```
d[j] = i;
#include <ext/pb_ds/tree_policy.hpp>
                                                             long long get(long long x, Node* &node, long long 1, long long r)
#include <ext/pb_ds/detail/standard_policies.hpp>
class LiChaoTree
                                                                                                                           for (int j = 0; j < m; ++j) {
                                                                 long long m = (1 + r) / 2;
                                                                                                                                while (!st.empty() && d[st.top()] <= d[j])</pre>
   long long L,R;
                                                                 if(r - 1 == 1)
                                                                                                                                   st.pop();
   bool minimize;
                                                                                                                                d1[j] = st.empty() ? -1 : st.top();
   int lines;
                                                                     return f(node->line, x);
                                                                                                                               st.push(j);
   struct Node
                                                                 else if (x < m)
                                                                                                                           while (!st.empty())
       complex<long long> line;
                                                                                                                               st.pop();
       Node *children[2];
                                                                     if(node->children[0]==0) return f(node->line, x); for (int j = m.-1; j >= 0; --j) {
   if(minimize) return min(f(node->line, x), get(x, node->children[0]t, pop(n););
   else return max(f(node->line, x), get(x, node->children[0]t, pop(n););
       Node(complex<long long> ln= {0,100000000000000000})
           line=ln;
           children[0]=0;
                                                                                                                               d2[j] = st.empty() ? m : st.top();
                                                                 else
           children[1]=0;
                                                                                                                               st.push(j);
                                                                     if(node->children[1]==0) return f(node->line, x);
   } *root:
                                                                     if (minimize) return min(f(node->line, x), get(x, node->children[1], m, r));
                                                                     else return max(f(node->line, x), get(x, node->children[hep(m; r)); < m; ++j)
   long long dot(complex<long long> a, complex<long long> b)
       return (conj(a) * b).real();
                                                                                                                          ans = \max(ans, (i - d[j]) * (d2[j] - d1[j] - 1))
                                                          public:
   long long f(complex<long long> a, long long x)
                                                             LiChaoTree(long long l=-1000000001,long long r=1000000001,boot mn=false)
       return dot(a, \{x, 1\});
                                                                 L=1;
                                                                                                                        \mathbf{DS}
                                                                 R=r:
   void clear(Node* &node)
                                                                 root=0;
                                                                                                                         BIT 2D
                                                                 minimize=mn:
       if(node->children[0])
                                                                 lines=0;
                                                                                                                    const int mx = 1002, my = 1002;
                                                                                                                    long long bit[4] [mx] [my];
           clear(node->children[0]);
                                                             void AddLine(pair<long long,long long> ln)
                                                                                                                    void update( int x, int y, int val, int i ) {
                                                                                                                     int y1;
       if(node->children[1])
                                                                  add_line({ln.first,ln.second},root,L,R);
                                                                                                                     while( x<=mx ) {</pre>
                                                                 lines++;
                                                                                                                       y1=y;
           clear(node->children[1]);
                                                                                                                        while( v1<=mv)
                                                             int number_of_lines()
                                                                                                                          bit[i][x][v1] += val, v1 += (v1&-v1);
       delete node;
                                                                                                                       x += (x\&-x):
                                                                 return lines;
                                                          long long l, long long r)
   void add_line(complex<long long> nw, Node* &node,
                                                              Tong Tong getOptimumValue(long long x)
                                                                                                                    long long query( int x, int y, int i ) {
       if (node==0)
                                                                                                                     long long ans=0; int v1;
                                                                 return get(x,root,L,R);
                                                                                                                     while( x>0 ) {
           node=new Node(nw);
                                                                                                                       y1 = y;
           return;
                                                              ~LiChaoTree()
                                                                                                                        while( y1>0 )
                                                                                                                          ans += bit[i][x][y1], y1 -= (y1&-y1);
       long long m = (1 + r) / 2;
bool lef = (f(nw, 1) < f(node->line, 1) & minimize) | ((!minimize) & (nw, 1) > f(node->line, 1));
                                                                                                                       x = (x\&-x);
       bool mid = (f(nw, m) < f(node->line, m)&&minimize)||((!minimize)&&f(nw, m) > f(node->line, m));
                                                                                                                     return ans;
                                                          int main()
                                                                                                                    // add value k from (x1,y1) to (x2,y2) inclusive
           swap(node->line, nw);
                                                             return 0;
                                                                                                                    void add( int x1, int y1, int x2, int y2, int k) {
                                                                                                                     update(x1,y1,k,0);
       if(r - 1 == 1)
                                                                                                                     update(x1, y2+1, -k, 0);
                                                          1.4 zero_m atrix
                                                                                                                     update(x2+1,y1,-k,0);
           return;
                                                                                                                     update(x2+1, y2+1, k, 0);
                                                          int zero_matrix(vector<vector<int>> a) {
                                                                                                                     update(x1,y1,k*(1-y1),1);
       else if(lef != mid)
                                                             int n = a.size();
                                                                                                                     update(x1,y2+1,k*y2,1);
                                                             int m = a[0].size();
                                                                                                                     update(x2+1,y1,k*(y1-1),1);
           add_line(nw, node->children[0], 1, m);
                                                             int ans = 0;
                                                                                                                     update(x2+1,y2+1,-y2*k,1);
                                                             vector\langle int \rangle d(m, -1), d1(m), d2(m);
                                                                                                                     update(x1,y1,k*(1-x1),2);
       else
                                                             stack<int> st;
                                                             for (int i = 0; i < n; ++i) {
                                                                                                                     update(x1,y2+1,k*(x1-1),2);
                                                                 for (int j = 0; j < m; ++j) {
           add_line(nw, node->children[1], m, r);
                                                                                                                     update(x2+1,y1,k*x2,2);
                                                                     if (a[i][i] == 1)
                                                                                                                     update(x2+1,y2+1,-x2*k,2);
```

```
update(x1,y1,(x1-1)*(y1-1)*k,3);
                                                         for (int c : adj[v]) {
 update(x1,y2+1,-y2*(x1-1)*k,3);
                                                             if (c != parent[v]) {
 update(x2+1,y1,-x2*(y1-1)*k,3);
                                                                 parent[c] = v, depth[c] = depth[v] + 1;
 update(x2+1,y2+1,x2*y2*k,3);
                                                                 int c_size = dfs(c, adj);
                                                                 size += c_size;
// get value from (x1,y1) to (x2,y2) inclusive
                                                                 if (c_size > max_c_size)
                                                                     max_c_size = c_size, heavy[v] = c;}}
long long get( int x1, int y1, int x2, int y2 ) {
                                                          return size;}
 LL v1=query(x2,y2,0)*x2*y2 +
                                                      decompose(int v,int h,vector<vector<int>> const& adj){
       query(x2, y2, 1)*x2 +
                                                         head[v] = h, pos[v] = cur_pos++;
       query(x2, y2, 2)*y2 +
                                                          if (heavy[v] != -1)
       query(x2,y2,3);
                                                             decompose(heavy[v], h, adj);
 LL v2=query(x2,y1-1,0)*x2*(y1-1) +
                                                          for (int c : adj[v]) {
       query(x2,y1-1,1)*x2 +
                                                             if (c != parent[v] && c != heavy[v])
       query(x2,y1-1,3) +
                                                                 decompose(c, c, adj);}
       query(x2,y1-1,2)*(y1-1);
 LL v3=query(x1-1,y2,0)*(x1-1)*y2 +
                                                      void init(vector<vector<int>> const& adj) {
       query(x1-1,y2,2)*y2+
                                                          int n = adj.size();
       query(x1-1,y2,1)*(x1-1) +
                                                          parent = vector<int>(n);
       query(x1-1,y2,3);
                                                          depth = vector<int>(n);
 LL v4=query(x1-1,y1-1,0)*(x1-1)*(y1-1) +
                                                          heavy = vector\langle int \rangle (n, -1);
       query(x1-1,y1-1,1)*(x1-1) +
                                                          head = vector<int>(n);
       query(x1-1,y1-1,2)*(y1-1) +
                                                          pos = vector<int>(n);
       query (x1-1,y1-1,3);
                                                          cur_pos = 0;
 LL ans=v1-v2-v3+v4;
                                                          dfs(0, adj);
 return ans;
                                                          decompose(0, 0, adj);
     CD - hellbent
                                                      int query(int a, int b) {
                                                        int res = 0;
vector <int> g[N]; int n, child[N], done[N];
                                                        for (; head[a] != head[b]; b = parent[head[b]]) {
void dfs_size(int u, int par) {
                                                            if (depth[head[a]] > depth[head[b]])
 child[u] = 1;
                                                               swap(a, b);
 for (int v: g[u]) {
  if (done[v] or v == par) continue;
                                                      int cur_heavy_path_max =
                                                      segment_tree_query(pos[head[b]], pos[b]);
   dfs_size(v, u); child[u] += child[v];
                                                           res = max(res, cur_heavy_path_max);
                                                        if (depth[a] > depth[b])
int dfs_find_centroid(int u, int par, int sz) {
                                                            swap(a, b);
 for (int v: g[u]) {
                                                        int last_heavy_path_max =
   if (!done[v] and v != par and child[v] > sz) {
                                                        segment_tree_query(pos[a], pos[b]);
     return dfs_find_centroid(v,u,sz);
                                                        res = max(res, last_heavy_path_max);
                                                        return res;
 return u;
                                                      2.4 Implicit Treap
void solve (int u) {/**problem specific things */}
                                                      #include<bits/stdc++.h>
void dfs_decompose(int u) {
                                                      #include<math.h>
 dfs_size(u, -1);
                                                      #include<vector>
 int centroid=dfs_find_centroid(u,-1,child[u]/2);
                                                      #include<stdlib.h>
 solve(centroid);
                                                      using namespace std;
 done[centroid] = 1;
                                                      #define MAX 200005
 for (int v : g[centroid]) {
  if (!done[v]) dfs_decompose(v);
                                                      #define MOD 998244353
                                                      template <class T>
                                                      class implicit_treap
                                                          struct item
2.3 Hld - cpalgo
vector<int> parent, depth, heavy, head, pos;
                                                              int prior, cnt;
int cur_pos;
                                                             T value;
int dfs(int v, vector<vector<int>> const& adj) {
                                                             bool rev:
```

item *1,*r;

item(T v)

int size = 1;

int max_c_size = 0;

```
value=v:
           rev=false;
           l=NULL;
           r=NULL
           cnt=1;
           prior=rand();
    } *root,*node;
    int cnt (item * it)
       return it ? it->cnt : 0;
    void upd_cnt (item * it)
       if (it)
           it->cnt = cnt(it->1) + cnt(it->r) + 1;
    void push (item * it)
       if (it && it->rev)
           it->rev = false;
           swap (it->1, it->r);
           if (it->1) it->1->rev ^= true;
           if (it->r) it->r->rev ^= true;
   }
    void merge (item * & t, item * 1, item * r)
       push (1);
       push (r);
       if (!1 || !r)
t = 1 ? 1 : r;
       else if (l->prior > r->prior)
           merge (1->r, 1->r, r), t = 1;
           merge (r->1, 1, r->1), t = r;
       upd_cnt (t);
    void split (item * t, item * & 1, item * & r, int
       if (!t)
           return void( 1 = r = 0 ):
       push (t);
       int cur_key = add + cnt(t->1);
       if (key <= cur_key)</pre>
           split (t->1, 1, t->1, key, add), r = t;
           split (t->r, t->r, r, key, add + 1 + cnt(t-
11 = t;
       upd_cnt (t);
    void insert(item * &t,item * element,int key)
       item *1,*r;
       split(t,1,r,key);
       merge(1,1,element);
       merge(t,l,r);
       1=NŬLL;
```

```
r=NULL;
     elementAt(item * &t,int key)
       push(t);
       T ans;
       if(cnt(t->1)==key) ans=t->value;
       else if(cnt(t->1)>key) ans=elementAt(t->1,key);
       else ans=elementAt(t->r,key-1-cnt(t->l));
       return ans;
   void erase (item * & t, int key)
       push(t);
       if(!t) return;
       if (key == cnt(t->1))
           merge (t, t->1, t->r);
       else if(key<cnt(t->1))
           erase(t->1,key);
       else
           erase(t->r,key-cnt(t->1)-1);
       upd_cnt(t);
   void reverse (item * &t, int 1, int r)
       item *t1, *t2, *t3;
       split (t, t1, t2, 1);
       split (t2, t2, t3, r-l+1);
       t2->rev ^= true:
       merge (t, t1, t2);
       merge (t, t, t3);
   void cyclic_shift(item * &t,int L,int R)
       if(L==R) return;
       item *1,*r,*m;
       split(t,t,l,L);
       split(1,1,m,R-L+1);
       split(1,1,r,R-L);
       merge(t,t,r);
       merge(t,t,1);
       merge(t,t,m);
       1=NULL;
       r=NULL:
       m=NULL;
   void output (item * t,vector<T> &arr)
       if (!t) return;
       push (t);
       output (t->1,arr);
       arr.push_back(t->value);
       output (t->r,arr);
public:
   implicit_treap()
       root=NULL;
   void insert(T value,int position)
       node=new item(value);
       insert(root, node, position);
```

```
void erase(int position)
   erase(root, position);
void reverse(int l,int r)
   reverse(root,1,r);
 elementAt(int position)
   return elementAt(root, position);
void cyclic_shift(int L,int R)
   cyclic_shift(root,L,R);
int size()
   return cnt(root);
void output(vector<T> &arr)
   output(root,arr);
```

Mo Algorithm

```
#include<bits/stdc++.h>
using namespace std;
#define MOD 998244353
#define MAX 200005
#define MAX_BIT 50
#define PRECISION 0.0000000001
#define INF 200000000
void remove(int idx); // TODO: remove value at idx ftom dataisem(φτψή)e
void add(int idx); // TODO: add value at idx from data structure
int get_answer(); // TODO: extract the current answer of the datkeystructure
int block_size;
struct Query {
    int 1, r,k, idx;
    bool operator<(Query other) const
        if(1/block_size!=other.1/block_size) return (1<othernon;,*node;</pre>
        return (1/block_size&1)? (r<other.r) : (r>other.r)int cnt (item * it)
vector<int> mo_s_algorithm(vector<Query> queries) {
    vector<int> answers(queries.size());
    sort(queries.begin(), queries.end());
    // TODO: initialize data structure
    int cur 1 = 0:
    int cur_r = -1;
    for (Query q : queries) {
        while (cur_1 > q.1) {
            cur_1--;
            add(cur_1);
        while (cur_r < q.r) {</pre>
            cur_r++;
            add(cur_r);
```

```
while (cur_1 < q.1) {</pre>
           remove(cur_l);
           cur_1++;
       while (cur_r > q.r) {
           remove(cur_r);
           cur_r--;
       answers[q.idx] = get_answer();
   return answers;
int main()
   return 0;
```

