Portfolio Management and Risk Allocation

Walk-Forward Backtesting of Allocation Strategies

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1 Introduction

This project implements a full portfolio management and backtesting framework to study classical and risk-based asset allocation strategies under realistic market assumptions. The objective is to compare the risk–return characteristics, turnover, and drawdown behavior of different allocation approaches using a consistent walk-forward methodology with transaction costs.

The framework is designed to be fully reproducible and modular, with cached data loading, rolling covariance estimation, constrained optimization, and comprehensive performance evaluation.

2 Data and Returns

The investment universe consists of liquid exchange-traded funds representing multiple asset classes:

- SPY (US equities)
- QQQ (US technology equities)
- EEM (emerging market equities)
- TLT (long-duration US Treasuries)
- IEF (intermediate Treasuries)
- GLD (gold)
- VNQ (US real estate)

Daily adjusted closing prices are downloaded via `yfinance` and cached locally. Log returns are computed as

$$r_t = \log\left(\frac{P_t}{P_{t-1}}\right),$$

and all annualized statistics use 252 trading days per year.

3 Covariance Estimation

Portfolio construction relies on rolling estimates of the return covariance matrix using a 252-day lookback window. The following estimators are implemented:

- **Sample covariance**
- **EWMA covariance** with decay parameter $\lambda = 0.94$

- **Shrinkage covariance** toward the diagonal to reduce estimation noise

Shrinkage improves numerical stability and reduces excessive portfolio turnover, particularly in constrained optimization problems.

4 Portfolio Strategies

All strategies are long-only and fully invested unless otherwise stated.

4.1 Equal Weight

A baseline strategy allocating equal capital to each asset.

4.2 Minimum Variance

Weights are chosen to minimize portfolio variance:

$$\min_{w} \; w^\top \Sigma w \quad \text{s.t.} \quad \sum_i w_i = 1, \; w_i \geq 0.$$

4.3 Mean–Variance

A classical Markowitz formulation:

$$\max_{w} \; \mu^\top w - \frac{\gamma}{2} w^\top \Sigma w,$$

where expected returns μ are estimated using rolling historical means and γ controls risk aversion.

4.4 Risk Parity

Weights are chosen such that each asset contributes equally to total portfolio risk. An iterative solver enforces equal risk contributions under long-only constraints.

4.5 Volatility Targeting

A volatility targeting overlay rescales portfolio exposure to target a fixed annualized volatility (10%), with leverage capped at 1.5. Any remaining allocation is assigned to cash (assumed zero return).

5 Backtesting Methodology

A walk-forward backtest is performed with monthly rebalancing. At each rebalance date:

- Covariance and expected returns are estimated using only past data.
- Portfolio weights are recomputed.
- Transaction costs are applied based on turnover:

  $$\text{Cost}_t = c \sum_i |w_i^{(t)} - w_i^{(t-1)}|,$$

  where $c = 5$ basis points.

Daily portfolio returns are computed using lagged weights to avoid look-ahead bias.

6 Performance Metrics

For each strategy, the following metrics are computed:

- CAGR
- Annualized volatility
- Sharpe and Sortino ratios
- Maximum drawdown
- Calmar ratio
- Average turnover
- Value-at-Risk (95%) and Conditional VaR (95%)

The full metrics table is saved as `reports/metrics.csv` and loaded by both the CLI and the report.

7 Results

7.1 Cumulative Performance

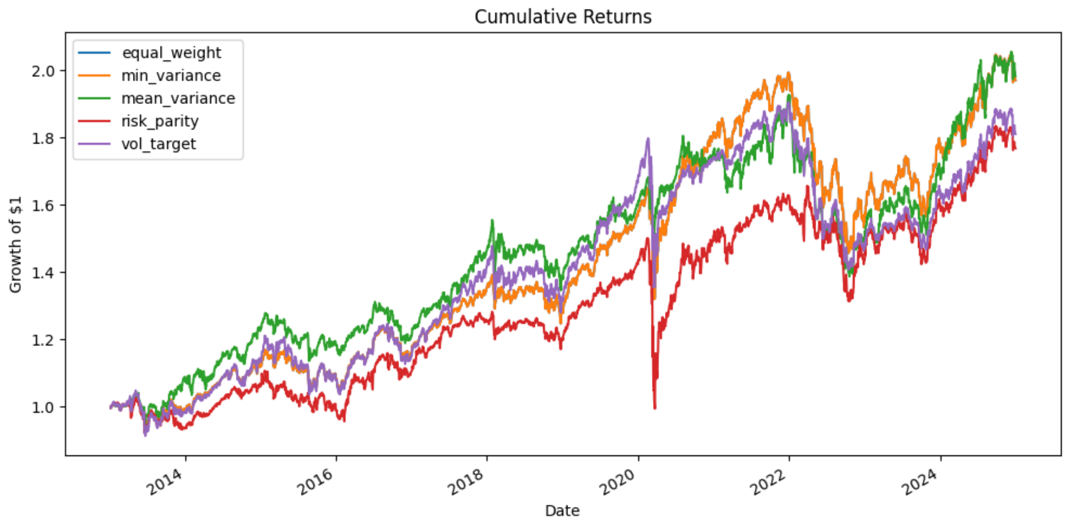

Figure 1: Cumulative portfolio growth for all strategies.

