1 DP

1.1 divide-and-conquer-optimization

```
int m, n;
vector<long long> dp_before(n), dp_cur(n);
long long C(int i, int j);
// compute dp_cur[l], ... dp_cur[r] (inclusive)
void compute(int l, int r, int optl, int optr){
    if (l > r)
        return;
    int mid = (l + r) >> 1;
    pair<long long, int> best = {LLONG_MAX, -1};
    for (int k = optl; k <= min(mid, optr); k++){
        best = min(best, {(k ? dp_before[k - 1] : 0) +
            C(k, mid), k});
    }
    dp_cur[mid] = best.first;
    int opt = best.second;
    compute(l, mid - 1, optl, opt);
    compute(mid + 1, r, opt, optr);
}
int solve(){
    for (int i = 0; i < n; i++)
        dp_before[i] = C(0, i);
    for (int i = 1; i < m; i++){
        compute(0, n - 1, 0, n - 1);
        dp_before = dp_cur;
    }
    return dp_before[n - 1];
}
```

1.2 knuth-optimization

```
int solve() {
    int N;
    ... // read N and input
    int dp[N][N], opt[N][N];
    auto C = [&](int i, int j) {
        ... // Implement cost function C.
    };
    for (int i = 0; i < N; i++) {
        opt[i][i] = i;
        ... // Initialize dp[i][i] according to the
            problem
    }
    for (int i = N-2; i >= 0; i--) {
        for (int j = i+1; j < N; j++) {
            int mn = INT_MAX;
            int cost = C(i, j);
            for (int k = opt[i][j-1]; k <= min(j-1,
                opt[i+1][j]); k++) {
                if (mn >= dp[i][k] + dp[k+1][j] + cost) {
                    opt[i][j] = k;
                    mn = dp[i][k] + dp[k+1][j] + cost;
                }
            }
            dp[i][j] = mn;
        }
    }
    cout << dp[0][N-1] << endl;
}
```

1.3 li-chao-tree

```
typedef long long ll;
class LiChaoTree{
    ll L,R;
    bool minimize;
    int lines;
    struct Node{
        complex<ll> line;
        Node *children[2];
        Node(complex<ll> ln= {0,10000000000000000000}){
            line=ln;
            children[0]=0;
            children[1]=0;
        }
    } *root;
    ll dot(complex<ll> a, complex<ll> b){
        return (conj(a) * b).real();
    }
    ll f(complex<ll> a, ll x){
        return dot(a, {x, 1});
    }
    void clear(Node* &node){
        if(node->children[0]){
            clear(node->children[0]);
        }
        if(node->children[1]){
            clear(node->children[1]);
        }
        delete node;
    }
    void add_line(complex<ll> nw, Node* &node, ll l, ll
         r){
        if(node==0){
            node=new Node(nw);
            return;
        }
        ll m = (l + r) / 2;
        bool lef = (f(nw, l) < f(node->line,
             l)&&minimize)||((!minimize)&&f(nw, l) >
             f(node->line, l));
        bool mid = (f(nw, m) < f(node->line,
             m)&&minimize)||((!minimize)&&f(nw, m) >
             f(node->line, m));
        if(mid){
            swap(node->line, nw);
        }
        if(r - l == 1){
            return;
        }
        else if(lef != mid){
            add_line(nw, node->children[0], l, m);
        }
        else{
            add_line(nw, node->children[1], m, r);
        }
    }
    ll get(ll x, Node* &node, ll l, ll r){
        ll m = (l + r) / 2;
        if(r - l == 1){
            return f(node->line, x);
        }
        else if(x < m){
            if(node->children[0]==0) return f(node->line,
                 x);
            if(minimize) return min(f(node->line, x),
                 get(x, node->children[0], l, m));
            else return max(f(node->line, x), get(x,
                 node->children[0], l, m));
        }
        else{
            if(node->children[1]==0) return f(node->line,
                 x);
            if(minimize) return min(f(node->line, x),
                 get(x, node->children[1], m, r));
            else return max(f(node->line, x), get(x,
                 node->children[1], m, r));
        }
    }
public:
    LiChaoTree(ll l=-1000000001,ll r=1000000001,bool
         mn=false){
        L=l;
        R=r;
        root=0;
        minimize=mn;
        lines=0;
    }
    void AddLine(pair<ll,ll> ln){
        add_line({ln.first,ln.second},root,L,R);
        lines++;
    }
    int number_of_lines(){
        return lines;
    }
    ll getOptimumValue(ll x){
        return get(x,root,L,R);
    }
    ~LiChaoTree(){
        if(root!=0) clear(root);
    }
};
```

