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1 DP	
1.1 DSU on Tree	
<pre>vector<int> *pvec[MAX]; vector<int> G[MAX]; int sz[MAX],color[MAX],color_counter[MAX]; pair<ll,int> Info[MAX]; pair<ll,int>dfs(int u,int p=-1,bool keep=false) { int i,j,k,child,hchild=-1; for(i=0; i<G[u].size(); i++) { if(G[u][i]==p) continue; if(hchild==-1 sz[hchild]<sz[G[u][i]]) { hchild=G[u][i]; } } for(i=0; i<G[u].size(); i++) { if(G[u][i]==p G[u][i]==hchild) continue; dfs(G[u][i],u,false); } if(hchild!=-1) { Info[u]=dfs(hchild,u,true); pvec[u]=pvec[hchild]; } else { pvec[u]=new vector<int> (); } pvec[u]->push_back(u); color_counter[color[u]]++; if(color_counter[color[u]]>Info[u].second) { Info[u].second=color_counter[color[u]]; Info[u].first=color[u]; } else if(color_counter[color[u]]==Info[u].second) { Info[u].first=Info[u].first+color[u]; } for(i=0; i<G[u].size(); i++) { if(G[u][i]==p G[u][i]==hchild) continue; child=G[u][i]; for(j=0; j<(*pvec[child]).size(); j++) { k=(*pvec[child])[j];</pre>	

```
pvec[u]->push_back(k);
color_counter[color[k]]++;
if(color_counter[color[k]]>Info[u].second)
{
    Info[u].second=color_counter[color[k]];
    Info[u].first=color[k];
}
else if(color_counter[color[k]]==Info[u].second)
{
    Info[u].first=Info[u].first+color[k];
}
}
}
if(!keep)
{
    for(j=0; j<(*pvec[u]).size(); j++)
    {
        k=(*pvec[u])[j];
        color_counter[color[k]]--;
    }
}
return Info[u];
}
```

1.2 Divide and Conquer Optimization

```
int m, n;
vector<long long> dp_before(n), dp_cur(n);
long long C(int i, int j);
// compute dp_cur[l], ... dp_cur[r] (inclusive)
void compute(int l, int r, int optl, int optr) {
    if (l > r)
        return;
    int mid = (l + r) >> 1;
    pair<long long, int> best = {LLONG_MAX, -1};
    for (int k = optl; k <= min(mid, optr); k++) {
        best = min(best, {(k ? dp_before[k - 1] : 0) + C(k, mid)});
    }
    dp_cur[mid] = best.first;
    int opt = best.second;
    compute(l, mid - 1, optl, opt);
    compute(mid + 1, r, opt, optr);
}
int solve() {
    for (int i = 0; i < n; i++)
        dp_before[i] = C(0, i);
    for (int i = 1; i < m; i++) {
        compute(0, n - 1, 0, n - 1);
        dp_before = dp_cur;
    }
    return dp_before[n - 1];
}
```

1.3 Li Chao Tree

```
#include <bits/stdc++.h>
#include <vector>
#include <math.h>
#include <string.h>
using namespace std;
#define MAX 200005
#define MOD 1000000007
#define INF 10000000000
#define EPS 0.0000000001
#define FASTIO ios_base::sync_with_stdio(false);cin.tie(0);
#include <ext/pb_ds/assoc_container.hpp>
```

<pre>#include <ext/pb_ds/tree_policy.hpp> #include <ext/pb_ds/detail/standard_policies.hpp> class LiChaoTree { long long L,R; bool minimize; int lines; struct Node { complex<long long> line; Node *children[2]; Node(complex<long long> ln= {0,1000000000000000000000000}) { line=ln; children[0]=0; children[1]=0; } } *root; long long dot(complex<long long> a, complex<long long> b) { return (conj(a) * b).real(); } long long f(complex<long long> a, long long x) { return dot(a, {x, 1}); } void clear(Node* &node) { if(node->children[0]) { clear(node->children[0]); } if(node->children[1]) { clear(node->children[1]); } delete node; } void add_line(complex<long long> nw, Node* &node, { if(node==0) { node=new Node(nw); return; } long long m = (l + r) / 2; bool lef = (f(nw, l) < f(node->line, l)&&minimize) ((!minimize)&&f(nw, l) > f(node->line, l)); bool mid = (f(nw, m) < f(node->line, m)&&minimize) ((!minimize)&&f(nw, m) > f(node->line, m)); if(mid) { swap(node->line, nw); } if(r - l == 1) { return; } else if(lef != mid) { add_line(nw, node->children[0], l, m); } else { add_line(nw, node->children[1], m, r); } }</pre>	<pre> } long long get(long long x, Node* &node, long long l, long long r) { long long m = (l + r) / 2; if(r - l == 1) { return f(node->line, x); } else if(x < m) { if(node->children[0]==0) return f(node->line, x); if(minimize) return min(f(node->line, x), get(x, node->children[0], l, m)); else return max(f(node->line, x), get(x, node->children[0], l, m)); } else { if(node->children[1]==0) return f(node->line, x); if(minimize) return min(f(node->line, x), get(x, node->children[1], m, r)); else return max(f(node->line, x), get(x, node->children[1], m, r)); } } public: LiChaoTree(long long l=-1000000001,long long r=1000000001,bool mn=false) { L=l; R=r; root=0; minimize=mn; lines=0; } void AddLine(pair<long long,long long> ln) { add_line({ln.first,ln.second},root,L,R); lines++; } int number_of_lines() { return lines; } long long l, long long r) long long getOptimumValue(long long x) { return get(x,root,L,R); } ~LiChaoTree() { if(root!=0) clear(root); } int main() { return 0; } }</pre>	<pre> d[j] = i; for (int j = 0; j < m; ++j) { while (!st.empty() && d[st.top()] <= d[j]) st.pop(); d1[j] = st.empty() ? -1 : st.top(); st.push(j); } while (!st.empty()) st.pop(); for (int j = m - 1; j >= 0; --j) { while (!st.empty() && d[st.top()] <= d[j]) st.pop(); d2[j] = st.empty() ? m : st.top(); st.push(j); } while (!st.empty()) st.pop(); for (int j = 0; j < m; ++j) ans = max(ans, (i - d[j]) * (d2[j] - d1[j] - 1)); return ans; } }</pre>
<div>2 DS</div> <div>2.1 BIT 2D</div>		
<pre>const int mx = 1002,my = 1002; long long bit[4][mx][my]; void update(int x, int y, int val, int i) { int y1; while(x<=mx) { y1=y; while(y1<=my) bit[i][x][y1] += val, y1 += (y1&-y1); x += (x&-x); } } long long query(int x, int y, int i) { long long ans=0; int y1; while(x>0) { y1 = y; while(y1>0) ans += bit[i][x][y1], y1 -= (y1&-y1); x -= (x&-x); } return ans; } // add value k from (x1,y1) to (x2,y2) inclusive void add(int x1, int y1, int x2, int y2, int k) { update(x1,y1,k,0); update(x1,y2+1,-k,0); update(x2+1,y1,-k,0); update(x2+1,y2+1,k,0); update(x1,y1,k*(1-y1),1); update(x1,y2+1,k*y2,1); update(x2+1,y1,k*(y1-1),1); update(x2+1,y2+1,-y2*k,1); update(x1,y1,k*(1-x1),2); update(x1,y2+1,k*(x1-1),2); update(x2+1,y1,k*x2,2); update(x2+1,y2+1,-x2*k,2); }</pre>		
<div>1.4 zero_matrix</div>		
<pre>int zero_matrix(vector<vector<int>> a) { int n = a.size(); int m = a[0].size(); int ans = 0; vector<int> d(m, -1), d1(m), d2(m); stack<int> st; for (int i = 0; i < n; ++i) { for (int j = 0; j < m; ++j) { if (a[i][j] == 1)</pre>		

```
update(x1,y1,(x1-1)*(y1-1)*k,3);
update(x1,y2+1,-y2*(x1-1)*k,3);
update(x2+1,y1,-x2*(y1-1)*k,3);
update(x2+1,y2+1,x2*y2*k,3);
}
// get value from (x1,y1) to (x2,y2) inclusive
long long get( int x1, int y1, int x2, int y2 ) {
    LL v1=query(x2,y2,0)*x2*y2 +
        query(x2,y2,1)*x2 +
        query(x2,y2,2)*y2 +
        query(x2,y2,3);
    LL v2=query(x2,y1-1,0)*x2*(y1-1) +
        query(x2,y1-1,1)*x2 +
        query(x2,y1-1,3) +
        query(x2,y1-1,2)*(y1-1);
    LL v3=query(x1-1,y2,0)*(x1-1)*y2 +
        query(x1-1,y2,2)*y2+
        query(x1-1,y2,1)*(x1-1) +
        query(x1-1,y2,3);
    LL v4=query(x1-1,y1-1,0)*(x1-1)*(y1-1) +
        query(x1-1,y1-1,1)*(x1-1) +
        query(x1-1,y1-1,2)*(y1-1) +
        query(x1-1,y1-1,3);
    LL ans=v1-v2-v3+v4;
    return ans;
}
```

2.2 CD - hellbent

```
vector<int> g[N]; int n, child[N], done[N];
void dfs_size(int u, int par) {
    child[u] = 1;
    for (int v: g[u]) {
        if (done[v] or v == par) continue;
        dfs_size(v, u); child[u] += child[v];
    }
}
int dfs_find_centroid(int u, int par, int sz) {
    for (int v: g[u]) {
        if (!done[v] and v != par and child[v] > sz) {
            return dfs_find_centroid(v,u,sz);
        }
    }
    return u;
}
void solve (int u) {/**problem specific things */}
void dfs_decompose(int u) {
    dfs_size(u, -1);
    int centroid=dfs_find_centroid(u,-1,child[u]/2);
    solve(centroid);
    done[centroid] = 1;
    for (int v : g[centroid]) {
        if (!done[v]) dfs_decompose(v);
    }
}
```

2.3 Hld - cpalgo

```
vector<int> parent, depth, heavy, head, pos;
int cur_pos;
int dfs(int v, vector<vector<int>> const& adj) {
    int size = 1;
    int max_c_size = 0;
```

```
for (int c : adj[v]) {
    if (c != parent[v]) {
        parent[c] = v, depth[c] = depth[v] + 1;
        int c_size = dfs(c, adj);
        size += c_size;
        if (c_size > max_c_size)
            max_c_size = c_size, heavy[v] = c;}}
return size;}
decompose(int v,int h,vector<vector<int>> const& adj){
    head[v] = h, pos[v] = cur_pos++;
    if (heavy[v] != -1)
        decompose(heavy[v], h, adj);
    for (int c : adj[v]) {
        if (c != parent[v] && c != heavy[v])
            decompose(c, c, adj);}
}
void init(vector<vector<int>> const& adj) {
    int n = adj.size();
    parent = vector<int>(n);
    depth = vector<int>(n);
    heavy = vector<int>(n, -1);
    head = vector<int>(n);
    pos = vector<int>(n);
    cur_pos = 0;
    dfs(0, adj);
    decompose(0, 0, adj);
}
int query(int a, int b) {
    int res = 0;
    for (; head[a] != head[b]; b = parent[head[b]]) {
        if (depth[head[a]] > depth[head[b]])
            swap(a, b);
        int cur_heavy_path_max =
            segment_tree_query(pos[head[b]], pos[b]);
        res = max(res, cur_heavy_path_max);
    }
    if (depth[a] > depth[b])
        swap(a, b);
    int last_heavy_path_max =
        segment_tree_query(pos[a], pos[b]);
    res = max(res, last_heavy_path_max);
    return res;
}
```

2.4 Implicit Treap

```
#include<bits/stdc++.h>
#include<math.h>
#include<vector>
#include<stdlib.h>
using namespace std;
#define MAX 200005
#define MOD 998244353
#define NINF -1000000000000000000
template <class T>
class implicit_treap
{
    struct item
    {
        int prior, cnt;
        T value;
        bool rev;
        item *l,*r;
        item(T v)
```

```
{
    value=v;
    rev=false;
    l=NULL;
    r=NULL;
    cnt=1;
    prior=rand();
}
} *root,*node;
int cnt (item * it)
{
    return it ? it->cnt : 0;
}
void upd_cnt (item * it)
{
    if (it)
        it->cnt = cnt(it->l) + cnt(it->r) + 1;
}
void push (item * it)
{
    if (it && it->rev)
    {
        it->rev = false;
        swap (it->l, it->r);
        if (it->l) it->l->rev ^= true;
        if (it->r) it->r->rev ^= true;
    }
}
void merge (item * &t, item * l, item * r)
{
    push (l);
    push (r);
    if (!l || !r)
        t = l ? l : r;
    else if (l->prior > r->prior)
        merge (l->r, l->r, r), t = l;
    else
        merge (r->l, l, r->l), t = r;
    upd_cnt (t);
}
void split (item * t, item * &l, item * &r, int
{
    if (!t)
        return void( l = r = 0 );
    push (t);
    int cur_key = add + cnt(t->l);
    if (key <= cur_key)
        split (t->l, l, t->l, key, add), r = t;
    else
        split (t->r, t->r, r, key, add + 1 + cnt(t->l = t;
    upd_cnt (t);
}
void insert(item * &t,item * element,int key)
{
    item *l,*r;
    split(t,l,r,key);
    merge(l,l,element);
    merge(t,l,r);
    l=NULL;
```

```
    r=NULL;
}
T elementAt(item * &t,int key)
{
    push(t);
    T ans;
    if(cnt(t->l)==key) ans=t->value;
    else if(cnt(t->l)>key) ans=elementAt(t->l,key);
    else ans=elementAt(t->r,key-1-cnt(t->l));
    return ans;
}
void erase (item * & t, int key)
{
    push(t);
    if(!t) return;
    if (key == cnt(t->l))
        merge (t, t->l, t->r);
    else if(key<cnt(t->l))
        erase(t->l,key);
    else
        erase(t->r,key-cnt(t->l)-1);
    upd_cnt(t);
}
void reverse (item * &t, int l, int r)
{
    item *t1, *t2, *t3;
    split (t, t1, t2, l);
    split (t2, t2, t3, r-l+1);
    t2->rev ^= true;
    merge (t, t1, t2);
    merge (t, t, t3);
}
void cyclic_shift(item * &t,int L,int R)
{
    if(L==R) return;
    item *l,*r,*m;
    split(t,t,l,L);
    split(l,l,m,R-L+1);
    split(l,l,r,R-L);
    merge(t,t,r);
    merge(t,t,l);
    merge(t,t,m);
    l=NULL;
    r=NULL;
    m=NULL;
}
void output (item * t,vector<T> &arr)
{
    if (!t) return;
    push (t);
    output (t->l,arr);
    arr.push_back(t->value);
    output (t->r,arr);
}
public:
implicit_treap()
{
    root=NULL;
}
void insert(T value,int position)
{
    node=new item(value);
    insert(root,node,position);
}
```

```
    }
    void erase(int position)
    {
        erase(root,position);
    }
    void reverse(int l,int r)
    {
        reverse(root,l,r);
    }
    T elementAt(int position)
    {
        return elementAt(root,position);
    }
    void cyclic_shift(int L,int R)
    {
        cyclic_shift(root,L,R);
    }
    int size()
    {
        return cnt(root);
    }
    void output(vector<T> &arr)
    {
        output(root,arr);
    }
};
```

