

## 1 DP

### 1.1 divide-and-conquer-optimization

```
int m, n;
vector<long long> dp_before(n), dp_cur(n);
long long C(int i, int j);
// compute dp_cur[l], ... dp_cur[r] (inclusive)
void compute(int l, int r, int optl, int optr){
    if (l > r)
        return;
    int mid = (l + r) >> 1;
    pair<long long, int> best = {LLONG_MAX, -1};
    for (int k = optl; k <= min(mid, optr); k++){
        best = min(best, {(k ? dp_before[k - 1] : 0) + C(k, mid), k});
    }
    dp_cur[mid] = best.first;
    int opt = best.second;
    compute(l, mid - 1, optl, opt);
    compute(mid + 1, r, opt, optr);
}
int solve(){
    for (int i = 0; i < n; i++)
        dp_before[i] = C(0, i);
    for (int i = 1; i < m; i++){
        compute(0, n - 1, 0, n - 1);
        dp_before = dp_cur;
    }
    return dp_before[n - 1];
}
```

### 1.2 knuth-optimization

```
int solve() {
    int N;
    ... // read N and input
    int dp[N][N], opt[N][N];
    auto C = [&](int i, int j) {
        ... // Implement cost function C.
    };
    for (int i = 0; i < N; i++) {
        opt[i][i] = i;
        ... // Initialize dp[i][i] according to the problem
    }
    for (int i = N-2; i >= 0; i--) {
        for (int j = i+1; j < N; j++) {
            int mn = INT_MAX;
            int cost = C(i, j);
            for (int k = opt[i][j-1]; k <= min(j-1, opt[i+1][j]); k++) {
                if (mn >= dp[i][k] + dp[k+1][j] + cost) {
                    opt[i][j] = k;
                    mn = dp[i][k] + dp[k+1][j] + cost;
                }
            }
            dp[i][j] = mn;
        }
    }
    cout << dp[0][N-1] << endl;
}
```

### 1.3 li-chao-tree

```
typedef long long ll;
class LiChaoTree{
    ll L,R;
    bool minimize;
    int lines;
    struct Node{
        complex<ll> line;
        Node *children[2];
        Node(complex<ll> ln= {0,1000000000000000000000}){
            line=ln;
            children[0]=0;
            children[1]=0;
        }
    } *root;
    ll dot(complex<ll> a, complex<ll> b){
        return (conj(a) * b).real();
    }
};
```

```
}
ll f(complex<ll> a, ll x){
    return dot(a, {x, 1});
}
void clear(Node* &node){
    if(node->children[0]){
        clear(node->children[0]);
    }
    if(node->children[1]){
        clear(node->children[1]);
    }
    delete node;
}
void add_line(complex<ll> nw, Node* &node, ll l, ll r){
    if(node==0){
        node=new Node(nw);
        return;
    }
    ll m = (l + r) / 2;
    bool lef = (f(nw, l) < f(node->line, l)&&minimize)||(!minimize)&&f(nw, l) > f(node->line, l));
    bool mid = (f(nw, m) < f(node->line, m)&&minimize)||(!minimize)&&f(nw, m) > f(node->line, m));
    if(mid){
        swap(node->line, nw);
    }
    if(r - l == 1){
        return;
    }
    else if(lef != mid){
        add_line(nw, node->children[0], l, m);
    }
    else{
        add_line(nw, node->children[1], m, r);
    }
}
ll get(ll x, Node* &node, ll l, ll r){
    ll m = (l + r) / 2;
    if(r - l == 1){
        return f(node->line, x);
    }
    else if(x < m){
        if(node->children[0]==0) return f(node->line, x);
        if(minimize) return min(f(node->line, x), get(x, node->children[0], l, m));
        else return max(f(node->line, x), get(x, node->children[0], l, m));
    }
    else{
        if(node->children[1]==0) return f(node->line, x);
        if(minimize) return min(f(node->line, x), get(x, node->children[1], m, r));
        else return max(f(node->line, x), get(x, node->children[1], m, r));
    }
}
public:
    LiChaoTree(ll l=-10000000001, ll r=10000000001, bool mn=false){
        L=l;
        R=r;
        root=0;
        minimize=mn;
        lines=0;
    }
    void AddLine(pair<ll,ll> ln){
        add_line({ln.first,ln.second},root,L,R);
        lines++;
    }
    int number_of_lines(){
        return lines;
    }
    ll getOptimumValue(ll x){
        return get(x,root,L,R);
    }
    ~LiChaoTree(){
        if(root!=0) clear(root);
    }
};
```

### 1.4 zero-matrix

```
int zero_matrix(vector<vector<int>> a) {
    int n = a.size();
    int m = a[0].size();
    int ans = 0;
    vector<int> d(m, -1), d1(m), d2(m);
    stack<int> st;
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < m; ++j) {
            if (a[i][j] == 1)
                d[j] = i;
        }
        for (int j = 0; j < m; ++j) {
            while (!st.empty() && d[st.top()] <= d[j])
                st.pop();
            d1[j] = st.empty() ? -1 : st.top();
            st.push(j);
        }
        while (!st.empty())
            st.pop();
        for (int j = m - 1; j >= 0; --j) {
            while (!st.empty() && d[st.top()] <= d[j])
                st.pop();
            d2[j] = st.empty() ? m : st.top();
            st.push(j);
        }
        while (!st.empty())
            st.pop();
        for (int j = 0; j < m; ++j)
            ans = max(ans, (i - d[j]) * (d2[j] - d1[j] - 1));
    }
    return ans;
}
```

## 2 DS

### 2.1 Heavy light decomposition

```
int value[N], Tree[N];
int parent[N], depth[N], heavy[N], head[N], pos[N];
int cur_pos, n;
vector<int> adj[N];
int dfs(int v) {
    int size = 1, max_c_size = 0;
    for (int c : adj[v]) {
        if (c != parent[v]) {
            parent[c] = v, depth[c] = depth[v] + 1;
            int c_size = dfs(c);
            size += c_size;
            if (c_size > max_c_size)
                max_c_size = c_size, heavy[v] = c;
        }
    }
    return size;
}
void update(int idx, int x, int nn){
    while(idx<=nn){
        Tree[idx] += x; idx += (idx & -idx);
    }
}
void decompose(int v, int h) {
    head[v] = h, pos[v] = cur_pos;
    update(cur_pos, value[v], n+1);
    cur_pos++;
    if (heavy[v] != -1) decompose(heavy[v], h);
    for (int c : adj[v]) {
        if (c != parent[v] && c != heavy[v])
            decompose(c, c);
    }
}
int query_bit(int idx){
    int sum=0;
    while(idx>0){
        sum+=Tree[idx]; idx=(idx&-idx);
    }
    return sum;
}
void init_hld(int root, int n){
```

```

memset(Tree, 0, sizeof Tree);
memset(heavy, -1, sizeof heavy);
cur_pos = 1; parent[root] = -1;
assert(dfs(root) == n);
decompose(root, root);
}
int segment_tree_query(int x, int y) {
    if (y < x) swap(x, y);
    return query_bit(y) - query_bit(x - 1);
}
int query_hld(int a, int b) {
    int res = 0;
    for (; head[a] != head[b]; b = parent[head[b]]) {
        if (depth[head[a]] > depth[head[b]])
            swap(a, b);
        int cur_heavy_path_max =
            segment_tree_query(pos[head[b]], pos[b]);
        res += cur_heavy_path_max;
    }
    if (depth[a] > depth[b])
        swap(a, b);
    /// now a is the lca or quert(a,b)
    int last_heavy_path_max = segment_tree_query(pos[a],
        pos[b]);
    res += last_heavy_path_max;
    return res;
}

```

## 2.2 MO with update

```

const int N = 1e5 + 5;
const int P = 2000; // block size = (2*n^2)^(1/3)
struct query {
    int t, l, r, k, i;
};
vector<query> q;
vector<array<int, 3>> upd;
vector<int> ans; a;
void add(int x); void rem(int x); int get_answer();
void mos_algorithm() {
    sort(q.begin(), q.end(), [](const query &a, const query &b) {
        if (a.t / P != b.t / P) return a.t < b.t;
        if (a.l / P != b.l / P) return a.l < b.l;
        if ((a.l / P) & 1) return a.r < b.r;
        return a.r > b.r;
    });
    for (int i = upd.size() - 1; i >= 0; --i) a[upd[i][0]] = upd[i][1];
    int L = 0, R = -1, T = 0;
    auto apply = [&](int i, int fl) {
        int p = upd[i][0], x = upd[i][fl + 1];
        if (L <= p && p <= R) { rem(a[p]); add(x); }
        a[p] = x;
    };
    ans.clear(); ans.resize(q.size());
    for (auto qr : q) {
        int t = qr.t, l = qr.l, r = qr.r, k = qr.k;
        while (T < t) apply(T++, 1);
        while (T > t) apply(--T, 0);
        while (R < r) add(a[++R]);
        while (L > l) add(a[--L]);
        while (R > r) rem(a[R--]);
        while (L < l) rem(a[L++]);
        ans[qr.i] = get_answer();
    }
}
void TEST_CASES(int cas) {
    cin >> n >> m; a.resize(n); for (int i = 0; i < n; i++) cin >> a[i];
    for (int i = 0; i < m; i++) { int tp; scanf("%d", &tp);
        if (tp == 1) { int l, r, k; cin >> l >> r >> k;
            q.push_back({upd.size(), l - 1, r - 1, k, q.size()}); }
        else { int p, x; cin >> p >> x; --p;
            upd.push_back({p, a[p], x}); a[p] = x;
        }
    }
    mos_algorithm();
}

```

## 2.3 bipartite-disjoint-set-union

```
void make_set(int v) {
```

```

parent[v] = make_pair(v, 0); rank[v] = 0; bipartite[v] = true; }
pair<int, int> find_set(int v) { if (v != parent[v].first) {
    int parity = parent[v].second; parent[v] = find_set(
        parent[v].first); parent[v].second ^= parity; }
    return parent[v];
}
void add_edge(int a, int b) {
    pair<int, int> pa = find_set(a);
    a = pa.first; int x = pa.second;
    pair<int, int> pb = find_set(b); b = pb.first;
    int y = pb.second;
    if (a == b) {
        if (x == y) bipartite[a] = false;
    } else {
        if (rank[a] < rank[b]) swap(a, b);
        parent[b] = make_pair(a, x ^ y ^ 1);
        bipartite[a] ^= bipartite[b];
        if (rank[a] == rank[b]) ++rank[a];
    }
}
bool is_bipartite(int v) { return bipartite[find_set(v).first]; }

```

## 2.4 bitset

```

int LEN; // length of Bitset array t
struct Bitset {
    ull t[N/64+5]; Bitset() { memset(t, 0, sizeof t); }
    void set(int p) { t[p>>6] |= 1llu << (p&63); }
    void shift() { ull last = 0llu;
        for (int i = 0; i < LEN; i++) {
            ull curr = t[i] >> 63llu; (t[i] <= 1) ? last = curr;
        }
    }
    int count() { int ret = 0;
        for (int i = 0; i < LEN; i++) ret += __builtin_popcountll(t[i]);
        return ret;
    }
    Bitset &operator = (Bitset const &b) {
        memcpy(t, b.t, sizeof(t)); return *this;
    }
    Bitset &operator |= (Bitset &b) {
        for (int i = 0; i < LEN; i++) t[i] |= b.t[i]; return *this;
    }
    Bitset &operator &= (Bitset &b) {
        for (int i = 0; i < LEN; i++) t[i] &= b.t[i]; return
            *this;
    }
    Bitset &operator ^= (Bitset &b) {
        for (int i = 0; i < LEN; i++) t[i] ^= b.t[i]; return
            *this;
    }
};
Bitset operator - (const Bitset &a, const Bitset &b) {
    Bitset tmp; ull last = 0;
    for (int i = 0; i < LEN; i++) { ull curr = (a.t[i] < b.t[i] +
        last);
        tmp.t[i] = a.t[i] - b.t[i] - last; last = curr;
    }
    return tmp;
}
// https://loj.ac/p/6564 , string s, t, m_old = 0
// lcs formula: let x = m_old | Occurance set of s [char or t]
// m_new = ((x - ((m_old < 1) + 1)) ^ x) & x; finally m_new.count()

```

## 2.5 centroid decomposition

```

set<int> g[N];
int par[N], sub[N], level[N], ans[N]; int DP[LOGN][N];
int n, m; int nn;
void dfs1(int u, int p) {
    sub[u] = 1; nn++;
    for (auto it = g[u].begin(); it != g[u].end(); it++)
        if (*it != p) {
            dfs1(*it, u); sub[u] += sub[*it];
        }
}
int dfs2(int u, int p) {
    for (auto it = g[u].begin(); it != g[u].end(); it++)
        if (*it != p && sub[*it] > nn/2)

```

```

        return dfs2(*it, u);
    }
    return u;
}
void decompose(int root, int p) {
    nn = 0; dfs1(root, root); int centroid = dfs2(root, root);
    if (p == -1) p = centroid; par[centroid] = p;
    for (auto it = g[centroid].begin(); it != g[centroid].end(); it++) {
        g[*it].erase(centroid); decompose(*it, centroid);
    }
    g[centroid].clear();
}

```

## 2.6 dsu-rollback

```

struct dsu_save {
    int v, rnkv, u, rnku; dsu_save() {}
    dsu_save(int _v, int _rnkv, int _u, int _rnku)
        : v(_v), rnkv(_rnkv), u(_u), rnku(_rnku) {}
};
struct dsu_with_rollback {
    vector<int> p, rnk; int comps; stack<dsu_save> op;
    dsu_with_rollback() {}
    dsu_with_rollback(int n) { p.resize(n); rnk.resize(n);
        for (int i = 0; i < n; i++) { p[i] = i; rnk[i] = 0; }
        comps = n;
    }
    int find_set(int v) { return (v == p[v]) ? v : find_set(p[v]); }
    bool unite(int v, int u) { v = find_set(v); u = find_set(u);
        if (v == u) return false; comps--;
        if (rnk[v] > rnk[u]) swap(v, u);
        op.push(dsu_save(v, rnk[v], u, rnk[u])); p[v] = u;
        if (rnk[u] == rnk[v]) rnk[u]++; return true;
    }
    void rollback() { if (op.empty()) return;
        dsu_save x = op.top(); op.pop(); comps++; p[x.v] = x.v;
        rnk[x.v] = x.rnk; p[x.u] = x.u; rnk[x.u] = x.rnk;
    }
};
struct query {
    int v, u; bool united;
    query(int _v, int _u) : v(_v), u(_u) {}
};
struct QueryTree {
    vector<vector<query>> t; dsu_with_rollback dsu; int T;
    QueryTree() {}
    QueryTree(int _T, int n) : T(_T) {
        dsu = dsu_with_rollback(n); t.resize(4 * T + 4);
        void add_to_tree(int v, int l, int r, int ul, int ur, query &q) {
            if (ul > ur) return;
            if (l == ul && r == ur) { t[v].push_back(q); return; }
            int mid = (l + r) / 2;
            add_to_tree(2 * v, l, mid, ul, min(ur, mid), q);
            add_to_tree(2 * v + 1, mid + 1, r, max(ul, mid + 1), ur, q);
        }
        void add_query(query q, int l, int r) {
            add_to_tree(1, 0, T - 1, l, r, q);
        }
        void dfs(int v, int l, int r, vector<int> &ans) {
            for (query &q : t[v]) q.united = dsu.unite(q.v, q.u);
            if (l == r) ans[l] = dsu.comps;
            else { int mid = (l + r) / 2;
                dfs(2 * v, l, mid, ans); dfs(2 * v + 1, mid + 1, r, ans);
                for (query q : t[v]) { if (q.united) dsu.rollback(); }
            }
        }
        vector<int> solve() {
            vector<int> ans(T); dfs(1, 0, T - 1, ans); return ans;
        }
    }
};

```

## 2.7 link cut tree

```

const int MOD = 998244353;
int sum(int a, int b) {
    return a + b >= MOD ? a + b - MOD : a + b;
}
int mul(int a, int b) {
    return (a * 1LL * b) % MOD;
}
typedef pair<int, int> Linear;
Linear compose(const Linear &p, const Linear &q) {

```

```

return Linear(mul(p.first, q.first), sum(mul(q.second,
p.first), p.second));
}
struct SplayTree {
    struct Node {
        int ch[2] = {0, 0}, p = 0;
        long long self = 0, path = 0; // Path aggregates
        long long sub = 0, vir = 0; // Subtree aggregates
        bool flip = 0; // Lazy tags
        int size = 1;
        Linear_self[1, 0], _path_shoja[1, 0], _path_ulta[1, 0];
    };
    vector<Node> T;
    SplayTree(int n) : T(n + 1) {
        T[0].size = 0;
    }
    void push(int x) {
        if (!x || !T[x].flip) return;
        int l = T[x].ch[0], r = T[x].ch[1];
        T[l].flip ^= 1, T[r].flip ^= 1;
        swap(T[x].ch[0], T[x].ch[1]);
        T[x].flip = 0;
        swap(T[x]._path_shoja, T[x]._path_ulta);
    }
    void pull(int x) {
        int l = T[x].ch[0], r = T[x].ch[1]; push(l); push(r);
        T[x].size = T[l].size + T[r].size + 1;
        T[x].path = T[l].path + T[x].self + T[r].path;
        T[x].sub = T[x].vir + T[l].sub + T[r].sub + T[x].self;
        T[x]._path_shoja = compose(T[r]._path_shoja,
            compose(T[x]._self, T[l]._path_shoja));
        T[x]._path_ulta = compose(T[l]._path_ulta,
            compose(T[x]._self, T[r]._path_ulta));
    }
    void set(int x, int d, int y) {
        T[x].ch[d] = y; T[y].p = x; pull(x);
    }
    void splay(int x) {
        auto dir = [&](int x) {
            int p = T[x].p; if (!p) return -1;
            return T[p].ch[0] == x ? 0 : T[p].ch[1] == x ? 1 :
                -1;
        };
        auto rotate = [&](int x) {
            int y = T[x].p, z = T[y].p, dx = dir(x), dy =
                dir(y);
            set(y, dx, T[x].ch[!dx]);
            set(x, !dx, y);
            if (~dy) set(z, dy, x);
            T[x].p = z;
        };
        for (push(x); ~dir(x); ) {
            int y = T[x].p, z = T[y].p;
            push(z); push(y); push(x);
            int dx = dir(x), dy = dir(y);
            if (~dy) rotate(dx != dy ? x : y);
            rotate(x);
        }
    }
    int KthNext(int x, int k) {
        assert(k > 0);
        splay(x);
        x = T[x].ch[1];
        if (T[x].size < k) return -1;
        while (true) {
            push(x);
            int l = T[x].ch[0], r = T[x].ch[1];
            if (T[l].size+1 == k) return x;
            if (k <= T[l].size) x = l;
            else k -= T[l].size+1, x = r;
        }
    }
};
struct LinkCut : SplayTree {
    LinkCut(int n) : SplayTree(n) {}
    int access(int x) {
        int u = x, v = 0;
        for (; u; v = u, u = T[u].p) {

```

```

            splay(u);
            int& ov = T[u].ch[1];
            T[u].vir += T[ov].sub;
            T[u].vir -= T[v].sub;
            ov = v; pull(u);
        }
        splay(x);
        return v;
    }
    void reroot(int x) {
        access(x); T[x].flip ^= 1; push(x);
    }
    //makes v parent of u (optional: u must be a root)
    void Link(int u, int v) {
        reroot(u); access(v);
        T[v].vir += T[u].sub;
        T[u].p = v; pull(v);
    }
    //removes edge between u and v
    void Cut(int u, int v) {
        int _u = FindRoot(u);
        reroot(u); access(v);
        T[v].ch[0] = T[u].p = 0; pull(v);
        reroot(_u);
    }
    // Rooted tree LCA. Returns 0 if u and v arent connected.
    int LCA(int u, int v) {
        if (u == v) return u;
        access(u); int ret = access(v);
        return T[u].p ? ret : 0;
    }
    // Query subtree of u where v is outside the subtree.
    long long Subtree(int u, int v) {
        int _v = FindRoot(v);
        reroot(v); access(u);
        long long ans = T[u].vir + T[u].self;
        reroot(_v);
        return ans;
    }
    // Query path [u..v]
    long long Path(int u, int v) {
        int _u = FindRoot(u);
        reroot(u); access(v);
        long long ans = T[v].path;
        reroot(_u);
        return ans;
    }
    Linear _Path(int u, int v) {
        reroot(u); access(v); return T[v]._path_shoja;
    }
    // Update vertex u with value v
    void Update(int u, long long v) {
        access(u); T[u].self = v; pull(u);
    }
    // Update vertex u with value v
    void _Update(int u, Linear v) {
        access(u); T[u]._self = v; pull(u);
    }
    int FindRoot(int u) {
        access(u);
        while (T[u].ch[0]) {
            u = T[u].ch[0];
            push(u);
        }
        access(u);
        return u;
    }
    //k-th node (0-indexed) on the path from u to v
    int KthOnPath(int u, int v, int k) {
        if (u == v) return k == 0 ? u : -1;
        int _u = FindRoot(u);
        reroot(u); access(v);
        int ans = KthNext(u, k);
        reroot(_u);
        return ans;
    }
};
int main() {
    cin >> n >> q;

```

```

LinkCut lct(n);
for (int i = 1; i <= n; i++) {
    Linear l;
    cin >> l.first >> l.second;
    lct._Update(i, l);
}
for (int i = 1; i < n; i++) {
    int u, v;
    cin >> u >> v;
    lct.Link(u+1, v+1);
}
while (q--) {
    int op;
    cin >> op;
    if (op == 0) {
        int u, v, w, x;
        cin >> u >> v >> w >> x;
        lct.Cut(u+1, v+1);
        lct.Link(w+1, x+1);
    } else if (op == 1) {
        int p; Linear l;
        cin >> p >> l.first >> l.second;
        lct._Update(p+1, l);
    } else {
        int u, v, x;
        cin >> u >> v >> x;
        Linear l = lct._Path(u+1, v+1);
        cout << sum(mul(l.first, x), l.second) << "\n";
    }
}
return 0;
}

