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A model for project selection, adjustment and scheduling considering reinvestment strategy and financing

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Abstract

Developing mathematical models for project selection and scheduling has been one of the most attractive topics for research in the last three decades. So far, many mathematical models are presented to help organizations select the best portfolio of projects, maximize income, or achieve other goals they have in mind.

A comprehensive mathematical model is developed for project selection and scheduling in this research so that organizations can use it to maximize their income. Each project is divided into several phases, with a specific cost and revenue. Prerequisite and incompatibility relationship between projects is also considered. In addition, it is possible to adjust the project. So, they can be upgraded or sold with a time-depended value. It is possible to consider and adjust pre-existing projects as well. Finally, we attempted to minimize the average delay from the delivery time using a second objective function that increases the mathematical model efficiency and creates a trade-off between response variables. So, the decision-makers can reach a balance between income and delay.

Research aim

Providing a model for Project selection, adjustment, and scheduling projects considering reinvestment.

Research Method

A mathematical model (using operations research) is represented and solved using the GAMS software. To solve the mathematical model, we used the CPLEX algorithm. After adding the second objective function, the Augmented Epsilon Constraint method suggests five solutions considering the two objective functions.

Findings

The results of solving the mathematical model indicate the possibility of adjustment in the project selection problem significantly increases income and reduces constraints. In addition, there is a certain threshold for investment and the optimal number of selected projects. So, increasing the budget will not necessarily results in selecting more projects.

Conclusion

The results of solving the mathematical model in different scenarios show that there is a certain threshold for investment, and increasing the budget may not be profitable. Furthermore, decision-makers should select a certain number of projects (since there is a threshold for the number of profitable projects) unless there are managerial justifications that the mathematical model fails to consider.

Keywords

Project Selection, Project Scheduling, Project Adjustment, Project Portfolio Optimization

3. Modeling and solution method

3-1- Assumptions of the model

1- Some projects are available, and it is possible to select and schedule a number of them.

2- Each project consists of a certain number of different phases, each phase having a prerequisite relationship with the next phase.

3- It is possible to interrupt the project at the end of each phase (it is not possible to interrupt it during the implementation of a specific phase) and one or more periods of the project can be postponed.

4- Each project can be adjusted (upgrade or abandoning the project) at most once, and otherwise, it can continue without changes until the end.

5- Adjusting or continuing projects without change is known as the mode of doing the project.

6- mode zero is used to run the project in normal mode, mode 1 is used to display the upgrade, and mode 2 is used to display the abandonment of the project.

7- If we abandon the project at the end of each phase, the project will be sold with its value at that moment.

8- Prerequisite or incompatibility between projects is considered.

9- The planning horizon is divided into several periods.

10- Project scheduling is based on implementation phases.

11- The source of implementing projects is money and at the beginning of zero time, it is available in the form of an initial budget.

12- There is a certain constraint for the initial budget.

13- The cash flow related to each phase of each project includes an expense at the beginning of the phase and an income at the end of it.

14- The cost and income related to different modes are considered in the parameters of the model.

15- The budget surplus of each period can be invested in a period with a risk-free interest rate.

16- It is possible to finance by selling projects.

17- The interest rate is constant during the time horizon.

3-2. Notation

In general, there are four groups of notations:

- indices
- parameters
- Binary Decision variables
- Non-negative decision variables

1- The indices are as follows:

i, j: project indices ($i, j = 1,2, \ldots, N$)

k: index of each phase of project ($k = 1,2, \ldots, K$)

m: mode index ($m = 0$ for running the project in normal mode, $m = 1$ for upgrading, and $m = 2$ for abandoning the project)

t: index of the period ($t = 0, \ldots., T$)

2- The parameters are as follows:

I: Set of all projects

H_i: The set of prerequisite projects of i ($H_i \subset I$)

E_i: set of incompatible projects with i ($E_i \subset I$, $E_i \cap H_i = \emptyset$)

T: Length of the horizon

f_i: number of phases in the project i

R_{ikm}: final income of phase k of project i in mode m ($i \in I$)

c_{ikm}: The initial cost of phase k of project i in mode m ($i \in I$)

d_{ikm}: time required to complete phase k of project i in mode m ($i \in I$)

SV_{ik}: The value of project i at the beginning of phase k

v: Starting budget at the beginning of the period ($t = 0$)

r: periodic interest rate

L: A very large number

3- Binary variables are:

$$x_{ikmt} = \begin{cases} 1 & \text{If phase } k \text{ of project } i \text{ starts in mode } m \text{ in period } t \\ 0 & \text{Otherwise} \end{cases}$$

$$y_i = \begin{cases} 1 & \text{If project } i \text{ is selected} \\ 0 & \text{Otherwise} \end{cases}$$

4- non-negative decision variables:

BS_t: budget surplus in period t

w_0: Initial budget

3-3. Mathematical model

$$Max\ Z = BS_T \qquad (1)$$

s.t:

$$\sum_{m=0}^{1}\sum_{t=0}^{T} tx_{i1mt} + L(1 - y_i) \geq \sum_{m=0}^{1}\sum_{t=0}^{T} x_{j.f_j.mt}\left(t + d_{j.f_j.m}\right) \qquad i = 1 \dots .N \quad \forall j \in H_i \qquad (2)$$

$$\sum_{m=0}^{1}\sum_{t=0}^{T}x_{i1mt} \leq L(1-\sum_{k=1}^{K}\sum_{t=0}^{T}x_{jk2t}) \qquad \begin{matrix} i = 1 \dots . N \\ \forall\, j \in H_i \end{matrix} \tag{3}$$

$$\sum_{m=0}^{1}\sum_{t=0}^{T}x_{i1mt} + \sum_{m=0}^{1}\sum_{t=0}^{T}x_{j1mt} \leq 1 \qquad \begin{matrix} i \in\ I \\ \forall\, j \in E_i \end{matrix} \tag{4}$$

$$\begin{aligned}\sum_{m=0}^{2}\sum_{t=0}^{T}tx_{ikmt} & \\ &\geq \sum_{m=0}^{1}\sum_{t=0}^{T}x_{i.k-1.mt}(t \\ &+ d_{i.k-1.m}) \\ &- \sum_{g=1}^{k-1}\sum_{t=0}^{T}x_{ig2t}\end{aligned} \qquad \begin{matrix} i = 1 \dots . N \\ k = 2.3 \dots . K \end{matrix} \tag{5}$$

$$\sum_{k=1}^{K}\sum_{t=1}^{T}x_{ik2t} \leq y_i \qquad i = 1 \dots . N \tag{6}$$

$$\sum_{m=0}^{2}\sum_{t=0}^{T}x_{ikmt} \leq y_i \qquad \begin{matrix} i = 1 \dots . N \\ k = 1 \dots . K \end{matrix} \tag{7}$$

$$\begin{aligned}\sum_{k=u+1}^{K}\sum_{m=0}^{2}\sum_{t=0}^{T}x_{ikmt} & \\ &\leq L(1-\sum_{t=0}^{T}x_{iu2t})\end{aligned} \qquad \begin{aligned} i &= 1 \dots . N \\ u & \\ &= \{0.1. \dots . K \\ &- 1\}\end{aligned} \tag{8}$$

$$\sum_{t=0}^{T}(x_{ik1t}) \geq \sum_{t=0}^{T} x_{iu1t} - L \sum_{t=0}^{T} \sum_{u'=1}^{u-1} x_{iu'2t} \qquad i = 1 \dots . N \quad \forall\, k \quad \forall\, u > k \tag{9}$$

$$\sum_{i=1}^{N} \sum_{m=1}^{2} x_{i1m0}\, c_{i1m} + BS_0 \leq w_0 \tag{10}$$

$$BS_{t-1}(1+r) + \sum_{i=1}^{N} \sum_{k=1}^{K} \sum_{m=0}^{1} x_{ikm.t-d_{ikm}+1} R_{ikm} + \sum_{i=1}^{N} \sum_{k=1}^{K} x_{ik2t} SV_{ik} = BS_t + \sum_{i=1}^{N} \sum_{k=1}^{K} \sum_{m=0}^{1} x_{ikmt} c_{ikm} \qquad \forall\, t \tag{11}$$

$$\sum_{k=1}^{K} \sum_{m=0}^{2} \sum_{t=0}^{T} x_{ikmt}(d_{ikm} + t) \leq T \qquad i = 1 \dots . N \tag{12}$$

$$w_0 \leq v \tag{13}$$

$$w_0 \geq 0 \tag{14}$$

$$BS_t \geq 0 \qquad \forall\, t \tag{15}$$

$$x_{ikmt} \cdot y_i \in \{0.1\} \qquad i = 1 \dots . N \qquad (16)$$

Equation 1: Objective Function

Equation 2: Ensuring prerequisites between projects. If project i is not selected, the constraint will be omitted.

Equation 3: If project j (prerequisite) is sold (not completed), project i will not be implemented.

Equation 4: Ensuring incompatibility of projects

Equation 5: Meeting the prerequisites between different phases of a project. If the project is abandoned, this constraint will be omitted.

Equation 6: Each project can be abandoned only once if ever selected.

Equation 7: If a project is not selected, the variables corresponding to the start of its different phases as well as the abandonment of the project will be zero.

Equation 8: If a project is abandoned in a certain phase, subsequent phases cannot be completed.

Equation 9: If a project is upgraded, subsequent phases will be implemented upgraded (if the project is not abandoned)

Equation 10: How to allocate the initial budget. In order to make the solution space feasible, “less than or equal to” is used instead of “equal”.

Equation 11: Budget balance equation in different periods

Equation 12: It ensures that the project is completed within the planned horizon.

Equation 13: Compliance with the upper limit of the available initial budget

Equation 14: The initial dedicated budget will not accept a negative value.

Equation 15: The budget surplus of each period will not accept a negative value.

Equation 16: Decision variables accept only values 0 and 1.