1.4 zero-matrix

```
int zero_matrix(vector<vector<int>> a) {
    int n = a.size();
    int m = a[0].size();
    int ans = 0;
    vector<int> d(m, -1), d1(m), d2(m);
    stack<int> st;
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < m; ++j) {
            if (a[i][j] == 1)
                d[j] = i;
        }
        for (int j = 0; j < m; ++j) {
            while (!st.empty() && d[st.top()] <= d[j])
                st.pop();
            d1[j] = st.empty() ? -1 : st.top();
            st.push(j);
        }
        while (!st.empty())
            st.pop();
        for (int j = m - 1; j >= 0; --j) {
            while (!st.empty() && d[st.top()] <= d[j])
                st.pop();
            d2[j] = st.empty() ? m : st.top();
            st.push(j);
        }
        while (!st.empty())
            st.pop();
        for (int j = 0; j < m; ++j)
            ans = max(ans, (i - d[j]) * (d2[j] - d1[j] -
                1));
    }
    return ans;
}
```

2 DS

2.1 $MO_{with_{update}}$

```
const int N = 1e5 +5;
const int P = 2000; //block size = (2*n^2)^(1/3)
struct query{
    int t, l, r, k, i;
};
vector<query> q;
```

```
vector<array<int, 3>> upd;
vector<int> ans;
vector<int>a;
void add(int x);
void rem(int x);
int get_answer();
void mos_algorithm(){
    sort(q.begin(), q.end(), [](const query &a, const
        query &b){
        if (a.t / P != b.t / P)
            return a.t < b.t;
        if (a.l / P != b.l / P)
            return a.l < b.l;
        if ((a.l / P) & 1)
            return a.r < b.r;
        return a.r > b.r;
    });
    for (int i = upd.size() - 1; i >= 0; --i)
        a[upd[i][0]] = upd[i][1];
    int L = 0, R = -1, T = 0;
    auto apply = [&](int i, int fl){
        int p = upd[i][0];
        int x = upd[i][fl + 1];
        if (L <= p && p <= R){
            rem(a[p]);
            add(x);
        }
        a[p] = x;
    };
    ans.clear();
    ans.resize(q.size());
    for (auto qr : q){
        int t = qr.t, l = qr.l, r = qr.r, k = qr.k;
        while (T < t)
            apply(T++, 1);
        while (T > t)
            apply(--T, 0);
        while (R < r)
            add(a[++R]);
        while (L > l)
            add(a[--L]);
        while (R > r)
            rem(a[R--]);
        while (L < l)
            rem(a[L++]);
        ans[qr.i] = get_answer();
    }
}
void TEST_CASES(int cas){
    int n, m;
    cin>>n>>m;
    a.resize(n);
    for(int i=0;i<n;i++){
        cin>>a[i];
    }
    for(int i=0;i<m;i++){
        int tp;
        scanf("%d", &tp);
        if (tp == 1){
            int l, r, k;
            cin>>l>>r>>k;
            q.push_back({upd.size(), l - 1, r - 1, k,
                q.size()});
        }
        else{
            int p, x;
            cin>>p>>x;
            --p;
            upd.push_back({p, a[p], x});
            a[p] = x;
        }
    }
    mos_algorithm();
}
```

2.2 bipartite-disjoint-set-union

```
void make_set(int v) {
    parent[v] = make_pair(v, 0);
    rank[v] = 0;
    bipartite[v] = true;
}
pair<int, int> find_set(int v) {
    if (v != parent[v].first) {
        int parity = parent[v].second;
        parent[v] = find_set(parent[v].first);
        parent[v].second ^= parity;
    }
    return parent[v];
}
void add_edge(int a, int b) {
    pair<int, int> pa = find_set(a);
    a = pa.first;
    int x = pa.second;

    pair<int, int> pb = find_set(b);
    b = pb.first;
    int y = pb.second;

    if (a == b) {
        if (x == y)
            bipartite[a] = false;
    } else {
        if (rank[a] < rank[b])
            swap (a, b);
        parent[b] = make_pair(a, x^y^1);
        bipartite[a] &= bipartite[b];
        if (rank[a] == rank[b])
            ++rank[a];
    }
}
bool is_bipartite(int v) {
    return bipartite[find_set(v).first];
}
```

