ 1].flow -= tr;
     return tr;
   return 0;
 long long flow(int _s,int _t) {
    s=_s; t=_t;
    long long f = 0;
   while (true)
     fill(level.begin(), level.end(), -1);
     level[s] = 0; q.push(s);
     if (!bfs()) break;
     fill(ptr.begin(), ptr.end(), 0);
     while (long long pushed = dfs(s, flow_inf)){
       f += pushed:
   flow_through.assign(maxid+1, 0);
   for(int i = 0; i < n; i++){
     for(auto j : adj[i]) {
       int idx = j;
       FlowEdge e = edges[idx];
       if(e.id >= 0)flow_through[e.id] = e.flow;
   return f;
/*for bipartite graph*/
class Minimum_node_cover
public:
 map<pair<int.int>.bool>matched:
 vector<vector<int> >adj;
  vector<int>minimum_vertex,maximum_set,l,r;
  vector<bool>vis;
  /*number of nodes in dinic without source and destination src
      = 0 . dest = sz+1
d.flow() should be called before constructor calling*/
  Minimum_node_cover(int sz, Dinic &d){
   adj.resize(sz+5); vis.resize(sz+5);
   for(auto it:d.edges){
     if(it.u>0 && it.u <=sz && it.v>0 && it.v<=sz &&
          it.cap==1){
       if(it.flow==1){
         adj[it.u].push_back(it.v);
         matched[ {it.u,it.v}]=1;
       else adi[it.v].push back(it.u):
   for(auto it:d.edges){
     if(it.v==0 && it.cap==1) l.push_back(it.u);
     if(it.u==sz+1 && it.cap==1) r.push_back(it.v);
   sort(l.begin(),l.end());sort(r.begin(),r.end());
   1.resize(distance(1.begin(),unique(1.begin(),1.end())) );
   r.resize(distance(r.begin(),unique(r.begin(),r.end())) );
   for(auto it:d.edges){
     if(it.v==0 && it.cap==1 && it.flow==0){
       if(!vis[it.u]) dfs2(it.u, 1);
     if(!vis[i]) minimum_vertex.push_back(i);
     else maximum_set.push_back(i);
```

```
for(int i:r){
     if(vis[i]) minimum vertex.push back(i):
     else maximum_set.push_back(i);
 void dfs2(int s, bool bam){
   vis[s] = 1;
if(bam){
     for(int i:adj[s]){
       if(vis[i]) continue;
       if(matched[\{s,i\}]==0) dfs2(i,0);
   else{
     for(int i:adj[s]){
       if(vis[i]) continue;
       if(matched[{s,i}]==1) dfs2(i,1);
//flow through \lceil i \rceil = extra flow beyond 'low' sent through edge i
struct LR_Flow{
 Dinic F:int n. s. t:
 struct edge{
   int u, v, 1, r, id;
 véctor<edge> edges;
 LR Flow() {}
 LR_Flow(int _n){
   n = n + 2; s = n - 2, t = n - 1;;
   edges.clear();
 void add_edge(int u, int v, int l, int r, int id = -1){
   assert(0 <= 1 && 1 <= r);
   edges.push_back({u, v, 1, r, id});
 bool feasible(int _s = -1, int _t = -1, int L = -1, int R =
   if (L != -1)
     edges.push_back({_t, _s, L, R, -1});
   F = Dinic(n):
   long long target = 0;
   for (auto e : edges){
     int u = e.u, v = e.v, l = e.l, r = e.r, id = e.id;
     if (1 != 0){
       F.add_edge(s, v, 1); F.add_edge(u, t, 1);
       target += 1;
     F.add_edge(u, v, r - 1, id);
   auto ans = F.flow(s, t);
   if (L != -1)edges.pop_back();
   if (ans < target)return 0; //not feasible
 int max_flow(int _s, int _t){ //-1 means flow is not feasible
   int m\bar{x} = 1e5 + 9;
  if (!feasible(_s, _t, 0, mx))return -1; return F.flow(_s, _t);
 int min_flow(int _s, int _t){ //-1 means flow is not feasible
   int mx = 1e9; int ans = -1, 1 = 0, r = mx;
   while (1 <= r){
     int mid = 1 + r >> 1;
    if (feasible(_s, _t, 0, mid))
ans = mid, r = mid - 1;
else l = mid + 1;
   return ans;
```

15 Diophantine

```
int gcd(int a, int b, int& x, int& y) {
  if (b == 0) {
    x = 1;
    y = 0;
```

```
return a;
int x1, y1;
int d = gcd(b, a \% b, x1, y1);
x = y1;
y = x1 - y1 * (a / b);
return d;
bool find_any_solution(int a, int b, int c, int &x0, int &y0,
  int &g) {
g = gcd(abs(a), abs(b), x0, y0);
if (c % g) {
 return false;
x0 *= c / g;
y0 *= c / \bar{g};
if (a < 0) x0 = -x0;
if (b < 0) y0 = -y0;
return true:
void shift_solution(int & x, int & y, int a, int b, int cnt) {
x += cnt * b;
y -= cnt * a;
int find_all_solutions(int a, int b, int c, int minx, int
    maxx, int miny, int maxy) {
int x, y, g;
if (!find_any_solution(a, b, c, x, y, g))
 return 0;
a /= g;
b /= g;
int sign_a = a > 0 ? +1 : -1;
int sign_b = b > 0 ? +1 : -1;
shift_solution(x, y, a, b, (minx - x) / b);
if (x < minx)
 shift_solution(x, y, a, b, sign_b);
if (x > maxx)
return 0;
int lx1 = x;
shift_solution(x, y, a, b, (maxx - x) / b);
if (x > maxx)
 shift_solution(x, y, a, b, -sign_b);
int rx\bar{1} = x;
shift_solution(x, y, a, b, -(miny - y) / a);
if (v < minv)
 shift_solution(x, y, a, b, -sign_a);
if (y > maxy)
 return 0;
shift_solution(x, y, a, b, -(maxy - y) / a);
if (y > maxy)
 shift_solution(x, y, a, b, sign_a);
int rx\bar{2} = x:
if (1x2 > rx2)
 swap(lx2, rx2);
int lx = max(lx1, lx2);
int rx = min(rx1, rx2);
if (lx > rx)
 return 0;
return (rx - lx) / abs(b) + 1:
```

16 DiscreteLog

```
// Returns minimum x for which a ^ x % m = b % m. O(sqrt(m) )
int solve(int a, int b, int m){
// if (a == 0)
// return b == 0 ? 1 : -1;
a %= m, b %= m;int k = 1, add = 0, g;
while ((g = __gcd(a, m)) > 1) {
if (b == k)return add;
if (b % g) return -1;
b /= g, m /= g, ++add; k = (k * 111 * a / g) % m;
```

```
}
int n = sqrt(m) + 1; int an = 1;
for (int i = 0; i < n; ++i) an = (an * 111 * a) % m;
unordered_map<int, int> vals;
for (int q = 0, cur = b; q <= n; ++q) {
  vals[cur] = q; cur = (cur * 111 * a) % m;
}
for (int p = 1, cur = k; p <= n; ++p) {
  cur = (cur * 111 * an) % m;
  if (vals.count(cur)) { int ans = n * p - vals[cur] + add;
    return ans;
}
}
return -1;
}</pre>
```

17 EulerTour

```
vector<multiset<int> >adj; vector<int>ans;
void euler_circuit(int src){
    stack<int>st; st.push(src);
    while(!st.empty()){
        int v = st.top();
        if(adj[v].size()==0){
            ans.push_back(v); st.pop();
        }
        else{
        int f = *adj[v].begin();
        adj[v].erase(adj[v].begin()); adj[f].erase(adj[f].find(v));
        st.push(f);
    }
}
```

$\overline{18}$ FFT

```
struct CD {
double x, y;
 CD(double x=0, double y=0) :x(x), y(y) {}
 CD operator+(const CD& o) { return {x+o.x, y+o.y};}
CD operator-(const CD& o) { return {x-o.x, y-o.y};} CD operator*(const CD& o) { return {x*o.x-y*o.y,
      x*o.y+o.x*y};}
void operator /= (double d) { x/=d; y/=d;}
double real() {return x;}
 double imag() {return y;}
CD conj(const CD &c) {return CD(c.x, -c.y);}
typedef long long LL;
const double PI = acos(-1.0L);
namespace FFT {
  int N;

 vector<int> perm;
 vector<CD> wp[2];
 void precalculate(int n) {
  assert((n & (n-1)) == 0);
  perm = vector<int> (N, 0);
  for (int k=1: k<N: k<<=1) {
  for (int i=0; i<k; i++) {
   perm[i] <<= 1;
   perm[i+k] = 1 + perm[i];
  wp[0] = wp[1] = vector < CD > (N);
  for (int i=0: i<N: i++) {
   wp[0][i] = CD(cos(2*PI*i/N), sin(2*PI*i/N));
   \operatorname{wp}[1][i] = \operatorname{CD}(\cos(2*\operatorname{PI}*i/N), -\sin(2*\operatorname{PI}*i/N));
 void fft(vector<CD> &v, bool invert = false) {
  if (v.size() != perm.size()) precalculate(v.size());
  for (int i=0; i < \overline{N}; i++)
   if (i < perm[i])</pre>
   swap(v[i], v[perm[i]]);
  for (int len = 2; len <= N; len *= 2) {
  for (int i=0, d = N/len; i<N; i+=len) {
   for (int j=0, idx=0; j<len/2; j++, idx += d) {
```

```
CD x = v[i+j];
     CD y = wp[invert][idx]*v[i+j+len/2];
v[i+j] = x+y;
     v[i+j+len/2]' = x-v;
  if (invert) {
  for (int i=0; i<N; i++) v[i]/=N;
 void pairfft(vector<CD> &a, vector<CD> &b, bool invert =
  int N = a.size();
 vector<CD> p(N); for (int i=0; i<N; i++) p[i] = a[i] + b[i] * CD(0, 1);
 fft(p, invert);
 p.push_back(p[0]);
 for (int i=0: i<N: i++) {
  if (invert) {
   a[i] = CD(p[i].real(), 0);
b[i] = CD(p[i].imag(), 0);
  else {
   a[i] = (p[i]+conj(p[N-i]))*CD(0.5, 0);
b[i] = (p[i]-conj(p[N-i]))*CD(0, -0.5);
vector<LL> multiply(const vector<LL> &a, const vector<LL> &b)
  int \hat{n} = 1;
 while (n < a.size()+ b.size()) n<<=1;
vector<CD> fa(a.begin(), a.end()), fb(b.begin(), b.end());
 fa.resize(n); fb.resize(n);
// fft(fa); fft(fb);
  pairfft(fa, fb);
  for (int i=0; i<n; i++) fa[i] = fa[i] * fb[i];
  fft(fa, true);
  vector<LL> ans(n);
  for (int i=0; i<n; i++) ans[i] = round(fa[i].real());
const int M = 1e9+7, B = sqrt(M)+1;
vector<LL> anyMod(const vector<LL> &a, const vector<LL> &b) {
 int n = 1;
  while (n < a.size()+ b.size()) n<<=1;
 vector<CD> al(n), ar(n), bl(n), br(n);
for (int i=0; i<a.size(); i++) al[i] = a[i]%M/B, ar[i] =</pre>
       a[i]%M%B;
  for (int i=0; i<b.size(); i++) bl[i] = b[i]%M/B, br[i] =
       b[i]%M%B;
pairfft(al, ar); pairfft(bl, br);
// fft(al); fft(ar); fft(bl); fft(br);
 for (int i=0; i<n; i++) {
   CD ll = (al[i] * bl[i]), lr = (al[i] * br[i]);
  CD rl = (ar[i] * bl[i]), rr = (ar[i] * br[i]);
al[i] = ll; ar[i] = lr;
bl[i] = rl; br[i] = rr;
 pairfft(al, ar, true); pairfft(bl, br, true);
   fft(al, true); fft(ar, true); fft(bl, true); fft(br, true);
  vector<LL> ans(n);
  for (int i=0; i<n; i++) {
  LL right = round(br[i].real()), left = round(al[i].real());;
LL mid = round(round(bl[i].real()) + round(ar[i].real()));
  ans[i] = ((left\%M)*B*B + (mid\%M)*B + right)\%M;
 return ans:
19 FWHT
```

```
const int inv2 = (mod + 1) >> 1;
const int M = (1 << 20);
const int OR = 0;
const int AND = 1;</pre>
```

```
const int XOR = 2:
struct FWHT{
int P1[M], P2[M];
void wt(int *a, int n, int flag = XOR) {
 if (n == 0)
  return:
 int m = n / 2
 wt(a, m, flag);
 wt(a + m, m, flag);
 for (int i = 0; i < m; i++) {
  int x = a[i], y = a[i + m];
  if (flag == OR)
   a[i] = x, a[i + m] = (x + y) \% mod;
  if (flag == AND)
   a[i] = (x + y) \% mod, a[i + m] = y;
  if (flag == XOR)
   a[i] = (x + y) \% \mod, a[i + m] = (x - y + mod) \% \mod;
void iwt(int* a, int n, int flag = XOR) {
 if (n == 0)
  return;
 int m = n / 2:
 iwt(a, m, flag);
 iwt(a + m, m, flag);
 for (int i = 0; i < m; i++) {
  int x = a[i], y = a[i + m];
  if (flag == OR)
   a[i] = x, a[i + m] = (y - x + mod) % mod;
  if (flag == AND)
   a[i] = (x - y + mod) \% mod, a[i + m] = y;
  if (flag == XOR)
   a[i] = 1LL * (x + y) * inv2 % mod, a[i + m] = 1LL * (x - y)
        + mod) * inv2 % mod; // replace inv2 by >>1 if not
        required
vector<int> multiply(int n, vector<int> A, vector<int> B, int
     flag = XOR) {
 assert(__builtin_popcount(n) == 1);
 A.resize(n):
 B.resize(n);
 for (int i = 0; i < n; i++)
P1[i] = A[i];
 for (int i = 0; i < n; i++)
  P2[i] = B[i];
 wt(P1, n, flag);
 wt(P2, n, flag);
 for (int i = 0; i < n; i++)
  P1[i] = 1LL * P1[i] * P2[i] % mod;
 iwt(P1, n, flag);
 return vector<int> (P1, P1 + n);
vector<int> pow(int n, vector<int> A, long long k, int flag =
     XOR) {
 assert(__builtin_popcount(n) == 1);
 A.resize(n);
 for (int i = 0; i < n; i++)
  P1[i] = A[i];
 wt(P1, n, flag);
for(int i = 0; i < n; i++)
  P1[i] = POW(P1[i], k);
 iwt(P1, n, flag);
 return vector<int> (P1, P1 + n):
int32_t main(){
int \bar{n};
cin >> n;
vector<int> a(M, 0);
for(int i = 0; i < n; i++) {
 int. k:
 cin_>> k;
 a[k]++;
vector<int> v = t.pow(M, a, n+1, AND);
int ans = 1;
for(int i = 1; i < M; i++)
 ans += v[i] > 0;
```

```
cout << ans << '\n';
return 0;
}</pre>
```

20 Fibonacci Shortcut

```
pair<int, int> fib (int n) {
  if (n == 0) return {0, 1};
  auto p = fib(n >> 1);
  int c = p.first * (2 * p.second - p.first);
  int d = p.first * p.first + p.second * p.second;
  if (n & 1) return {d, c + d};
  else return {c, d};
}
```

21 FloorCeilChange

```
vector<int> where_floor_changes(int n){
  int now=1; vector<int>v;
  while(now<=n) {
    v.push_back(now); now=n/(n/now)+1;
  }
  return v;
}

vector<pair<int,int>> where_ceil_changes(int m){
  vector<pair<int,int>>v; int l=1;
  while(1<=m) {
    if(1==m) { v.push_back({m,m}); break;}
    int cl=(m+l-1)/1;
    int r=(m+cl-2)/(cl-1)-1;
    r=min(r,m); r=max(r,l);
    v.push_back({1,r});
    if(r==m) break;
    l=r+1;
  }
  return v;
}</pre>
```

22 Floorsum

```
// floor( (a*i+b)/m ) for 0 <= i <= n-1
ll floor_sum(ll n, ll m, ll a, ll b) {
    ll ans = 0;
    if (a >= m) {
        ans += (n - 1) * n * (a / m) / 2;
        a %= m;
    }
    if (b >= m) {
        ans += n * (b / m);
        b %= m;
    }
    ll y_max = (a * n + b) / m, x_max = (y_max * m - b);
    if (y_max == 0) return ans;
    ans += (n - (x_max + a - 1) / a) * y_max;
    ans += floor_sum(y_max, a, m, (a - x_max % a) % a);
    return ans;
}
```

23 Gauss

```
const double EPS = 1e-9;
const int INF = 2;
int gauss (vector < vector < double > > a, vector < double > & ans) {
int n = (int) a.size();
int m = (int) a[0].size() - 1;
vector<int> where (m, -1);
for (int col=0, row=0; col<m && row<n; ++col) {
  int sel = row;
 for (int i=row; i<n; ++i)
  if (abs (a[i][col]) > abs (a[sel][col]))
   sel = i:
  if (abs (a[sel][col]) < EPS)
  continue;
  for (int i=col; i<=m; ++i)
  swap (a[sel][i], a[row][i]);
  where[col] = row;
  for (int i=0; i<n; ++i)
  if (i != row) {
   double c = a[i][col] / a[row][col];
```

```
for (int j=col; j<=m; ++j)
    a[i][j] -= a[row][j] * c;
}
++row;
}
ans.assign (m, 0);
for (int i=0; i<m; ++i)
    if (where[i]!= -1)
    ans[i] = a[where[i]][m] / a[where[i]][i];
for (int i=0; i<n; ++i) {
    double sum = 0;
    for (int j=0; j<m; ++j)
        sum += ans[j] * a[i][j];
    if (abs (sum - a[i][m]) > EPS)
    return 0;
}
for (int i=0; i<m; ++i)
    if (where[i] == -1)
    return INF;
return 1;
}</pre>
```

24 Geo 2D