3-3-2. Developed mathematical model

Additional notation and constraint:

$TD(i,k)$: Total deviation from the due date of phase k of project i

$Due\_date(i,k)$: Due date for phase k of project i

$$TD(i,k) = \max\{0, \sum_{k=1}^{K}\sum_{m=0}^{1}(t+d_{i,f(i),m})\, x_{i,f(i),m,t} - Due\_date(i,k)\} \qquad \forall\, i$$

$$Min\ Z2 = \sum_{i=1}^{N}\frac{TD(i)}{N'}$$

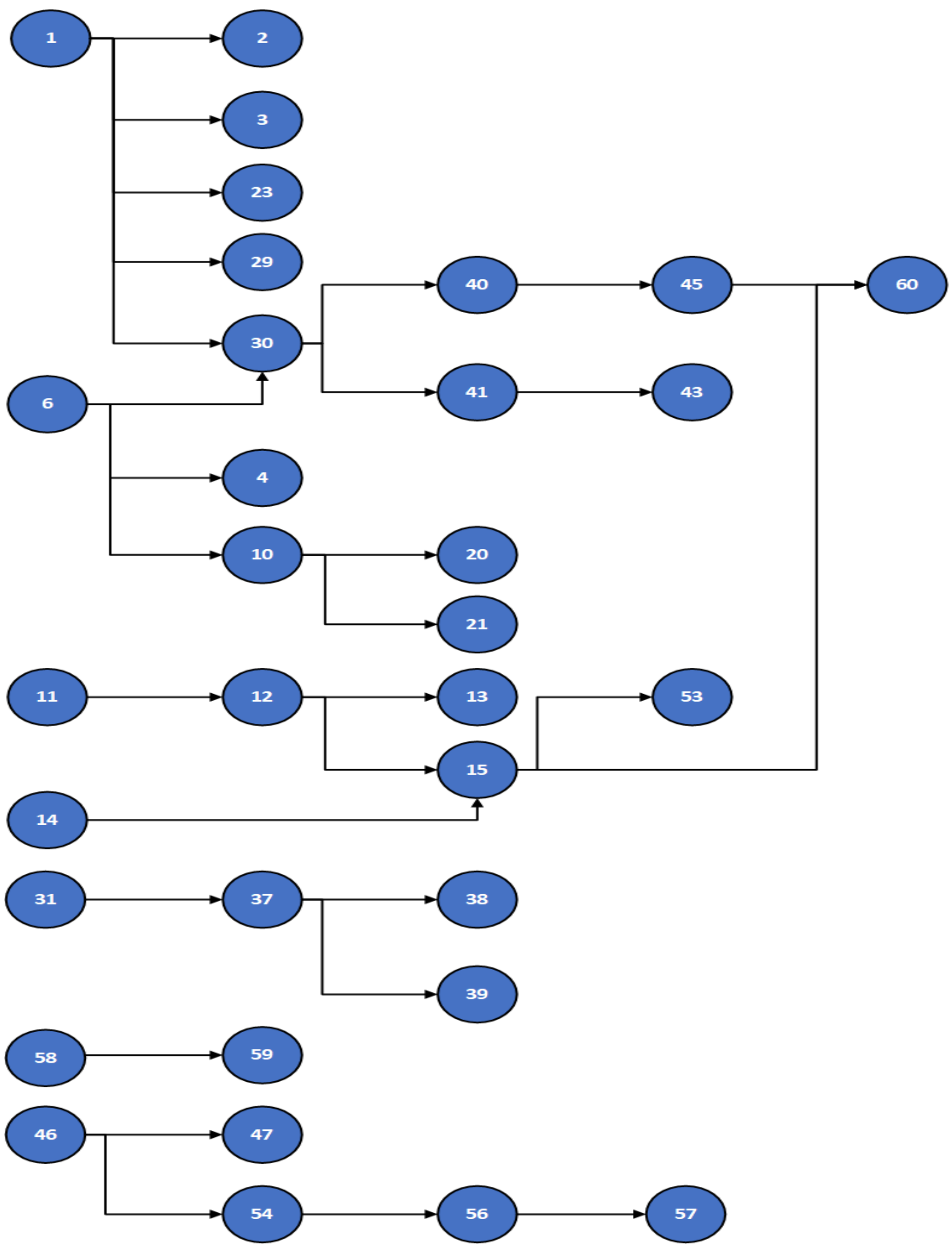

Figure 1- Prerequisite relationship between projects

Table 3- Characteristics of sensitivity analysis scenarios

possibility of sale	Possibility of upgrading	Normal execution	The name of the scenario	Scenario number
has it	has it	has it	Complete execution	1
has it	does not have	has it	Run without upgrades	2
does not have	does not have	has it	Execution without Adjustment	3

- As before, with the increase of the initial budget, the income also increases in all responses.
- The yield percentage decreases with the increase of interest rate in all responses.
- The amount of delay in all responses is constant as the initial budget changes.
- As the interest rate increases, the income, and percentage of income increase.
- As the interest rate increases, the percentage of the selected project and phases decrease.
- As the interest rate increases, the delay decreases because fewer projects are selected.