```

## 2.8 sparse table 2d

```

const int N=500; const int K = 8 ; // k >= ceil(lg22(n)) +1
int arr[N][N]; int st[K+1][K+1][N][N]; int lg2[N+1];
void ini(){ lg2[1] = 0;
    for (int i = 2; i <= N; i++) lg2[i] = lg2[i/2] + 1; }
int f(int i, int j){ return max(i, j); }
void pre( int n, int m){
    for(int x=0; x<=K; x++){
        for(int y=0; y<=K; y++){
            for(int i=0; i<n; i++){
                for(int j=0; j<m; j++){
                    if(i+(1<<x)>n or j+(1<<y)>m) continue;
                    if(x>0) st[x][y][i][j] = f(st[x-1][y][i][j],
                        st[x-1][y][i+(1<<(x-1))][j]);
                    else if(y>0) st[x][y][i][j] = f(st[x][y-1][i][j],
                        st[x][y-1][i][j+(1<<(y-1))]);
                    else st[x][y][i][j] = f(arr[i][j], arr[i][j]);
                } } } }
    int getf( int R1, int C1, int R2, int C2){
        if(R1>R2) swap(R1,R2); if(C1>C2) swap(C1,C2);
        int x = lg2[R2 - R1 + 1]; int y = lg2[C2 - C1 + 1];
        return f(f(st[x][y][R1][C1], st[x][y][R2-(1<<x)+1][C1]),
            st[x][y][R1][C2-(1<<y)+1], st[x][y][R2-(1<<x)+1][C2-(1<<y)+1]);
    }
}

```

## 2.9 treap

```

template <class T>
class treap{
    struct item{
        int prior, cnt;
        T key;
        item *l,*r;
        item(T v) {
            key=v;
            l=NULL;
            r=NULL;
            cnt=1;
            prior=rand();
        }
    } *root,*node;
    int cnt (item * it){
        return it ? it->cnt : 0;
    }
    void upd_cnt (item * it){

```

```

    if (it) it->cnt = cnt(it->l) + cnt(it->r) + 1;
}
void split (item * t, T key, item * &l, item * &r){
    if (!t)
        l = r = NULL;
    else if (key < t->key)
        split (t->l, key, l, t->l), r = t;
    else
        split (t->r, key, t->r, r), l = t;
    upd_cnt(t);
}
void insert (item * &t, item * it){
    if (!t)
        t = it;
    else if (it->prior > t->prior)
        split (t, it->key, it->l, it->r), t = it;
    else
        insert (it->key < t->key ? t->l : t->r, it);
    upd_cnt(t);
}
// keys(l) < keys(r)
void merge (item * &t, item * l, item * r){
    if (!l || !r)
        t = l ? l : r;
    else if (l->prior > r->prior)
        merge (l->r, l->r, r), t = l;
    else
        merge (r->l, l, r->l), t = r;
    upd_cnt(t);
}
void erase (item * &t, T key){
    if (t->key == key)
        merge (t, t->l, t->r);
    else
        erase (key < t->key ? t->l : t->r, key);
    upd_cnt(t);
}
T elementAt(item * &t, int key){
    T ans;
    if(cnt(t->l)==key) ans=t->key;
    else if(cnt(t->l)>key) ans=elementAt(t->l, key);
    else ans=elementAt(t->r, key-1-cnt(t->l));
    upd_cnt(t);
    return ans;
}
item * unite (item * l, item * r){
    if (!l || !r) return l ? l : r;
    if (l->prior < r->prior) swap (l, r);
    item * lt, * rt;
    split (r, l->key, lt, rt);
    l->l = unite (l->l, lt);
    l->r = unite (l->r, rt);
    upd_cnt(l);
    upd_cnt(r);
    return l;
}
void heapify (item * t){
    if (!t) return;
    item * max = t;
    if (t->l != NULL && t->l->prior > max->prior)
        max = t->l;
    if (t->r != NULL && t->r->prior > max->prior)
        max = t->r;
    if (max != t)
    {
        swap (t->prior, max->prior);
        heapify (max);
    }
}
item * build (T * a, int n){
    if (n == 0) return NULL;
    int mid = n / 2;
    item * t = new item (a[mid], rand ());
    t->l = build (a, mid);
    t->r = build (a + mid + 1, n - mid - 1);
    heapify (t);
    return t;
}
void output (item * t, vector<T> &arr){

```

```

    if (!t) return;
    output (t->l, arr);
    arr.push_back(t->key);
    output (t->r, arr);
}
public:
    treap(){
        root=NULL;
    }
    treap(T *a, int n){
        build(a, n);
    }
    void insert(T value){
        node=new item(value);
        insert(root, node);
    }
    void erase(T value){
        erase(root, value);
    }
    T elementAt(int position){
        return elementAt(root, position);
    }
    int size(){
        return cnt(root);
    }
    void output(vector<T> &arr){
        output(root, arr);
    }
    int range_query(T l, T r){ //(l,r]
        item *previous, *next, *current;
        split(root, l, previous, current);
        split(current, r, current, next);
        int ans=cnt(current);
        merge(root, previous, current);
        merge(root, root, next);
        previous=NULL;
        current=NULL;
        next=NULL;
        return ans;
    }
};
template <class T>
class implicit_treap{
    struct item{
        int prior, cnt;
        T value;
        bool rev;
        item *l, *r;
        item(T v){
            value=v;
            rev=false;
            l=NULL;
            r=NULL;
            cnt=1;
            prior=rand();
        }
    } *root, *node;
    int cnt (item * it){
        return it ? it->cnt : 0;
    }
    void upd_cnt (item * it){
        if (it)
            it->cnt = cnt(it->l) + cnt(it->r) + 1;
    }
    void push (item * it){
        if (it && it->rev){
            it->rev = false;
            swap (it->l, it->r);
            if (it->l) it->l->rev ^= true;
            if (it->r) it->r->rev ^= true;
        }
    }
    void merge (item * &t, item * l, item * r){
        push (l);
        push (r);
        if (!l || !r)
            t = l ? l : r;
        else if (l->prior > r->prior)
            merge (l->r, l->r, r), t = l;
        else

```

```

            merge (r->l, l, r->l), t = r;
        upd_cnt (t);
    }
    void split (item * t, item * &l, item * &r, int key, int add = 0){
        if (!t)
            return void( l = r = 0 );
        push (t);
        int cur_key = add + cnt(t->l);
        if (key <= cur_key)
            split (t->l, l, t->l, key, add), r = t;
        else
            split (t->r, t->r, r, key, add + 1 + cnt(t->l)), l = t;
        upd_cnt (t);
    }
    void insert(item * &t, item * element, int key){
        item *l, *r;
        split(t, l, r, key);
        merge(l, l, element);
        merge(t, l, r);
        l=NULL;
        r=NULL;
    }
    T elementAt(item * &t, int key){
        push(t);
        T ans;
        if(cnt(t->l)==key) ans=t->value;
        else if(cnt(t->l)>key) ans=elementAt(t->l, key);
        else ans=elementAt(t->r, key-1-cnt(t->l));
        return ans;
    }
    void erase (item * &t, int key){
        push(t);
        if(!t) return;
        if (key == cnt(t->l))
            merge (t, t->l, t->r);
        else if(key<cnt(t->l))
            erase(t->l, key);
        else
            erase(t->r, key-cnt(t->l)-1);
        upd_cnt(t);
    }
    void reverse (item * &t, int l, int r){
        item *t1, *t2, *t3;
        split (t, t1, t2, l);
        split (t2, t2, t3, r-1+1);
        t2->rev ^= true;
        merge (t, t1, t2);
        merge (t, t, t3);
    }
    void cyclic_shift(item * &t, int L, int R){
        if(L==R) return;
        item *l, *r, *m;
        split(t, t, l, L);
        split(l, l, m, R-L+1);
        split(l, l, r, R-L);
        merge(t, t, r);
        merge(t, t, l);
        merge(t, t, m);
        l=NULL;
        r=NULL;
        m=NULL;
    }
    void output (item * t, vector<T> &arr){
        if (!t) return;
        push (t);
        output (t->l, arr);
        arr.push_back(t->value);
        output (t->r, arr);
    }
}
public:
    implicit_treap(){
        root=NULL;
    }
    void insert(T value, int position){
        node=new item(value);
        insert(root, node, position);
    }
}

```



```

void erase(int position){
    erase(root,position);
}
void reverse(int l,int r){
    reverse(root,l,r);
}
T elementAt(int position){
    return elementAt(root,position);
}
void cyclic_shift(int L,int R){
    cyclic_shift(root,L,R);
}
int size(){
    return cnt(root);
}
void output(vector<T> &arr){
    output(root,arr);
}
};

```

## 2.10 wavelet tree

```

#include <bits/stdc++.h>
using namespace std;
#define fo(i,n) for(i=0;i<n;i++)
#define ll long long
#define pb push_back
#define mp make_pair
typedef pair<int, int> pii;
typedef pair<ll, ll> pll;
typedef vector<int> vi;
const int N = 3e5, M = N;
//=====
const int MAX = 1e6;
int a[N];
struct wavelet_tree{
#define vi vector<int>
#define pb push_back
    int lo, hi;
    wavelet_tree *l=0, *r=0;
    vi b;
    vi c; // c holds the prefix sum of elements
    //nos are in range [x,y]
    //array indices are [from, to]
    wavelet_tree(int *from, int *to, int x, int y){
        lo = x, hi = y;
        if( from >= to)
            return;
        if( hi == lo ){
            b.reserve(to-from+1);
            b.pb(0);
            c.reserve(to-from+1);
            c.pb(0);
            for(auto it = from; it != to; it++){
                b.pb(b.back() + 1);
                c.pb(c.back()+*it);
            }
            return;
        }
        int mid = (lo+hi)/2;
        auto f = [mid](int x){
            return x <= mid;
        };
        b.reserve(to-from+1);
        b.pb(0);
        c.reserve(to-from+1);
        c.pb(0);
        for(auto it = from; it != to; it++){
            b.pb(b.back() + f(*it));
            c.pb(c.back() + *it);
        }
        //see how lambda function is used here
        auto pivot = stable_partition(from, to, f);
        l = new wavelet_tree(from, pivot, lo, mid);
        r = new wavelet_tree(pivot, to, mid+1, hi);
    }
    // swap a[i] with a[i+1] , if a[i]!=a[i+1] call
    swapadjacent(i)
    void swapadjacent(int i){
        if(lo == hi)

```

```

        return;
        b[i] = b[i-1] + b[i+1] - b[i];
        c[i] = c[i-1] + c[i+1] - c[i];
        if( b[i+1]-b[i] == b[i] - b[i-1]){
            if(b[i]-b[i-1])
                return this->l->swapadjacent(b[i]);
            else
                return this->r->swapadjacent(i-b[i]);
        }
        else
            return;
    }
    //kth smallest element in [l, r]
    int kth(int l, int r, int k){
        if(l > r)
            return 0;
        if(lo == hi)
            return lo;
        int inLeft = b[r] - b[l-1];
        int lb = b[l-1]; //amt of nos in first (l-1) nos that
        go in left
        int rb = b[r]; //amt of nos in first (r) nos that go in
        left
        if(k <= inLeft)
            return this->l->kth(lb+1, rb, k);
        return this->r->kth(l-lb, r-rb, k-inLeft);
    }
    //count of nos in [l, r] Less than or equal to k
    int LTE(int l, int r, int k){
        if(l > r or k < lo)
            return 0;
        if(hi <= k)
            return r - l + 1;
        int lb = b[l-1], rb = b[r];
        return this->l->LTE(lb+1, rb, k) + this->r->LTE(l-lb,
            r-rb, k);
    }
    //count of nos in [l, r] equal to k
    int count(int l, int r, int k){
        if(l > r or k < lo or k > hi)
            return 0;
        if(lo == hi)
            return r - l + 1;
        int lb = b[l-1], rb = b[r], mid = (lo+hi)/2;
        if(k <= mid)
            return this->l->count(lb+1, rb, k);
        return this->r->count(l-lb, r-rb, k);
    }
    //sum of nos in [l, r] less than or equal to k
    int sumk(int l, int r, int k){
        if(l > r or k < lo)
            return 0;
        if(hi <= k)
            return c[r] - c[l-1];
        int lb = b[l-1], rb = b[r];
        return this->l->sumk(lb+1, rb, k) + this->r->sumk(l-lb,
            r-rb, k);
    }
}
~wavelet_tree(){
    if(l)
        delete l;
    if(r)
        delete r;
}
};
int main(){
    ios_base::sync_with_stdio(false);
    cin.tie(NULL);
    srand(time(NULL));
    int i,n,k,j,q,l,r;
    cin >> n;
    fo(i, n) cin >> a[i+1];
    wavelet_tree T(a+1, a+n+1, 1, MAX);
    cin >> q;
    while(q--){
        int x;
        cin >> x;
        cin >> l >> r >> k;
        if(x == 0){
            //kth smallest
            cout << "Kth smallest: ";

```

```

        cout << T.kth(l, r, k) << endl;
    }
    if(x == 1){
        //less than or equal to K
        cout << "LTE: ";
        cout << T.LTE(l, r, k) << endl;
    }
    if(x == 2){
        //count occurrence of K in [l, r]
        cout << "Occurrence of K: ";
        cout << T.count(l, r, k) << endl;
    }
    if(x == 3){
        //sum of elements less than or equal to K in [l, r]
        cout << "Sum: ";
        cout << T.sumk(l, r, k) << endl;
    }
}
return 0;
}

```

## 3 Geo

### 3.1 3dGeo

```

int dcmp(double x) { return abs(x) < EPS ? 0 : (x<0 ? -1 : 1); }
double degreeToRadian(double rad) { return rad*PI/180; }
struct Point {
    double x, y, z;
    Point() : x(0), y(0), z(0) {}
    Point(double X, double Y, double Z) : x(X), y(Y), z(Z) {}
    Point operator + (const Point& u) const {
        return Point(x + u.x, y + u.y, z + u.z);
    }
    Point operator - (const Point& u) const {
        return Point(x - u.x, y - u.y, z - u.z);
    }
    Point operator * (const double u) const {
        return Point(x * u, y * u, z * u);
    }
    Point operator / (const double u) const {
        return Point(x / u, y / u, z / u);
    }
    friend std::ostream &operator << (std::ostream &os, const
        Point &p) {
        return os << p.x << " " << p.y << " " << p.z;
    }
    friend std::istream &operator >> (std::istream &is, Point
        &p) {
        return is >> p.x >> p.y >> p.z;
    }
};
double dot(Point a, Point b) {
    return a.x * b.x + a.y * b.y + a.z * b.z;
}
Point cross(Point a, Point b) {
    return Point(a.y*b.z - a.z*b.y, a.z*b.x - a.x*b.z, a.x*b.y
        - a.y*b.x);
}
double length(Point a) {
    return sqrt(dot(a, a));
}
double distance(Point a, Point b) {
    return length(a-b);
}
Point unit(const Point &p) {
    return p/length(p);
}
// Rotate p around axis x, with angle radians.
Point rotate(Point p, Point axis, double angle) {
    axis = unit(axis);
    Point comp1 = p * cos(angle);
    Point comp2 = axis * (1-cos(angle)) * dot(axis, p);
    Point comp3 = cross(axis, p) * sin(angle);
    return comp1 + comp2 + comp3;
}
struct Line {Point a, v;}; //a+tv
// returns the distance from point a to line l
double distancePointLine(Point p, Line l) {

```

```

    return length(cross(l.v, p - l.a)) / length(l.v);
}
// distance from Line ab to Line cd
double distanceLineLine(Line a, Line b) {
    Point cr = cross(a.v, b.v);
    double crl = length(cr);
    if (dcmp(crl) == 0) return distancePointLine(a.a, b);
    return abs(dot(cr, a.a-b.a))/crl;
}
struct Plane {
    Point normal; // Normal = (A, B, C)
    double d; // dot(Normal) = d <--> Ax + By + Cz = d
    Point P; // anyPoint on the plane, optional
    Plane(Point normal, double d) {
        double len = length(normal);
        assert(dcmp(len) > 0);
        normal = normal / len;
        d = d / len;
        if (dcmp(normal.x)) P = Point(d/normal.x, 0, 0);
        else if (dcmp(normal.y)) P = Point(0, d/normal.y, 0);
        else P = Point(0, 0, d/normal.z);
    }
    //Plane given by three Non-Collinear Points
    Plane(Point a, Point b, Point c) {
        normal = unit(cross(b-a, c-a));
        d = dot(normal, a);
        P = a;
    }
    bool onPlane(Point a) {
        return dcmp(dot(normal, a) - d) == 0;
    }
    double distance(Point a) {
        return abs(dot(normal, a) - d);
    }
    double isParallel(Line l) {
        return dcmp(dot(l.v, normal)) == 0;
    }
    //return t st l.a + t*l.v is a point on the plane, check
    parallel first
    double intersectLine(Line l) {
        return dot(P-l.a, normal)/dot(l.v, normal);
    }
};

```

### 3.2 Circle Cover

```

//Check if the all of the area of circ(0, R) in
//Circ(00, RR) is covered by some other circle
bool CoverCircle(PT O, double R, vector<PT> &cen,
    vector<double> &rad, PT OO, double RR) {
    int n = cen.size();
    vector<pair<double, double>> arcs;
    for (int i=0; i<n; i++) {
        PT P = cen[i]; double r = rad[i];
        if (i!=0 && R + sqrt(dist2(O, P))<R) return 1;
        if (i==0 && r + sqrt(dist2(O, P))<R) return 1;
        vector<PT> inter =
            CircleCircleIntersection(O, P, R, r);
        if (inter.size() <= 1) continue;
        PT X = inter[0], Y = inter[1];
        if (cross(O, X, Y) < 0) swap(X, Y);
        if (!(cross(O, X, P) >= 0 &&
            cross(O, Y, P) <= 0)) swap(X, Y);
        if (i==0) swap(X, Y);
        X = X-O; Y=Y-O;
        double ll = atan2(X.y, X.x);
        double rr = atan2(Y.y, Y.x);
        if (rr < ll) rr += 2*PI;
        arcs.emplace_back(ll, rr);
    }
    if (arcs.empty()) return false;
    sort(arcs.begin(), arcs.end());
    double st = arcs[0].ff, en = arcs[0].ss, ans = 0;
    for (int i=1; i<arcs.size(); i++) {
        if (arcs[i].first <= en + EPS)
            en = max(en, arcs[i].second);
        else st = arcs[i].first, en = arcs[i].second;
        ans = max(ans, en-st);
    }
}

```

```

    return ans >= 2*PI;
}

```

### 3.3 Circle Union Area

```

struct Point {
    LD x,y;
    LD operator*(const Point &a)const {
        return x*a.y-y*a.x;
    }
    LD operator/(const Point &a)const {
        return sqrt((a.x-x)*(a.x-x)+(a.y-y)*(a.y-y));
    }
}po[N];
LD r[N];
int sgn(LD x) {return fabs(x)<EPS?0:(x>0.0?1:-1);}
pair<LD,bool> ARG[2*N];
LD cir_union(Point c[],LD r[],int n) {
    LD sum = 0.0, sum1 = 0.0, d,p1,p2,p3;
    for(int i = 0; i < n; i++) {
        bool f = 1;
        for(int j = 0; f&&j<n; j++)
            if(i!=j && sgn(r[j]-r[i]-c[j]/c[j])!=1)f=0;
        if(!f) swap(r[i],r[-n]),swap(c[i-],c[n]);
    }
    for(int i = 0; i < n; i++) {
        int k = 0, cnt = 0;
        for(int j = 0; j < n; j++) {
            if(i!=j&&sgn((d=c[i]/c[j])-r[i]-r[j])<=0){
                p3=acos((r[i]*r[i]+d*d-r[j]*r[j])/(
                    (2.0*r[i]*d)));
                p2=atan2(c[j].y-c[i].y,c[j].x-c[i].x);
                p1 = p2-p3; p2 = p2+p3;
                if(sgn(p1+PI)==-1) p1+=2*PI,cnt++;
                if(sgn(p2-PI)==1) p2-=2*PI,cnt++;
                ARG[k++] = make_pair(p1,0);
                ARG[k++] = make_pair(p2,1);
            }
        }
        if(k) {
            sort(ARG,ARG+k);
            p1 = ARG[k-1].first-2*PI;
            p3 = r[i]*r[i];
            for(int j = 0; j < k; j++) {
                p2 = ARG[j].first;
                if(cnt==0) {
                    sum+=(p2-p1-sin(p2-p1))*p3;
                    sum1+=(c[i]+Point(cos(p1),sin(p1))*
                        r[i])*(c[i]+
                        Point(cos(p2),sin(p2))*r[i]);
                }
                p1 = p2;
                ARG[j].second ? cnt--:cnt++;
            }
            else sum += 2*PI*r[i]*r[i];
        }
        return (sum+fabs(sum1))*0.5;
    }
}

```

### 3.4 basic-area-geometry

```

struct point2d {
    ftype x, y;
    point2d() {}
    point2d(ftype x, ftype y): x(x), y(y) {}
    point2d& operator+=(const point2d &t) {
        x += t.x;
        y += t.y;
        return *this;
    }
    point2d& operator-=(const point2d &t) {
        x -= t.x;
        y -= t.y;
        return *this;
    }
    point2d& operator*=(ftype t) {
        x *= t;
        y *= t;
        return *this;
    }
}

