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Summary

The present study examines revenue management in the hotel business. In simple language, revenue management is the use of management strategies and mathematical methods and principles for pricing or capacity allocation.
For example, in the airline industry, decision makers do not make all the seats available to the public for reservation at once, but depending on the demand and price, they make a part of it depended on the period.

Revenue management first appeared in American airlines, and then its application spread to other industries, including the hotel industry. Similar to the airline industry, decision makers in hotels do not make all the rooms available at once and set different prices on the rooms depending on the type of room the time of reservation.

In addition, by using different forecasting methods, they also calculate the probability of passengers not showing up or canceling their requests, and in this way they manage to overbook and thus increase their income.
In short, the important topics that are raised in the revenue management in hotel industry are demand forecasting, pricing, how to allocate room capacity, using mathematical modeling and operational research to calculate how to price or allocate rooms. The models presented in this research are classified into two categories: capacity allocation models and pricing models.

1-3- Levers

Table 2- Comparison of revenue management levers

purpose	Lever	Row
Redefining the pricing strategy and formulating pricing monitoring tactics	Pricing	1
How to optimize or allocate capacity	Inventor	2
Price promotion	Marketing	3
The price is suitable for the customers of each channel	Communication channels	4

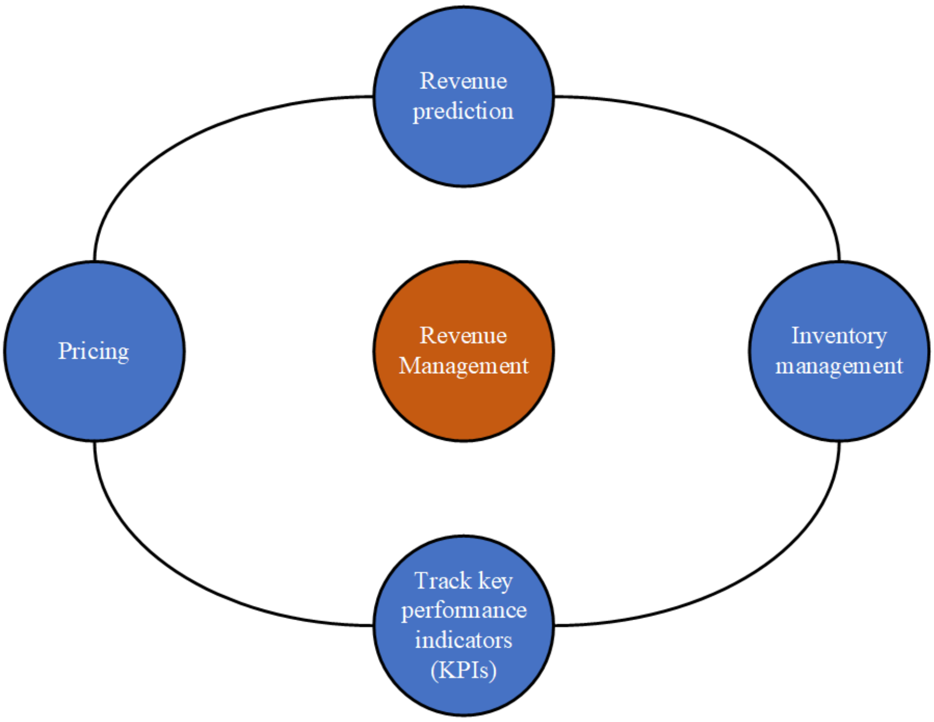

Figure 1- Elements involved in revenue management

1-4- Revenue management process

Table 3- Summary of revenue management processes

Description	Step	Row
The first step in a revenue management process Data collection to obtain appropriate information	Data collection	1
Segmentation of customers based on their responses to different prices	Segmentation	2
Anticipating and Quantitative display of influential elements including demand, available inventory, market share and total market	Anticipation	3
The pinnacle of the revenue management process Determining how the company responds to Anticipations	Optimization	4
Continuous evaluation and improvement	Dynamic reassessment	5

