Heston Stochastic Volatility: Pricing and Calibration (Synthetic Study)

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1 Overview

This report documents a small quantitative finance project implementing the Heston stochastic volatility model for European option pricing, implied volatility (IV) inversion, and parameter calibration on synthetic option data. The workflow includes: (i) Heston semi-closed-form pricing (via characteristic function integration), (ii) robust IV inversion with no-arbitrage checks, and (iii) calibration of Heston parameters to a synthetic option surface with diagnostics and visual comparisons.

2 Model

Under the Heston model (risk-neutral measure), the spot price S_t and instantaneous variance v_t evolve as:

$$dS_t = (r - q)S_t \, dt + \sqrt{v_t} \, S_t \, dW_t^{(1)}, \tag{1}$$

$$dv_t = \kappa(\theta - v_t) \, dt + \sigma \sqrt{v_t} \, dW_t^{(2)}, \tag{2}$$

$$dW_t^{(1)} \, dW_t^{(2)} = \rho \, dt, \tag{3}$$

where r is the risk-free rate, q is the dividend yield, κ the mean reversion speed, θ the long-run variance, σ the volatility of variance, and ρ the correlation (driving the skew).

Feller condition. A sufficient condition to keep v_t strictly positive is:

$$2\kappa\theta \geq \sigma^2. \tag{4}$$

In practice, the Feller condition can be violated while pricing remains well-defined; empirical calibrations often do not strictly satisfy it.

3 Pricing and Implied Volatility

European call/put prices are computed via the standard Heston characteristic-function approach ("Little Heston Trap" style formulation) with consistent probability integrals. Implied volatilities are obtained by inverting the Black–Scholes price using a robust solver with no-arbitrage clipping to handle numerical edge cases.

3.1 Heston implied volatility smile

Figure 1 shows the implied volatility smile at $T = 1$ year for a representative parameter set. Negative correlation $\rho < 0$ produces an equity-like skew: higher IV for low strikes and lower IV for high strikes.

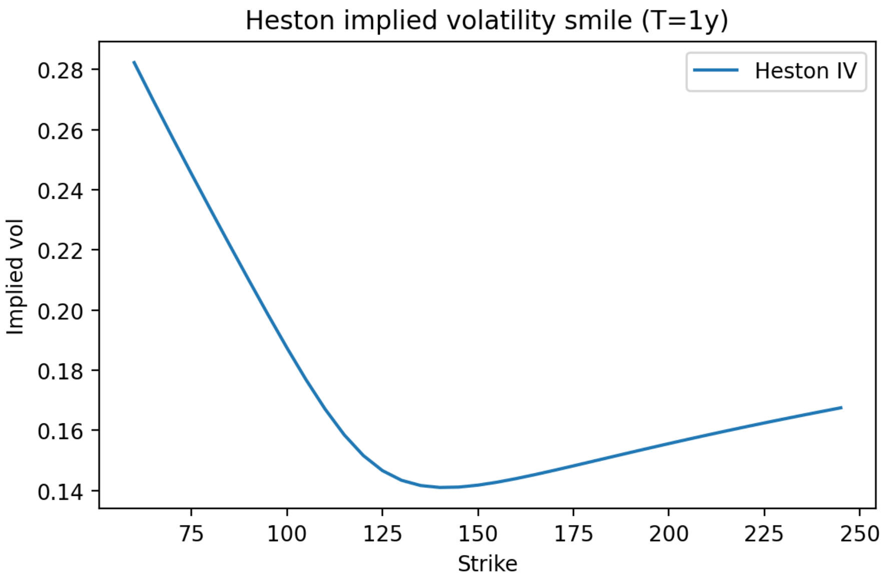


Figure 1: Heston implied volatility smile at $T = 1$ year.

3.2 Heston call price surface

Figure 2 shows the call price surface across strikes and maturities. Prices decrease with strike and increase with maturity, as expected, and remain within no-arbitrage bounds.

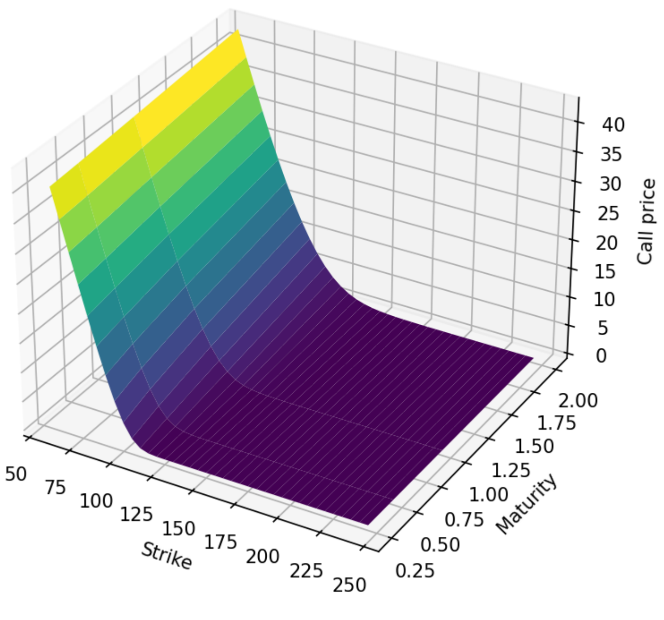


Figure 2: Heston call price surface over strikes and maturities.