2.6Treap

```
#include<bits/stdc++.h>
                                                      #include<math.h>
                                                      #include<vector>
                                                      #include<stdlib.h>
                                                      using namespace std;
                                                      #define MAX 400005
                                                      #define MOD 998244353
                                                      #define INF 2000000000
                                                      template <class T>
                                                      class treap
                                                          struct item
                                                             int prior, cnt;
                                                                 1=NULL;
                                                                 cnt=1;
                                                                 prior=rand();
                                                             return it ? it->cnt : 0;
                                                         void upd_cnt (item * it)
                                                             if (it)
                                                                 it \rightarrow cnt = cnt(it \rightarrow l) + cnt(it \rightarrow r) + 1;
// invariant: data structure will always reflect the raidesplint_(i,temr*rt, T key, item * & 1, item * &
                                                             if (!t)
                                                                 1 = r = NULL;
                                                             else if (key < t->key)
                                                                 split (t->1, key, 1, t->1), r = t;
                                                                 split (t->r, key, t->r, r), l = t;
```

upd_cnt(t);

```
void insert (item * & t, item * it)
   if (!t)
       t = it:
    else if (it->prior > t->prior)
        split (t, it->key, it->l, it->r), t = it;
        insert (it->key < t->key ? t->l : t->r, i \ddagger);
   upd_cnt(t);
}
void merge (item * & t, item * 1, item * r)
   if (!1 || !r)
       t = 1 ? 1 : r;
    else if (l->prior > r->prior)
        merge (1->r, 1->r, r), t = 1;
        merge (r->1, 1, r->1), t = r;
   upd_cnt(t);
void erase (item * & t, T key)
   if (t->key == key)
       merge (t, t->1, t->r);
        erase (key \langle t-\ranglekey ? t->1 : t-\rangler, key);
   upd_cnt(t);
  elementAt(item * &t,int key)
   if(cnt(t->1)==key) ans=t->key;
   else if(cnt(t->1)>key) ans=elementAt(t->1,key);
   else ans=elementAt(t->r,key-1-cnt(t->l));
   upd_cnt(t);
   return ans;
item * unite (item * 1, item * r)
    if (!1 || !r) return 1 ? 1 : r;
    if (l->prior < r->prior) swap (l, r);
   item * lt, * rt;
   split (r, l->key, lt, rt);
   1->1 = unite (1->1, 1t);
   1->r = unite (1->r, rt);
   upd_cnt(1);
   upd_cnt(r);
   return 1;
void heapify (item * t)
    if (!t) return;
   item * max = t:
    if (t->l != NULL && t->l->prior > max->prior)
        \max = t->1;
    if (t->r != NULL && t->r->prior > max->prior)
        \max = t->r;
    if (max != t)
```

```
swap (t->prior, max->prior);
          heapify (max);
   item * build (T * a, int n)
       if (n == 0) return NULL;
       int mid = n / 2;
       item * t = new item (a[mid], rand ());
       t->1 = build (a, mid);
       t->r = build (a + mid + 1, n - mid - 1);
       heapify (t);
       return t;
   void output (item * t,vector<T> &arr)
       if (!t) return;
       output (t->1,arr);
       arr.push_back(t->key);
       output (t->r,arr);
public:
   treap()
       root=NULL;
   treap(T *a, int n)
       build(a,n);
   void insert(T value)
       node=new item(value);
       insert(root, node);
   void erase(T value)
       erase(root, value);
     elementAt(int position)
       return elementAt(root, position);
   int size()
       return cnt(root);
   void output(vector<T> &arr)
       output(root,arr);
   int range_query(T 1,T r) //(1,r]
       item *previous,*next,*current;
       split(root,1,previous,current);
       split(current,r,current,next);
       int ans=cnt(current):
       merge(root,previous,current);
       merge(root,root,next);
       previous=NULL;
       current=NULL;
       next=NULL;
```

```
return ans;
      sparse table 2d
int st[K][K][N][N]; int lg[N];
void pre() {
  lg[\bar{1}] = 0;
  for (int i=2; i<N; i++) lg[i] = lg[i/2]+1;
int query(int 11, int r1, int 12, int r2) {
  int xx = lg[12-11+1], yy = lg[r2-r1+1];
  return max(max(st[xx][yy][11][r1]
             st[xx][yy][12-(1<<xx)+1][r1]),
         \max(\text{st}[xx][yy][11][r2-(1<<yy)+1],
         st[xx][yy][12-(1<<xx)+1][r2-(1<<yy)+1]);
void build() {
 for (int x=0; x<K; x++) {
   for (int y=0; y<K; y++) {</pre>
     for (int i=1; i<=n; i++) {
        for (int j=1; j<=m; j++) {</pre>
          if (i+(1<< x)-1>n | j+(1<< y)-1>m)
          if (!x&&!y) st[0][0][i][j]=flag[i][j];
          else if (x>0) st[x][y][i][j] =
\max(\text{st}[x-1][y][i][j], \text{st}[x-1][y][i+(1<<(x-1))][j]);
          else if (y>0) st[x][y][i][j] =
\max(\text{st}[x][y-1][i][j], \text{st}[x][y-1][i][j+(1<<(y-1))]);
     Flow
3.1 Dinic's Algorithm
#include<bits/stdc++.h>
#include<vector>
using namespace std;
#define MAX 100
#define HUGE_FLOW 1000000000
#define BEGIN 1
#define DEFAULT_LEVEL 0
struct FlowEdge {
    int v, u;
```

FlowEdge(int v, int u, long long cap) : v(v), u(u)

Dinic(int n, int s, int t) : n(n), s(s), t(t) {

long long cap, flow = 0;

vector<FlowEdge> edges;

vector<int> level, ptr;

adj.resize(n);

ptr.resize(n);

level.resize(n);

vector<vector<int>> adj;

const long long flow_inf = 1e18;

struct Dinic {

int n, m = 0;

queue<int> q;

int s, t;

```
return 0;
                                                                                                              memset(inb,0,sizeof(inb));
                                                                                                              mark_blossom(lca,u);
   void add_edge(int v, int u, long long cap) {
                                                                                                              mark_blossom(lca,v);
                                                          Edmond's Blossom Algorithm
      edges.emplace_back(v, u, cap);
                                                                                                              if (base[u]!=lca)
                                                     /***Copied from https://codeforces.com/blog/entry/49402***/ father[u]=v; if (base[v]!=lca
      edges.emplace_back(u, v, 0);
      adj[v].push_back(m);
                                                                                                                 (base[v]!=lca)
      adj[u].push_back(m + 1);
                                                                                                                  father[v]=u;
                                                                                                              for (int u=0; u<V; u++)</pre>
                                                     V->number of vertices
                                                                                                                  if (inb[base[u]])
                                                     E->number of edges
   bool bfs() {
                                                     pair of vertices as edges (vertices are 1..V)
                                                                                                                     base[u]=lca;
      while (!q.empty()) {
                                                                                                                     if (!inq[u])
                                                     GIVES:
          int v = q.front();
                                                                                                                         inq[q[++qt]=u]=true;
          q.pop();
                                                     output of edmonds() is the maximum matching
                                                     match[i] is matched pair of i (-1 if there isn't a matched pair)
          for (int id : adj[v]) {
              if (edges[id].cap - edges[id].flow < 1) */</pre>
                                                                                                          int find_augmenting_path(int s)
                  continue:
                                                     #include <bits/stdc++.h>
              if (level[edges[id].u] != -1)
                                                     using namespace std;
                                                                                                              memset(inq,0,sizeof(inq));
                  continue
                                                     const int M=500;
                                                                                                              memset(father,-1,sizeof(father));
              level[edges[id].u] = level[v] + 1;
                                                     struct struct_edge
                                                                                                              for (int i=0; i<V; i++) base[i]=i;</pre>
              q.push(edges[id].u);
                                                                                                              inq[q[qh=qt=0]=s]=true;
                                                        int v;
                                                                                                              while (qh<=qt)</pre>
                                                        struct_edge* n;
      return level[t] != -1;
                                                                                                                  int u=q[qh++];
                                                     typedef struct_edge* edge;
                                                                                                                  for (edge e=adj[u]; e; e=e->n)
                                                     struct_edge pool[M*M*2];
   long long dfs(int v, long long pushed) {
                                                     edge top=pool,adj[M];
      if (pushed == 0)
                                                                                                                     int v=e->v:
                                                     int V,E,match[M],qh,qt,q[M],father[M],base[M];
                                                                                                                     if (base[u]!=base[v]&&match[u]!=v)
          return 0;
                                                     bool inq[M],inb[M],ed[M][M];
      if (v == t)
                                                                                                                         if ((v==s)||(match[v]!=-1 && father[match[v]]
                                                     void add_edge(int u,int v)
                                                                                                                             blossom_contraction(s,u,v);
          return pushed;
      else if (father[v]==-1)
                                                        top->v=u, top->n=adj[v], adj[v]=top++;
                                                                                                                            father[v]=u;
          int u = edges[id].u;
                                                                                                                            if (match[v] == -1)
          return v;
                                                                                                                            else if (!ing[match[v]])
          long long tr = dfs(u, min(pushed, edges[id].cap - edges[id].flow));
if (tr == 0)
static bool inp[M];
                                                                                                                                inq[q[++qt]=match[v]]=true;
                                                        memset(inp,0,sizeof(inp));
              continue;
                                                        while(1)
          edges[id].flow += tr;
          edges[id ^ 1].flow -= tr;
                                                                                                              return -1;
                                                            inp[u=base[u]]=true;
          return tr;
                                                            if (u==root) break;
                                                                                                          int augment_path(int s,int t)
      return 0;
                                                            u=father[match[u]];
                                                                                                              int u=t,v,w;
                                                        while(1)
   long long flow() {
   long long f = 0;
                                                                                                              while (u!=-1)
                                                            if (inp[v=base[v]]) return v;
      while (true) {
                                                                                                                  v=father[u];
                                                            else v=father[match[v]];
          fill(level.begin(), level.end(), -1);
                                                                                                                  w=match[v];
          level[s] = 0;
                                                                                                                 match[v]=u;
          q.push(s);
                                                                                                                  match[u]=v;
                                                     void mark_blossom(int lca,int u)
                                                                                                                  u=w;
          if (!bfs())
              break;
                                                        while (base[u]!=lca)
                                                                                                              return t!=-1;
          fill(ptr.begin(), ptr.end(), 0);
          while (long long pushed = dfs(s, flow_inf)) {
                                                            int v=match[u];
                                                                                                          int edmonds()
              f += pushed;
                                                            inb[base[u]]=inb[base[v]]=true;
                                                            u=father[v]
                                                                                                              int matchc=0:
                                                            if (base[u]!=lca) father[u]=v;
                                                                                                              memset(match,-1,sizeof(match));
      return f;
                                                                                                              for (int u=0; u<V; u++)</pre>
   }
                                                                                                                  if (match[u] == -1)
                                                     void blossom_contraction(int s,int u,int v)
                                                                                                                     matchc+=augment_path(u,find_augmenting_path
int main()
                                                                                                              return matchc;
                                                        int lca=LCA(s,u,v);
```

```
int main()
   FILE *in=stdin;
   int u,v;
   fscanf(in, "%d", &V);
   while(fscanf(in, "%d %d", &u, &v)!=EOF)
       if (!ed[u-1][v-1])
           add_edge(u-1,v-1);
           ed[u-1][v-1]=ed[v-1][u-1]=true;
   printf("%d\n",2*edmonds());
   for (int i=0; i<V; i++)</pre>
       if (i<match[i])</pre>
           printf("%d %d\n",i+1,match[i]+1);
   return 0:
3.3 Hungarian Algorithm
#include <bits/stdc++.h>
#include <vector>
#include<math.h>
#include<string.h>
using namespace std;
#define MAX 300005
#define MOD 100000007
#define GMAX 19
```

```
#define INF 20000000000000000
#define EPS 0.000000001
#define FASTIO ios_base::sync_with_stdio(false);cin.tie(NULL)
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
#include <ext/pb_ds/detail/standard_policies.hpp>
class HungarianAlgorithm
   int N,inf,n,max_match;
   int *lx,*ly,*xy,*yx,*slack,*slackx,*prev;
   int **cost;
   bool *S, *T
   void init_labels()
       for(int x=0; x<n; x++) lx[x]=0;
       for(int y=0; y<n; y++) ly[y]=0;
       for (int x = 0; x < n; x++)
          for (int y = 0; y < n; y++)
              lx[x] = max(lx[x], cost[x][y]);
   void update_labels()
      int x, y, delta = inf; //init delta as infinit///augmenting path exists!
       for (y = 0; y < n; y++) //calculate delta using slack</pre>
           if (!T[y])
              delta = min(delta, slack[y]);
       for (x = 0; x < n; x++) //update X labels
           if (S[x]) lx[x] = delta;
       for (y = 0; y < n; y++) //update Y labels
           if (T[y]) ly[y] += delta;
       for (y = 0; y < n; y++) //update slack array
           if (!T[y])
```

```
slack[v] -= delta;
                                                                                                                  if (y < n) break; //augmenting path found!
                                                                                                                  update_labels(); //augmenting path not foun
      void add_to_tree(int x, int prevx)
                                                                                                                  wr = rd = 0;
//x - current vertex, prevx - vertex from X before x in the alternating path y < n; y++)
//so we add edges (prevx, xy[x]), (xy[x], x)
                                                                                                                        //in this cycle we add edges that were
            S[x] = true; //add x to S
           prev[x] = prevx; //we need this when augmenting/result of improving the labeling, we add edge (slack
                  (int y = 0; y < n; y++) //update slacks, because why aid new vertextack [y] == 0, also with this exif [1x[x] + 1y[y] - cost[x][y] < slack[y] //([y], [yx][y]) or augment the matching, if [y] was exposed
            for (int y = 0; y < n; y++) //update slacks,
                                                                                                                        if (!T[y] \&\& slack[y] == 0)
                         slack[y] = lx[x] + ly[y] - cost[x][y];
                                                                                                                               if (yx[y] == -1) //exposed vertex in
                         slackx[y] = x;
                                                                                                                                      x = slackx[y];
                                                                                                                                      break:
      void augment() //main function of the algorithm
            if (max_match == n) return; //check wether matching is already perfect
            int x, y, root; //just counters and root vertex
            int x, y, root; //just counters and root vertex T[y] = true; //else just add y t int q[N], wr = 0, rd = 0; //q - queue for bfs, wr,rd - write and read (!S[yx[y]])
            //memset(S, false, sizeof(S)); //init set S
                                                                                                                                            q[wr++] = yx[y]; //add vertex
            for(int i=0;i<n;i++) S[i]=false;</pre>
                                                                                               //y, to the queue
            //memset(T, false, sizeof(T)); //init set T
                                                                                                                                            add_to_tree(yx[y], slackx[y])
            for(int i=0;i<n;i++) T[i]=false;</pre>
            //memset(prev, -1, sizeof(prev)); //init set prev for the alternating tree
            for(int i=0;i<n;i++) prev[i]=-1;</pre>
            for (x = 0; x < n; x++) //finding root of the tree
                  if (xy[x] == -1)
                                                                                                                  if (y < n) break; //augmenting path found!</pre>
                         q[wr++] = root = x;
                                                                                                           if (y < n) //we found augmenting path!
                         prev[x] = -2;
                         S[x] = true;
                                                                                                                  max_match++; //increment matching
                         break;
                                                                                               //in this cycle we inverse edges along augmenting path
                                                                                                                  for (int cx = x, cy = y, ty; cx != -2; cx =
            for (y = 0; y < n; y++) //initializing slack array
                                                                                                                        tv = xv[cx]:
                                                                                                                        yx[cy] = cx;
                   slack[y] = lx[root] + ly[y] - cost[root][y];
                                                                                                                        xy[cx] = cy;
                   slackx[y] = root;
                                                                                                                  augment(); //recall function, go to step 1
            while (true) //main cycle
                  while (rd < wr) //building tree with bfs cycle}//end of augment() function
                                                                                                     HungarianAlgorithm(int vv,int inf=1000000000)
                         x = q[rd++]; //current vertex from X part
                         for (y = 0; y < n; y++) //iterate through all <u>wedges</u> in equality graph
                                                                                                           n=N;
                                if (cost[x][y] == lx[x] + ly[y] &&|!T[y]) max_match=0;
                                                                                                           this->inf=inf;
                                                                                                         veloctenewininkt [Mound, so
                                      if (yx[y] == -1) break; //an exposed
                                                                                                           ly=new int[N];//labels of X and Y parts
                                                                                                           xy=new int[N];//xy[x] - vertex that is matched
                                      T[y] = true; //else just add y to T,
                                      q[wr++] = yx[y]; //add vertex yx[y], why is in the interpolation of the decomposition of th
                                                                                                            slack=new int[N];//as in the algorithm descript
//with y, to the queue
                                      add_to_tree(yx[y], x); //add edges (x, x) awkk=(ne, yxi[xt][N]tx//tshackxxe[y] such a vertex, the
                                                                                                           prev=new int[N];//array for memorizing alternat
                                                                                                           S=new bool[N];
                                                                                                           T=new bool[N];//sets S and T in algorithm
                         if (y < n) break; //augmenting path found!
                                                                                                           cost=new int*[N];//cost matrix
```

```
for(int i=0; i<N; i++)</pre>
           cost[i]=new int[N];
    ~HungarianAlgorithm()
       delete []lx;
       delete []ly;
       delete []xy;
       delete []yx;
       delete []slack;
       delete []slackx;
       delete []prev;
       delete []S;
       delete []T;
       int i;
       for(i=0; i<N; i++)</pre>
           delete [](cost[i]);
       delete []cost;
   void setCost(int i,int j,int c)
       cost[i][j]=c;
   int* matching(bool first=true)
       int *ans;
       ans=new int[N];
       for(int i=0;i<N;i++)</pre>
           if(first) ans[i]=xy[i];
           else ans[i]=yx[i];
       return ans;
   int hungarian()
       int ret = 0; //weight of the optimal matching #include <ext/pb_ds/assoc_container.hpp>
       max_match = 0; //number of vertices in curren # matching ext/pb_ds/tree_policy.hpp>
       for(int x=0; x<n; x++) xy[x]=-1;
       for(int y=0;y<n;y++) yx[y]=-1;
       init_labels(); //step 0
       augment(); //steps 1-3
       for (int x = 0; x < n; x++) //forming answer there
           ret += cost[x][xy[x]];
       return ret;
int main()
   int t,T=1;
   scanf("%d",&T);
   for(t=0;t<T;t++)</pre>
       int n,i,j;
scanf("%d",&n);
       HungarianAlgorithm h(n);
       int own[n],opposite[n];
       for(i=0;i<n;i++)</pre>
```

```
scanf("%d", &own[i]);
   for(j=0;j<n;j++)
       scanf("%d",&opposite[j]);
    for(i=0;i<n;i++)
       for(j=0;j<n;j++)
           int v;
           if(own[i] == opposite[j]) v=1;
           else if(own[i]>opposite[j]) v=2;
           else v=0;
           h.setCost(i,j,v);
       }
    int ans=h.hungarian();
   printf("Case %d: %d\n",t+1,ans);
return 0;
```