```

```

point2d& operator/=(ftype t) {
    x /= t;
    y /= t;
    return *this;
}
point2d operator+(const point2d &t) const {
    return point2d(*this) += t;
}
point2d operator-(const point2d &t) const {
    return point2d(*this) -= t;
}
point2d operator*(ftype t) const {
    return point2d(*this) *= t;
}
point2d operator/(ftype t) const {
    return point2d(*this) /= t;
}
};
point2d operator*(ftype a, point2d b) {
    return b * a;
}
struct point3d {
    ftype x, y, z;
    point3d() {}
    point3d(ftype x, ftype y, ftype z): x(x), y(y), z(z) {}
    point3d& operator+=(const point3d &t) {
        x += t.x;
        y += t.y;
        z += t.z;
        return *this;
    }
    point3d& operator-=(const point3d &t) {
        x -= t.x;
        y -= t.y;
        z -= t.z;
        return *this;
    }
    point3d& operator*=(ftype t) {
        x *= t;
        y *= t;
        z *= t;
        return *this;
    }
    point3d& operator/=(ftype t) {
        x /= t;
        y /= t;
        z /= t;
        return *this;
    }
    point3d operator+(const point3d &t) const {
        return point3d(*this) += t;
    }
    point3d operator-(const point3d &t) const {
        return point3d(*this) -= t;
    }
    point3d operator*(ftype t) const {
        return point3d(*this) *= t;
    }
    point3d operator/(ftype t) const {
        return point3d(*this) /= t;
    }
}
};
point3d operator*(ftype a, point3d b) {
    return b * a;
}
ftype dot(point2d a, point2d b) {
    return a.x * b.x + a.y * b.y;
}
ftype dot(point3d a, point3d b) {
    return a.x * b.x + a.y * b.y + a.z * b.z;
}
ftype norm(point2d a) {
    return dot(a, a);
}
double abs(point2d a) {
    return sqrt(norm(a));
}
double proj(point2d a, point2d b) {
    return dot(a, b) / abs(b);
}

```

```

double angle(point2d a, point2d b) {
    return acos(dot(a, b) / abs(a) / abs(b));
}
point3d cross(point3d a, point3d b) {
    return point3d(a.y * b.z - a.z * b.y,
        a.z * b.x - a.x * b.z,
        a.x * b.y - a.y * b.x);
}
ftype triple(point3d a, point3d b, point3d c) {
    return dot(a, cross(b, c));
}
ftype cross(point2d a, point2d b) {
    return a.x * b.y - a.y * b.x;
}
point2d intersect(point2d a1, point2d d1, point2d a2, point2d
    d2) {
    return a1 + cross(a2 - a1, d2) / cross(d1, d2) * d1;
}
point3d intersect(point3d a1, point3d n1, point3d a2, point3d
    n2, point3d a3, point3d n3) {
    point3d x(n1.x, n2.x, n3.x);
    point3d y(n1.y, n2.y, n3.y);
    point3d z(n1.z, n2.z, n3.z);
    point3d d(dot(a1, n1), dot(a2, n2), dot(a3, n3));
    return point3d(triple(d, y, z),
        triple(x, d, z),
        triple(x, y, d)) / triple(n1, n2, n3);
}
int signed_area_parallelogram(point2d p1, point2d p2, point2d
    p3) {
    return cross(p2 - p1, p3 - p1);
}
double triangle_area(point2d p1, point2d p2, point2d p3) {
    return abs(signed_area_parallelogram(p1, p2, p3)) / 2.0;
}
bool clockwise(point2d p1, point2d p2, point2d p3) {
    return signed_area_parallelogram(p1, p2, p3) < 0;
}
bool counter_clockwise(point2d p1, point2d p2, point2d p3) {
    return signed_area_parallelogram(p1, p2, p3) > 0;
}
double area(const vector<point>& fig) {
    double res = 0;
    for (unsigned i = 0; i < fig.size(); i++) {
        point p = i ? fig[i - 1] : fig.back();
        point q = fig[i];
        res += (p.x - q.x) * (p.y + q.y);
    }
    return fabs(res) / 2;
}
//Pick: S = I + B/2 - 1
int count_lattices(Fraction k, Fraction b, long long n) {
    auto fk = k.floor();
    auto fb = b.floor();
    auto cnt = 0LL;
    if (k >= 1 || b >= 1) {
        cnt += (fk * (n - 1) + 2 * fb) * n / 2;
        k -= fk;
        b -= fb;
    }
    auto t = k * n + b;
    auto ft = t.floor();
    if (ft >= 1) {
        cnt += count_lattices(1 / k, (t - t.floor()) / k,
            t.floor());
    }
    return cnt;
}

```

### 3.5 delaunay-voronoi

```

typedef long long ll;
bool ge(const ll& a, const ll& b) { return a >= b; }
bool le(const ll& a, const ll& b) { return a <= b; }
bool eq(const ll& a, const ll& b) { return a == b; }
bool gt(const ll& a, const ll& b) { return a > b; }
bool lt(const ll& a, const ll& b) { return a < b; }
int sgn(const ll& a) { return a >= 0 ? a ? 1 : 0 : -1; }
struct pt {

```

```

    ll x, y;
    pt() {}
    pt(ll _x, ll _y) : x(_x), y(_y) {}
    pt operator-(const pt& p) const {
        return pt(x - p.x, y - p.y);
    }
    ll cross(const pt& p) const {
        return x * p.y - y * p.x;
    }
    ll cross(const pt& a, const pt& b) const {
        return (a - *this).cross(b - *this);
    }
    ll dot(const pt& p) const {
        return x * p.x + y * p.y;
    }
    ll dot(const pt& a, const pt& b) const {
        return (a - *this).dot(b - *this);
    }
    ll sqrLength() const {
        return this->dot(*this);
    }
    bool operator==(const pt& p) const {
        return eq(x, p.x) && eq(y, p.y);
    }
};
const pt inf_pt = pt(1e18, 1e18);
struct QuadEdge {
    pt origin;
    QuadEdge* rot = nullptr;
    QuadEdge* onext = nullptr;
    bool used = false;
    QuadEdge* rev() const {
        return rot->rot;
    }
    QuadEdge* lnext() const {
        return rot->rev()->onext->rot;
    }
    QuadEdge* oprev() const {
        return rot->onext->rot;
    }
    pt dest() const {
        return rev()->origin;
    }
};
QuadEdge* make_edge(pt from, pt to) {
    QuadEdge* e1 = new QuadEdge;
    QuadEdge* e2 = new QuadEdge;
    QuadEdge* e3 = new QuadEdge;
    QuadEdge* e4 = new QuadEdge;
    e1->origin = from;
    e2->origin = to;
    e3->origin = e4->origin = inf_pt;
    e1->rot = e3;
    e2->rot = e4;
    e3->rot = e2;
    e4->rot = e1;
    e1->onext = e1;
    e2->onext = e2;
    e3->onext = e4;
    e4->onext = e3;
    return e1;
}
void splice(QuadEdge* a, QuadEdge* b) {
    swap(a->onext->rot->onext, b->onext->rot->onext);
    swap(a->onext, b->onext);
}
void delete_edge(QuadEdge* e) {
    splice(e, e->oprev());
    splice(e->rev(), e->rev()->oprev());
    delete e->rev()->rot;
    delete e->rev();
    delete e->rot;
    delete e;
}
QuadEdge* connect(QuadEdge* a, QuadEdge* b) {
    QuadEdge* e = make_edge(a->dest(), b->origin);
    splice(e, a->lnext());
    splice(e->rev(), b);
    return e;
}

```

```

bool left_of(pt p, QuadEdge* e) {
    return gt(p.cross(e->origin, e->dest()), 0);
}
bool right_of(pt p, QuadEdge* e) {
    return lt(p.cross(e->origin, e->dest()), 0);
}
template <class T>
T det3(T a1, T a2, T a3, T b1, T b2, T b3, T c1, T c2, T c3) {
    return a1 * (b2 * c3 - c2 * b3) - a2 * (b1 * c3 - c1 * b3)
        + a3 * (b1 * c2 - c1 * b2);
}
bool in_circle(pt a, pt b, pt c, pt d) {
    // If there is __int128, calculate directly.
    // Otherwise, calculate angles.
    #if defined(__LP64__) || defined(_WIN64)
        __int128 det = -det3<__int128>(b.x, b.y, b.sqrLength(),
            c.x, c.y,
            c.sqrLength(), d.x, d.y,
            d.sqrLength());
        det += det3<__int128>(a.x, a.y, a.sqrLength(), c.x, c.y,
            c.sqrLength(), d.x,
            d.y, d.sqrLength());
        det -= det3<__int128>(a.x, a.y, a.sqrLength(), b.x, b.y,
            b.sqrLength(), d.x,
            d.y, d.sqrLength());
        det += det3<__int128>(a.x, a.y, a.sqrLength(), b.x, b.y,
            b.sqrLength(), c.x,
            c.y, c.sqrLength());
        return det > 0;
    #else
        auto ang = [](pt l, pt mid, pt r) {
            ll x = mid.dot(l, r);
            ll y = mid.cross(l, r);
            long double res = atan2((long double)x, (long double)y);
            return res;
        };
        long double kek = ang(a, b, c) + ang(c, d, a) - ang(b, c,
            d) - ang(d, a, b);
        if (kek > 1e-8)
            return true;
        else
            return false;
    #endif
}
pair<QuadEdge*, QuadEdge*> build_tr(int l, int r, vector<pt>&
    p) {
    if (r - l + 1 == 2) {
        QuadEdge* res = make_edge(p[l], p[r]);
        return make_pair(res, res->rev());
    }
    if (r - l + 1 == 3) {
        QuadEdge* a = make_edge(p[l], p[l + 1]), *b =
            make_edge(p[l + 1], p[r]);
        splice(a->rev(), b);
        int sg = sgn(p[l].cross(p[l + 1], p[r]));
        if (sg == 0)
            return make_pair(a, b->rev());
        QuadEdge* c = connect(b, a);
        if (sg == 1)
            return make_pair(a, b->rev());
        else
            return make_pair(c->rev(), c);
    }
    int mid = (l + r) / 2;
    QuadEdge* ldo, *ldi, *rdo, *rdi;
    tie(ldo, ldi) = build_tr(l, mid, p);
    tie(rdi, rdo) = build_tr(mid + 1, r, p);
    while (true) {
        if (left_of(rdi->origin, ldi)) {
            ldi = ldi->lnext();
            continue;
        }
        if (right_of(ldi->origin, rdi)) {
            rdi = rdi->rev()->onext;
            continue;
        }
        break;
    }
}

```

```

QuadEdge* basel = connect(rdi->rev(), ldi);
auto valid = [&basel](QuadEdge* e) { return
    right_of(e->dest(), basel); };
if (ldi->origin == ldo->origin)
    ldo = basel->rev();
if (rdi->origin == rdo->origin)
    rdo = basel;
while (true) {
    QuadEdge* lcand = basel->rev()->onext;
    if (valid(lcand)) {
        while (in_circle(basel->dest(), basel->origin,
            lcand->dest(),
            lcand->onext->dest())) {
            QuadEdge* t = lcand->onext;
            delete_edge(lcand);
            lcand = t;
        }
    }
    QuadEdge* rcand = basel->oprev();
    if (valid(rcand)) {
        while (in_circle(basel->dest(), basel->origin,
            rcand->dest(),
            rcand->oprev()->dest())) {
            QuadEdge* t = rcand->oprev();
            delete_edge(rcand);
            rcand = t;
        }
    }
    if (!valid(lcand) && !valid(rcand))
        break;
    if (!valid(lcand) ||
        (valid(rcand) && in_circle(lcand->dest(),
            lcand->origin,
            rcand->origin,
            rcand->dest())))
        basel = connect(rcand, basel->rev());
    else
        basel = connect(basel->rev(), lcand->rev());
}
return make_pair(ldo, rdo);
}
vector<tuple<pt, pt, pt>> delaunay(vector<pt> p) {
    sort(p.begin(), p.end(), [](const pt& a, const pt& b) {
        return lt(a.x, b.x) || (eq(a.x, b.x) && lt(a.y, b.y));
    });
    auto res = build_tr(0, (int)p.size() - 1, p);
    QuadEdge* e = res.first;
    vector<QuadEdge*> edges = {e};
    while (lt(e->onext->dest().cross(e->dest(), e->origin), 0))
        e = e->onext;
    auto add = [&p, &e, &edges]() {
        QuadEdge* curr = e;
        do {
            curr->used = true;
            p.push_back(curr->origin);
            edges.push_back(curr->rev());
            curr = curr->lnext();
        } while (curr != e);
    };
    add();
    p.clear();
    int kek = 0;
    while (kek < (int)edges.size()) {
        if (!(e = edges[kek++])->used)
            add();
    }
    vector<tuple<pt, pt, pt>> ans;
    for (int i = 0; i < (int)p.size(); i += 3) {
        ans.push_back(make_tuple(p[i], p[i + 1], p[i + 2]));
    }
    return ans;
}

```

### 3.6 half-plane-intersection

```

class HalfPlaneIntersection{
    static double eps, inf;
public:
    struct Point{
        double x, y;

```

```

        explicit Point(double x = 0, double y = 0) : x(x), y(y)
        {}
        // Addition, subtraction, multiply by constant, cross
        // product.
        friend Point operator + (const Point& p, const Point&
            q){
            return Point(p.x + q.x, p.y + q.y);
        }
        friend Point operator - (const Point& p, const Point&
            q){
            return Point(p.x - q.x, p.y - q.y);
        }
        friend Point operator * (const Point& p, const double&
            k){
            return Point(p.x * k, p.y * k);
        }
        friend double cross(const Point& p, const Point& q){
            return p.x * q.y - p.y * q.x;
        }
    };
    // Basic half-plane struct.
    struct Halfplane{
        // 'p' is a passing point of the line and 'pq' is the
        // direction vector of the line.
        Point p, pq;
        double angle;
        Halfplane() {}
        Halfplane(const Point& a, const Point& b) : p(a), pq(b
            - a){
            angle = atan2l(pq.y, pq.x);
        }
        // Check if point 'r' is outside this half-plane.
        // Every half-plane allows the region to the LEFT of
        // its line.
        bool out(const Point& r){
            return cross(pq, r - p) < -eps;
        }
        // Comparator for sorting.
        // If the angle of both half-planes is equal, the
        // leftmost one should go first.
        bool operator < (const Halfplane& e) const{
            if (fabsl(angle - e.angle) < eps) return cross(pq,
                e.p - p) < 0;
            return angle < e.angle;
        }
        // We use equal comparator for std::unique to easily
        // remove parallel half-planes.
        bool operator == (const Halfplane& e) const{
            return fabsl(angle - e.angle) < eps;
        }
        // Intersection point of the lines of two half-planes.
        // It is assumed they're never parallel.
        friend Point inter(const Halfplane& s, const Halfplane&
            t){
            double alpha = cross((t.p - s.p), t.pq) /
                cross(s.pq, t.pq);
            return s.p + (s.pq * alpha);
        }
    };
    static vector<Point> hp_intersect(vector<Halfplane>& H){
        Point box[4] = // Bounding box in CCW order{
            Point(inf, inf),
            Point(-inf, inf),
            Point(-inf, -inf),
            Point(inf, -inf)
        };
        for(int i = 0; i < 4; i++) // Add bounding box
            half-planes.{
                Halfplane aux(box[i], box[(i+1) % 4]);
                H.push_back(aux);
            }
        // Sort and remove duplicates
        sort(H.begin(), H.end());
        H.erase(unique(H.begin(), H.end(), H.end());
            deque<Halfplane> dq;
            int len = 0;
            for(int i = 0; i < int(H.size()); i++){

```

```

                // Remove from the back of the deque while last
                // half-plane is redundant
                while (len > 1 && H[i].out(inter(dq[len-1],
                    dq[len-2]))){
                    dq.pop_back();
                    --len;
                }
                // Remove from the front of the deque while first
                // half-plane is redundant
                while (len > 1 && H[i].out(inter(dq[0], dq[1]))){
                    dq.pop_front();
                    --len;
                }
                // Add new half-plane
                dq.push_back(H[i]);
                ++len;
            }
        }
        // Final cleanup: Check half-planes at the front
        // against the back and vice-versa
        while (len > 2 && dq[0].out(inter(dq[len-1],
            dq[len-2]))){
            dq.pop_back();
            --len;
        }
        while (len > 2 && dq[len-1].out(inter(dq[0], dq[1]))){
            dq.pop_front();
            --len;
        }
        // Report empty intersection if necessary
        if (len < 3) return vector<Point>();
        // Reconstruct the convex polygon from the remaining
        // half-planes.
        vector<Point> ret(len);
        for(int i = 0; i+1 < len; i++){
            ret[i] = inter(dq[i], dq[i+1]);
        }
        ret.back() = inter(dq[len-1], dq[0]);
        return ret;
    }
};
double HalfPlaneIntersection::eps=1e-9;
double HalfPlaneIntersection::inf=1e9;

```

### 3.7 heart-of-geometry-2d

```

typedef double ftype;
const double EPS = 1E-9;
struct pt{
    ftype x, y;
    int id;
    pt() {}
    pt(ftype _x, ftype _y):x(_x), y(_y) {}
    pt operator+(const pt & p) const{
        return pt(x + p.x, y + p.y);
    }
    pt operator-(const pt & p) const{
        return pt(x - p.x, y - p.y);
    }
    ftype cross(const pt & p) const{
        return x * p.y - y * p.x;
    }
    ftype dot(const pt & p) const{
        return x * p.x + y * p.y;
    }
    ftype cross(const pt & a, const pt & b) const{
        return (a - *this).cross(b - *this);
    }
    ftype dot(const pt & a, const pt & b) const{
        return (a - *this).dot(b - *this);
    }
    ftype sqrLen() const{
        return this->dot(*this);
    }
    bool operator<(const pt& p) const{
        return x < p.x - EPS || (abs(x - p.x) < EPS && y < p.y
            - EPS);
    }
    bool operator==(const pt& p) const{
        return abs(x-p.x)<EPS && abs(y-p.y)<EPS;
    }
}

```



```

    }
};
int sign(double x) { return (x > EPS) - (x < -EPS); }
inline int orientation(pt a, pt b, pt c) { return
    sign(a.cross(b,c)); }
bool is_point_on_seg(pt a, pt b, pt p) {
    if (fabs(b.cross(p,a)) < EPS) {
        if (p.x < min(a.x, b.x) - EPS || p.x > max(a.x, b.x) +
            EPS) return false;
        if (p.y < min(a.y, b.y) - EPS || p.y > max(a.y, b.y) +
            EPS) return false;
        return true;
    }
    return false;
}
bool is_point_on_polygon(vector<pt> &p, const pt& z) {
    int n = p.size();
    for (int i = 0; i < n; i++) {
        if (is_point_on_seg(p[i], p[(i + 1) % n], z)) return 1;
    }
    return 0;
}
int winding_number(vector<pt> &p, const pt& z) { // 0(n)
    if (is_point_on_polygon(p, z)) return 1e9;
    int n = p.size(), ans = 0;
    for (int i = 0; i < n; ++i) {
        int j = (i + 1) % n;
        bool below = p[i].y < z.y;
        if (below != (p[j].y < z.y)) {
            auto orient = orientation(z, p[j], p[i]);
            if (orient == 0) return 0;
            if (below == (orient > 0)) ans += below ? -1 : 1;
        }
    }
    return ans;
}
double dist_sqr(pt a, pt b) {
    return ((a.x-b.x)*(a.x-b.x) + (a.y-b.y)*(a.y-b.y));
}
double dist(pt a, pt b) {
    return sqrt((a.x-b.x)*(a.x-b.x) + (a.y-b.y)*(a.y-b.y));
}
double angle(pt a, pt b, pt c) {
    if (b==a || b==c) return 0;
    double A2 = dist_sqr(b,c);
    double C2 = dist_sqr(a,b);
    double B2 = dist_sqr(c,a);
    double A = sqrt(A2), C = sqrt(C2);
    double ans = (A2 + C2 - B2)/(A*C*2);
    if (ans < -1) ans = acos(-1);
    else if (ans > 1) ans = acos(1);
    else ans = acos(ans);
    return ans;
}
bool cmp(pt a, pt b) {
    return a.x < b.x || (a.x == b.x && a.y < b.y);
}
bool ccw(pt a, pt b, pt c, bool include_collinear=false) {
    int o = orientation(a, b, c);
    return o > 0 || (include_collinear && o == 0);
}
bool cw(pt a, pt b, pt c, bool include_collinear=false) {
    int o = orientation(a, b, c);
    return o < 0 || (include_collinear && o == 0);
}
bool collinear(pt a, pt b, pt c) { return orientation(a, b, c)
    == 0; }
double area(pt a, pt b, pt c) {
    return (a.x*(b.y-c.y)+b.x*(c.y-a.y)+c.x*(a.y-b.y))/2;
}
struct cmp_x {
    bool operator()(const pt &a, const pt &b) const {
        return a.x < b.x || (a.x == b.x && a.y < b.y);
    }
};
struct cmp_y {
    bool operator()(const pt &a, const pt &b) const {
        return a.y < b.y || (a.y == b.y && a.x < b.x);
    }
};

```

```

    }
};
struct circle : pt {
    ftype r;
};
bool insideCircle(circle c, pt p) {
    return dist_sqr(c,p) <= c.r*c.r + EPS;
}
struct line {
    ftype a, b, c;
    line() {}
    line(pt p, pt q) {
        a = p.y - q.y;
        b = q.x - p.x;
        c = -a * p.x - b * p.y;
        norm();
    }
    void norm() {
        double z = sqrt(a * a + b * b);
        if (abs(z) > EPS)
            a /= z, b /= z, c /= z;
    }
    line getParallel(pt p) {
        line ans = *this;
        ans.c = -(ans.a*p.x+ans.b*p.y);
        return ans;
    }
    ftype getValue(pt p) {
        return a*p.x+b*p.y+c;
    }
    line getPerpend(pt p) {
        line ans;
        ans.a=this->b;
        ans.b=-(this->a);
        ans.c = -(ans.a*p.x+ans.b*p.y);
        return ans;
    }
    //dist formula is wrong but don't change
    double dist(pt p) const { return a * p.x + b * p.y + c; }
};
double sqr (double a) {
    return a * a;
}
double det(double a, double b, double c, double d) {
    return a*d - b*c;
}
bool intersect(line m, line n, pt & res) {
    double zn = det(m.a, m.b, n.a, n.b);
    if (abs(zn) < EPS)
        return false;
    res.x = -det(m.c, m.b, n.c, n.b) / zn;
    res.y = -det(m.a, m.c, n.a, n.c) / zn;
    return true;
}
bool parallel(line m, line n) {
    return abs(det(m.a, m.b, n.a, n.b)) < EPS;
}
bool equivalent(line m, line n) {
    return abs(det(m.a, m.b, n.a, n.b)) < EPS
        && abs(det(m.a, m.c, n.a, n.c)) < EPS
        && abs(det(m.b, m.c, n.b, n.c)) < EPS;
}
double det(double a, double b, double c, double d) {
    return a * d - b * c;
}
inline bool betw(double l, double r, double x) {
    return min(l, r) <= x + EPS && x <= max(l, r) + EPS;
}
inline bool intersect_1d(double a, double b, double c, double
    d) {
    if (a > b)
        swap(a, b);
    if (c > d)
        swap(c, d);
    return max(a, c) <= min(b, d) + EPS;
}
bool intersect_segment(pt a, pt b, pt c, pt d, pt& left, pt&
    right) {
    if (!intersect_1d(a.x, b.x, c.x, d.x) ||
        !intersect_1d(a.y, b.y, c.y, d.y))