4 Synthetic Calibration Setup

A synthetic option chain is generated from a chosen set of "true" Heston parameters over a grid of strikes and maturities. Small noise may be added to prices to mimic market imperfections. Calibration then solves:

$$\hat{\vartheta} = \arg\min_{\vartheta} \mathcal{L}(\vartheta), \tag{5}$$

where $\vartheta = (\kappa, \theta, \sigma, \rho, v_0)$ and $\mathcal{L}$ is an objective defined on either option prices or implied volatilities (depending on the target).

4.1 Synthetic price distribution

Figure 3 visualizes synthetic option prices pooled across maturities as a function of strike.

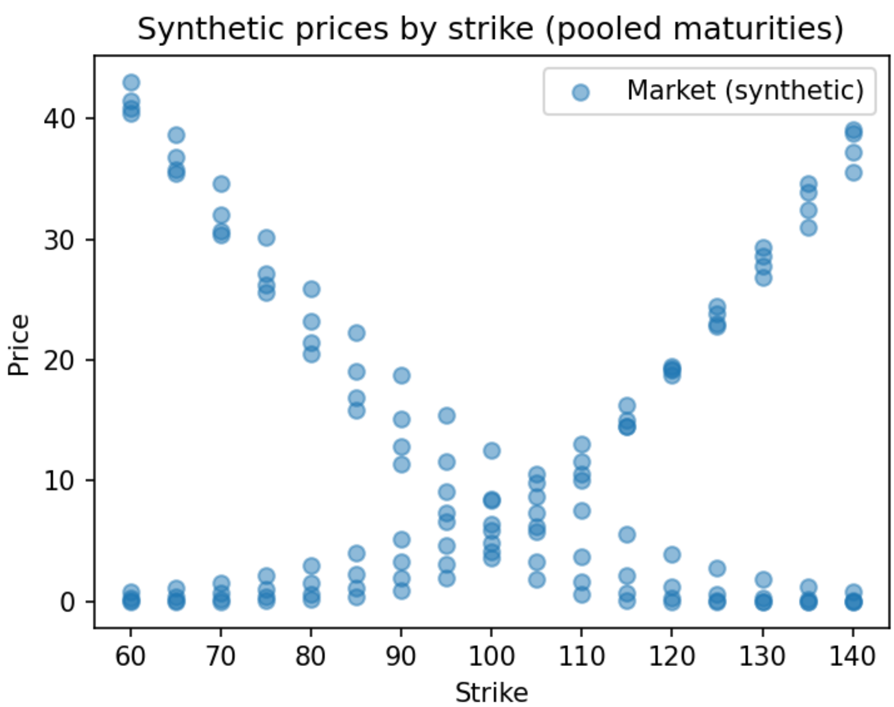


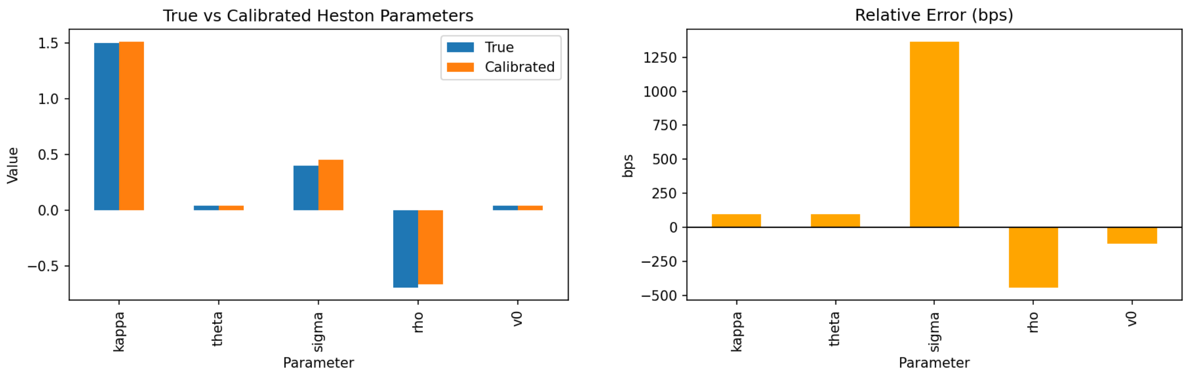
Figure 3: Synthetic option prices versus strike (pooled maturities).

5 Calibration Results

This section compares recovered parameters to the true parameters and evaluates the fit in implied volatility space.

5.1 True vs calibrated parameters

Figure 4 shows the calibrated parameters alongside the true parameters used in the synthetic generator. Relative errors are also shown in basis points to make small differences visible.



(a) True vs calibrated values. (b) Relative error (basis points).

Figure 4: Parameter recovery from synthetic calibration.

