جدول انتگرال ها

انتگرال های پایه ای

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1 \tag{1}$$

$$\int \frac{1}{x} dx = \ln \left|x\right| \tag{2}$$

$$\int udv = uv - \int vdu \tag{3}$$

$$\int \frac{1}{ax+b} dx = \frac{1}{a} \ln \left|ax+b\right| \Big| \tag{4}$$

انتگرال های کسری

$$\int \frac{1}{(x+a)^2} dx = -\frac{1}{x+a} \tag{5}$$

$$\int (x+a)^n dx = \frac{(x+a)^{n+1}}{n+1}, n \neq -1 \tag{6}$$

$$\int x(x+a)^n dx = \frac{(x+a)^{n+1}((n+1)x-a)}{(n+1)(n+2)} \tag{7}$$

$$\int \frac{1}{1+x^2} dx = \tan^{-1} x \tag{8}$$

$$\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} \tag{9}$$

$$\int \frac{x}{a^2+x^2} dx = \frac{1}{2} \ln \left|a^2+x^2\right| \tag{10}$$

$$\int \frac{x^2}{} dx = x - a \tan^{-1} \frac{x}{} \tag{11}$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a}$$

$$\int \frac{x^3}{a^2 + x^2}\, dx = \frac{1}{2} x^2 - \frac{1}{2} a^2 \ln \left| a^2 + x^2 \right| \tag{12}$$

$$\int \frac{1}{ax^2 + bx + c}\, dx = \frac{2}{\sqrt{4ac - b^2}} \tan^{-1} \frac{2ax + b}{\sqrt{4ac - b^2}} \tag{13}$$

$$\int \frac{1}{(x + a)(x + b)}\, dx = \frac{1}{b - a} \ln \frac{a + x}{b + x}\ ,\ a \neq b \tag{14}$$

$$\int \frac{x}{(x + a)^2}\, dx = \frac{a}{a + x} + \ln \left| a + x \right| \tag{15}$$

$$\int \frac{x}{ax^2 + bx + c}\, dx = \frac{1}{2a} \ln \left| ax^2 + bx + c \right| - \frac{b}{a\sqrt{4ac - b^2}} \tan^{-1} \frac{2ax + b}{\sqrt{4ac - b^2}} \tag{16}$$

انتگرال های رادیکالی

$$\int \sqrt{x - a}\ \ dx = \frac{2}{3} (x - a)^{3/2} \tag{17}$$

$$\int \frac{1}{\sqrt{x \pm a}}\ \ dx = 2\sqrt{x \pm a} \tag{18}$$

$$\int \frac{1}{\sqrt{a - x}}\ \ dx = -2\sqrt{a - x} \tag{19}$$

$$\int x\sqrt{x - a}\ \ dx = \begin{cases} \frac{2a}{3} (x - a)^{3/2} + \frac{2}{5} (x - a)^{5/2}\ , \text{ or} \\ \frac{2}{3} x(x - a)^{3/2} - \frac{4}{15} (x - a)^{5/2}\ , \text{ or} \\ \frac{2}{15} (2a + 3x)(x - a)^{3/2} \end{cases} \tag{20}$$

$$\int \sqrt{ax + b}\ \ dx = (\frac{2b}{3a} + \frac{2x}{3})\sqrt{ax + b} \tag{21}$$

$$\int (ax+b)^{3/2}\,dx = \frac{2}{5a}(ax+b)^{5/2} \tag{22}$$

$$\int \frac{x}{\sqrt{x \pm a}}\,dx = \frac{2}{3}(x \mp 2a)\sqrt{x \pm a} \tag{23}$$

$$\int \sqrt{\frac{x}{a-x}}\,dx = -\sqrt{x(a-x)} - a\tan^{-1}\frac{\sqrt{x(a-x)}}{x-a} \tag{24}$$

$$\int \sqrt{\frac{x}{a+x}}\,dx = \sqrt{x(a+x)} - a\ln\left[\sqrt{x} + \sqrt{x+a}\right] \tag{25}$$

$$\int x\sqrt{ax+b}\,dx = \frac{2}{15a^2}(-2b^2 + abx + 3a^2x^2)\sqrt{ax+b} \tag{26}$$

$$\int \sqrt{x(ax+b)}\,dx = \frac{1}{4a^{3/2}}\left[(2ax+b)\sqrt{ax(ax+b)} - b^2\ln\left|a\sqrt{x} + \sqrt{a(ax+b)}\right|\right] \tag{27}$$

$$\int \sqrt{x^3(ax+b)}\,dx = \left[\frac{b}{12a} - \frac{b^2}{8a^2x} + \frac{x}{3}\right]\sqrt{x^3(ax+b)} + \frac{b^3}{8a^{5/2}}\ln\left|a\sqrt{x} + \sqrt{a(ax+b)}\right| \tag{28}$$

$$\int \sqrt{x^2 \pm a^2}\,dx = \frac{1}{2}x\sqrt{x^2 \pm a^2} \pm \frac{1}{2}a^2\ln\left|x + \sqrt{x^2 \pm a^2}\right| \tag{29}$$