```
const double pi = 4 * atan(1);const double eps = 1e-6;
inline int dcmp (double x) { if (fabs(x) < eps) return 0; else
return x < 0 ? -1 : 1; }
double fix_acute(double th) {return th<-pi ? (th+2*pi): th>pi
     ? (th-2*pi) : th;}
inline double getDistance (double x, double y) { return sqrt(x
    * x + y * y); }
inline double torad(double deg) { return deg / 180 * pi; }
struct Pt {
Pt (double x = 0, double y = 0): x(x), y(y) {}
void read () { scanf("%lf%lf", &x, &y); }
void write () { printf("%lf %lf", x, y); }
bool operator == (const Pt& u) const { return dcmp(x - u.x)
      == 0 \&\& dcmp(y - u.y) == 0; }
 bool operator != (const Pt& u) const { return !(*this == u); }
 bool operator < (const Pt& u) const { return dcmp(x - u.x) <
      0' \mid | (dcmp(x-u.x) == 0 \&\& dcmp(y-u.y) < 0); }
 bool operator > (const Pt& u) const { return u < *this; }
 bool operator <= (const Pt& u) const { return *this < u ||
      *this == u: }
 bool operator >= (const Pt& u) const { return *this > u ||
      *this == u; }
Pt operator + (const Pt& u) { return Pt(x + u.x, y + u.y); }
Pt operator - (const Pt& u) { return Pt(x - u.x, y - u.y); }
Pt operator * (const double u) { return Pt(x * u, y * u); }
Pt operator / (const double u) { return Pt(x / u, y / u); }
 double operator * (const Pt& u) { return x*u.y - y*u.x; }
typedef Pt Vector;
typedef vector<Pt> Polygon;
struct Line {
 double a, b, c;
 Line (double a = 0, double b = 0, double c = 0); a(a), b(b),
      c(c) {}
struct Segment{
 Pt a;Pt b;
 Segment(Pt aa, Pt bb) {a=aa, b=bb;}
struct DirLine {
 Pt p; Vector v;
 double ang;
 DirLine () {}
 DirLine (Pt p, Vector v): p(p), v(v) { ang = atan2(v.y, v.x);
 bool operator < (const DirLine& u) const { return ang <
      u.ang; }
namespace Punctual {
double getDistance (Pt a, Pt b) { double x=a.x-b.x,
      y=a.y-b.y; return sqrt(x*x + y*y); }
```

```
namespace Vectorial {
double getDot (Vector a, Vector b) { return a.x * b.x + a.y *
     b.y; }
double getCross (Vector a, Vector b) { return a.x * b.y - a.y
     * b.x: }
double getLength (Vector a) { return sqrt(getDot(a, a)); }
double getPLength (Vector a) { return getDot(a, a); }
double getAngle (Vector u) { return atan2(u.y, u.x); }
double getSignedAngle (Vector a, Vector b) {return
      getAngle(b)-getAngle(a);}
Vector rotate (Vector a, double rad) { return
      Vector(a.x*cos(rad)-a.y*sin(rad),
      a.x*sin(rad)+a.y*cos(rad)); }
Vector ccw(Vector a, double co, double si) {return
      Vector(a.x*co-a.y*si, a.y*co+a.x*si);}
Vector cw (Vector a, double co, double si) {return
      Vector(a.x*co+a.y*si, a.y*co-a.x*si);}
Vector scale(Vector a, double s = 1.0) {return a /
      getLength(a) * s;}
Vector getNormal (Vector a) { double l = getLength(a); return
      Vector(-a.y/1, a.x/1); }
namespace ComplexVector {
typedef complex<double> Pt;
typedef Pt Vector;
double getDot(Vector a, Vector b) { return real(conj(a)*b); }
double getCross(Vector a, Vector b) { return imag(conj(a)*b);
Vector rotate(Vector a, double rad) { return a*exp(Pt(0,
     rad)); }
namespace Linear {
using namespace Vectorial;
Line getLine (double x1, double y1, double x2, double y2) {
return Line(y2-y1, x1-x2, y1*x2-x1*y2); }
Line getLine (double a, double b, Pt u) { return Line(a, -b,
      u.y * b - u.x * a); }
bool getIntersection (Line p, Line q, Pt& o) {
 if (fabs(p.a * q.b - q.a * p.b) < eps)
  return false:
 o.x = (q.c * p.b - p.c * q.b) / (p.a * q.b - q.a * p.b);
o.y = (q.c * p.a - p.c * q.a) / (p.b * q.a - q.b * p.a);
 return true:
bool getIntersection (Pt p, Vector v, Pt q, Vector w, Pt& o) {
 if (dcmp(getCross(v, w)) == 0) return false;
 Vector u = p - q;
 double k = getCross(w, u) / getCross(v, w);
 o = p + v * k;
return true:
double getDistanceToLine (Pt p, Pt a, Pt b) { return
   fabs(getCross(b-a, p-a) / getLength(b-a)); }
double getDistanceToSegment (Pt p, Pt a, Pt b) {
 if (a == b) return getLength(p-a);
 Vector v1 = b - a, v2 = p - a, v3 = p - b;
 if (dcmp(getDot(v1, v2)) < 0) return getLength(v2);</pre>
  else if (dcmp(getDot(v1, v3)) > 0) return getLength(v3);
 else return fabs(getCross(v1, v2) / getLength(v1));
 double getDistanceSegToSeg (Pt a,Pt b,Pt c,Pt d){
 double Ans=INT_MAX;
 Ans=min(Ans,getDistanceToSegment(a,c,d));
 Ans=min(Ans,getDistanceToSegment(b,c,d));
 Ans=min(Ans,getDistanceToSegment(c,a,b));
 Ans=min(Ans,getDistanceToSegment(d,a,b));
 return Ans:
Pt getPtToLine (Pt p, Pt a, Pt b) { Vector v = b-a; return
     a+v*(getDot(v, p-a) / getDot(v,v)); }
bool onSegment (Pt p, Pt a, Pt b) { return dcmp(getCross(a-p, b-p)) == 0 && dcmp(getDot(a-p, b-p)) <= 0; }
bool haveIntersection (Pt a1, Pt a2, Pt b1, Pt b2) {
 if(onSegment(a1,b1,b2)) return true;
 if(onSegment(a2,b1,b2)) return true;
 if(onSegment(b1,a1,a2)) return true;
 if(onSegment(b2,a1,a2)) return true; //Case of touch
```

```
double c1=getCross(a2-a1, b1-a1), c2=getCross(a2-a1, b2-a1),
      c3=getCross(b2-b1, a1-b1), c4=getCross(b2-b1,a2-b1);
 return dcmp(c1)*dcmp(c2) < 0 && dcmp(c3)*dcmp(c4) < 0;
bool onLeft(DirLine 1, Pt p) { return dcmp(1.v * (p-1.p)) >=
     0: }
namespace Triangular {
using namespace Vectorial;
double getAngle (double a, double b, double c) { return
     acos((a*a+b*b-c*c) / (2*a*b));}
double getArea (double a, double b, double c) { double s
     =(a+b+c)/2; return sqrt(s*(s-a)*(s-b)*(s-c)); }
double getArea (double a, double h) { return a * h / 2; ]
double getArea (Pt a, Pt b, Pt c) { return fabs(getCross(b -
a, c - a)) / 2; }
double getDirArea (Pt a, Pt b, Pt c) { return getCross(b - a,
     c - a) / 2;}
//ma/mb/mc = length of median from side a/b/c
double getArea_(double ma, double mb, double mc) {double
     s=(ma+mb+mc)/2; return 4/3.0 *
      sqrt(s*(s-ma)*(s-mb)*(s-mc));
 //ha/hb/hc = length of perpendicular from side a/b/c
double get_Area(double ha, double hb, double hc){
 double H=(1/ha+1/hb+1/hc)/2; double A_=4 * sqrt(H *
       (H-1/ha)*(H-1/hb)*(H-1/hc)); return 1.0/_A_;
bool PtInTriangle(Pt a, Pt b, Pt c, Pt p){
 double s1 = getArea(a,b,c);
 double s2 = getArea(p,b,c) + getArea(p,a,b) + getArea(p,c,a);
 return dcmp(s1 - s2) == 0;
namespace Polygonal {
using namespace Vectorial:
using namespace Linear:
using namespace Triangular;
double getSignedArea (Pt* p, int n) {
 double ret = 0;
 for (int i = 0; i < n-1; i++)
  ret += (p[i]-p[0]) * (p[i+1]-p[0]);
 return ret/2;
int getConvexHull (Pt* p. int n. Pt* ch) {
 sort(p, p + n);
  // preparing lower hull
  int^m = 0;
 for (int i = 0; i < n; i++){
while (m > 1 \&\& dcmp(getCross(ch[m-1]-ch[m-2],
       p[i]-ch[m-1])) <= 0) m--;
  ch[m++] = p[i];
  // preparing upper hull
 int k = m;
for (int i = n-2; i >= 0; i--){
  while (m > k && dcmp(getCross(ch[m-1]-ch[m-2],
       p[i]-ch[m-2]) <= 0) m--;
  ch[m++] = p[i];
  if (n > 1) m--;
 return m;
int isPtInPolygon (Pt o, Pt* p, int n) {
 int wn = 0;
for (int i = 0; i < n; i++) {
  int j = (i + 1) \% n;
  if (onSegment(o, p[i], p[j]) || o == p[i]) return 0;
int k = dcmp(getCross(p[j] - p[i], o-p[i]));
  int d1 = dcmp(p[i].y - o.y);
int d2 = dcmp(p[j].y - o.y);
if (k > 0 && d1 <= 0 && d2 > 0) wn++;
  if (k < 0 \&\& d2 <= 0 \&\& d1 > 0) wn--:
 return wn ? -1 : 1;
void rotatingCalipers(Pt *p, int n, vector<Segment>& sol) {
 sol.clear();
 int j = 1; p[n] = p[0];
```

```
for (int i = 0; i < n; i++) {
  while (getCross(p[j+1]-p[i+1], p[i]-p[i+1]) >
     getCross(p[j]-p[i+1], p[i]-p[i+1]))
          = (j+1) \% n;
    sol.push_back(Segment(p[i],p[j]));
    sol.push_back(Segment(p[i + 1], p[j + 1]));
void rotatingCalipersGetRectangle (Pt *p, int n, double&
           area, double& perimeter) {
  p[n] = p[0];
pln] = plUJ;
int l = 1, r = 1, j = 1;
area = perimeter = 1e20;
for (int i = 0; i < n; i++) {
    Vector v = (p[i+1]-p[i]) / getLength(p[i+1]-p[i]);
    while (dcmp(getDot(v, p[r%n]-p[i]) - getDot(v, p[(r+1)%n]-p[i])) < 0) r++;
    retired (i < r i | demr(getCrose(v, p[i*n]-p[i]) - retired (i < r i | demr(getCrose(v, p[i*n]-p[i]) - retired (i < r i | demr(getCrose(v, p[i*n]-p[i]) - retired (i < r i | demr(getCrose(v, p[i*n]-p[i]) - retired (i < r i | demr(getCrose(v, p[i*n]-p[i]) - retired (i < r i ) demr(getCrose(v, p[i*n]-p[i]) - retired (i < r i ) demr(getCrose(v, p[i*n]-p[i]) - retired (i < r i ) demr(getCrose(v, p[i*n]-p[i]) - retired (i < r i ) demr(getCrose(v, p[i*n]-p[i]) - retired (i < r i ) demr(getCrose(v, p[i*n]-p[i]) - retired (i < r i ) demr(getCrose(v, p[i*n]-p[i]) - retired (i < r i ) demr(getCrose(v, p[i*n]-p[i]) - retired (i < r i ) demr(getCrose(v, p[i*n]-p[i]) - retired (i < r i ) demr(getCrose(v, p[i*n]-p[i]) - retired (i < r i ) demr(getCrose(v, p[i*n]-p[i]) - retired (i < r i ) demr(getCrose(v, p[i*n]-p[i]) - retired (i < r i ) demr(getCrose(v, p[i*n]-p[i]) - retired (i < r i ) demr(getCrose(v, p[i*n]-p[i]) - retired (i < r i ) demr(getCrose(v, p[i*n]-p[i]) - retired (i < r i ) demr(getCrose(v, p[i*n]-p[i]) - retired (i < r i ) demr(getCrose(v, p[i*n]-p[i]) - retired (i < r i ) demr(getCrose(v, p[i*n]-p[i]) - retired (i < r i ) demr(getCrose(v, p[i*n]-p[i]) - retired (i < r i ) demr(getCrose(v, p[i*n]-p[i]) - retired (i < r i ) demr(getCrose(v, p[i*n]-p[i]) - retired (i < r i ) demr(getCrose(v, p[i*n]-p[i]) - retired (i < r i ) demr(getCrose(v, p[i*n]-p[i]) - retired (i < r i ) demr(getCrose(v, p[i*n]-p[i]) - retired (i < r i ) demr(getCrose(v, p[i*n]-p[i]) - retired (i < r i ) demr(getCrose(v, p[i*n]-p[i]) - retired (i < r i ) demr(getCrose(v, p[i*n]-p[i]) - retired (i < r i ) demr(getCrose(v, p[i*n]-p[i]) - retired (i < r i ) demr(getCrose(v, p[i*n]-p[i]) - retired (i < r i ) demr(getCrose(v, p[i*n]-p[i]) - retired (i < r i ) demr(getCrose(v, p[i*n]-p[i]) - retired (i < r i ) demr(g
    while (j < r || dcmp(getCross(v, p[j%n]-p[i]) -
    getCross(v,p[(j+1)%n]-p[i])) < 0) j++;</pre>
    while (1 < j \mid | dcmp(getDot(v, p[1/n]-p[i]) - getDot(v, p[(1+1)/n]-p[i])) > 0) 1++;
    double w = getDot(v, p[r%n]-p[i])-getDot(v, p[1%n]-p[i]);
double h = getDistanceToLine (p[j%n], p[i], p[i+1]);
    area = min(area, w * h);
    perimeter = min(perimeter, 2 * w + 2 * h);
Polygon cutPolygon (Polygon u, Pt a, Pt b) {
  Polygon ret;
  int n = u.size();
  for (int i = 0; i < n; i++) {
Pt c = u[i], d = u[(i+1)%n];
   if (dcmp((b-a)*(c-a)) >= 0) ret.push_back(c);
if (dcmp((b-a)*(d-c)) != 0) {
      Pt t:
      getIntersection(a, b-a, c, d-c, t);
      if (onSegment(t, c, d))
        ret.push_back(t);
 return ret:
int halfPlaneIntersection(DirLine* li, int n, Pt* poly) {
  sort(li, li + n);
  int first, last;
  Pt* p = new Pt[n];
  DirLine* q = new DirLine[n];
  q[first=last=0] = li[0];
  for (int i = 1; i < n; i++) {
    while (first < last && !onLeft(li[i], p[last-1])) last--;
    while (first < last && !onLeft(li[i], p[first])) first++;
    q[++last] = li[i];
    if (dcmp(q[last].v * q[last-1].v) == 0) {
      if (onLeft(q[last], li[i].p)) q[last] = li[i];
    if (first < last)
      getIntersection(q[last-1].p, q[last-1].v, q[last].p,
                q[last].v, p[last-1]);
  while (first < last && !onLeft(q[first], p[last-1])) last--;
  if (last - first <= 1) { delete [] p; delete [] q; return 0;
  getIntersection(q[last].p, q[last].v, q[first].p,
            q[first].v, p[last]);
  for (int i = first; i \le last; i++) poly[m++] = p[i];
  delete [] p; delete [] q;
  return m;
Polygon simplify (const Polygon& poly) {
  Polygon ret;
  int n = poly.size();
  for (int i = 0; i < n; i++) {
   Pt a = poly[i];
    Pt b = polv[(i+1)%n]:
    Pt c = poly[(i+2)\%n];
```

```
if (dcmp((b-a)*(c-b)) != 0 \&\& (ret.size() == 0 || b !=
       ret[ret.size()-1]))
   ret.push_back(b);
 return ret:
Pt ComputeCentroid( Pt* p,int n){
 Pt c(0,0);
  double scale = 6.0 * getSignedArea(p,n);
 for (int i = 0; i < n; i++){
  int j = (i+1) % n;
c = c + (p[i]+p[j])*(p[i].x*p[j].y - p[j].x*p[i].y);
 return c / scale:
// pt must be in ccw order with no three collinear Pts
// returns inside = 1, on = 0, outside = -1
int PtInConvexPolygon(Pt* pt, int n, Pt p){
 assert(n >= 3):
 int lo = 1, hi = n - 1;
  while(hi - 1o > 1){
  int mid = (lo + hi) / 2;
  if(getCross(pt[mid] - pt[0], p - pt[0]) > 0) lo = mid;
  else hi = mid;
 bool in = PtInTriangle(pt[0], pt[lo], pt[hi], p);
 if(!in) return -1;
 if(getCross(pt[lo] - pt[lo-1], p - pt[lo-1]) == 0) return 0;
 if (getCross(pt[hi] - pt[lo]) = 0) return 0;
if (getCross(pt[hi] - pt[(hi+1)\%n]) = 0)
      return 0;
 return 1;
 // Calculate [ACW, CW] tangent pair from an external Pt
#define CW
#define ACW
int direction(Pt st, Pt ed, Pt q) {return dcmp(getCross(ed -
     st, q - ed));}
bool isGood(Pt u, Pt v, Pt Q, int dir) {return direction(Q,
     u, v) != -dir;}
Pt better(Pt u, Pt v, Pt Q, int dir) {return direction(Q, u,
     v) == dir ? u : v;}
Pt tangents(Pt* pt, Pt Q, int dir, int lo, int hi){
 while(hi - lo > 1){
  int mid = (lo + hi)/2
  bool pvs = isGood(pt[mid], pt[mid - 1], Q, dir);
  bool nxt = isGood(pt[mid], pt[mid + 1], Q, dir);
  if(pvs && nxt) return pt[mid];
  if(!(pvs || nxt)){
   Pt p1 = tangents(pt, Q, dir, mid+1, hi);
   Pt p2 = tangents(pt, Q, dir, lo, mid - 1);
return better(p1, p2, Q, dir);
  if(!pvs){
   if(direction(Q, pt[mid], pt[lo]) == dir) hi = mid - 1;
   else if(better(pt[lo], pt[hi], Q, dir) == pt[lo]) hi = mid
   - 1;
else lo = mid + 1;
  if(!nxt){
   if(direction(Q, pt[mid], pt[lo]) == dir) lo = mid + 1;
   else if(better(pt[lo], pt[hi], Q, dir) == pt[lo]) hi = mid
   - 1;
else lo = mid + 1:
  }
 Pt ret = pt[lo];
 for(int i = lo + 1; i <= hi; i++) ret = better(ret, pt[i],
      Q, dir);
 return ret:
// [ACW, CW] Tangent
pair<Pt, Pt> get_tangents(Pt* pt, int n, Pt Q){
 Pt acw_tan = tangents(pt, Q, ACW, 0, n - 1);
 Pt cw_tan = tangents(pt, Q, CW, 0, n - 1);
 return make_pair(acw_tan, cw_tan);
struct Circle {
```