5.2 IV fit at $T = 1$ year

Figure 5 compares the synthetic "market" IV smile at $T = 1$ year to the IV smile implied by model prices under the calibrated parameters. Close overlap indicates a successful calibration in volatility space.

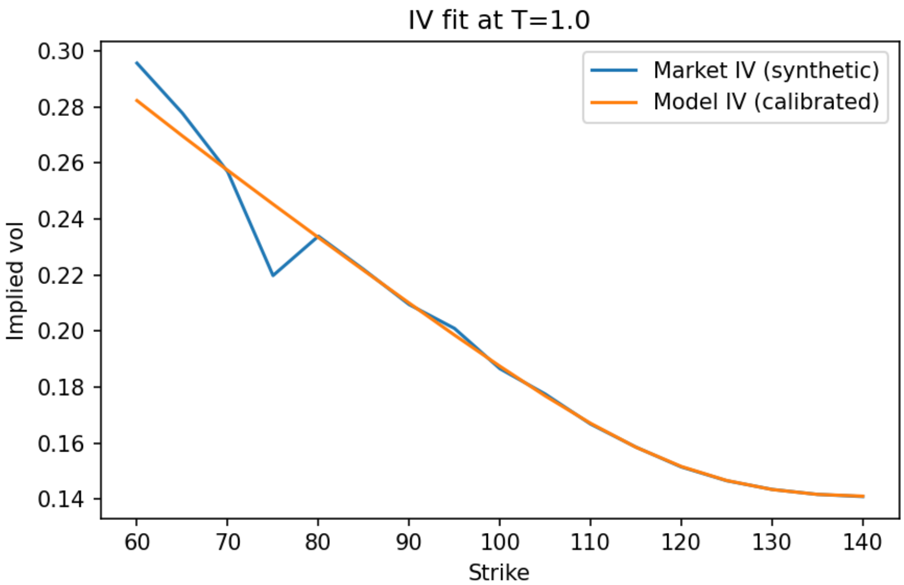


Figure 5: IV fit at $T = 1$ year: synthetic market IV vs calibrated model IV.

5.3 IV error heatmap

Figure 6 shows the IV error across the strike–maturity grid:

$$\Delta\sigma_{\mathrm{IV}} = \sigma_{\mathrm{model}} - \sigma_{\mathrm{market}}.$$

This highlights localized regions where the model under/over-estimates the synthetic surface. Small residual patterns can arise from noise, numerical integration tolerance, and IV inversion sensitivity (especially deep ITM/OTM).

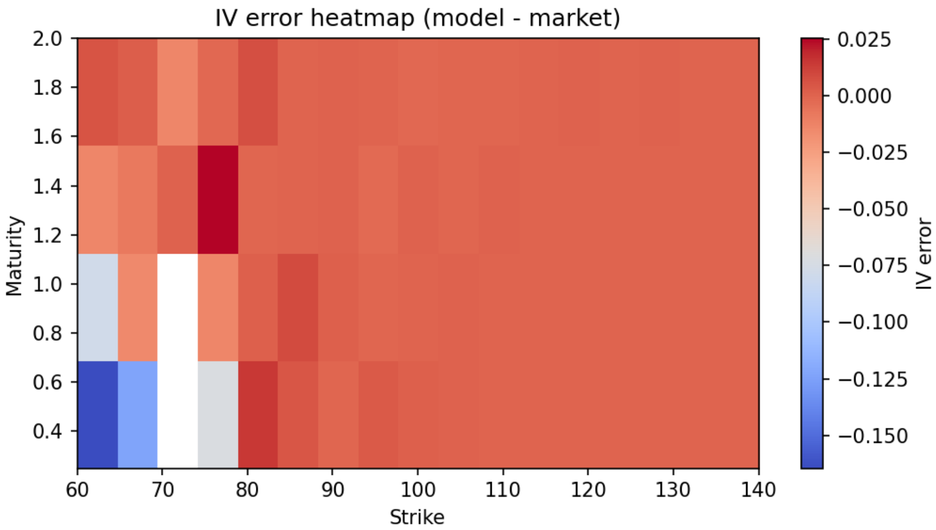


Figure 6: IV error heatmap across strikes and maturities (model minus market).

6 Discussion

Interpretation. The calibrated parameters closely match the true generating parameters in the synthetic experiment, and the IV fit indicates strong agreement. The negative correlation ρ drives the skew commonly observed in equity option markets.

Numerical stability. The project includes no-arbitrage checks (price bounds), robust IV inversion, and sanity tests such as the Black–Scholes limiting regime (near-constant volatility). These checks are important in practice to avoid silent numerical failures.

7 Limitations and Next Steps

- **Synthetic data:** Real market calibration would require real option chains (bid/ask handling, filtering, weighting).
- **Objective choices:** Calibrating to price vs IV can change sensitivity and parameter identifiability.
- **Model risk:** Heston may not fit all maturities simultaneously without extensions (jumps, time-dependent parameters).
- **Next steps:** Add weighting by vega / bid-ask, incorporate constraints (including optional Feller), and run calibration on real option data.