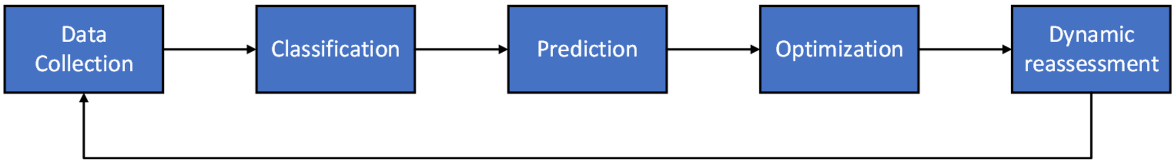

Figure 2 - Revenue management process

1-5- Revenue management in hotel business

Earning the best income from available resources is a vital goal for every production and service organization. One of the differences between a product and a service is that the product can be stored, while the service does not have such a feature.

Hotels are a service organization that every night, the room is considered as its output, and if proper scheduling and management is not done on this output, the hotel will lose a huge amount of income[2].

Today, revenue management is considered as a common method in the hotel industry and it helps managers in making decisions related to room prices, reservations, and allocation of demand for specific rooms.

Revenue management is defined as a technique that enables managers to sell each room (which is defined as a perishable asset) at the highest price and thus generate the highest level of revenue [3]. As mentioned before, the main elements of the revenue management process are:

- Data collection
- Classification
- Prediction
- Optimization
- Re-evaluation, which is also considered a kind of control.

In general, three groups of decisions have been identified in the field of income management, which are structural, price and quantity decisions [2].

2- The second chapter: revenue management models in hotel business

2-2-4-5- Proposed model of scheduling with reservation periods

Model inputs:

1- The price of the rooms
2- Number of rooms
3- The price of reservation periods
4- Reservation information
5- The possibility of canceling the request
6- The possibility of non-attendance
7- Possibility of early departure
8- The possibility of late departure

Therefore, the model will be as follows:

$$\begin{aligned}
\text{Max}\, z &= \sum_{s=1}^{S} p_s \sum_{h=1}^{H} \sum_{m=1}^{M} \sum_{i=1}^{T-1} \sum_{j=i+1}^{T} R_{i,j,m}^{h,s} \,(X_{ijm}^{h} \\
&- \sum_{e=i+1}^{j-1} E_{iej} + \sum_{v=i+1}^{j-1} V_{ijv}) \\
&- \lambda \sum_{s=1}^{S} p_s |\min \Bigg\{ 0, \sum_{h=1}^{H} \sum_{m=1}^{M} \sum_{i=1}^{T-1} \sum_{j=i+1}^{T} R_{i,j,m}^{h,s} \,(X_{ijm}^{h} \\
&- \sum_{e=i+1}^{j-1} E_{iej} + \sum_{v=i+1}^{j-1} V_{ijv}) \\
&- \sum_{s=1}^{S} p_s \sum_{h=1}^{H} \sum_{m=1}^{M} \sum_{i=1}^{T-1} \sum_{j=i+1}^{T} R_{i,j,m}^{h,s} \,(X_{ijm}^{h} \\
&- \sum_{e=i+1}^{j-1} E_{iej} + \sum_{v=i+1}^{j-1} V_{ijv}) \Bigg\} | \\
&- \sum_{s=1}^{S} p_s \sum_{h=1}^{H} \sum_{m=1}^{M} \sum_{i=1}^{T-1} \sum_{j=i+1}^{T} w_{i,j,m}^{h} \min \Bigg\{ 0, U_{i,j,m}^{h,s} \\
&- (X_{ijm}^{h} - \sum_{e=i+1}^{j-1} E_{iej} + \sum_{v=i+1}^{j-1} V_{ijv}) \Bigg\}
\end{aligned} \tag{60}$$