```
Pt o; double r;
Circle () {}
Circle (Pt o, double r = 0): o(o), r(r) {}
void read () { o.read(), scanf("%lf", &r); }
Pt pt(double rad) { return Pt(o.x + cos(rad)*r, o.y +
     sin(rad)*r); }
double getArea (double rad) { return rad * r * r / 2; }
//area of the circular sector cut by a chord with central
     angle alpha
double sector(double alpha) {return r * r * 0.5 * (alpha -
     sin(alpha));}
namespace Circular {
using namespace Linear
using namespace Vectorial;
using namespace Triangular;
int getLineCircleIntersection (Pt p, Pt q, Circle O, double&
     t1, double& t2, vector<Pt>& sol) {
 Vector v = q - p;
 //sol.clear();
 double a = v.x, b = p.x - 0.o.x, c = v.y, d = p.y - 0.o.y; double e = a*a+c*c, f = 2*(a*b+c*d), g = b*b+d*d-0.r*0.r; double delta = f*f - 4*e*g;
 if (dcmp(delta) < 0) return 0;
  if (dcmp(delta) == 0) {
  t1 = t\dot{2} = -f / (2 * e);
  sol.push_back(p + v * t1);
  return 1;
  t1 = (-f - sqrt(delta)) / (2 * e); sol.push_back(p + v * t1);
 t2 = (-f + sort(delta)) / (2 * e) : sol.push back(p + v * t2) :
 return 2;
// signed area of intersection of circle(c.o, c.r) and
// triangle(c.o. s.a. s.b) \lceil cross(a-o, b-o)/2 \rceil
double areaCircleTriIntersection(Circle c, Segment s){
 using namespace Linear;
 double OA = getLength(c.o - s.a);
 double OB = getLength(c.o - s.b);
  // sector
  if (dcmp(getDistanceToSegment(c.o, s.a, s.b) - c.r) >= 0)
  return fix_acute(getSignedAngle(s.a - c.o, s.b - c.o)) *
       (c.r*c.r) / 2.0;
  // triangle
  if (dcmp(OA - c.r) \le 0 \&\& dcmp(OB - c.r) \le 0)
  return getCross(c.o-s.b,s.a-s.b) / 2.0;
  // three part: (A, a) (a, b) (b, B)
 vector<Pt>Sect; double t1,t2;
  getLineCircleIntersection(s.a, s.b, c, t1, t2, Sect);
  return areaCircleTriIntersection(c, Segment(s.a, Sect[0]))
  + areaCircleTriIntersection(c, Segment(Sect[0], Sect[1]))
  + areaCircleTriIntersection(c, Segment(Sect[1], s.b));
// area of intersection of circle(c.o, c.r) and simple
     polyson(p[])
double areaCirclePolygon(Circle c, Polygon p){
 double res = .0;
  int n = p.size();
 for (int i = 0; i < n; ++ i)
  res += areaCircleTriIntersection(c, Segment(p[i],
       p[(i+1)%n]));
 return fabs(res):
// interior (d < R - r) \longrightarrow -2
// interior tangents (d = R - r) \longrightarrow -1
// concentric (d = 0)
 // secants (R - r < d < R + r) ----> 0
 // exterior tangents (d = R + r) \longrightarrow 1
 // exterior (d > R + r) ----> 2
int getPos(Circle o1, Circle o2) {
 using namespace Vectorial;
 double d = getLength(o1.o - o2.o);
  int in = dcmp(d - fabs(o1.r - o2.r)), ex = dcmp(d - (o1.r + o2.r))
 o2.r));
return in<0 ? -2 : in==0? -1 : ex==0 ? 1 : ex>0? 2 : 0:
int getCircleCircleIntersection (Circle o1, Circle o2,
     vector<Pt>& sol) {
```

```
double d = getLength(o1.o - o2.o);
 if (dcmp(d) == 0) {
 if (dcmp(o1.r - o2.r) == 0) return -1; return 0:
 if (dcmp(o1.r + o2.r - d) < 0) return 0;
 if (dcmp(fabs(o1.r-o2.r) - d) > 0) return 0;
 Vector v = o2.o - o1.o;
 double co = (o1.r*o1.r + getPLength(v) - o2.r*o2.r) / (2 *
     o1.r * getLength(v));
 double si = sqrt(fabs(1.0 - co*co));
Pt p1 = scale(cw(v,co, si), o1.r) + o1.o;
Pt p2 = scale(ccw(v,co, si), o1.r) + o1.o;
 sol.push_back(p1);
 if (p1 == p2) return 1;
 sol.push_back(p2);
return 2;
double areaCircleCircle(Circle o1, Circle o2){
Vector AB = o2.o - o1.o;double d = getLength(AB);
if(d \ge 01.r + 02.r) return 0;
if (d + o1.r <= o2.r) return pi * o1.r * o1.r;
if(d + o2.r <= o1.r) return pi * o2.r * o2.r;
 double alpha1 = acos((o1.r * o1.r + d * d - o2.r * o2.r))
      (2.0* o1.r*d):
 double alpha2 = acos((o2.r * o2.r + d * d - o1.r * o1.r) /
     (2.0* o2.r*d):
return o1.sector(2*alpha1) + o2.sector(2*alpha2);
int getTangents (Pt p, Circle o, Vector* v) {
Vector u = 0.0 - p;
double d = getLength(u);
if (d < o.r) return 0;
 else if (dcmp(d - o.r) == 0) {
 v[0] = rotate(u, pi / 2);
 return 1:
} else {
 double ang = asin(o.r / d);
 v[0] = rotate(u, -ang);
 v[1] = rotate(u, ang);
 return 2:
int getTangentPts (Pt p, Circle o, vector<Pt>& v) {
Vector u = p - o.o :double d = getLength(u):
if (d < o.r) return 0;
 else if (dcmp(d - o.r) == 0) {
 v.push back(o.o+u):
 return 1;
} else {
 double ang = acos(o.r / d);
 u = u / getLength(u) * o.r;
 v.push_back(o.o+rotate(u, -ang));
 v.push_back(o.o+rotate(u, ang));
 return 2;
int getTangents (Circle o1, Circle o2, Pt* a, Pt* b) {
int cnt = 0:
 if (dcmp(o1.r-o2.r) < 0) \{ swap(o1, o2); swap(a, b); \}
double d2 = getPLength(o1.o - o2.o);
double rdif = o1.r - o2.r, rsum = o1.r + o2.r; if (dcmp(d2 - rdif * rdif) < 0) return 0;
 if (dcmp(d2) == 0 \&\& dcmp(o1.r - o2.r) == 0) return -1;
 double base = getAngle(o2.o - o1.o);
if (dcmp(d2 - rdif * rdif) == 0) {
 a[cnt] = o1.pt(base); b[cnt] = o2.pt(base); cnt++;
 return cnt;
 double ang = acos((o1.r - o2.r) / sqrt(d2));
 a[cnt] = o1.pt(base+ang); b[cnt] = o2.pt(base+ang); cnt++;
 a[cnt] = o1.pt(base-ang); b[cnt] = o2.pt(base-ang); cnt++;
 if (dcmp(d2 - rsum * rsum) == 0) {
 a[cnt] = o1.pt(base); b[cnt] = o2.pt(pi+base); cnt++;
 else if (dcmp(d2 - rsum * rsum) > 0) {
 double ang = acos((o1.r + o2.r) / sqrt(d2));
```

```
a[cnt] = o1.pt(base+ang); b[cnt] = o2.pt(pi+base+ang);
  a[cnt] = o1.pt(base-ang); b[cnt] = o2.pt(pi+base-ang);
        cnt++;
 return cnt;
Circle CircumscribedCircle(Pt p1, Pt p2, Pt p3) {
 double Bx = p2.x - p1.x, By = p2.y - p1.y;
double Cx = p3.x - p1.x, Cy = p3.y - p1.y;
double D = 2 * (Bx * Cy - By * Cx);
 double cx = (Cy * (Bx * Bx + By * By) - By * (Cx * Cx + Cy * By))
      Cy)) / D + p1.x;
  By)) / D + p1.y;
 Pt p = Pt(cx, cy);
 return Circle(p, getLength(p1 - p));
Circle InscribedCircle(Pt p1, Pt p2, Pt p3) {
 double a = getLength(p2 - p3); double b = getLength(p3 - p1);
 double c = getLength(p1 - p2);
 Pt p = (p1 * a + p2 * b + p3 * c) / (a + b + c);
 return Circle(p, getDistanceToLine(p, p1, p2));
//distance From P : distance from Q = rp : rq
Circle getApolloniusCircle(const Pt& P,const Pt& Q, double
 rp, double rq ){
rq *= rq ;rp *= rp ;
 double a = rq - rp;
 assert(dcmp(a));
 double g = rq * P.x - rp * Q.x ; g /= a ;
 double h = rq * P.y - rp * Q.y ; h /= a ;
 double c = rq*P.x*P.x-rp*Q.x*Q.x+rq*P.y*P.y-rp*Q.y*Q.y;
 Pt o(g,h);
 double R = g*g + h*h - c;
 R = sqrt(R);
 return Circle(o,R);
//Polar Sort
inline bool up (Pt p) {
 return p.y > 0 or (p.y == 0 \text{ and } p.x >= 0);
sort(v.begin(), v.end(), [] (Pt a, Pt b) {
  return up(a) == up(b) ? a.x * b.y > a.y * b.x : up(a) <</pre>
       up(\bar{b}):
});
```

25 Geo 3D

```
const double pi = 4 * atan(1);
const double eps = 1e-10;
inline int dcmp (double x) { if (fabs(x) < eps) return 0; else
     return x < 0 ? -1 : 1; }
inline double torad(double deg) { return deg / 180 * pi; }
struct Point{
double x, y
Point (double x = 0, double y = 0): x(x), y(y) {}
Point operator + (const Point& u) { return Point(x + u.x, y +
     u.\bar{y}); }
Point operator - (const Point& u) { return Point(x - u.x. v -
     \mathbf{u}.\bar{\mathbf{v}}); }
Point operator * (const double u) { return Point(x * u, y *
     u); }
Point operator / (const double u) { return Point(x / u, y /
     u): }
double operator * (const Point& u) { return x*u.y - y*u.x; }
struct Pt3D{
double x, y, z;
Pt3D() {}
void read () {cin>>x>>y>>z;}
void write () {cout<<x<<" --- "<<y<\" --- "<<z<<"\n";}</pre>
Pt3D(double x, double y, double z): x(x), y(y), z(z) {}
Pt3D(const Pt3D &p) : x(p.x), y(p.y), z(p.z) {}
Pt3D operator +(Pt3D b) {return Pt3D(x+b.x,y+b.y, z+b.z);}
Pt3D operator -(Pt3D b) {return Pt3D(x-b.x,y-b.y, z-b.z);}
```

```
Pt3D operator *(double b) {return Pt3D(x*b,y*b, z*b);}
Pt3D operator /(double b) {return Pt3D(x/b,y/b, z/b);}
bool operator <(Pt3D b) {return make_pair(make_pair(x,y),z) <</pre>
      make_pair(make_pair(b.x,b.y),b.z);}
 bool operator ==(Pt3D b) {return dcmp(x-b.x)==0 \&\&
      dcmp(y-b.y) == 0 && dcmp(z-b.z) == 0;
typedef Pt3D Vector3D;
typedef vector<Point> Polygon;
typedef vector<Pt3D> Polyhedron;
namespace Vectorial{
double getDot (Vector3D a, Vector3D b) {return
      a.x*b.x+a.y*b.y+a.z*b.z;}
 Vector3D getCross(Vector3D a, Vector3D b) {return
      Pt3D(a.y*b.z-a.z*b.y, a.z*b.x-a.x*b.z, a.x*b.y-a.y*b.x);}
double getLength (Vector3D a) {return sqrt(getDot(a, a)); }
double getPLength (Vector3D a) {return getDot(a, a); }
Vector3D unitVector(Vector3D v) {return v/getLength(v);}
double getUnsignedAngle(Vector3D u, Vector3D v){
  double cosTheta = getDot(u,v)/getLength(u)/getLength(v);
  cosTheta = max(-1.0,min(1.0,cosTheta));
  return acos(cosTheta);
 Vector3D rotate(Vector3D v, Vector3D a, double rad){
 a = unitVector(a):
 return v * cos(rad) + a * (1 - cos(rad)) * getDot(a,v) +
       getCross(a,v) * sin(rad);
struct Line3D{
Vector3D v; Pt3D o;
Line3D() {};
Line3D(Vector3D v,Pt3D o):v(v),o(o){}
Pt3D getPoint(double t) {return o + v*t;}
namespace Linear{
using namespace Vectorial;
double getDistSq(Line3D 1, Pt3D p) {return
getPLength(getCross(1.v,p-1.o))/getPLength(1.v);}
double getDistLinePoint(Line3D 1, Pt3D p) {return
      sqrt(getDistSq(1,p));}
 bool cmp(Line3D 1,Pt3D p, Pt3D q) {return getDot(1.v,p) <
getDot(1.v.q);}
Pt3D projection(Line3D 1,Pt3D p) {return 1.o + 1.v *
getDot(1.v,p-1.o)/getPLength(1.v);}
Pt3D reflection(Line3D 1,Pt3D p) {return
projection(1,p)+projection(1,p)-p;}
double getAngle(Line3D 1,Line3D m) {return
    getUnsignedAngle(1.v,m.v);}
 bool isParallel(Line3D p,Line3D q) {return
dcmp(getPLength(getCross(p.v,q.v))) == 0;}
bool isPerpendicular(Line3D p,Line3D q) {return
      dcmp(getDot(p.v,q.v)) = 0;
 double getDist(Line3D 1, Line3D m){
 Vector3D n = getCross(1.v, m.v);
  if(getPLength(n) == 0) return getDistLinePoint(1,m.o);
  else return fabs(getDot(m.o-1.o , n)) / getLength(n);
Pt3D getClosestPointOnLine1(Line3D 1,Line3D m){
  Vector3D n = getCross(1.v, m.v);
  Vector3D n2 = getCross(m.v, n);
 return 1.o + 1.v * getDot(m.o-1.o, n2) / getDot(1.v, n2);
struct Plane{
Vector3D n; //normal n
double d; //getDot(n,p) = d for any point p on the plane
Plane() {}
Plane(Vector3D n. double d): n(n), d(d) {}
Plane(Vector3D n, Pt3D p) : n(n), d(Vectorial :: getDot(n,p))
Plane(const Plane &p): n(p.n), d(p.d) {}
namespace Planar{
using namespace Vectorial;
Plane getPlane(Pt3D a.Pt3D b.Pt3D c) {return
      Plane(getCross(b-a,c-a),a);}
```

```
Plane translate(Plane p, Vector3D t) {return Plane(p.n,
      p.d+getDot(p.n,t));}
Plane shiftUp(Plane p, double dist) {return Plane(p.n,
      p.d+dist*getLength(p.n));}
Plane shiftDown(Plane p,double dist) {return Plane(p.n,
      p.d-dist*getLength(p.n));}
double getSide(Plane p,Pt3D a) {return getDot(p.n,a)-p.d;}
double getDistance(Plane p,Pt3D a) {return
fabs(getSide(p,a))/getLength(p.n);}
Pt3D projection(Plane p,Pt3D a) {return
a-p.n*getSide(p,a)/getPLength(p.n);}
Pt3D reflection(Plane p,Pt3D a) {return
a-p.n*getSide(p,a)/getPLength(p.n)*2;}
bool intersect(Plane p, Line3D 1, Pt3D& a){
  if(dcmp(getDot(p.n,l.v)) == 0) return false;
 a = 1.0 - 1.v * getSide(p,1.o) / getDot(p.n,1.v);
 return true;
bool intersect(Plane p,Plane q,Line3D& 1){
 1.v = getCross(p.n,q.n);
if(dcmp(getPLength(1.v)) == 0) return false;
 l.o = getCross(q.n*p.d - p.n*q.d , l.v) / getPLength(l.v);
return true;
double getAngle(Plane p,Plane q) {return
      getUnsignedAngle(p.n,q.n);}
bool isParallel(Plane p,Plane q) {return
dcmp(getPLength(getCross(p.n,q.n))) == 0;}
bool isPerpendicular(Plane p,Plane q) {return
      dcmp(getDot(p.n,q.n)) == 0;
bool getAngle(Plane p,Line3D 1) {return pi/2.0 -
    getUnsignedAngle(p.n,1.v);}
bool isParallel(Plane p,Line3D 1) {return
      dcmp(getDot(p.n,l.v)) == 0;
bool isPerpendicular(Plane p,Line3D 1) {return
      dcmp(getPLength(getCross(p.n,l.v))) == 0;}
Line3D perpThrough(Plane p,Pt3D a) {return Line3D(p.n,a);}
Plane perpThrough(Line3D 1,Pt3D a) {return Plane(1.v,a);}
//Modify p.n if necessary with respect to the reference point Vector3D rotateCCW90(Plane p,Vector3D d) {return
      getCross(p.n,d);}
Vector3D rotateCW90(Plane p, Vector3D d) {return
      getCross(d,p.n);}
pair<Pt3D, Pt3D> TwoPointsOnPlane(Plane p){
 if(dcmp(N.x) == 0 && dcmp(N.y) == 0) return
 {Pt3D(1,0,D/N.z), Pt3D(0,1,D/N.z)};
if(dcmp(N.y) == 0 && dcmp(N.z) == 0) return
       \{Pt3D(D/N.x,1,0), Pt3D(D/N.x,0,1)\};
 if(dcmp(N.z) == 0 \&\& dcmp(N.x) == 0) return
        \{Pt3D(1,D/N.y,0), Pt3D(0,D/N.y,1)\};
 if(dcmp(N.x) == 0) return \{Pt3D(0,1,D/N.z), Pt3D(0,0,D/N.z)\}; if(dcmp(N.y) == 0) return \{Pt3D(0,1,D/N.z), Pt3D(D/N.x,0,0)\};
 if(dcmp(N.z) == 0) return \{Pt3D(D/N.x,0,1), Pt3D(0,D/N.y,0)\};
 if (dcmp(D)!=0) return \{Pt3D(D/N.x,0,0), Pt3D(0,D/N.y,0)\};
 return {Pt3D(N.y,-N.x,0), Pt3D(-N.y,N.x,0)};
Point From3Dto2D(Plane p, Pt3D a){
 assert( dcmp(getSide(p,a)) == 0 );
 auto Pair = TwoPointsOnPlane(p);
 Pt3D A = Pair.first;
Pt3D B = Pair.second:
 Vector3D Z = p.n; Z = Z / getLength(Z);
Vector3D X = B - A; X = X / getLength(X);
 Vector3D Y = getCross(Z,X);
 Vector3D v = a - A;
 assert( dcmp(getDot(v,Z)) == 0);
 return Point(getDot(v,X),getDot(v,Y));
Pt3D From2Dto3D(Plane p, Point a){
 auto Pair = TwoPointsOnPlane(p):
 Pt3D A = Pair.first;
Pt3D B = Pair.second;
Vector3D Z = p.n; Z = Z / getLength(Z);
 Vector3D X = B - A; X = X / getLength(X);
 Vector3D Y = getCross(Z,X);
```