3.4 Maximum Bipartite Matching

```
/** Source: https://iq.opengenus.org/hopcroft-karp-algorithmpa**/u[u] = NIL;
#include <bits/stdc++.h>
#include <vector>
#include<math.h>
|#include<string.h>
using namespace std;
#define MAX 300005
#define BEGIN 1
#define MOD 100000007
#define INF INT_MAX
#define EPS 0.000000001
#define CHAINS 18
#define NIL 0
#define NOT_VISITED 0
#define VISTTING 1
#define VISITED 2
#define FASTIO ios_base::sync_with_stdio(false);cin.tie(NULL)
#include <ext/pb_ds/detail/standard_policies.hpp>
// A class to represent Bipartite graph for
// Hopcroft Karp implementation
class BGraph
   // m and n are number of vertices on left
   // and right sides of Bipartite Graph
   // adj[u] stores adjacents of left side
   // vertex 'u'. The value of u ranges from 1 to m.
   // 0 is used for dummy vertex
   std::list<int> *adj;
   // pointers for hopcroftKarp()
   int *pair_u, *pair_v, *dist;
public:
   BGraph(int m, int n); // Constructor
   void addEdge(int u, int v); // To add edge
   // Returns true if there is an augmenting path
```

```
bool bfs();
   // Adds augmenting path if there is one beginning
   bool dfs(int u);
   // Returns size of maximum matching
   int hopcroftKarpAlgorithm();
// Returns size of maximum matching
int BGraph::hopcroftKarpAlgorithm()
   // pair_u[u] stores pair of u in matching on left
   // If u doesn't have any pair, then pair_u[u] is N
   pair_u = new int[m + 1];
   // pair_v[v] stores pair of v in matching on right
   // If v doesn't have any pair, then pair_u[v] is N
   pair_v = new int[n + 1];
   // dist[u] stores distance of left side vertices
   dist = new int[m + 1];
   // Initialize NIL as pair of all vertices
   for (int u = 0; u \le m; u++)
   for (int v = 0; v \le n; v++)
       pair_v[v] = NIL;
   // Initialize result
   int result = 0;
   // Keep updating the result while there is an
   // augmenting path possible.
   while (bfs())
       // Find a free vertex to check for a matching
       for (int u = 1; u <= m; u++)
           // If current vertex is free and there is
           // an augmenting path from current vertex
           // then increment the result
           if (pair_u[u] == NIL && dfs(u))
              result++;
   return result;
// Returns true if there is an augmenting path availab
bool BGraph::bfs()
   std::queue<int> q; //an integer queue for bfs
   // First layer of vertices (set distance as 0)
   for (int u = 1; u <= m; u++)
       // If this is a free vertex, add it to queue
       if (pair_u[u] == NIL)
           // u is not matched so distance is 0
           dist[u] = 0:
           q.push(u);
       // Else set distance as infinite so that this v
```

```
else
                                                                                                                                       dist[u] = INF;
                        dist[u] = INF;
                                                                                                                                      return false;
       }
                                                                                                                                                                                                                                                      int flow = 0;
                                                                                                                               return true;
                                                                                                                                                                                                                                                      int cost = 0
        // Initialize distance to NIL as infinite
                                                                                                                                                                                                                                                      vector<int> d, p;
       dist[NIL] = INF;
                                                                                                                             Constructor for initialization
                                                                                                                                                                                                                                                      while (flow < K) {</pre>
       // q is going to contain vertices of left side on BGraph::BGraph(int m, int n)
                                                                                                                                                                                                                                                              shortest_paths(N, s, d, p);
       while (!q.empty())
                                                                                                                                                                                                                                                              if (d[t] == INF)
                                                                                                                               this->m = m;
                                                                                                                                                                                                                                                                      break;
                                                                                                                               this->n = n;
               // dequeue a vertex
                                                                                                                               adj = new std::list<int>[m + 1];
                                                                                                                                                                                                                                                              // find max flow on that path
               int u = q.front();
                                                                                                                                                                                                                                                              int f = K - flow;
               q.pop();
                                                                                                                                                                                                                                                              int cur = t;
               // If this node is not NIL and can provide a shortware tien to wid edge from u to v if (dist[u] < dist[NIL])
                                                                                                                                                                                                                                                              while (cur != s) {
               if (dist[u] < dist[NIL])</pre>
                                                                                                                                                                                                                                                                      cur = p[cur];
                        // Get all the adjacent vertices of the dequeuadj wertpush back(v); // Add v to us list.
                        std::list<int>::iterator it;
                                                                                                                                                                                                                                                              // apply flow
                        for (it = adj[u].begin(); it != adj[u].end(); ++it)
                                                                                                                                                                                                                                                              flow += f;
                                                                                                                       3.5 Minimum Cost Maximum Flow
                                                                                                                                                                                                                                                              cost += f * d[t]:
                                int v = *it:
                                                                                                                                                                                                                                                              cur = t;
                                                                                                                       struct Edge
                                                                                                                                                                                                                                                              while (cur != s) {
                                // If pair of v is not considered so far
                                                                                                                                                                                                                                                                      capacity[p[cur]][cur] -= f;
                                // i.e. (v, pair_v[v]) is not yet explorediadgerom, to, capacity, cost;
                                                                                                                                                                                                                                                                      capacity[cur][p[cur]] += f;
                                if (dist[pair_v[v]] == INF)
                                                                                                                                                                                                                                                                      cur = p[cur];
                                       // Consider the pair and push it tovector vector adj, cost, capacity;
                                        dist[pair_v[v]] = dist[u] + 1;
                                                                                                                       const int INF = 1e9;
                                        q.push(pair_v[v]);
                                                                                                                       void shortest_paths(int n, int v0, vector<int>& d, vector<int>&uph +1;
                                                                                                                              d.assign(n, INF);
               }
                                                                                                                              d[v0] = 0;
                                                                                                                                                                                                                                                              return cost;
                                                                                                                              vector<bool> inq(n, false);
                                                                                                                               queue<int> q;
                                                                                                                                                                                                                                                        Geo
       // If we could come back to NIL using alternating
                                                                                                                       pata pishisojņēt
       // vertices then there is an augmenting path available assign(n, -1);
                                                                                                                                                                                                                                                        Convex Hull
       return (dist[NIL] != INF);
                                                                                                                               while (!q.empty()) {
                                                                                                                                                                                                                                              struct pt {double x, y;};
                                                                                                                                       int u = q.front();
                                                                                                                                                                                                                                              bool cmp(pt a, pt b) {
// Returns true if there is an augmenting path beginning with free yeartex u
bool BGraph::dfs(int u)
                                                                                                                                       inq[u] = false;
                                                                                                                                       for (int v : adi[u]) {
                                                                                                                                                                                                                                              bool cw(pt a, pt b, pt c) {
       if (u != NIL)
                                                                                                                                               if (capacity[u][v] > 0 && d[v] > d[u] + c\phi st_{v} = hv_{v} = hv_
                                                                                                                                                       d[v] = d[u] + cost[u][v];
               std::list<int>::iterator it;
                                                                                                                                                       p[v] = u;
                                                                                                                                                                                                                                              bool ccw(pt a, pt b, pt c) {
               for (it = adj[u].begin(); it != adj[u].end(); ++it)
                                                                                                                                                       if (!inq[v]) {
                                                                                                                                                               inq[v] = true;
                        // Adjacent vertex of u
                                                                                                                                                               q.push(v);
                                                                                                                                                                                                                                              vector<pt> a;
                        int v = *it;
                                                                                                                                                                                                                                              void convex_hull(vector<pt>& a) {
                        // Follow the distances set by BFS search
                                                                                                                                               }
                                                                                                                                                                                                                                                      if (a.size() == 1)
                        if (dist[pair_v[v]] == dist[u] + 1)
                                                                                                                                                                                                                                                              return;
                                                                                                                                                                                                                                                      sort(a.begin(), a.end(), &cmp);
                                // If dfs for pair of v also return the then
                                                                                                                                                                                                                                                      pt p1 = a[0], p2 = a.back();
                                if (dfs(pair_v[v]) == true)
                                { // new matching possible, store the matching possible, store the matching possible, store the matching possible and the 
                                                                                                                               adj.assign(N, vector<int>());
                                                                                                                                                                                                                                                      up.push_back(p1);
                                        pair_v[v] = u;
                                                                                                                               cost.assign(N, vector<int>(N, 0));
                                                                                                                                                                                                                                                      down.push_back(p1);
                                        pair_u[u] = v;
                                                                                                                               capacity.assign(N, vector<int>(N, 0));
                                                                                                                                                                                                                                                      for (int i = 1; i < (int)a.size(); i++) {</pre>
                                        return true;
                                                                                                                                                                                                                                                              if (i == a.size() - 1 || cw(p1, a[i], p2)) {
                                                                                                                              for (Edge e : edges) {
                                                                                                                                                                                                                                                                      while (up.size() >= 2 &&
                                                                                                                                       adj[e.from].push_back(e.to);
                                                                                                                                       adj[e.to].push_back(e.from);
                                                                                                                                                                                                                                                              !cw(up[up.size()-2], up[up.size()-1], a[i]))
                                                                                                                                       cost[e.from][e.to] = e.cost;
                                                                                                                                                                                                                                                                              up.pop_back();
               // If there is no augmenting path beginning with u theost[e.to][e.from] = -e.cost;
                                                                                                                                                                                                                                                                      up.push_back(a[i]);
```

```
capacity[e.from][e.to] = e.capacity;
          f = min(f, capacity[p[cur]][cur]);
   return a.x < b.x \mid | (a.x == b.x && a.y < b.y);
 return a.x*(b.y-c.y)+b.x*(c.y-a.y)+c.x*(a.y-b.y)>0;
vector<pair<double,pair<double,double> > > pp;
```

```
if (i == a.size() - 1 || ccw(p1, a[i], p2)) {
           while(down.size() >= 2 &&!
ccw(down[down.size()-2], down[down.size()-1], a[i]))
              down.pop_back();
           down.push_back(a[i]);
   }
   a.clear():
for(int i=down.size()-2;i>0;i--)a.push_back(down[i])
```

```
Half Plane Intersection
#define MAX 200005
#define MOD 1009
#define SMOD 998244353
#define ROOT 318
#define GMAX 19
#define EPS 0.00000001
#define NIL 0
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
#include <ext/pb_ds/detail/standard_policies.hpp>
class HalfPlaneIntersection{
   static double eps, inf;
public:
   struct Point{
       double x, y;
explicit Point(double x=0,double y=0):x(x), y(y) {}
friend Point operator+(const Point&p,const Point&g){
   return Point(p.x + q.x, p.y + q.y);
friend Point operator-(const Point&p,const Point&q){
   return Point(p.x - q.x, p.y - q.y);
friend Point operator*(const Point&p,double&k){
   return Point(p.x * k, p.y * k);
friend double cross(const Point& p, const Point& q){
   return p.x * q.y - p.y * q.x;
  Basic half-plane struct.
   struct Halfplane
   {// 'p' is a passing point of the line and
       //'pq' is the direction vector of the line.
       Point p, pq; double angle;
       Halfplane() {}
       Halfplane(const Point&a, const Point&b):p(a)
       ,pq(\bar{b}-a){
           angle = atan21(pq.y, pq.x);
// Check if point 'r' is outside this half-plane.
// Every half-plane allows the region to the LEFT
// of its line.
       bool out(const Point& r){
          return cross(pq, r - p) < -eps;</pre>
//Comparator for sorting. If the angle of both half-
//planes is equal, the leftmost one should go first.
```

```
bool operator < (const Halfplane& e) const{</pre>
                                                                  if (fabsl(angle - e.angle) < eps)</pre>
                                                                      return cross(pq, e.p - p) < 0;
                                                                  return angle < e.angle;
                                                              }// We use equal comparator for std::unique
                                                              //to easily remove parallel half-planes.
                                                              bool operator == (const Halfplane& e) const{
                                                                  return fabsl(angle - e.angle) < eps;</pre>
                                                              }// Intersection point of the lines of two
for(int i=0;i<(int)up.size();i++)a.push_back(up[i]); //half-planes. It is assumed they're never parallel.</pre>
                                                              friend Point inter(const Halfplane& s,
                                                                                const Halfplane& t){
                                                       double alpha=cross((t.p-s.p),t.pq)/cross(s.pq,t.pq);
                                                       return s.p + (s.pq * alpha);
                                                       vector<Point>hp_intersect(vector<Halfplane>&H)
                                                              Point box [4] = // Bounding box in CCW order
                                                                  Point(inf, inf)
                                                                  Point(-inf, inf)
                                                                  Point(-inf, -inf),
                                                                  Point(inf, -inf)
                                                       for(int i=0;i<4;i++){//Add bounding box half-planes.</pre>
                                                                  Halfplane aux(box[i], box[(i+1) \% 4]);
                                                                  H.push_back(aux);
                                                              // Sort and remove duplicates
                                                              sort(H.begin(), H.end());
                                                              H.erase(unique(H.begin(), H.end()), H.end());
                                                              deque < Halfplane > dq;
                                                              int len = 0;
                                                              for(int i = 0; i < int(H.size()); i++){</pre>
                                                       // Remove from the back of the deque while last
                                                       // half-plane is redundant
                                                       while(len>1&&H[i].out(inter(dq[len-1], dq[len-2]))){
                                                                      dq.pop_back();
                                                                      --len;
                                                                  }// Remove from the front of the deque
                                                              //while first half-plane is redundant
                                                           while (len > 1 && H[i].out(inter(dq[0],dq[1]))){
                                                                      dq.pop_front();
--len;
                                                                  } // Add new half-plane
                                                                  dq.push_back(H[i]); ++len;
                                                              } // Final cleanup: Check half-planes at the
                                                              //front against the back and vice-versa
                                                       while(len>2&&dq[0].out(inter(dq[len-1],dq[len-2]))){
                                                                  dq.pop_back(); --len;
                                                       while (len > 2 && dq[len-1].out(inter(dq[0],dq[1]))) {
                                                                  dq.pop_front(); --len;
                                                              // Report empty intersection if necessary
                                                              if (len < 3) return vector<Point>();
                                                       // Reconstruct the convex polygon from
                                                       //the remaining half-planes.
                                                              vector<Point> ret(len);
                                                              for(int i = 0; i+1 < len; i++){
```

```
ret[i] = inter(dq[i], dq[i+1]);
       ret.back() = inter(dq[len-1], dq[0]);
       return ret;
double HalfPlaneIntersection::eps=1e-9;
double HalfPlaneIntersection::inf=1e9;
   vector<HalfPlaneIntersection::Halfplane> V;
    vector<HalfPlaneIntersection::Point> P;
   for(i=0; i<n; i++){
       int c;
       scanf("%d",&c);
       HalfPlaneIntersection::Halfplane h;
       HalfPlaneIntersection::Point p;
       for(j=0; j<c; j++){
    scanf("%lf %lf",&p.x,&p.y);</pre>
           P.push_back(p);
       for(j=0; j<c; j++){</pre>
h=HalfPlaneIntersection::Halfplane(P[j],P[(j+1)%c]);
           V.push_back(h);
       P.clear();
   P=HalfPlaneIntersection::hp_intersect(V);
   double ans=0;
   n=P.size();
   for(i=0; i<n; i++){</pre>
ans=ans+P[i].x*P[(i+1)%n].y-P[i].y*P[(i+1)%n].x;
   ans=ans/2;
```