2.5 Mo Algorithm

```
#include<bits/stdc++.h>
using namespace std;
#define MOD 998244353
#define MAX 200005
#define MAX_BIT 50
#define PRECISION 0.000000000001
#define INF 2000000000
void remove(int idx); // TODO: remove value at idx from data structure
void add(int idx); // TODO: add value at idx from data structure
int get_answer(); // TODO: extract the current answer of the data structure
int block_size;
struct Query {
    int l, r,k, idx;
    bool operator<(Query other) const
    {
        if(l/block_size!=other.l/block_size) return (l<other.l)?true:false;
        return (l/block_size&1)? (r<other.r) : (r>other.r);
    }
}
vector<int> mo_s_algorithm(vector<Query> queries) {
    vector<int> answers(queries.size());
    sort(queries.begin(), queries.end());

    // TODO: initialize data structure

    int cur_l = 0;
    int cur_r = -1;
    // invariant: data structure will always reflect the range [cur_l, cur_r]
    for (Query q : queries) {
        while (cur_l > q.l) {
            cur_l--;
            add(cur_l);
        }
        while (cur_r < q.r) {
            cur_r++;
            add(cur_r);
        }
    }
}
```

```
    }
    while (cur_l < q.l) {
        remove(cur_l);
        cur_l++;
    }
    while (cur_r > q.r) {
        remove(cur_r);
        cur_r--;
    }
    answers[q.idx] = get_answer();
}
return answers;
}
int main()
{
    return 0;
}
```

2.6 Treap

```
#include<bits/stdc++.h>
#include<math.h>
#include<vector>
#include<stdlib.h>
using namespace std;
#define MAX 400005
#define MOD 998244353
#define INF 2000000000
template <class T>
class treap
{
    struct item
    {
        int prior, cnt;
        T key;
        item *l,*r;
        item(T key):key(key),l(NULL),r(NULL),cnt=1,prior=rand(){}
    }
    item *root,*node;
    int cnt (item * it)
    {
        return it ? it->cnt : 0;
    }
    void upd_cnt (item * it)
    {
        if (it)
            it->cnt = cnt(it->l) + cnt(it->r) + 1;
    }
    void split (item * t, T key, item * & l, item * & r)
    {
        if (!t)
            l = r = NULL;
        else if (key < t->key)
            split (t->l, key, l, t->l), r = t;
        else
            split (t->r, key, t->r, r), l = t;
        upd_cnt(t);
    }
}
```

<pre> } void insert (item * &t, item * it) { if (!t) t = it; else if (it->prior > t->prior) split (t, it->key, it->l, it->r), t = it; else insert (it->key < t->key ? t->l : t->r, it); upd_cnt(t); } void merge (item * &t, item * l, item * r) { if (!l !r) t = l ? l : r; else if (l->prior > r->prior) merge (l->r, l->r, r), t = l; else merge (r->l, l, r->l), t = r; upd_cnt(t); } void erase (item * &t, T key) { if (t->key == key) merge (t, t->l, t->r); else erase (key < t->key ? t->l : t->r, key); upd_cnt(t); } T elementAt(item * &t,int key) { T ans; if(cnt(t->l)==key) ans=t->key; else if(cnt(t->l)>key) ans=elementAt(t->l,key); else ans=elementAt(t->r,key-1-cnt(t->l)); upd_cnt(t); return ans; } item * unite (item * l, item * r) { if (!l !r) return l ? l : r; if (l->prior < r->prior) swap (l, r); item * lt, * rt; split (r, l->key, lt, rt); l->l = unite (l->l, lt); l->r = unite (l->r, rt); upd_cnt(l); upd_cnt(r); return l; } void heapify (item * t) { if (!t) return; item * max = t; if (t->l != NULL && t->l->prior > max->prior) max = t->l; if (t->r != NULL && t->r->prior > max->prior) max = t->r; if (max != t) { swap (t->prior, max->prior); heapify (max); } } item * build (T * a, int n) { if (n == 0) return NULL; int mid = n / 2; item * t = new item (a[mid], rand ()); t->l = build (a, mid); t->r = build (a + mid + 1, n - mid - 1); heapify (t); return t; } void output (item * t,vector<T> &arr) { if (!t) return; output (t->l,arr); arr.push_back(t->key); output (t->r,arr); } public: treap() { root=NULL; } treap(T *a,int n) { build(a,n); } void insert(T value) { node=new item(value); insert(root,node); } void erase(T value) { erase(root,value); } T elementAt(int position) { return elementAt(root,position); } int size() { return cnt(root); } void output(vector<T> &arr) { output(root,arr); } int range_query(T l,T r) //(l,r] { item *previous,*next,*current; split(root,l,previous,current); split(current,r,current,next); int ans=cnt(current); merge(root,previous,current); merge(root,root,next); previous=NULL; current=NULL; next=NULL; }</pre>	<pre> return ans; } }; 2.7 sparse table 2d int st[K][K][N][N]; int lg[N]; void pre() { lg[1] = 0; for (int i=2; i<N; i++) lg[i] = lg[i/2]+1; } int query(int l1, int r1, int l2, int r2) { int xx = lg[l2-l1+1], yy = lg[r2-r1+1]; return max(max(st[xx][yy][l1][r1], st[xx][yy][l2-(1<<xx)+1][r1]), max(st[xx][yy][l1][r2-(1<<yy)+1], st[xx][yy][l2-(1<<xx)+1][r2-(1<<yy)+1])); } void build() { for (int x=0; x<K; x++) { for (int y=0; y<K; y++) { for (int i=1; i<=n; i++) { for (int j=1; j<=m; j++) { if (i+(1<<x)-1>n j+(1<<y)-1>m) continue; if (!x&&!y) st[0][0][i][j]=flag[i][j]; else if (x>0) st[x][y][i][j] = max(st[x-1][y][i][j],st[x-1][y][i+(1<<(x-1))][j]); else if (y>0) st[x][y][i][j] = max(st[x][y-1][i][j],st[x][y-1][i][j+(1<<(y-1))]); } } } } }</pre>	<pre>3 Flow 3.1 Dinic's Algorithm #include<bits/stdc++.h> #include<vector> using namespace std; #define MAX 100 #define HUGE_FLOW 1000000000 #define BEGIN 1 #define DEFAULT_LEVEL 0 struct FlowEdge { int v, u; long long cap, flow = 0; FlowEdge(int v, int u, long long cap) : v(v), u(u) }; struct Dinic { const long long flow_inf = 1e18; vector<FlowEdge> edges; vector<vector<int>>> adj; int n, m = 0; int s, t; vector<int> level, ptr; queue<int> q; Dinic(int n, int s, int t) : n(n), s(s), t(t) { adj.resize(n); level.resize(n); ptr.resize(n); }</pre>
--	---	--


```

}

void add_edge(int v, int u, long long cap) {
    edges.emplace_back(v, u, cap);
    edges.emplace_back(u, v, 0);
    adj[v].push_back(m);
    adj[u].push_back(m + 1);
    m += 2;
}

bool bfs() {
    while (!q.empty()) {
        int v = q.front();
        q.pop();
        for (int id : adj[v]) {
            if (edges[id].cap - edges[id].flow < 1)
                continue;
            if (level[edges[id].u] != -1)
                continue;
            level[edges[id].u] = level[v] + 1;
            q.push(edges[id].u);
        }
    }
    return level[t] != -1;
}

long long dfs(int v, long long pushed) {
    if (pushed == 0)
        return 0;
    if (v == t)
        return pushed;
    for (int& cid = ptr[v]; cid < (int)adj[v].size(); cid++) {
        int id = adj[v][cid];
        int u = edges[id].u;
        if (level[v] + 1 != level[u] || edges[id].cap - edges[id].flow < 1)
            continue;
        long long tr = dfs(u, min(pushed, edges[id].cap - edges[id].flow));
        if (tr == 0)
            continue;
        edges[id].flow += tr;
        edges[id ^ 1].flow -= tr;
        return tr;
    }
    return 0;
}

long long flow() {
    long long f = 0;
    while (true) {
        fill(level.begin(), level.end(), -1);
        level[s] = 0;
        q.push(s);
        if (!bfs())
            break;
        fill(ptr.begin(), ptr.end(), 0);
        while (long long pushed = dfs(s, flow_inf))
            f += pushed;
    }
    return f;
}

int main()
{

```

```

        return 0;
    }

3.2 Edmond's Blossom Algorithm

/**Copied from https://codeforces.com/blog/entry/49402**/
/*
GETS:
V->number of vertices
E->number of edges
pair of vertices as edges (vertices are 1..V)
GIVES:
output of edmonds() is the maximum matching
match[i] is matched pair of i (-1 if there isn't a matched pair)
*/

#include <bits/stdc++.h>
using namespace std;
const int M=500;
struct struct_edge
{
    int v;
    struct_edge* n;
};
typedef struct_edge* edge;
struct_edge pool[M*M*2];
edge top=pool,adj[M];
int V,E,match[M],qh,qt,q[M],father[M],base[M];
bool inq[M],inb[M],ed[M][M];
void add_edge(int u,int v)
{
    top->v=v,top->n=adj[u],adj[u]=top++;
    top->v=u,top->n=adj[v],adj[v]=top++;
}

int LCA(int root,int u,int v)
{
    if (edges[id].cap - edges[id].flow < 1)
        continue;
    static bool inp[M];
    memset(inp,0,sizeof(inp));
    while(1)
    {
        inp[u=base[u]]=true;
        if (u==root) break;
        u=father[match[u]];
    }
    while(1)
    {
        if (inp[v=base[v]]) return v;
        else v=father[match[v]];
    }
}

void mark_blossom(int lca,int u)
{
    while (base[u]!=lca)
    {
        int v=match[u];
        inb[base[u]]=inb[base[v]]=true;
        u=father[v];
        if (base[u]!=lca) father[u]=v;
    }
}

void blossom_contraction(int s,int u,int v)
{
    int lca=LCA(s,u,v);

```

```

    memset(inb,0,sizeof(inb));
    mark_blossom(lca,u);
    mark_blossom(lca,v);
    if (base[u]!=lca)
        father[u]=v;
    if (base[v]!=lca)
        father[v]=u;
    for (int u=0; u<V; u++)
        if (inb[base[u]])
        {
            base[u]=lca;
            if (!inq[u])
                inq[q[++qt]=u]=true;
        }
}

int find_augmenting_path(int s)
{
    memset(inq,0,sizeof(inq));
    memset(father,-1,sizeof(father));
    for (int i=0; i<V; i++) base[i]=i;
    inq[q[qh=qt=0]=s]=true;
    while (qh<=qt)
    {
        int u=q[qh++];
        for (edge e=adj[u]; e; e=e->n)
        {
            int v=e->v;
            if (base[u]!=base[v]&&match[u]!=v)
                if ((v==s)|| (match[v]==-1 && father[match[v]]!=v))
                    blossom_contraction(s,u,v);
            else if (father[v]==-1)
            {
                father[v]=u;
                if (match[v]==-1)
                    return v;
                else if (!inq[match[v]])
                    inq[q[++qt]=match[v]]=true;
            }
        }
    }
    return -1;
}

int augment_path(int s,int t)
{
    int u=t,v,w;
    while (u!=-1)
    {
        v=father[u];
        w=match[v];
        match[v]=u;
        match[u]=v;
        u=w;
    }
    return t!=-1;
}

int edmonds()
{
    int matchc=0;
    memset(match,-1,sizeof(match));
    for (int u=0; u<V; u++)
        if (match[u]==-1)
            matchc+=augment_path(u,find_augmenting_path(s));
    return matchc;
}

```

```

}
int main()
{
    FILE *in=stdin;
    int u,v;
    fscanf(in,"%d",&V);
    while(fscanf(in,"%d %d",&u,&v)!=EOF)
    {
        if (!ed[u-1][v-1])
        {
            add_edge(u-1,v-1);
            ed[u-1][v-1]=ed[v-1][u-1]=true;
        }
    }
    printf("%d\n",2*edmonds());
    for (int i=0; i<V; i++)
        if (i<match[i])
            printf("%d %d\n",i+1,match[i]+1);
    return 0;
}

```

3.3 Hungarian Algorithm

```

#include <bits/stdc++.h>
#include <vector>
#include <math.h>
#include <string.h>
using namespace std;
#define MAX 300005
#define MOD 1000000007
#define GMAX 19
#define INF 2000000000000000000
#define EPS 0.000000001
#define FASTIO ios_base::sync_with_stdio(false);cin.tie(NULL)
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
#include <ext/pb_ds/detail/standard_policies.hpp>
class HungarianAlgorithm
{
    int N,inf,n,max_match;
    int *lx,*ly,*xy,*yx,*slack,*slackx,*prev;
    int **cost;
    bool *S,*T;
    void init_labels()
    {
        for(int x=0;x<n;x++) lx[x]=0;
        for(int y=0;y<n;y++) ly[y]=0;
        for (int x = 0; x < n; x++)
            for (int y = 0; y < n; y++)
                lx[x] = max(lx[x], cost[x][y]);
    }
    void update_labels()
    {
        int x, y, delta = inf; //init delta as infinity
        for (y = 0; y < n; y++) //calculate delta using slack
            if (!T[y])
                delta = min(delta, slack[y]);
        for (x = 0; x < n; x++) //update X labels
            if (S[x]) lx[x] -= delta;
        for (y = 0; y < n; y++) //update Y labels
            if (T[y]) ly[y] += delta;
        for (y = 0; y < n; y++) //update slack array
            if (!T[y])