$S.t$:

$$\sum_{m=1}^{M} \sum_{j=1}^{T} (X_{ijm}^{h} - \sum_{e=i+1}^{j-1} E_{iej} + \sum_{v=i+1}^{j-1} V_{ivj}) \,(1 - p_a - p_b) \leq C^{h} \quad \forall k = 1, \ldots, K \tag{61}$$

$$\sum_{m=1}^{M}\sum_{i=1}^{k}\sum_{j=k+1}^{T}(X_{ijm}^{h}-\sum_{e=i+1}^{j-1}E_{iej}+\sum_{v=i+1}^{j-1}V_{ivj})(1-p_a-p_b)\leq C^h\quad\forall h=1,\dots,H;\ \forall k=1,\dots,T-1 \tag{62}$$

$$0\leq x_{i,j,m}^{h}(1-p_a-p_b)\leq \max\{U_{i,j,m}^{h,s}\},\ intege\ i=1,\dots,T-1;\ j=2,\dots,T;\ m=1,\dots,M;\ h=1,\dots,H;\ s=1,\dots,S \tag{63}$$

$$0\leq i<j\leq T \tag{64}$$

Indice and decision variables:

$R_{i,j,t}^{h}$: Income from room type h that $h=1,2,\dots,H$
To enter at time i that $i=1,\dots,T-1$
and departure at time j that $j=2,\dots,T$
and the reservation period m that $m=1,\dots,M$

$R_{i,j,t}^{h,s}$: Income under the scenario of s

$U_{i,j,m}^{h}$: Demand for room type h, which is check-in at time i and check-out at time j and booked in period m.

$U_{i,j,m}^{h,s}$: Demand for room type h, which is check-in at time i and check-out at time j and booked in period m under scenario s

C^h: Capacity (number) of rooms of type h

$x_{i,j,m}^{h}$: The number of $h-type$ room reservations that are entered at time i and exited at time j and have been booked in period m.

p_a: The possibility of canceling the request

p_b: Probability of not attending

Y_{ij}: Actual number of passengers arriving on day i and departing on day j.

E_{iej}: The number of passengers who planned to arrive on day i and leave on day j according to the original plan, but left earlier.
V_{ijv}: The number of passengers who planned to arrive on day i and leave on day j according to the original plan, but left later.

From the past formulas for early exit we have:

$$E_{iej} = \begin{cases} k_1 \times 0 & & j = i + 1 \\ k_2 \times p_1 \times x_{ij} & j = i + 2 & e = j - 1 \\ k_3 \times p_1 \times x_{ij} & j \geq i + 3 & e = j - 1 \\ k_4 \times p_2 \times x_{ij} & j \geq i + 3 & e = j - 2 \end{cases} \tag{65}$$

$$\sum_{k=1}^{K} k_i = 1 \quad , k = 0 \; or \; 1 \tag{66}$$

$$E_{iej} = \begin{cases} k_1 \times 0 & & j = T \\ k_2 \times p_1 \times x_{ij} & j = T - 1 & v = j + 1 \\ k_3 \times p_1 \times x_{ij} & j \leq T - 2 & v = j + 1 \\ k_4 \times p_2 \times x_{ij} & j \leq T - 2 & v = j + 2 \end{cases} \tag{67}$$

$$\sum_{k=1}^{K} k_i = 1 \quad , k = 0 \; or \; 1 \tag{68}$$

Therefore, the actual number of passengers arriving on day i and leaving on day j can be calculated as follows:

$$Y_{ijm}^{h} = X_{ijm}^{h} - \sum_{e=i+1}^{j-1} E_{iej} + \sum_{v=i+1}^{j-1} V_{ivj} \tag{69}$$