```
return A + X * a.x + Y * a.v:
struct Sphere{
Pt3D c;
double r;
Sphere() {}
Sphere(Pt3D c, double r) : c(c), r(r) {}
 //Spherical cap with polar angle theta
 double Height(double alpha) {return r*(1-cos(alpha));}
double BaseRadius(double alpha) {return r*sin(alpha);}
double Volume(double alpha) {double h = Height(alpha); return
      pi*h*h*(3*r-h)/3.0;
 double SurfaceArea(double alpha) {double h = Height(alpha);
      return 2*pi*r*h:}
namespace Spherical{
using namespace Vectorial;
using namespace Planar;
using namespace Linear;
Sphere CircumscribedSphere(Pt3D a,Pt3D b,Pt3D c,Pt3D d){
 assert( dcmp(getSide(getPlane(a,b,c), d)) != 0);
 Plane U = Plane(a-b, (a+b)/2);
Plane V = Plane(b-c, (b+c)/2);
Plane W = Plane(c-d, (c+d)/2);
Line3D 11,12;
 bool ret1 = intersect(U,V,l1);
bool ret2 = intersect(V,W,l2);
  assert(ret1 == true && ret2 == true):
  assert( dcmp(getDist(11,12)) == 0);
  Pt3D C = getClosestPointOnLine1(11,12);
 return Sphere(C, getLength(C-a));
pair < double, double > Sphere Sphere Intersection (Sphere s1, Sphere
  double d = getLength(s1.c-s2.c);
  if(dcmp(d - s1.r -s2.r) >= 0) return {0,0};
double R1 = max(s1.r,s2.r); double R2 = min(s1.r,s2.r);
  double y = R1 + R2 - d;
 double y = 11 1 12 d,

double x = (R1*R1 - R2*R2 + d*d) / (2*d);

double h1 = R1 - x;

double h2 = y - h1;
  double Volume = pi*h1*h1*(3*R1-h1)/3.0 +
       pi*h2*h2*(3*R2-h2)/3.0;
  double SurfaceArea = 2*pi*R1*h1 + 2*pi*R2*h2;
 return make pair(SurfaceArea, Volume):
 Pt3D getPointOnSurface(double r,double Lat,double Lon){
 Lat = torad(Lat); //North-South
Lon = torad(Lon); //East-West
 return Pt3D(r*cos(Lat)*cos(Lon), r*cos(Lat)*sin(Lon),
       r*sin(Lat));
 int intersect(Sphere s,Line3D 1, vector<Pt3D>& ret){
 double h2 = s.r*s.r - getDistSq(1,s.c);
 if(dcmp(h2)<0) return 0:
 Pt3D p = projection(1,s.c);
 if(dcmp(h2) == 0) {ret.push_back(p); return 1;}
Vector3D h = 1.v * sqrt(h2) / getLength(1.v);
 ret.push_back(p-h); ret.push_back(p+h); return 2;
 double GreatCircleDistance(Sphere s,Pt3D a,Pt3D b){
 return s.r * getUnsignedAngle(a-s.c, b-s.c);
namespace Poly{
using namespace Vectorial;
Sphere SmallestEnclosingSphere(Polyhedron p){
 int n = p.size();
 Pt3D C(0,0,0);
 for(int i=0; i < n; i++) C = C + p[i];
 C = C / n;
double P = 0.1;
  int pos = 0;
  int Accuracy = 70000:
  for (int i = 0; i < Accuracy; i++) {
  pos = 0;
   for (int j = 1; j < n; j++){
```

```
if(getPLength(C - p[j]) > getPLength(C - p[pos])) pos = j;
}
C = C + (p[pos] - C)*P;
P *= 0.998;
}
return Sphere(C, getPLength(C - p[pos]));
}
```

26 HLD

```
struct HLD{
vector<int> parent, depth, heavy, head, pos;
int cur_pos; segtree seg;
int dfs(int v, vector<vector<int>> const& adj) {
  int size = 1; int max_c_size = 0;
  for (int c : adj[v]) \{
  if (c != parent[v]) {
  parent[c] = v, depth[c] = depth[v] + 1;
    int c_size = dfs(c, adj);
    size += c_size;
    if (c_size > max_c_size) max_c_size = c_size, heavy[v] = c;
  return size;
 void decompose(int v, int h, vector<vector<int>> const& adj) {
 head[v] = h, pos[v] = cur_pos++;
if (heavy[v] != -1)
  decompose(heavy[v], h, adj);
  for (int c : adj[v]) {
  if (c != parent[v] && c != heavy[v])
    decompose(c, c, adj);
void init(vector<vector<int>> const& adj, vector<ll>&a) {
 int n = adj.size();
  parent = vector<int>(n); depth = vector<int>(n);
  heavy = vector<int>(n, -1); head = vector<int>(n);
  pos = vector<int>(n); cur_pos = 0;
dfs(0, adj); decompose(0, 0, adj);
  vector<ll>tmp(n);
 for(int i=0; i<n; i++) {
  tmp[pos[i]] = a[i];</pre>
  seg.init(n,tmp);
 int query(int a, int b) {
  11 res = 0;
 for (; head[a] != head[b]; b = parent[head[b]]) {
  if (depth[head[a]] > depth[head[b]])
   int cur_heavy_path_max = seg.query(pos[head[b]], pos[b]);
  res += cur_heavy_path_max;
  if (depth[a] > depth[b])
  swap(\bar{a}, b);
  int last_heavy_path_max = seg.query(pos[a], pos[b]);
  res += last_heavy_path_max;
 return res;
 void update(int a, int b, int x) {
 for (; head[a] != head[b]; b = parent[head[b]]) {
  if (depth[head[a]] > depth[head[b]]) swap(a, b);
   seg.update(pos[head[b]], pos[b], x);
  if (depth[a] > depth[b]) swap(a, b);
  seg.update(pos[a], pos[b],x);
```

27 Hackenbush

```
struct hackenbush {
  int n;
  vector<vector<int>> adj;
  hackenbush(int n) : n(n), adj(n) { }
  void add_edge(int u, int v) {
```

```
adj[u].push_back(v);
 if (u != v) adj[v].push_back(u);
// r is the only root connecting to the ground
int grundy(int r) {
 vector<int> num(n). low(n):
 int t = 0;
 function<int(int, int)> dfs = [&](int p, int u) {
  num[u] = low[u] = ++t;
  int ans = 0;
  for (int v : adj[u]) {
  if (v == p) { p += 2 * n; continue; }
if (num[v] == 0) {
   int res = dfs(u, v);
low[u] = min(low[u], low[v]);
if (low[v] > num[u]) ans ^= (1 + res) ^ 1; // bridge
    else ans ^= res; // non bridge
   } else low[u] = min(low[u], num[v]);
  if (p > n) p = 2 * n;
  for (int v : adj[u])
   if (v != p && num[u] <= num[v]) ans ^= 1;
  return ans:
return dfs(-1, r);
int main() {
int cases; scanf("%d", &cases);
for (int icase = 0; icase < cases; ++icase) {
  int n; scanf("%d", &n);</pre>
 vector<int> ground(n);
 int r;
 for (int i = 0; i < n; ++i) {
  scanf("%d", &ground[i]);</pre>
  if (ground[i] == 1) r = i;
 int ans = 0:
 hackenbush g(n);
 for (int i = 0; i < n - 1; ++i) {
 int u, v;
scanf("%d %d", &u, &v);
  if (ground[u]) u = r;
  if (ground[v]) v = r;
  if (u == v) ans = 1;
  else g.add_edge(u, v);
int res = ans ^ g.grundy(r);
printf("%d\n", res != 0);
```

28 Hashing 2D

```
int mods[2] = {1000000007, 1000000009};
int bases[2] = {137, 281};
int pwbase[2] [MAX];
void Preprocess(){
  pwbase[0][0] = pwbase[1][0] = 1;
  for(int i = 0; i < 2; i++) {
    for(int j = 1; j < MAX; j++) {
      pwbase[i][j] = (pwbase[i][j - 1] *1ll* bases[i]) % mods[i];
    }
}
struct Hashing{
  int hsh[2][MAX];
  string str;
  void setstr(string &_str) {
    str = str;
    hsh[0][str.size()] = 0;
    hsh[1][str.size()] = 0;
    hsh[1][str.size()] = 0;
    Build();
}
void Build() {
  for(int i = str.size() - 1; i >= 0; i--) {
    for(int j = 0; j < 2; j++) {</pre>
```

29 Hopcroft

```
// If input graph is not given in L-R manner, make it so by coloring.
// Input graph must be bipartite const int N=200*200+5;
struct HopcroftKarp
static const int inf = 1e9; int n;
vector<int> 1, r, d; vector<vector<int>> g;
HopcroftKarp(int _n, int _m) {
 n = n; int p = n + m + 1;
 g.resize(p); l.resize(p, 0); r.resize(p, 0); d.resize(p, 0);
void add_edge(int u, int v) {
 g[u].push_back(v + n); //right id is increased by n, so is
bool bfs() {
 queue<int> q;
 for (int u = 1; u <= n; u++) {
  if (!l[u])
   d[u] = 0, q.push(u);
  else
   d[u] = inf:
 d[0] = inf;
  while (!q.empty()) {
  int u = q.front(); q.pop();

for (auto v : g[u]) { if (d[r[v]] == inf) {

   d[r[v]] = d[u] + 1; q.push(r[v]);
 return d[0] != inf;
bool dfs(int u) {
 if (!u) return true
 for (auto v : g[u]){ if(d[r[v]] == d[u] + 1 && dfs(r[v])){
   l[u] = v; r[v] = u;
return true;
 d[u] = inf;
 return false;
int maximum_matching(){
 int ans = \overline{0}:
 while (bfs()){ for(int u = 1; u <= n; u++)
   if (!l[u] && dfs(u)) ans++;
 return ans;
```

0 Hungarian

namespace wm{

```
bool vis[N]; int U[N], V[N], P[N]
int way [N], minv [N], match [N], ar [N] [N]
///n=no of row, m=no of col,1 based,flag=MAXIMIZE/MINIMIZE
///match[i]=the column to which row i is matched
int hungarian(int n,int m,int mat[N][N],int flag){
  clr(U), clr(V), clr(P), clr(ar), clr(way);
  for (int i = 1; i \le n; i++){
    for (int j = 1; j <= m; j++){
    ar[i][j] = mat[i][j];
      if (flag == MAXIMIZE) ar[i][j] = -ar[i][j];
  if (n > m) m = n;
  int i, j, a, b, c, d, r, w;
  for (i = 1; i <= n; i++){
P[0] = i, b = 0;
    for (j=0; j \le m; j++) \min v[j] = \inf, vis[j] = 0;
      vis[b] = true; a = P[b], d = 0, w = inf;
      for (j = 1; j \le m; j++){
        if (!vis[j]){
         r = ar[\tilde{a}][j] - U[a] - V[j];
          if (r < minv[j]) minv[j] = r, way[j]=b;</pre>
          if (minv[i] < w) w = minv[i], d = j;
      for (j = 0; j <= m; j++){
   if (vis[j]) U[P[j]] += w, V[j] -= w;
        else minv[i] -= w:
      \dot{b} = d;
    } while (P[b] != 0);
      d = way[b]; P[b] = P[d], b = d;
    } while (b != 0);
  for (j = 1; j <= m; j++) match[P[j]] = j;
  return (flag == MINIMIZE) ? -V[0]: V[0];
```

31 IntersectingSegmentSweepLine

```
struct seg {
pt p, q; int id;
double get_y(double x) const {
 if (abs(p.x - q.x) < EPS)return p.y;
 return p.y + (q.y - p.y) * (x - p.x) / (q.x - p.x);
bool intersect1d(double 11, double r1, double 12, double r2) {
if (11 > r1) swap(11, r1);
if (12 > r2) swap(12, r2);
return max(11, 12) \le min(r1, r2) + EPS
int vec(const pt& a, const pt& b, const pt& c) {
double s = (b.x - a.x) * (c.y - a.y) - (b.y - a.y) * (c.x - a.y)
     a.x);
return abs(s) < EPS ? 0 : s > 0 ? +1 : -1;
bool intersect(const seg& a, const seg& b) {
return intersect1d(a.p.x, a.q.x, b.p.x, b.q.x) &&
    intersect1d(a.p.y, a.q.y, b.p.y, b.q.y) && vec(a.p, a.q, b.p) * vec(a.p, a.q, b.q) <= 0 &&
    vec(b.p, b.q, a.p) * vec(b.p, b.q, a.q) <= 0;
bool operator<(const seg& a, const seg& b){
double x = max(min(a.p.x, a.q.x), min(b.p.x, b.q.x));
return a.get_y(x) < b.get_y(x) - EPS;
struct event {
double x; int tp, id;
event() {}
event(double x, int tp, int id) : x(x), tp(tp), id(id) {}
bool operator<(const event& e) const {
```

```
if (abs(x - e.x) > EPS)return x < e.x;
return tp > e.tp;
sét<seg> s;
vector < set < seg >:: iterator > where;
set<seg>::iterator prev(set<seg>::iterator it) {
return it == s.begin() ? s.end() : --it;
set<seg>::iterator next(set<seg>::iterator it) {
pair<int, int> solve(const vector<seg>& a) {
int n = (int)a.size();vector<event> e;
for (int i = 0; i < n; ++i) {
 e.push_back(event(min(a[i].p.x, a[i].q.x), +1, i));
e.push_back(event(max(a[i].p.x, a[i].q.x), -1, i));
sort(e.begin(), e.end());s.clear();
where.resize(a.size());
int id = e[i].id;
if (e[i].tp == +1) {
  set<seg>::iterator nxt = s.lower_bound(a[id]), prv =
       prev(nxt);
  if (nxt != s.end() && intersect(*nxt, a[id]))
   return make_pair(nxt->id, id);
  if (prv != s.end() && intersect(*prv, a[id]))
   return make_pair(prv->id, id);
  where[id] = s.insert(nxt, a[id]);
 } else {
  set<seg>::iterator nxt = next(where[id]), prv =
       prev(where[id]);
  if (nxt != s.end() && prv != s.end() && intersect(*nxt,
   return make_pair(prv->id, nxt->id);
  s.erase(where[id]);
return make_pair(-1, -1);
```

32 KnuthDP

$\overline{33}$ LCA

```
template <class T>
struct RMQ { // O-based
vector<vector<T>> rmq;
T kInf = numeric_limits<T>::max();
void build(const vector<T>& V) {
  int n = V.size(), on = 1, dep = 1;
  while (on < n) on *= 2, ++dep;
  rmq.assign(dep, V);
  for (int i = 0; i < dep - 1; ++i)
  for (int j = 0; j < n; ++j) {
    rmq[i + 1][j] = min(rmq[i][j], rmq[i][min(n - 1, j + (1 << i)]);
    reconstruction</pre>
```

```
T query(int a, int b) \{ // [a, b) \}
 if (b <= a) return kInf;</pre>
 int dep = 31 - \_builtin\_clz(b - a); // log(b - a)
 return min(rmq[dep][a], rmq[dep][b - (1 << dep)]);
struct LCA { // O-based vector<int> enter, depth, exxit;
vector<vector<int>> G;
vector<pair<int, int>> linear;
RMQ<pair<int, int>> rmq;
int timer = 0;
LCA() {}
LCA(int n): enter(n, -1), exxit(n, -1), depth(n), G(n),
     linear(2 * n) {}
 void dfs(int node, int dep) {
 linear[timer] = {dep, node};
 enter[node] = timer++;
depth[node] = dep;
  for (auto vec : G[node])
 if (enter[vec] == -1) {
  dfs(vec, dep + 1);
  linear[timer++] = {dep, node};
 exxit[node] = timer;
void add_edge(int a, int b) {
 G[a].push_back(b);
 G[b].push_back(a);
void build(int root) {
 dfs(root, 0);
 rmq.build(linear);
int query(int a, int b) {
 a = enter[a], b = enter[b];
 return rmq.query(min(a, b), max(a, b) + 1).second;
int dist(int a, int b) {
 return depth[a] + depth[b] - 2 * depth[query(a, b)];
```

34 LCABinaryLift