4.3 Line Segment Intersection

```
struct pt {
   double x, y;
    bool operator<(const pt& p) const
  return x < p.x-EPS \mid | (abs(x-p.x) < EPS & y < p.y-EPS);
struct line {
    double a, b, c;
    line() {}
   line(pt p, pt q){
    a = p.y - q.y;
       b = q.x - p.x;
       c = -a * p.x - b * p.y;
       norm();
   void norm(){
       double z = sqrt(a * a + b * b);
       if (abs(z) > EPS) a /= z, b /= z, c /= z;
    double dist(pt p){ return a * p.x + b * p.y + c;}
double det(double a, double b, double c, double d){
   return a * d - b * c;
inline bool betw(double 1, double r, double x){
   return min(1, r) \leq x + EPS && x \leq max(1,r)+EPS;
bool intersect_1d(double a, double b, dbl c, dbl d)
```

```
if (a > b)
       swap(a, b);
   if (c > d)
       swap(c, d);
   return max(a, c) <= min(b, d) + EPS;</pre>
intersect(pt a,pt b,pt c,pt d,pt& left,pt& right){
   if (!intersect_1d(a.x, b.x, c.x, d.x) ||
     !intersect_1d(a.y, b.y, c.y, d.y))return false;
   line m(a, b); line n(c, d);
   double zn = det(m.a, m.b, n.a, n.b);
   if (abs(zn) < EPS) {</pre>
   if (abs(m.dist(c)) > EPS \mid | abs(n.dist(a)) > EPS)
           return false;
       if (b < a) swap(a, b);
       if (d < c) swap(c, d);
       left = max(a, c); right = min(b, d);
       return true;
left.x = right.x = -det(m.c, m.b, n.c, n.b) / zn;
left.y = right.y = -det(m.a, m.c, n.a, n.c) / zn;
return betw(a.x,b.x,left.x)&&betw(a.y,b.y,left.y) &&
   betw(c.x, d.x, left.x) && betw(c.y, d.y, left.y);
```

4.4 Minimum Perimeter Triangle

```
#define MAX 300005
#define MOD 1000000007
#define SMOD 998244353
#define EPS 0.000000001
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
#include <ext/pb_ds/detail/standard_policies.hpp>
struct pt{
   double x, y;
   int id:
struct cmp_x{
   bool operator()(const pt & a, const pt &b)const{ | struct pt{
       return a.x < b.x | | (a.x == b.x \&\& a.y < b.y);
};
struct cmp_y{
   bool operator()(const pt& a, const pt & b)const{
       return a.y < b.y;</pre>
int n; vector<pt> a; double mindist;
pair<int, pair<int, int> > best_pair;
void upd_ans(const pt & a, const pt &b,const pt&c){
   double distC = sqrt((a.x-b.x)*(a.x-b.x)
            +(a.y-b.y)*(a.y-b.y));
   double distA = sqrt((c.x - b.x)*(c.x - b.x)
            + (c.y - b.y)*(c.y - b.y));
   double distB = sqrt((a.x - c.x)*(a.x - c.x)
            + (a.y - c.y)*(a.y - c.y));
   if (distA + distB + distC < mindist){</pre>
   mindist = distA + distB + distC;
   best_pair=make_pair(a.id,make_pair(b.id,c.id));
```

```
vector<pt> t;
void rec(int 1, int r){
   if (r - 1 \le 3 \&\&r - 1 \ge 2)
       for (int i = 1; i < r; ++i){
          for (int j = i + 1; j < r; ++j){
              for(int k=j+1;k<r;k++){</pre>
                  upd_ans(a[i],a[j],a[k]);
       sort(a.begin() + 1, a.begin() + r, cmp_y());
   int m = (l + r) >> 1; int midx = a[m].x;
   rec(1, m); rec(m, r);
   merge(a.begin() + 1, a.begin() + m, a.begin()
   + m, a.begin() + r, t.begin(), cmp_y());
   copy(t.begin(), t.begin() + r - 1, a.begin()+1); |public:
   int tsz = 0;
   for (int i = 1; i < r; ++i){
       if (abs(a[i].x - midx) < mindist/2){
          for (int j = tsz - 1; j \ge 0
          && a[i].y - t[j].y < mindist/2; --j){
              if(i+1<r) upd_ans(a[i],a[i+1],t[j]);
              if(j>0) upd_ans(a[i], t[j-1], t[j]);
          t[tsz++] = a[i]:
```

Minkowski

```
#define MAX 300005
#define BEGIN 1
#define CHAINS 18
#define NOT_VISITED 0
#define VISTTING 1
#define VISITED 2
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
#include <ext/pb_ds/detail/standard_policies.hpp>
   long long x, y;
   pt() {}
   pt(long long _x, long long _y):x(_x), y(_y) {}
   pt operator+(const pt & p) const{
       return pt(x + p.x, y + p.y);
   pt operator-(const pt & p) const{
       return pt(x - p.x, y - p.y);
   long long cross(const pt & p) const{
       return x * p.y - y^* * p.x;
   long long dot(const pt & p) const{
       return x * p.x + y * p.y;
   long long cross(const pt & a, const pt&b)const{
       return (a - *this).cross(b - *this);
   long long dot(const pt & a, const pt & b) const{
       return (a - *this).dot(b - *this);
```

```
long long sqrLen() const{
       return this->dot(*this);
class pointLocationInPolygon{
   bool lexComp(const pt & 1, const pt & r){
       return 1.x < r.x \mid | (1.x == r.x && 1.y < r.y);
   int sgn(long long val){
       return val > 0 ? 1 : (val == 0 ? 0 : -1);
   vector<pt> seq; int n; pt translate;
   bool pointInTriangle(pt a, pt b, pt c,pt point){
       long long s1 = abs(a.cross(b, c));
       long long s2 = abs(point.cross(a, b)) +
   abs(point.cross(b, c)) + abs(point.cross(c, a));
       return s1 == s2;
   pointLocationInPolygon(){}
   pointLocationInPolygon(vector<pt> & points){
       prepare(points);
   void prepare(vector<pt> & points){
       seq.clear();
       n = points.size();
       int pos = 0;
       for(int i = 1; i < n; i++){
           if(lexComp(points[i], points[pos]))
              pos = i;
       translate.x=points[pos].x;
       translate.y=points[pos].y;
       rotate(points.begin(), points.begin() +
       pos, points.end());
n--;
       seq.resize(n);
       for(int i = 0; i < n; i++)</pre>
           seq[i] = points[i + 1] - points[0];
   bool pointInConvexPolygon(pt point){
       point.x-=translate.x;
       point.y-=translate.y;
       if(seq[0].cross(point) != 0 && sgn(seq[0].
       cross(point))!=sgn(seq[0].cross(seq[n-1])))
           return false;
       if(seq[n-1].cross(point)!=0\&\&sgn(seq[n-1])
       .cross(point))!=sgn(seq[n-1].cross(seq[0])))
           return false;
       if(seq[0].cross(point) == 0)
           return seq[0].sqrLen()>= point.sqrLen();
       int 1 = 0. r = n - 1:
       while (r - 1 > 1)
           int mid = (1 + r)/2; int pos = mid;
           if(seq[pos].cross(point) >= 0)1 = mid;
           else r = mid;
       int pos = 1;
       return pointInTriangle(seq[pos],
       seq[pos+1],pt(0,0),point);
```

```
\~pointLocationInPolygon(){
       seq.clear();
class Minkowski{
   static void reorder_polygon(vector<pt> & P){
       size_t pos = 0;
       for(size_t i = 1; i < P.size(); i++){</pre>
           if(P[i].y < P[pos].y | |
       (P[i].y == P[pos].y && P[i].x < P[pos].x))
              pos = i;
       rotate(P.begin(), P.begin() + pos, P.end());
public:
   vector<pt> minkowski(vector<pt> P,vector<pt>Q){
       // the first vertex must be the lowest
       reorder_polygon(P);
       reorder_polygon(Q);
       // we must ensure cyclic indexing
       P.push_back(P[0]);
       P.push_back(P[1]);
       Q.push_back(Q[0]);
       Q.push_back(Q[1]);
       // main part
       vector<pt> result;
       size_t i = 0, j = 0;
       while(i < P.size() - 2 || j < Q.size() - 2){
           result.push_back(P[i] + Q[j]);
auto cross = (P[i + 1] - P[i]).cross(Q[j+1]-Q[j]);
           if(cross >= 0) ++i;
           if(cross <= 0) ++j;
       return result;
};
```

