Volatility Modelling and Option Pricing

A Quantitative Study Using Rolling, EWMA, and GARCH Models

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1 Data

We study daily adjusted close prices of the SPY ETF over the period 2015–2024. Prices are converted to log returns,

$$r_t = \log\left(\frac{P_t}{P_{t-1}}\right),$$

which form the basic input for all volatility models. Returns are treated as approximately mean-zero and are expressed in daily units unless stated otherwise. Annualization is performed using a factor of $\sqrt{252}$.

2 Volatility Models

We estimate time-varying volatility using three standard approaches:

- **Rolling (Historical) Volatility:** Standard deviation of returns over a fixed window (20 trading days).
- **EWMA Volatility:** Exponentially weighted moving average variance,

$$\sigma_t^2 = \lambda\sigma_{t-1}^2 + (1-\lambda)r_{t-1}^2, \quad \lambda = 0.94.$$

- **GARCH(1,1):** Conditional variance model,

$$\sigma_t^2 = \omega + \alpha r_{t-1}^2 + \beta\sigma_{t-1}^2,$$

  with parameters estimated via maximum likelihood.

All models produce one-step-ahead volatility forecasts.

3 Evaluation Methodology

Volatility forecasts are evaluated using a walk-forward (out-of-sample) framework. At each time t, models are fitted using data available up to t and used to forecast volatility for $t+1$.

To reduce noise in the target, realized volatility is computed using windowed realized variance,

$$\mathrm{RV}_t^{(w)} = \sqrt{\frac{252}{w}\sum_{i=0}^{w-1} r_{t-i}^2},$$

with windows $w = 5$ (weekly) and $w = 21$ (monthly).

Forecast accuracy is assessed using standard metrics:

- Mean Squared Error (MSE) on variance,
- Mean Absolute Error (MAE) on volatility,
- QLIKE loss, commonly used in volatility forecasting.

4 Results

Forecast Accuracy

Table 1 summarizes the out-of-sample performance for each model. Smoothed realized volatility leads to more stable and interpretable comparisons.

Table 1: Out-of-sample volatility forecast evaluation using windowed realized volatility

Model	Window	MSE (variance)	MAE (vol)	QLIKE
Historical	5	0.009359	0.052850	-2.713700
Historical	21	0.000178	0.007565	-2.767174
EWMA	5	0.008596	0.056341	-2.683947
EWMA	21	0.000927	0.021133	-2.732824
GARCH(1,1)	5	0.001597	0.032931	-2.838724
GARCH(1,1)	21	0.004505	0.033791	-2.679181

Forecasted vs Realized Volatility

Figure 1 compares forecasted volatility to windowed realized volatility. All models capture volatility clustering, with GARCH reacting most strongly to shocks and exhibiting clear mean reversion.

