

Theoretical Computer Science Cheat Sheet

Definitions		Series					
$f(n) = O(g(n))$	iff \exists positive c, n_0 such that $0 \leq f(n) \leq cg(n) \forall n \geq n_0$.	$\sum_{i=1}^n i = \frac{n(n+1)}{2}, \quad \sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{i=1}^n i^3 = \frac{n^2(n+1)^2}{4}.$					
$f(n) = \Omega(g(n))$	iff \exists positive c, n_0 such that $f(n) \geq cg(n) \geq 0 \forall n \geq n_0$.	In general: $\sum_{i=1}^n i^m = \frac{1}{m+1} (n+1)^{m+1} - 1 - \sum_{i=1}^m \frac{B_{m+1-i}}{m+1-i} n^{m+1-i}$					
$f(n) = \Theta(g(n))$	iff $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$.	$\sum_{i=1}^n i^m = \frac{1}{m+1} n^{m+1} + \frac{1}{2} n^m + \frac{1}{6} n^{m-1} + \dots$					
$f(n) = o(g(n))$	iff $\lim_{n \rightarrow \infty} f(n)/g(n) = 0$.	Geometric series: $\sum_{i=0}^{\infty} c^i = \frac{c^{n+1} - 1}{c - 1}, c \neq 1, \quad \sum_{i=0}^{\infty} c^i = \frac{1}{1 - c}, \quad \sum_{i=1}^{\infty} \frac{c^i}{i} = -\ln(1 - c), c < 1.$					
$\lim_{n \rightarrow \infty} a_n = a$	iff $\forall \epsilon > 0, \exists n_0$ such that $ a_n - a < \epsilon, \forall n \geq n_0$.	Harmonic series: $H_n = \sum_{i=1}^n \frac{1}{i}, \quad H_n = \frac{n(n+1)}{2} H_{n-1} - \frac{n(n-1)}{4}$					
$\sup S$	least $b \in \mathbb{R}$ such that $b \geq s, \forall s \in S$.	$\sum_{i=1}^n H_i = (n+1)H_n - n$					
$\inf S$	greatest $b \in \mathbb{R}$ such that $b \leq s, \forall s \in S$.	$\sum_{i=1}^n \frac{1}{i^2} = \frac{n(n+1)}{2} H_n - \frac{n(n-1)}{4}$					
$\liminf_{n \rightarrow \infty} a_n$	$\lim_{n \rightarrow \infty} \inf\{a_i \mid i \geq n, i \in \mathbb{N}\}$.	$\sum_{i=1}^n H_i = (n+1)H_n - n$					
$\limsup_{n \rightarrow \infty} a_n$	$\lim_{n \rightarrow \infty} \sup\{a_i \mid i \geq n, i \in \mathbb{N}\}$.	$\sum_{i=1}^n \frac{1}{i^2} = \frac{n(n+1)}{2} H_n - \frac{n(n-1)}{4}$					
$\binom{n}{k}$	Combinations: Size k subsets of a size n set.	$\sum_{i=1}^n H_i = (n+1)H_n - n$					
$!n$	Stirling numbers (1st kind): Arrangements of an n element set into k cycles.	$\sum_{i=1}^n H_i = (n+1)H_n - n$					
$S(n, k)$	Stirling numbers (2nd kind): Partitions of an n element set into k non-empty sets.	$\sum_{i=1}^n H_i = (n+1)H_n - n$					
$P(n, k)$	1st order Eulerian numbers: Permutations $\pi_1 \pi_2 \dots \pi_n$ on $\{1, 2, \dots, n\}$ with k ascents.	$\sum_{i=1}^n H_i = (n+1)H_n - n$					
$E(n, k)$	2nd order Eulerian numbers.	$\sum_{i=1}^n H_i = (n+1)H_n - n$					
C_n	Catalan Numbers: Binary trees with $n+1$ vertices.	$\sum_{i=1}^n H_i = (n+1)H_n - n$					
14.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	1.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	2.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	3.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
15.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	4.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	5.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	6.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
16.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	7.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	8.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	9.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
17.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	10.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	11.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	12.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
18.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	13.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	14.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	15.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
19.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	16.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	17.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	18.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
20.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	19.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	20.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	21.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
21.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	22.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	23.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	24.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
22.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	25.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	26.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	27.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
23.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	28.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	29.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	30.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
24.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	31.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	32.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	33.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
25.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	34.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	35.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	36.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
26.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	37.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	38.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	39.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
27.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	40.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	41.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	42.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
28.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	43.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	44.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	45.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
29.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	46.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	47.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	48.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
30.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	49.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	50.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	51.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
31.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	52.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	53.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	54.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
32.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	55.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	56.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	57.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