```

```

        slack[y] -= delta;
    }
    void add_to_tree(int x, int prevx)
    //x - current vertex,prevx - vertex from X before x in the alternating path,
    //so we add edges (prevx, xy[x]), (xy[x], x)
    {
        S[x] = true; //add x to S
        prev[x] = prevx; //we need this when augmenting
        for (int y = 0; y < n; y++) //update slacks,
            if (lx[x] + ly[y] - cost[x][y] < slack[y])
            {
                slack[y] = lx[x] + ly[y] - cost[x][y];
                slackx[y] = x;
            }
    }
    void augment() //main function of the algorithm
    {
        if (max_match == n) return; //check whether matching is already perfect
        int x, y, root; //just counters and root vertex
        int q[N], wr = 0, rd = 0; //q - queue for bfs, wr,rd - write and read
        //pos in queue
        //memset(S, false, sizeof(S)); //init set S
        for(int i=0;i<n;i++) S[i]=false;
        //memset(T, false, sizeof(T)); //init set T
        for(int i=0;i<n;i++) T[i]=false;
        //memset(prev, -1, sizeof(prev)); //init set prev
        for(int i=0;i<n;i++) prev[i]=-1;
        for (x = 0; x < n; x++) //finding root of the tree
        {
            if (xy[x] == -1)
            {
                q[wr++] = root = x;
                prev[x] = -2;
                S[x] = true;
                break;
            }
        }
        for (y = 0; y < n; y++) //initializing slack array
        {
            slack[y] = lx[root] + ly[y] - cost[root][y];
            slackx[y] = root;
        }
        while (true) //main cycle
        {
            while (rd < wr) //building tree with bfs cycle
            {
                x = q[rd++]; //current vertex from X part
                for (y = 0; y < n; y++) //iterate through all edges in equality graph
                {
                    if (cost[x][y] == lx[x] + ly[y] && !T[y])
                    {
                        if (yx[y] == -1) break; //an exposed vertex exists!
                        T[y] = true; //else just add y to T,
                        q[wr++] = yx[y]; //add vertex yx[y], which is introduced
                        //with y, to the queue
                        add_to_tree(yx[y], x); //add edges (x, yx[y]), (yx[y], x)
                    }
                }
            }
            if (y < n) break; //augmenting path found!
        }
    }

```

```

        if (y < n) break; //augmenting path found!
        update_labels(); //augmenting path not found
        wr = rd = 0;
        for (y = 0; y < n; y++)
        {
            //in this cycle we add edges that were a
            //result of improving the labeling, we add edge (slack
            //and only if !T[y] && slack[y] == 0, also with this e
            //because we add new vertex to y,
            //if (y, yx[y]) or augment the matching, if y was exposed
            if (!T[y] && slack[y] == 0)
            {
                if (yx[y] == -1) //exposed vertex in
                {
                    x = slackx[y];
                    break;
                }
                else
                {
                    T[y] = true; //else just add y to T,
                    if (!S[yx[y]])
                    {
                        q[wr++] = yx[y]; //add vertex
                        add_to_tree(yx[y], slackx[y])
                    }
                }
            }
            if (y < n) break; //augmenting path found!
        }
        if (y < n) //we found augmenting path!
        {
            max_match++; //increment matching
            //in this cycle we inverse edges along augmenting path
            for (int cx = x, cy = y, ty; cx != -2; cx =
            {
                ty = xy[cx];
                yx[cy] = cx;
                xy[cx] = cy;
            }
            augment(); //recall function, go to step 1
        }
    } //end of augment() function
public:
    HungarianAlgorithm(int vv,int inf=1000000000)
    {
        N=vv;
        n=N;
        max_match=0;
        this->inf=inf;
        int Nund, so
        ly=new int[N]; //labels of X and Y parts
        xy=new int[N]; //xy[x] - vertex that is matched
        T=new int[N]; //xy[x] - vertex that is matched
        q=new int[N]; //as in the algorithm descript
        slack=new int[N]; //slackx[y] - vertex that is matched
        prev=new int[N]; //array for memorizing alternat
        S=new bool[N];
        T=new bool[N]; //sets S and T in algorithm
        cost=new int*[N]; //cost matrix
    }

```

```

for(int i=0; i<N; i++)
{
    cost[i]=new int[N];
}
}
HungarianAlgorithm()
{
    delete []lx;
    delete []ly;
    delete []xy;
    delete []yx;
    delete []slack;
    delete []slackx;
    delete []prev;
    delete []S;
    delete []T;
    int i;
    for(i=0; i<N; i++)
    {
        delete [](cost[i]);
    }
    delete []cost;
}

```

```

void setCost(int i,int j,int c)
{
    cost[i][j]=c;
}

```

```

int* matching(bool first=true)
{

```

```

    int *ans;
    ans=new int[N];
    for(int i=0;i<N;i++)
    {
        if(first) ans[i]=xy[i];
        else ans[i]=yx[i];
    }
    return ans;
}

```

```

int hungarian()
{

```

```

    int ret = 0; //weight of the optimal matching
    max_match = 0; //number of vertices in current matching
    for(int x=0;x<n;x++) xy[x]=-1;
    for(int y=0;y<n;y++) yx[y]=-1;
    init_labels(); //step 0
    augment(); //steps 1-3
    for (int x = 0; x < n; x++) //forming answer
        ret += cost[x][xy[x]];
    return ret;
}

```

```

};
int main()
{

```

```

    int t,T=1;
    scanf("%d",&T);
    for(t=0;t<T;t++)
    {
        int n,i,j;
        scanf("%d",&n);
        HungarianAlgorithm h(n);
        int own[n],opposite[n];
        for(i=0;i<n;i++)
        {

```

```

            scanf("%d",&own[i]);
        }
        for(j=0;j<n;j++)
        {
            scanf("%d",&opposite[j]);
        }
        for(i=0;i<n;i++)
        {
            for(j=0;j<n;j++)
            {
                int v;
                if(own[i]==opposite[j]) v=1;
                else if(own[i]>opposite[j]) v=2;
                else v=0;
                h.setCost(i,j,v);
            }
        }
        int ans=h.hungarian();
        printf("Case %d: %d\n",t+1,ans);
    }
    return 0;
}

```

3.4 Maximum Bipartite Matching

```

/** Source: https://iq.opengenus.org/hopcroft-karp-algorithm/
#include <bits/stdc++.h>
#include <vector>
#include <math.h>
#include <string.h>
using namespace std;
#define MAX 300005
#define BEGIN 1
#define MOD 1000000007
#define INF INT_MAX
#define EPS 0.0000000001
#define CHAINS 18
#define NIL 0
#define NOT_VISITED 0
#define VISITING 1
#define VISITED 2
#define FASTIO ios_base::sync_with_stdio(false);cin.tie(NULL);
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
#include <ext/pb_ds/detail/standard_policies.hpp>
// A class to represent Bipartite graph for
// Hopcroft Karp implementation
class BGraph
{
    // m and n are number of vertices on left
    // and right sides of Bipartite Graph
    int m, n;

    // adj[u] stores adjacents of left side
    // vertex 'u'. The value of u ranges from 1 to m.
    // 0 is used for dummy vertex
    std::list<int> *adj;

    // pointers for hopcroftKarp()
    int *pair_u, *pair_v, *dist;

public:
    BGraph(int m, int n); // Constructor
    void addEdge(int u, int v); // To add edge

    // Returns true if there is an augmenting path

```

```

    bool bfs();
    // Adds augmenting path if there is one beginning
    // with u
    bool dfs(int u);
    // Returns size of maximum matching
    int hopcroftKarpAlgorithm();
};

// Returns size of maximum matching
int BGraph::hopcroftKarpAlgorithm()
{
    // pair_u[u] stores pair of u in matching on left
    // If u doesn't have any pair, then pair_u[u] is NIL
    pair_u = new int[m + 1];

    // pair_v[v] stores pair of v in matching on right
    // If v doesn't have any pair, then pair_u[v] is NIL
    pair_v = new int[n + 1];

    // dist[u] stores distance of left side vertices
    dist = new int[m + 1];

    // Initialize NIL as pair of all vertices
    for (int u = 0; u <= m; u++)
        pair_u[u] = NIL;
    for (int v = 0; v <= n; v++)
        pair_v[v] = NIL;

    // Initialize result
    int result = 0;

    // Keep updating the result while there is an
    // augmenting path possible.
    while (bfs())
    {
        // Find a free vertex to check for a matching
        for (int u = 1; u <= m; u++)
        {
            // If current vertex is free and there is
            // an augmenting path from current vertex
            // then increment the result
            if (pair_u[u] == NIL && dfs(u))
                result++;
        }
        return result;
    }

    // Returns true if there is an augmenting path available
    bool BGraph::bfs()
    {
        std::queue<int> q; //an integer queue for bfs

        // First layer of vertices (set distance as 0)
        for (int u = 1; u <= m; u++)
        {
            // If this is a free vertex, add it to queue
            if (pair_u[u] == NIL)
            {
                // u is not matched so distance is 0
                dist[u] = 0;
                q.push(u);
            }

            // Else set distance as infinite so that this v

```


<pre> else dist[u] = INF; } // Initialize distance to NIL as infinite dist[NIL] = INF; // q is going to contain vertices of left side only while (!q.empty()) { // dequeue a vertex int u = q.front(); q.pop(); // If this node is not NIL and can provide a shorter path to adjacent vertices if (dist[u] < dist[NIL]) { // Get all the adjacent vertices of the dequeued vertex u std::list<int>::iterator it; for (it = adj[u].begin(); it != adj[u].end(); ++it) { int v = *it; // If pair of v is not considered so far // i.e. (v, pair_v[v]) is not yet explored if (dist[pair_v[v]] == INF) { // Consider the pair and push it to queue dist[pair_v[v]] = dist[u] + 1; q.push(pair_v[v]); } } } } // If we could come back to NIL using alternating path of distinct // vertices then there is an augmenting path available return (dist[NIL] != INF); } // Returns true if there is an augmenting path beginning with free vertex u bool BGraph::dfs(int u) { if (u != NIL) { std::list<int>::iterator it; for (it = adj[u].begin(); it != adj[u].end(); ++it) { // Adjacent vertex of u int v = *it; // Follow the distances set by BFS search if (dist[pair_v[v]] == dist[u] + 1) { // If dfs for pair of v also return true then if (dfs(pair_v[v]) == true) { // new matching possible, store the matching pair_v[v] = u; pair_u[u] = v; return true; } } } } // If there is no augmenting path beginning with u then</pre>	<pre> dist[u] = INF; return false; } return true; } // Constructor for initialization BGraph::BGraph(int m, int n) { this->m = m; this->n = n; adj = new std::list<int>[m + 1]; } // function to add edge from u to v void BGraph::addEdge(int u, int v) { adj[u].push_back(v); // Add v to us list. } 3.5 Minimum Cost Maximum Flow struct Edge { int from, to, capacity, cost; }; vector<vector<int>> adj, cost, capacity; const int INF = 1e9; void shortest_paths(int n, int v0, vector<int>& d, vector<int>& p) { d.assign(n, INF); d[v0] = 0; vector<bool> inq(n, false); queue<int> q; q.push(v0); p.assign(n, -1); while (!q.empty()) { int u = q.front(); q.pop(); inq[u] = false; for (int v : adj[u]) { if (capacity[u][v] > 0 && d[v] > d[u] + cost[u][v]) { d[v] = d[u] + cost[u][v]; p[v] = u; if (!inq[v]) { inq[v] = true; q.push(v); } } } } } int min_cost_flow(int N, vector<Edge> edges, int K, int s, int t) { adj.assign(N, vector<int>()); cost.assign(N, vector<int>(N, 0)); capacity.assign(N, vector<int>(N, 0)); for (Edge e : edges) { adj[e.from].push_back(e.to); adj[e.to].push_back(e.from); cost[e.from][e.to] = e.cost; cost[e.to][e.from] = -e.cost;</pre>	<pre> capacity[e.from][e.to] = e.capacity; } int flow = 0; int cost = 0; vector<int> d, p; while (flow < K) { shortest_paths(N, s, d, p); if (d[t] == INF) break; // find max flow on that path int f = K - flow; int cur = t; while (cur != s) { f = min(f, capacity[p[cur]][cur]); cur = p[cur]; } // apply flow flow += f; cost += f * d[t]; cur = t; while (cur != s) { capacity[p[cur]][cur] -= f; capacity[cur][p[cur]] += f; cur = p[cur]; } if (flow < K) return cost; } } 4 Geo 4.1 Convex Hull struct pt {double x, y;}; bool cmp(pt a, pt b) { return a.x < b.x (a.x == b.x && a.y < b.y); } bool cw(pt a, pt b, pt c) { return a.x*(b.y-c.y)+b.x*(c.y-a.y)+c.x*(a.y-b.y)<0; } bool ccw(pt a, pt b, pt c) { return a.x*(b.y-c.y)+b.x*(c.y-a.y)+c.x*(a.y-b.y)>0; } vector<pt> a; vector<pair<double,pair<double,double>>> pp; void convex_hull(vector<pt>& a) { if (a.size() == 1) return; sort(a.begin(), a.end(), &cmp); pt p1 = a[0], p2 = a.back(); vector<pt> up, down; up.push_back(p1); down.push_back(p1); for (int i = 1; i < (int)a.size(); i++) { if (i == a.size() - 1 cw(p1, a[i], p2)) { while (up.size() >= 2 && !cw(up[up.size()-2], up[up.size()-1], a[i])) up.pop_back(); up.push_back(a[i]); } } }</pre>
--	--	---

```

    }
    if (i == a.size() - 1 || ccw(p1, a[i], p2)) {
        while(down.size() >= 2 &&!
            ccw(down[down.size()-2], down[down.size()-1], a[i]))
            down.pop_back();
        down.push_back(a[i]);
    }
    a.clear();
    for(int i=0; i<(int)up.size(); i++) a.push_back(up[i]);
    for(int i=down.size()-2; i>0; i--) a.push_back(down[i]);
}

```

4.2 Half Plane Intersection

```

#define MAX 200005
#define MOD 1009
#define SMOD 998244353
#define ROOT 318
#define GMAX 19
#define INF 1000000000000000000
#define EPS 0.000000001
#define NIL 0
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
#include <ext/pb_ds/detail/standard_policies.hpp>
class HalfPlaneIntersection{
    static double eps, inf;
public:
    struct Point{
        double x, y;
    explicit Point(double x=0, double y=0):x(x), y(y) {}
    friend Point operator+(const Point&p, const Point&q){
        return Point(p.x + q.x, p.y + q.y);
    }
    friend Point operator-(const Point&p, const Point&q){
        return Point(p.x - q.x, p.y - q.y);
    }
    friend Point operator*(const Point&p, double&k){
        return Point(p.x * k, p.y * k);
    }
    friend double cross(const Point&p, const Point&q){
        return p.x * q.y - p.y * q.x;
    }
};

// Basic half-plane struct.
struct Halfplane
{
    // 'p' is a passing point of the line and
    // 'pq' is the direction vector of the line.
    Point p, pq; double angle;
    Halfplane() {}
    Halfplane(const Point&a, const Point&b):p(a), pq(b-a){
        angle = atan2l(pq.y, pq.x);
    }
    // Check if point 'r' is outside this half-plane.
    // Every half-plane allows the region to the LEFT
    // of its line.
    bool out(const Point&r){
        return cross(pq, r - p) < -eps;
    }
    //Comparator for sorting. If the angle of both half-
    //planes is equal, the leftmost one should go first.