```

```

        return false;
    line m(a, b);
    line n(c, d);
    double zn = det(m.a, m.b, n.a, n.b);
    if (abs(zn) < EPS) {
        if (abs(m.dist(c)) > EPS || abs(n.dist(a)) > EPS)
            return false;
        if (b < a)
            swap(a, b);
        if (d < c)
            swap(c, d);
        left = max(a, c);
        right = min(b, d);
        return true;
    } else {
        left.x = right.x = -det(m.c, m.b, n.c, n.b) / zn;
        left.y = right.y = -det(m.a, m.c, n.a, n.c) / zn;
        return betw(a.x, b.x, left.x) && betw(a.y, b.y, left.y)
            &&
            betw(c.x, d.x, left.x) && betw(c.y, d.y, left.y);
    }
}
void tangents (pt c, double r1, double r2, vector<line> & ans)
{
    double r = r2 - r1;
    double z = sqr(c.x) + sqr(c.y);
    double d = z - sqr(r);
    if (d < -EPS) return;
    d = sqrt (abs (d));
    line l;
    l.a = (c.x * r + c.y * d) / z;
    l.b = (c.y * r - c.x * d) / z;
    l.c = r1;
    ans.push_back (l);
}
vector<line> tangents (circle a, circle b) {
    vector<line> ans;
    for (int i=-1; i<=1; i+=2)
        for (int j=-1; j<=1; j+=2)
            tangents (b-a, a.r*i, b.r*j, ans);
    for (size_t i=0; i<ans.size(); ++i)
        ans[i].c -= ans[i].a * a.x + ans[i].b * a.y;
    return ans;
}
class pointLocationInPolygon {
    bool lexComp(const pt &l, const pt &r) {
        return l.x < r.x || (l.x == r.x && l.y < r.y);
    }
    int sgn(ftype val) {
        return val > 0 ? 1 : (val == 0 ? 0 : -1);
    }
    vector<pt> seq;
    int n;
    pt translate;
    bool pointInTriangle(pt a, pt b, pt c, pt point) {
        ftype s1 = abs(a.cross(b, c));
        ftype s2 = abs(point.cross(a, b)) + abs(point.cross(b,
            c)) + abs(point.cross(c, a));
        return s1 == s2;
    }
public:
    pointLocationInPolygon() {}
    pointLocationInPolygon(vector<pt> & points) {
        prepare(points);
    }
    void prepare(vector<pt> & points) {
        seq.clear();
        n = points.size();
        int pos = 0;
        for (int i = 1; i < n; i++) {
            if (lexComp(points[i], points[pos]))
                pos = i;
        }
        translate.x = points[pos].x;
        translate.y = points[pos].y;
        rotate(points.begin(), points.begin() + pos,
            points.end());
        n--;
    }

```

```

    seq.resize(n);
    for(int i = 0; i < n; i++){
        seq[i] = points[i + 1] - points[0];
    }
    bool pointInConvexPolygon(pt point){
        point.x -= translate.x;
        point.y -= translate.y;
        if(seq[0].cross(point) != 0 && sgn(seq[0].cross(point))
            != sgn(seq[0].cross(seq[n - 1])))
            return false;
        if(seq[n - 1].cross(point) != 0 && sgn(seq[n - 1].cross(point))
            != sgn(seq[n - 1].cross(seq[0])))
            return false;
        if(seq[0].cross(point) == 0)
            return seq[0].sqrLen() >= point.sqrLen();
        int l = 0, r = n - 1;
        while(r - l > 1){
            int mid = (l + r) / 2;
            int pos = mid;
            if(seq[pos].cross(point) >= 0) l = mid;
            else r = mid;
        }
        int pos = l;
        return pointInTriangle(seq[pos], seq[pos + 1], pt(0, 0), point);
    }
    pointLocationInPolygon(){
        seq.clear();
    }
};

class Minkowski{
    static void reorder_polygon(vector<pt> & P){
        size_t pos = 0;
        for(size_t i = 1; i < P.size(); i++){
            if(P[i].y < P[pos].y || (P[i].y == P[pos].y &&
                P[i].x < P[pos].x))
                pos = i;
        }
        rotate(P.begin(), P.begin() + pos, P.end());
    }
public:
    static vector<pt> minkowski(vector<pt> P, vector<pt> Q){
        // the first vertex must be the lowest
        reorder_polygon(P);
        reorder_polygon(Q);
        // we must ensure cyclic indexing
        P.push_back(P[0]);
        P.push_back(P[1]);
        Q.push_back(Q[0]);
        Q.push_back(Q[1]);
        // main part
        vector<pt> result;
        size_t i = 0, j = 0;
        while(i < P.size() - 2 || j < Q.size() - 2){
            result.push_back(P[i] + Q[j]);
            auto cross = (P[i + 1] - P[i]).cross(Q[j + 1] - Q[j]);
            if(cross >= 0)
                ++i;
            if(cross <= 0)
                ++j;
        }
        return result;
    }
};

vector<pt> circle_line_intersections(circle cir, line l){
    double r = cir.r, a = l.a, b = l.b, c = l.c + l.a*cir.x + l.b*cir.y;
    vector<pt> ans;
    double x0 = -a*c/(a*a+b*b), y0 = -b*c/(a*a+b*b);
    if(c*c > r*r*(a*a+b*b)+EPS);
    else if( abs(c*c - r*r*(a*a+b*b)) < EPS){
        pt p;
        p.x = x0;
        p.y = y0;
        ans.push_back(p);
    }
    else{
        double d = r*r - c*c/(a*a+b*b);

```

```

        double mult = sqrt(d / (a*a+b*b));
        double ax, ay, bx, by;
        ax = x0 + b * mult;
        bx = x0 - b * mult;
        ay = y0 + a * mult;
        by = y0 - a * mult;
        pt p;
        p.x = ax;
        p.y = ay;
        ans.push_back(p);
        p.x = bx;
        p.y = by;
        ans.push_back(p);
    }
    for(int i=0; i<ans.size(); i++){
        ans[i] = ans[i] + cir;
    }
    return ans;
}

double circle_polygon_intersection(circle c, vector<pt> &V){
    int n = V.size();
    double ans = 0;
    for(int i=0; i<n; i++){
        line l(V[i], V[(i+1)%n]);
        vector<pt> lpts = circle_line_intersections(c, l);
        int sz = lpts.size();
        for(int j=sz-1; j>=0; j--){
            if(!is_point_on_seg(V[i], V[(i+1)%n], lpts[j])){
                swap(lpts.back(), lpts[j]);
                lpts.pop_back();
            }
        }
        lpts.push_back(V[i]);
        lpts.push_back(V[(i+1)%n]);
        sort(lpts.begin(), lpts.end());
        sz = lpts.size();
        if(V[(i+1)%n] < V[i])
            reverse(lpts.begin(), lpts.end());
        for(int j=1; j<sz; j++){
            if(insideCircle(c, lpts[j-1])
                && insideCircle(c, lpts[j]))
                ans = ans + area(lpts[j-1], lpts[j], c);
            else{
                double ang = angle(lpts[j-1], c, lpts[j]);
                double aa = c.r*c.r*ang/2;
                if(ccw(lpts[j-1], lpts[j], c))
                    ans = ans+aa;
                else
                    ans = ans-aa;
            }
        }
    }
    ans = abs(ans);
    return ans;
}

void convex_hull(vector<pt> &a, bool include_collinear = false){
    pt p0 = *min_element(a.begin(), a.end(), [](pt a, pt b){
        return make_pair(a.y, a.x) < make_pair(b.y, b.x);
    });
    sort(a.begin(), a.end(), [&p0](const pt& a, const pt& b){
        int o = orientation(p0, a, b);
        if(o == 0)
            return (p0.x-a.x)*(p0.x-a.x) + (p0.y-a.y)*(p0.y-a.y)
                < (p0.x-b.x)*(p0.x-b.x) + (p0.y-b.y)*(p0.y-b.y);
        return o < 0;
    });
    if(include_collinear){
        int i = (int)a.size()-1;
        while(i >= 0 && collinear(p0, a[i], a.back())) i--;
        reverse(a.begin()+i+1, a.end());
    }
    vector<pt> st;
    for(int i = 0; i < (int)a.size(); i++){
        while(st.size() > 1 && !cw(st[st.size()-2], st.back(), a[i], include_collinear))
            st.pop_back();
        st.push_back(a[i]);
    }
    a = st;
}

```

```

    int m = a.size();
    for(int i = 0; i < m-1; i++){
        swap(a[i], a[m-1-i]);
    }
}

double mindist;
pair<int, pair<int, int>> best_pair;
void upd_ans(const pt &a, const pt &b, const pt &c){
    double distC = sqrt((a.x - b.x)*(a.x - b.x) + (a.y - b.y)*(a.y - b.y));
    double distA = sqrt((c.x - b.x)*(c.x - b.x) + (c.y - b.y)*(c.y - b.y));
    double distB = sqrt((a.x - c.x)*(a.x - c.x) + (a.y - c.y)*(a.y - c.y));
    if(distA + distB + distC < mindist){
        mindist = distA + distB + distC;
        best_pair = make_pair(a.id, make_pair(b.id, c.id));
    }
}

vector<pt> t;
//Min possible triplet distance
void rec(int l, int r){
    if(r - l <= 3 && r - l >= 2){
        for(int i = l; i < r; ++i){
            for(int j = i + 1; j < r; ++j){
                for(int k = j + 1; k < r; k++){
                    upd_ans(a[i], a[j], a[k]);
                }
            }
        }
        sort(a.begin() + l, a.begin() + r, cmp_y());
        return;
    }
    int m = (l + r) >> 1;
    int midx = a[m-1].x;
    /*
    * Got WA in a team contest
    * For putting midx = a[m].x;
    * Don't know why. Maybe due to
    * floating point numbers.
    */
    rec(l, m);
    rec(m, r);
    merge(a.begin() + l, a.begin() + m, a.begin() + m,
        a.begin() + r, t.begin(), cmp_y());
    copy(t.begin(), t.begin() + r - l, a.begin() + l);
    int tsz = 0;
    for(int i = l; i < r; ++i){
        if(abs(a[i].x - midx) < mindist/2){
            for(int j = tsz - 1; j >= 0 && a[i].y - t[j].y <
                mindist/2; --j){
                if(i+1 < r) upd_ans(a[i], a[i+1], t[j]);
                if(j > 0) upd_ans(a[i], t[j-1], t[j]);
            }
            t[tsz++] = a[i];
        }
    }
}

```

### 3.8 intersecting-segments-pair

```

const double EPS = 1E-9;
struct pt {
    double x, y;
};

struct seg {
    pt p, q;
    int id;
    double get_y(double x) const {
        if(abs(p.x - q.x) < EPS)
            return p.y;
        return p.y + (q.y - p.y) * (x - p.x) / (q.x - p.x);
    }
};

bool intersectId(double l1, double r1, double l2, double r2) {
    if(l1 > r1)
        swap(l1, r1);
    if(l2 > r2)
        swap(l2, r2);
}

```

```

    return max(l1, l2) <= min(r1, r2) + EPS;
}
int vec(const pt& a, const pt& b, const pt& c) {
    double s = (b.x - a.x) * (c.y - a.y) - (b.y - a.y) * (c.x - a.x);
    return abs(s) < EPS ? 0 : s > 0 ? +1 : -1;
}
bool intersect(const seg& a, const seg& b) {
    return intersect1d(a.p.x, a.q.x, b.p.x, b.q.x) &&
        intersect1d(a.p.y, a.q.y, b.p.y, b.q.y) &&
        vec(a.p, a.q, b.p) * vec(a.p, a.q, b.q) <= 0 &&
        vec(b.p, b.q, a.p) * vec(b.p, b.q, a.q) <= 0;
}
bool operator<(const seg& a, const seg& b) {
    double x = max(min(a.p.x, a.q.x), min(b.p.x, b.q.x));
    return a.get_y(x) < b.get_y(x) - EPS;
}
struct event {
    double x;
    int tp, id;
    event() {}
    event(double x, int tp, int id) : x(x), tp(tp), id(id) {}
    bool operator<(const event& e) const {
        if (abs(x - e.x) > EPS)
            return x < e.x;
        return tp > e.tp;
    }
};
set<seg> s;
vector<set<seg>::iterator> where;
set<seg>::iterator prev(set<seg>::iterator it) {
    return it == s.begin() ? s.end() : --it;
}
set<seg>::iterator next(set<seg>::iterator it) {
    return ++it;
}
pair<int, int> solve(const vector<seg>& a) {
    int n = (int)a.size();
    vector<event> e;
    for (int i = 0; i < n; ++i) {
        e.push_back(event(min(a[i].p.x, a[i].q.x), +1, i));
        e.push_back(event(max(a[i].p.x, a[i].q.x), -1, i));
    }
    sort(e.begin(), e.end());
    s.clear();
    where.resize(a.size());
    for (size_t i = 0; i < e.size(); ++i) {
        int id = e[i].id;
        if (e[i].tp == +1) {
            set<seg>::iterator nxt = s.lower_bound(a[id]), prv = prev(nxt);
            if (nxt != s.end() && intersect(*nxt, a[id]))
                return make_pair(nxt->id, id);
            if (prv != s.end() && intersect(*prv, a[id]))
                return make_pair(prv->id, id);
            where[id] = s.insert(nxt, a[id]);
        } else {
            set<seg>::iterator nxt = next(where[id]), prv = prev(where[id]);
            if (nxt != s.end() && prv != s.end() && intersect(*nxt, *prv))
                return make_pair(prv->id, nxt->id);
            s.erase(where[id]);
        }
    }
    return make_pair(-1, -1);
}

```

### 3.9 point-location

```

typedef long long ll;
bool ge(const ll& a, const ll& b) { return a >= b; }
bool le(const ll& a, const ll& b) { return a <= b; }
bool eq(const ll& a, const ll& b) { return a == b; }
bool gt(const ll& a, const ll& b) { return a > b; }
bool lt(const ll& a, const ll& b) { return a < b; }
int sgn(const ll& x) { return le(x, 0) ? eq(x, 0) ? 0 : -1 : 1; }
struct pt {

```

```

    ll x, y;
    pt() {}
    pt(ll _x, ll _y) : x(_x), y(_y) {}
    pt operator-(const pt& a) const { return pt(x - a.x, y - a.y); }
    ll dot(const pt& a, const pt& b) const { return x * a.x + y * a.y; }
    ll dot(const pt& a, const pt& b) const { return (a - *this).dot(b - *this); }
    ll cross(const pt& a, const pt& b) const { return x * a.y - y * a.x; }
    ll cross(const pt& a, const pt& b) const { return (a - *this).cross(b - *this); }
    bool operator==(const pt& a) const { return a.x == x && a.y == y; }
};
struct Edge {
    pt l, r;
};
bool edge_cmp(Edge* edge1, Edge* edge2) {
    const pt a = edge1->l, b = edge1->r;
    const pt c = edge2->l, d = edge2->r;
    int val = sgn(a.cross(b, c)) + sgn(a.cross(b, d));
    if (val != 0)
        return val > 0;
    val = sgn(c.cross(d, a)) + sgn(c.cross(d, b));
    return val < 0;
}
enum EventType { DEL = 2, ADD = 3, GET = 1, VERT = 0 };
struct Event {
    EventType type;
    int pos;
    bool operator<(const Event& event) const { return type < event.type; }
};
vector<Edge*> sweepline(vector<Edge*> planar, vector<pt> queries) {
    using pt_type = decltype(pt::x);
    // collect all x-coordinates
    auto s = set<pt_type, std::function<bool(const pt_type&, const pt_type&)>>(lt);
    for (pt p : queries)
        s.insert(p.x);
    for (Edge* e : planar) {
        s.insert(e->l.x);
        s.insert(e->r.x);
    }
    // map all x-coordinates to ids
    int cid = 0;
    auto id = map<pt_type, int, std::function<bool(const pt_type&, const pt_type&)>>(lt);
    for (auto x : s)
        id[x] = cid++;
    // create events
    auto t = set<Edge*, decltype(*edge_cmp)>(edge_cmp);
    auto vert_cmp = [] (const pair<pt_type, int>& l, const pair<pt_type, int>& r) {
        if (!eq(l.first, r.first))
            return lt(l.first, r.first);
        return l.second < r.second;
    };
    auto vert = set<pair<pt_type, int>, decltype(vert_cmp)>(vert_cmp);
    vector<vector<Event>> events(cid);
    for (int i = 0; i < (int)queries.size(); i++) {
        int x = id[queries[i].x];
        events[x].push_back(Event{GET, i});
    }
    for (int i = 0; i < (int)planar.size(); i++) {
        int lx = id[planar[i]->l.x], rx = id[planar[i]->r.x];
        if (lx > rx) {
            swap(lx, rx);
            swap(planar[i]->l, planar[i]->r);
        }
        if (lx == rx) {
            events[lx].push_back(Event{VERT, i});
        } else {

```

```

            events[lx].push_back(Event{ADD, i});
            events[rx].push_back(Event{DEL, i});
        }
    }
    // perform sweep line algorithm
    vector<Edge*> ans(queries.size(), nullptr);
    for (int x = 0; x < cid; x++) {
        sort(events[x].begin(), events[x].end());
        vert.clear();
        for (Event event : events[x]) {
            if (event.type == DEL) {
                t.erase(planar[event.pos]);
            }
            if (event.type == VERT) {
                vert.insert(make_pair(
                    min(planar[event.pos]->l.y, planar[event.pos]->r.y),
                    event.pos));
            }
            if (event.type == ADD) {
                t.insert(planar[event.pos]);
            }
            if (event.type == GET) {
                auto jt = vert.upper_bound(
                    make_pair(queries[event.pos].y, planar.size()));
                if (jt != vert.begin()) {
                    --jt;
                    int i = jt->second;
                    if (ge(max(planar[i]->l.y, planar[i]->r.y), queries[event.pos].y)) {
                        ans[event.pos] = planar[i];
                        continue;
                    }
                }
                Edge* e = new Edge;
                e->l = e->r = queries[event.pos];
                auto it = t.upper_bound(e);
                if (it != t.begin())
                    ans[event.pos] = *(--it);
                delete e;
            }
        }
        for (Event event : events[x]) {
            if (event.type != GET)
                continue;
            if (ans[event.pos] != nullptr && eq(ans[event.pos]->l.x, ans[event.pos]->r.x))
                continue;
            Edge* e = new Edge;
            e->l = e->r = queries[event.pos];
            auto it = t.upper_bound(e);
            delete e;
            if (it == t.begin())
                e = nullptr;
            else
                e = *(--it);
            if (ans[event.pos] == nullptr) {
                ans[event.pos] = e;
                continue;
            }
            if (e == nullptr)
                continue;
            if (e == ans[event.pos])
                continue;
            if (id[ans[event.pos]->r.x] == x) {
                if (id[e->l.x] == x) {
                    if (gt(e->l.y, ans[event.pos]->r.y))
                        ans[event.pos] = e;
                }
            }
            else {
                ans[event.pos] = e;
            }
        }
    }
    return ans;
}
struct DCEL {
    struct Edge {

```

```

    pt origin;
    Edge* nxt = nullptr;
    Edge* twin = nullptr;
    int face;
};
vector<Edge*> body;
};
vector<pair<int, int>> point_location(DCEL planar, vector<pt>
queries){
    vector<pair<int, int>> ans(queries.size());
    vector<Edge*> planar2;
    map<intptr_t, int> pos;
    map<intptr_t, int> added_on;
    int n = planar.body.size();
    for (int i = 0; i < n; i++) {
        if (planar.body[i]->face > planar.body[i]->twin->face)
            continue;
        Edge* e = new Edge;
        e->l = planar.body[i]->origin;
        e->r = planar.body[i]->twin->origin;
        added_on[(intptr_t)e] = i;
        pos[(intptr_t)e] =
            lt(planar.body[i]->origin.x,
                planar.body[i]->twin->origin.x)
            ? planar.body[i]->face
            : planar.body[i]->twin->face;
        planar2.push_back(e);
    }
    auto res = sweepline(planar2, queries);
    for (int i = 0; i < (int)queries.size(); i++) {
        if (res[i] == nullptr) {
            ans[i] = make_pair(1, -1);
            continue;
        }
        pt p = queries[i];
        pt l = res[i]->l, r = res[i]->r;
        if (eq(p.cross(l, r), 0) && le(p.dot(l, r), 0)) {
            ans[i] = make_pair(0, added_on[(intptr_t)res[i]]);
            continue;
        }
        ans[i] = make_pair(1, pos[(intptr_t)res[i]]);
    }
    for (auto e : planar2)
        delete e;
    return ans;
}

```

### 3.10 vertical-decomposition

```

typedef double dbl;
const dbl eps = 1e-9;
inline bool eq(dbl x, dbl y){
    return fabs(x - y) < eps;
}
inline bool lt(dbl x, dbl y){
    return x < y - eps;
}
inline bool gt(dbl x, dbl y){
    return x > y + eps;
}
inline bool le(dbl x, dbl y){
    return x < y + eps;
}
inline bool ge(dbl x, dbl y){
    return x > y - eps;
}
struct pt{
    dbl x, y;
    inline pt operator - (const pt & p) const{
        return pt{x - p.x, y - p.y};
    }
    inline pt operator + (const pt & p) const{
        return pt{x + p.x, y + p.y};
    }
    inline pt operator * (dbl a) const{
        return pt{x * a, y * a};
    }
    inline dbl cross(const pt & p) const{
        return x * p.y - y * p.x;
    }
}

