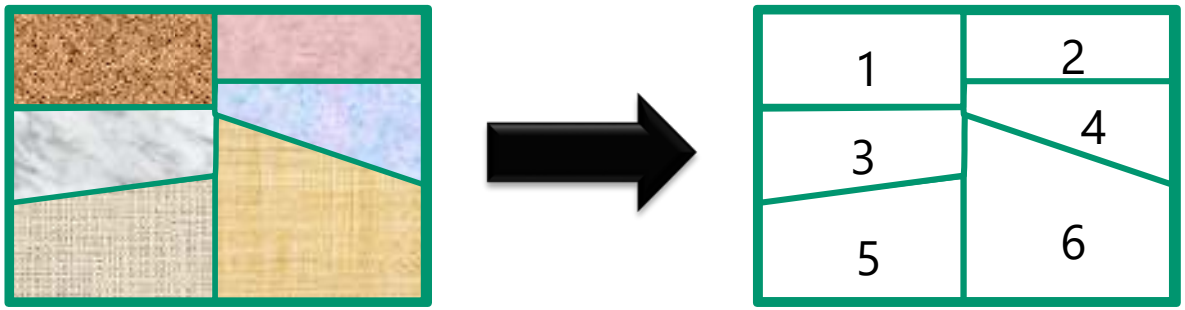
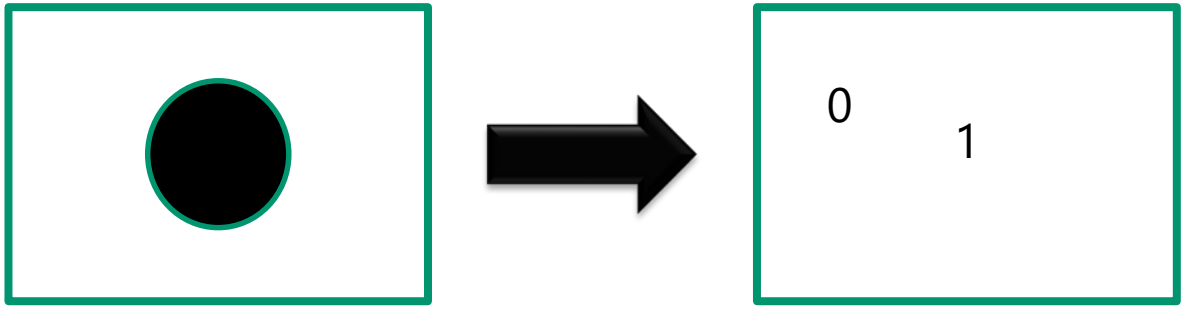
Segmentation

Segmentation



Foreground
+
Background



Binary Image

Segmentation

- Intensity - based
- Similarity - based → مثلاً دیتای مدرن
- Discontinuity (dissimilarity) - based