```

```

    bool operator < (const Halfplane& e) const{
        if (fabsl(angle - e.angle) < eps)
            return cross(pq, e.p - p) < 0;
        return angle < e.angle;
    }
    // We use equal comparator for std::unique
    //to easily remove parallel half-planes.
    bool operator == (const Halfplane& e) const{
        return fabsl(angle - e.angle) < eps;
    }
    // Intersection point of the lines of two
    //half-planes. It is assumed they're never parallel.
    friend Point inter(const Halfplane& s,
        const Halfplane& t){
        double alpha=cross((t.p-s.p), t.pq)/cross(s.pq, t.pq);
        return s.p + (s.pq * alpha);
    }
};

vector<Point> hp_intersect(vector<Halfplane>&H)
{
    Point box[4] = // Bounding box in CCW order
    {
        Point(inf, inf),
        Point(-inf, inf),
        Point(-inf, -inf),
        Point(inf, -inf)
    };
    for(int i=0; i<4; i++){//Add bounding box half-planes.
        Halfplane aux(box[i], box[(i+1) % 4]);
        H.push_back(aux);
    }
    // Sort and remove duplicates
    sort(H.begin(), H.end());
    H.erase(unique(H.begin(), H.end()), H.end());
    deque<Halfplane> dq;
    int len = 0;
    for(int i = 0; i < int(H.size()); i++){
        // Remove from the back of the deque while last
        // half-plane is redundant
        while(len>1&&H[i].out(inter(dq[len-1], dq[len-2]))){
            dq.pop_back();
            --len;
        }
        // Remove from the front of the deque
        //while first half-plane is redundant
        while (len > 1 && H[i].out(inter(dq[0], dq[1]))){
            dq.pop_front();
            --len;
        }
        // Add new half-plane
        dq.push_back(H[i]); ++len;
    }
    // Final cleanup: Check half-planes at the
    //front against the back and vice-versa
    while(len>2&&dq[0].out(inter(dq[len-1], dq[len-2]))){
        dq.pop_back(); --len;
    }
    while(len > 2 && dq[len-1].out(inter(dq[0], dq[1]))){
        dq.pop_front(); --len;
    }
    // Report empty intersection if necessary
    if (len < 3) return vector<Point>();
    // Reconstruct the convex polygon from
    //the remaining half-planes.
    vector<Point> ret(len);
    for(int i = 0; i+1 < len; i++){

```

```

        ret[i] = inter(dq[i], dq[i+1]);
    }
    ret.back() = inter(dq[len-1], dq[0]);
    return ret;
};

double HalfPlaneIntersection::eps=1e-9;
double HalfPlaneIntersection::inf=1e9;
vector<HalfPlaneIntersection::Halfplane> V;
vector<HalfPlaneIntersection::Point> P;
for(i=0; i<n; i++){
    int c;
    scanf("%d", &c);
    HalfPlaneIntersection::Halfplane h;
    HalfPlaneIntersection::Point p;
    for(j=0; j<c; j++){
        scanf("%lf %lf", &p.x, &p.y);
        P.push_back(p);
    }
    for(j=0; j<c; j++){
        h=HalfPlaneIntersection::Halfplane(P[j], P[(j+1)%c]);
        V.push_back(h);
    }
    P.clear();
    P=HalfPlaneIntersection::hp_intersect(V);
    double ans=0;
    n=P.size();
    for(i=0; i<n; i++){
        ans=ans+P[i].x*P[(i+1)%n].y-P[i].y*P[(i+1)%n].x;
    }
    ans=ans/2;
}

```

4.3 Line Segment Intersection

```

struct pt {
    double x, y;
    bool operator<(const pt& p) const
    {
        return x<p.x-EPS|| (abs(x-p.x)<EPS && y < p.y- EPS);
    }
};

struct line {
    double a, b, c;
    line() {}
    line(pt p, pt q){
        a = p.y - q.y;
        b = q.x - p.x;
        c = -a * p.x - b * p.y;
        norm();
    }
    void norm(){
        double z = sqrt(a * a + b * b);
        if (abs(z) > EPS) a /= z, b /= z, c /= z;
    }
    double dist(pt p){ return a * p.x + b * p.y + c; }
};

double det(double a, double b, double c, double d){
    return a * d - b * c;
}

inline bool betw(double l, double r, double x){
    return min(l, r) <= x + EPS && x <= max(l, r)+EPS;
}

bool intersect_1d(double a, double b, dbl c, dbl d)

```

```

{
    if (a > b)
        swap(a, b);
    if (c > d)
        swap(c, d);
    return max(a, c) <= min(b, d) + EPS;
}bool
intersect(pt a,pt b,pt c,pt d,pt& left,pt& right){
    if (!intersect_1d(a.x, b.x, c.x, d.x) ||
        !intersect_1d(a.y, b.y, c.y, d.y))return false;
    line m(a, b); line n(c, d);
    double zn = det(m.a, m.b, n.a, n.b);
    if (abs(zn) < EPS) {
        if (abs(m.dist(c)) > EPS || abs(n.dist(a)) > EPS)
            return false;
        if (b < a) swap(a, b);
        if (d < c) swap(c, d);
        left = max(a, c); right = min(b, d);
        return true;
    } else {
        left.x = right.x = -det(m.c, m.b, n.c, n.b) / zn;
        left.y = right.y = -det(m.a, m.c, n.a, n.c) / zn;
        return betw(a.x,b.x,left.x)&&betw(a.y,b.y,left.y) &&
            betw(c.x, d.x, left.x) && betw(c.y, d.y, left.y);
    }
}

```

```

}
vector<pt> t;
void rec(int l, int r){
    if (r - l <= 3 && r - l >= 2){
        for (int i = l; i < r; ++i){
            for (int j = i + 1; j < r; ++j){
                for(int k=j+1;k<r;k++){
                    upd_ans(a[i],a[j],a[k]);
                }
            }
        }
        sort(a.begin() + l, a.begin() + r, cmp_y());
        return;
    }
    int m = (l + r) >> 1; int midx = a[m].x;
    rec(l, m); rec(m, r);
    merge(a.begin() + l, a.begin() + m, a.begin()
        + m, a.begin() + r, t.begin(), cmp_y());
    copy(t.begin(), t.begin() + r - l, a.begin()+l);
    int tsz = 0;
    for (int i = l; i < r; ++i){
        if (abs(a[i].x - midx) < mindist/2){
            for (int j = tsz - 1; j >= 0
                && a[i].y - t[j].y < mindist/2; --j){
                if(i+1<r) upd_ans(a[i],a[i+1],t[j]);
                if(j>0) upd_ans(a[i], t[j-1], t[j]);
            }
            t[tsz++] = a[i];
        }
    }
}

```

```

}
long long sqrLen() const{
    return this->dot(*this);
}
};
class pointLocationInPolygon{
    bool lexComp(const pt & l, const pt & r){
        return l.x < r.x || (l.x == r.x && l.y<r.y);
    }
    int sgn(long long val){
        return val > 0 ? 1 : (val == 0 ? 0 : -1);
    }
    vector<pt> seq; int n; pt translate;
    bool pointInTriangle(pt a, pt b, pt c,pt point){
        long long s1 = abs(a.cross(b, c));
        long long s2 = abs(point.cross(a, b)) +
            abs(point.cross(b, c)) + abs(point.cross(c, a));
        return s1 == s2;
    }
public:
    pointLocationInPolygon(){
        pointLocationInPolygon(vector<pt> & points){
            prepare(points);
        }
        void prepare(vector<pt> & points){
            seq.clear();
            n = points.size();
            int pos = 0;
            for(int i = 1; i < n; i++){
                if(lexComp(points[i], points[pos]))
                    pos = i;
            }
            translate.x=points[pos].x;
            translate.y=points[pos].y;
            rotate(points.begin(), points.begin() +
                pos, points.end());
            n--;
            seq.resize(n);
            for(int i = 0; i < n; i++)
                seq[i] = points[i + 1] - points[0];
        }
        bool pointInConvexPolygon(pt point){
            point.x-=translate.x;
            point.y-=translate.y;
            if(seq[0].cross(point) != 0 && sgn(seq[0].
                cross(point))!=sgn(seq[0].cross(seq[n-1])))
                return false;
            if(seq[n-1].cross(point)!=0&&sgn(seq[n-1]
                .cross(point))!=sgn(seq[n-1].cross(seq[0])))
                return false;
            if(seq[0].cross(point) == 0)
                return seq[0].sqrLen()>= point.sqrLen();
            int l = 0, r = n - 1;
            while(r - l > 1){
                int mid = (l + r)/2; int pos = mid;
                if(seq[pos].cross(point) >= 0)l = mid;
                else r = mid;
            }
            int pos = l;
            return pointInTriangle(seq[pos],
                seq[pos+1],pt(0,0),point);
        }
    }
}

```

4.4 Minimum Perimeter Triangle

```

#define MAX 300005
#define MOD 1000000007
#define SMOD 998244353
#define INF 6000000000000000000
#define EPS 0.0000000001
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
#include <ext/pb_ds/detail/standard_policies.hpp>
struct pt{
    double x, y;
    int id;
};
struct cmp_x{
    bool operator()(const pt & a, const pt & b)const{
        return a.x < b.x || (a.x == b.x && a.y<b.y);
    }
};
struct cmp_y{
    bool operator()(const pt& a, const pt & b)const{
        return a.y < b.y;
    }
};
int n; vector<pt> a; double mindist;
pair<int,pair<int, int> > best_pair;
void upd_ans(const pt & a, const pt & b,const pt&c){
    double distC = sqrt((a.x-b.x)*(a.x-b.x)
        +(a.y-b.y)*(a.y-b.y));
    double distA = sqrt((c.x - b.x)*(c.x - b.x)
        + (c.y - b.y)*(c.y - b.y));
    double distB = sqrt((a.x - c.x)*(a.x - c.x)
        + (a.y - c.y)*(a.y - c.y));
    if (distA + distB + distC < mindist){
        mindist = distA + distB + distC;
        best_pair=make_pair(a.id,make_pair(b.id,c.id));
    }
}

```

4.5 Minkowski

```

#define MAX 300005
#define BEGIN 1
#define CHAINS 18
#define NOT_VISITED 0
#define VISITING 1
#define VISITED 2
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
#include <ext/pb_ds/detail/standard_policies.hpp>
struct pt{
    long long x, y;
    pt() {}
    pt(long long _x, long long _y):x(_x), y(_y) {}
    pt operator+(const pt & p) const{
        return pt(x + p.x, y + p.y);
    }
    pt operator-(const pt & p) const{
        return pt(x - p.x, y - p.y);
    }
    long long cross(const pt & p) const{
        return x * p.y - y * p.x;
    }
    long long dot(const pt & p) const{
        return x * p.x + y * p.y;
    }
    long long cross(const pt & a, const pt&b)const{
        return (a - *this).cross(b - *this);
    }
    long long dot(const pt & a, const pt & b) const{
        return (a - *this).dot(b - *this);
    }
}

```

```

~pointLocationInPolygon(){
    seq.clear();
}
};
class Minkowski{
    static void reorder_polygon(vector<pt> & P){
        size_t pos = 0;
        for(size_t i = 1; i < P.size(); i++){
            if(P[i].y < P[pos].y ||
            (P[i].y == P[pos].y && P[i].x < P[pos].x))
                pos = i;
        }
        rotate(P.begin(), P.begin() + pos, P.end());
    }
public:
    vector<pt> minkowski(vector<pt> P,vector<pt>Q){
        // the first vertex must be the lowest
        reorder_polygon(P);
        reorder_polygon(Q);
        // we must ensure cyclic indexing
        P.push_back(P[0]);
        P.push_back(P[1]);
        Q.push_back(Q[0]);
        Q.push_back(Q[1]);
        // main part
        vector<pt> result;
        size_t i = 0, j = 0;
        while(i < P.size() - 2 || j < Q.size() - 2){
            result.push_back(P[i] + Q[j]);
            auto cross = (P[i + 1] - P[i]).cross(Q[j+1]-Q[j]);
            if(cross >= 0) ++i;
            if(cross <= 0) ++j;
        }
        return result;
    };
};

```

4.6 Pair of Intersecting Segments

```

#define MAX 100009
#define MAX_NODES 100005
struct pt {
    double x, y;
};
struct seg {
    pt p, q; int id;
    double get_y(double x) const {
        if (abs(p.x - q.x) < EPS)
            return p.y;
        return p.y+(q.y-p.y)*(x-p.x)/(q.x - p.x);
    }
};
bool intersect1d(double l1, dbl r1, dbl l2, dbl r2){
    if (l1 > r1) swap(l1, r1);
    if (l2 > r2) swap(l2, r2);
    return max(l1, l2) <= min(r1, r2) + EPS;
}
int vec(const pt& a, const pt& b, const pt& c) {
    double s = (b.x-a.x)*(c.y-a.y)-(b.y-a.y)*(c.x-a.x);
    return abs(s) < EPS ? 0 : s > 0 ? +1 : -1;
}
bool intersect(const seg& a, const seg& b){
    return intersect1d(a.p.x,a.q.x,b.p.x,b.q.x) &&