```

```

}
inline dbl dot(const pt & p) const{
    return x * p.x + y * p.y;
}
inline bool operator == (const pt & p) const{
    return eq(x, p.x) && eq(y, p.y);
}
};
struct Line{
    pt p[2];
    Line(){
        Line(pt a, pt b):p{a, b}{}
    }
    pt vec() const{
        return p[1] - p[0];
    }
    pt& operator [] (size_t i){
        return p[i];
    }
};
inline bool lexComp(const pt & l, const pt & r){
    if (fabs(l.x - r.x) > eps){
        return l.x < r.x;
    }
    else return l.y < r.y;
}
vector<pt> interSegSeg(Line l1, Line l2){
    if (eq(l1.vec().cross(l2.vec()), 0)){
        if (!eq(l1.vec().cross(l2[0] - l1[0]), 0))
            return {};
        if (!lexComp(l1[0], l1[1]))
            swap(l1[0], l1[1]);
        if (!lexComp(l2[0], l2[1]))
            swap(l2[0], l2[1]);
        pt l = lexComp(l1[0], l2[0]) ? l2[0] : l1[0];
        pt r = lexComp(l1[1], l2[1]) ? l1[1] : l2[1];
        if (l == r)
            return {l};
        else return lexComp(l, r) ? vector<pt>{l, r} :
            vector<pt>();
    }
    else{
        dbl s = (l2[0] - l1[0]).cross(l2.vec()) /
            l1.vec().cross(l2.vec());
        pt inter = l1[0] + l1.vec() * s;
        if (ge(s, 0) && le(s, 1) && le((l2[0] - inter).dot(l2[1]
            - inter), 0))
            return {inter};
        else
            return {};
    }
}
inline char get_segtype(Line segment, pt other_point){
    if (eq(segment[0].x, segment[1].x))
        return 0;
    if (!lexComp(segment[0], segment[1]))
        swap(segment[0], segment[1]);
    return (segment[1] - segment[0]).cross(other_point -
        segment[0]) > 0 ? 1 : -1;
}
dbl union_area(vector<tuple<pt, pt, pt>> triangles){
    vector<Line> segments(3 * triangles.size());
    vector<char> segtype(segments.size());
    for (size_t i = 0; i < triangles.size(); i++){
        pt a, b, c;
        tie(a, b, c) = triangles[i];
        segments[3 * i] = lexComp(a, b) ? Line(a, b) : Line(b,
            a);
        segtype[3 * i] = get_segtype(segments[3 * i], c);
        segments[3 * i + 1] = lexComp(b, c) ? Line(b, c) :
            Line(c, b);
        segtype[3 * i + 1] = get_segtype(segments[3 * i + 1],
            a);
        segments[3 * i + 2] = lexComp(c, a) ? Line(c, a) :
            Line(a, c);
        segtype[3 * i + 2] = get_segtype(segments[3 * i + 2],
            b);
    }
    vector<dbl> k(segments.size(), b(segments.size()));
}

```

```

for (size_t i = 0; i < segments.size(); i++){
    if (segtype[i]){
        k[i] = (segments[i][1].y - segments[i][0].y) /
            (segments[i][1].x - segments[i][0].x);
        b[i] = segments[i][0].y - k[i] * segments[i][0].x;
    }
}
dbl ans = 0;
for (size_t i = 0; i < segments.size(); i++){
    if (!segtype[i])
        continue;
    dbl l = segments[i][0].x, r = segments[i][1].x;
    vector<pair<dbl, int>> evts;
    for (size_t j = 0; j < segments.size(); j++){
        if (!segtype[j] || i == j)
            continue;
        dbl l1 = segments[j][0].x, r1 = segments[j][1].x;
        if (ge(l1, r) || ge(l, r1))
            continue;
        dbl common_l = max(l, l1), common_r = min(r, r1);
        auto pts = interSegSeg(segments[i], segments[j]);
        if (pts.empty()){
            dbl y1 = k[j] * common_l + b[j];
            dbl y1 = k[i] * common_l + b[i];
            if (lt(y1, y1) == (segtype[i] == 1)){
                int evt_type = -segtype[i] * segtype[j];
                evts.emplace_back(common_l, evt_type);
                evts.emplace_back(common_r, -evt_type);
            }
        }
        else if (pts.size() == 1u){
            dbl y1 = k[i] * common_l + b[i], y11 = k[j] *
                common_l + b[j];
            int evt_type = -segtype[i] * segtype[j];
            if (lt(y1, y1) == (segtype[i] == 1)){
                evts.emplace_back(common_l, evt_type);
                evts.emplace_back(pts[0].x, -evt_type);
            }
            y1 = k[i] * common_r + b[i], y11 = k[j] *
                common_r + b[j];
            if (lt(y11, y1) == (segtype[i] == 1)){
                evts.emplace_back(pts[0].x, evt_type);
                evts.emplace_back(common_r, -evt_type);
            }
        }
        else{
            if (segtype[j] != segtype[i] || j > i){
                evts.emplace_back(common_l, -2);
                evts.emplace_back(common_r, 2);
            }
        }
    }
    evts.emplace_back(l, 0);
    sort(evts.begin(), evts.end());
    size_t j = 0;
    int balance = 0;
    while (j < evts.size()){
        size_t ptr = j;
        while (ptr < evts.size() && eq(evts[j].first,
            evts[ptr].first)){
            balance += evts[ptr].second;
            ++ptr;
        }
        if (!balance && !eq(evts[j].first, r)){
            dbl next_x = ptr == evts.size() ? r :
                evts[ptr].first;
            ans -= segtype[i] * (k[i] * (next_x +
                evts[j].first) + 2 * b[i]) * (next_x -
                evts[j].first);
        }
        j = ptr;
    }
    return ans/2;
}
pair<dbl, dbl> union_perimeter(vector<tuple<pt, pt, pt>>
    triangles){
    //Same as before
}

```



```

pair<dbl,dbl> ans = make_pair(0,0);
for(size_t i = 0; i < segments.size(); i++){
    //Same as before
    double
    dist=(segments[i][1].x-segments[i][0].x)*(segments[i][1].x-segments[i][0].x)+(segments[i][1].y-segments[i][0].y)*(segments[i][1].y-segments[i][0].y);
    dist=sqrt(dist);
    while(j < evts.size()){
        size_t ptr = j;
        while(ptr < evts.size() && eq(evts[j].first, evts[ptr].first)){
            balance += evts[ptr].second;
            ++ptr;
        }
        if(!balance && !eq(evts[j].first, r)){
            dbl next_x = ptr == evts.size() ? r : evts[ptr].first;
            ans.first += dist * (next_x - evts[j].first) / (r-1);
            if(eq(segments[i][1].y,segments[i][0].y))
                ans.second+=(next_x - evts[j].first);
            j = ptr;
        }
    }
    return ans;
}

```

## 4 Graph

### 4.1 DMST with solution

```

// not tested yet
const int INF = 1029384756;
#define MAXN 1000
#define FOR(i,x) for(auto i :x )
struct edge_t {
    int u,v,w;
    set< pair<int,int> > add, sub;
    edge_t() : u(-1), v(-1), w(0) {}
    edge_t(int _u, int _v, int _w) {
        u = _u;
        v = _v;
        w = _w;
        add.insert({u, v});
    }
    edge_t& operator += (const edge_t& obj) {
        w += obj.w;
        for (auto it : obj.add) {
            if (!sub.count(it)) add.insert(it);
            else sub.erase(it);
        }
        for (auto it : obj.sub) {
            if (!add.count(it)) sub.insert(it);
            else add.erase(it);
        }
        return *this;
    }
    edge_t& operator -= (const edge_t& obj) {
        w -= obj.w;
        for (auto it : obj.sub) {
            if (!sub.count(it)) add.insert(it);
            else sub.erase(it);
        }
        for (auto it : obj.add) {
            if (!add.count(it)) sub.insert(it);
            else add.erase(it);
        }
        return *this;
    }
} eg[MAXN*MAXN],prv[MAXN],EDGE_INF(-1,-1,INF);
int N,M;
int cid,incyc[MAXN],contracted[MAXN];
vector<int> E[MAXN];
edge_t dmst(int rt) {
    edge_t cost;
    for (int i=0; i<N; i++) {
        contracted[i] = incyc[i] = 0;
        prv[i] = EDGE_INF;
    }
    cid = 0;
    int u,v;

```

```

while (true) {
    for (v=0; v<N; v++) {
        if (v != rt && !contracted[v] && prv[v].w == INF)
            break;
        if (v >= N) break; // end
        for (int i=0; i<M; i++) {
            if (eg[i].v == v && eg[i].w < prv[v].w)
                prv[v] = eg[i];
        }
        if (prv[v].w == INF) // not connected
            return EDGE_INF;
        cost += prv[v];
        for (u=prv[v].u; u!=v && u!=-1; u=prv[u].u);
        if (u == -1) continue;
        incyc[v] = ++cid;
        for (u=prv[v].u; u!=v; u=prv[u].u) {
            contracted[u] = 1;
            incyc[u] = cid;
        }
        for (int i=0; i<M; i++) {
            if (incyc[eg[i].u] != cid && incyc[eg[i].v] == cid) {
                eg[i].w -= prv[eg[i].v];
            }
        }
        for (int i=0; i<M; i++) {
            if (incyc[eg[i].u] == cid) eg[i].u = v;
            if (incyc[eg[i].v] == cid) eg[i].v = v;
            if (eg[i].u == eg[i].v) eg[i].w = eg[i].w;
        }
        for (int i=0; i<N; i++) {
            if (contracted[i]) continue;
            if (prv[i].u>=0 && incyc[prv[i].u] == cid)
                prv[i].u = v;
        }
        prv[v] = EDGE_INF;
    }
    return cost;
}
#define F first
#define S second
void solve() {
    edge_t cost = dmst(0);
    for (auto it : cost.add) { // find a solution
        E[it.F].push_back(it.S);
        prv[it.S] = edge_t(it.F,it.S,0);
    }
}

```

### 4.2 DMST

```

// tested on https://lightoj.com/problem/teleport
const int inf = 1e9;
struct edge {
    int u, v, w;
    edge() {}
    edge(int a,int b,int c) : u(a), v(b), w(c) {}
    bool operator < (const edge& o) const {
        if (u == o.u)
            if (v == o.v) return w < o.w;
            else return v < o.v;
        return u < o.u;
    }
};
int dmst(vector<edge> &edges, int root, int n) {
    int ans = 0;
    int cur_nodes = n;
    while (true) {
        vector<int> lo(cur_nodes, inf), pi(cur_nodes, inf);
        for (int i = 0; i < edges.size(); ++i) {
            int u = edges[i].u, v = edges[i].v, w = edges[i].w;
            if (w < lo[v] and u != v) {
                lo[v] = w;
                pi[v] = u;
            }
        }
        lo[root] = 0;
        for (int i = 0; i < lo.size(); ++i) {

```

```

        if (i == root) continue;
        if (lo[i] == inf) return -1;
    }
    int cur_id = 0;
    vector<int> id(cur_nodes, -1), mark(cur_nodes, -1);
    for (int i = 0; i < cur_nodes; ++i) {
        ans += lo[i];
        int u = i;
        while (u != root and id[u] < 0 and mark[u] != i) {
            mark[u] = i;
            u = pi[u];
        }
        if (u != root and id[u] < 0) { // Cycle
            for (int v = pi[u]; v != u; v = pi[v]) id[v] = cur_id;
            cur_id;
            id[u] = cur_id++;
        }
        if (cur_id == 0) break;
        for (int i = 0; i < cur_nodes; ++i)
            if (id[i] < 0) id[i] = cur_id++;
        for (int i = 0; i < edges.size(); ++i) {
            int u = edges[i].u, v = edges[i].v, w = edges[i].w;
            edges[i].u = id[u];
            edges[i].v = id[v];
            if (id[u] != id[v]) edges[i].w -= lo[v];
        }
        cur_nodes = cur_id;
        root = id[root];
    }
    return ans;
}

```

### 4.3 Flow With Demands

**Finding an arbitrary flow** Consider flow networks, where we additionally require the flow of each edge to have a certain amount, i.e. we bound the flow from below by a **demand** function  $d(e)$ :

$$d(e) \leq f(e) \leq c(e)$$

So next each edge has a minimal flow value, that we have to pass along the edge.

We make the following changes in the network. We add a new source  $s'$  and a new sink  $t'$ , a new edge from the source  $s'$  to every other vertex, a new edge for every vertex to the sink  $t'$ , and one edge from  $t$  to  $s$ . Additionally we define the new capacity function  $c'$  as:

- $c'((s',v)) = \sum_{u \in V} d((u,v))$  for each edge  $(s',v)$ .
- $c'((v,t')) = \sum_{w \in V} d((v,w))$  for each edge  $(v,t')$ .
- $c'((u,v)) = c((u,v)) - d((u,v))$  for each edge  $(u,v)$  in the old network.
- $c'((t,s)) = \infty$

If the new network has a saturating flow (a flow where each edge outgoing from  $s'$  is completely filled, which is equivalent to every edge incoming to  $t'$  is completely filled), then the network with demands has a valid flow, and the actual flow can be easily reconstructed from the new network. Otherwise there doesn't exist a flow that satisfies all conditions. Since a saturating flow has to be a maximum flow, it can be found by any maximum flow algorithm.

**Minimal flow** Note that along the edge  $(t, s)$  (from the old sink to the old source) with the capacity  $\infty$  flows the entire flow of the corresponding old network. I.e. the capacity of this edge effects the flow value of the old network. By giving this edge a sufficient large capacity (i.e.  $\infty$ ), the flow of the old network is unlimited. By limiting this edge by smaller capacities, the flow value will decrease. However if we limit this edge by a too small value, than the network will not have a saturated solution, e.g. the corresponding solution for the original network will not satisfy the demand of the edges. Obviously here can use a binary search to find the lowest value with which all constraints are still satisfied. This gives the minimal flow of the original network.

#### 4.4 articulation-vertex

```
int n; // number of nodes
vector<vector<int>>> adj; // adjacency list of graph
vector<bool> visited;
vector<int> tin, low;
int timer;
void dfs(int v, int p = -1) {
    visited[v] = true;
    tin[v] = low[v] = timer++;
    int children=0;
    for (int to : adj[v]) {
        if (to == p) continue;
        if (visited[to]) {
            low[v] = min(low[v], tin[to]);
        } else {
            dfs(to, v);
            low[v] = min(low[v], low[to]);
            if (low[to] >= tin[v] && p != -1)
                IS_CUTPOINT(v);
            ++children;
        }
    }
    if (p == -1 && children > 1)
        IS_CUTPOINT(v);
}
void find_cutpoints() {
    timer = 0;
    visited.assign(n, false);
    tin.assign(n, -1);
    low.assign(n, -1);
    for (int i = 0; i < n; ++i) {
        if (!visited[i])
            dfs(i);
    }
}
```

#### 4.5 bellman-ford

```
struct Edge {
    int a, b, cost;
};
int n, m;
vector<Edge> edges;
const int INF = 1000000000;
void solve() {
    vector<int> d(n);
    vector<int> p(n, -1);
    int x;
    for (int i = 0; i < n; ++i) {
        x = -1;
        for (Edge e : edges) {
            if (d[e.a] + e.cost < d[e.b]) {
                d[e.b] = d[e.a] + e.cost;
                p[e.b] = e.a;
                x = e.b;
            }
        }
    }
    if (x == -1) {
        cout << "No negative cycle found.";
    }
}
```

```
} else {
    for (int i = 0; i < n; ++i)
        x = p[x];
    vector<int> cycle;
    for (int v = x; v = p[v]) {
        cycle.push_back(v);
        if (v == x && cycle.size() > 1)
            break;
    }
    reverse(cycle.begin(), cycle.end());
    cout << "Negative cycle: ";
    for (int v : cycle)
        cout << v << ' ';
    cout << endl;
}
}
```

#### 4.6 bridge

```
const int vmax = 2e5+10, emax = 2e5+10;
namespace Bridge { //edge, nodes, comps 1 indexed
    vector<int> adj[vmax]; // edge-id
    pair<int, int> edges[emax]; // (u, v)
    bool isBridge[emax];
    int visited[vmax]; // 0-unvis, 1-vising, 2-vis
    int st[vmax], low[vmax], clk = 0, edgeId = 0;
    // For bridge tree components
    int who[vmax], compId = 0;
    vector<int> stk;
    // For extra end time calc
    int en[vmax];
    void dfs(int u, int parEdge) {
        visited[u] = 1; low[u] = st[u] = ++clk;
        stk.push_back(u);
        for (auto e : adj[u]) {
            if (e == parEdge) continue;
            int v = edges[e].first, edges[e].second;
            if (visited[v] == 1) {
                low[u] = min(low[u], st[v]);
            } else if (visited[v] == 0) {
                dfs(v, e); low[u] = min(low[u], low[v]);
            }
        }
        visited[u] = 2;
        if (st[u] == low[u]) { // found
            ++compId; int cur;
            do {
                cur = stk.back(); stk.pop_back();
                who[cur] = compId;
            } while (cur != u);
            if (parEdge != -1) { isBridge[parEdge] = true; }
        }
        en[u] = clk;
    }
    void clearAll(int n) {
        for (int i = 0; i < n; ++i) {
            adj[i].clear(); visited[i] = st[i] = 0;
        }
        for (int i = 0; i <= edgeId; ++i) isBridge[i] = 0;
        clk = compId = edgeId = 0;
    }
    void findBridges(int n) {
        for (int i = 1; i <= n; ++i) {
            if (visited[i] == 0) dfs(i, -1);
        }
    }
    bool isReplacable(int eid, int u, int v) {
        if (!isBridge[eid]) return true;
        int a = edges[eid].first, b = edges[eid].second;
        if (st[a] > st[b]) swap(a, b);
        return (st[b] <= st[u] && st[u] <= en[b])
            != (st[b] <= st[v] && st[v] <= en[b]);
    }
    void addEdge(int u, int v) {
        edgeId++; edges[edgeId] = {u, v};
        adj[u].emplace_back(edgeId);
        adj[v].emplace_back(edgeId);
    }
}
```

#### 4.7 edmond-blossom

```
/**Copied from https://codeforces.com/blog/entry/49402**/
/*
GETS:
V->number of vertices
E->number of edges
pair of vertices as edges (vertices are 1..V)
GIVES:
output of edmonds() is the maximum matching
match[i] is matched pair of i (-1 if there isn't a matched pair)
*/
const int M=500;
struct struct_edge
{
    int v;
    struct_edge* n;
};
typedef struct_edge* edge;
struct struct_pool {
    struct_edge pool[M*M*2];
    edge top=pool, adj[M];
    int V,E,match[M],qh,qt,q[M],father[M],base[M];
    bool inq[M],inb[M],ed[M][M];
    void add_edge(int u,int v)
    {
        top->v=v,top->n=adj[u],adj[u]=top++;
        top->v=u,top->n=adj[v],adj[v]=top++;
    }
    int LCA(int root,int u,int v)
    {
        static bool inp[M];
        memset(inp,0,sizeof(inp));
        while(1)
        {
            inp[u=base[u]]=true;
            if (u==root) break;
            u=father[match[u]];
        }
        while(1)
        {
            if (inp[v=base[v]]) return v;
            else v=father[match[v]];
        }
    }
    void mark_blossom(int lca,int u)
    {
        while (base[u]!=lca)
        {
            int v=match[u];
            inb[base[u]]=inb[base[v]]=true;
            u=father[v];
            if (base[u]!=lca) father[u]=v;
        }
    }
    void blossom_contraction(int s,int u,int v)
    {
        int lca=LCA(s,u,v);
        memset(inb,0,sizeof(inb));
        mark_blossom(lca,u);
        mark_blossom(lca,v);
        if (base[u]!=lca)
            father[u]=v;
        if (base[v]!=lca)
            father[v]=u;
        for (int u=0; u<V; u++)
            if (inb[base[u]])
            {
                base[u]=lca;
                if (!inq[u])
                    inq[q[++qt]=u]=true;
            }
    }
    int find_augmenting_path(int s)
    {
        memset(inq,0,sizeof(inq));
        memset(father,-1,sizeof(father));
        for (int i=0; i<V; i++) base[i]=i;
        inq[q[qh=qt=0]=s]=true;
    }
}
```

```

while (qh<=qt)
{
    int u=q[qh++];
    for (edge e=adj[u]; e; e=e->n)
    {
        int v=e->v;
        if (base[u]!=base[v]&&match[u]!=v)
            if ((v==s)|| (match[v]==-1 &&
                father[match[v]]!=-1))
                blossom_contraction(s,u,v);
            else if (father[v]==-1)
            {
                father[v]=u;
                if (match[v]==-1)
                    return v;
                else if (!inq[match[v]])
                    inq[q[++qt]=match[v]]=true;
            }
    }
}
return -1;
}
int augment_path(int s,int t)
{
    int u=t,v,w;
    while (u!=-1)
    {
        v=father[u];
        w=match[v];
        match[v]=u;
        match[u]=v;
        u=w;
    }
    return t!=-1;
}
int edmonds()//Gives number of matchings
{
    int matchc=0;
    memset(match,-1,sizeof(match));
    for (int u=0; u<V; u++)
        if (match[u]==-1)
            matchc+=augment_path(u,find_augmenting_path(u));
    return matchc;
}
//To add edge add_edge(u-1,v-1);
    ed[u-1][v-1]=ed[v-1][u-1]=true;
}

```

#### 4.8 euler-path

```

int main() {
    int n;
    vector<vector<int>> g(n, vector<int>(n));
    // reading the graph in the adjacency matrix
    vector<int> deg(n);
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < n; ++j)
            deg[i] += g[i][j];
    }
    int first = 0;
    while (first < n && !deg[first])
        ++first;
    if (first == n) {
        cout << -1;
        return 0;
    }
    int v1 = -1, v2 = -1;
    bool bad = false;
    for (int i = 0; i < n; ++i) {
        if (deg[i] & 1) {
            if (v1 == -1)
                v1 = i;
            else if (v2 == -1)
                v2 = i;
            else
                bad = true;
        }
    }
    if (v1 != -1)
        ++g[v1][v2], ++g[v2][v1];
    stack<int> st;
    st.push(first);
}