We already have:

$$-y_s = \min\left\{0, \sum_{h=1}^{H}\sum_{m=1}^{M}\sum_{i=1}^{T-1}\sum_{j=i+1}^{T} R_{i,j,m}^{h}\, x_{i,j,m}^{h} - \sum_{s=1}^{S} p_s \sum_{h=1}^{H}\sum_{m=1}^{M}\sum_{i=1}^{T-1}\sum_{j=i+1}^{T} R_{i,j,m}^{h,s}\, x_{i,j,m}^{h}\right\} \tag{70}$$

$$-z_{i,j,m}^{h,s} = \min\{0, U_{i,j,m}^{h,s} - x_{i,j,m}^{h}\} \tag{71}$$

And

$$y_s \geq 0\,, z_{i,j,m}^{h,s} \geq 0 \tag{72}$$

On the other hand, the issue that remains unclear is that in case of cancellation of the request or non-attendance of hotel guests, an amount must be returned to them. This amount is determined as a percentage of the amount paid.

For example, if the request is canceled, 80% of the amount will be returned to the passengers. To enter this number in the model, it is necessary to subtract the return amount from the average income in the objective function. We display the complement of this number from 100 percent, i.e. 20 percent, with g and it is a number between 0 and 1.

If no amount is returned for cancellation of reservation or non-attendance, this number is zero, and if the entire amount is returned, this number is 1.

$$Rx - [(p_a x + p_b x) \times Rg] = Rv. \tag{73}$$

The equation (73) shows the earned income. By simplifying it, we get the following equation:

$$Rx[1 - g(p_a + p_b)] = Rv. \tag{74}$$

The value of g can also be different depending on the type of reservation or the type of check-in and check-out. Also, the type of room is also influential in this issue, so this number can be shown as follows and assuming it is constant:

$g_{i,j,m}^{h}$

Since the actual number of requests (Y) also replaces the reservations (x) and by simplifying and linearizing the model as mentioned in the previous model, the proposed model will be as follows:

$$\begin{aligned}\text{Max}\, z &= \sum_{s=1}^{S} p_s \sum_{h=1}^{H}\sum_{m=1}^{M}\sum_{i=1}^{T-1}\sum_{j=i+1}^{T} [1 - g_{i,j,m}^{h}(p_a + p_b)] R_{i,j,m}^{h,s}\, Y_{i,j,m}^{h} \\ &- \lambda \sum_{s=1}^{S} p_s\, y_s \\ &- \sum_{s=1}^{S} p_s \sum_{h=1}^{H}\sum_{m=1}^{M}\sum_{i=1}^{T-1}\sum_{j=i+1}^{T} w_{i,j,m}^{h} z_{i,j,m}^{h,s}\end{aligned} \tag{75}$$

$S.t$:

$$\sum_{t=1}^{M}\sum_{j=1}^{T} Y_{1jm}^{h}\,(1 - p_a - p_b) \le C^h \quad \forall h = 1, \dots, H \tag{76}$$

$$\sum_{m=1}^{M}\sum_{i=1}^{k}\sum_{j=k+1}^{T} Y_{i,j,m}^{h}\,(1 - p_a - p_b) \le C^h \quad \forall h = 1, \dots, H; \quad \forall k = 1, \dots, T-1 \tag{77}$$

$$\sum_{h=1}^{H}\sum_{m=1}^{M}\sum_{i=1}^{T-1}\sum_{j=i+1}^{T} R_{i,j,m}^{h,s} Y_{i,j,m}^{h} - \sum_{s=1}^{S} p_s \sum_{h=1}^{H}\sum_{m=1}^{M}\sum_{i=1}^{T-1}\sum_{j=i+1}^{T} R_{i,j,m}^{h,s} Y_{i,j,m}^{h} + y_s \geq 0 \quad s = 1, \dots, S \tag{78}$$

$$y_s \geq 0 \quad s = 1, \dots, S$$

$$U_{i,j,m}^{h,s} - Y_{i,j,m}^{h} + z_{i,j,m}^{h,s} \geq 0 \quad i = 1, \dots, T-1;\ j = 2, \dots, T;\ m = 1, \dots, M; h = 1, \dots, H;\ s = 1, \dots, S \tag{79}$$