```

```

    intersect1d(a.p.y,a.q.y,b.p.y,b.q.y) &&
    vec(a.p,a.q,b.p)*vec(a.p,a.q,b.q) <= 0 &&
    vec(b.p, b.q, a.p)*vec(b.p,b.q,a.q) <= 0;
}
bool operator<(const seg& a, const seg& b){
    double x=max(min(a.p.x,a.q.x),min(b.p.x,b.q.x));
    return a.get_y(x) < b.get_y(x) - EPS;
}
struct event {
    double x; int tp, id;
    event() {}
    event(double x, int tp, int id):x(x),tp(tp),id(id){}
    bool operator<(const event& e) const {
        if (abs(x - e.x) > EPS) return x < e.x;
        return tp > e.tp;
    }
};
set<seg> s; vector<set<seg>::iterator> where;
set<seg>::iterator prev(set<seg>::iterator it) {
    return it == s.begin() ? s.end() : --it;
}
set<seg>::iterator next(set<seg>::iterator it) {
    return ++it;
}
pair<int, int> solve(const vector<seg>& a) {
    int n = (int)a.size(); vector<event> e;
    for (int i = 0; i < n; ++i) {
        e.push_back(event(min(a[i].p.x,a[i].q.x),+1,i));
        e.push_back(event(max(a[i].p.x,a[i].q.x),-1,i));
    }
    sort(e.begin(), e.end()); s.clear();
    where.resize(a.size());
    for (size_t i = 0; i < e.size(); ++i) {
        int id = e[i].id;
        if (e[i].tp == +1) {
            set<seg>::iterator nxt=
            s.lower_bound(a[id]),prv=prev(nxt);
            if(nxt!=s.end()&&intersect(*nxt,a[id]))
                return make_pair(nxt->id, id);
            if(prv!=s.end()&&intersect(*prv,a[id]))
                return make_pair(prv->id, id);
            where[id] = s.insert(nxt, a[id]);
        } else {
            set<seg>::iterator nxt = next(where[id])
            , prv = prev(where[id]);
            if (nxt != s.end() && prv != s.end() &&
            intersect(*nxt, *prv))
                return make_pair(prv->id, nxt->id);
            s.erase(where[id]);
        }
    }
    return make_pair(-1, -1);
}

```

4.7 Vertical Decomposition

```

#define MAX 300005
#define MOD 1000000007
#define GMAX 19
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
#include <ext/pb_ds/detail/standard_policies.hpp>
typedef double dbl;

```

```

const dbl eps = 1e-9;
inline bool eq(dbl x, dbl y){
    return fabs(x - y) < eps;
}
inline bool lt(dbl x, dbl y){
    return x < y - eps;
}
inline bool gt(dbl x, dbl y){
    return x > y + eps;
}
inline bool le(dbl x, dbl y){
    return x < y + eps;
}
inline bool ge(dbl x, dbl y){
    return x > y - eps;
}
struct pt{
    dbl x, y;
    inline pt operator - (const pt & p)const{
        return pt{x - p.x, y - p.y};
    }
    inline pt operator + (const pt & p)const{
        return pt{x + p.x, y + p.y};
    }
    inline pt operator * (dbl a)const{
        return pt{x * a, y * a};
    }
    inline dbl cross(const pt & p)const{
        return x * p.y - y * p.x;
    }
    inline dbl dot(const pt & p)const{
        return x * p.x + y * p.y;
    }
    inline bool operator == (const pt & p)const{
        return eq(x, p.x) && eq(y, p.y);
    }
};
struct Line{
    pt p[2];
    Line(){} Line(pt a, pt b):p{a, b}{}
    pt vec()const{
        return p[1] - p[0];
    }
    pt& operator [] (size_t i){
        return p[i];
    }
};
inline bool lexComp(const pt & l, const pt & r){
    if(fabs(l.x - r.x) > eps){
        return l.x < r.x;
    }
    else return l.y < r.y;
}
vector<pt> interSegSeg(Line l1, Line l2){
    if(eq(l1.vec().cross(l2.vec()), 0)){
        if(!eq(l1.vec().cross(l2[0] - l1[0]), 0))
            return {};
        if(!lexComp(l1[0], l1[1]))
            swap(l1[0], l1[1]);
        if(!lexComp(l2[0], l2[1]))
            swap(l2[0], l2[1]);
        pt l = lexComp(l1[0], l2[0]) ? l2[0] : l1[0];
    }
}

```



```

    pt r = lexComp(l1[1], l2[1]) ? l1[1] : l2[1];
    if(l == r)
        return {l};
    else
        return lexComp(l,r)?vector<pt>{l,r}:vector<pt>();
    }
    else{
        dbl s = (l2[0] - l1[0]).cross(l2.vec()) /
            l1.vec().cross(l2.vec());
        pt inter = l1[0] + l1.vec() * s;
        if(ge(s, 0) && le(s, 1) && le((l2[0] -
            inter).dot(l2[1] - inter), 0))
            return {inter};
        else
            return {};
    }
}
char get_segtype(Line segment, pt other_point){
    if(eq(segment[0].x, segment[1].x))
        return 0;
    if(!lexComp(segment[0], segment[1]))
        swap(segment[0], segment[1]);
    return (segment[1]-segment[0]).cross(other_point
        - segment[0]) > 0 ? 1 : -1;
}
dbl union_area(vector<tuple<pt, pt,pt> > triangles){
    vector<Line> segments(3 * triangles.size());
    vector<char> segtype(segments.size());
    for(size_t i = 0; i < triangles.size(); i++){
        pt a, b, c; tie(a, b, c) = triangles[i];
        segments[3*i]=lexComp(a,b)?Line(a,b):Line(b,a);
        segtype[3 * i] = get_segtype(segments[3 * i], c);
        segments[3*i+1]=lexComp(b,c)?Line(b,c):Line(c,b);
        segtype[3*i+1]=get_segtype(segments[3*i+1],a);
        segments[3*i+2]=lexComp(c,a)?Line(c,a):Line(a,c);
        segtype[3*i+2]=get_segtype(segments[3*i+2],b);
    }
    vector<dbl> k(segments.size()), b(segments.size());
    for(size_t i = 0; i < segments.size(); i++){
        if(segtype[i]){
            k[i]=(segments[i][1].y-segments[i][0].y)
                /(segments[i][1].x-segments[i][0].x);
            b[i]=segments[i][0].y-k[i]*segments[i][0].x;
        }
    }
    dbl ans = 0;
    for(size_t i = 0; i < segments.size(); i++){
        if(!segtype[i]) continue;
        dbl l = segments[i][0].x,r=segments[i][1].x;
        vector<pair<dbl, int> > evts;
        for(size_t j = 0; j < segments.size(); j++){
            if(!segtype[j] || i == j)
                continue;
            dbl l1 = segments[j][0].x, r1 = segments[j][1].x;
            if(ge(l1, r) || ge(l, r1))
                continue;
            dbl common_l = max(l, l1), common_r = min(r, r1);
            auto pts = interSegSeg(segments[i], segments[j]);
            if(pts.empty()){
                dbl y1l = k[j] * common_l + b[j];
                dbl yl = k[i] * common_l + b[i];
                if(lt(y1l, yl) == (segtype[i] == 1)){

```

```

                    int evt_type = -segtype[i] * segtype[j];
                    evts.emplace_back(common_l, evt_type);
                    evts.emplace_back(common_r, -evt_type);
                }
            }
            else if(pts.size() == 1u){
                dbl yl=k[i]*common_l+b[i],y1l=k[j]*common_l+b[j];
                int evt_type = -segtype[i] * segtype[j];
                if(lt(y1l, yl) == (segtype[i] == 1)){
                    evts.emplace_back(common_l, evt_type);
                    evts.emplace_back(pts[0].x, -evt_type);
                }
                yl =k[i]*common_r+b[i],y1l=k[j]*common_r+b[j];
                if(lt(y1l, yl) == (segtype[i] == 1)){
                    evts.emplace_back(pts[0].x, evt_type);
                    evts.emplace_back(common_r, -evt_type);
                }
            }
            else{
                if(segtype[j] != segtype[i] || j>i){
                    evts.emplace_back(common_l, -2);
                    evts.emplace_back(common_r, 2);
                }
            }
        }
        evts.emplace_back(l, 0);
        sort(evts.begin(), evts.end());
        size_t j = 0; int balance = 0;
        while(j < evts.size()){
            size_t ptr = j;
            while(ptr < evts.size() &&
                eq(evts[j].first, evts[ptr].first)){
                balance += evts[ptr].second;
                ++ptr;
            }
            if(!balance && !eq(evts[j].first, r)){
                if(!balance && !eq(evts[j].first, r)){
                    ans -= segtype[i] * (k[i] * (next_x
                        +evts[j].first)+2*b[i])*(next_x-evts[j].first);
                }
                j = ptr;
            }
        }
        return ans/2;
    }
}

```

4.8 common tangent

```

struct pt {
    double x, y;
    pt operator- (pt p) {
        pt res = { x-p.x, y-p.y };
        return res;
    }
};
struct circle : pt {
    double r;
};
struct line {
    double a, b, c;
};
const double EPS = 1E-9;
double sqr (double a) {

```

```

    return a * a;
}
void tangents (pt c, double r1, double r2,
    vector<line> & ans) {
    double r = r2 - r1;
    double z = sqr(c.x) + sqr(c.y);
    double d = z - sqr(r);
    if (d < -EPS) return;
    d = sqrt (abs (d));
    line l; l.a = (c.x * r + c.y * d) / z;
    l.b = (c.y * r - c.x * d) / z; l.c = r1;
    ans.push_back (l);
}
vector<line> tangents (circle a, circle b) {
    vector<line> ans;
    for (int i=-1; i<=1; i+=2)
        for (int j=-1; j<=1; j+=2)
            tangents (b-a, a.r*i, b.r*j, ans);
    for (size_t i=0; i<ans.size(); ++i)
        ans[i].c -= ans[i].a * a.x + ans[i].b * a.y;
    return ans;
}

```

5 Graph

5.1 Articulation Vertex

```

int n; // number of nodes
vector<vector<int>> adj; // adjacency list of graph

vector<bool> visited;
vector<int> tin, low;
int timer;

void dfs(int v, int p = -1) {
    visited[v] = true;
    tin[v] = low[v] = timer++;
    int children=0;
    for (int to : adj[v]) {
        if (to == p) continue;
        if (visited[to]) {
            low[v] = min(low[v], tin[to]);
        } else {
            dfs(to, v);
            low[v] = min(low[v], low[to]);
            if (low[to] >= tin[v] && p!=-1)
                IS_CUTPOINT(v);
            ++children;
        }
    }
    if(p == -1 && children > 1)
        IS_CUTPOINT(v);
}

void find_cutpoints() {
    timer = 0;
    visited.assign(n, false);
    tin.assign(n, -1);
    low.assign(n, -1);
    for (int i = 0; i < n; ++i) {
        if (!visited[i])
            dfs (i);
    }
}

```


5.2 Strongly Connected Components

```
vector<vector<int>> adj, adj_rev;
vector<bool> used;
vector<int> order, component;

void dfs1(int v) {
    used[v] = true;
    for (auto u : adj[v])
        if (!used[u])
            dfs1(u);
    order.push_back(v);
}

void dfs2(int v) {
    used[v] = true;
    component.push_back(v);
    for (auto u : adj_rev[v])
        if (!used[u])
            dfs2(u);
}

int main() {
    int n;
    // ... read n ...

    for (;;) {
        int a, b;
        // ... read next directed edge (a,b) ...
        adj[a].push_back(b);
        adj_rev[b].push_back(a);
    }

    used.assign(n, false);
    for (int i = 0; i < n; i++)
        if (!used[i])
            dfs1(i);

    used.assign(n, false);
    reverse(order.begin(), order.end());
    for (auto v : order)
        if (!used[v]) {
            dfs2(v);
            // ... processing next component ...
            component.clear();
        }
}
```

6 Math

6.1 Discrete Root

```
#define MAX 100000
int prime[MAX+1], Phi[MAX+1];
void sieve(){
    int i, j;
    for(i=2; i*i<=MAX; i++){
        if(prime[i]) continue;
        for(j=i; j*i<=MAX; j++){
            if(prime[i*j]==0) prime[i*j]=i;
        }
    }
}
```

```
}
}

void PhiWithSieve(){
    int i;
    for(i=2; i<=MAX; i++){
        if(prime[i]==0){
            Phi[i]=i-1;
        }
        else if((i/prime[i])%prime[i]==0){
            Phi[i]=Phi[i/prime[i]]*prime[i];
        }
        else{
            Phi[i]=Phi[i/prime[i]]*(prime[i]-1);
        }
    }
}

int gcd(int a, int b){
    if(b==0) return a;
    else return gcd(b, a%b);
}

int powmod (int a, int b, int p) {
    int res = 1;
    while (b)
        if (b & 1)
            res = int (res * 111 * a % p), --b;
        else
            a = int (a * 111 * a % p), b >>= 1;
    return res;
}

int PrimitiveRoot(int p){
    vector<int> fact;
    int phi=Phi[p];
    int n=phi;
    while(n>1){
        if(prime[n]==0){
            fact.push_back(n);
            n=1;
        }
        else{
            int f=prime[n];
            while(n%f==0){
                n=n/f;
            }
            fact.push_back(f);
        }
    }
    int res;
    for(res=p-1; res>1; --res){
        for(n=0; n<fact.size(); n++){
            if(powmod(res, phi/fact[n], p)==1){
                break;
            }
        }
        if(n>=fact.size()) return res;
    }
    return -1;
}

int DiscreteLog(int a, int b, int m) {
    a %= m, b %= m;
    int n = sqrt(m) + 1;
    map<int, int> vals;
    for (int p = 1; p <= n; ++p)
        vals[powmod(a, (int) (111 * p * n) % m, m)] = p;
}
```

```
for (int q = 0; q <= n; ++q) {
    int cur = (powmod(a, q, m) * 111 * b) % m;
    if (vals.count(cur)) {
        int ans = vals[cur] * n - q;
        return ans;
    }
}
return -1;
}

vector<int> DiscreteRoot(int n, int a, int k){
    int g = PrimitiveRoot(n);
    vector<int> ans;
    int any_ans = DiscreteLog(powmod(g, k, n), a, n);
    if (any_ans == -1){
        return ans;
    }
    int delta = (n-1) / gcd(k, n-1);
    for(int cur=any_ans%delta; cur<n-1; cur+=delta)
        ans.push_back(powmod(g, cur, n));
    sort(ans.begin(), ans.end());
    return ans;
}
```