```
int n, 1;
vector<vector<int>> adi:
vector<int> tin, tout; vector<vector<int>> up
void dfs(int v, int p){
tin[v] = ++timer;
up[v][0] = p;
for (int i = 1; i <= 1; ++i)
 up[v][i] = up[up[v][i-1]][i-1];
for (int u : adj[v]) {
 if (u != p)
  dfs(u, v);
tout[v] = ++timer:
bool is_ancestor(int u, int v){
return tin[u] <= tin[v] && tout[u] >= tout[v];
int lca(int u, int v){
if (is_ancestor(u, v))
 return u;
if (is_ancestor(v, u))
 return v;
for (int i = 1; i \ge 0; --i) {
 if (!is_ancestor(up[u][i], v))
  u = up[u][i];
return up[u][0];
void preprocess(int root) {
tin.resize(n);
```

```
tout.resize(n);
timer = 0;
l = ceil(log2(n));
up.assign(n, vector<int>(l + 1));
dfs(root, root);
}
```

```
LCT rooted
typedef pair< int, int >Linear;
Linear compose(const Linear &p, const Linear &q)
return Linear(mul(p.first, q.first),
      sum(mul(q.second, p.first), p.second));
struct SplayTree
struct Node {
 int ch[2] = \{0, 0\}, p = 0;
 long long self = 0, path = 0; //Path aggregates
  long long sub = 0, vir = 0;//Subtree aggregate
  int size = 1; bool flip = 0; // Lazy tags
  Linear _{self\{1, 0\}}, _{shoja\{1, 0\}}, _{ulta\{1, 0\}};
 vector<Node> T:
SplayTree(int \hat{n}) : T(n + 1) {
 T[0].size = 0;
void push(int x) {
  if (!x || !T[x].flip)
  return;
 int 1 = T[x].ch[0], r = T[x].ch[1];
T[1].flip ^= 1, T[r].flip ^= 1;
swap(T[x].ch[0], T[x].ch[1]); T[x].flip = 0;
  swap(T[x].shoja, T[x].ulta);
void pull(int x) {
  int l=T[x].ch[0],r=T[x].ch[1];
  push(1):
  push(r):
 T[x].size = T[1].size + T[r].size + 1;
T[x].path = T[1].path + T[x].self + T[r].path;
 T[x].sub=T[x].vir+T[]].sub+T[r].sub+T[x].self;
T[x].shoja = compose(T[r].shoja,
        compose(T[x]._self, T[1].shoja));
  T[x].ulta = compose(T[1].ulta,
       compose(T[x]._self, T[r].ulta));
 void set(int x, int d, int y) {
 T[x].ch[d] = y; T[y].p = x; pull(x);
void splay(int x) {
  auto dir = [&](int x)
  int p = T[x].p;
   if (!p) return -1;
  return T[p].ch[0] == x?0:T[p].ch[1] == x?1:-1;
  auto rotate = [&](int x)
  int y = T[x].p,z=T[y].p,dx=dir(x),dy=dir(y);
set(y, dx, T[x].ch[!dx]); set(x, !dx, y);
   if (^*dy) set(z, dy, x);
  T[x].p = z;
  for (push(x); "dir(x); )
  int y = T[x].p,z = T[y].p;
push(z); push(y); push(x);
   int dx = dir(x), dy = dir(y);
   if (~dy) rotate(dx!=dy?x:y);
  rotate(x);
 int KthNext(int x, int k) {
 assert(k > 0); splay(x);
 x = T[x].ch[1];
  if (T[x].size < k) return -1;
```

```
while (true)
  push(x); int 1 = T[x].ch[0], r = T[x].ch[1];
  if (T[1].size+1 == k) return x;
  if (k <= T[1].size) x = 1;
  else k \rightarrow T[1].size+1, x = r;
struct LinkCut : SplayTree
LinkCut(int n) : SplayTree(n) {}
int access(int x) {
 int u = x, v = 0;
 for (; u; v = u, u = T[u].p)
  splay(u); int & ov = T[u].ch[1];
  T[u].vir += T[ov].sub; T[u].vir -= T[v].sub;
  ov = v; pull(u);
 splay(x);
 return v;
void reroot(int x) {
 access(x); T[x].flip ^= 1; push(x);
l'/lmakes v parent of u !(u must be a root)
void Link(int u, int v) {
 reroot(u); access(v); T[v].vir += T[u].sub;
 T[u].p = v; pull(v);
\c///removes edge between u and v
void Cut(int u, int v) {
 int _u = FindRoot(u); reroot(u);
access(v); T[v].ch[0] = T[u].p = 0;
 pull(v); reroot(_u);
//Rooted tree LCA.Returns 0 if u v not connected
int LCA(int u. int v) {
 if (u == v) return u;
 access(u); int ret = access(v);
 return T[u].p ? ret : 0;
//Query subtree of u where v is outside the sbtr
long long Subtree(int u, int v) {
 int _v = FindRoot(v); reroot(v); access(u);
 long long ans = T[u].vir + T[u].self;
 reroot(_v);
 return ans
long long Path(int u, int v) {
  int _u = FindRoot(u); reroot(u); access(v);
 long long ans = T[v].path;
 reroot(_u);
 return ans;
Linear _Path(int u, int v) {
 reroot(u); access(v);
return T[v].shoja;
void Update(int u, long long v) {
 access(u); T[u].self = v; pull(u);
void _Update(int u, Linear v) {
 access(u); T[u]._self = v;
 pull(u);
 int FindRoot(int u) {
 access(11)
 while (T[u].ch[0]) { u = T[u].ch[0]; push(u);}
 access(u);
 return u:
\dot{/}//k-th node (0-indexed) on the path from u to v
int KthOnPath(int u, int v, int k) {
 if (u == v) return k == 0? u : -1;
 int u = FindRoot(u):
 reroot(u); access(v);
 int ans = KthNext(u, k); reroot(_u);
```

```
return ans;
};
```

12

36 LIS

```
int lis(vector<int> const& a) {
  int n = a.size();
  const int INF = 1e9;
  vector<int> d(n+1, INF);
  d[0] = -INF;

for (int i = 0; i < n; i++) {
  int j = upper_bound(d.begin(), d.end(), a[i]) - d.begin();
  if (d[j-1] < a[i] && a[i] < d[j])
  d[j] = a[i];
  }
  int ans = 0;
  for (int i = 0; i <= n; i++) {
    if (d[i] < INF)
    ans = i;
  }
  return ans;
}</pre>
```

37 MO

```
struct query{int l,r,idx;};
int block;
bool comp1(query p,query q){
   if (p.1 / block != q.1 / block) {
      if(p.1==q.1) return p.r<q.r;
      return p.l < q.l;
   }
   return (p.1 / block & 1) ? (p.r < q.r) : (p.r > q.r);
}
void mos_algorithm(int n, vector<query>&queries){
   vector<int> answers(queries.size());
   block = (int)sqrt(n);
   sort(queries.begin(), queries.end(),comp1);
   int cur_l = 0;
   int cur_r = -1;
   for (query q : queries) {
      while (cur_l > q.l) {cur_l--; add(cur_l);}
      while (cur_l < q.r) {cur_r++;add(cur_r);}
      while (cur_r > q.r) {Remove(cur_l);cur_l++;}
      while (cur_r > q.r) {Remove(cur_r);cur_r---;}
      answers[q.idx] = get_answer();
   }
   for(int i:answers) {cout<<i<<"\n";}
}</pre>
```

38 Manacher

```
vector<int> d1(n);
for (int i = 0, l = 0, r = -1; i < n; i++) {
   int k = (i > r) ? 1 : min(d1[l + r - i], r - i + 1);
   while (0 <= i - k && i + k < n && s[i - k] == s[i + k]) {
     k++;
   }
d1[i] = k--;
   if (i + k > r) {
     l = i - k;
     r = i + k;
   }
}
vector<int> d2(n);
for (int i = 0, l = 0, r = -1; i < n; i++) {
   int k = (i > r) ? 0 : min(d2[l + r - i + 1], r - i + 1);
   while (0 <= i - k - 1 && i + k < n && s[i - k - 1] == s[i + k]) {
     k++;
   }
d2[i] = k--;
   if (i + k > r) {
     l = i - k - 1;
     r = i + k;
}
}
```

39 MatrixDeterminant

```
double det = 1;
for (int i=0; i<n; ++i) {
  int k = i;
  for (int j=i+1; j<n; ++j)
  if (abs (a[j][i]) > abs (a[k][i]))
    k = j;
  if (abs (a[k][i]) < EPS) {
    det = 0;
    break;
}
swap (a[i], a[k]);
if (i != k)
    det = -det;
    det *= a[i][i];
for (int j=i+1; j<n; ++j)
    a[i][j] /= a[i][i];
for (int j=0; j<n; ++j)
    if (j != i &k abs (a[j][i]) > EPS)
    for (int k=i+1; k<n; ++k)
    a[j][k] -= a[i][k] * a[j][i];
}</pre>
```

40 MatrixExpo

```
/* try to avoid vector. Possibly use STL array or pointers */
void multiply(vector<vector<int> >&a, vector<vector<int> >&b){
  int n = a.size(), m= a[0].size(), 1 = b[0].size();
 vector<vector<int> >ret(n.vector<int>(1));
 for(int i=0; i<n; i++) {
  for(int k=0; k<m; k++) {</pre>
   for(int j=0; j<1; j++) {
    ret[i][j] = add(ret[i][j], gun(a[i][k],b[k][j],mod),mod);
 swap(ret,a);
void bigmod(vector<vector<int> >&a, int p){
 int n = a.size();
 assert(a.size()==a[0].size());
 vector<vector<int> >res(n, vector<int> (n));
 for(int i=0; i<n; i++) {
  for(int j=0; j<n; j++) {
  res[i][j] = 0;
   if(i==j)
    res[i][j]=1;
 while(p) {
  if(p&1) {
   multiply(res,a);
  multiply(a,a);
  p>>=1;
 swap(a, res);
```

41 MillerRabin

```
using u64 = uint64_t;
using u128 = __uint128_t;
u64 binpower(u64 base, u64 e, u64 mod) {
u64 result = 1;
base %= mod;
while (e) {
   if (e & 1)
      result = (u128)result * base % mod;
   base = (u128)base * base % mod;
   e >>= 1;
}
return result;
}
bool check_composite(u64 n, u64 a, u64 d, int s) {
u64 x = binpower(a, d, n);
```

```
if (x == 1 | | x == n - 1)
 return false;
for (int r = 1; r < s; r++) {
 x = (u128)x * x % n;
 if (x == n - 1)
 return false:
return true;
bool MillerRabin(u64 n) { // returns true if n is prime, else
    returns false.
if (n < 2)
 return false;
int r = 0:
\bar{u}64 \ \bar{d} = \bar{n}' - 1;
while ((d \& 1)) == 0) {
 d >>= 1:
 r++;
for (int a: {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37}) {
 if (n == a)
  return true;
 if (check_composite(n, a, d, r))
  return false;
return true;
```

42 MinCostFlow1

#define fst first

```
#define snd second
#define all(c) ((c).begin()), ((c).end())
#define TEST(s) if (!(s)) { cout << __LINE__ << " " << #s <<
     endl: exit(-1): }
const long long INF = 1e9;
struct graph {
  typedef int flow_type;
 typedef int cost_type;
 struct edge {
 int src, dst;
flow_type capacity, flow;
cost_type cost;
size_t rev;
 void add_edge(int src, int dst, flow_type cap, cost_type
 adj[src].push_back({src, dst, cap, 0, cost, adj[dst].size()});
 adj[dst].push_back({dst, src, 0, 0, -cost,
      adj[src].size()-1});
 int n;
  vector<vector<edge>> adj;
  graph(int n) : n(n), adj(n) { }
pair<flow_type, cost_type> min_cost_max_flow(int s, int t) {
flow_type flow = 0;
 cost_type cost = 0;
for (int u = 0; u < n; ++u)
  for (auto &e: adj[u]) e.flow = 0;
 vector<cost_type> p(n, 0);
 auto rcost = [&](edge e) { return e.cost + p[e.src] -
      p[e.dst]; };
 for (int iter = 0; ; ++iter) {
  vector<int> prev(n, -1); prev[s] = 0;
  vector<cost_type> dist(n, INF); dist[s] = 0;
  if (iter == 0) {
  vector<int> count(n); count[s] = 1;
  queue<int> que;
  for (que.push(s); !que.empty(); ) {
   int u = que.front(); que.pop();
count[u] = -count[u];
for (auto &e: adj[u]) {
   if (e.capacity > e.flow && dist[e.dst] > dist[e.src] +
        rcost(e)) {
    dist[e.dst] = dist[e.src] + rcost(e);
prev[e.dst] = e.rev;
```

```
if (count[e.dst] <= 0) {
   count[e.dst] = -count[e.dst] + 1:
   que.push(e.dst);
  }
 typedef pair<cost_type, int> node;
priority_queue<node, vector<node>, greater<node>> que;
 que.push(\{0, s\});
 while (!que.empty()) {
   node a = que.top(); que.pop();
  if (a.snd == t) break;
if (dist[a.snd] > a.fst) continue;
for (auto e: adj[a.snd]) {
  if (e.capacity > e.flow && dist[e.dst] > a.fst + rcost(e)) {
    dist[e.dst] = dist[e.src] + rcost(e);
prev[e.dst] = e.rev;
    que.push({dist[e.dst], e.dst});
  if (prev[t] == -1) break:
 for (int u = 0; u < n; ++u)
if (dist[u] < dist[t]) p[u] += dist[u] - dist[t];</pre>
  function<flow_type(int,flow_type)> augment = [&](int u,
 flow_type cur) {
if (u == s) return cur;
 edge &r = adj[u][prev[u]], &e = adj[r.dst][r.rev];
 flow_type f = augment(e.src, min(e.capacity - e.flow, cur));
e.flow += f; r.flow -= f;
 return f:
  flow_type f = augment(t, INF);
  cost += f * (p[t] - p[s]);
return {flow, cost};
```

43 MinCostFlow2

```
struct Edge{
int u. v:
long long cap, cost;
Edge(int _u, int _v, long long _cap, long long _cost) {
u = _u;
v = _v;
cap = _cap;
cost = _cost;
struct MinCostFlow{
int n, s, t;
long long flow, cost;
vector<vector<int> > graph;
vector<Edge> e:
/* if cost is double, dist should be double*/
vector<long long> dist;
vector<int> parent;
MinCostFlow(int _n) {
 /* 0-based indexing*/
n = _n;
 graph.assign(n, vector<int> ());
void addEdge(int u, int v, long long cap, long long cost,
      bool directed = true)
  graph[u].push_back(e.size());
 e.push_back(Edge(u, v, cap, cost));
  graph[v].push_back(e.size());
 e.push_back(Edge(v, u, 0, -cost));
 if(!directed)
```

```
addEdge(v, u, cap, cost, true);
pair<long long, long long> getMinCostFlow(int _s, int _t) {
flow = 0, cost = 0;
 while(SPFA()) {
 flow += sendFlow(t, 1LL<<62);
return make_pair(flow, cost);
bool SPFA() {
parent.assign(n, -1);
dist.assign(n, 1LL<<62);</pre>
 dist[s] = 0:
 vector<int> queuetime(n, 0);
 queuetime[s] = 1;
 vector<bool> inqueue(n, 0);
 inqueue[s] = true;
 queue<int> q;
 q.push(s);
 bool negativecycle = false;
 while(!q.empty() && !negativecycle) {
 int u = q.front();
 q.pop();
  inqueue[u] = false;
 for(int i = 0; i < graph[u].size(); i++) {
  int eIdx = graph[u][i];
  int v = e[eIdx].v;</pre>
  11 w = e[eIdx].cost, cap = e[eIdx].cap;
   if(dist[u] + w < dist[v] && cap > 0) {
   dist[v] = dist[u] + w;
parent[v] = eIdx;
    if(!inqueue[v]) {
    q.push(v);
     queuetime[v]++;
     inqueue[v] = true;
     if(queuetime[v] == n+2) {
     negativecycle = true;
     break:
return dist[t] != (1LL<<62);
long long sendFlow(int v, long long curFlow) {
 if(parent[v] == -1)
 return curflow:
 int eIdx = parent[v];
 int u = e[eIdx].u:
ll w = e[eIdx].cost;
long long f = sendFlow(u, min(curFlow, e[eIdx].cap));
 e[eIdx].cap -= f;
 e[eIdx^1].cap += f;
return f:
```

MinimumStack

```
void small_left(vector<int>& v, vector<int>& res){
stack<pair<int, int> > stk;
stk.push(make_pair(INT_MIN, v.size()));//initial value
for (int i = v.size()-1; i >= 0; i--) {
 while (stk.top().first > v[i]) {
  res[stk.top().second] = i;
```

```
stk.pop();
stk.push(make_pair(v[i], i));
while (stk.top().second < v.size()) {
res[stk.top().second] = -1;
stk.pop();
```

Minkowski

```
void reorder_polygon(vector<pt> & P){
    size_t pos = 0;
 for(size_t i = 1; i < P.size(); i++){
 if(P[i].y < P[pos].y || (P[i].y == P[pos].y && P[i].x <
       P[pos].x))
rotate(P.begin(), P.begin() + pos, P.end());
vector<pt> minkowski(vector<pt> P, vector<pt> Q){
 reorder_polygon(P);reorder_polygon(Q);
P.push_back(P[0]); P.push_back(P[1]);
Q.push_back(Q[0]);Q.push_back(Q[1]);
vector<pt> result;
 size_t i = 0, j = 0;
while(i < P.size() - 2 || j < Q.size() - 2){
  result.push_back(P[i] + Q[j]);</pre>
  auto cross = (P[i + 1] - P[i]).cross(Q[j + 1] - Q[j]);
 if(cross >= 0) ++i;
 if(cross <= 0) ++j;
return result;
```

return hash1 ^ hash2;