$$z_{i,j,m}^{h,s} \geq 0 \quad i = 1, \dots, T-1;\ j = 2, \dots, T;\ m = 1, \dots, M; h = 1, \dots, H;\ s = 1, \dots, S \tag{80}$$

$$0 \leq Y_{ijm}^{h}(1 - p_a - p_b) \leq \max\{U_{i,j,m}^{h,s}\}, \quad integer\ i = 1, \dots, T-1;\ j = 2, \dots, T;\ m = 1, \dots, M;\ h = 1, \dots, H;\ s = 1, \dots, S \tag{81}$$

$$0 \leq i < j \leq T \tag{82}$$

$$Constant\ g_{i,j,m}^{h} \in [0,1] \tag{83}$$

Table 11 - Summary of the final model with reservation periods

weakness	strength	Objective function	Decision variable	Purpose	category
Model complexity	close to reality Considering fines and deviation from demand or deviation from average income Considering cancellation of reservation and non-attendance Considering relocation at the time of departure	Maximize revenue	Allocation amount	How to allocate a room with possible demand	capacity Allocation

2-3- Pricing models

2-3-1- Dynamic pricing model

Dynamic pricing model assumptions:

In the presented model, staying in a hotel is determined by parameters a, l, and k. To formulate this model, it is necessary to define additional parameters and auxiliary variables, which will be explained further. There are several types of rooms represented by the k parameter.

$$
\begin{aligned}
& Max = \sum_{l=1}^{\max l} P_{l,K} \times O_{l,K} \\
& S.t: \\
& O_{l,k} \leq C_{l,k} \quad \forall l \\
& P_{l,k} \geq 0 \quad \forall l
\end{aligned}
\tag{84}
$$

The goal of this model is to maximize the total revenue of the hotel, therefore, its objective function is defined as a set of values obtained by multiplying the price of a specific night by the number of rooms reserved for the same night.

In addition, the only limitation of this model is that the total number of reservations made in each night should not be more than the capacity of the hotel in the same night.
In the following, the model of dynamic income management is explained.

Indice and decision variables:
1- allocated prices found at the end of each night for room type k:

$$P_{l,k}$$

2- The auxiliary variable in this model is also calculated as follows:
The number of rooms that are assigned to reservation requests and the type related to each class:

$$X_{a,L,k}$$

$$X_{a,L,k} = d_{a,L,k}\left(\frac{\sum_{l=a}^{a+L-1}\sum_{k=1}^{K} P_{l,k}}{L \times P_{nom}}\right)^{e} \tag{85}$$

3- The number of rooms reserved for the end of each night:

$$O_{l,k}$$

$$O_{l,k} = \sum_{a,l,k \epsilon N_l} X_{a,L,k} \tag{86}$$

4- The nominal price which is often equal to the average price of the rooms.

$$P_{nom}$$

This price can be extracted from the hotel records.
5- Price elasticity of demand:

$$e$$

The price elasticity of demand measures the percentage change in the demand in a specific time period relative to the percentage change in the price. Since price and demand have an inverse relationship, the price elasticity coefficient of demand will be a negative number.

According to the law of demand, when the price increases, the amount of demand will decrease, so that price and demand move in the opposite direction and their relationship is generally inverse. Therefore, price elasticity of demand is generally considered as a negative indicator.

In economics, for ease of work, its negative sign is ignored and its absolute value is used in calculations, or its sign is changed by placing a negative sign at the beginning of the price elasticity of demand formula.

The formula for calculating the price elasticity of demand is as follows, where ΔQ is the change in demand and ΔP is the change in price:

$$e = -\frac{\frac{\Delta Q}{Q}}{\frac{\Delta P}{P}} \tag{87}$$

6- Hotel capacity at the end of each night

$$C_{l,k}$$

The output of this model is the optimal prices for each night to determine the appropriate pricing policy for the hotel. The point worth mentioning is that the prices obtained in this model can be considered continuously and used similarly in many online reservation systems.