33.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	58.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	59.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	60.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
34.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	61.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	62.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	63.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
35.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	64.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	65.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	66.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
36.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	67.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	68.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	69.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
37.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	70.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	71.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	72.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
38.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	73.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	74.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	75.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
39.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	76.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	77.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	78.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
40.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	79.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	80.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	81.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
41.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	82.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	83.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	84.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
42.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	85.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	86.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	87.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
43.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	88.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	89.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	90.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
44.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	91.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	92.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	93.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
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46.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	97.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	98.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	99.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
47.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	100.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	101.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	102.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
48.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	103.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	104.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	105.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
49.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	106.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	107.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	108.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
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56.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	127.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	128.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	129.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
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60.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	139.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	140.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	141.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
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62.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	145.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	146.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	147.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
63.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	148.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	149.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	150.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
64.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	151.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	152.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	153.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
65.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	154.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	155.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	156.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
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67.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	160.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	161.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	162.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
68.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	163.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	164.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	165.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
69.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	166.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	167.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	168.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
70.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	169.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	170.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	171.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
71.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	172.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	173.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	174.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
72.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	175.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	176.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	177.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
73.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	178.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	179.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	180.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
74.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	181.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	182.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	183.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
75.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	184.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	185.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	186.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
76.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	187.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	188.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	189.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
77.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	190.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	191.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	192.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
78.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	193.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	194.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	195.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
79.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	196.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	197.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	198.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
80.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	199.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	200.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	201.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
81.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	202.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	203.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	204.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
82.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	205.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	206.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	207.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
83.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	208.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	209.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	210.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
84.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	211.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	212.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	213.	$\sum_{k=0}^n \binom{n}{k} = 2^n$
85.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	214.	$\sum_{k=0}^n \binom{n}{k} = 2^n$	215.	$\sum_{k=0}^n \binom{n}{k} = 2^n$		

Theoretical Computer Science Cheat Sheet		
Identities Cont.		Trees
<p>38. $\sum_{k=0}^n \binom{n}{k} = 2^n$</p> <p>40. $\sum_{k=0}^n \binom{n}{k} (-1)^k = 0$</p> <p>42. $\sum_{k=0}^n \binom{n}{k} = 2^n$</p> <p>44. $\sum_{k=0}^n \binom{n}{k} (-1)^k = 0$</p> <p>46. $\sum_{k=0}^n \binom{n}{k} = 2^n$</p> <p>48. $\sum_{k=0}^n \binom{n}{k} = 2^n$</p>	<p>39. $\sum_{k=0}^n \binom{n}{k} x^k = (1+x)^n$</p> <p>41. $\sum_{k=0}^n \binom{n}{k} (-1)^k = 0$</p> <p>43. $\sum_{k=0}^n \binom{n}{k} = 2^n$</p> <p>45. $\sum_{k=0}^n \binom{n}{k} (-1)^k = 0$</p> <p>47. $\sum_{k=0}^n \binom{n}{k} = 2^n$</p> <p>49. $\sum_{k=0}^n \binom{n}{k} = 2^n$</p>	<p>Every tree with n vertices has $n - 1$ edges.</p> <p>Kraft inequality: If the depths of the leaves of a binary tree are d_1, \dots, d_n:</p> $\sum_{i=1}^n 2^{-d_i} \leq 1,$ <p>and equality holds only if every internal node has 2 sons.</p>
Recurrences		
<p>Master method:</p> <p>$T(n) = aT(n/b) + f(n), a \geq 1, b > 1$</p> <p>If $\exists c > 0$ such that $f(n) = O(n^{\log_b a - c})$ then</p> $T(n) = \Theta(n^{\log_b a}).$ <p>If $f(n) = \Theta(n^{\log_b a})$ then</p> $T(n) = \Theta(n^{\log_b a} \log_2 n).$ <p>If $\exists c > 0$ such that $f(n) = \Omega(n^{\log_b a + c})$, and $\exists c < 1$ such that $af(n/b) \leq cf(n)$ for large n, then</p> $T(n) = \Theta(f(n)).$ <p>Substitution (example): Consider the following recurrence</p> $T_{i+1} = 2^{2^i} \cdot T_i, T_1 = 2.$ <p>Note that T_i is always a power of two. Let $t_i = \log_2 T_i$. Then we have</p> $t_{i+1} = 2^i + t_i, t_1 = 1.$ <p>Let $u_i = t_i / 2^i$. Dividing both sides of the previous equation by 2^{i+1} we get</p> $\frac{t_{i+1}}{2^{i+1}} = \frac{2^i}{2^{i+1}} + \frac{t_i}{2^i}.$ <p>Substituting we find</p> $u_{i+1} = \frac{1}{2} + u_i, u_1 = \frac{1}{2},$ <p>which is simply $u_i = i/2$. So we find that T_i has the closed form $T_i = 2^{i2^{i-1}}$.</p> <p>Summing factors (example): Consider the following recurrence</p> $T(n) = 3T(n/2) + n, T(1) = 1.$ <p>Rewrite so that all terms involving T are on the left side</p> $T(n) - 3T(n/2) = n.$ <p>Now expand the recurrence, and choose a factor which makes the left side “telescope”</p>	<p>Let $m = \log_2 n$. Summing the left side we get $T(n) - 3^m T(1) = T(n) - 3^m = T(n) - n^k$ where $k = \log_2 3 \approx 1.58496$.</p> <p>Summing the right side we get</p> $\sum_{i=0}^{m-1} \frac{n}{2^i} 3^i = n \sum_{i=0}^{m-1} \left(\frac{3}{2}\right)^i.$ <p>Let $c = \frac{3}{2}$. Then we have</p> $\sum_{i=0}^{m-1} c^i = n \frac{c^m - 1}{c - 1}$ $= 2n(c^{\log_2 n} - 1)$ $= 2n(c^{(\log_2 n - 1) \log_2 3} - 1)$ $= 2n^k - 2n,$ <p>and so $T(n) = 3n^k - 2n$. Full history recurrences can often be changed to limited history ones (example): Consider</p> $T_i = 1 + \sum_{j=0}^{i-1} T_j, T_0 = 1.$ <p>Note that</p> $T_{i+1} = 1 + \sum_{j=0}^i T_j.$ <p>Subtracting we find</p> $T_{i+1} - T_i = 1 + T_i - 1 - \sum_{j=0}^{i-1} T_j$ $= T_i.$ <p>And so $T_{i+1} = 2T_i = 2^{i+1}$.</p>	<p>Generating functions:</p> <ol style="list-style-type: none"> Multiply both sides of the equation by x^i. Sum both sides over all i for which the equation is valid. Choose a generating function $G(x)$. Usually $G(x) = \sum_{i=0}^{\infty} x^i g_i$. Rewrite the equation in terms of the generating function $G(x)$. Solve for $G(x)$. The coefficient of x^i in $G(x)$ is g_i. <p>Example:</p> $g_{i+1} = 2g_i + 1, g_0 = 0.$ <p>Multiply and sum:</p> $\sum_{i \geq 0} g_{i+1} x^i = \sum_{i \geq 0} 2g_i x^i + \sum_{i \geq 0} x^i.$ <p>We choose $G(x) = \sum_{i \geq 0} x^i g_i$. Rewrite in terms of $G(x)$:</p> $\frac{G(x) - g_0}{x} = 2G(x) + \frac{1}{1-x}.$ <p>Simplify:</p> $\frac{G(x)}{x} = 2G(x) + \frac{1}{1-x}.$ <p>Solve for $G(x)$:</p> $G(x) = \frac{x}{(1-x)(1-2x)}.$ <p>Expand this using partial fractions:</p> $G(x) = x \left(\frac{2}{1-2x} - \frac{1}{1-x} \right)$ $= x \sum_{i \geq 0} 2^{i+1} x^i - \sum_{i \geq 0} x^i$ $= \sum_{i \geq 0} (2^{i+1} - 1) x^{i+1}.$ <p>So $g_i = 2^i - 1$.</p>