روی مرز

ناپیوستگی

Segmentation

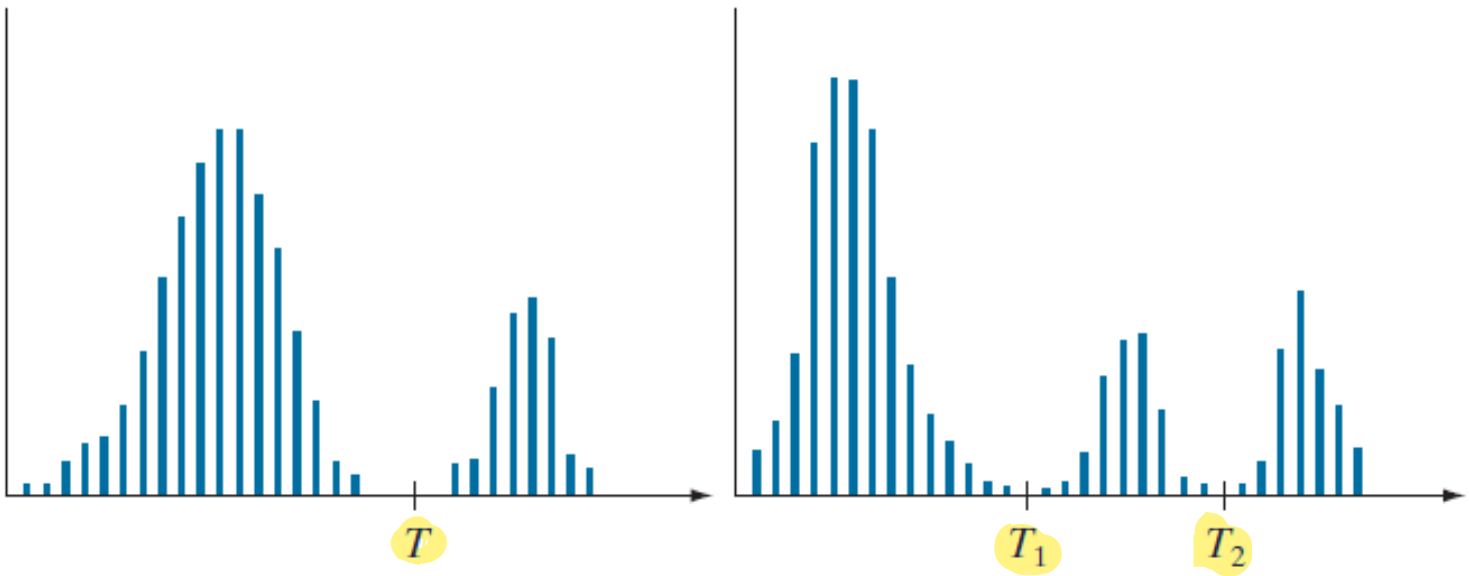
INTENSITY – BASED SEGMENTATION

Background – Foreground:

$$I \longrightarrow J_{Binary}$$

$$J_{Binary}(x,y) = \begin{cases} 1 & \text{if } I(x,y) > T \\ 0 & \text{if } I(x,y) \leq T \end{cases}$$