```
46 Miscellaneous
for(int i=a._Find_first(); i < a.size(); i = a._Find_next(i))
mt19937 rng(chrono::steady_clock::now().</pre>
     time_since_epoch().count());
int getrand(int a, int b){
int x = uniform_int_distribution<int>(a, b)(rng);
return x:
11.splice(l1.end(), 12);
merge(a.begin(), a.end(),
   b.begin(), b.end(),
   back_inserter(c));
#pragma GCC optimize("Ofast")
#pragma GCC target("sse,sse2,sse3,sse4,popcnt,abm,mmx,
#pragma GCC optimize("unroll-loops")
#pragma GCC optimize("03")
struct custom hash {
 static uint64_t splitmix64(uint64_t x) {
 x += 0x9e3779b97f4a7c15:
 x = (x^{(x)} > 30)) * 0xbf58476d1ce4e5b9;

x = (x^{(x)} > 27)) * 0x94d049bb133111eb;
 return x \hat{} (x >> 31);
 size_t operator()(uint64_t x) const {
  static const uint64_t FIXED_RANDOM =
       chrono::steady_clock::now().
  time_since_epoch().count();
 return splitmix64(x + FIXED_RANDOM);
unordered_map<long long, int, custom_hash> safe_map;
struct hash_pair {
 template <class T1, class T2>
 size_t operator()(const pair<T1, T2>& p) const {
  auto hash1 = hash<T1>{}(p.first);
  auto hash2 = hash<T2>{}(p.second);
```

```
unordered_map<pair<int, int>, int, hash_pair> mp;
//mod inverse for all m
inv[1] = 1;
for(int i = 2; i < m; ++i)
inv[i] = m - (m/i) * inv[m%i] % m;
//ordered set
//submask supermask
for (int s=m; s; s=(s-1)&m)
for (int mask=need; mask < (1<<n); mask = (mask+1)|need)
string str="abc def gh", buf; stringstream ss(str);
while(ss >> buf) cout << buf << endl;
```

47 Mo with update

```
const int N = 1e5 + 5;
const int P = 2000; //block \ size = (2*n^2)^(1/3)
struct query{int t, 1, r, k, i;}; vector<query> q;vector<array<int, 3>> upd;
vector<int> ans: vector<int>a:
void mos_algorithm(){
sort(q.begin(), q.end(), [](const query &a, const query &b) {
   if (a.t / P != b.t / P) return a.t < b.t;
   if (a.l / P != b.l / P) return a.l < b.l;</pre>
 if ((a.1 / P) & 1) return a.r < b.r:
 return a.r > b.r;
});
for (int i = upd.size() - 1; i \ge 0; --i)
a[upd[i][0]] = upd[i][1];
int L = 0, R = -1, T = 0;
auto apply = [&](int i, int fl) {
  int p = upd[i][0]; int x = upd[i][fl + 1];
 if (\hat{L} \leq p \&\& p \leq R) \{ rem(a[p]); add(x); \}
 a[p] = x;
 ans.clear();ans.resize(q.size());
for (auto qr : q) {
  int t = qr.t, 1 = qr.l, r = qr.r, k = qr.k;
  while (T < t) apply (T++, 1);
 while (T > t) apply(--T, 0);
while (R < r) add(a[++R]);
while (L > 1) add(a[--L]);
  while (R > r) rem(a[R--]);
while (L < 1) rem(a[L++]);
  ans[qr.i] = get_answer();
void TEST_CASES(int cas){
int n, m; cin>>n>m;
a.resize(n);
 for(int i=0;i<n;i++) { cin>>a[i]; }
 for(int i=0; i < m; i++)
  int tp;scanf("%d", &tp);
  if (tp == 1) {
   int 1, r, k;cin>>l>>r>>k;
   q.push_back({upd.size(), 1 - 1, r - 1, k, q.size()});
   int \bar{p}, x;cin>>p>>x;
   --p;upd.push_back({p, a[p], x});
   a[p] = x;
mos_algorithm();
```

Monotone Queue

```
vector<int>dp(n,0);
vector<int>newdp(n);
for(int i=1; i<=m; i++){
int koto = d*(a[i].t - a[i-1].t);
deque<pair<int,int> >dq;
```

```
int 1 = 0 , r= -1;
for(int j=0; j<n; j++){
  int eil = max(Oll, j-koto) , eir = min(n-1,j+koto);
  while(!dq.empty() && dq.front().first < eil) {
      dq.pop_front();
    }
  while(r!= eir) {
      r++;
    int val = dp[r];
    int idx = r;
    while(!dq.empty() && dq.back().second <= val) {
      dq.pop_back();
    }
    dq.push_back({idx,val});
    }
    newdp[j] = a[i].b - abs(a[i].a - j-1) + dq.front().second ;
}
swap(dp,newdp);
}
cout<<*max_element(dp.begin(), dp.end())<<"\n";</pre>
```

49 NTT

```
namespace NTT {
 vector<int> perm, wp[2];
const int mod = 998244353, G = 3; ///G is the primitive root
  int root, inv, N, invN;
int power(int a, int p) {
     int ans = 1:
      while (p) {
       if (p & 1) ans = (1LL*ans*a)%mod;
       a = (1LL*a*a) \% mod;
      p >>= 1;
     return ans;
  void precalculate(int n) {
     assert( (n&(n-1)) == 0 && (mod-1)%n==0);
     invN = power(N, mod-2);
     perm = \overline{wp[0]} = wp[1] = vector < int > (N);
     perm[0] = 0;
      for (int k=1: k<N: k<<=1)
       for (int i=0; i < k; i++) {
          perm[i] <<= 1;
          perm[i+k] = 1 + perm[i];
     root = power(G, (mod-1)/N);
      inv = power(root, mod-2);
      wp[0][0]=wp[1][0]=1;
     wp[1] in it is in it in it is in i
  void fft(vector<int> &v, bool invert = false) {
     if (v.size() != perm.size()) precalculate(v.size());
     for (int i=0; i<\bar{N}; i++)
       if (i < perm[i])</pre>
          swap(v[i], v[perm[i]]);
     for (int len = 2; len <= N; len *= 2) {
  for (int i=0, d = N/len; i<N; i+=len) {</pre>
          for (int j=0, idx=0; j<len/2; j++, idx += d) {
            int x = v[i+j];
             int y = (wp[invert][idx]*1LL*v[i+j+len/2])%mod;
            v[i+j] = (x+y)=mod ? x+y-mod : x+y);

v[i+j+len/2] = (x-y)=0 ? x-y : x-y+mod);
      if (invert) {
       for (int &x : v) x = (x*1LL*invN)\mbox{mod};
  vector<int> multiply(vector<int> a, vector<int> b) {
```

```
int n = 1;
while (n < a.size()+ b.size()) n <<=1;
a.resize(n);
b.resize(n);
fft(a);
fft(b);
for (int i=0; i < n; i++) a[i] = (a[i] * 1LL * b[i]) % mod;
fft(a, true);
return a;
};
};</pre>
```

50 Nearest Point Pair

```
#define x first
#define y second
long long dist2(pair<int, int> a, pair<int, int> b) {
return 1LL * (a.x - b.x) * (a.x - b.x) + 1LL * (a.y - b.y) *
      (a.y - b.y);
pair<int, int> closest_pair(vector<pair<int, int>> a) {
int n = a.size():
assert(n >= 2);
vector<pair<pair<int, int>, int>> p(n);
for (int i = 0; i < n; i++) p[i] = {a[i], i};</pre>
sort(p.begin(), p.end());
int 1 = 0, r = 2;
long long ans = dist2(p[0].x, p[1].x);
pair<int, int> ret = \{0, 1\};
while (r < n) {
  while (1 < r && 1LL * (p[r].x.x - p[l].x.x) * (p[r].x.x -
      p[1].x.x) >= ans) 1++;
 for (int i = 1; i < r; i++) {
  long long nw = dist2(p[i].x, p[r].x);
  if (nw < ans) {
   ret = \{p[i].y, p[r].y\};
 r++:
return ret:
```

51 PalindromicTree

```
/*-> diff(v) = len(v) - len(link(v))
-> series link will lead from the vertex v to the vertex u
     corresponding
  to the maximum suffix palindrome of v which satisfies diff(v)
       != diff(u)
-> path within series links to the root contains only O(log n)
     vertices
-> cnt contains the number of palindromic suffixes of the node*,
struct PalindromicTree {
 struct node {
 int nxt[26], len, st, en, link, diff, slink, cnt, oc;
string s;vector<node> t;
int sz, last;
 PalindromicTree() {}
PalindromicTree(string _s) {
   s = _s;
 int n = s.size();
  t.clear();
 t.resize(n + 9); sz = 2, last = 2;
  t[1].len = -1, t[1].link = 1;
 t[2].len = 0, t[2].link = 1;

t[1].diff = t[2].diff = 0;
 t[1].slink = 1;t[2].slink = 2;
 int extend(int pos) { // returns 1 if it creates a new
      palindrome
 int cur = last, curlen = 0;
int ch = s[pos] - 'a';
  while (1) {curlen = t[cur].len;
  if (pos - 1 - curlen) = 0 && s[pos - 1 - curlen] == s[pos])
        break;
  cur = t[cur].link:
```

```
if (t[cur].nxt[ch]) {last = t[cur].nxt[ch];
  t[last].oc++:
  return 0;
 sz++;last = sz;
 t[sz].oc = 1;t[sz].len = t[cur].len + 2;
 t[cur].nxt[ch] = sz;t[sz].en = pos;
 t[sz].st = pos - t[sz].len + 1;
 if (t[sz].len == 1) {
  t[sz].link = 2;t[sz].cnt = 1;
  t[sz].diff = 1;t[sz].slink = 2;
  return 1:
 while (1) {
  cur = t[cur].link;curlen = t[cur].len;
  if (pos - 1 - curlen) = 0 \&\& s[pos - 1 - curlen] == s[pos])
   t[sz].link = t[cur].nxt[ch];
   break:
 t[sz].cnt = 1 + t[t[sz].link].cnt;
  \begin{array}{l} t[sz].diff = t[sz].len - t[t[sz].link].len; \\ if (t[sz].diff == t[t[sz].link].diff) \ t[sz].slink = \\ \end{array} 
       t[t[sz].link].slink:
 else t[sz].slink = t[sz].link;
 return 1;
void calc_occurrences() {
for (int i = sz: i \ge 3: i--) t[t[i].link].oc += t[i].oc:
vector<array<int, 2>> minimum_partition() { //(even, odd), 1
      indexeď
 int n = s.size();
 vector\langle array \langle int, 2 \rangle \rangle ans(n + 1, \{0, 0\}), series_ans(n + 5, 0)
 {0, 0});
ans[0][1] = series_ans[2][1] = 1e9;
 for (int i = 1; i <= n; i++) {
  extend(i - 1):
  for (int k = 0; k < 2; k++) {
   ans[i][k] = 1e9;
   for (int v = last; t[v].len > 0; v = t[v].slink) {
    series_ans[v][!k] = ans[i - (t[t[v].slink].len +
           t[[v].diff)][!k];
     if (t[v].diff == t[t[v].link].diff) series_ans[v][!k] =
    min(series_ans[v][!k], series_ans[t[v].link][!k]);
ans[i][k] = min(ans[i][k], series_ans[v][!k] + 1);
 return ans;
int32 t main() {
string s; cin >> s
PalindromicTree t(s);
for (int i = 0; i < s.size(); i++) t.extend(i);</pre>
t.calc_occurrences();
long long ans = 0;
for (int i = 3; i <= t.sz; i++) ans += t.t[i].oc;
cout << ans << '\n':
//auto ans = t.minimum_partition();
// for (int i = 1; i <= s.size(); i++) {
// cout << (ans[i][1] == 1e9 ? -1 : ans[i][1]) << ',';
// cout << (ans[i][0] == 1e9 ? -2 : ans[i][0]) << '\n';
// }
```

52 Partition

```
int p[MAX];
inline int Add(int a,int b){return (a+b)%mod;}
int PartitionFunction(){
  p[0] = 1;
  for(int i = 1; i < MAX; i++){
    int j = 1, r = 1;
    while(i - (3*j*j - j) / 2 >= 0){
      p[i] = Add(p[i], p[i - (3*j*j - j) / 2] * r);
    if(i - (3*j*j + j) / 2 >= 0)
```

```
p[i] = Add(p[i], p[i - (3*j*j + j) / 2] * r);
    j += 1;
    r *= -1;
    if(p[i]<0) p[i]+=mod;
}
</pre>
```

53 Phi

```
int phi[N+1];
void phi_1_to_n(int n) {
  phi[0] = 0; phi[1] = 1;
  for (int i = 2; i <= n; i++) phi[i] = i;
  for (int i = 2; i <= n; i++) {
    if (phi[i] == i) { for (int j = i; j <= n; j += i)
        phi[j] -= phi[j] / i;
    }
}</pre>
```

54 PointOrderingByAngle

```
typedef pair<int,int> pii;
struct point{
int x, y;
bool operator <(point &p) const{
  if(x=p.x){
  return y > p.y;
  return x<p.x;
struct line{
point p1, p2;
line(){}
line(point &p, point &q){
  p2 = q;
 bool operator <(line &p){
  11 a = (p1.x - p2.x);
 11 b = (p1.y-p2.y);
  11 c = (p.p1.x-p.p2.x);
  11 d = (p.p1.y-p.p2.y);
  return a*d < b*c:
void TEST_CASES(int cas){
int n;
scanf("%d",&n);
map<pii, int>mp;
vector<point>v(n);
for(int i=0; i<n; i++){
  v[i].read();
sort(v.begin(),v.end());
for(int i=0; i<n; i++){
mp[ {v[i].x, v[i].y}] = i;
 vector<line>lines:
for(int i=0; i<n; i++){
 for(int j=i+1; j<n; j++){
lines.emplace_back(v[i],v[j]);
sort(lines.begin(),lines.end());
for(line &1: lines){
 point p1= 1.p1;
point p2 = 1.p2;
int idx1 = mp[ {p1.x,p1.y}];
int idx2 = mp[ {p2.x, p2.y}];
  //Do your work
  //Swap indexes
  v[idx1] = 1.p2;
  v[idx2] = 1.p1;
  mp[ {v[idx1].x, v[idx1].y}] = idx1;
mp[ {v[idx2].x, v[idx2].y}] = idx2;
```

55 PollardRho

```
typedef long long LL;
typedef unsigned long long ULL;
namespace Rho {
ULL mult(ULL a, ULL b, ULL mod) {
LL ret = a * b - mod * (ULL)(1.0L / mod * a * b)
  return ret + mod * (ret < 0) - mod * (ret >= (LL) mod):
 ULL power(ULL x, ULL p, ULL mod){
 ULL s=1, m=x; while(p) {
   if(p&1) s = mult(s, m, mod);
   p>>=1;
   \hat{m} = \text{mult}(m, m, \text{mod});
  return s;
 vector<LL> bases = {2, 325, 9375, 28178, 450775, 9780504,
      1795265022};
 bool isprime(LL n) {
  if (n<2) return 0:
  if (n\%2==0) return n==2;
  ULL s = __builtin_ctzll(n-1), d = n>>s;
  for (ULL x: bases) {
   ULL p = power(x%n, d, n), t = s;
   while (p!=1 \&\& p!=n-1 \&\& x\%n \&\& t--) p = mult(p, p, n);
   if (p!=n-1 && t != s) return 0;
 return 1:
 mt19937_64 rng(chrono::system_clock::now().
 time_since_epoch().count());
 ULL FindFactor(ULL n)
  if (n == 1 || isprime(n)) return n;
  ULL c = 1, x = 0, y = 0, t = 0, prod = 2, x0 = 1, q;
auto f = [&](ULL X) { return mult(X, X, n) + c;};
  while (t++ % 128 or gcd(prod, n) == 1) {
if (x == y) c = rng()%(n-1)+1, x = x0, y = f(x);
   if ((q = mult(prod, max(x, y) - min(x, y), n))) prod = q;
   x = f(x), y = f(f(y));
 return gcd(prod, n);
 vector<ULL> factorize(ULL x) {
  if (x == 1) return \{\};
  ULL a = FindFactor(x), b = x/a:
  if (a == x) return \{a\};
 vector<ULL> L = factorize(a), R = factorize(b);
L.insert(L.end(), R.begin(), R.end());
 return L;
```

56 Popcount Trick

```
int cnt[1<<16];
void preprocess(){
    for(int mask=0; mask<(1<<16); mask++) {
        for(int i=0; i<16; i++) {
            if(mask&(1<<ii)) cnt[mask]++;
        }
    }
int query(ll x){
    int ans=0,G=(1<<16)-1;
    ans+=cnt[x&G]; x>>=16;
    ans+=cnt[x&G]; x>>=16;
    ans+=cnt[x&G]; x>>=16;
    ans+=cnt[x&G]; x>>=16;
    ans+=cnt[x&G]; x>>=16;
    ans+=cnt[x&G]; x>>=16;
    ans+=cnt[x&G]; x>=16;
    ans+=cnt[x&G];
}
```

57 PrefixFunction

```
vector<int> prefix_function(string s) {
  int n = (int)s.length();
  vector<int> pi(n);
  for (int i = 1; i < n; i++) {
    int j = pi[i-1];
    while (j > 0 && s[i] != s[j])
    j = pi[j-1];
    if (s[i] == s[j])
    j++;
    pi[i] = j;
  }
  return pi;
}
```

58 PrimitiveRoot

```
// Finds the primitive root modulo p
int generator(int p) {
vector<int> fact;
int phi = p-1, n = phi;
for (int i = 2; i * i <= n; ++i) {
   if (n % i == 0) {</pre>
  fact.push_back(i);
  while (n \% i == 0)
   n /= i;
if (n > 1) fact.push_back(n);
for (int res = 2; res <= p; ++res) {
 bool ok = true:
 for (int factor : fact) {
  if (powmod(res, phi / factor, p) == 1) {
  ok = false; break;
 if (ok) return res;
return -1;
// This program finds all numbers x such that x^k = a \pmod{n}
int main() {
int n, k, a;
scanf("%d %d %d", &n, &k, &a);
if (a == 0) {
 puts("1\n0");
 return 0;
int g = generator(n);
int sq = (int) sqrt (n + .0) + 1;
vector<pair<int, int>> dec(sq);
for (int i = 1; i <= sq; ++i)
dec[i-1] = {powmod(g, i * sq * k % (n - 1), n), i};
sort(dec.begin(), dec.end());
int any_ans = -1;
for (int i = 0; i < sq; ++i) {
 int my = powmod(g, i * k % (n - 1), n) * a % n;
 auto it = lower_bound(dec.begin(), dec.end(), make_pair(my,
 if (it != dec.end() && it->first == my) {
  any_ans = it->second * sq - i;
  break;
if (any_ans == -1) {
 puts("0");return 0;
// Print all possible answers
int delta = (n-1) / gcd(k, n-1);
vector<int> ans;
for (int cur = any_ans % delta; cur < n-1; cur += delta)
 ans.push_back(powmod(g, cur, n));
sort(ans.begin(), ans.end());
```

59 SCC