In addition, in order to prevent customer dissatisfaction due to price fluctuations, the revenue management system can only provide the average price of the rooms.

Table 12 summarizes the dynamic pricing model.

Table 12 – Summary of dynamic pricing model

Weakness	Strength	Objective Function	Decision Variable	Purpose	Category
Increasing parameters and becoming more complex	Considering the price elasticity of demand	Maximize revenue	Allocated prices at the end of each night	Determine the optimal prices for each night	Pricing models

Conclusion and suggestion

The models presented in this research can be summarized as follows:

1- Capacity allocation models which include classic model, basic model and model with reservation periods.
2- Dynamic pricing model

The classic model is one of the practical capacity allocation models, which, despite its simplicity, has good capabilities to maximize income due to the consideration of room type, arrival day, and length of stay. This model can be used in all types of hotels and its simplicity makes it efficient.
Basic scheduling models, take into account the day of arrival and departure, so there is more freedom for the decision makers They increase the use of the model.
The advantage that the classic model has over this model is that it takes into acount the length of stay instead of the day of departure, and this, in addition to being more simple, makes sccheduling easier.

Sccheduling models with reservation periods are the most complicated types of models presented. These models consider various factors:

- room type
- day of arrival
- departure day
- Reservation registration period
- money back
- Possible demand
- Penalty for passing demand
- risk aversion
- Other factors, all of which make sccheduling difficult and complicated, but the model is very close to reality and its efficiency increases.

This type of model is mostly used by big hotels.
Finally, the dynamic pricing model, considering the price elasticity of demand, makes the model more practical and sets a reasonable limit for pricing.

In this research, an effort was made to examine all possible factors in all possible cases, and the presented models have the possibility of being used in the hotel industry, but the research about revenue management in the hotel field is still debatable. Also, many algorithms can be considered to solve its problems.

In the following, the models related to capacity allocation are summarized and compared in tables.

Table 13 - Comparison of capacity allocation models

Disadvantage	Advantage	Application	Model Name	Row
Not considering all factors	Simplicity	Small to large hotels	Classic model	1
Not considering all factors	Simplicity	Small to large hotels	Basic sccheduling model	2
Very complex	Considering all aspects	Usually medium to large Hotels	Final Scheduling model with reservation periods	3

Table 14 - Comparison of basic sccheduling models

More assumptions	Application	Model Name	Row
The date of arrival and departure of the passenger	Small to large hotels	simple basic sccheduling model	1
-The date of arrival and departure of the passenger - room type	Small to large hotels	Basic sccheduling model with Room Type	2

Table 15 - Comparison of sccheduling models with reservation periods

Advantages	Application	Model Name	Row
The date of arrival and departure of the passenger room type Reservation period	Usually medium to large	Scheduling model with reservation periods with definite demand simple model	1

The date of arrival and departure of the passenger room type Reservation period The possibility of cancellation of the application or non-attendance	Usually medium to large	Scheduling model with reservation periods with definite demand Considering the possibility of non-attendance and canceling the application	2
The date of arrival and departure of the passenger room type Early or late departure	Usually medium to large	Scheduling model with reservation periods with definite demand Consider leaving early or late	3
The date of arrival and departure of the passenger room type risk taking Penalty weight Deviation from average income Deviation from demand	Usually medium to large	Scheduling model with reservation periods with probabilistic demand Basic model	4
The date of arrival and departure of the passenger room type risk taking Penalty weight Deviation from average income Deviation from demand Possibility of cancellation of reservation or non-attendance Early or late departure	Usually medium to large	Scheduling model with reservation periods with probabilistic demand The final model	5