Segmentation: Histogram-based methods



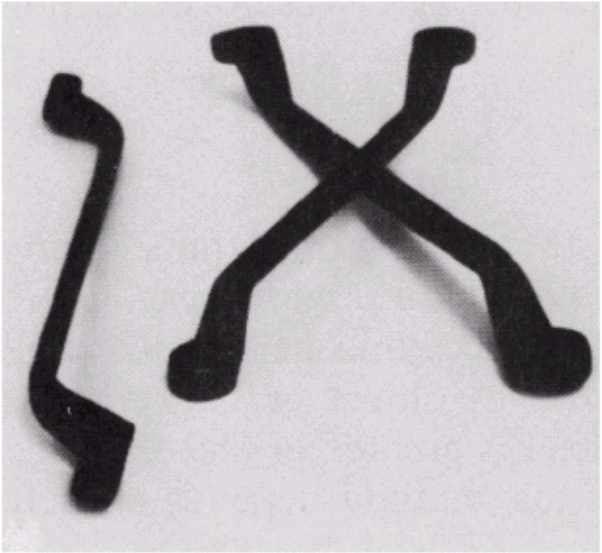
By single threshold

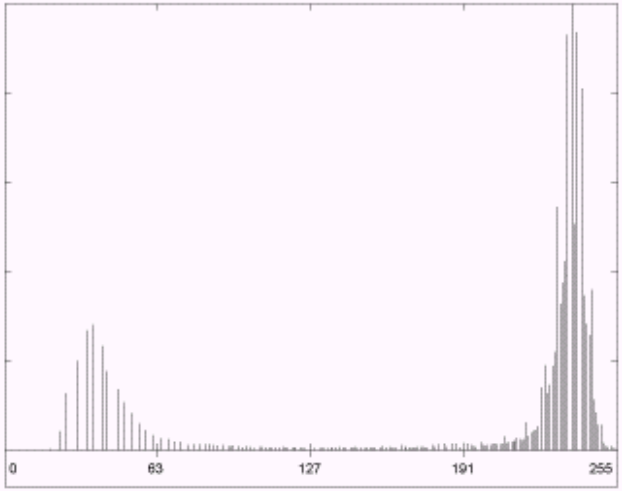
By multiple thresholds

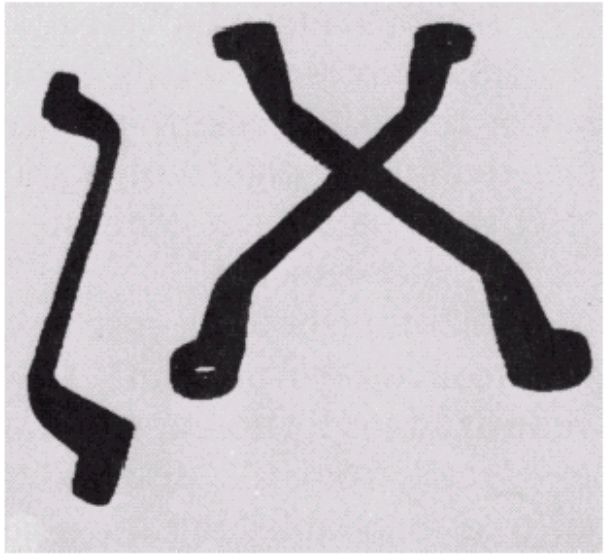
حالت باینری

دو یا چند تا تشخیص می‌دهیم

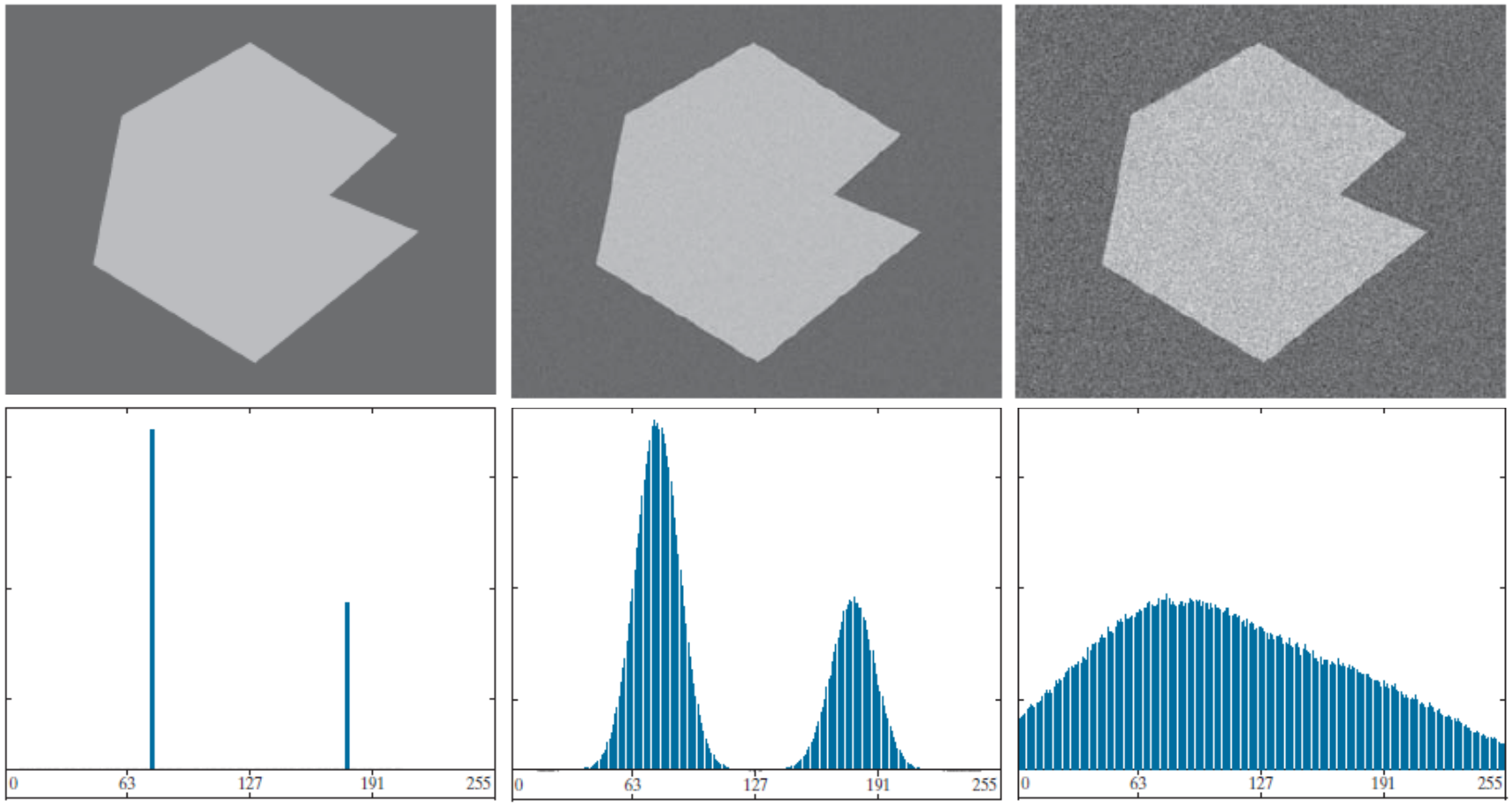
Manual Global Thresholding