6.2 Fast Fourier Transform

```
#define MOD 1000000007
#define MAX 200005
#define PMAX 55
#define PRECISION 0.000001
#define INF 2000000000
using cd = complex<double>;
const double PI = acos(-1);
void fft(vector<cd>& a, bool invert){
    int n = a.size();
    for(int i = 1, j = 0; i < n; i++){
        int bit = n>>1;
        for(; j&bit; bit>>=1){
            j^=bit;
        }
        j ^= bit;
        if(i < j) swap(a[i], a[j]);
    }
    for(int len = 2; len <= n; len <= 1){
        double ang = 2*PI/len*(invert ? -1 : 1);
        cd wlen(cos(ang), sin(ang));
        for(int i = 0; i < n; i += len){
            cd w(1);
            for(int j = 0; j < len/2; j++){
                cd u = a[i+j], v = a[i+j+len/2]*w;
                a[i+j] = u+v;
                a[i+j+len/2] = u-v;
                w *= wlen;
            }
        }
    }
    if(invert){
        for(cd &x: a) x /= n;
    }
}

vector<int> multiply(vector<int>&a, vector<int>&b){
    vector<cd> fa(a.begin(), a.end());
    vector<cd> fb(b.begin(), b.end());
    int n = 1;
    while(n < a.size()+b.size()) n <= 1;
    fa.resize(n); fb.resize(n);
}
```

```

fft(fa, false);
fft(fb, false);
for(int i = 0; i < n; i++)
    fa[i] *= fb[i];
fft(fa, true);
vector<int> result(n);
for(int i = 0; i < n; i++)
    result[i] = round(fa[i].real());
return result;
}

//Number Theoretic Transformation
ll gcd(ll a, ll b){
    if(b==0) return a;
    else return gcd(b, a%b);
}

ll egcd(ll a, ll b, ll & x, ll & y) {
    if (a == 0) {
        x = 0; y = 1;
        return b;
    }
    ll x1, y1; ll d = egcd(b % a, a, x1, y1);
    x = y1 - (b / a) * x1; y = x1;
    return d;
}

ll ModuloInverse(ll a, ll n){
    ll x, y; x=gcd(a, n);
    a=a/x; n=n/x;
    ll res = egcd(a, n, x, y); x=(x%n+n)%n;
    return x;
}

const int mod = 998244353;
const int root = 15311432;
const int root_1 = 469870224;
const int root_pw = 1 << 23;
void fft(vector<int> & a, bool invert) {
    int n = a.size();
    for (int i = 1, j = 0; i < n; i++) {
        int bit = n >> 1;
        for (; j & bit; bit >>= 1)
            j ^= bit;
        if (i < j)
            swap(a[i], a[j]);
    }
    for (int len = 2; len <= n; len <= 1) {
        int wlen = invert ? root_1 : root;
        for (int i = len; i < root_pw; i <= 1)
            wlen = (int)(1LL * wlen * wlen % mod);
        for (int i = 0; i < n; i += len) {
            int w = 1;
            for (int j = 0; j < len / 2; j++) {
                int u = a[i+j], v = (int)(1LL * a[i+j+len/2] * w % mod);
                a[i+j] = u + v < mod ? u + v : u + v - mod;
                a[i+j+len/2] = u - v >= 0 ? u - v : u - v + mod;
                w = (int)(1LL * w * wlen % mod);
            }
        }
    }
    if (invert) {
        int n_1 = (int) ModuloInverse(n, mod);
        for (int & x : a)
            x = (int)(1LL * x * n_1 % mod);
    }
}

```

```

}
vector<int> multiply(vector<int> &a, vector<int> &b){
    vector<int> fa(a.begin(), a.end());
    vector<int> fb(b.begin(), b.end());
    int n = 1;
    while(n < a.size()+b.size())
        n <= 1;
    fa.resize(n); fb.resize(n);
    fft(fa, false); fft(fb, false);
    for(int i = 0; i < n; i++)
        fa[i] = (int) (1LL * fa[i] * fb[i] % mod);
    fft(fa, true);
    vector<int> result(n);
    for(int i = 0; i < n; i++)
        result[i] = fa[i];
    return result;
}

```

6.3 Polynomial Algebra

```

namespace algebra {
    const int inf = 1e9; const int magic = 500;
    // threshold for sizes to run the naive algo
    namespace fft {
        const int maxn = 1 << 18;
        typedef double ftype;
        typedef complex<ftype> point;
        point w[maxn];
        const ftype pi = acos(-1);
        bool initiated = 0;
        void init() {
            if(!initiated) {
                for(int i = 1; i < maxn; i *= 2) {
                    for(int j = 0; j < i; j++) {
                        w[i + j] = polar(ftype(1), pi * j / i);
                    }
                }
                initiated = 1;
            }
        }
    }
    template<typename T>
    void fft(T *in, point *out, int n, int k = 1) {
        if(n == 1) {
            *out = *in;
        } else {
            n /= 2;
            fft(in, out, n, 2 * k);
            fft(in + k, out + n, n, 2 * k);
            for(int i = 0; i < n; i++) {
                auto t = out[i + n] * w[i + n];
                out[i + n] = out[i] - t;
                out[i] += t;
            }
        }
    }
    template<typename T>
    void mul_slow(vector<T> &a, const vector<T> &b){
        vector<T> res(a.size() + b.size() - 1);
        for(size_t i = 0; i < a.size(); i++) {
            for(size_t j = 0; j < b.size(); j++) {
                res[i + j] += a[i] * b[j];
            }
        }
        a = res;
    }
}

```

```

}
template<typename T>
void mul(vector<T> &a, const vector<T> &b) {
    if(min(a.size(), b.size()) < magic) {
        mul_slow(a, b);
        return;
    }
    init();
    static const int shift = 15, mask = (1 << shift) - 1;
    size_t n = a.size() + b.size() - 1;
    while(__builtin_popcount(n) != 1) {
        n++;
    }
    a.resize(n);
    static point A[maxn], B[maxn];
    static point C[maxn], D[maxn];
    for(size_t i = 0; i < n; i++) {
        A[i] = point(a[i] & mask, a[i] >> shift);
        if(i < b.size()) {
            B[i] = point(b[i] & mask, b[i] >> shift);
        } else {
            B[i] = 0;
        }
    }
    fft(A, C, n); fft(B, D, n);
    for(size_t i = 0; i < n; i++) {
        point c0 = C[i] + conj(C[(n - i) % n]);
        point c1 = C[i] - conj(C[(n - i) % n]);
        point d0 = D[i] + conj(D[(n - i) % n]);
        point d1 = D[i] - conj(D[(n - i) % n]);
        A[i] = c0 * d0 - point(0, 1) * c1 * d1;
        B[i] = c0 * d1 + d0 * c1;
    }
    fft(A, C, n); fft(B, D, n);
    reverse(C + 1, C + n);
    reverse(D + 1, D + n);
    int t = 4 * n;
    for(size_t i = 0; i < n; i++) {
        int64_t A0 = llround(real(C[i]) / t);
        T A1 = llround(imag(D[i]) / t);
        T A2 = llround(imag(C[i]) / t);
        a[i] = A0 + (A1 << shift) + (A2 << 2 * shift);
    }
    return;
}
}
template<typename T>
T bpow(T x, size_t n) {
    return n ? n % 2 ? x * bpow(x, n - 1) : bpow(x * x, n / 2) : T(1);
}
template<typename T>
T bpow(T x, size_t n, T m) {
    return n ? n % 2 ? x * bpow(x, n - 1, m) :
        bpow(x * x, n / 2, m) : T(1);
}
template<typename T>
T gcd(const T &a, const T &b) {
    return b == T(0) ? a : gcd(b, a % b);
}
template<typename T>
T nCr(T n, int r) { // runs in O(r)
    T res(1);
    for(int i = 0; i < r; i++) {

```

```

    res *= (n - T(i));
    res /= (i + 1);
}
return res;
}
template<int m>
struct modular {
    int64_t r;
    modular() : r(0) {}
    modular(int64_t rr) : r(rr) {
        if(abs(r) >= m) r %= m; if(r < 0) r += m;
        modular inv() const {return bpow(*this, m - 2);}
        modular operator*(modular&t){return(r*t.r)%m;}
        modular operator/(modular&t){return*this*t.inv();}
        modular operator+=(modular&t){r+=t.r;
        if(r>=m) r-=m; return *this;}
        modular operator -= (modular &t) {
            r -= t.r; if(r < 0) r += m; return *this;}
        modular operator + (modular &t) {
            return modular(*this) += t;}
        modular operator - (modular &t) {
            return modular(*this) -= t;}
        modular operator *= (modular &t) {
            return *this = *this * t;}
        modular operator /= (modular &t) {
            return *this = *this / t;}
        bool operator==(modular&t){return r==t.r;}
        bool operator!=(modular&t){return r != t.r;}
        operator int64_t() const {return r;}
};
template<int T>
istream& operator>> (istream &in, modular<T> &x){
    return in >> x.r;
}
template<typename T>
struct poly {
    vector<T> a;
    void normalize() { // get rid of leading zeroes
        while(!a.empty() && a.back() == T(0)) {
            a.pop_back();
        }
    }
    poly(){}
    poly(T a0) : a{a0}{normalize();}
    poly(vector<T> t) : a(t){normalize();}
    poly operator += (const poly &t) {
        a.resize(max(a.size(), t.a.size()));
        for(size_t i = 0; i < t.a.size(); i++) {
            a[i] += t.a[i];
        }
        normalize();
        return *this;
    }
    poly operator -= (const poly &t) {
        a.resize(max(a.size(), t.a.size()));
        for(size_t i = 0; i < t.a.size(); i++) {
            a[i] -= t.a[i];
        }
        normalize();
        return *this;
    }
    poly operator+(poly &t){return poly(*this)+=t;}
    poly operator-(poly &t){return poly(*this)-=t;}

```

```

    poly mod_xk(size_t k){
        // get same polynomial mod x^k
        k = min(k, a.size());
        return vector<T>(begin(a), begin(a) + k);
    }
    poly mul_xk(size_t k) const { // multiply by x^k
        poly res(*this);
        res.a.insert(begin(res.a), k, 0);
        return res;
    }
    poly div_xk(size_t k) const {
        // divide by x^k, dropping coefficients
        k = min(k, a.size());
        return vector<T>(begin(a) + k, end(a));
    }
    poly substr(size_t l, size_t r) const {
        // return mod_xk(r).div_xk(l)
        l = min(l, a.size());
        r = min(r, a.size());
        return vector<T>(begin(a) + l, begin(a) + r);
    }
    poly inv(size_t n) const {
        // get inverse series mod x^n
        assert(!is_zero());
        poly ans = a[0].inv();
        size_t a = 1;
        while(a < n) {
            poly C = (ans*mod_xk(2*a)).substr(a, 2 * a);
            ans -= (ans * C).mod_xk(a).mul_xk(a);
            a *= 2;
        }
        return ans.mod_xk(n);
    }
    poly operator *= (const poly &t)
    {fft::mul(a, t.a); normalize(); return *this;}
    poly operator * (const poly &t) const
    {return poly(*this) *= t;}
    poly reverse(size_t n, bool rev = 0) const {
        // reverses and leaves only n terms
        poly res(*this);
        if(rev) { // If rev = 1 then tail goes to head
            res.a.resize(max(n, res.a.size()));
        }
        std::reverse(res.a.begin(), res.a.end());
        return res.mod_xk(n);
    }
    pair<poly, poly> divmod_slow(const poly &b)
    const { // when divisor or quotient is small
        vector<T> A(a);
        vector<T> res;
        while(A.size() >= b.a.size()) {
            res.push_back(A.back() / b.a.back());
            if(res.back() != T(0)) {
                for(size_t i = 0; i < b.a.size(); i++) {
                    A[A.size() - i - 1] -= res.back() *
                        b.a[b.a.size() - i - 1];
                }
            }
            A.pop_back();
        }
        std::reverse(begin(res), end(res));
        return {res, A};
    }

```

```

}
pair<poly, poly> divmod(const poly &b) const
{ // returns quotient and remainder of a mod b
    if(deg() < b.deg()) {
        return {poly{0}, *this};
    }
    int d = deg() - b.deg();
    if(min(d, b.deg()) < magic) {
        return divmod_slow(b);
    }
    poly D = (reverse(d + 1) * b.reverse(d + 1).
        inv(d + 1)).mod_xk(d + 1).reverse(d + 1, 1);
    return {D, *this - D * b};
}
poly operator/(poly&t){return divmod(t).first;}
poly operator%(poly&t){return divmod(t).second;}
poly operator *= (T &x) {
    for(auto &it: a) {
        it *= x;
    }
    normalize();
    return *this;
}
poly operator /= (const T &x) {
    for(auto &it: a) {
        it /= x;
    }
    normalize();
    return *this;
}
poly operator *(T &x){return poly(*this) *= x;}
poly operator /(T &x){return poly(*this) /= x;}
void print() const {
    for(auto it: a) {
        cout << it << ' ';
    }
    cout << endl;
}
T eval(T x){ // evaluates in single point x
    T res(0);
    for(int i = int(a.size()) - 1; i >= 0; i--) {
        res *= x;
        res += a[i];
    }
    return res;
}
T& lead() { // leading coefficient
    return a.back();
}
int deg() const { // degree
    return a.empty() ? -inf : a.size() - 1;
}
bool is_zero() const { // is polynomial zero
    return a.empty();
}
T operator [] (int idx) const {
    return idx>=(int)a.size()||idx<0?T(0):a[idx];
}
T& coef(size_t idx) {
    // mutable reference at coefficient
    return a[idx];
}
bool operator == (poly &t) {return a == t.a;}

```

```

bool operator != ( poly &t) {return a != t.a;}
poly deriv() { // calculate derivative
    vector<T> res;
    for(int i = 1; i <= deg(); i++) {
        res.push_back(T(i) * a[i]);
    }
    return res;
}
poly integr() { // calculate integral with C = 0
    vector<T> res = {0};
    for(int i = 0; i <= deg(); i++) {
        res.push_back(a[i] / T(i + 1));
    }
    return res;
}
size_t leading_xk() const {
    // Let p(x) = x^k * t(x), return k
    if(is_zero()) {
        return inf;
    }
    int res = 0;
    while(a[res] == T(0)) {
        res++;
    }
    return res;
}
poly log(size_t n){//calculate log p(x) mod x^n
    assert(a[0] == T(1));
    return (deriv().mod_xk(n)*inv(n))
        .integr().mod_xk(n);
}
poly exp(size_t n){//calculate exp p(x) mod x^n
    if(is_zero()) {
        return T(1);
    }
    assert(a[0] == T(0));
    poly ans = T(1); size_t a = 1;
    while(a < n) {
        poly C=ans.log(2*a).div_xk(a)-substr(a,2*a);
        ans -= (ans * C).mod_xk(a).mul_xk(a);
        a *= 2;
    }
    return ans.mod_xk(n);
}
poly pow_slow(size_t k,size_t n){//if k is small
    return k%2?(*this*pow_slow(k-1,n)).mod_xk(n)
:(*this* *this).mod_xk(n).pow_slow(k/2,n):T(1);
}
poly pow(size_t k,size_t n){
    //calculate p^k(n) mod x^n
    if(is_zero()) return *this;
    if(k < magic) return pow_slow(k, n);
    int i = leading_xk();
    T j = a[i];
    poly t = div_xk(i) / j;
    return bpow(j,k)*(t.log(n)*T(k))
        .exp(n).mul_xk(i*k).mod_xk(n);
}
poly mulx(T x){
    //component-wise multiplication with x^k
    T cur = 1; poly res(*this);
    for(int i = 0; i <= deg(); i++) {
        res.coef(i) *= cur;