Mean–variance allocation achieves the highest long-term growth, while risk parity exhibits lower returns and larger drawdowns in this universe.

7.2 Rolling Volatility

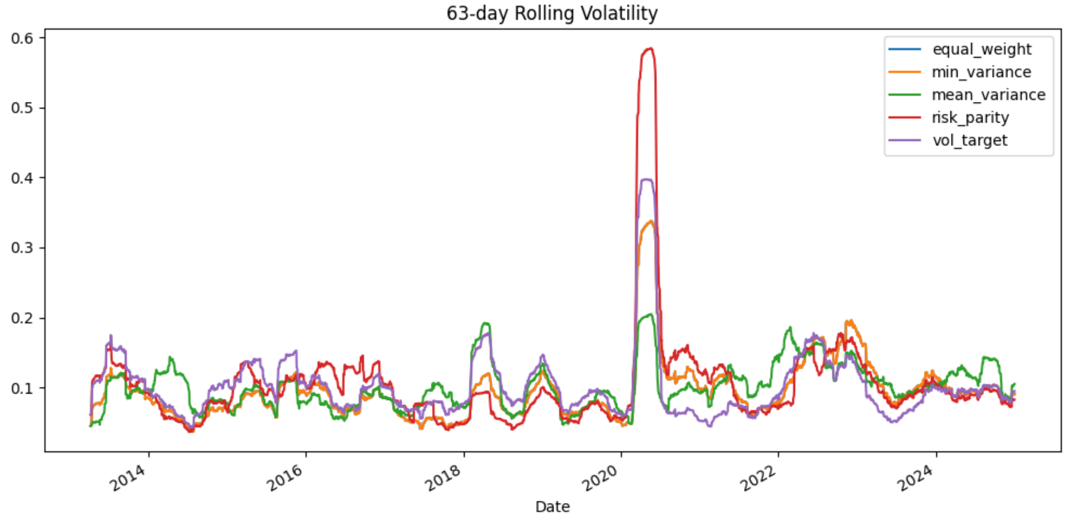

Figure 2: 63-day rolling annualized volatility.

Volatility spikes during the 2020 crisis are clearly visible. The volatility targeting strategy successfully caps realized risk relative to other approaches.

7.3 Turnover

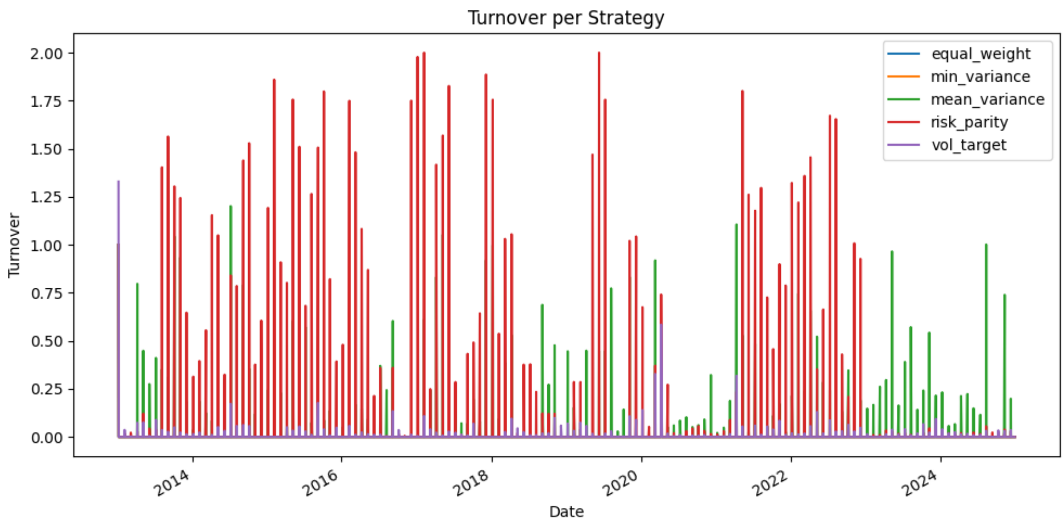

Figure 3: Portfolio turnover by strategy.