2.3 dsu-rollback

```
struct dsu_save {
    int v, rnkv, u, rnku;
    dsu_save() {}
    dsu_save(int _v, int _rnkv, int _u, int _rnku)
        : v(_v), rnkv(_rnkv), u(_u), rnku(_rnku) {}
};
struct dsu_with_rollbacks {
    vector<int> p, rnk;
    int comps;
    stack<dsu_save> op;
    dsu_with_rollbacks() {}
    dsu_with_rollbacks(int n) {
        p.resize(n);
        rnk.resize(n);
        for (int i = 0; i < n; i++) {
            p[i] = i;
            rnk[i] = 0;
        }
        comps = n;
    }
    int find_set(int v) {
        return (v == p[v]) ? v : find_set(p[v]);
    }
    bool unite(int v, int u) {
        v = find_set(v);
        u = find_set(u);
        if (v == u)
            return false;
        comps--;
        if (rnk[v] > rnk[u])
            swap(v, u);
        op.push(dsu_save(v, rnk[v], u, rnk[u]));
        p[v] = u;
        if (rnk[u] == rnk[v])
            rnk[u]++;
        return true;
    }
    void rollback() {
        if (op.empty())
            return;
        dsu_save x = op.top();
        op.pop();
        comps++;
        p[x.v] = x.v;
        rnk[x.v] = x.rnkv;
        p[x.u] = x.u;
        rnk[x.u] = x.rnku;
    }
};
struct query {
    int v, u;
    bool united;
    query(int _v, int _u) : v(_v), u(_u) {
    }
};
struct QueryTree {
    vector<vector<query>> t;
    dsu_with_rollbacks dsu;
    int T;
    QueryTree() {}
    QueryTree(int _T, int n) : T(_T) {
        dsu = dsu_with_rollbacks(n);
        t.resize(4 * T + 4);
    }
    void add_to_tree(int v, int l, int r, int ul, int
        ur, query& q) {
        if (ul > ur)
            return;
        if (l == ul && r == ur) {
            t[v].push_back(q);
            return;
        }
        int mid = (l + r) / 2;
        add_to_tree(2 * v, l, mid, ul, min(ur, mid), q);
        add_to_tree(2 * v + 1, mid + 1, r, max(ul, mid +
            1), ur, q);
    }
    void add_query(query q, int l, int r) {
        add_to_tree(1, 0, T - 1, l, r, q);
    }
    void dfs(int v, int l, int r, vector<int>& ans) {
        for (query& q : t[v]) {
            q.united = dsu.unite(q.v, q.u);
        }
        if (l == r)
            ans[l] = dsu.comps;
        else {
            int mid = (l + r) / 2;
            dfs(2 * v, l, mid, ans);
            dfs(2 * v + 1, mid + 1, r, ans);
        }
        for (query q : t[v]) {
            if (q.united)
                dsu.rollback();
```

```
        }
    }
    vector<int> solve() {
        vector<int> ans(T);
        dfs(1, 0, T - 1, ans);
        return ans;
    }
};
```

2.4 mo

```
struct Query {
    int l, r,k, idx;
    bool operator<(Query other) const
    {
        if(l/block_size!=other.l/block_size) return
            (l<other.l);
        return (l/block_size&1)? (r<other.r) :
            (r>other.r);
    }
};
vector<int> mo_s_algorithm(vector<Query> queries) {
    vector<int> answers(queries.size());
    sort(queries.begin(), queries.end());
    // TODO: initialize data structure
    int cur_l = 0;
    int cur_r = -1;
    // invariant: data structure will always reflect the
        range [cur_l, cur_r]
    for (Query q : queries) {
        while (cur_l > q.l) {
            cur_l--;
            add(cur_l);
        }
        while (cur_r < q.r) {
            cur_r++;
            add(cur_r);
        }
        while (cur_l < q.l) {
            remove(cur_l);
            cur_l++;
        }
        while (cur_r > q.r) {
            remove(cur_r);
            cur_r--;
        }
        answers[q.idx] = get_answer();
    }
    return answers;
}
```