4.6 Pair of Intersecting Segments

```
#define MAX 100009
#define MAX_NODES 100005
struct pt {
   double x, y;
struct seg {
   pt p, q; int id;
   double get_y(double x) const {
       if (abs(p.x - q.x) < EPS)
           return p.y;
       return p.y+(q.y-p.y)*(x-p.x)/(q.x - p.x);
};
bool intersect1d(double 11, dbl r1, dbl 12, dbl r2){
   if (11 > r1) swap(11, r1);
   if (12 > r2) swap(12, r2);
   return max(11, 12) \leq min(r1, r2) + EPS;
int vec(const pt& a, const pt& b, const pt& c) {
 double s = (b.x-a.x)*(c.y-a.y)-(b.y-a.y)*(c.x-a.x); #define GMAX 19
   return abs(s) < EPS ? 0 : s > 0 ? +1 : -1;
bool intersect(const seg& a, const seg& b){
   return intersect1d(a.p.x,a.q.x,b.p.x,b.q.x) &&
```

```
intersect1d(a.p.y,a.q.y,b.p.y,b.q.y) &&
          vec(a.p,a.q,b.p)*vec(a.p,a.q,b.q) <= 0 &&
          vec(b.\bar{p}, b.\bar{q}, \bar{a}.p)*vec(\bar{b}.p,\bar{b}.q,\bar{a}.q) \le 0;
bool operator<(const seg& a, const seg& b){
    double x=max(min(a.p.x,a.q.x),min(b.p.x,b.q.x));
    return a.get_v(x) < b.get_v(x) - EPS;
struct event {
   double x; int tp, id;
    event() {}
event(double x, int tp, int id):x(x),tp(tp),id(id){}
    bool operator<(const event& e) const {
       if (abs(x - e.x) > EPS) return x < e.x;
       return tp > e.tp;
set<seg> s; vector<set<seg>::iterator> where;
set<seg>::iterator prev(set<seg>::iterator it) {
   return it == s.begin() ? s.end() : --it;
set<seg>::iterator next(set<seg>::iterator it) {
   return ++it:
pair<int, int> solve(const vector<seg>& a) {
    int n = (int)a.size(); vector<event> e;
    for (int i = 0; i < n; ++i) {
   e.push_back(event(min(a[i].p.x,a[i].q.x),+1,i));
    e.push_back(event(max(a[i].p.x,a[i].q.x),-1,i));
    sort(e.begin(), e.end()); s.clear();
    where.resize(a.size());
    for (size_t i = 0; i < e.size(); ++i) {</pre>
       int id = e[i].id;
       if (e[i].tp == +1) {
           set<seg>::iterator nxt=
           s.lower_bound(a[id]),prv=prev(nxt);
           if(nxt!=s.end()&&intersect(*nxt,a[id]))
               return make_pair(nxt->id, id);
           if(prv!=s.end()&&intersect(*prv,a[id]))
               return make_pair(prv->id, id);
           where [id] = s.insert(nxt, a[id]):
           set<seg>::iterator nxt = next(where[id])
           , prv = prev(where[id]);
           if (nxt != s.end() && prv != s.end() &&
           intersect(*nxt, *prv))
               return make_pair(prv->id, nxt->id);
           s.erase(where[id]);
   return make_pair(-1, -1);
```

1.7 Vertical Decomposition

```
#define MAX 300005
#define MOD 1000000007
#define GMAX 19
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
#include <ext/pb_ds/detail/standard_policies.hpp>
typedef double dbl;
```

```
const dbl eps = 1e-9;
inline bool eq(dbl x, dbl y){
   return fabs(x - y) < eps;
inline bool lt(dbl x, dbl y){
   return x < y - eps;
inline bool gt(dbl x, dbl y){
   return x > y + eps;
inline bool le(dbl x, dbl y){
   return x < y + eps;
inline bool ge(dbl x, dbl y){
   return x > y - eps;
struct pt{
   dbl x, y;
   inline pt operator - (const pt & p)const{
       return pt\{x - p.x, y - p.y\};
   inline pt operator + (const pt & p)const{
       return pt\{x + p.x, y + p.y\};
   inline pt operator * (dbl a)const{
       return pt\{x * a, y * a\};
   inline dbl cross(const pt & p)const{
       return x * p.y - y * p.x;
   inline dbl dot(const pt & p)const{
       return x * p.x + \hat{y} * p.\hat{y};
   inline bool operator == (const pt & p)const{
       return eq(x, p.x) && eq(y, p.y);
struct Line{
   pt p[2];
   \overline{\text{Line}}(){} Line(pt a, pt b):p{a, b}{}
   pt vec()const{
       return p[1] - p[0];
   pt& operator [](size_t i){
       return p[i];
inline bool lexComp(const pt & 1, const pt & r){
   if(fabs(1.x - r.x) > eps){
       return 1.x < r.x;
   else return l.y < r.y;</pre>
vector<pt> interSegSeg(Line 11, Line 12){
   if(eq(11.vec().cross(12.vec()), 0)){
       if(!eq(11.vec().cross(12[0] - 11[0]), 0))
           return {};
       if(!lexComp(l1[0], l1[1]))
           swap(11[0], 11[1]);
       if(!lexComp(12[0], 12[1]))
           swap(12[0], 12[1]);
       pt 1 = lexComp(11[0], 12[0]) ? 12[0] : 11[0];
```

```
pt r = lexComp(11[1], 12[1]) ? 11[1] : 12[1];
       if(1 == r)
           return {1};
 return lexComp(1,r)?vector<pt>{1,r}:vector<pt>();
   else{
       dbl s = (12[0] - 11[0]).cross(12.vec()) /
       11.vec().cross(12.vec());
       pt inter = 11[0] + 11.vec() * s;
       if(ge(s, 0) && le(s, 1) && le((12[0] -
       inter).dot(12[1] - inter), 0))
           return {inter};
       else
           return {};
   }
char get_segtype(Line segment,pt other_point){
   if(eq(segment[0].x, segment[1].x))
       return 0;
   if(!lexComp(segment[0], segment[1]))
       swap(segment[0], segment[1]);
   return (segment[1]-segment[0]).cross(other_point
    - segment[0]) > 0 ? 1 : -1;
dbl union_area(vector<tuple<pt, pt,pt> > triangles){
   vector<Line> segments(3 * triangles.size());
   vector<char> segtype(segments.size());
   for(size_t i = 0; i < triangles.size(); i++){</pre>
       pt a, b, c; tie(a, b, c) = triangles[i];
 segments[3*i]=lexComp(a,b)?Line(a,b):Line(b,a);
 segtype[3 * i] = get_segtype(segments[3 * i], c);
 segments[3*i+1]=lexComp(b,c)?Line(b,c):Line(c,b);
 segtype[3*i+1] = get_segtype(segments[3*i+1],a);
 segments [3*i+2] = lexComp(c,a)? Line(c,a): Line(a,c);
 segtype[3*i+2] = get_segtype(segments[3*i+2],b);
 vector<dbl>k(segments.size()),b(segments.size());
   for(size_t i = 0; i < segments.size(); i++){</pre>
       if(segtype[i]){
           k[i] = (segments[i][1].y-segments[i][0].y)
             /(segments[i][1].x-segments[i][0].x);
       b[i]=segments[i][0].y-k[i]*segments[i][0].x;
   dbl ans = 0;
   for(size_t i = 0; i < segments.size(); i++){</pre>
       if(!segtype[i]) continue;
       dbl l = segments[i][0].x,r=segments[i][1].x;
       vector<pair<dbl, int> > evts;
       for(size_t j = 0; j < segments.size(); j++){</pre>
           if(!segtype[j] || i == j)
 dbl 11 = segments[j][0].x, r1 = segments[j][1].x;
           if(ge(l1, r) || ge(l, r1))
              continue;
 dbl common_l = max(l, l1), common_r = min(r, r1);
 auto pts = interSegSeg(segments[i], segments[j]);
           if(pts.empty()){
              dbl yl1 = k[j] * common_l + b[j];
              dbl vl = k[i] * common_l + b[i];
             if(lt(yl1, yl) == (segtype[i] == 1)){
```

```
int evt_type = -segtype[i] * segtype[j];
       evts.emplace_back(common_1, evt_type);
       evts.emplace_back(common_r, -evt_type);
          else if(pts.size() == 1u){
 dbl yl=k[i]*common_l+b[i],yl1=k[j]*common_l+b[j];
 int evt_type = -segtype[i] * segtype[j];
 if(lt(yl1, yl) == (segtype[i] == 1)){
     evts.emplace_back(common_l, evt_type);
     evts.emplace_back(pts[0].x, -evt_type);
 yl = k[i] * common_r + b[i], yl1 = k[j] * common_r + b[j];
 if(lt(yl1, yl) == (segtype[i] == 1)){
     evts.emplace_back(pts[0].x, evt_type);
     evts.emplace_back(common_r, -evt_type);
          else{
              if(segtype[j] != segtype[i] || j>i){
                  evts.emplace_back(common_1, -2);
                  evts.emplace_back(common_r, 2);
       evts.emplace_back(1, 0);
       sort(evts.begin(), evts.end());
       size_t j = 0; int balance = 0;
      while(j < evts.size()){</pre>
          size_t ptr = j;
          while(ptr < evts.size() &&</pre>
          eq(evts[j].first, evts[ptr].first)){
              balance += evts[ptr].second;
              ++ptr;
          if(!balance && !eq(evts[j].first, r)){
dbl next_x = ptr == evts.size() ?r:evts[ptr].first;
              ans -= segtype[i] * (k[i] * (next_x
+evts[j].first)+2*b[i])*(next_x-evts[j].first);
          j = ptr;
   return ans/2;
    common tangent
   double x, y;
   pt operator- (pt p) {
```

```
struct pt {
       pt res = \{x-p.x, y-p.y\};
       return res;
struct circle : pt {
    double r;
struct line {
    double a, b, c;
const double EPS = 1E-9;
double sqr (double a) {
```

```
return a * a;
void tangents (pt c, double r1, double r2,
         vector<line> & ans) {
   double r = r2 - r1;
   double z = sqr(c.x) + sqr(c.y);
   double d = z - sqr(r);
   if (d < -EPS) return;</pre>
   d = sqrt (abs (d));
   line 1; l.a = (c.x * r + c.y * d) / z;
   1.b = (c.v * r - c.x * d) / z; 1.c = r1;
   ans.push_back (1);
vector<line> tangents (circle a, circle b) {
   vector<line> ans;
   for (int i=-1; i<=1; i+=2)
       for (int j=-1; j<=1; j+=2)
           tangents (b-a, a.r*i, b.r*j, ans);
   for (size_t i=0; i<ans.size(); ++i)</pre>
       ans[i].c = ans[i].a * a.x + ans[i].b * a.y;
   return ans;
```

Graph

Articulation Vertex

```
int n; // number of nodes
vector<vector<int>> adj; // adjacency list of graph
vector<bool> visited;
vector<int> tin, low;
int timer;
void dfs(int v, int p = -1) {
   visited[v] = true;
   tin[v] = low[v] = timer++:
   int children=0;
   for (int to : adj[v]) {
       if (to == p) continue;
       if (visited[to]) {
           low[v] = min(low[v], tin[to]);
       } else {
           dfs(to, v);
           low[v] = min(low[v], low[to]);
           if (low[to] >= tin[v] && p!=-1)
              IS_CUTPOINT(v);
           ++children;
   if(p == -1 \&\& children > 1)
       IS_CUTPOINT(v);
void find_cutpoints() {
   timer = 0;
   visited.assign(n, false);
   tin.assign(n, -1);
   low.assign(n, -1);
   for (int i = 0; i < n; ++i) {
       if (!visited[i])
           dfs (i);
```

```
Strongly Connected Components
vector<vector<int>> adj, adj_rev;
vector<bool> used;
vector<int> order, component;
void dfs1(int v) {
   used[v] = true;
   for (auto u : adj[v])
       if (!used[u])
           dfs1(u):
   order.push_back(v);
void dfs2(int v) {
   used[v] = true:
   component.push_back(v);
   for (auto u : adj_rev[v])
       if (!used[u])
           dfs2(u);
int main() {
   int n:
   // ... read n ...
   for (;;) {
       int a, b;
       // ... read next directed edge (a,b) ...
       adj[a].push_back(b);
       adj_rev[b].push_back(a);
   used.assign(n, false);
   for (int i = 0; i < n; i++)
       if (!used[i])
           dfs1(i);
   used.assign(n, false);
   reverse(order.begin(), order.end());
   for (auto v : order)
       if (!used[v]) {
           dfs2 (v);
           // ... processing next component ...
           component.clear();
```

6 Math

6.1 Combinatrics

```
#include<bits/stdc++.h>
using namespace std;
#define MAX 100000
#define MOD 1000000007
long long int fact[MAX+1],fact_inv[MAX+1];
long long int gcd(long long int a,long long int b)
{
   if(b==0) return a;
   else return gcd(b,a%b);
```

```
long long int egcd(long long int a, long long int b,
   if (a == 0) {
       x = 0:
       y = 1;
       return b;
   long long int x1, y1;
   long long int d = egcd(b % a, a, x1, y1);
   x = v1 - (b / a) * x1:
   y = x1;
   return d;
long long int ModuloInverse(long long int a, long long int n)
   long long int x,y;
   x=\gcd(a,n);
   a=a/x;
   n=n/x:
   long long int res = egcd(a,n,x,y);
   x=(x%n+n)%n;
   return x;
void precal()
   fact[0]=fact_inv[0]=1;
   for(i=1:i<=MAX:i++)</pre>
       fact[i]=(fact[i-1]*i)%MOD;
   i=MAX:
   fact_inv[i]=ModuloInverse(fact[i],MOD);
   for(i=MAX-1;i>0;i--)
       fact_inv[i]=(fact_inv[i+1]*(i+1))%MOD;
long long int C(int n,int r)
   long long int res=fact[n];
   res=(res*fact_inv[n-r])%MOD;
   res=(res*fact_inv[r])%MOD;
   return res;
int main()
   precal();
   while(true)
       int n,r;
scanf("%d %d",&n,&r);
       long long int res=C(n,r);
       long long int mod_inv=ModuloInverse(n,MOD);
       printf("%lld %lld\n",res,mod_inv);
   return 0;
```

6.2 Discrete Root

```
#include<bits/stdc++.h>
#include<math.h>
using namespace std;
```

```
#define MAX 100000
longprime [MAX+&] xP,hi [MAX+bhg int & y) {
void sieve()
    int i,j;
    for(i=2; i*i<=MAX; i++)</pre>
       if(prime[i]) continue;
       for(j=i; j*i<=MAX; j++)</pre>
           if(prime[i*j]==0) prime[i*j]=i;
void PhiWithSieve()
    for(i=2; i<=MAX; i++)</pre>
        if(prime[i]==0)
           Phi[i]=i-1:
        else if((i/prime[i])%prime[i]==0)
           Phi[i]=Phi[i/prime[i]]*prime[i];
       else
           Phi[i]=Phi[i/prime[i]]*(prime[i]-1);
int gcd(int a,int b)
    if(b==0) return a;
    else return gcd(b,a%b);
int powmod (int a, int b, int p) {
    int res = 1:
    while (b)
       if (b & 1)
           res = int (res * 111 * a % p), --b;
           a = int (a * 111 * a % p), b >>= 1;
   return res:
int PrimitiveRoot(int p)
    vector<int>fact:
    int phi=Phi[p];
    int n=phi;
    while(n>1)
       if(prime[n]==0)
           fact.push_back(n);
           int f=prime[n];
           while (n\%f==0)
```

```
n=n/f;
           fact.push_back(f);
   }
int res;
   for(res=p-1; res>1; --res)
       for(n=0; n<fact.size(); n++)</pre>
           if(powmod(res,phi/fact[n],p)==1)
               break:
       if(n>=fact.size()) return res;
   return -1;
int DiscreteLog(int a, int b, int m) {
   a %= m, b %= m;
   int n = sqrt(m) + 1;
   map<int, int> vals;
   for (int p = 1; p <= n; ++p)
       vals[powmod(a,(int) (111* p * n) %m , m)] = p;
   for (int q = 0; q \le n; ++q) {
       int cur = (powmod(a, q, m) * 111 * b) % m;
       if (vals.count(cur)) {
           int ans = vals[cur] * n - q;
           return ans;
   return -1;
vector<int> DiscreteRoot(int n,int a,int k)
   int g = PrimitiveRoot(n);
   vector<int> ans;
   int any_ans = DiscreteLog(powmod(g,k,n),a,n);
   if (any_ans == -1)
       return ans;
   int delta = (n-1) / gcd(k, n-1);
   for (int cur = any_ans % delta; cur < n-1; cur +=
       ans.push_back(powmod(g, cur, n));
   sort(ans.begin(), ans.end());
   return ans;
```