Risk parity and mean–variance strategies exhibit higher turnover, explaining their sensitivity to transaction costs.

7.4 Portfolio Weights

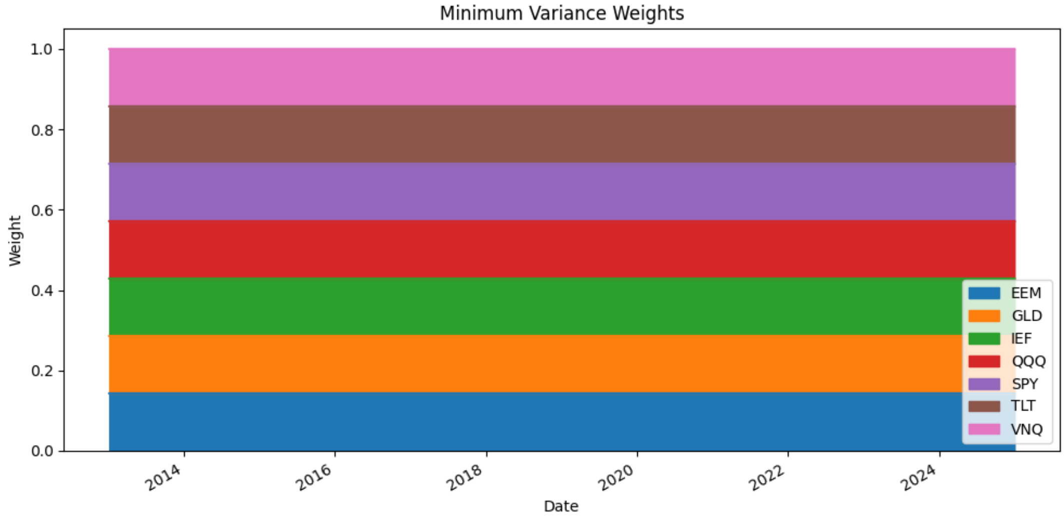

Figure 4: Minimum variance portfolio weights.

The minimum variance portfolio closely resembles the equal-weight benchmark due to strong cross-asset correlations and covariance shrinkage.

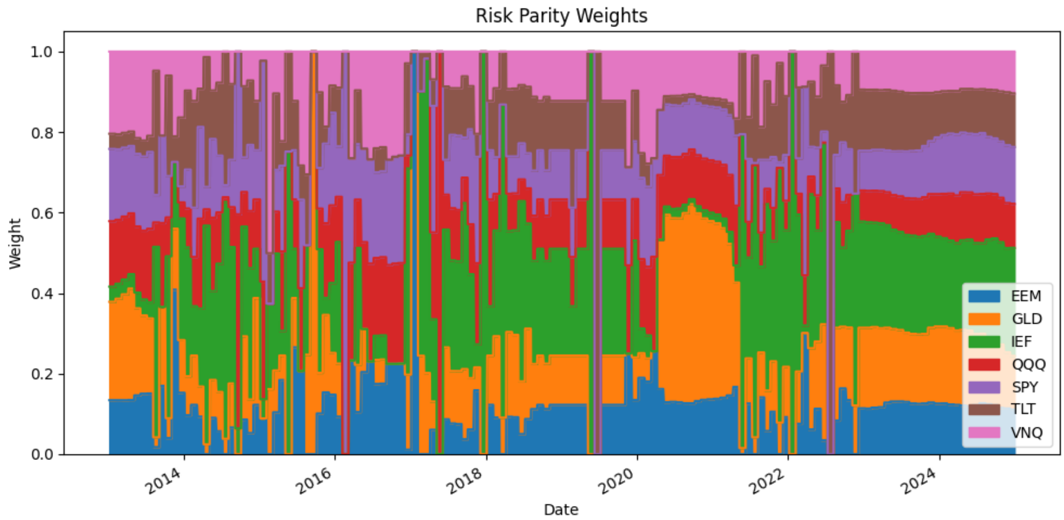

Figure 5: Risk parity portfolio weights.

Risk parity allocations are highly dynamic, especially during periods of elevated volatility.

8 Discussion

The results highlight several important insights:

- Covariance shrinkage stabilizes optimization but can collapse minimum variance solutions toward equal weighting.
- Mean–variance allocation improves returns at the cost of higher turnover.

- Risk parity without leverage may underperform in equity-dominated regimes.
- Volatility targeting provides effective downside risk control.

9 Limitations

The framework assumes:

- No short-selling
- Constant transaction costs
- No estimation uncertainty beyond rolling windows

Extensions could include regime detection, Bayesian return estimation, or derivative-based risk overlays.

10 Conclusion

This project demonstrates a complete, production-style portfolio management pipeline, combining sound financial theory with robust numerical implementation and realistic backtesting discipline.