```

```

        cur *= x;
    }
    return res;
}
poly mulx_sq(T x){
    //component-wise multiplication with x^{k^2}
    T cur = x; T total = 1; T xx = x * x;
    poly res(*this);
    for(int i = 0; i <= deg(); i++) {
        res.coef(i) *= total;
        total *= cur;
        cur *= xx;
    }
    return res;
}
vector<T> chirpz_even(T z, int n) {
    // P(1), P(z^2), P(z^4), ..., P(z^{2(n-1)})
    int m = deg();
    if(is_zero()) {
        return vector<T>(n, 0);
    }
    vector<T> vv(m + n);
    T zi = z.inv(); T zz = zi * zi;
    T cur = zi; T total = 1;
    for(int i = 0; i <= max(n - 1, m); i++) {
        if(i <= m) {vv[m - i] = total;}
        if(i < n) {vv[m + i] = total;}
        total *= cur;
        cur *= zz;
    }
}
poly w=(mulx_sq(z)*vv).substr(m,m+n).mulx_sq(z);
vector<T> res(n);
for(int i = 0; i < n; i++) {
    res[i] = w[i];
}
return res;
}
vector<T> chirpz(T z, int n) {
    // P(1), P(z), P(z^2), ..., P(z^{(n-1)})
    auto even = chirpz_even(z, (n + 1) / 2);
    auto odd = mulx(z).chirpz_even(z, n / 2);
    vector<T> ans(n);
    for(int i = 0; i < n / 2; i++) {
        ans[2 * i] = even[i];
        ans[2 * i + 1] = odd[i];
    }
    if(n % 2 == 1) {
        ans[n - 1] = even.back();
    }
    return ans;
}
template<typename iter>
vector<T> eval(vector<poly>&tree,int v,iter l,iter
r,iter ly,iter ry){//auxiliary evaluation function
    if(r - l == 1) {
        return {*ly / a[0]};
    } else {
        auto m = 1 + (r - l) / 2;
        auto my = ly + (ry - ly) / 2;
        auto A = (*this % tree[2 * v]).
            inter(tree, 2 * v, l, m, ly, my);
        auto B = (*this % tree[2 * v + 1]).
            inter(tree, 2 * v + 1, m, r, my, ry);
        return A * tree[2 * v + 1] + B * tree[2*v];
    }
}
};
template<typename T>
poly<T> operator * (const T& a,const poly<T>&b){
    return b * a;
}
template<typename T>
poly<T> xk(int k) { // return x^k
    return poly<T>{1}.mul_xk(k);
}
template<typename T>
T resultant(poly<T>a,poly<T>b){
    //computes resultant of a and b
    if(b.is_zero()) {
        return 0;
    } else if(b.deg() == 0) {
        return bpow(b.lead(), a.deg());
    } else {
        int pw = a.deg();
        a %= b; pw -= a.deg();
        T mul=bpow(b.lead(),pw)*T((b.deg()&a.deg()&1)
?-1:1); T ans= resultant(b, a);
        return ans * mul;
    }
}
template<typename iter>
poly<typename iter::value_type> kmul(iter L,
iter R) { // computes
//(x-a1)(x-a2)...(x-an) without building tree
    if(R - L == 1) {
        return vector<typename iter::value_type>{-*L,1};
    } else {
        iter M = L + (R - L) / 2;
        return kmul(L, M) * kmul(M, R);
    }
}

```

```

}
vector<T> eval(vector<T> x) {
    // evaluate polynomial in (x1, ..., xn)
    int n = x.size();
    if(is_zero()) {
        return vector<T>(n, T(0));
    }
    vector<poly> tree(4 * n);
    build(tree, 1, begin(x), end(x));
    return eval(tree, 1, begin(x), end(x));
}
template<typename iter>
poly inter(vector<poly>&tree,int v,iter l,iter
r,iter ly,iter ry){//axiliary interpolation function
    if(r - l == 1) {
        return {*ly / a[0]};
    } else {
        auto m = 1 + (r - l) / 2;
        auto my = ly + (ry - ly) / 2;
        auto A = (*this % tree[2 * v]).
            inter(tree, 2 * v, l, m, ly, my);
        auto B = (*this % tree[2 * v + 1]).
            inter(tree, 2 * v + 1, m, r, my, ry);
        return A * tree[2 * v + 1] + B * tree[2*v];
    }
}
};
template<typename T>
poly<T> operator * (const T& a,const poly<T>&b){
    return b * a;
}
template<typename T>
poly<T> xk(int k) { // return x^k
    return poly<T>{1}.mul_xk(k);
}
template<typename T>
T resultant(poly<T>a,poly<T>b){
    //computes resultant of a and b
    if(b.is_zero()) {
        return 0;
    } else if(b.deg() == 0) {
        return bpow(b.lead(), a.deg());
    } else {
        int pw = a.deg();
        a %= b; pw -= a.deg();
        T mul=bpow(b.lead(),pw)*T((b.deg()&a.deg()&1)
?-1:1); T ans= resultant(b, a);
        return ans * mul;
    }
}
template<typename iter>
poly<typename iter::value_type> kmul(iter L,
iter R) { // computes
//(x-a1)(x-a2)...(x-an) without building tree
    if(R - L == 1) {
        return vector<typename iter::value_type>{-*L,1};
    } else {
        iter M = L + (R - L) / 2;
        return kmul(L, M) * kmul(M, R);
    }
}

```



```

template<typename T, typename iter>
poly<T> build(vector<poly<T>> &res, int v, iter L,
              iter R){//
// builds evaluation tree for (x-a1)(x-a2)...(x-an)
if(R - L == 1) {
    return res[v] = vector<T>{-*L, 1};
} else {
    iter M = L + (R - L) / 2;
    return res[v] = build(res, 2 * v, L, M)
        * build(res, 2 * v + 1, M, R);
}
}
template<typename T>
poly<T> inter(vector<T>x,vector<T>y){//interpo-
//lates minimum polynomial from (xi, yi) pairs
int n = x.size();
vector<poly<T>> tree(4 * n);
return build(tree, 1, begin(x), end(x)).deriv()
.inter(tree, 1, begin(x), end(x), begin(y), end(y));
};
using namespace algebra;
const int mod = 1e9 + 7;
typedef modular<mod> base;
typedef poly<base> polyn;
using namespace algebra;
signed main() {
    int n = 100000;
    polyn a; vector<base> x;
    for(int i = 0; i <= n; i++) {
        a.a.push_back(1 + rand() % 100);
        x.push_back(1 + rand() % (2 * n));
    }
    sort(begin(x), end(x));
    x.erase(unique(begin(x), end(x)), end(x));
    auto b = a.eval(x);
    cout << clock() / double(CLOCKS_PER_SEC) << endl;
    auto c = inter(x, b);
    polyn md = kmul(begin(x), end(x));
    cout << clock() / double(CLOCKS_PER_SEC) << endl;
    assert(c == a % md);
}

```

6.4 all comb

```

vector<int> ans;
void gen(int n, int k, int idx, bool rev) {
    if (k > n || k < 0) return;
    if (!n) {
        for (int i = 0; i < idx; ++i) {
            if (ans[i])
                cout << i + 1;
        }
        cout << "\n";
        return;
    }
    ans[idx] = rev;
    gen(n - 1, k - rev, idx + 1, false);
    ans[idx] = !rev;
    gen(n - 1, k - !rev, idx + 1, true);
}
void all_combinations(int n, int k) {
    ans.resize(n);
    gen(n, k, 0, false);
}

```

```

}
6.5 gauss elimination
/* n rows/equations, m+1 columns, m variables
calculates determinant, rank and ans[] ->value
for variables
returns {0, 1, INF} -> number of solutions */
const double EPS = 1e-9;
#define MAX 105
#define INF 100000000
int where[MAX], Rank;
double Det;
int gauss(double a[MAX][MAX],
           double ans[MAX], int n, int m) {
    Det = 1.0, Rank = 0;
    memset(where, -1, sizeof(where));
    for(int col=0, row = 0; col<m&&row < n; ++col) {
        int sel = row;
        for(int i = row+1; i < n; ++i)
            if(fabs(a[i][col])>fabs(a[sel][col])) sel=i;
        if(fabs(a[sel][col])<EPS) {Det=0.0; continue;}
        for(int j=0;j<=m;++j)swap(a[sel][j],a[row][j]);
        if(row != sel) Det = -Det;
        Det *= a[row][col];
        where[col] = row;
        double s = (1.0 / a[row][col]);
        for(int j = 0; j <= m; ++j) a[row][j] *= s;
        for(int i = 0; i < n; ++i) if (i != row &&
            fabs(a[i][col]) > EPS) {
                double t = a[i][col];
                for(int j = 0; j <= m; ++j)
                    a[i][j] -= a[row][j] * t;
            }
        ++row, ++Rank;
    }
    for(int i = 0; i < m; ++i)
        ans[i] = (where[i] == -1) ? 0.0 : a[where[i]][m];
    for(int i = Rank; i < n; ++i)
        if(fabs(a[i][m]) > EPS) return 0;
    for(int i = 0; i < m; ++i)
        if(where[i] == -1) return INF;
    return 1;
}
// calculates gauss modulo a prime
long long Det;
long long bigmod(long long x,
                 long long pow, long long mod) {
    long long ret = 1;
    while(pow > 0) {
        if(pow & 1) ret = (ret * x) % mod;
        x = (x * x) % mod;
        pow >>= 1;
    }
    return ret;
}
#define INVERSE(a, m) bigmod(a, m-2, m)
int gauss(long long a[MAX][MAX],
           long long ans[MAX],int n,int m,long long mod){
    Det = 1, Rank = 0;
    memset(where, -1, sizeof(where));
    for(int col = 0, row = 0; col<m&&row<n;++col){
        int sel = row;
        for(int i = row+1; i < n; ++i)

```

```

        if(fabs(a[i][col]) > fabs(a[sel][col])) sel=i;
        if(!a[sel][col]) { Det = 0; continue; }
        for(int j=0;j<=m;++j) swap(a[sel][j],a[row][j]);
        if(row != sel) Det = -Det;
        Det = (Det * a[row][col]) % mod;
        where[col] = row;
        // inverse of a[row][col]
        long long s = INVERSE(a[row][col], mod);
        for(int j = 0; j <= m; ++j)
            a[row][j] = (a[row][j] * s) % mod;
        for(int i = 0; i < n; ++i) if (i != row &&
            a[i][col] > 0) {
                long long t = a[i][col];
                for(int j = 0; j <= m; ++j) a[i][j] =
                    (a[i][j] - (a[row][j]*t) % mod + mod)%mod;
            }
        ++row, ++Rank;
    }
    for(int i = 0; i < m; ++i)
        ans[i] = (where[i] == -1) ? 0 : a[where[i]][m];
    for(int i = Rank; i < n; ++i)
        if(a[i][m]) return 0;
    for(int i = 0; i < m; ++i)
        if(where[i] == -1) return INF;
    return 1;
}
// calculates 32 times faster for modulo 2
int Det; // number of variables (must be defined)
int gauss(vector < bitset<MAX> > &a,
           bitset<MAX> &ans, int n, int m) {
    Det = 1, Rank = 0;
    memset(where, -1, sizeof(where));
    for(int col=0,row=0; col < m && row < n;++col){
        int sel = row;
        for(int i = row; i < n; ++i)
            if(a[i][col]) { sel = i; break; }
        if(!a[sel][col]) { Det = 0; continue; }
        swap(a[sel], a[row]);
        if(row != sel) Det = -Det;
        Det &= a[row][col];
        where[col] = row;
        for(int i = 0; i < n; ++i)
            if (i != row&&a[i][col] > 0)a[i]^=a[row];
        ++row, ++Rank;
    }
    for(int i = 0; i < m; ++i)
        ans[i] = (where[i] == -1)?0:a[where[i]][m];
    for(int i = Rank;i<n;++i)if(a[i][m]) return 0;
    for(int i = 0; i < m; ++i)
        if(where[i] == -1) return INF;
    return 1;
}

```

6.6 linear sieve

```

vector<int> lp(N+1);
vector<int> pr;
for (int i=2; i <= N; ++i) {
    if (lp[i] == 0) {
        lp[i] = i;pr.push_back(i);}
    for (int j=0; j < (int)pr.size()
        && pr[j] <= lp[i] && i*pr[j] <= N; ++j)
        lp[i * pr[j]] = pr[j];
}

```


7 String

7.1 Aho Corasick

```
const int K = 26;
struct Vertex {
    int next[K];
    bool leaf = false;
    int p = -1;
    char pch;
    int link = -1;
    int go[K];
    Vertex(int p=-1, char ch='$') : p(p), pch(ch) {
        fill(begin(next), end(next), -1);
        fill(begin(go), end(go), -1);
    }
};
vector<Vertex> t(1);
void add_string(string const& s) {
    int v = 0;
    for (char ch : s) {
        int c = ch - 'a';
        if (t[v].next[c] == -1) {
            t[v].next[c] = t.size();
            t.emplace_back(v, ch);
        }
        v = t[v].next[c];
    }
    t[v].leaf = true;
}
int go(int v, char ch) {
    int c = ch - 'a';
    if (t[v].go[c] == -1) {
        if (v == 0 || t[v].p == 0)
            t[v].link = 0;
        else
            t[v].link = go(get_link(t[v].p), t[v].pch);
    }
    return t[v].link;
}
int go(int v, char ch) {
    int c = ch - 'a';
    if (t[v].go[c] == -1) {
        if (t[v].next[c] != -1)
            t[v].go[c] = t[v].next[c];
        else
            t[v].go[c] = v == 0?0: go(get_link(v), ch);
    }
    return t[v].go[c];
}
```