```
vector<vector<int>> adj, adj_rev;
vector<bool> used;
vector<int> order, component;
```

```
void dfs1(int v) {
used[v] = true;
for (auto u : adj[v])
 if (!used[u]) dfs1(u);
order.push_back(v);
void dfs2(int v) {
used[v] = true; component.push_back(v);
for (auto u : adj_rev[v]) if (!used[u])
  dfs2(u);
int main() {
used.assign(n, false);
for (int i = 0; i < n; i++)
 if (!used[i]) dfs1(i);
used.assign(n, false);
reverse(order.begin(), order.end());
for (auto v : order) if (!used[v]) {
  dfs2 (v):
  //processing next component
  component.clear();
```

SOSDP

```
//memory optimized, super easy to code.
for(int i = 0; i < (1 << N); ++i)
F[i] = A[i]:
for(int i = 0; i < N; ++i){
for(int mask = 0; mask < (1<<N); ++mask) {
 if(mask & (1<<i))
  F[mask] += F[mask^(1<<i)];
//sunermask
for(int i = 0; i < N; ++i){
for(int mask = (1<<N)-1; mask >=0; --mask) {
 if(!(mask & (1<<i)))
  dp[mask] += dp[mask|(1<<i)];
```

SegSegIntersection

```
inline double cross(PT a, PT b) { return a.x * b.y - a.y *
    b.x;}
inline double cross2(PT a, PT b, PT c) { return cross(b - a, c
     - a); }
bool is_point_on_seg(PT a, PT b, PT p) {
if (fabs(cross(p - b, a - b)) < eps) {
 if (p.x < min(a.x, b.x) \mid\mid p.x > max(a.x, b.x)) return false;
 if (p.y < min(a.y, b.y) \mid | p.y > max(a.y, b.y)) return false;
 return true;
return false:
bool seg_seg_intersection(PT a, PT b, PT c, PT d, PT &ans) {
double oa = cross2(c, d, a), ob = cross2(c, d, b);
double oc = cross2(a, b, c), od = cross2(a, b, d); if (oa * ob < 0 && oc * od < 0){
 ans = (a * ob - b * oa) / (ob - oa);
 return 1;
else return 0;
set<PT> seg_seg_intersection_inside(PT a, PT b, PT c, PT d) {
PT ans;
if (seg_seg_intersection(a, b, c, d, ans)) return {ans};
if (is_point_on_seg(c, d, a)) se.insert(a);
if (is_point_on_seg(c, d, b)) se.insert(b);
if (is_point_on_seg(a, b, c)) se.insert(c);
if (is_point_on_seg(a, b, d)) se.insert(d);
return se;
```

SegmentedSieve

```
vector<char> segmentedSieve(long long L, long long R) {
// generate all primes up to sqrt(R)
long long lim = sqrt(R);
vector<char> mark(lim + 1, false);
 vector<long long> primes;
for (long long i = 2; i <= lim; ++i) {
    if (!mark[i]) {
  primes.emplace_back(i);
  for (long long j = i * i; j <= lim; j += i)
   mark[i] = true:
vector<char> isPrime(R - L + 1, true);
for (long long i : primes)
 for (long long j = \max(i * i, (L + i - 1) / i * i); j <= R;
      j += i)
  isPrime[j - L] = false;
if (L == 1)
 isPrime[0] = false;
return isPrime;
63 Segtree 2D
```

void build_y(int vx, int lx, int rx, int vy, int ly, int ry) {

```
if (ly == ry) {
 if (1x == rx)
  t[vx][vy] = a[lx][ly];
 t[vx][vy] = t[vx*2][vy] + t[vx*2+1][vy];
} else {
 int my = (ly + ry) / 2;
 build_y(vx, lx, rx, vy*2, ly, my);
 build_y(vx, lx, rx, vy*2+1, my+1, ry)
 t[vx][vy] = t[vx][vy*2] + t[vx][vy*2+1];
void build_x(int vx, int lx, int rx) {
if (lx != rx) {
 int mx = (lx + rx) / 2;
 build_x(vx*2, lx, mx);
 build_x(vx*2+1, mx+1, rx);
build_y(vx, lx, rx, 1, 0, m-1);
int sum_y(int vx, int vy, int tly, int try_, int ly, int ry) {
if (ly > ry)
 return 0;
if (ly == tly && try_ == ry)
 return t[vx][vv];
int tmy = (tly + try_) / 2;
return sum_y(vx, vy*2, tly, tmy, ly, min(ry, tmy))
  + sum_y(vx, vy*2+1, tmy+1, try_, max(ly, tmy+1), ry);
int sum_x(int vx, int tlx, int trx, int lx, int rx, int ly,
    int ry) {
if (lx > rx)
 return 0;
if (lx == tlx && trx == rx)
 return sum_y(vx, 1, 0, m-1, ly, ry);
int tmx = (tlx + trx) / 2;
return sum_x(vx*2, tlx, tmx, lx, min(rx, tmx), ly, ry)
  + sum_x(vx*2+1, tmx+1, trx, max(lx, tmx+1), rx, ly, ry);
void update_y(int vx, int lx, int rx, int vy, int ly, int ry,
    int x, int y, int new_val) {
if (ly == ry) {
 if (1x == rx)
  t[vx][vy] = new_val;
  t[vx][vy] = t[vx*2][vy] + t[vx*2+1][vy];
} else {
 int my = (ly + ry) / 2;
 if (v \le mv)
  update_v(vx, lx, rx, vy*2, ly, my, x, y, new_val);
```

```
update_y(vx, lx, rx, vy*2+1, my+1, ry, x, y, new_val);
 t[vx][vy] = t[vx][vy*2] + t[vx][vy*2+1];
void update_x(int vx, int lx, int rx, int x, int y, int
    new_val) {
if (lx != rx) {
 int mx = (1x + rx) / 2;
 if (x \le mx)
 update_x(vx*2, lx, mx, x, y, new_val);
  update_x(vx*2+1, mx+1, rx, x, y, new_val);
update_y(vx, lx, rx, 1, 0, m-1, x, y, new_val);
```

Segtree Lazv

```
void push(int v) +
t[v*2] += lazv[v]
lazv[v*2] += lazv[v]:
t[v*2+1] += lazv[v];
lazy[v*2+1] += lazy[v];
lazv[v] = 0:
void update(int v, int tl, int tr, int l, int r, int addend) {
if (1 > r)
return;
if (l == tl && tr == r) {
 t[v] += addend:
 lazy[v] += addend;
} else {
 push(v);
 int tm = (tl + tr) / 2;
 update(v*2, tl, tm, l, min(r, tm), addend);
 update(v*2+1, tm+1, tr, max(1, tm+1), r, addend);
 t[v] = max(t[v*2], t[v*2+1]);
int query(int v, int tl, int tr, int l, int r) {
if (1 > r)
 return -INF;
if (1 <= t1 && tr <= r)
return t[v];
push(v);
int tm = (tl + tr) / 2;
return max(query(v*2, t1, tm, 1, min(r, tm)),
     query(v*2+1, tm+1, tr, max(1, tm+1), r));
```

Segtree Persistent

```
const 11 INF = 4e18;
//Point update and range min
struct Node {
int l=-1,r=-1; pii val;
Node(pii v, int l=-1, int r=-1) {
 this ->1 = 1;
 this->r = r;
 val = v:
véctor<Node>nodes;
inline void Merge(Node &a, Node &b, Node &c) {
a.val = min(b.val,c.val);
int build(int tl, int tr) {
if (tl == tr) {
 nodes.emplace_back(make_pair(INF, -1));
 return (int)nodes.size()-1;
int tm = (tl + tr) / 2;
int Left = build(t1, tm);
int Right = build(tm+1, tr);
nodes.emplace_back(make_pair(INF, -1),Left,Right);
Merge(nodes.back(), nodes[Left], nodes[Right]);
```

```
return (int)nodes.size()-1;
int update(int v, int tl, int tr, int pos, pii val) {
if (tl == tr) {
 nodes.emplace_back(val);
 return (int)nodes.size()-1;
int tm = (tl + tr) / 2; int Left, Right;
if (pos <= tm) {
 Left = update(nodes[v].1, tl, tm, pos,val);
 Right = nodes[v].r;
 Left = nodes[v].1;
 Right = update(nodes[v].r, tm+1, tr, pos,val);
nodes.emplace_back(make_pair(INF, -1),Left,Right);
Merge(nodes.back(), nodes[Left], nodes[Right]);
return (int)nodes.size()-1:
pii query(int v, int tl, int tr, int a, int b) {
  if (a>tr|| tl>b || tl > tr) return {INF, -1};
if(t1 >=a && tr <=b) return nodes[v].val;
int mid= (tl+tr)/2;
return min(query(nodes[v].1,tl, mid, a,b), query(nodes[v].r,
     mid+1, tr, a, b));
```

66 Segtree beats

```
struct info{
  int maxi = 0,smaxi = -1e9, cnt = 0, lazy = 0;
 bool has=0:11 sum = 0:
struct segtree{
 int n;
 vector<info>t;
 segtree(int n,vector<int>&a)
  this->n = n;t.resize(n*4);
  build(1,0,n-1,a);
 void Merge(info &node, info &l, info &r){
 node.maxi = max(1.maxi, r.maxi);
node.cnt = (node.maxi==1.maxi ? 1.cnt : 0) +
       (node.maxi==r.maxi ? r.cnt : 0);
  node.sum = 1.sum + r.sum:
  if(l.maxi != r.maxi)
  node.smaxi =max({ min(l.maxi, r.maxi) ,l.smaxi, r.smaxi});
  node.smaxi = max(l.smaxi, r.smaxi);
 void build(int node, int l,int r, vector<int>&a) {
  if(l==r){
  t[node].maxi = a[1];t[node].cnt = 1;t[node].sum = a[1];
   return;
  int mid= (1+r)/2;
  build(node*2, 1, mid,a);
build(node *2 +1, mid+1, r,a);
Merge(t[node], t[node*2], t[node *2+1]);
 void dop(int node, int add){
  if(t[node].maxi <= add)</pre>
  t[node].sum -= t[node].maxi *111* t[node].cnt:
 t[node].maxi = add;
t[node].sum += t[node].maxi *111* t[node].cnt;
t[node].lazy= add;
 t[node].has = 1:
 void push_down(int node){
  if(t[node].has){
   dop(node*2, t[node].lazy);
   dop(node*2+1, t[node].lazy);
   t[node].lazy =0;
   t[node].has = 0:
```

```
void update(int node, int l.int r. int i. int i. int add){
if(1>j || r<i || t[node].maxi <= add){
 return:
 if(l>=i && r<=j && t[node].smaxi < add){
 int x = t[node].maxi - add:
 t[node].sum -= t[node].maxi *1ll* t[node].cnt;
  t[node].maxi = add;
  t[node].sum += t[node].maxi *1ll* t[node].cnt;
  t[node].lazy= add;
 t[node].has = 1;
 return:
 int mid = (1+r)/2;
 push_down(node);
update(node *2, 1,mid, i, j, add);
update(node *2 +1, mid+1, r, i, j, add);
Merge(t[node], t[node *2], t[node*2+1]);
void update(int l.int r. int add){
update(1,0,n-1,1,r,add);
pair<ll,int> query(int node, int 1, int r, int i, int j){
if(l>i || r<i){
 return make_pair(0,-1e9);
 if(l>=i && r<=i){
 return make_pair(t[node].sum,t[node].maxi);
 int mid = (1+r)/2;
 push down(node):
 pair<ll,int> x = query(node *2, 1,mid, i, j);
pairlint> y = query(node *2 +1, mid+1, r, i, j);
return make_pair(x.first+y.first , max(x.second,y.second));
pair<ll,int> query(int 1, int r){
return query(1,0,n-1,1,r);
```

67 Simplex

```
* Note: Simplex algorithm on augmented matrix a of dimension
       (m+1)x(n+1)
* returns 1 if feasible, 0 if not feasible, -1 if unbounded
* returns solution in b[] in original var order, max(f) in ret
 * form: maximize sum_j(a_mj*x_j)-a_mn \ s.t.
 sum_j(a_ij*x_j) \le a_in
* in standard form.
 * To convert into standard form:
 * 1. if exists equality constraint, then replace by both >=
 * 2. if variable x doesn't have nonnegativity constraint, then
 replace by * difference of 2 variables like x1-x2, where x1>=0, x2>=0
* 3. for a>=b constraints, convert to -a<=-b

* note: watch out for -0.0 in the solution, algorithm may cycle
 * EPS = 1e-7 may give wrong answer, 1e-10 is better
typedef vector<ld> vd;
typedef vector<vd> vvd;
typedef vector<int> vi;
const ld EPS = 1e-10;
struct LPSolver{
 int m, n; vi B, N; vvd D;
LPSolver(const vvd &A, const vd &b, const vd &c): m(b.size()), n(c.size()), N(n + 1), B(m), D(m + 2), vd(n + 1)
  for (int i = 0; i < m; i++)
  for (int j = 0; j < n; j++)
D[i][j] = A[i][j];
 for (int i = 0; i < m; i++) {
    B[i] = n + i; D[i][n] = -1;
    D[i][n + 1] = b[i];
 for (int j = 0; j < n; j++) {
N[j] = j; D[m][j] = -c[j];
```

```
N[n] = -1: D[m + 1][n] = 1:
 void Pivot(int r, int s) {
   ld inv = 1.0 / D[r][s];
   for (int i = 0; i < m + 2; i++) if (i != r)
       for (int j = 0; j < n + 2; j++)
if (j != s) D[i][j] -= D[r][j] * D[i][s] * inv;
   for (int j = 0; j < n + 2; j++) if (j != s)
       D[r][j] *= inv;
   for (int i = 0; i < m + 2; i++) if (i != r)
       D[i][s] *= -inv;
   D[r][s] = inv; swap(B[r], N[s]);
 bool Simplex(int phase) {
   int x = phase == 1 ? m + 1 : m;
   while (true) {
      int. s = -1:
      for (int j = 0; j <= n; j++) {
       if (phase == 2 \&\& N[j] == -1) continue;
        if (s == -1 \mid | D[x][j] < D[x][s] \mid | D[x][j] == D[x][s] &&
                   N[i] < N[s]
          s = j;
      if (D[x][s] > -EPS) return true;
      for (int i = 0; i < m; i++) {
       if (D[i][s] < EPS) continue;
if (r == -1 || D[i][n + 1] / D[i][s] < D[r][n + 1] /
                   D[r][s] ||
              (D[i][n+1] / D[i][s]) == (D[r][n+1] / D[r][s]) &&
                        B[i] < B[r])
      if (r == -1) return false:
      Pivot(r, s);
 ld Solve(vd &x) {
   int r = 0:
   for (int i = 1; i < m; i++)
     if (D[i][n + 1] < D[r][n + 1])
   if(D[r][n + 1] < -EPS) {
     Pivot(r, n);
      if (!Simplex(1) || D[m + 1][n + 1] < -EPS)
       return -numeric_limits<ld>::infinity();
      for (int i = 0; i < m; i++)
       if (B[i] == -1) {
          int s = -1;
         for (int j = 0; j <= n; j++)
if (s == -1 | D[i][j] < D[i][s] || D[i][j] == D[i][s] &&
                        N[j] < N[s]
         s = j;
Pivot(i, s);
   if (!Simplex(2))
     return numeric limits<ld>::infinitv():
   x = vd(n);
  for (int i = 0; i < m; i++)
if (B[i] < n) x[B[i]] = D[i][n + 1];
return D[m][n + 1];
 /st Equations are of the matrix form Ax<=b, and we want to
the function c. We are given coeffs of A, b and c. In case of
           minimizing,
we negate the coeffs of c and maximize it. Then the negative of
           returned
 'value' is the answer.
All the constraints should be in <= form. So we may need to
          negate the
coeffs.
int main(){
 const int m = 4: const int n = 3:
 1d _A[m][n] = \{ \{ 6, -1, 0 \}, \{ -1, -5, 0 \}, \{ 1, 5, 1 \}, \{ -1, -5, 0 \}, \{ 1, 5, 1 \}, \{ -1, -5, 0 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5, 1 \}, \{ 1, 5
             -1, -5, -1 }
```

```
};
Id _b[m] = { 10, -4, 5, -5 };
Id _c[n] = { 1, -1, 0 };
vvd A(m);
vd b(_b, _b + m);
vd c(_c, _c + n);
for (int i = 0; i < m; i++)
    A[i] = vd(_A[i], _A[i] + n);
LPSolver solver(A, b, c);
vd x;
Id value = solver.Solve(x);
cerr << "VALUE: " << value << endl; /* VALUE: 1.29032*/
cerr << "SOLUTION:"; /*SOLUTION: 1.74194 0.451613 1*/
for (size_t i = 0; i < x.size(); i++)
}
cerr << " " << x[i];
}
</pre>
```

68 Simpson

69 StableMarriage