Fast Fourier Transform

```
#include <bits/stdc++.h>
using namespace std;
#define MOD 1000000007
#define MAX 200005
#define PMAX 55
#define PRECISION 0.000001
#define INF 200000000
#define FASTIO ios_base::sync_with_stdio(false);cin.tie(gul2)g int gcd(long long int a,long long int b)
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
#include <ext/pb_ds/detail/standard_policies.hpp>
```

```
using cd = complex<double>;
const double PI = acos(-1);
void fft(vector<cd>& a, bool invert)
   int n = a.size();
   for(int i = 1, j = 0; i < n; i++){
       int bit = n > 1;
       for(; j&bit; bit>>=1){
           j^=bit;
       i ^= bit:
       if(i < j)
           swap(a[i], a[j]);
   for(int len = 2; len <= n; len <<= 1){
       double ang = 2*PI/len*(invert ? -1 : 1);
       cd wlen(cos(ang), sin(ang));
       for(int i = 0; i < n; i += len){</pre>
           cd w(1);
           for(int j = 0; j < len/2; j++){
               cd u = a[i+j], v = a[i+j+len/2]*w;
               a[i+j] = u+v;
               a[i+j+len/2] = u-v;
               w \neq wlen;
   }
   if(invert){
       for(cd &x: a)
           x /= n;
vector<int> multiply(vector<int> const& a, vector<int> const&b) swap(a[i], a[j]);
   vector<cd> fa(a.begin(), a.end());
   vector<cd> fb(b.begin(), b.end());
   int n = 1;
   while(n < a.size()+b.size())</pre>
       n <<= 1;
   fa.resize(n):
delfa) resize(n);
(fa, false);
   fft(fb, false);
   for(int i = 0; i < n; i++)</pre>
       fa[i] *= fb[i]:
   fft(fa, true);
   vector<int> result(n);
   for(int i = 0; i < n; i++)</pre>
       result[i] = round(fa[i].real());
   return result;
//Number Theoretic Transformation
   if(b==0) return a;
   else return gcd(b,a%b);
```

```
15
if (a == 0) {
                                                          x = 0:
                                                          y = 1;
                                                          return b;
                                                      long long int x1, y1;
                                                      long long int d = \operatorname{egcd}(b \% a, a, x1, y1);
                                                      x = y1 - (b / a) * x1;
                                                      y = x1;
                                                      return d;
                                                   long long int ModuloInverse(long long int a, long long
                                                      long long int x,y;
                                                      x=\gcd(a,n);
                                                      a=a/x;
                                                      n=n/x:
                                                      long long int res = egcd(a,n,x,y);
                                                      x=(x^n+n)^n;
                                                      return x;
                                                   const int mod = 998244353;
                                                   const int root = 15311432
                                                   const int root_1 = 469870224;
                                                   const int root_pw = 1 << 23;</pre>
                                                   void fft(vector<int> & a, bool invert) {
                                                      int n = a.size():
                                                      for (int i = 1, j = 0; i < n; i++) {
                                                          int bit = n >> 1;
                                                          for (; j & bit; bit >>= 1)
    j ^= bit;
j ^= bit;
                                                          if (i < j)
                                                      for (int len = 2; len <= n; len <<= 1) {
                                                          int wlen = invert ? root_1 : root;
                                                          for (int i = len; i < root_pw; i <<= 1)
                                                             wlen = (int)(1LL * wlen * wlen % mod);
                                                          for (int i = 0; i < n; i += len) {
                                                             int w = 1;
                                                             for (int j = 0; j < len / 2; j++) {
                                                                 int u = a[i+j], v = (int)(1LL * a[i+j+le])
                                                                 a[i+j] = u + v < mod ? u + v : u + v - v
                                                                 a[i+j+len/2] = u - v >= 0 ? u - v : u -
                                                                 w = (int)(1LL * w * wlen % mod);
                                                      if (invert) {
                                                          int n_1 = (int) ModuloInverse(n, mod);
                                                          for (int & x : a)
                                                             x = (int)(1LL * x * n_1 \% mod);
                                                   vector<int> multiply(vector<int> const& a, vector<int>
```

```
vector<int> fa(a.begin(), a.end());
                                                         template<typename T>
                                                                                                            template<typename T>
   vector<int> fb(b.begin(), b.end());
                                                         void mul_slow(vector<T> &a, const vector<T> &b)
                                                                                                            T gcd(const T &a, const T &b) {
   int n = 1;
                                                          vector<T> res(a.size() + b.size() - 1);
   while(n < a.size()+b.size())</pre>
                                                          for(size_t i = 0; i < a.size(); i++) {</pre>
      n <<= 1:
                                                            for(size_t j = 0; j < b.size(); j++) {</pre>
   fa.resize(n);
                                                              res[i + j] += a[i] * b[j];
                                                                                                            template<typename T>
   fb.resize(n);
                                                                                                            T \ nCr(T \ n, int \ r) \ \{ // runs in O(r) \}
   fft(fa, false);
                                                                                                              T res(1):
                                                          \dot{a} = res;
   fft(fb, false);
                                                                                                              for(int i = 0; i < r; i++) {
                                                                                                                res *= (n - T(i));
   for(int i = 0; i < n; i++)
                                                                                                                res /= (i + 1);
      fa[i] = (int) (1LL*fa[i]*fb[i]%mod);
                                                         template<typename T>
                                                                                                              return res:
                                                         void mul(vector<T> &a, const vector<T> &b) {
   fft(fa, true);
                                                          if(min(a.size(), b.size()) < magic) {</pre>
   vector<int> result(n);
                                                            mul_slow(a, b);
                                                                                                            template<int m>
   for(int i = 0; i < n; i++)
                                                            return;
                                                                                                            struct modular {
  int64_t r;
      result[i] = fa[i];
   return result;
                                                          init();
                                                                                                              modular() : r(0) {}
                                                          static const int shift = 15, mask = (1 << shift) -</pre>
                                                          n++;
    Polynomial Algebra
                                                          a.resize(n);
#include <bits/stdc++.h>
                                                          static point A[maxn], B[maxn];
using namespace std;
                                                          static point C[maxn], D[maxn];
namespace algebra {
                                                          for(size_t i = 0; i < n; i++) {</pre>
 const int inf = 1e9;
                                                            A[i] = point(a[i] & mask, a[i] >> shift);
 const int magic = 500; // threshold for sizes to run the nagical sosize()) {
                                                              B[i] = point(b[i] & mask, b[i] >> shift);
 namespace fft {
                                                            } else {
   const int maxn = 1 << 18;
                                                              B[i] = 0;
   typedef double ftype;
   typedef complex<ftype> point;
                                                          fft(A, C, n); fft(B, D, n);
                                                                                                            template<int T>
   point w[maxn];
                                                          for(size_t i = 0; i < n; i++) {</pre>
   const ftype pi = acos(-1);
                                                            point c0 = C[i] + conj(C[(n - i) \% n]);
                                                                                                              return in >> x.r;
   bool initiated = 0;
                                                            point c1 = C[i] - conj(C[(n - i) \% n]);
   void init() {
                                                            point d0 = D[i] + conj(D[(n - i) \% n]);
     if(!initiated) {
                                                            point d1 = D[i] - conj(D[(n - i) \% n]);
                                                                                                            template<typename T>
      for(int i = 1; i < maxn; i *= 2) {</pre>
                                                            A[i] = c0 * d0 - point(0, 1) * c1 * d1;
                                                                                                            struct poly {
        for(int j = 0; j < i; j++) {
                                                            B[i] = c0 * d1 + \bar{d}0 * c1;
                                                                                                              vector<T> a;
          w[i + j] = polar(ftype(1), pi * j / i);
                                                          fft(A, C, n); fft(B, D, n);
                                                          reverse(C + 1, C + n);
      initiated = 1;
                                                          reverse(D + 1, D + n);
                                                                                                                  a.pop_back();
                                                          int t = 4 * n;
                                                          for(size_t i = 0; i < n; i++) {</pre>
   template<typename T>
                                                            int64_t A0 = llround(real(C[i]) / t);
   void fft(T *in, point *out, int n, int k = 1) {
                                                                                                              poly(){}
                                                            T A1 = llround(imag(D[i]) / t);
     if(n == 1) {
                                                                                                              poly(T a0) : a{a0}{normalize();}
                                                            T A2 = 11round(imag(C[i]) / t);
       *out = *in;
                                                            a[i] = A0 + (A1 << shift) + (A2 << 2 * shift);
     } else {
      n /= 2;
                                                          return:
      fft(in, out, n, 2 * k);
      fft(in + k, out + n, n, 2 * k);
      for(int i = 0; i < n; i++) {
                                                                                                                  a[i] += t.a[i];
                                                       template<typename T>
         auto t = out[i + n] * w[i + n];
                                                       T bpow(T x, size_t n) {
         out[i + n] = out[i] - t;
                                                        return n? n % 2 ? x * bpow(x, n - 1) : bpow(x * x, n /normal ize()); return *thi's;
         out[i] += t;
                                                       template<typename T>
                                                       T bpow(T x, size_t n, T m) {
                                                        return n ? n % 2 ? x * bpow(x, n - 1, m) % m : bpow(x * x * gsize(max (a. size()));
```

```
return b == T(0) ? a : gcd(b, a \% b);
  hodular(int64_t rr) : r(rr) {if(abs(r) >= m) r %= :
 modular/iny()/const {return bpow(*this, m - 2);}
  modular operator * (const modular &t) const {retur
 modular operator / (const modular &t) const {retur
 modular operator += (const modular &t) {r += t.r;
 modular operator -= (const modular &t) {r -= t.r;
 modular operator + (const modular &t) const {retur
 modular operator - (const modular &t) const {retur
 modular operator *= (const modular &t) {return *th
 modular operator /= (const modular &t) {return *th
  bool operator == (const modular &t) const {return
  bool operator != (const modular &t) const {return
 operator int64_t() const {return r;}
istream& operator >> (istream &in, modular<T> &x) {
 void normalize() { // get rid of leading zeroes
   while(!a.empty() && a.back() == T(0)) {
 poly(vector<T> t) : a(t){normalize();}
 poly operator += (const poly &t) {
   a.resize(max(a.size(), t.a.size()));
   for(size_t i = 0; i < t.a.size(); i++) {</pre>
 poly operator -= (const poly &t) {
```

```
for(size_t i = 0; i < t.a.size(); i++) {</pre>
                                                        return {res, A};
                                                                                                              T& coef(size_t idx) { // mutable reference at coef
   a[i] = t.a[i];
                                                                                                               return a[idx];
                                                       pair<poly, poly> divmod(const poly &b) const { // returns questiend and (remaindery of t) works { return a =
  normalize();
                                                         if(deg() < b.deg()) {
 return *this;
                                                                                                              bool operator != (const poly &t) const {return a !
                                                          return {poly{0}, *this};
poly operator + (const poly &t) const {return poly(*this) += t;}
                                                                                                              poly deriv() { // calculate derivative
poly operator - (const poly &t) const {return poly(*this)t-d= 7;deg() - b.deg();
                                                                                                                vector<T> res;
                                                         if(min(d, b.deg()) < magic) {</pre>
                                                                                                                for(int i = 1; i <= deg(); i++) {
poly mod_xk(size_t k) const { // get same polynomial mod return divmod_slow(b);
                                                                                                                 res.push_back(T(i) * a[i]);
 k = min(k, a.size());
  return vector<T>(begin(a), begin(a) + k);
                                                        return \{D, *this - D * b\};
poly mul_xk(size_t k) const { // multiply by x^k
                                                                                                              poly integr() { // calculate integral with C = 0
  poly res(*this);
                                                                                                               vector<T> res = {0};
                                                       poly operator / (const poly &t) const {return divmod(t)f.dri(rist;}i = 0; i <= deg(); i++) {
  res.a.insert(begin(res.a), k, 0);
                                                       poly operator % (const poly &t) const {return divmod(t).seesondush_back(a[i] / T(i + 1));
  return res;
                                                       poly operator /= (const poly &t) {return *this = |divmod}(t).first;}
poly div_xk(size_t k) const { // divide by x^k, droppolycopeffatorents (const poly &t) {return *this = divmod(et)\frac{1386}{1386}.}
 k = min(k, a.size());
                                                       poly operator *= (const T &x) {
                                                                                                              size_t leading_xk() const { // Let p(x) = x^k * t(
 return vector<T>(begin(a) + k, end(a));
                                                        for(auto &it: a) {
                                                                                                                if(is_zero()) {
                                                          it *= x;
                                                                                                                 return inf;
poly substr(size_t 1, size_t r) const { // return mod_xk(r).div_xk(1)
                                                        normalize();
 l = min(l, a.size());
                                                                                                                int res = 0;
                                                        return *this;
 r = min(r, a.size());
                                                                                                                while(a[res] == T(0)) {
 return vector<T>(begin(a) + 1, begin(a) + r);
                                                                                                                 res++;
                                                       poly operator /= (const T &x) {
poly inv(size_t n) const { // get inverse series | mod x fpr(auto &it: a) {
                                                                                                                return res;
                                                          it /= x;
  assert(!is_zero());
  poly ans = a[0].inv();
                                                                                                              poly log(size_t n) { // calculate log p(x) mod x^n
                                                        normalize();
  size_t a = 1;
                                                                                                                assert(a[0] == T(1));
                                                        return *this:
  while(a < n) {
                                                                                                                return (deriv().mod_xk(n) * inv(n)).integr().mod
   poly C = (ans * mod_xk(2 * a)).substr(a, 2 * a);
                                                      poly operator * (const T &x) const {return poly(*this) *= x;}
poly operator / (const T &x) const {return poly(*this) oly exp(size_t n) { // calculate exp p(x) mod x^n
if(is_zero()) {
   ans -= (ans * C).mod_xk(a).mul_xk(a);
   a *= 2;
                                                                                                                 return T(1);
                                                       void print() const {
 return ans.mod_xk(n);
                                                        for(auto it: a) {
                                                                                                                assert(a[0] == T(0));
                                                           cout << it << '''
                                                                                                                poly ans = T(1);
poly operator *= (const poly &t) {fft::mul(a, t.a); normalize(); return *this;}
                                                                                                                size_t a = 1;
poly operator * (const poly &t) const {return poly(*this9ut*=<\f.\frac{endl}{endl};
                                                                                                                while(a < n)  {
                                                                                                                 poly C = ans.log(2 * a).div_xk(a) - substr(a, 2)
poly reverse(size_t n, bool rev = 0) const { // reverseevah(Tlxhvesnshlf n/ternsuates in single point |x
                                                                                                                 ans -= (ans * C).mod_xk(a).mul_xk(a);
  poly res(*this);
                                                        T res(0);
                                                                                                                 a *= 2:
                                                        for(int i = int(a.size()) - 1; i >= 0; i--) {
  if(rev) { // If rev = 1 then tail goes to head
                                                          res *= x;
   res.a.resize(max(n, res.a.size()));
                                                          res += a[i];
                                                                                                                return ans.mod_xk(n);
  std::reverse(res.a.begin(), res.a.end());
                                                         return res;
  return res.mod_xk(n);
                                                                                                              poly pow_slow(size_t k, size_t n) { // if k is sma
                                                                                                                return k ? k % 2 ? (*this * pow_slow(k - 1, n)).
                                                   { /T& lead() { // leading coefficient freturn a.back(); quotient is small
pair<poly, poly> divmod_slow(const poly &b) const
                                                                                                              poly pow(size_t k, size_t n) { // calculate p^k(n)
  vector<T> A(a);
                                                                                                                if(is_zero()) {
  vector<T> res
                                                       int deg() const { // degree
                                                                                                                 return *this;
  while(A.size() >= b.a.size()) {
                                                        return a.empty() ? -inf : a.size() - 1;
   res.push_back(A.back() / b.a.back());
                                                                                                                if(k < magic) {</pre>
    if(res.back() != T(0)) {
                                                       bool is_zero() const { // is polynomial zero
                                                                                                                 return pow_slow(k, n);
     for(size_t i = 0; i < b.a.size(); i++) {</pre>
       A[A.size() - i - 1] -= res.back() * b.a[b|a.size()turn_i a.emp;ty();
                                                                                                               int i = leading_xk();
                                                       T operator [](int idx) const {
                                                                                                                 i = a[i];
                                                                                                          a[idx]oiy t = div_xk(i) / j;
                                                        return idx >= (int)a.size() || idx < 0 ? T(0)
   A.pop_back();
                                                                                                                return bpow(j, k) * (t.log(n) * T(k)).exp(n).mul
  std::reverse(begin(res), end(res));
```

```
poly mulx(T x) { // component-wise multiplication with x auto B = (*this % tree[2 * v + 1]).eval(tree, 2 * if(R +, Im;=r)) {
   T cur = 1;
                                                                                      A.insert(end(A), begin(B), end(B));
                                                                                                                                                                   return res[v] = vector<T>{-*L, 1};
   poly res(*this);
                                                                                                                                                                 } else {
                                                                                      return A;
   for(int i = 0; i <= deg(); i++) {
                                                                                                                                                                    iter M = L + (R - L) / 2;
                                                                                                                                                                    return res[v] = build(res, 2 * v, L, M) * build()
     res.coef(i) *= cur;
                                                                                vector<T> eval(vector<T> x) { // evaluate polynomial in (x1, ..., xn)
     cur *= x;
                                                                                   int n = x.size();
   return res;
                                                                                                                                                              template<typename T>
                                                                                   if(is_zero()) {
poly mulx_sq(T x) { // component-wise multiplication with x^{k^2}
                                                                                                                                                              poly<T> inter(vector<T> x, vector<T> y) { // interpo
                                                                                                                                                                 int n = x.size();
                                                                                   vector<poly> tree(4 * n);
                                                                                                                                                                 vector<poly<T>> tree(4 * n);
   T total = 1;
                                                                                                                                                                 return build(tree, 1, begin(x), end(x)).deriv().in
                                                                                   build(tree, 1, begin(x), end(x));
   T xx = x * x;
                                                                                   return eval(tree, 1, begin(x), end(x));
   poly res(*this);
   for(int i = 0; i <= deg(); i++) {
                                                                                template<typename iter>
     res.coef(i) *= total;
                                                                                poly inter(vector<poly> &tree, int v, iter 1, itersing the first algebra // auxiliary interpolation fu
     total *= cur;
                                                                                   if(r - 1 == 1) {
     cur *= xx;
                                                                                                                                                            const int mod = 1e9 + 7;
                                                                                      return {*ly / a[0]};
                                                                                                                                                            typedef modular<mod> base;
  return res;
                                                                                   } else {
                                                                                                                                                            typedef poly<br/>base> polyn;
                                                                                      auto m = 1 + (r - 1) / 2:
                                                                                                                                                            using namespace algebra;
vector<T> chirpz_even(T z, int n) { // P(1), P(z^2), P(z^2), P(z^2), my = P(z^2) (myl)) / 2;
   int m = deg();
                                                                                      auto A = (*this % tree[2 * v]).inter(tree, 2
                                                                                                                                                           signed fibity, my);
10s:"sync, With still (V));;
                                                                                      auto B = (*this % tree[2 * v + 1]).inter(tree,
   if(is_zero()) {
     return vector<T>(n, 0);
                                                                                      return A * tree[2 * v + 1] + B * tree[2 * v];
                                                                                                                                                              cin.tie(0);
                                                                                                                                                              int n = 100000;
   vector < T > vv(m + n);
                                                                                                                                                              polyn a;
   T zi = z.inv();
                                                                                                                                                              vector<base> x;
   T zz = zi * zi;
                                                                              témplate<typename T>
                                                                                                                                                              for(int i = 0; i <= n; i++) {
   T cur = zi;
                                                                              poly<T> operator * (const T& a, const poly<T>& b)
                                                                                                                                                                 a.a.push_back(1 + rand() \% 100);
   T total = 1;
                                                                                return b * a;
                                                                                                                                                                 x.push_back(1 + rand() \% (2 * n));
   for(int i = 0; i \le max(n - 1, m); i++) {
     if(i <= m) {vv[m - i] = total;}</pre>
                                                                                                                                                              sort(begin(x), end(x));
     if(i < n) {vv[m + i] = total;}</pre>
                                                                              template<typename T>
                                                                                                                                                              x.erase(unique(begin(x), end(x)), end(x));
     total *= cur;
                                                                              poly<T> xk(int k) { // return x^k
                                                                                                                                                              auto b = a.eval(x);
     cur *= zz;
                                                                                return poly<T>{1}.mul_xk(k);
                                                                                                                                                              cout << clock() / double(CLOCKS_PER_SEC) << endl;</pre>
   poly w = (mulx_sq(z) * vv).substr(m, m + n).mulk_sq(z);
                                                                                                                                                              auto c = inter(x, b);
   vector<T> res(n);
                                                                              template<typename T>
                                                                                                                                                              polyn md = kmul(begin(x), end(x));
   for(int i = 0; i < n; i++) {
                                                                             T resultant(poly<T> a, poly<T> b) { // computes resultant($\delta\text{clank}()\) / double(CLOCKS_PER_SEC) << endl;
     res[i] = w[i];
                                                                                if(b.is_zero()) {
                                                                                                                                                              assert(c == a % md):
                                                                                   return 0;
                                                                                                                                                              return 0;
  return res:
                                                                                } else if(b.deg() == 0) {
                                                                                   return bpow(b.lead(), a.deg());
vector<T> chirpz(T z, int n) { // P(1), P(z), P(z^2), elseP(z^4(n-1))
                                                                                                                                                           6.5 all comb
   auto even = chirpz_{even}(z, (n + 1) / 2);
                                                                                   int pw = a.deg();
                                                                                                                                                            vector<int> ans;
   auto odd = mulx(z).chirpz_even(z, n / 2);
                                                                                   a %= b;
   vector<T> ans(n);
                                                                                                                                                           void gen(int n, int k, int idx, bool rev) {
                                                                                   pw -= a.deg();
                                                                                   T \text{ mul} = bpow(b.lead(), pw) * T((b.deg() & a.deg() & if)(k > 1n: | 1)k < 0) return;
   for(int i = 0; i < n / 2; i++) {
                                                                                                                                                                 if (!n) {
     ans[2 * i] = even[i];
                                                                                   T ans = resultant(b, a);
     ans [2 * i + 1] = odd[i]:
                                                                                   return ans * mul;
                                                                                                                                                                      for (int i = 0; i < idx; ++i) {</pre>
                                                                                                                                                                            if (ans[i])
   if(n % 2 == 1) {
                                                                                                                                                                                 cout << i + 1;
     ans[n-1] = even.back();
                                                                              template<typename iter>
                                                                             poly<typename iter::value_type> kmul(iter L, iter R) { // contutés "(x=a1)(x-a2)...(x-an) without building return:
  return ans;
                                                                                if(R - L == 1)  {
                                                                                   return vector<typename iter::value_type>{-*L, 1};
                                                                                                                                                                 ans[idx] = rev;
template<typename iter>
vector<T> eval(vector<poly> &tree, int v, iter 1,
                                                                                                                                                                 gen(n-1, k-rev, idx + 1, false);
                                                                            iter real M/= auxil/pary revaluation function
                                                                                                                                                                 ans[idx] = !rev;
  if(r - 1 == 1) {
                                                                                   return kmul(L, M) * kmul(M, R);
                                                                                                                                                                 gen(n-1, k-!rev, idx + 1, true);
     return {eval(*1)};
   } else {
                                                                                                                                                           void all_combinations(int n, int k) {
     auto m = 1 + (r - 1) / 2;
                                                                              template<typename T, typename iter>
     auto A = (*this % tree[2 * v]).eval(tree, 2 *
                                                                           V poly (T) build (vector < poly < T>> & res, int v, iter L, iter and restant the end of the control of the con
                                                                                                                                                                 gen(n, k, 0, false);
```