7.2 Hashing

```
const int MAX = 3000009;
ll mods[2] = {1000000007, 1000000009};
//Some back-up primes: 1072857881, 1066517951, 1040160883
ll bases[2] = {137, 281};
ll pwbase[3][MAX];

void Preprocess(){
    pwbase[0][0] = pwbase[1][0] = 1;
    for(ll i = 0; i < 2; i++){
        for(ll j = 1; j < MAX; j++){
```

```
pwbase[i][j] = (pwbase[i][j - 1] * bases[i])%mods[i];
    }
}

ll fmod(ll a, ll b, int md=mods[0]) {
    unsigned long long x = (long long) a * b;
    unsigned xh = (unsigned) (x >> 32), xl = (unsigned) (x << 32);
    asm(
        "divl %4; \n\t"
        : "=a" (d), "=d" (m)
        : "d" (xh), "a" (xl), "r" (md)
    );
    return m;
}

struct Hashing{
    vector<vector<ll>> hsh;
    Hashing(){}
    Hashing(string &_str) {
        hsh.push_back(vector<ll>(_str.size()+5, 0));
        hsh.push_back(vector<ll>(_str.size()+5, 0));
        Build1(_str);
    }

    void Build(const string &str){Build1(str);Build2(str);}
    void Build1(const string &str) {
        int j = 0;
        for(ll i = str.size() - 1; i >= 0; i--){
            hsh[j][i] = fmod(hsh[j][i + 1], bases[j], mods[j]);
            if (hsh[j][i] > mods[j])
                hsh[j][i] -= mods[j];
        }
    }
    void Build2(const string &str) {
        int j = 1;
        for(ll i = str.size() - 1; i >= 0; i--){
            hsh[j][i] = fmod(hsh[j][i + 1], bases[j], mods[j]);
            if (hsh[j][i] > mods[j])
                hsh[j][i] -= mods[j];
        }
    }

    pair<ll,ll> GetHash(ll i, ll j){
        assert(i <= j);
        ll tmp1 = (hsh[0][i] - fmod(hsh[0][j + 1], pwbase[0][j - i + 1], mods[0]));
        ll tmp2 = (hsh[1][i] - fmod(hsh[1][j + 1], pwbase[1][j - i + 1], mods[1]));
        if(tmp1 < 0) tmp1 += mods[0];
        if(tmp2 < 0) tmp2 += mods[1];
        return make_pair(tmp1, tmp2);
    }

    ll getSingleHash(ll i, ll j) {
        assert(i <= j);
        ll tmp1 = (hsh[0][i] - fmod(hsh[0][j + 1], pwbase[0][j - i + 1], mods[0]));
        if(tmp1 < 0) tmp1 += mods[0];
        return tmp1;
    }
};
```

7.3 Manacher's Algorithm

```
vector<int> d1(n);
for (int i = 0, l = 0, r = -1; i < n; i++){
```

```
int t = mods[i]?1: min(d1[l + r - i], r - i + 1);
while(0<=i-k&& i+k<n&& s[i-k]==s[i+k]) k++;
d1[i] = k--;
if (i + k > r){
    l = i - k; r = i + k;
}
}
}

vector<int> d2(n);
for (int i = 0, l = 0, r = -1; i < n; i++){
    int k=(i>r)?0:min(d2[l+r-i+1],r-i+1);
    while(0<=i-k-1&& i+k<n&& s[i-k-1]==s[i+k]) k++;
    d2[i]=k--;
    if(i+k>r){
        l=i-k-1;
        r=i+k;
    }
}
}
```

7.4 Suffix Array

```
vector<int> sort_cyclic_shifts(char *s) {
    int n = strlen(s);
    const int alphabet = 256;
    vector<int> p(n), c(n), cnt(max(alphabet, n),0);
    for (int i = 0; i < n; i++) cnt[s[i]]++;
    for (int i = 1; i < alphabet; i++) cnt[i] += cnt[i-1];
    for (int i = 0; i < n; i++) p[--cnt[s[i]]] = i;
    c[p[0]] = 0;
    int classes = 1;
    for (int i = 1; i < n; i++) {
        for (int j = p[i-1]; j < p[i]; j++) classes++;
        if (s[p[i-1]] != s[p[i]]) classes++;
        c[p[i]] = classes - 1;
    }
    vector<int> pn(n), cn(n);
    for (int h = 0; (1 << h) < n; ++h) {
        for (int i = 0; i < n; i++) {
            pn[i] = p[i] - (1 << h);
            if (pn[i] < 0)
                pn[i] += n;
            fill(cnt.begin(), cnt.begin() + classes, 0);
            for (int i = 0; i < n; i++) cnt[c[pn[i]]]++;
            for(int i=1;i<classes;i++) cnt[i]+=cnt[i-1];
            for (int i = n-1; i >= 0; i--)
                p[--cnt[c[pn[i]]]] = pn[i];
            cn[p[0]] = 0;
            for (int i = 1; i < n; i++) {
                int ind=p[i] + (1 << h);
                if(ind>=n) ind=ind-n;
                pair<int, int> cur = {c[p[i]], c[ind]};
                ind=p[i-1] + (1 << h);
                if(ind>=n) ind=ind-n;
                pair<int, int> prev = {c[p[i-1]], c[ind]};
                if (cur != prev)
                    ++classes;
                cn[p[i]] = classes - 1;
            }
            c.swap(cn);
        }
        return p;
    }
}

vector<int> suffix_array_construction(char *s) {
```

```

int n=strlen(s);
s[n]='#';
vector<int> sorted_shifts=sort_cyclic_shifts(s);
sorted_shifts.erase(sorted_shifts.begin());
return sorted_shifts;
}
vector<int> lcp_construction(char *s,
                           vector<int> const& p) {
    int n = strlen(s);
    vector<int> rank(n, 0);
    for (int i = 0; i < n; i++)
        rank[p[i]] = i;
    int k = 0;
    vector<int> lcp(n-1, 0);
    for (int i = 0; i < n; i++) {
        if (rank[i] == n - 1) {
            k = 0;
            continue;
        }
        int j = p[rank[i] + 1];
        while (i + k < n && j + k < n && s[i+k] == s[j+k])
            k++;
        lcp[rank[i]] = k;
        if (k)
            k--;
    }
    return lcp;
}
int lcp(int i, int j) {
    int ans = 0;
    for (int k = log_n; k >= 0; k--) {
        if (c[k][i] == c[k][j]) {
            ans += 1 << k;
            i += 1 << k;
            j += 1 << k;
        }
    }
    return ans;
}

```

7.5 Suffix Automaton

```

class SuffixAutomaton{
    bool complete; int last; set<char> alphabet;
    struct state{
        // shortest_non_appearing_string -> snas
        // length_of_substrings -> los
        int len, link, endpos, first_pos, snas, height;
        ll substrings, los;
        bool is_clone;
        map<char, int> next; vector<int> inv_link;
        state(int leng=0, int li=0){
            len=leng; link=li; first_pos=-1;
            substrings=0; los=0;
            endpos=1; snas=0;
            is_clone=false; height=0;
        }
    };
    vector<state> st;
    void process(int node){
        map<char, int> ::iterator mit;
        st[node].substrings=1; st[node].snas=st.size();
        if((int) st[node].next.size()
            <(int) alphabet.size()) st[node].snas=1;
        for(mit=st[node].next.begin();

```

```

            mit!=st[node].next.end(); ++mit){
            if(st[mit->second].substrings==0)
                process(mit->second);
            st[node].height=max(st[node].height,
                1+st[mit->second].height);
            st[node].substrings=st[node].substrings
                +st[mit->second].substrings;
            st[node].los=st[node].los
                +st[mit->second].los+st[mit->second].substrings;
            st[node].snas=min(st[node].snas,
                1+st[mit->second].snas);
        }
        if(st[node].link!=-1){
            st[st[node].link].inv_link.push_back(node);
        }
    }
    void set_suffix_links(int node){
        int i;
        for(i=0; i<st[node].inv_link.size(); i++){
            set_suffix_links(st[node].inv_link[i]);
            st[node].endpos=st[node].endpos+st[st[node].
                inv_link[i]].endpos;
        }
    }
    void output_all_occurrences(int v,
        int P_length, vector<int> &pos){
        if (!st[v].is_clone)
            pos.push_back(st[v].first_pos - P_length+1);
        for (int u : st[v].inv_link)
            output_all_occurrences(u, P_length, pos);
    }
    void kth_smallest(int node, int k, vector<char> &str){
        if(k==0) return;
        map<char, int> ::iterator mit;
        for(mit=st[node].next.begin();
            mit!=st[node].next.end(); ++mit){
            if(st[mit->second].substrings<k)
                k=k-st[mit->second].substrings;
            else{
                str.push_back(mit->first);
                kth_smallest(mit->second, k-1, str);
                return;
            }
        }
    }
    int find_occurrence_index(int node, int index,
        vector<char> &str){
        if(index==str.size()) return node;
        if(!st[node].next.count(str[index])) return -1;
        else return find_occurrence_index(st[node].
            next[str[index]], index+1, str);
    }
    void klen_smallest(int node, int k, vector<char> &str){
        if(k==0) return;
        map<char, int> ::iterator mit;
        for(mit=st[node].next.begin();
            mit!=st[node].next.end(); ++mit){
            if(st[mit->second].height>=k-1){
                str.push_back(mit->first);
                klen_smallest(mit->second, k-1, str);
                return;
            }
        }
    }

```

```

    }
}
void minimum_non_existing_string(int node,
    vector<char> &str){
    map<char, int> ::iterator mit;
    set<char> ::iterator sit;
    for(mit=st[node].next.begin(), sit=
        alphabet.begin(); sit!=alphabet.end(); ++sit, ++mit){
        if(mit==st[node].next.end() || mit->first!=(*sit))
            str.push_back(*sit);
        return;
    }
    else if(st[node].snas==1+st[mit->second].snas)
    {
        str.push_back(*sit);
        minimum_non_existing_string(mit->second, str);
        return;
    }
}
void find_substrings(int node, int index, vector<char>
    &str, vector<pair<ll, ll> > &sub_info){
    sub_info.push_back(make_pair(st[node].substrings,
        st[node].los+st[node].substrings*index));
    if(index==str.size()) return;
    if(st[node].next.count(str[index]))
    {
        find_substrings(st[node].next[str[index]],
            index+1, str, sub_info);
        return;
    }
    else
    {
        sub_info.push_back(make_pair(0, 0));
    }
}
void check()
{
    if(!complete)
    {
        process(0);
        set_suffix_links(0);
        int i;
        complete=true;
    }
}
public:
SuffixAutomaton(set<char> &alpha)
{
    st.push_back(state(0, -1));
    last=0;
    complete=false;
    set<char> ::iterator sit;
    for(sit=alpha.begin(); sit!=alpha.end(); sit++)
    {
        alphabet.insert(*sit);
    }
    st[0].endpos=0;
}
void sa_extend(char c){
    int cur = st.size();

```

```

//printf("New node (%d,%c)\n",cur,c);
st.push_back(state(st[last].len + 1));
st[cur].first_pos=st[cur].len-1;
int p = last;
while (p != -1 && !st[p].next.count(c)){
    st[p].next[c] = cur;
    //printf("Set edge %d -> %d (%c)\n",p,cur,c);
    p = st[p].link;
}
if (p == -1){
    st[cur].link = 0;
    //printf("Set link %d -> %d\n",cur,0);
}
else{
    int q = st[p].next[c];
    if (st[p].len + 1 == st[q].len){
        st[cur].link = q;
        //printf("Set link %d -> %d\n",cur,q);
    }
    else{
        int clone = st.size();
        //printf("Create clone node %d from %d\n",clone,q);
        //printf("Set link %d -> %d\n",clone,st[q].link);
        st.push_back(state(st[p].len+1,st[q].link));
        st[clone].next = st[q].next;
        st[clone].is_clone=true;
        st[clone].endpos=0;
        st[clone].first_pos=st[q].first_pos;
        while (p != -1 && st[p].next[c] == q){
            //printf("Change transition %d -> %d : %d -> %d (%c)
            //\n",p,q,p,clone,c);
            st[p].next[c] = clone;
            p = st[p].link;
        }
        //printf("Change link %d -> %d : %d -> %d\n",q,st[q]
        // .link,q,clone);
        //printf("Set link %d -> %d\n",cur,clone);
        st[q].link = st[cur].link = clone;
    }
}
last = cur;
complete=false;
}
SuffixAutomaton(){
    int i;
    for(i=0; i<st.size(); i++){
        st[i].next.clear();
        st[i].inv_link.clear();
    }
}

```

```

}
st.clear();
alphabet.clear();
}
void kth_smallest(int k,vector<char> &str){
    check();
    kth_smallest(0,k,str);
}
int FindFirstOccurrenceIndex(vector<char> &str){
    check();
    int ind=find_occurrence_index(0,0,str);
    if(ind==0) return -1;
    else if(ind==-1) return st.size();
    else return st[ind].first_pos+1-(int)str.size();
}
void FindAllOccurrenceIndex(vector<char> &str,
    vector<int> &pos){
    check();
    int ind=find_occurrence_index(0,0,str);
    if(ind!=-1)
        output_all_occurrences(ind,str.size(),pos);
}
int Occurrences(vector<char> &str){
    check();
    int ind=find_occurrence_index(0,0,str);
    if(ind==0) return 1;
    else if(ind==-1) return 0;
    else return st[ind].endpos;
}
void klen_smallest(int k,vector<char> &str){
    check();
    if(st[0].height>=k) klen_smallest(0,k,str);
}
void minimum_non_existing_string(vector<char> &str){
    check();
    int ind=find_occurrence_index(0,0,str);
    if(ind!=-1) minimum_non_existing_string(ind,str);
}
ll cyclic_occurrence(vector<char> &str){
    check();
    int i,j,len; ll ans=0;
    int n=str.size(); set<int> S;
    set<int>::iterator it;
    for(i=0,j=0,len=0; i<n*2-1; i++){
        //printf("%d->%c\n",i,str[i%n]);
        if(st[j].next.count(str[i%n])){
            len++;
            j=st[j].next[str[i%n]];
        }
    }
}

```

```

else{
    while(j!=-1&&(!st[j].next.count(str[i%n])))
        j=st[j].link;
    if(j!=-1){
        len=st[j].len+1;
        j=st[j].next[str[i%n]];
    }
    else{
        len=0;
        j=0;
    }
}
while(st[j].link!=-1&&st[st[j].link].len>=n){
    j=st[j].link; len=st[j].len;
}
if(len>=n) S.insert(j);
}
for(it=S.begin();it!=S.end();++it){
    ans=ans+st[*it].endpos;
}
return ans;
}; // main
vector<char> X;
int i;
set<char> alpha;
for(i=0; i<26; i++){
    alpha.insert('a'+i);
}
SuffixAutomaton sa(alpha);
char c;
for(i=0;; i++){
    scanf("%c",&c);
    if(!('a'<=c&&c<='z')) break;
    sa.sa_extend(c);
}
int n,j;
scanf("%d",&n);
for(j=0; j<n; j++){
    for(i=0;; i++){
        scanf("%c",&c);
        if(!('a'<=c&&c<='z')) break;
        X.push_back(c);
    }
    ll ans=sa.cyclic_occurrence(X);
    X.clear();
    printf("%I64d\n",ans);
}
}

```