2.5 treap

```
template <class T>
class treap{
    struct item{
        int prior, cnt;
        T key;
        item *l,*r;
        item(T v)
        {
            key=v;
            l=NULL;
            r=NULL;
            cnt=1;
            prior=rand();
        }
    } *root,*node;
    int cnt (item * it){
        return it ? it->cnt : 0;
    }
    void upd_cnt (item * it){
        if (it) it->cnt = cnt(it->l) + cnt(it->r) + 1;
    }
    void split (item * t, T key, item * & l, item * & r){
        if (!t)
            l = r = NULL;
        else if (key < t->key)
            split (t->l, key, l, t->l), r = t;
        else
            split (t->r, key, t->r, r), l = t;
        upd_cnt(t);
    }
    void insert (item * & t, item * it){
        if (!t)
            t = it;
        else if (it->prior > t->prior)
            split (t, it->key, it->l, it->r), t = it;
        else
            insert (it->key < t->key ? t->l : t->r, it);
        upd_cnt(t);
    }
    void merge (item * & t, item * l, item * r){
        if (!l || !r)
            t = l ? l : r;
        else if (l->prior > r->prior)
            merge (l->r, l->r, r), t = l;
        else
            merge (r->l, l, r->l), t = r;
        upd_cnt(t);
    }
    void erase (item * & t, T key){
        if (t->key == key)
            merge (t, t->l, t->r);
        else
            erase (key < t->key ? t->l : t->r, key);
        upd_cnt(t);
    }
    T elementAt(item * &t,int key){
        T ans;
        if(cnt(t->l)==key) ans=t->key;
        else if(cnt(t->l)>key) ans=elementAt(t->l,key);
        else ans=elementAt(t->r,key-1-cnt(t->l));
        upd_cnt(t);
        return ans;
    }
    item * unite (item * l, item * r){
        if (!l || !r) return l ? l : r;
        if (l->prior < r->prior) swap (l, r);
        item * lt, * rt;
        split (r, l->key, lt, rt);
        l->l = unite (l->l, lt);
        l->r = unite (l->r, rt);
        upd_cnt(l);
        upd_cnt(r);
        return l;
    }
    void heapify (item * t){
        if (!t) return;
        item * max = t;
        if (t->l != NULL && t->l->prior > max->prior)
            max = t->l;
        if (t->r != NULL && t->r->prior > max->prior)
            max = t->r;
        if (max != t)
        {
            swap (t->prior, max->prior);
            heapify (max);
        }
    }
    item * build (T * a, int n){
        if (n == 0) return NULL;
        int mid = n / 2;
        item * t = new item (a[mid], rand ());
        t->l = build (a, mid);
        t->r = build (a + mid + 1, n - mid - 1);
        heapify (t);
        return t;
    }
    void output (item * t,vector<T> &arr){
        if (!t) return;
        output (t->l,arr);
        arr.push_back(t->key);
        output (t->r,arr);
    }
public:
    treap(){
        root=NULL;
    }
    treap(T *a,int n){
        build(a,n);
    }
    void insert(T value){
        node=new item(value);
        insert(root,node);
    }
    void erase(T value){
        erase(root,value);
    }
    T elementAt(int position){
        return elementAt(root,position);
    }
    int size(){
        return cnt(root);
    }
    void output(vector<T> &arr){
        output(root,arr);
    }
    int range_query(T l,T r){ //(l,r]
        item *previous,*next,*current;
        split(root,l,previous,current);
        split(current,r,current,next);
        int ans=cnt(current);
        merge(root,previous,current);
        merge(root,root,next);
        previous=NULL;
        current=NULL;
        next=NULL;
        return ans;
    }
};
template <class T>
class implicit_treap{
    struct item{
        int prior, cnt;
        T value;
        bool rev;
        item *l,*r;
        item(T v){
            value=v;
            rev=false;
            l=NULL;
            r=NULL;
            cnt=1;
            prior=rand();
        }
    } *root,*node;
    int cnt (item * it){
        return it ? it->cnt : 0;
    }
    void upd_cnt (item * it){
        if (it)
            it->cnt = cnt(it->l) + cnt(it->r) + 1;
```

```
    }