6.6 gauss elimination /* n rows/equations, m+1 columns, m variables calculates determinant, rank and ans[] ->value for variables returns {0, 1, INF} -> number of solutions */ const double EPS = 1e-9; #define MAX 105 #define INF 10000000 int where[MAX], Rank; double Det; int gauss(double a[MAX][MAX], double ans[MAX], int n, int m) { Det = 1.0, Rank = 0; memset(where, -1, sizeof(where)); for(int col=0,row = 0;col<m&&row < n; ++col) { int sel = row; for(int i = row+1; i < n; ++i)</pre> if(fabs(a[i][col])>fabs(a[sel][col])) sel=i; if(fabs(a[sel][col]) < EPS) {Det=0.0; continue;}</pre> for(int j=0;j<=m;++j)swap(a[sel][j],a[row][j]);</pre> if(row != sel) Det = -Det; Det *= a[row][col]; where[col] = row; double s = (1.0 / a[row][col]);for(int j = 0; j <= m; ++j) a[row][j] *= s;</pre> for(int i = 0; i < n; ++i) if (i != row && fabs(a[i][col]) > EPS) { double t = a[i][col]; for(int j = 0; j <= m; ++j) a[i][j] -= a[row][j] * t;++row, ++Rank; for(int i = 0; i < m; ++i) ans[i] = (where[i] == -1) ?0.0:a[where[i]][m];for(int i = Rank; i < n; ++i)</pre> if(fabs(a[i][m]) > EPS) return 0; for(int i = 0; i < m; ++i)</pre> if(where[i] == -1) return INF; // calculates gauss modulo a prime long long Det; long long bigmod(long long x, long long pow, long long mod) { long long ret = 1: while(pow > 0) { if(pow & 1) ret = (ret * x) % mod; x = (x * x) % mod;pow >>= 1; return ret; #define INVERSE(a, m) bigmod(a, m-2, m) int gauss(long long a[MAX][MAX], long long ans[MAX],int n,int m,long long mod){ Det = 1, Rank = 0; memset(where, -1, sizeof(where)); for(int col = 0, row = 0; col<m&&row<n;++col){ int sel = row; for(int i = row+1; i < n; ++i)</pre>

```
if(abs(a[i][col]) > abs(a[sel][col])) sel=i;
if(!a[sel][col]) { Det = 0; continue; }
  for(int j=0;j<=m;++j) swap(a[sel][j],a[row][j]);</pre>
  if(row != sel) Det = -Det;
  Det = (Det * a[row][col]) % mod;
  where[col] = row;
                       // inverse of a[row][col]
  long long s = INVERSE(a[row][col], mod);
 for (int j = 0; j \le m; ++j)
   a[row][j] = (a[row][j] * s) % mod;
  for(int i = 0; i < n; ++i) if (i != row &&
                                  a[i][col] > 0) {
     long long t = a[i][col];
for(int j = 0; j <= m; ++j) a[i][j] =</pre>
      (a[i][j] - (a[row][j]*t)^{mod} + mod)^{mod};
  ++row, ++Rank;
for(int i = 0; i < m; ++i)
 ans[i] = (where[i] == -1) ? 0 : a[where[i]][m];
 for(int i = Rank; i < n; ++i)
 if(a[i][m]) return 0;
for(int i = 0; i < m; ++i)
  if(where[i] == -1) return INF;</pre>
return 1;
// calculates 32 times faster for modulo 2
int Det; // number of variables (must be defined)
int gauss(vector < bitset<MAX> > &a,
          bitset<MAX> &ans, int n, int m) {
  Det = 1, Rank = 0;
  memset(where, -1, sizeof(where));
   for(int col=0,row=0; col < m && row < n;++col){</pre>
      int sel = row;
      for(int i = row; i < n; ++i)</pre>
      if(a[i][col]) { sel = i; break; }
if(!a[sel][col]) { Det = 0; continue; }
      swap(a[sel], a[row]);
      if(row != sel) Det = -Det;
      Det &= a[row][col];
      where[col] = row;
      for(int i = 0; i < n; ++i)
        if (i != row&&a[i][col] > 0)a[i]^=a[row];
      ++row, ++Rank;
  for(int i = 0; i < m; ++i)
  ans[i] = (where[i] == -1)?0:a[where[i]][m];</pre>
   for(int i = Rank;i<n;++i)if(a[i][m]) return 0;</pre>
  for(int i = 0; i < m; ++i)</pre>
   if(where[i] == -1) return INF;
  return 1;
6.7 linear sieve
vector<int> lp(N+1);
vector<int> pr;
for (int i=2; i <= N; ++i) {
    if (lp[i] == 0) {
       lp[i] = i;pr.push_back(i);}
for (int j=0; j < (int)pr.size()</pre>
&& pr[j] <= lp[i] && i*pr[j] <= N; ++j)
    lp[i * pr[j]] = pr[j];
```

```
String
7.1 Aho Corasick
const int K = 26:
struct Vertex {
   int next[K];
   bool leaf = false;
   int p = -1;
   char pch;
   int link' = -1;
   int go[K];
   Vertex(int p=-1, char ch='$') : p(p), pch(ch) {
       fill(begin(next), end(next), -1);
       fill(begin(go), end(go), -1);
vector<Vertex> t(1);
void add_string(string const& s) {
   int v = 0;
   for (char ch : s) {
       int c = ch - 'a';
       if (t[v].next[c] == -1) {
          t[v].next[c] = t.size();
          t.emplace_back(v, ch);
       v = t[v].next[c];
   t[v].leaf = true:
int go(int v, char ch);
int get_link(int v) {
   if (t[v].link == -1) {
       if (v == 0 || t[v].p == 0)
          t[v].link = 0;
          t[v].link = go(get_link(t[v].p), t[v].pch);
   return t[v].link;
int go(int v, char ch) {
   int c = ch - 'a';
   if (t[v].go[c] == -1) {
       if (t[v].next[c] != -1)
          t[v].go[c] = t[v].next[c];
          t[v].go[c] = v == 0 ? 0 : go(get_link(v), c)
   return t[v].go[c];
    Manacher's Algorithm
#include<bits/stdc++.h>
```