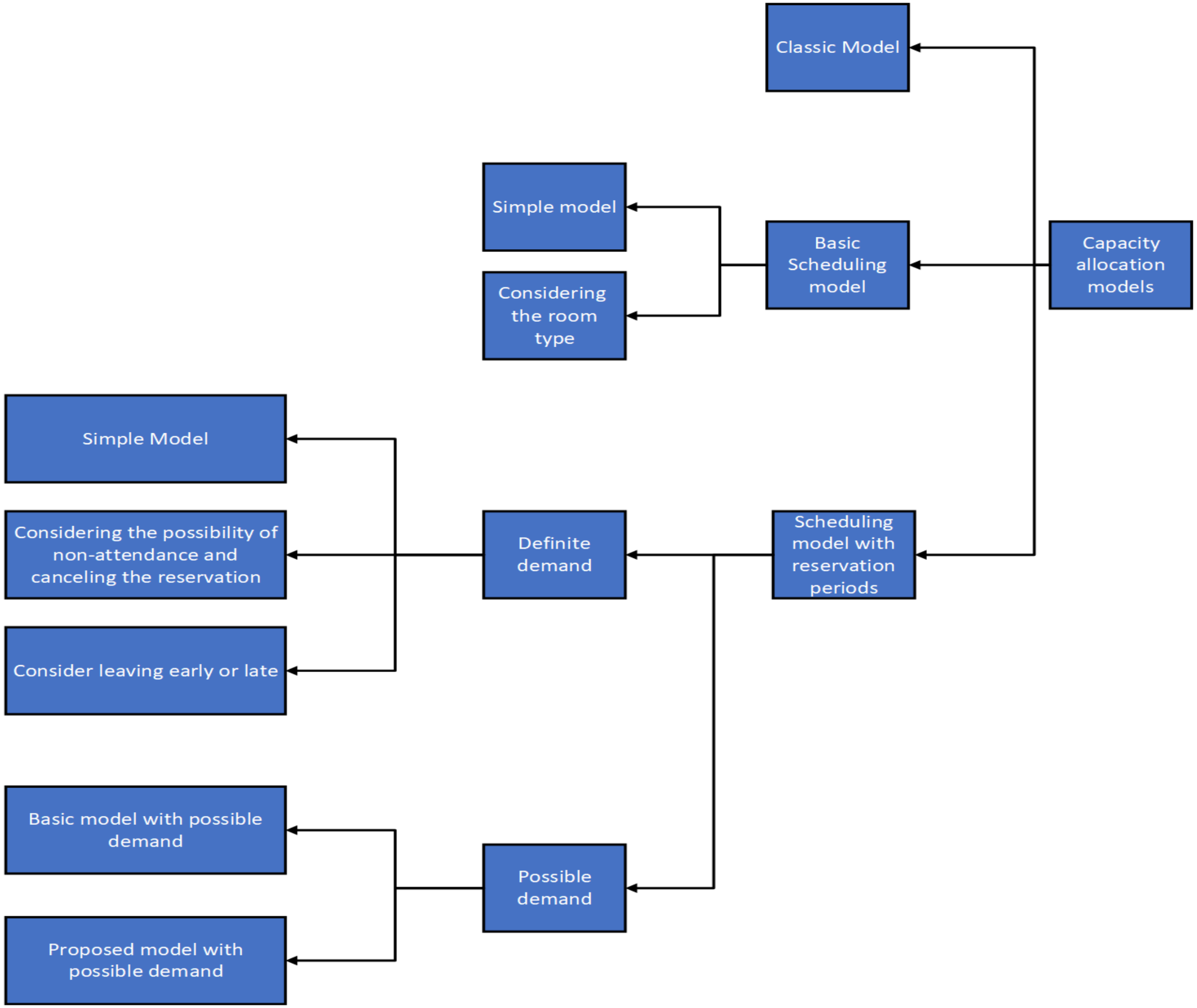

Figure 6 – Types of revenue management capacity allocation models

It is suggested to present different scenarios for pricing in future researches. This means that the price should be variable like the demand so that the model is closer to the reality because the pricing can change depending on different conditions.

Also, when it is not possible to accept passengers due to overbooking, in addition to the financial penalties that are deducted from the income, other costs such as damage to the hotel's brand and other such factors can also be considered.