    void push (item * it){
        if (it && it->rev){
            it->rev = false;
            swap (it->l, it->r);
            if (it->l) it->l->rev ^= true;
            if (it->r) it->r->rev ^= true;
        }
    }
    void merge (item * & t, item * l, item * r){
        push (l);
        push (r);
        if (!l || !r)
            t = l ? l : r;
        else if (l->prior > r->prior)
            merge (l->r, l->r, r), t = l;
        else
            merge (r->l, l, r->l), t = r;
        upd_cnt (t);
    }
    void split (item * t, item * & l, item * & r, int
         key, int add = 0){
        if (!t)
            return void( l = r = 0 );
        push (t);
        int cur_key = add + cnt(t->l);
        if (key <= cur_key)
            split (t->l, l, t->l, key, add), r = t;
        else
            split (t->r, t->r, r, key, add + 1 +
                cnt(t->l)), l = t;
        upd_cnt (t);
    }
    void insert(item * &t,item * element,int key){
        item *l,*r;
        split(t,l,r,key);
        merge(l,l,element);
        merge(t,l,r);
        l=NULL;
        r=NULL;
    }
    T elementAt(item * &t,int key){
        push(t);
        T ans;
        if(cnt(t->l)==key) ans=t->value;
        else if(cnt(t->l)>key) ans=elementAt(t->l,key);
        else ans=elementAt(t->r,key-1-cnt(t->l));
        return ans;
    }
    void erase (item * & t, int key){
        push(t);
        if(!t) return;
        if (key == cnt(t->l))
            merge (t, t->l, t->r);
        else if(key<cnt(t->l))
            erase(t->l,key);
        else
            erase(t->r,key-cnt(t->l)-1);
        upd_cnt(t);
    }
    void reverse (item * &t, int l, int r){
        item *t1, *t2, *t3;
        split (t, t1, t2, l);
        split (t2, t2, t3, r-l+1);
        t2->rev ^= true;
        merge (t, t1, t2);
        merge (t, t, t3);
    }
    void cyclic_shift(item * &t,int L,int R){
        if(L==R) return;
        item *l,*r,*m;
        split(t,t,l,L);
        split(l,l,m,R-L+1);
        split(l,l,r,R-L);
        merge(t,t,r);
        merge(t,t,l);
        merge(t,t,m);
        l=NULL;
        r=NULL;
        m=NULL;
    }
    void output (item * t,vector<T> &arr){
        if (!t) return;
        push (t);
        output (t->l,arr);
        arr.push_back(t->value);
        output (t->r,arr);
    }
public:
    implicit_treap(){
        root=NULL;
    }
    void insert(T value,int position){
        node=new item(value);
        insert(root,node,position);
    }
    void erase(int position){
        erase(root,position);
    }
    void reverse(int l,int r){
        reverse(root,l,r);
    }
    T elementAt(int position){
        return elementAt(root,position);
    }
    void cyclic_shift(int L,int R){
        cyclic_shift(root,L,R);
    }
    int size(){
        return cnt(root);
    }
    void output(vector<T> &arr){
        output(root,arr);
    }
};
```

3 header

```
#define FastIO ios::sync_with_stdio(false);
    cin.tie(0);cout.tie(0)

#include <ext/pb_ds/assoc_container.hpp> // Common file
using namespace __gnu_pbds;

/*
find_by_order(k) --> returns iterator to the kth largest
    element counting from 0
order_of_key(val) --> returns the number of items in a
    set that are strictly smaller than our item
*/
typedef tree<
int,
null_type,
less<int>,
rb_tree_tag,
tree_order_statistics_node_update>
ordered_set;


#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
gp_hash_table<int, int> table;
struct custom_hash {
    static uint64_t splitmix64(uint64_t x) {
        // http://xorshift.di.unimi.it/splitmix64.c
        x += 0x9e3779b97f4a7c15;
        x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
        x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
        return x ^ (x >> 31);
    }
    size_t operator()(uint64_t x) const {
        static const uint64_t FIXED_RANDOM =
            chrono::steady_clock::now().time_since_epoch().co
        return splitmix64(x + FIXED_RANDOM);
    }
};
gp_hash_table<long long, int, custom_hash>
    safe_hash_table;
```