```
//order[i][j]=indexOfMan i in j-th women'sListOfPreference
//prefer[i]=listOfWomen inOrderOf decreasingPreference int n;
int pre[N][N], order[N][N], nxt[N];
queue<int>q;
int future wife[N].future husband[N]
void engage(int man , int woman){
int m1 = future husband[woman]:
if(m1==0) {
 future wife[man] = woman:future husband[woman] = man:
else{
 future_wife[man] = woman;future_husband[woman] = man;
 future_wife[m1] =0;q.push(m1);
void TEST_CASES(int cas){
while(!q.empty())q.pop();
for(int i=1:i<=n:i++) {
 for(int j=1;j<=n;j++) {
  cin>>pre[i][j]; pre[i][j]-= n;
 nxt[i] = 1;future_wife[i] = 0;q.push(i);
for(int i=1;i<=n;i++) {
 for(int j=1;j <=n;j++) {
  int x; cin>>x;
order[i][x] = j;
 future husband[i] = 0:
while(!q.empty()) {
  int man = q.front(); q.pop();
 int woman = pre[man][nxt[man]++];
  if(future_husband[woman] == 0) {
  engage(man , woman);
  else if(order[woman][man] <</pre>
      order[woman][future husband[woman]]) {
  engage(man , woman);
  else{ q.push(man); }
```

```
for(int i=1;i<=n;i++) {
  cout<<" ("<<i<<" "<<future_wife[i]+n<<")";
}

70 Stirling</pre>
```

```
NTT ntt(mod);
vector<ll>v[MAX];
//Stirling1\ (n,k) = co-eff\ of\ x^k\ in\ x*(x+1)*(x+2)*...(x+n-1)
int Stirling1(int n, int r) {
int nn = 1;
while(nn < n) nn <<=1;
for(int i = 0; i < n; ++i) {v[i].push_back(i);
     v[i].push_back(1);}
for(int i = n; i < nn; ++i) v[i].push_back(1);
for(int j = nn; j > 1; j >>= 1) {
  int hn = j >> 1;
 for(int i = 0; i < hn; ++i) ntt.multiply(v[i], v[i + hn],
      v[i]);
return v[0][r];
NTT ntt(mod);
vector<int>a,b,res;
//Stirling2 (n,k) = co-eff of x^k in product of polynomials A &
//where A(i) = (-1)^i / i! and B(i) = i^n / i!
int Stirling2(int n, int r) {
a.resize(n+1); b.resize(n+1);
for(int i = 0; i \le n; i++){
 a[i] = invfct[i];
 if(i \% 2 == 1) a[i] = mod - a[i];
for(int i = 0: i \le n: i++){
 b[i] = bigMod(i, n, mod);
 b[i] = (b[i] *111* invfct[i]) % mod;
NTT ntt(mod);
ntt.multiply(a,b,res);
return res[r];
```

71 StressTest

```
#!/bin/sh
echo "Enter the name of first File : "read file
echo "Enter the name of second File :
read file2
g++ -o test_gen test_gen.cpp
g++ -o $file $file.cpp
g++ -o $file2 $file2.cpp
while true
do
   ./test_gen
   ./$file <input.txt> out1.txt
   ./$file2 <input.txt> out2.txt
if cmp -s "out1.txt" "out2.txt"; then
 echo "Test Case OK"
   else
 echo "ERROR ENCOUNTERED"
done
```

72 Suffix Automata

```
struct state{
  int len, link;
  map<char, int> next;
  ll dp=-1; //number of paths
  ll cnt=0; //endpos size
  bool is_cloned=false;
  vector<int>inv_link;
```

```
struct SA {
vector<state> st;
int sz, last;
void sa_init() {
st[0].len = 0;
st[0].link = -1;
 sz=0;
sz++;
last = 0;
void sa_extend(char c) {
 int cur = sz++;
 st[cur].len = st[last].len + 1;
 int p = last;
while (p != -1 && !st[p].next.count(c)) {
  st[p].next[c] = cur;
  p = st[p].link;
 if (p == -1) {
  st[cur].link = 0;
  int q = st[p].next[c];
  if (st[p].len + 1 == st[q].len){
   st[cur].link = q;
  else
   int clone = sz++;
   st[clone].len = st[p].len + 1;
   st[clone].next = st[q].next;
   st[clone].link = st[q].link;
   while (p != -1 && st[p].next[c] == q) {
    st[p].next[c] = clone;
    p = st[p].link;
   st[q].link = st[cur].link = clone;
   st[clone].is_cloned=true;
 last = cur;
ll run(int idx) {
 if(st[idx].dp!=-1)
  return st[idx].dp;
 if(idx!=0)
  st[idx].dp=st[idx].cnt;
 else st[idx].dp=0;
 for(char c='a';c<='z';c++) {
  if(!st[idx].next.count(c))
  int u=st[idx].next[c];
  st[idx].dp+=run(u);
 return st[idx].dp;
void dfs_in_tree(int idx) {
 if(st[idx].is_cloned==false) {
  st[idx].cnt=1;
 for(int u:st[idx].inv_link) {
  dfs_in_tree(u);
  st[idx].cnt+=st[u].cnt;
void build(string &s) {
 st.resize(2*(int)s.size());
 sa init():
 for(char c:s) {
  sa_extend(c);
 for(int i=1;i<sz;i++) {
  st[st[i].link].inv_link.push_back(i);
 dfs_in_tree(0);
```

3 SuffixArray

```
const int LOG = 20;
const int Loud = 20,
int sa[N],Data[N],rnk[N],height[N];
int wa[N],wb[N],wws[N],wv[N];
int lg[N], rmq[N][LOG];
void prelg() {
 lg[0] = lg[1] = 0;
 for(int i = 2; i < N; i++) {
lg[i] = lg[i/2] + 1;
struct SuffixArray {
 int n:
 int cmp(int *r,int a,int b,int 1) {
  return (r[a]==r[b]) && (r[a+1]==r[b+1]);
 void DA(int *r,int *sa,int n,int m) {
  int i,j,p,*x=wa,*y=wb,*t;
for(i=0; i<m; i++) wws[i]=0;
  for(i=0; i<n; i++) wws[x[i]=r[i]]++;
for(i=1; i<m; i++) wws[i]+=wws[i-1];
for(i=n-1; i>=0; i--) sa[--wws[x[i]]]=i;
  for(j=1,p=1; p<n; j*=2,m=p)
    for(p=0,i=n-j; i<n; i++) y[p++]=i;
   for(i=0; i<n; i++)
if(sa[i]>=j) y[p++]=sa[i]-j;
    for(i=0; i<n; i++) wv[i]=x[y[i]];
    for(i=0; i<m; i++) wws[i]=0;
   for(i=0; i<n; i++) wws[wv[i]]++;
for(i=1; i<m; i++) wws[i]+=wws[i-1];
for(i=n-1; i>=0; i--) sa[--wws[wv[i]]]=y[i];
    for(t=x,x=y,y=t,p=1,x[sa[0]]=0,i=1; i<n; i++)
     x[sa[i]] = cmp(y,sa[i-1],sa[i],j)?p-1:p++;
 void calheight(int *r,int *sa,int n) {
  int i,j,k=0;
  for(i=1; i<=n; i++) rnk[sa[i]]=i;
for(i=0; i<n; height[rnk[i++]]=k)</pre>
   for(k?k--:0, j=sa[rnk[i]-1]; r[i+k]==r[j+k]; k++);
 void suffix_array (string &A) {
  n = A.size();
  Data[n]=0;
   int cnt =0;
  for (int i = 0; i < n; i++){
    Data[i] = A[i]-'a'+1; //careful
   cnt = max(cnt, Data[i]);
  DA(Data,sa,n+1,cnt+1);
calheight(Data,sa,n);
  for(int i = 0; i < n; i++)
sa[i] = sa[i+1], height[i] = height[i+1], rnk[sa[i]] = i;</pre>
  range_lcp_init();
 /** LCP for range : build of rmg table **/
 void range_lcp_init() {
  for(int i = 0; i < n; i++)
rmq[i][0] = height[i];
  for(int j = 1; j < LOG; j++) {
  for(int i = 0; i < n; i++) {</pre>
     if (i+(1<< j)-1 < n)
      rmq[i][j] = min(rmq[i][j-1], rmq[i+(1<<(j-1))][j-1]);
     else break:
 /** lcp between l'th to r'th suffix in suffix array **/
 int query_lcp(int 1, int r) {
    assert(1 <= r);    assert(1>=0 && l<n && r>=0 && r<n);
   if(l == r) return n-sa[l];
  int k = \lg[r-l+1];
  return min(rmg[l][k],rmg[r-(1<<k)+1][k]);
 //i and j position in original string
 int getsuff(int i, int j) {
  i= rnk[i]; j = rnk[j];
```

```
return query_lcp(min(i,j),max(i,j));
} SA;
```

74 Treap

```
mt19937 rng(chrono::steady_clock::now().
time_since_epoch().count();
int getrand(int a, int b){
int x = uniform_int_distribution<int>(a, b)(rng);
return x;
struct treap{
int prior, val, subtreeSize;
treap *1;treap *r;treap *parent;
int sum, lazy;
treap(int data) {
 val= data; prior = getrand(-2e9, 2e9);
 subtreeSize = 1;
 1=NULL; r=NULL; parent = NULL; lazy = 0; sum = data;
typedef treap* ptreap;
int Size(ptreap t){
if(t)return t->subtreeSize;
return 0:
void update_size(ptreap t){
if(t) t->subtreeSize = 1+ Size(t->1) + Size(t->r);
void push(ptreap t){
if(!t || !t->lazy) {return;}
t->val += t->lazy;t->sum += t->lazy * Size(t);
if(t->1) t->1->lazy += t->lazy;
if(t->r) t->r->lazy += t->lazy;
t\rightarrow lazy = 0;
void reset(ptreap t){
if(t) t -> sum = t -> val:
void combine(ptreap &t, ptreap 1, ptreap r){
t \rightarrow sum = 1 \rightarrow sum + r \rightarrow sum;
void operation(ptreap t){
if(!t)return;
reset(t); push(t->1); push(t->r);
combine(t,t,t->1); combine(t,t,t->r);
void split(ptreap t, ptreap &l, ptreap &r, int pos, int add
    =Ö){
if(!t) {1 = NULL;r = NULL;return;}
push(t):
int curr = add + Size(t->1);
if(curr<=pos) {</pre>
 split(t-\bar{r}, t-r, r, pos, curr+1);
 if(t->r != NULL) t->r->parent = t;
 if(r!=NULL) r->parent = NULL;
 1 = t;
 split(t->1, 1, t->1,pos, add);
 if(t->1 != NULL) {t->1->parent = t;}
 if(1!=NULL) {1->parent = NULL;}
 r =t;
update_size(t); operation(t);
void Merge(ptreap &t, ptreap 1, ptreap r){
push(1);push(r);
if(!1 || !r) {if(1) t= 1;else t = r;}
 else if(l->prior > r->prior) {
 Merge(1->r, 1->r, r);
 if(1->r != NULL) \{1->r -> parent = 1;\}
 t= 1;
else {
```

```
Merge(r->1, 1, r->1);
 if(r\rightarrow l != NULL) {r\rightarrow l\rightarrow parent = r;}
 t = r;
update_size(t);operation(t);
int range_query(ptreap t, int 1, int r){
ptreap t1, t2, t3;
split(t,t1,t2,1-1); split(t2,t2,t3,r-1);
int ans = t2->sum;
Merge(t,t1,t2); Merge(t,t,t3);
return ans:
void range_update(ptreap t, int l,int r, int val){
  ptreap t1, t2, t3;
split(t,t1,t2,l-1);split(t2,t2,t3,r-1);
 t2->lazy += val; Merge(t,t1,t2); Merge(t,t,t3);
ptreap goup(ptreap t){
  if(t==NULL || t->parent==NULL) return t;
return goup(t->parent);
void output2 (ptreap t){
if (!t) return;
push (t);output2 (t->1);
 cout<<t->val<<" ";output2 (t->r);
```

75 Vertical Decomposition

```
inline bool le(dbl x, dbl y){return x < y + eps;}
inline bool ge(dbl x, dbl y){return x > y - eps;}
struct Line{
pt p[2];
Line(){}
Line(pt a, pt b):p{a, b}{}
pt vec()const{
 return p[1] - p[0];
pt& operator [](size_t i){
 return p[i];
inline bool lexComp(const pt & 1, const pt & r){
if(fabs(1.x - r.x) > eps)\{return 1.x < r.x;\}
else return l.y < r.y;
vector<pt> interSegSeg(Line 11, Line 12){
if(eq(11.vec().cross(12.vec()), 0)){
 if(!eq(11.vec().cross(12[0] - 11[0]), 0))
  return {};
 if(!lexComp(l1[0], l1[1])) swap(l1[0], l1[1]);
 if(!lexComp(12[0], 12[1])) swap(12[0], 12[1]);

pt 1 = lexComp(11[0], 12[0]) ? 12[0] : 11[0];

pt r = lexComp(11[1], 12[1]) ? 11[1] : 12[1];
 if(1 == r)
  return {1}:
 else return lexComp(1, r) ? vector<pt>{1, r} : vector<pt>();
else{
 dbl s = (12[0] - 11[0]).cross(12.vec()) /
      11.vec().cross(12.vec());
 pt inter = 11[0] + 11.vec() * s
 if(ge(s, 0) \&\& le(s, 1) \&\& le((l2[0] - inter).dot(l2[1] -
       inter), 0))
  return {inter};
 else
  return {};
inline char get_segtype(Line segment, pt other_point){
if(eq(segment[0].x, segment[1].x))
 return 0;
if(!lexComp(segment[0], segment[1]))
swap(segment[0], segment[1]);
return (segment[1] - segment[0]).cross(other_point -
    segment[0]) > 0 ? 1 : -1;
```

```
dbl union_area(vector<tuple<pt, pt, pt> > triangles){
vector<Line> segments(3 * triangles.size());
vector<char> segtype(segments.size());
for(size_t i = 0; i < triangles.size(); i++){</pre>
 pt a, b, c;
 tie(a, b, c) = triangles[i];
  segments[3 * i] = lexComp(a, b) ? Line(a, b) : Line(b, a);
  segtype[3 * i] = get_segtype(segments[3 * i], c);
  segments[3 * i + 1] = lexComp(b, c) ? Line(b, c) : Line(c,
  segtype[3 * i + 1] = get_segtype(segments[3 * i + 1], a);
  segments [3 * i + 2] = lexComp(c, a)? Line(c, a): Line(a,
  segtype[3 * i + 2] = get_segtype(segments[3 * i + 2], b);
vector<dbl> k(segments.size()), b(segments.size());
for(size_t i = 0; i < segments.size(); i++){</pre>
 if(segtype[i]){
  k[i] = (segments[i][1].y - segments[i][0].y) /
        (segments[i][1].x - segments[i][0].x);
  b[i] = segments[i][0].y - k[i] * segments[i][0].x;
dbl ans = 0;
for(size_t i = 0; i < segments.size(); i++){
 if(!segtype[i])
  continue;
  dbl l = segments[i][0].x, r = segments[i][1].x;
 vector<pair<dbl, int> > evts;
 for(size_t j = 0; j < segments.size(); j++){
  if(!segtype[j] || i == j)</pre>
   continue;
  dbl 11 = segments[j][0].x, r1 = segments[j][1].x;
  if(ge(l1, r) || ge(l, r1))
   continue;
  dbl common_l = max(l, l1), common_r = min(r, r1);
  auto pts = interSegSeg(segments[i], segments[j]);
  if(pts.empty()){
   dbl yl1 = k[j] * common_l + b[j];
dbl yl = k[i] * common_l + b[i];
   if(lt(yl1, yl) == (segtype[i] == 1)){
    int evt_type = -segtype[i] * segtype[j];
    evts.emplace_back(common_1, evt_type);
```

```
evts.emplace_back(common_r, -evt_type);
  else if(pts.size() == 1u){
   dbl yl = k[i] * common_l + b[i], yl1 = k[j] * common_l +
   int evt_type = -segtype[i] * segtype[j];
if(lt(yl1, yl) == (segtype[i] == 1)){
    evts.emplace_back(common_l, evt_type);
    evts.emplace_back(pts[0].x, -evt_type);
   yl = k[i] * common_r + b[i], yl1 = k[j] * common_r + b[j];
if(lt(yl1, yl) == (segtype[i] == 1)){
  evts.emplace_back(pts[0].x, evt_type);
    evts.emplace_back(common_r, -evt_type);
   if(segtype[j] != segtype[i] || j > i){
    evts.emplace_back(common_1, -2);
    evts.emplace_back(common_r, 2);
 evts.emplace_back(1, 0);
 sort(evts.begin(), evts.end());
 size_t j = 0;
 int balance = 0:
 while(j < evts.size()){</pre>
  size_t ptr = j;
  while(ptr < evts.size() && eq(evts[j].first,
        evts[ptr].first)){
   balance += evts[ptr].second;
   ++ptr;
  if(!balance && !eq(evts[j].first, r)){
   dbl next_x = ptr == evts.size() ? r : evts[ptr].first;
   ans -= segtype[i] * (k[i] * (next_x + evts[j].first) + 2 * b[i]) * (next_x - evts[j].first);
    = ptr;
return ans/2:
```

76 Voronoi

```
const Tf INF = 1e10;
vector<Polygon> voronoi(vector<PT> site, Tf bsq) {
  int n = site.size();
  vector<Polygon> region(n);
  PT A(-bsq, -bsq), B(bsq, -bsq),
  C(bsq, bsq), D(-bsq, bsq);
  for(int i = 0; i < n; ++i) {
    vector<DirLine> li(n - 1);
    for(int j = 0, k = 0; j < n; ++j) {
        if(i == j) continue;
        li[k++] = DirLine((site[i] + site[j]) / 2,
        rotate90(site[j] - site[i]));
    }
  li.emplace_back(A,B-A); li.emplace_back(B,C-B);
  li.emplace_back(C,D-C); li.emplace_back(D,A-D);
    region[i] = halfPlaneIntersection(li);
  }
  return region;
}</pre>
```

77 XORTrick

```
vector<int> basis[N];
int sz[N],a[N],LOGK = 21;
void insert_vector(vector<int>&basis ,int &sz, int mask){
  for(int i=0;i<LOGK;i++) {
    if(!mask&(1<i))) continue;
    if(!basis[i]) {
      basis[i] = mask; sz++; return;
    }
    mask^=basis[i];
  }
  bool check(vector<int>&basis , int mask){
    for(int i=0;i<LOGK;i++) {
      if(!mask&(1<<i))) continue;
      if(!basis[i]) { return 0; }
      mask^= basis[i];
    }
  return 1;
}</pre>
```