```

```

vector<int> res;
while (!st.empty()) {
    int v = st.top();
    int i;
    for (i = 0; i < n; ++i)
        if (g[v][i])
            break;
    if (i == n) {
        res.push_back(v);
        st.pop();
    } else {
        --g[v][i];
        --g[i][v];
        st.push(i);
    }
}
if (v1 != -1) {
    for (size_t i = 0; i + 1 < res.size(); ++i) {
        if ((res[i] == v1 && res[i + 1] == v2) ||
            (res[i] == v2 && res[i + 1] == v1)) {
            vector<int> res2;
            for (size_t j = i + 1; j < res.size(); ++j)
                res2.push_back(res[j]);
            for (size_t j = 1; j <= i; ++j)
                res2.push_back(res[j]);
            res = res2;
            break;
        }
    }
}
for (int i = 0; i < n; ++i) {
    for (int j = 0; j < n; ++j) {
        if (g[i][j])
            bad = true;
    }
}
if (bad) {
    cout << -1;
} else {
    for (int x : res)
        cout << x << " ";
}
}

```

#### 4.9 hopcraft-karp

```

/** Source: https://iq.opengenus.org/hopcroft-karp-algorithm/
**/
// A class to represent Bipartite graph for
// Hopcroft Karp implementation
class BGraph{
    // m and n are number of vertices on left
    // and right sides of Bipartite Graph
    int m, n;
    // adj[u] stores adjacents of left side
    // vertex 'u'. The value of u ranges from 1 to m.
    // 0 is used for dummy vertex
    std::list<int> *adj;
    // pointers for hopcroftKarp()
    int *pair_u, *pair_v, *dist;
public:
    BGraph(int m, int n); // Constructor
    void addEdge(int u, int v); // To add edge
    // Returns true if there is an augmenting path
    bool bfs();
    // Adds augmenting path if there is one beginning
    // with u
    bool dfs(int u);
    // Returns size of maximum matching
    int hopcroftKarpAlgorithm();
};
// Returns size of maximum matching
int BGraph::hopcroftKarpAlgorithm(){
    // pair_u[u] stores pair of u in matching on left side of
    // Bipartite Graph.
    // If u doesn't have any pair, then pair_u[u] is NIL
    pair_u = new int[m + 1];
    // pair_v[v] stores pair of v in matching on right side of
    // Bipartite Graph.
}

```

```

// If v doesn't have any pair, then pair_u[v] is NIL
pair_v = new int[n + 1];
// dist[u] stores distance of left side vertices
dist = new int[m + 1];
// Initialize NIL as pair of all vertices
for (int u = 0; u <= m; u++)
    pair_u[u] = NIL;
for (int v = 0; v <= n; v++)
    pair_v[v] = NIL;
// Initialize result
int result = 0;
// Keep updating the result while there is an
// augmenting path possible.
while (bfs()){
    // Find a free vertex to check for a matching
    for (int u = 1; u <= m; u++)
        // If current vertex is free and there is
        // an augmenting path from current vertex
        // then increment the result
        if (pair_u[u] == NIL && dfs(u))
            result++;
}
return result;
}
// Returns true if there is an augmenting path available, else
// returns false
bool BGraph::bfs(){
    std::queue<int> q; //an integer queue for bfs
    // First layer of vertices (set distance as 0)
    for (int u = 1; u <= m; u++){
        // If this is a free vertex, add it to queue
        if (pair_u[u] == NIL){
            // u is not matched so distance is 0
            dist[u] = 0;
            q.push(u);
        }
        // Else set distance as infinite so that this vertex is
        // considered next time for availability
        else
            dist[u] = INF;
    }
    // Initialize distance to NIL as infinite
    dist[NIL] = INF;
    // q is going to contain vertices of left side only.
    while (!q.empty()){
        // dequeue a vertex
        int u = q.front();
        q.pop();
        // If this node is not NIL and can provide a shorter
        // path to NIL then
        if (dist[u] < dist[NIL]){
            // Get all the adjacent vertices of the dequeued
            // vertex u
            std::list<int>::iterator it;
            for (it = adj[u].begin(); it != adj[u].end(); ++it){
                int v = *it;
                // If pair of v is not considered so far
                // i.e. (v, pair_v[v]) is not yet explored edge.
                if (dist[pair_v[v]] == INF){
                    // Consider the pair and push it to queue
                    dist[pair_v[v]] = dist[u] + 1;
                    q.push(pair_v[v]);
                }
            }
        }
    }
    // If we could come back to NIL using alternating path of
    // distinct
    // vertices then there is an augmenting path available
    return (dist[NIL] != INF);
}
// Returns true if there is an augmenting path beginning with
// free vertex u
bool BGraph::dfs(int u){
    if (u != NIL){
        std::list<int>::iterator it;
        for (it = adj[u].begin(); it != adj[u].end(); ++it){
            // Adjacent vertex of u
            int v = *it;
        }
    }
}

```

```

// Follow the distances set by BFS search
if (dist[pair_v[v]] == dist[u] + 1){
    // If dfs for pair of v also return true then
    if (dfs(pair_v[v]) == true){ // new matching
        possible, store the matching
        pair_v[v] = u;
        pair_u[u] = v;
        return true;
    }
}
// If there is no augmenting path beginning with u then.
dist[u] = INF;
return false;
}
return true;
}
// Constructor for initialization
BGraph::BGraph(int m, int n){
    this->m = m;
    this->n = n;
    adj = new std::list<int>[m + 1];
}
// function to add edge from u to v
void BGraph::addEdge(int u, int v){
    adj[u].push_back(v); // Add v to us list.
}

```

#### 4.10 hungarian-algorithm

```

class HungarianAlgorithm{
    int N,inf,n,max_match;
    int *lx,*ly,*xy,*yx,*slack,*slackx,*prev;
    int **cost;
    bool *S,*T;
    void init_labels(){
        for(int x=0;x<n;x++) lx[x]=0;
        for(int y=0;y<n;y++) ly[y]=0;
        for (int x = 0; x < n; x++){
            for (int y = 0; y < n; y++){
                lx[x] = max(lx[x], cost[x][y]);
            }
        }
        void update_labels(){
            int x, y, delta = inf; //init delta as infinity
            for (y = 0; y < n; y++) //calculate delta using slack
                if (!T[y])
                    delta = min(delta, slack[y]);
            for (x = 0; x < n; x++) //update X labels
                if (S[x]) lx[x] -= delta;
            for (y = 0; y < n; y++) //update Y labels
                if (T[y]) ly[y] += delta;
            for (y = 0; y < n; y++) //update slack array
                if (!T[y])
                    slack[y] -= delta;
        }
        void add_to_tree(int x, int prevx)
        //x - current vertex,prevx - vertex from X before x in the
        //alternating path,
        //so we add edges (prevx, xy[x]), (xy[x], x){
            S[x] = true; //add x to S
            prev[x] = prevx; //we need this when augmenting
            for (int y = 0; y < n; y++) //update slacks, because we
            //add new vertex to S
                if (lx[x] + ly[y] - cost[x][y] < slack[y]){
                    slack[y] = lx[x] + ly[y] - cost[x][y];
                    slackx[y] = x;
                }
        }
        void augment() //main function of the algorithm{
            if (max_match == n) return; //check wether matching is
            //already perfect
            int x, y, root; //just counters and root vertex
            int q[N], wr = 0, rd = 0; //q - queue for bfs, wr,rd -
            //write and read
            //pos in queue
            //memset(S, false, sizeof(S)); //init set S
            for(int i=0;i<n;i++) S[i]=false;
            //memset(T, false, sizeof(T)); //init set T
            for(int i=0;i<n;i++) T[i]=false;

```

```

//memset(prev, -1, sizeof(prev)); //init set prev - for
//the alternating tree
for(int i=0;i<n;i++) prev[i]=-1;
for (x = 0; x < n; x++) //finding root of the tree{
    if (xy[x] == -1){
        q[wr++] = root = x;
        prev[x] = -2;
        S[x] = true;
        break;
    }
}
for (y = 0; y < n; y++) //initializing slack array{
    slack[y] = lx[root] + ly[y] - cost[root][y];
    slackx[y] = root;
}
while (true) //main cycle{
    while (rd < wr) //building tree with bfs cycle{
        x = q[rd++]; //current vertex from X part
        for (y = 0; y < n; y++) //iterate through all
        //edges in equality graph{
            if (cost[x][y] == lx[x] + ly[y] && !T[y]){
                if (yx[y] == -1) break; //an exposed
                //vertex in Y found, so
                //augmenting path exists!
                T[y] = true; //else just add y to T,
                q[wr++] = yx[y]; //add vertex yx[y],
                //which is matched
                //with y, to the queue
                add_to_tree(yx[y], x); //add edges (x,y)
                //and (y,yx[y]) to the tree
            }
            if (y < n) break; //augmenting path found!
        }
        if (y < n) break; //augmenting path found!
        update_labels(); //augmenting path not found, so
        //improve labeling
        wr = rd = 0;
        for (y = 0; y < n; y++){
            //in this cycle we add edges that were added to
            //the equality graph as a
            //result of improving the labeling, we add edge (slackx[y], y)
            //to the tree if
            //and only if !T[y] && slack[y] == 0, also with this edge we
            //add another one
            //(y, yx[y]) or augment the matching, if y was exposed
            if (!T[y] && slack[y] == 0){
                if (yx[y] == -1) //exposed vertex in Y found
                //augmenting path exists!{
                    x = slackx[y];
                    break;
                }
                else{
                    T[y] = true; //else just add y to T,
                    if (!S[yx[y]]){
                        q[wr++] = yx[y]; //add vertex yx[y],
                        //which is matched with
                        //yx[y], to the queue
                        add_to_tree(yx[y], slackx[y]); //and
                        //add edges (x,y) and (y,
                        //yx[y]) to the tree
                    }
                }
            }
            if (y < n) break; //augmenting path found!
        }
        if (y < n) //we found augmenting path!{
            max_match++; //increment matching
            //in this cycle we inverse edges along augmenting path
            for (int cx = x, cy = y, ty; cx != -2; cx =
                prev[cx], cy = ty){
                ty = xy[cx];
                yx[cy] = cx;
                xy[cx] = cy;
            }
            augment(); //recall function, go to step 1 of the
            //algorithm
        }
    }
}

```

```

}
} //end of augment() function
public:
HungarianAlgorithm(int vv,int inf=1000000000){
    N=vv;
    n=N;
    max_match=0;
    this->inf=inf;
    lx=new int[N];
    ly=new int[N]; //labels of X and Y parts
    xy=new int[N]; //xy[x] - vertex that is matched with x,
    yx=new int[N]; //yx[y] - vertex that is matched with y
    slack=new int[N]; //as in the algorithm description
    slackx=new int[N]; //slackx[y] such a vertex, that
    //l(slackx[y]) + l(y) - w(slackx[y],y) = slack[y]
    prev=new int[N]; //array for memorizing alternating paths
    S=new bool[N];
    T=new bool[N]; //sets S and T in algorithm
    cost=new int*[N]; //cost matrix
    for(int i=0; i<N; i++){
        cost[i]=new int[N];
    }
}
HungarianAlgorithm(){
    delete []lx;
    delete []ly;
    delete []xy;
    delete []yx;
    delete []slack;
    delete []slackx;
    delete []prev;
    delete []S;
    delete []T;
    int i;
    for(i=0; i<N; i++){
        delete []cost[i];
    }
    delete []cost;
}
void setCost(int i,int j,int c){
    cost[i][j]=c;
}
int* matching(bool first=true){
    int *ans;
    ans=new int[N];
    for(int i=0;i<n;i++){
        if(first) ans[i]=xy[i];
        else ans[i]=yx[i];
    }
    return ans;
}
int hungarian(){
    int ret = 0; //weight of the optimal matching
    max_match = 0; //number of vertices in current matching
    for(int x=0;x<n;x++) xy[x]=-1;
    for(int y=0;y<n;y++) yx[y]=-1;
    init_labels(); //step 0
    augment(); //steps 1-3
    for (int x = 0; x < n; x++) //forming answer there
        ret += cost[x][xy[x]];
    return ret;
}
};

```

#### 4.11 max-flow-dinic

```

#include<bits/stdc++.h>
#include<vector>
using namespace std;
#define MAX 100
#define HUGE_FLOW 1000000000
#define BEGIN 1
#define DEFAULT_LEVEL 0
struct FlowEdge {
    int v, u;
    long long cap, flow = 0;
    FlowEdge(int v, int u, long long cap) : v(v), u(u),
        cap(cap) {}
};
struct Dinic {

```



```

const long long flow_inf = 1e18;
vector<FlowEdge> edges;
vector<vector<int>> adj;
int n, m = 0;
int s, t;
vector<int> level, ptr;
queue<int> q;
Dinic(int n, int s, int t) : n(n), s(s), t(t) {
    adj.resize(n);
    level.resize(n);
    ptr.resize(n);
}
void add_edge(int v, int u, long long cap) {
    edges.emplace_back(v, u, cap);
    edges.emplace_back(u, v, 0);
    adj[v].push_back(m);
    adj[u].push_back(m + 1);
    m += 2;
}
bool bfs() {
    while (!q.empty()) {
        int v = q.front();
        q.pop();
        for (int id : adj[v]) {
            if (edges[id].cap - edges[id].flow < 1)
                continue;
            if (level[edges[id].u] != -1)
                continue;
            level[edges[id].u] = level[v] + 1;
            q.push(edges[id].u);
        }
    }
    return level[t] != -1;
}
long long dfs(int v, long long pushed) {
    if (pushed == 0)
        return 0;
    if (v == t)
        return pushed;
    for (int& cid = ptr[v]; cid < (int)adj[v].size(); cid++) {
        int id = adj[v][cid];
        int u = edges[id].u;
        if (level[v] + 1 != level[u] || edges[id].cap - edges[id].flow < 1)
            continue;
        long long tr = dfs(u, min(pushed, edges[id].cap - edges[id].flow));
        if (tr == 0)
            continue;
        edges[id].flow += tr;
        edges[id ^ 1].flow -= tr;
        return tr;
    }
    return 0;
}
long long flow() {
    long long f = 0;
    while (true) {
        fill(level.begin(), level.end(), -1);
        level[s] = 0;
        q.push(s);
        if (!bfs())
            break;
        fill(ptr.begin(), ptr.end(), 0);
        while (long long pushed = dfs(s, flow_inf)) {
            f += pushed;
        }
    }
    return f;
}
};
int main() {
    return 0;
}

```

#### 4.12 min-cost-max-flow

```

struct Edge {
    int from, to, capacity, cost;
};

```

```

};
vector<vector<int>> adj, cost, capacity;
const int INF = 1e9;
void shortest_paths(int n, int v0, vector<int>& d,
    vector<int>& p) {
    d.assign(n, INF);
    d[v0] = 0;
    vector<bool> inq(n, false);
    queue<int> q;
    q.push(v0);
    p.assign(n, -1);
    while (!q.empty()) {
        int u = q.front();
        q.pop();
        inq[u] = false;
        for (int v : adj[u]) {
            if (capacity[u][v] > 0 && d[v] > d[u] + cost[u][v]) {
                d[v] = d[u] + cost[u][v];
                p[v] = u;
                if (!inq[v]) {
                    inq[v] = true;
                    q.push(v);
                }
            }
        }
    }
}
int min_cost_flow(int N, vector<Edge> edges, int K, int s, int t) {
    adj.assign(N, vector<int>());
    cost.assign(N, vector<int>(N, 0));
    capacity.assign(N, vector<int>(N, 0));
    for (Edge e : edges) {
        adj[e.from].push_back(e.to);
        adj[e.to].push_back(e.from);
        cost[e.from][e.to] = e.cost;
        cost[e.to][e.from] = -e.cost;
        capacity[e.from][e.to] = e.capacity;
    }
    int flow = 0;
    int cost = 0;
    vector<int> d, p;
    while (flow < K) {
        shortest_paths(N, s, d, p);
        if (d[t] == INF)
            break;
        // find max flow on that path
        int f = K - flow;
        int cur = t;
        while (cur != s) {
            f = min(f, capacity[p[cur]][cur]);
            cur = p[cur];
        }
        // apply flow
        flow += f;
        cost += f * d[t];
        cur = t;
        while (cur != s) {
            capacity[p[cur]][cur] -= f;
            capacity[cur][p[cur]] += f;
            cur = p[cur];
        }
        if (flow < K)
            return -1;
        else
            return cost;
    }
}

```

#### 4.13 online-bridge

```

vector<int> par, dsu_2ecc, dsu_cc, dsu_cc_size;
int bridges;
int lca_iteration;
vector<int> last_visit;
void init(int n) {
    par.resize(n);
    dsu_2ecc.resize(n);
}

```

```

dsu_cc.resize(n);
dsu_cc_size.resize(n);
lca_iteration = 0;
last_visit.assign(n, 0);
for (int i=0; i<n; ++i) {
    dsu_2ecc[i] = i;
    dsu_cc[i] = i;
    dsu_cc_size[i] = 1;
    par[i] = -1;
}
bridges = 0;
int find_2ecc(int v) {
    if (v == -1)
        return -1;
    return dsu_2ecc[v] == v ? v : dsu_2ecc[v] = find_2ecc(dsu_2ecc[v]);
}
int find_cc(int v) {
    v = find_2ecc(v);
    return dsu_cc[v] == v ? v : dsu_cc[v] = find_cc(dsu_cc[v]);
}
void make_root(int v) {
    v = find_2ecc(v);
    int root = v;
    int child = -1;
    while (v != -1) {
        int p = find_2ecc(par[v]);
        par[v] = child;
        dsu_cc[v] = root;
        child = v;
        v = p;
    }
    dsu_cc_size[root] = dsu_cc_size[child];
}
void merge_path (int a, int b) {
    ++lca_iteration;
    vector<int> path_a, path_b;
    int lca = -1;
    while (lca == -1) {
        if (a != -1) {
            a = find_2ecc(a);
            path_a.push_back(a);
            if (last_visit[a] == lca_iteration) {
                lca = a;
                break;
            }
            last_visit[a] = lca_iteration;
            a = par[a];
        }
        if (b != -1) {
            b = find_2ecc(b);
            path_b.push_back(b);
            if (last_visit[b] == lca_iteration) {
                lca = b;
                break;
            }
            last_visit[b] = lca_iteration;
            b = par[b];
        }
    }
    for (int v : path_a) {
        dsu_2ecc[v] = lca;
        if (v == lca)
            break;
        --bridges;
    }
    for (int v : path_b) {
        dsu_2ecc[v] = lca;
        if (v == lca)
            break;
        --bridges;
    }
}
void add_edge(int a, int b) {
    a = find_2ecc(a);
    b = find_2ecc(b);
    if (a == b)
        return;
    int ca = find_cc(a);
    int cb = find_cc(b);
}

```

```

if (ca != cb) {
    ++bridges;
    if (dsu_cc_size[ca] > dsu_cc_size[cb]) {
        swap(a, b);
        swap(ca, cb);
    }
    make_root(a);
    par[a] = dsu_cc[a] = b;
    dsu_cc_size[cb] += dsu_cc_size[a];
} else {
    merge_path(a, b);
}
}
}

```

#### 4.14 scc + 2 Sat

```

namespace SCC { //Everything 0-indexed.
const int N = 2e6+7; int which[N], vis[N], cc;
vector<int> adj[N], adjr[N]; vector<int> order;
void addEdge(int u, int v) {
    adj[u].push_back(v); adjr[v].push_back(u);
}
void dfs1(int u) {
    if (vis[u]) return; vis[u] = true;
    for(int v: adj[u]) dfs1(v); order.push_back(u);
}
void dfs2(int u, int id) {
    if(vis[u]) return; vis[u] = true;
    for(int v: adjr[u]) dfs2(v, id); which[u] = id;
}
int last = 0;
void findSCC(int n) {
    cc=0, last=n; order.clear(); fill(vis, vis+n, 0);
    for(int i=0; i<n; i++) if(!vis[i]) dfs1(i);
    reverse(order.begin(), order.end());
    fill(vis, vis+n, 0);
    for (int u: order) {
        if (vis[u]) continue; dfs2(u, cc); ++cc;
    }
}
void clear() {
    for (int i=0; i<last; i++)
        adj[i].clear(), adjr[i].clear();
}
}
struct TwoSat {
    int n; int vars = 0; vector<bool> ans;
    TwoSat(int n) : n(n), ans(n) {
        SCC::clear(); vars = 2*n;
    }
    void implies(int x, int y) {
        SCC::addEdge(x, y); SCC::addEdge(y^1, x^1);
    }
    void OR(int x, int y) {
        SCC::addEdge(x^1, y); SCC::addEdge(y^1, x);
    }
    void XOR(int x, int y) {
        implies(x, y^1); implies(x^1, y);
    }
    void atmostOne(vector<int> v) {
        int k = v.size();
        for (int i=0; i<k; i++) {
            if (i+1<k) implies(vars+2*i, vars+2*i+2);
            implies(v[i], vars+2*i);
            if (i>0) implies(v[i], vars+2*i-1);
        }
        vars += 2*k;
    }
    bool solve() {
        SCC::findSCC(vars); ans.resize(vars/2);
        for (int i=0; i<vars; i+=2) {
            if (SCC::which[i]==SCC::which[i+1]) return 0;
            if (i<2*n)
                ans[i/2] = SCC::which[i]>SCC::which[i+1];
        }
        return true;
    }
};

```

## 5 Math

### 5.1 BerleKampMassey

```

const int MOD = 998244353;
vector<LL> berlekampMassey(vector<LL> s) {
    if (s.empty()) return {};
    int n = s.size(), L = 0, m = 0;
    vector<LL> C(n), B(n), T;
    C[0] = B[0] = 1; LL b = 1;
    for (int i = 0; i < n; ++i) {
        ++m; LL d = s[i] % MOD;
        for (int j = 1; j <= L; ++j) d = (d + C[j] * s[i - j]) % MOD;
        if (!d) continue;
        T = C; LL coeff = d * bigMod(b, -1) % MOD;
        for (int j = m; j < n; ++j) C[j] = (C[j] - coeff * B[j - m]) % MOD;
        if (2*L > i) continue;
        L = i+1-L, B = T, b = d, m = 0;
    }
    C.resize(L + 1), C.erase(C.begin());
    for (LL &x : C) x = (MOD - x) % MOD;
    return C;
}