#include<vector>

int T,1;

int main()

using namespace std;

if (pn[i] < 0)

```
char s[MAX];
                                                                         pn[i] += n;
   gets(s);
   int n=strlen(s);
                                                                 fill(cnt.begin(), cnt.begin() + classes, 0);
   vector<int> d1(n);
                                                                 for (int i = 0; i < n; i++)
   for (int i = 0, l = 0, r = -1; i < n; i++)
                                                                     cnt[c[pn[i]]]++;
                                                                for (int i = 1; i < classes; i++)</pre>
       int k = (i > r) ? 1 : min(d1[1 + r - i], r - [i + 1);
                                                                    cnt[i] += cnt[i-1];
       while (0 \le i - k \&\& i + k \le n \&\& s[i - k] == s[i + k]dr (int i = n-1; i >= 0; i--)
                                                                    p[--cnt[c[pn[i]]]] = pn[i];
           k++;
                                                                 cn[\bar{p}[0]] = 0;
                                                                 classes = 1;
       d1[i] = k--;
                                                                 for (int i = 1; i < n; i++) {
       if(i + k > r)
                                                                    int ind=p[i] + (1 << h);</pre>
                                                                    if(ind>=n) ind=ind-n;
           l = i - k;
                                                                    pair<int, int> cur = {c[p[i]], c[ind]};
           r = i + k:
                                                                     ind=p[i-1] + (1 << h);
                                                                     if(ind>=n) ind=ind-n;
                                                                    pair<int, int> prev = {c[p[i-1]], c[ind]};#define VISITED 2
   vector<int> d2(n);
                                                                    if (cur != prev)
   for (int i = 0, l = 0, r = -1; i < n; i++)
                                                                         ++classes;
       int k = (i > r) ? 0 : min(d2[1 + r - i + 1], r - i + 1); cn[p[i]] = classes - 1; while (0 <= i - k - 1 && i + k < n && s[i - k] - 1] == \frac{1}{2}s[i + k];
           k++;
       d2[i] = k--;
                                                         vector<int> suffix_array_construction(char *s) {
       if (i + k > r)
                                                             int n=strlen(s);
           1 = i - k - 1;
           r = i + k;
                                                             vector<int> sorted_shifts = sort_cyclic_shifts(s);
                                                             sorted_shifts.erase(sorted_shifts.begin());
                                                             return sorted_shifts;
   return 0;
                                                         vector<int> lcp_construction(char *s, vector<int> const& p)
                                                             int n = strlen(s);
                                                             vector<int> rank(n, 0);
7.3 Suffix Array
                                                             for (int i = 0; i < n; i++)</pre>
#include<bits/stdc++.h>
                                                                rank[p[i]] = i;
#include<string.h>
                                                             int k = 0;
using namespace std;
                                                             vector<int> lcp(n-1, 0);
#define MAX 100000
                                                             for (int i = 0; i < n; i++) {
vector<int> sort_cyclic_shifts(char *s) {
                                                                 if (rank[i] == n - 1) {
   int n = strlen(s);
                                                                    k = 0:
   const int alphabet = 256;
                                                                     continue;
   vector<int> p(n), c(n), cnt(max(alphabet, n), 0)
   for (int i = 0; i < n; i++)
                                                                 int j = p[rank[i] + 1];
       cnt[s[i]]++;
                                                                while (i + k < n \&\& j + k < n \&\& s[i+k] == s[j+k]) roid process(int node)
   for (int i = 1; i < alphabet; i++)</pre>
                                                                    k++:
       cnt[i] += cnt[i-1];
                                                                lcp[rank[i]] = k;
   for (int i = 0; i < n; i++)
                                                                 if (k)
       p[--cnt[s[i]]] = i;
                                                                    k--:
   c[p[0]] = 0;
   int classes = 1;
                                                             return lcp;
   for (int i = 1; i < n; i++) {
       if (s[p[i]] != s[p[i-1]])
                                                         int lcp(int i, int j) {
           classes++;
                                                             int ans = 0;
       c[p[i]] = classes - 1;
                                                             for (int k = log_n; k \ge 0; k--) {
                                                                if (c[k][i] == c[k][j]) {
    ans += 1 << k;
   vector<int> pn(n), cn(n);
   for (int h = 0; (1 << h) < n; ++h) {
                                                                    i += 1 << k;
j += 1 << k;
       for (int i = 0; i < n; i++) {
           pn[i] = p[i] - (1 << h);
```

```
return ans;
7.4 Suffix Automaton
#include <bits/stdc++.h>
#include <vector>
#include<math.h>
#include<string.h>
using namespace std;
#define MAX 300005
#define BEGIN 1
#define MOD 1000000007
#define EPS 0.0000000001
#define CHAINS 18
#define NOT_VISITED 0
#define VISITING 1
#define FASTIO ios_base::sync_with_stdio(false);cin.ti
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
#include <ext/pb_ds/detail/standard_policies.hpp>
class SuffixAutomaton
    bool complete;
    int last
    set<char> alphabet;
    struct state
       int len, link, endpos, first_pos, shortest_non_a
       long long substrings,length_of_substrings;
       bool is_clone;
       map<char, int> next;
       vector<int> inv_link;
       state(int leng=0,int li=0)
           len=leng;
           link=li;
           first_pos=-1;
           substrings=0;
           length_of_substrings=0;
           endpos=1;
           shortest_non_appearing_string=0;
           is_clone=false;
           height=0;
    véctor<state> st;
       map<char, int> ::iterator mit;
       st[node].substrings=1;
       st[node].shortest_non_appearing_string=st.size
       if((int) st[node].next.size()<(int) alphabet.st</pre>
       for(mit=st[node].next.begin(); mit!=st[node].ne
           if(st[mit->second].substrings==0) process(m
           st[node].height=max(st[node].height,1+st[mi
           st[node].substrings=st[node].substrings+st[
```

st[node].length_of_substrings=st[node].leng

st[node].shortest_non_appearing_string=min(

if(st[node].link!=-1)

```
//printf("Set link %d -> %d\n",cur,0);
       st[st[node].link].inv_link.push_back(node)
                                                                  return:
                                                              else if (st[node].shortest_non_appearing_string==1 eds [mit->second].shortest_non_appearing_string)
void set_suffix_links(int node)
                                                                                                                     int q = st[p].next[c];
                                                                  str.push_back(*sit);
                                                                                                                     if (st[p].len + 1 == st[q].len)
                                                                  minimum_non_existing_string(mit->second,str);
   for(i=0; i<st[node].inv_link.size(); i++)</pre>
                                                                                                                         st[cur].link = q;
       set_suffix_links(st[node].inv_link[i]);
                                                                                                                         //printf("Set link %d -> %d\n",cur,q);
       st[node].endpos=st[node].endpos+st[st[node].inv_link[i]].endpos;
                                                       void find_substrings(int node,int index,vector<char> &str,veeter<pair<long long,long long> > &sub_info)
void output_all_occurrences(int v, int P_length, vector<instab_&invaso.push_back(make_pair(st[node].substrites,st[node].lengthcoloredestingzes) [node].substrings*
                                                                                                                         //printf("Create clone node %d from %d\r
                                                          if(index==str.size()) return;
                                                          if(st[node].next.count(str[index]))
   if (!st[v].is_clone)
                                                                                                                         //printf("Set link %d -> %d\n",clone,st
       pos.push_back(st[v].first_pos - P_length +
                                                   1);
                                                                                                                         st.push_back(state(st[p].len + 1,st[q]...
                                                              find_substrings(st[node].next[str[index]],index+1,str,substrings(st[node].next = st[q].next;
   for (int u : st[v].inv_link)
       output_all_occurrences(u, P_length, pos);
                                                                                                                         st[clone].is_clone=true;
                                                                                                                         st[clone].endpos=0;
                                                          else
void kth_smallest(int node,int k,vector<char> &str)
                                                                                                                         st[clone].first_pos=st[q].first_pos;
                                                                                                                         while (p != -1 \&\& st[p].next[c] == q)
                                                              sub_info.push_back(make_pair(0,0));
   if(k==0) return;
   map<char, int> ::iterator mit;
                                                                                                                             //printf("Change transition %d -> %d
   for(mit=st[node].next.begin(); mit!=st[node].hext/end(); ++mit)
                                                                                                                             st[p].next[c] = clone;
                                                                                                                             p = st[p].link;
       if(st[mit->second].substrings<k) k=k-st[mit->second].substrings;
                                                          if(!complete)
                                                                                                                         //printf("Change link %d -> %d : %d -> %
                                                                                                                         //printf("Set link %d -> %d\n",cur,clone
           str.push_back(mit->first);
                                                              process(0);
                                                                                                                         st[q].link = st[cur].link = clone;
                                                              set_suffix_links(0);
           kth_smallest(mit->second,k-1,str);
                                                              compléte=true;
                                                                                                                 last = cur;
                                                                                                                 complete=false;
int find_occurrence_index(int node,int index,vectoreist) &str)
                                                                                                               SuffixAutomaton()
                                                       SuffixAutomaton(set<char> &alpha)
   if(index==str.size()) return node;
                                                                                                                 int i;
                                                           st.push_back(state(0,-1));
   if(!st[node].next.count(str[index])) return -1;
                                                                                                                 for(i=0; i<st.size(); i++)</pre>
   else return find_occurrence_index(st[node].next[str[ihdext]];index+1,str);
                                                           complete=false;
                                                                                                                     st[i].next.clear();
                                                           set<char>::iterator sit;
void klen_smallest(int node,int k,vector<char> &str)
                                                                                                                     st[i].inv_link.clear();
                                                           for(sit=alpha.begin(); sit!=alpha.end(); sit++)
   if(k==0) return;
                                                                                                                 st.clear();
                                                              alphabet.insert(*sit);
   map<char, int> ::iterator mit;
                                                                                                                 alphabet.clear();
   for(mit=st[node].next.begin(); mit!=st[node].hext.end(); ++mit)
                                                           st[0].endpos=0;
                                                                                                              void kth_smallest(int k,vector<char> &str)
       if(st[mit->second].height>=k-1)
                                                       void sa_extend(char c)
                                                                                                                  check();
           str.push_back(mit->first);
                                                                                                                 kth_smallest(0,k,str);
                                                           int cur = st.size();
           klen_smallest(mit->second,k-1,str);
                                                           //printf("New node (%d,%c)\n",cur,c);
           return;
                                                                                                              int FindFirstOccurrenceIndex(vector<char> &str)
                                                          st.push_back(state(st[last].len + 1));
                                                          st[cur].first_pos=st[cur].len-1;
                                                                                                                 check();
                                                          int p = last;
                                                                                                                 int ind=find_occurrence_index(0,0,str);
void minimum_non_existing_string(int node, vector \( \frac{char}{char} \) & \( \frac{string}{le} \) (p != -1 && !st[p] .next.count(c))
                                                                                                                 if(ind==0) return -1;
                                                                                                                 else if(ind==-1) return st.size();
   map<char, int> ::iterator mit;
                                                              st[p].next[c] = cur;
                                                                                                                 else return st[ind].first_pos+1-(int) str.size
   set<char>::iterator sit;
                                                              //printf("Set_edge %d -> %d (%c)\n",p,cur,c); \
   for(mit=st[node].next.begin(),sit=alphabet.begin(); sit!=alphabet.end(); ++sit,++mit)
                                                                                                              void FindAllOccurrenceIndex(vector<char> &str,vect
       if(mit==st[node].next.end()||mit->first!=(*sit)) if (p == -1)
                                                                                                                 int ind=find_occurrence_index(0,0,str);
           str.push_back(*sit);
                                                              st[cur].link = 0;
```

```
if(ind!=-1) output_all_occurrences(ind,str.size(),pos);
                                                                   len++;
                                                                                                            vector<char> X;
                                                                   j=st[j].next[str[i%n]];
                                                                                                            int main()
int Occurrences(vector<char> &str)
                                                               else
                                                                                                               int i;
                                                                                                               set<char> alpha;
    check();
                                                                   while (j!=-1\&\&(!st[j].next.count(str[i])) for (i=0; i<26; i++)
   int ind=find_occurrence_index(0,0,str);
   if(ind==0) return 1;
                                                                                                                   alpha.insert('a'+i);
   else if(ind==-1) return 0;
                                                                       j=st[j].link;
   else return st[ind].endpos;
                                                                                                               SuffixAutomaton sa(alpha);
                                                                   if(j!=-1)
                                                                                                               char c;
void klen_smallest(int k,vector<char> &str)
                                                                                                               for(i=0;; i++)
                                                                       len=st[i].len+1;
   check():
                                                                       j=st[j].next[str[i%n]];
                                                                                                                   scanf("%c",&c);
   if(st[0].height>=k) klen_smallest(0,k,str);
                                                                                                                   if(!('a'<=c&&c<='z')) break;
                                                                   else
                                                                                                                   sa.sa_extend(c);
void minimum_non_existing_string(vector<char> &str)
                                                                       len=0;
                                                                                                               int n,j;
scanf("%d ",&n);
                                                                       j=0;
   int ind=find_occurrence_index(0,0,str);
                                                                                                               for(j=0; j<n; j++)
   if(ind!=-1) minimum_non_existing_string(ind,str);
                                                               while(st[j].link!=-1&&st[st[j].link].len>+n) {
                                                                                                                   for(i=0;; i++)
long long cyclic_occurrence(vector<char> &str)
                                                                    j=st[j].link;
                                                                                                                       scanf("%c",&c);
                                                                    len=st[j].len;
   check();
int i,j,len;
                                                                                                                       if(!('a'<=c&&c<='z')) break;
                                                                                                                       X.push_back(c);
                                                               if(len>=n) S.insert(j);
   long long ans=0;
    int n=str.size();
                                                                                                                   long long ans=sa.cyclic_occurrence(X);
   set<int> S;
                                                           for(it=S.begin();it!=S.end();++it)
   set<int>::iterator it:
                                                                                                                   X.clear();
   for(i=0,j=0,len=0; i<n*2-1; i++)</pre>
                                                                                                                   printf("\frac{164d}{n}",ans);
                                                               ans=ans+st[*it].endpos;
        //printf("%d->%c\n",i,str[i%n]);
                                                                                                               return 0;
                                                           return ans;
        if(st[j].next.count(str[i\( n \) ]))
```