$$\int \sqrt{a^2 - x^2}\,dx = \frac{1}{2}x\sqrt{a^2 - x^2} + \frac{1}{2}a^2\tan^{-1}\frac{x}{\sqrt{a^2 - x^2}} \tag{30}$$

$$\int x\sqrt{x^2 \pm a^2}\,dx = \frac{1}{3}(x^2 \pm a^2)^{3/2} \tag{31}$$

$$\int \frac{1}{\sqrt{x^2 \pm a^2}}\,dx = \ln\left|x + \sqrt{x^2 \pm a^2}\right| \tag{32}$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\frac{x}{a} \tag{33}$$

$$\int \frac{x}{\quad}\,dx = \sqrt{x^2 \pm a^2} \tag{34}$$

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} \qquad (34)$$

$$\int \frac{x}{\sqrt{a^2 - x^2}}\, dx = -\sqrt{a^2 - x^2} \qquad (35)$$

$$\int \frac{x^2}{\sqrt{x^2 \pm a^2}}\, dx = \frac{1}{2} x \sqrt{x^2 \pm a^2} \mp \frac{1}{2} a^2 \ln \left| x + \sqrt{x^2 \pm a^2} \right| \qquad (36)$$

$$\int \sqrt{ax^2 + bx + c}\, dx = \frac{b + 2ax}{4a} \sqrt{ax^2 + bx + c} + \frac{4ac - b^2}{8a^{3/2}} \ln \left| 2ax + b + 2\sqrt{a(ax^2 + bx^+ c)} \right| \qquad (37)$$

$$\int x\sqrt{ax^2 + bx + c}\, dx = \frac{1}{48a^{5/2}} \left(2\sqrt{a}\sqrt{ax^2 + bx + c}\, (-3b^2 + 2abx + 8a(c + ax^2)) + 3(b^3 - 4abc) \ln \left| b + 2ax + 2\sqrt{a}\sqrt{ax^2 + bx + c} \right| \right) \qquad (38)$$

$$\int \frac{1}{\sqrt{ax^2 + bx + c}}\, dx = \frac{1}{\sqrt{a}} \ln \left| 2ax + b + 2\sqrt{a(ax^2 + bx + c)} \right| \qquad (39)$$

$$\int \frac{x}{\sqrt{ax^2 + bx + c}}\, dx = \frac{1}{a} \sqrt{ax^2 + bx + c} - \frac{b}{2a^{3/2}} \ln \left| 2ax + b + 2\sqrt{a(ax^2 + bx + c)} \right| \qquad (40)$$

$$\int \frac{dx}{(a^2 + x^2)^{3/2}} = \frac{x}{a^2 \sqrt{a^2 + x^2}} \qquad (41)$$

انتگرا لهای لگاریتمی

$$\int \ln ax\, dx = x \ln ax - x \qquad (42)$$

$$\int x \ln x\, dx = \frac{1}{2} x^2 \ln x - \frac{x^2}{4} \qquad (43)$$

$$\int x^2 \ln x\, dx = \frac{1}{3} x^3 \ln x - \frac{x^3}{9} \qquad (44)$$

$$\int x^n \ln x\, dx = x^{n+1} \left(\frac{\ln x}{\quad} - \frac{1}{\quad} \right), \quad n \neq -1 \qquad (45)$$

$$\frac{}{n+1} \qquad (n+1)^2$$

$$\int \frac{\ln ax}{x}\, dx = \frac{1}{2}(\ln ax)^2 \tag{46}$$

$$\int \frac{\ln x}{x^2}\, dx = -\frac{1}{x} - \frac{\ln x}{x} \tag{47}$$

$$\int \ln(ax+b)\, dx = (x + \frac{b}{a}) \ln(ax+b) - x, a \neq 0 \tag{48}$$

$$\int \ln(x^2+a^2)\, dx = x \ln(x^2+a^2) + 2a \tan^{-1} \frac{x}{a} - 2x \tag{49}$$

$$\int \ln(x^2-a^2)\, dx = x \ln(x^2-a^2) + a \ln \frac{x+a}{x-a} - 2x \tag{50}$$

$$\int \ln(ax^2+bx+c)\, dx = \frac{1}{a} \sqrt{4ac-b^2}\, \tan^{-1} \frac{2ax+b}{\sqrt{4ac-b^2}} - 2x + (\frac{b}{2a} + x) \ln(ax^2+bx+c) \tag{51}$$

$$\int x \ln(ax+b)\, dx = \frac{bx}{2a} - \frac{1}{4}x^2 + \frac{1}{2}(x^2 - \frac{b^2}{a^2}) \ln(ax+b) \tag{52}$$

$$\int x \ln(a^2-b^2x^2)\, dx = -\frac{1}{2}x^2 + \frac{1}{2}(x^2 - \frac{a^2}{b^2}) \ln(a^2-b^2x^2) \tag{53}$$

$$\int (\ln x)^2\, dx = 2x - 2x \ln x + x(\ln x)^2 \tag{54}$$

$$\int (\ln x)^3\, dx = -6x + x(\ln x)^3 - 3x(\ln x)^2 + 6x \ln x \tag{55}$$

$$\int x(\ln x)^2\, dx = \frac{x^2}{4} + \frac{1}{2}x^2(\ln x)^2 - \frac{1}{2}x^2 \ln x$$