```

### 5.2 FloorSum

```

LL mod(LL a, LL m) {
    LL ans = a%m;
    return ans < 0 ? ans+m : ans;
}
//Sum(floor((ax+b)/m)) for i=0 to n-1, (n,m >= 0)
LL floorSum(LL n, LL m, LL a, LL b) {
    LL ra = mod(a, m), rb = mod(b, m), k = (ra*n+rb);
    LL ans = ((a-ra)/m) * n*(n-1)/2 + ((b-rb)/m) * n;
    if (k < m) return ans;
    return ans + floorSum(k/m, ra, m, k%m);
}

```

### 5.3 Stern Brocot Tree

```

//finds x/y with min y st: L <= (x/y) < R
pair<LL,LL> solve(LL L, LL R) {
    pair<LL,LL> l(0, 1), r(1, 1);
    if(L==0.0) return l; // corner case
    while(true) {
        pair<int, int> m(l.x+r.x, l.y+r.y);
        if(m.x<L*m.y){ // move to the right
            LL kl=1, kr=1;
            while(l.x+kr*r.x <= L*(l.y+kr*r.y)) kr*=2;
            while(kl!=kr){
                LL km = (kl+kr)/2;
                if(l.x+km*r.x < L*(l.y+km*r.y)) kl=km+1;
                else kr=km;
            }
            l={l.x+(kl-1)*r.x, l.y+(kl-1)*r.y};
        }
        else if(m.x>=R*m.y){ //move to left
            LL kl=1, kr=1;
            while(r.x+kr*1.x>=R*(r.y+kr*1.y)) kr*=2;
            while(kl!=kr){
                LL km = (kl+kr)/2;
                if(r.x+km*1.x>=R*(r.y+km*1.y)) kl = km+1;
                else kr = km;
            }
            r={r.x+(kl-1)*1.x, r.y+(kl-1)*1.y};
        }
        else return m;
    }
}

```

### 5.4 Sum Of Kth Power

```

LL mod;
LL S[105][105];
// Find 1^k+2^k+...+n^k % mod
void solve() {
    LL n, k;
    /*
    x^k = sum (i=1 to k) Stirling2(k, i) * i! * ncr(x, i)
    */
}

```

```

sum (x = 0 to n) x^k
= sum (i = 0 to k) Stirling2(k, i) * i! * sum (x = 0 to n) ncr(x, i)
= sum (i = 0 to k) Stirling2(k, i) * i! * ncr(n + 1, i + 1)
= sum (i = 0 to k) Stirling2(k, i) * i! * (n + 1)! / (i + 1)! / (n - i)!
= sum (i = 0 to k) Stirling2(k, i) * (n - i + 1) * (n - i + 2) * ... (n + 1) / (i + 1)

*/
S[0][0] = 1 % mod;
for (int i = 1; i <= k; i++) {
    for (int j = 1; j <= i; j++) {
        if (i == j) S[i][j] = 1 % mod;
        else S[i][j] = (j * S[i - 1][j] + S[i - 1][j - 1]) % mod;
    }
    LL ans = 0;
    for (int i = 0; i <= k; i++) {
        LL fact = 1, z = i + 1;
        for (LL j = n - i + 1; j <= n + 1; j++) {
            LL mul = j;
            if (mul % z == 0) {
                mul /= z;
                z /= z;
            }
            fact = (fact * mul) % mod;
        }
        ans = (ans + S[k][i] * fact) % mod;
    }
}

```

### 5.5 combination-generator

```

bool next_combination(vector<int>& a, int n) {
    int k = (int)a.size();
    for (int i = k - 1; i >= 0; i--) {
        if (a[i] < n - k + i + 1) {
            a[i]++;
            for (int j = i + 1; j < k; j++)
                a[j] = a[j - 1] + 1;
            return true;
        }
    }
    return false;
}
vector<int> ans;
void gen(int n, int k, int idx, bool rev) {
    if (k > n || k < 0)
        return;
    if (!n) {
        for (int i = 0; i < idx; ++i) {
            if (ans[i])
                cout << i + 1;
        }
        cout << "\n";
        return;
    }
    ans[idx] = rev;
    gen(n - 1, k - rev, idx + 1, false);
    ans[idx] = !rev;
    gen(n - 1, k - !rev, idx + 1, true);
}
void all_combinations(int n, int k) {
    ans.resize(n);
    gen(n, k, 0, false);
}

```

### 5.6 continued-fractions

```

auto fraction(int p, int q) {
    vector<int> a;
    while(q) {
        a.push_back(p / q);
        tie(p, q) = make_pair(q, p % q);
    }
    return a;
}
auto convergents(vector<int> a) {
    vector<int> p = {0, 1};
}

```

```
vector<int> q = {1, 0};
for(auto it: a) {
    p.push_back(p[p.size() - 1] * it + p[p.size() - 2]);
    q.push_back(q[q.size() - 1] * it + q[q.size() - 2]);
}
return make_pair(p, q);
}
```

### 5.7 crt anachor

```
/// Chinese remainder theorem (special case): find z st z%m1 =
r1, z%m2 = r2.
/// z is unique modulo M = lcm(m1, m2). Returns (z, M). On
failure, M = -1.
PLL CRT(LL m1, LL r1, LL m2, LL r2) {
    LL s, t;
    LL g = egcd(m1, m2, s, t);
    if (r1%g != r2%g) return PLL(0, -1);
    LL M = m1*m2;
    LL ss = ((s*r2)%m2)*m1;
    LL tt = ((t*r1)%m1)*m2;
    LL ans = ((ss+tt)%M+M)%M;
    return PLL(ans/g, M/g);
}
// expected: 23 105
// 11 12
PLL ans = CRT({3,5,7}, {2,3,2});
cout << ans.first << " " << ans.second << endl;
ans = CRT({4,6}, {3,5});
cout << ans.first << " " << ans.second << endl;
```

### 5.8 discrete-root

```
#define MAX 100000
int prime[MAX+1], Phi[MAX+1];
vector<int> pr;
void sieve(){
    for (int i=2; i <= N; ++i) {
        if (prime[i] == 0) {
            prime[i] = i;
            pr.push_back(i);
        }
        for (int j=0; j < (int)pr.size() && pr[j] <= prime[i]
            && i*pr[j] <= N; ++j) {
            prime[i * pr[j]] = pr[j];
        }
    }
}
void PhiWithSieve(){
    int i;
    for(i=2; i<=MAX; i++){
        if(prime[i]==i){
            Phi[i]=i-1;
        }
        else if((i/prime[i])%prime[i]==0){
            Phi[i]=Phi[i/prime[i]]*prime[i];
        }
        else{
            Phi[i]=Phi[i/prime[i]]*(prime[i]-1);
        }
    }
}
int powmod (int a, int b, int p) {
    int res = 1;
    while (b)
        if (b & 1)
            res = int (res * 1ll * a % p), --b;
        else
            a = int (a * 1ll * a % p), b >>= 1;
    return res;
}
int PrimitiveRoot(int p){
    vector<int> fact;
    int phi=Phi[p];
    int n=phi;
    while(n>1){
        if(prime[n]==n){
            fact.push_back(n);
            n=1;
        }
    }
}
```

```
else{
    int f=prime[n];
    while(n%f==0){
        n=n/f;
    }
    fact.push_back(f);
}
}
int res;
for(res=p-1; res>1; --res){
    for(n=0; n<fact.size(); n++){
        if(powmod(res,phi/fact[n],p)==1){
            break;
        }
    }
    if(n>=fact.size()) return res;
}
return -1;
}
int DiscreteLog(int a, int b, int m) {
    a %= m, b %= m;
    int n = sqrt(m) + 1;
    map<int, int> vals;
    for (int p = 1; p <= n; ++p)
        vals[powmod(a, (int) (1ll * p * n) % m, m)] = p;
    for (int q = 0; q <= n; ++q) {
        int cur = powmod(a, q, m) * 1ll * b % m;
        if (vals.count(cur)) {
            int ans = vals[cur] * n - q;
            return ans;
        }
    }
    return -1;
}
vector<int> DiscreteRoot(int n, int a, int k){
    int g = PrimitiveRoot(n);
    vector<int> ans;
    int any_ans = DiscreteLog(powmod(g, k, n), a, n);
    if (any_ans == -1){
        return ans;
    }
    int delta = (n-1) / gcd(k, n-1);
    for (int cur = any_ans % delta; cur < n-1; cur += delta)
        ans.push_back(powmod(g, cur, n));
    sort(ans.begin(), ans.end());
    return ans;
}
```

### 5.9 fast-fourier-transform

```
using cd = complex<double>;
const double PI = acos(-1);
typedef long long ll;
void fft(vector<cd>& a, bool invert){
    int n = a.size();
    for(int i = 1, j = 0; i < n; i++){
        int bit = n>>1;
        for(; j&bit; bit>>=1){
            j^=bit;
        }
        j ^= bit;
        if(i < j)
            swap(a[i], a[j]);
    }
    for(int len = 2; len <= n; len <= 1){
        double ang = 2*PI/len*(invert ? -1 : 1);
        cd wlen(cos(ang), sin(ang));
        for(int i = 0; i < n; i += len){
            cd w(1);
            for(int j = 0; j < len/2; j++){
                cd u = a[i+j], v = a[i+j+len/2]*w;
                a[i+j] = u+v;
                a[i+j+len/2] = u-v;
                w *= wlen;
            }
        }
    }
    if(invert){
        for(cd &x: a)
            x /= n;
    }
}
```

```
}
vector<int> multiply(vector<int> const& a, vector<int>
const& b){
    vector<cd> fa(a.begin(), a.end());
    vector<cd> fb(b.begin(), b.end());
    int n = 1;
    while(n < a.size()+b.size())
        n <= 1;
    fa.resize(n);
    fb.resize(n);
    fft(fa, false);
    fft(fb, false);
    for(int i = 0; i < n; i++)
        fa[i] *= fb[i];
    fft(fa, true);
    vector<int> result(n);
    for(int i = 0; i < n; i++)
        result[i] = round(fa[i].real());
    return result;
}
/*Number Theoretic Transformation
11 int gcd(11 int a, 11 int b){
    if(b==0) return a;
    else return gcd(b,a%b);
}
11 int egcd(11 int a, 11 int b, 11 int & x, 11 int & y) {
    if (a == 0) {
        x = 0;
        y = 1;
        return b;
    }
    11 int x1, y1;
    11 int d = egcd(b % a, a, x1, y1);
    x = y1 - (b / a) * x1;
    y = x1;
    return d;
}
11 int ModuloInverse(11 int a, 11 int n){
    11 int x, y;
    x=gcd(a,n);
    a=a/x;
    n=n/x;
    11 int res = egcd(a,n,x,y);
    x=(x*n+n)%n;
    return x;
}
const int mod = 998244353;
const int root = 15311432;
const int root_1 = 469870224;
const int root_pw = 1 << 23;
998244353 = 119 * 2^23 + 1, primitive root = 3
985661441 = 235 * 2^22 + 1, primitive root = 3
1012924417 = 483 * 2^21 + 1, primitive root = 5
void fft(vector<int> &a, bool invert) {
    int n = a.size();
    for (int i = 1, j = 0; i < n; i++) {
        int bit = n >> 1;
        for (; j & bit; bit >>= 1)
            j ^= bit;
        j ^= bit;
        if (i < j)
            swap(a[i], a[j]);
    }
    for (int len = 2; len <= n; len <= 1) {
        int wlen = invert ? root_1 : root;
        for (int i = len; i < root_pw; i <= 1)
            wlen = (int)(1LL * wlen * wlen % mod);
        for (int i = 0; i < n; i += len) {
            int w = 1;
            for (int j = 0; j < len / 2; j++) {
                int u = a[i+j], v = (int)(1LL * a[i+j+len/2] * w
                    % mod);
                a[i+j] = u + v < mod ? u + v : u + v - mod;
                a[i+j+len/2] = u - v >= 0 ? u - v : u - v + mod;
                w = (int)(1LL * w * wlen % mod);
            }
        }
    }
}
if (invert) {
```

```

    int n_1 = (int) ModuloInverse(n, mod);
    for (int & x : a)
        x = (int)(1LL * x * n_1 % mod);
}
}
vector<int> multiply(vector<int> const& a, vector<int>
    const&b){
    vector<int> fa(a.begin(), a.end());
    vector<int> fb(b.begin(), b.end());
    int n = 1;
    while(n < a.size()+b.size())
        n <= 1;
    fa.resize(n);
    fb.resize(n);
    fft(fa, false);
    fft(fb, false);
    for(int i = 0; i < n; i++)
        fa[i] = (int) (1LL*fa[i]*fb[i]%mod);
    fft(fa, true);
    vector<int> result(n);
    for(int i = 0; i < n; i++)
        result[i] = fa[i];
    return result;
}
*/

```

### 5.10 fast-walsh-hadamard

```

void FWHT(vector<LL> &p, bool inv) {
    int n = p.size(); assert((n&(n-1))==0);
    for (int len=1; 2*len<=n; len <= 1) {
        for (int i = 0; i < n; i += len+len){
            for (int j = 0; j < len; j++) {
                LL u = p[i+j], v = p[i+len+j];
                ///XOR p[i+j]=u+v; p[i+len+j]=u-v;
                ///OR if(!inv) p[i+j]=v, p[i+len+j]=u+v;
                ///OR else p[i+j]=-u+v, p[i+len+j]=u;
                ///AND if(!inv) p[i+j]=u+v, p[i+len+j]=u;
                ///AND else p[i+j]=v, p[i+len+j]=u-v;
            }
        }
    }
    ///XOR if(inv) for(int i=0;i<n;i++) p[i]/=n;
    vector<LL> convo(vector<LL> a, vector<LL> b) {
        int n = 1, sz = max(a.size(), b.size());
        while(n<sz) n*=2;
        a.resize(n); b.resize(n); vector<LL> res(n, 0);
        FWHT(a, 0); FWHT(b, 0);
        for(int i=0;i<n;i++) res[i] = a[i] * b[i];
        FWHT(res, 1);
        return res;
    }
}

```

### 5.11 find-root

```

double sqrt_newton(double n) {
    const double eps = 1E-15;
    double x = 1;
    for (;;) {
        double nx = (x + n / x) / 2;
        if (abs(x - nx) < eps)
            break;
        x = nx;
    }
    return x;
}
int isqrt_newton(int n) {
    int x = 1;
    bool decreased = false;
    for (;;) {
        int nx = (x + n / x) >> 1;
        if (x == nx || nx > x && decreased)
            break;
        decreased = nx < x;
        x = nx;
    }
    return x;
}

```

### 5.12 integer-factorization

```

long long pollards_p_minus_1(long long n) {
    int B = 10;
    long long g = 1;
    while (B <= 1000000 && g < n) {
        long long a = 2 + rand() % (n - 3);
        g = gcd(a, n);
        if (g > 1)
            return g;
        // compute a^M
        for (int p : primes) {
            if (p >= B)
                continue;
            long long p_power = 1;
            while (p_power * p <= B)
                p_power *= p;
            a = power(a, p_power, n);
            g = gcd(a - 1, n);
            if (g > 1 && g < n)
                return g;
        }
        B *= 2;
    }
    return 1;
}
long long mult(long long a, long long b, long long mod) {
    long long result = 0;
    while (b) {
        if (b & 1)
            result = (result + a) % mod;
        a = (a + a) % mod;
        b >>= 1;
    }
    return result;
}
long long f(long long x, long long c, long long mod) {
    return (mult(x, x, mod) + c) % mod;
}
long long rho(long long n, long long x0=2, long long c=1) {
    long long x = x0;
    long long y = x0;
    long long g = 1;
    while (g == 1) {
        x = f(x, c, n);
        y = f(y, c, n);
        y = f(y, c, n);
        g = gcd(abs(x - y), n);
    }
    return g;
}
long long brent(long long n, long long x0=2, long long c=1) {
    long long x = x0;
    long long g = 1;
    long long q = 1;
    long long xs, y;
    int m = 128;
    int l = 1;
    while (g == 1) {
        y = x;
        for (int i = 1; i < l; i++)
            x = f(x, c, n);
        int k = 0;
        while (k < l && g == 1) {
            xs = x;
            for (int i = 0; i < m && i < l - k; i++) {
                x = f(x, c, n);
                q = mult(q, abs(y - x), n);
            }
            g = gcd(q, n);
            k += m;
        }
        l *= 2;
    }
    if (g == n) {
        do {
            xs = f(xs, c, n);
            g = gcd(abs(xs - y), n);
        } while (g == 1);
    }
    return g;
}

```

### 5.13 integration-simpson

```

const int N = 1000 * 1000; // number of steps (already
    multiplied by 2)
double simpson_integration(double a, double b){
    double h = (b - a) / N;
    double s = f(a) + f(b); // a = x_0 and b = x_2n
    for (int i = 1; i <= N - 1; ++i) { // Refer to final
        // Simpson's formula
        double x = a + h * i;
        s += f(x) * ((i & 1) ? 4 : 2);
    }
    s *= h / 3;
    return s;
}

```

### 5.14 linear-diophantine-equation-gray-code

```

int gcd(int a, int b, int& x, int& y) {
    if (b == 0) {
        x = 1;
        y = 0;
        return a;
    }
    int x1, y1;
    int d = gcd(b, a % b, x1, y1);
    x = y1;
    y = x1 - y1 * (a / b);
    return d;
}
bool find_any_solution(int a, int b, int c, int &x0, int &y0,
    int &g) {
    g = gcd(abs(a), abs(b), x0, y0);
    if (c % g) {
        return false;
    }
    x0 *= c / g;
    y0 *= c / g;
    if (a < 0) x0 = -x0;
    if (b < 0) y0 = -y0;
    return true;
}
void shift_solution(int &x, int &y, int a, int b, int cnt) {
    x += cnt * b;
    y -= cnt * a;
}
int find_all_solutions(int a, int b, int c, int minx, int
    maxx, int miny, int maxy) {
    int x, y, g;
    if (!find_any_solution(a, b, c, x, y, g))
        return 0;
    a /= g;
    b /= g;
    int sign_a = a > 0 ? +1 : -1;
    int sign_b = b > 0 ? +1 : -1;
    shift_solution(x, y, a, b, (minx - x) / b);
    if (x < minx)
        shift_solution(x, y, a, b, sign_b);
    if (x > maxx)
        return 0;
    int lx1 = x;
    shift_solution(x, y, a, b, (maxx - x) / b);
    if (x > maxx)
        shift_solution(x, y, a, b, -sign_b);
    int rx1 = x;
    shift_solution(x, y, a, b, -(miny - y) / a);
    if (y < miny)
        shift_solution(x, y, a, b, -sign_a);
    if (y > maxy)
        return 0;
    int lx2 = x;
    shift_solution(x, y, a, b, -(maxy - y) / a);
    if (y > maxy)
        shift_solution(x, y, a, b, sign_a);
    int rx2 = x;
    if (lx2 > rx2)
        swap(lx2, rx2);
    int lx = max(lx1, lx2);
    int rx = min(rx1, rx2);
    if (lx > rx)

```



```

    return 0;
    return (rx - lx) / abs(b) + 1;
}
int g (int n) {
    return n ^ (n >> 1);
}
int rev_g (int g) {
    int n = 0;
    for (; g; g >= 1)
        n ^= g;
    return n;
}

```

### 5.15 linear-equation-system

```

const double EPS = 1e-9;
const int INF = 2; // it doesn't actually have to be infinity
// or a big number
int gauss (vector < vector<double> > a, vector<double> & ans) {
    int n = (int) a.size();
    int m = (int) a[0].size() - 1;
    vector<int> where (m, -1);
    for (int col=0, row=0; col<m && row<n; ++col) {
        int sel = row;
        for (int i=row; i<n; ++i)
            if (abs (a[i][col]) > abs (a[sel][col]))
                sel = i;
        if (abs (a[sel][col]) < EPS)
            continue;
        for (int i=col; i<=m; ++i)
            swap (a[sel][i], a[row][i]);
        where[col] = row;
        for (int i=0; i<n; ++i)
            if (i != row) {
                double c = a[i][col] / a[row][col];
                for (int j=col; j<=m; ++j)
                    a[i][j] -= a[row][j] * c;
            }
        ++row;
    }
    ans.assign (m, 0);
    for (int i=0; i<m; ++i)
        if (where[i] != -1)
            ans[i] = a[where[i]][m] / a[where[i]][i];
    for (int i=0; i<n; ++i) {
        double sum = 0;
        for (int j=0; j<m; ++j)
            sum += ans[j] * a[i][j];
        if (abs (sum - a[i][m]) > EPS)
            return 0;
    }
    for (int i=0; i<m; ++i)
        if (where[i] == -1)
            return INF;
    return 1;
}

```

### 5.16 matrix-determinant

```

const double EPS = 1E-9;
int n;
vector < vector<double> > a (n, vector<double> (n));
double det = 1;
for (int i=0; i<n; ++i) {
    int k = i;
    for (int j=i+1; j<n; ++j)
        if (abs (a[j][i]) > abs (a[k][i]))
            k = j;
    if (abs (a[k][i]) < EPS) {
        det = 0;
        break;
    }
    swap (a[i], a[k]);
    if (i != k)
        det = -det;
    det *= a[i][i];
    for (int j=i+1; j<n; ++j)
        a[i][j] /= a[i][i];
    for (int j=0; j<n; ++j)
        if (j != i && abs (a[j][i]) > EPS)
            for (int k=i+1; k<n; ++k)

```

```

        a[j][k] -= a[i][k] * a[j][i];
}
cout << det;