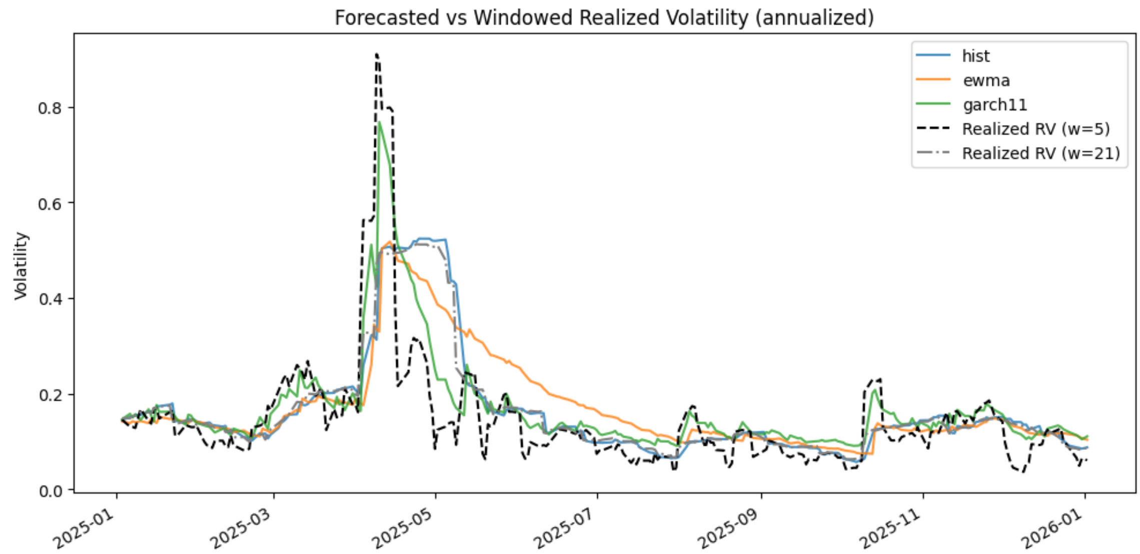

Figure 1: Forecasted volatility vs windowed realized volatility (annualized)

5 Option Pricing

Estimated volatility is used as input for European call option pricing.

- **Black–Scholes:** Closed-form pricing under constant volatility.
- **Monte Carlo:** Simulation of geometric Brownian motion under the risk-neutral measure,

$$dS_t = rS_t dt + \sigma S_t dW_t.$$

Monte Carlo pricing includes 95% confidence intervals, demonstrating convergence to the Black–Scholes price at the expected rate $\mathcal{O}(1/\sqrt{N})$.

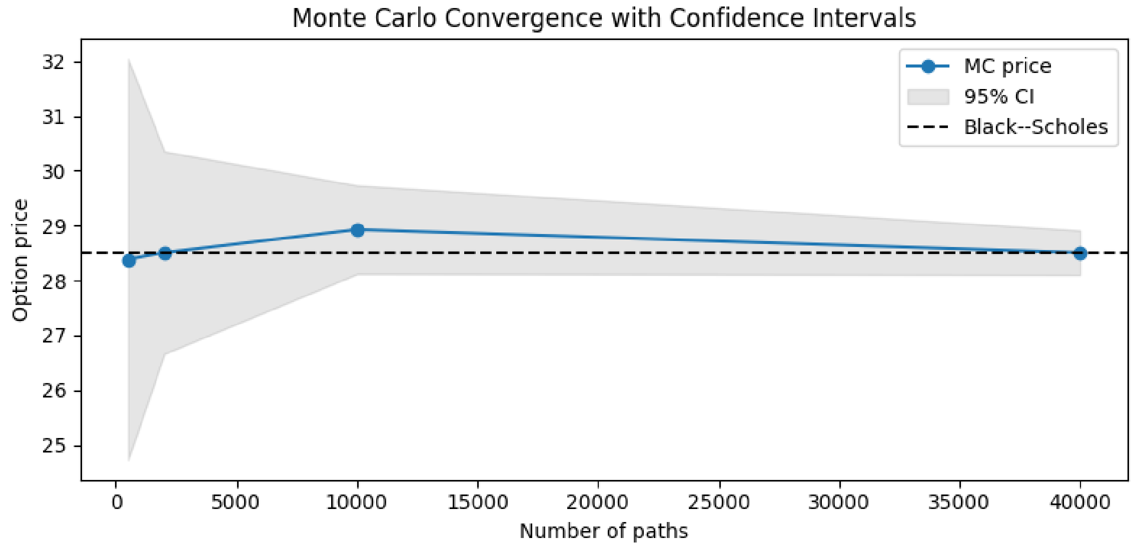

Figure 2: Monte Carlo option pricing with 95% confidence intervals

6 Limitations

Several limitations should be noted:

- Daily data limits the precision of realized volatility estimation.
- Windowed realized variance remains a proxy for latent volatility.
- Black–Scholes and Monte Carlo pricing assume lognormal dynamics and constant volatility.
- Transaction costs and market microstructure effects are not modeled.

Despite these limitations, the project demonstrates a complete and coherent volatility modelling and option pricing pipeline.