```

### 5.17 matrix-rank

```

const double EPS = 1E-9;
int compute_rank(vector<vector<double>> A) {
    int n = A.size();
    int m = A[0].size();
    int rank = 0;
    vector<bool> row_selected(n, false);
    for (int i = 0; i < m; ++i) {
        int j;
        for (j = 0; j < n; ++j) {
            if (!row_selected[j] && abs(A[j][i]) > EPS)
                break;
        }
        if (j != n) {
            ++rank;
            row_selected[j] = true;
            for (int p = i + 1; p < m; ++p)
                A[j][p] /= A[j][i];
            for (int k = 0; k < n; ++k) {
                if (k != j && abs(A[k][i]) > EPS) {
                    for (int p = i + 1; p < m; ++p)
                        A[k][p] -= A[j][p] * A[k][i];
                }
            }
        }
    }
    return rank;
}

```

### 5.18 nCr mod p<sup>a</sup>

```

LL F[1000009];
void pre(LL mod, LL pp) { // mod is pp^a, pp is prime
    F[0] = 1;
    REPL(1, 1, mod) {
        // we keep in F factorial with the terms which are coprime
        // with pp
        if (i % pp != 0) F[i] = (F[i-1] * i) % mod;
        else F[i] = F[i-1];
    }
}
LL fact2(LL nn, LL mod) {
    LL cycle = nn / mod;
    LL n2 = nn % mod;
    return (bigmod(F[mod], cycle, mod) * F[n2]) % mod;
}
// returns highest power of pp that divides N and the coprime
// with pp part of N! % mod
PLL fact(LL N, LL pp, LL mod) {
    LL nn = N; LL ord = 0;
    while (nn > 0) { nn /= pp; ord += nn; }
    LL ans = 1; nn = N;
    while (nn > 0) {
        ans = (ans * fact2(nn, mod)) % mod;
        nn /= pp;
    }
    return MP(ord, ans);
}
LL ncrp(ULL n, ULL r, LL prm, LL pr) { // ncr mod prm^pr
    LL mod = bigmod(prm, pr, INF), temp;
    pre(mod, prm);
    PLL
        x = fact(n, prm, mod), y = fact(r, prm, mod), z = fact(n-r, prm, mod);
    LL guun = x.second * modInverse(y.second, mod, prm);
    guun %= mod; guun *= modInverse(z.second, mod, prm);
    guun %= mod;
    LL guun2 = x.first - y.first - z.first;
    guun *= bigmod(prm, guun2, mod);
    guun %= mod;
    return guun;
}

```

### 5.19 primality-test

```

using u64 = uint64_t;
using u128 = __uint128_t;

```

```

u64 binpower(u64 base, u64 e, u64 mod) {
    u64 result = 1;
    base %= mod;
    while (e) {
        if (e & 1)
            result = (u128)result * base % mod;
        base = (u128)base * base % mod;
        e >>= 1;
    }
    return result;
}
bool check_composite(u64 n, u64 a, u64 d, int s) {
    u64 x = binpower(a, d, n);
    if (x == 1 || x == n - 1)
        return false;
    for (int r = 1; r < s; r++) {
        x = (u128)x * x % n;
        if (x == n - 1)
            return false;
    }
    return true;
}
bool MillerRabin(u64 n) { // returns true if n is prime, else
    returns false.
    if (n < 2)
        return false;
    int r = 0;
    u64 d = n - 1;
    while ((d & 1) == 0) {
        d >>= 1;
        r++;
    }
    for (int a : {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37}) {
        if (n == a)
            return true;
        if (check_composite(n, a, d, r))
            return false;
    }
    return true;
}
bool probablyPrimeFermat(int n, int iter=5) {
    if (n < 4)
        return n == 2 || n == 3;
    for (int i = 0; i < iter; i++) {
        int a = 2 + rand() % (n - 3);
        if (binpower(a, n - 1, n) != 1)
            return false;
    }
    return true;
}

```

### 5.20 prime counting function

```

#define MAXN 500
#define MAXM 100010
#define MAXP 666666
#define MAX 10000010
#define ll long long int
#define chkbit(ar, i) (((ar[(i) >> 6]) & (1 << (((i) >> 1) & 31)))
#define setbit(ar, i) (((ar[(i) >> 6]) |= (1 << (((i) >> 1) & 31)))
#define isprime(x) (( (x) && ((x)&1) && (!chkbit(ar, (x)))) || ((x) == 2))
namespace pc {
    long long dp[MAXN][MAXM];
    unsigned int ar[(MAX>>6)+5] = {0};
    int len=0, primes[MAXP], counter[MAX];
    void Sieve() {
        setbit(ar, 0), setbit(ar, 1);
        for (int i=3; (i*i)<MAX; i++) {
            if (!chkbit(ar, i)) {
                int k=i<<1;
                for (int j=(i*i); j<MAX; j+=k) setbit(ar, j);
            }
        }
        for (int i=1; i<MAX; i++) {
            counter[i] = counter[i-1];
            if (isprime(i)) primes[len++] = i, counter[i]++;
        }
    }
}

```

```

void init(){
    Sieve();
    for(int n=0;n<MAXN;n++){
        for(int m=0;m<MAXM;m++){
            if(!n) dp[n][m]=m;
            else dp[n][m]=dp[n-1][m]-dp[n-1][m/primes[n-1]];
        }
    }
}

ll phi(ll m,int n){
    if(n==0) return m;
    if(primes[n-1]>=m) return 1;
    if(m<MAXM && n<MAXN) return dp[n][m];
    return phi(m,n-1) - phi(m/primes[n-1],n-1);
}

ll Lehmer(long long m){
    if(m<MAX) return counter[m];
    ll w,res=0;
    int i,a,s,c,x,y;
    s=sqrt(0.9+m), y=cbrt(0.9+m);
    a=counter[y], res=phi(m,a)+a-1;
    for(i=a;primes[i]<=s;i++){
        res=res-Lehmer(m/primes[i])+Lehmer(primes[i])-1;
    }
    return res;
}

```

## 6 String

### 6.1 aho-corasick

```

const int K = 26;
struct Vertex {
    int next[K];
    bool leaf = false;
    int p = -1;
    char pch;
    int link = -1;
    int go[K];
    Vertex(int p=-1, char ch='$') : p(p), pch(ch) {
        fill(begin(next), end(next), -1);
        fill(begin(go), end(go), -1);
    }
};
vector<Vertex> t(1);
void add_string(string const& s) {
    int v = 0;
    for (char ch : s) {
        int c = ch - 'a';
        if (t[v].next[c] == -1) {
            t[v].next[c] = t.size();
            t.emplace_back(v, ch);
        }
        v = t[v].next[c];
    }
    t[v].leaf = true;
}

int go(int v, char ch);
int get_link(int v) {
    if (t[v].link == -1) {
        if (v == 0 || t[v].p == 0)
            t[v].link = 0;
        else
            t[v].link = go(get_link(t[v].p), t[v].pch);
    }
    return t[v].link;
}

int go(int v, char ch) {
    int c = ch - 'a';
    if (t[v].go[c] == -1) {
        if (t[v].next[c] != -1)
            t[v].go[c] = t[v].next[c];
        else
            t[v].go[c] = v == 0 ? 0 : go(get_link(v), ch);
    }
    return t[v].go[c];
}

```

### 6.2 manacher

```

int main(){
    char s[MAX];
    vector<int> d1(n);
    for (int i = 0, l = 0, r = -1; i < n; i++){
        int k = (i > r) ? 1 : min(d1[l + r - i + 1], r - i + 1);
        while (0 <= i - k && i + k < n && s[i - k] == s[i + k]){
            k++;
        }
        d1[i] = k--;
        if (i + k > r){
            l = i - k;
            r = i + k;
        }
    }
    vector<int> d2(n);
    for (int i = 0, l = 0, r = -1; i < n; i++){
        int k = (i > r) ? 0 : min(d2[l + r - i + 1], r - i + 1);
        while (0 <= i - k - 1 && i + k < n && s[i - k - 1] == s[i + k]){
            k++;
        }
        d2[i] = k--;
        if (i + k > r){
            l = i - k - 1;
            r = i + k;
        }
    }
    return 0;
}

```

### 6.3 palindromic tree

```

struct node {
    int next[26];
    int len;
    int sufflink;
    int num;
};
int len;
char s[MAXN];
node tree[MAXN];
int num; // node 1 - root with len -1, node 2 - root with len 0
int suff; // max suffix palindrome
long long ans;
bool addLetter(int pos) {
    int cur = suff, curlen = 0;
    int let = s[pos] - 'a';
    while (true) {
        curlen = tree[cur].len;
        if (pos - 1 - curlen >= 0 && s[pos - 1 - curlen] == s[pos])
            break;
        cur = tree[cur].sufflink;
    }
    if (tree[cur].next[let]) {
        suff = tree[cur].next[let];
        return false;
    }
    num++;
    suff = num;
    tree[num].len = tree[cur].len + 2;
    tree[cur].next[let] = num;
    if (tree[num].len == 1) {
        tree[num].sufflink = 2;
        tree[num].num = 1;
        return true;
    }
    while (true) {
        cur = tree[cur].sufflink;
        curlen = tree[cur].len;
        if (pos - 1 - curlen >= 0 && s[pos - 1 - curlen] == s[pos]) {
            tree[num].sufflink = tree[cur].next[let];
            break;
        }
    }
    tree[num].num = 1 + tree[tree[num].sufflink].num;
    return true;
}

void initTree() {
    num = 2; suff = 2; // memset tree must
    tree[1].len = -1; tree[1].sufflink = 1;
}

```

```

tree[2].len = 0; tree[2].sufflink = 1;
}

int main() {
    gets(s);
    len = strlen(s);
    initTree();
    for (int i = 0; i < len; i++) {
        addLetter(i);
        ans += tree[suff].num;
    }
    cout << ans << endl;
    return 0;
}

```

### 6.4 suffix array da

```

/*
sa => ith smallest suffix of the string
rak => rak[i] indicates the position of suffix(i) in the suffix array
height => height[i] indicates the LCP of i-1 and i th suffix
* LCP of suffix(i) & suffix(j) = { L = rak[i], R = rak[j]
  , min(height[L+1, R]);}*/
const int maxn = 5e5+5;
int wa[maxn],wb[maxn],wv[maxn],wc[maxn];
int r[maxn],sa[maxn],rak[maxn],
height[maxn],dp[maxn][22],jump[maxn], SIGMA = 0;
int cmp(int *r,int a,int b,int l){return r[a] == r[b] &&
r[a+l] == r[b+l];}

void da(int *r,int *sa,int n,int m){
    int i,j,p,*x=wa,*y=wb,*t;
    for(i=0;i<n;i++) wc[i]=0;
    for(i=0;i<n;i++) wc[x[i]=r[i]]++;
    for(i=1;i<n;i++) wc[i] += wc[i-1];
    for(i=n-1;i>=0;i--) sa[--wc[x[i]]] = i;
    for(j=1,p=1;p<n;j*=2,m=p){
        for(p=0,i=n-j;i<n;i++) y[p++] = i;
        for(i=0;i<n;i++) if(sa[i] >= j) y[p++] = sa[i] - j;
        for(i=0;i<m;i++) wc[i] = 0;
        for(i=0;i<n;i++) wc[wv[i]]++;
        for(i=1;i<m;i++) wc[i] += wc[i-1];
        for(i=n-1;i>=0;i--) sa[--wc[wv[i]]] = y[i];
        for(t=x,x=y,y=t,p=1,x[sa[0]] = 0,i=1;i<n;i++) x[sa[i]] =
            cmp(y,sa[i-1],sa[i],j) ? p-1:p++;
    }
}

void calheight(int *r,int *sa,int n){
    int i,j,k=0;
    for(i=1;i<n;i++) rak[sa[i]] = i;
    for(i=0;i<n;height[rak[i++]] = k) {
        for(k?k--:0, j=sa[rak[i]-1]; r[i+k] == r[j+k]; k++);
    }
}

void initRMQ(int n){
    for(int i=0;i<=n;i++) dp[i][0] = height[i];
    for(int j=1;(1<<j)<=n;j++){
        for(int i=0;i+(1<<j)-1<=n;i++){
            dp[i][j] = min(dp[i][j-1], dp[i+(1<<(j-1))][j-1]);
        }
    }
    for(int i=1;i<=n;i++){
        int k=0;
        while((1<<(k+1))<=i) k++;
        jump[i] = k;
    }
}

int askRMQ(int L,int R){
    int k = jump[R-L+1];
    return min(dp[L][k], dp[R - (1<<k) + 1][k]);
}

int main(){
    scanf("%s",s);
    int n = strlen(s);
    for(int i=0;i<n;i++){
        r[i] = s[i]-'a' + 1;
        SIGMA = max(SIGMA, r[i]);
    }
}

```

```

r[n] = 0;
da(r,sa,n+1,SIGMA + 1); // don't forget SIGMA + 1. It will
                        // ruin you.
calheight(r,sa,n);
}

```

## 6.5 suffix-automaton

```

class SuffixAutomaton{
bool complete;
int last;
set<char> alphabet;
struct state{
int len, link, endpos, first_pos, snas, height;
long long substrings, sublen;
bool is_clone;
map<char, int> next;
vector<int> inv_link;
state(int leng=0, int li=0){
len=leng;
link=li;
first_pos=-1;
substrings=0;
sublen=0; // length of all substrings
endpos=1;
snas=0; // shortest_non_appearing_string
is_clone=false;
height=0;
}
};
vector<state> st;
void process(int node){
map<char, int> ::iterator mit;
st[node].substrings=1;
st[node].snas=st.size();
if((int) st[node].next.size() < (int) alphabet.size())
st[node].snas=1;
for(mit=st[node].next.begin(); mit!=st[node].next.end();
++mit){
if(st[mit->second].substrings==0) process(mit->second);
st[node].height=max(st[node].height, 1+st[mit->second].height);
st[node].substrings+=st[mit->second].substrings;
st[node].sublen+=st[mit->second].sublen;
st[node].snas=min(st[node].snas,
1+st[mit->second].snas);
}
if(st[node].link!=-1){
st[st[node].link].inv_link.push_back(node);
}
}
void set_suffix_links(int node){
int i;
for(i=0; i<st[node].inv_link.size(); i++){
set_suffix_links(st[node].inv_link[i]);
st[node].endpos=st[node].endpos+st[st[node].inv_link[i]].endpos;
}
}
void output_all_occurrences(int v, int P_length, vector<int>
&pos){
if (!st[v].is_clone)
pos.push_back(st[v].first_pos - P_length + 1);
for (int u : st[v].inv_link)
output_all_occurrences(u, P_length, pos);
}
void kth_smallest(int node, int k, vector<char> &str){
if(k==0) return;
map<char, int> ::iterator mit;
for(mit=st[node].next.begin(); mit!=st[node].next.end();
++mit){
if(st[mit->second].substrings<k)
k=k-st[mit->second].substrings;
else{
str.push_back(mit->first);
kth_smallest(mit->second, k-1, str);
return;
}
}
}
}

```

```

int find_occurrence_index(int node, int index, vector<char>
&str){
if(index==str.size()) return node;
if(!st[node].next.count(str[index])) return -1;
else return
find_occurrence_index(st[node].next[str[index]], index+1, str);
}
void klen_smallest(int node, int k, vector<char> &str){
if(k==0) return;
map<char, int> ::iterator mit;
for(mit=st[node].next.begin(); mit!=st[node].next.end();
++mit){
if(st[mit->second].height>=k-1){
str.push_back(mit->first);
klen_smallest(mit->second, k-1, str);
return;
}
}
}
void minimum_non_existing_string(int node, vector<char> &str){
map<char, int> ::iterator mit;
set<char> ::iterator sit;
for(mit=st[node].next.begin(), sit=alphabet.begin();
sit!=alphabet.end(); ++sit, ++mit){
if(mit==st[node].next.end() || mit->first != (*sit)){
str.push_back(*sit);
return;
}
else if(st[node].snas==1+st[mit->second].snas){
str.push_back(*sit);
minimum_non_existing_string(mit->second, str);
return;
}
}
}
void find_substrings(int node, int index, vector<char>
&str, vector<pair<long long, long long> > &sub_info){
sub_info.push_back(make_pair(st[node].substrings, st[node].sublen+st[
node].sublen));
if(index==str.size()) return;
if(st[node].next.count(str[index])){
find_substrings(st[node].next[str[index]], index+1, str, sub_info);
return;
}
else{
sub_info.push_back(make_pair(0, 0));
}
}
void check(){
if(!complete){
process(0);
set_suffix_links(0);
int i;
complete=true;
}
}
public:
SuffixAutomaton(set<char> &alpha){
st.push_back(state(0, -1));
last=0;
complete=false;
set<char> ::iterator sit;
for(sit=alpha.begin(); sit!=alpha.end(); sit++){
alphabet.insert(*sit);
}
st[0].endpos=0;
}
void sa_extend(char c){
int cur = st.size();
st.push_back(state(st[last].len + 1));
st[cur].first_pos=st[cur].len-1;
int p = last;
while (p != -1 && !st[p].next.count(c)){
st[p].next[c] = cur;
p = st[p].link;
}
if (p == -1){
st[cur].link = 0;
}
else{
int q = st[p].next[c];
}
}
}

```

```

if (st[p].len + 1 == st[q].len){
st[cur].link = q;
}
else{
int clone = st.size();
st.push_back(state(st[p].len + 1, st[q].link));
st[clone].next = st[q].next;
st[clone].is_clone=true;
st[clone].endpos=0;
st[clone].first_pos=st[q].first_pos;
st[clone].first_pos+=st[q].first_pos;
while (p != -1 && st[p].next[c] == q){
st[p].next[c] = clone;
p = st[p].link;
}
st[q].link = st[cur].link = clone;
}
}
last = cur;
complete=false;
}
~SuffixAutomaton(){
int i;
for(i=0; i<st.size(); i++){
st[i].next.clear();
st[i].inv_link.clear();
}
st.clear();
alphabet.clear();
}
void kth_smallest(int k, vector<char> &str){
check();
klen_smallest(0, k, str);
}
int FindFirstOccurrenceIndex(vector<char> &str){
check();
int ind=find_occurrence_index(0, 0, str);
if(ind==0) return ind;
else return st[ind].first_pos+1-(int) str.size();
}
void FindAllOccurrenceIndex(vector<char> &str, vector<int>
&pos){
check();
int ind=find_occurrence_index(0, 0, str);
if(ind!=-1) output_all_occurrences(ind, str.size(), pos);
}
int Occurrences(vector<char> &str){
check();
int ind=find_occurrence_index(0, 0, str);
if(ind==0) return 1;
else if(ind==-1) return 0;
else return st[ind].endpos;
}
void klen_smallest(int k, vector<char> &str){
check();
if(st[0].height>=k) klen_smallest(0, k, str);
}
void minimum_non_existing_string(vector<char> &str){
check();
int ind=find_occurrence_index(0, 0, str);
if(ind!=-1) minimum_non_existing_string(ind, str);
}
long long cyclic_occurrence(vector<char> &str){
check();
int i, j, len;
long long ans=0;
int n=str.size();
set<int> S;
set<int> ::iterator it;
for(i=0, j=0, len=0; i<n*2-1; i++){
if(st[j].next.count(str[i%n])){
len++;
j=st[j].next[str[i%n]];
}
else{
while(j!=-1 && (!st[j].next.count(str[i%n])))
j=st[j].link;
if(j!=-1){
}
}
}
}

```

```

        len=st[j].len+1;
        j=st[j].next[st[i%n]];
    }
    else{
        len=0;
        j=0;
    }
}
while(st[j].link!=-1&&st[st[j].link].len>=n){
    j=st[j].link;
    len=st[j].len;
}
if(len>=n) S.insert(j);
}
for(it=S.begin();it!=S.end();++it){
    ans=ans+st[*it].endpos;
}
return ans;
};
}

```

## 6.6 z-algorithm

```

vector<int> z_function(string s) {
    int n = (int) s.length();
    vector<int> z(n);
    for (int i = 1, l = 0, r = 0; i < n; ++i) {
        if (i <= r)
            z[i] = min (r - i + 1, z[i - l]);
        while (i + z[i] < n && s[z[i]] == s[i + z[i]])
            ++z[i];
    }
}

```

```

        if (i + z[i] - 1 > r)
            l = i, r = i + z[i] - 1;
    }
    return z;
}

```

## 7 header

```

#define FastIO ios::sync_with_stdio(false);
cin.tie(0);cout.tie(0)
#include <ext/pb_ds/assoc_container.hpp> // Common file
using namespace __gnu_pbds;
/*
find_by_order(k) --> returns iterator to the kth largest
element counting from 0
order_of_key(val) --> returns the number of items in a set
that are strictly smaller than our item
*/
typedef tree<
int,
null_type,
less<int>,
rb_tree_tag,
tree_order_statistics_node_update>
ordered_set;
// #pragma GCC optimize("O3,unroll-loops")
// #pragma GCC target("avx2,bmi,bmi2,lzcnt")
// mt19937
rng(chrono::system_clock::now().time_since_epoch().count());
#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;

```

```

struct custom_hash {
    static uint64_t splitmix64(uint64_t x) {
        x += 0x9e3779b97f4a7c15; //Random
        x=(x^(x>>30))*0xbf58476d1ce4e5b9; //Random
        x=(x^(x>>27))*0x94d049bb133111eb; //Random
        return x^(x>>31);
    }
    const uint64_t FIXED_RANDOM = chrono::
        steady_clock::now().time_since_epoch().count();
    size_t operator()(uint64_t x) const {
        return splitmix64(x + FIXED_RANDOM);
    }
    size_t operator()(pair<int, int> x) const {
        return splitmix64((uint64_t(x.first)<<32) +
            x.second + FIXED_RANDOM);
    }
};
gp_hash_table<pair<int,int>,int,custom_hash> ht;
# stresstester GENERATOR SOL1 SOL2 ITERATIONS
for i in $(seq 1 "$4") ; do
    echo -en "\rAttempt $i/$4"
    $1 > in.txt
    $2 < in.txt > out1.txt
    $3 < in.txt > out2.txt
    diff -y out1.txt out2.txt > diff.txt
    if [ $? -ne 0 ] ; then
        echo -e "\nTestcase Found:"; cat in.txt
        echo -e "\nOutputs:"; cat diff.txt
        exit
    fi
done

```