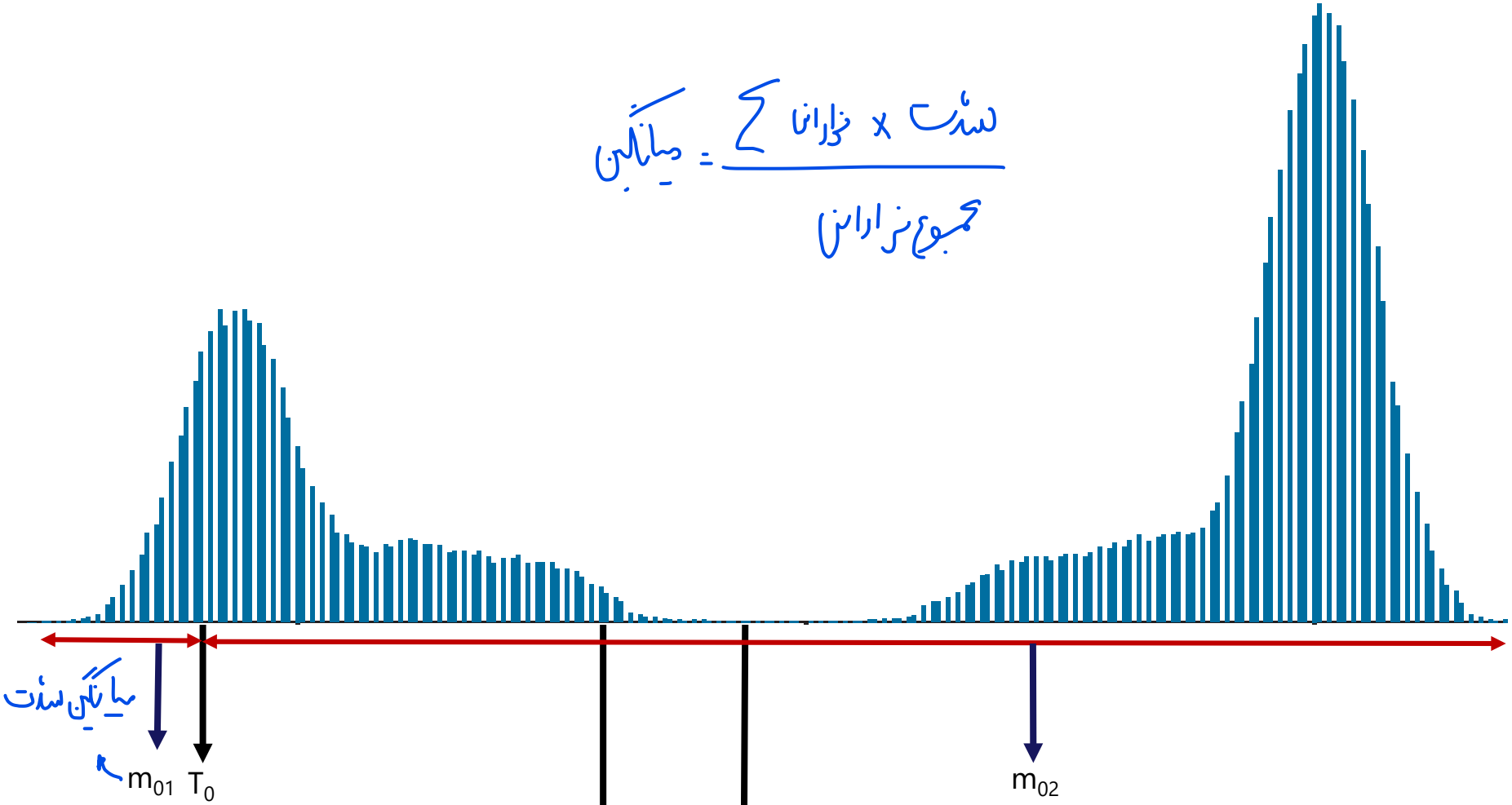


Noise in thresholding



Automatic Global thresholding

میانگین = $\frac{\sum \text{شدت} \times \text{فراوانی}}{\text{مجموع فراوانی}}$



$T_2 = \frac{m_{11}+m_{12}}{2}$

$\ldots T_{i+1} = \frac{1}{2}(m_{i1} + m_{i2})$

Automatic Global thresholding

K-means clustering:

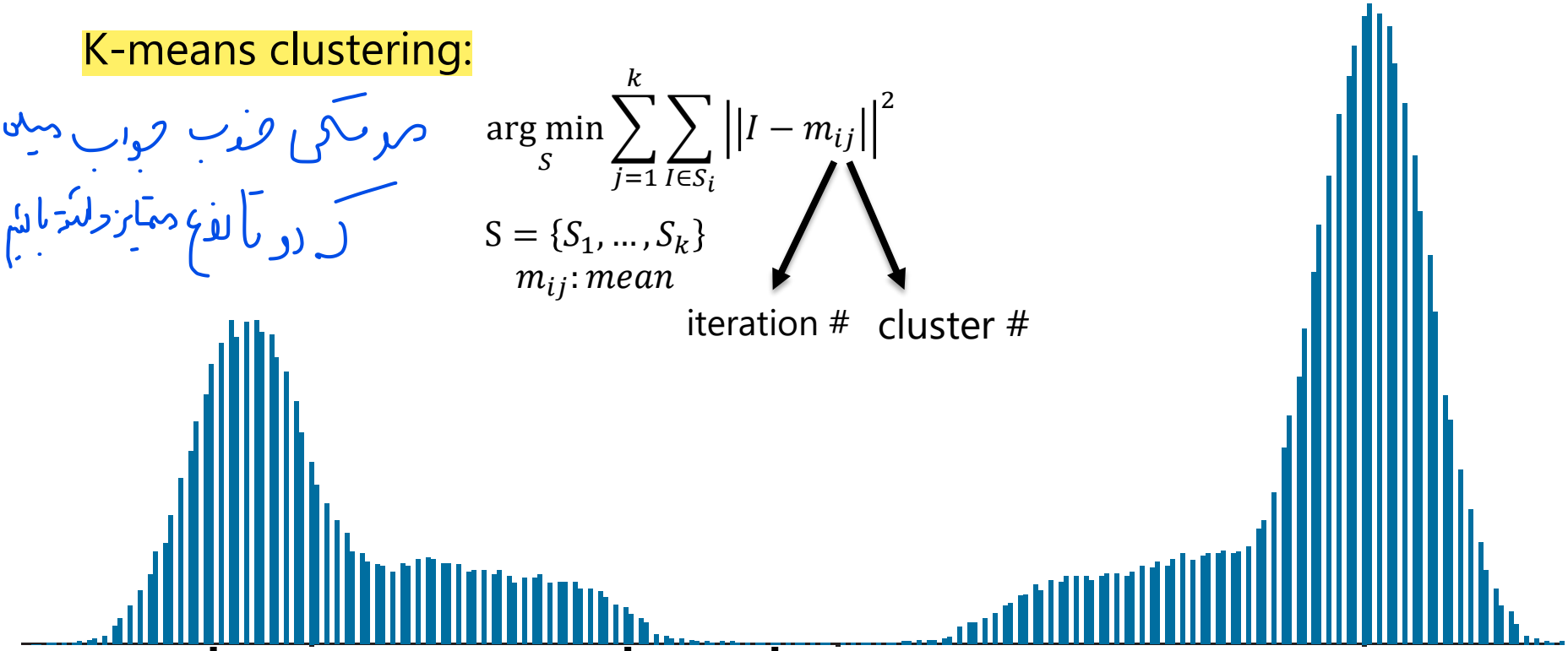
صورتی خوب جواب میده
که دو تا لوب متمایز داشته باشیم

$$\arg\min_{S} \sum_{j=1}^{k} \sum_{I \in S_i} \left\| I - m_{ij} \right\|^2$$

$S = \{S_1, \dots, S_k\}$

m_{ij}: *mean*

iteration # cluster #



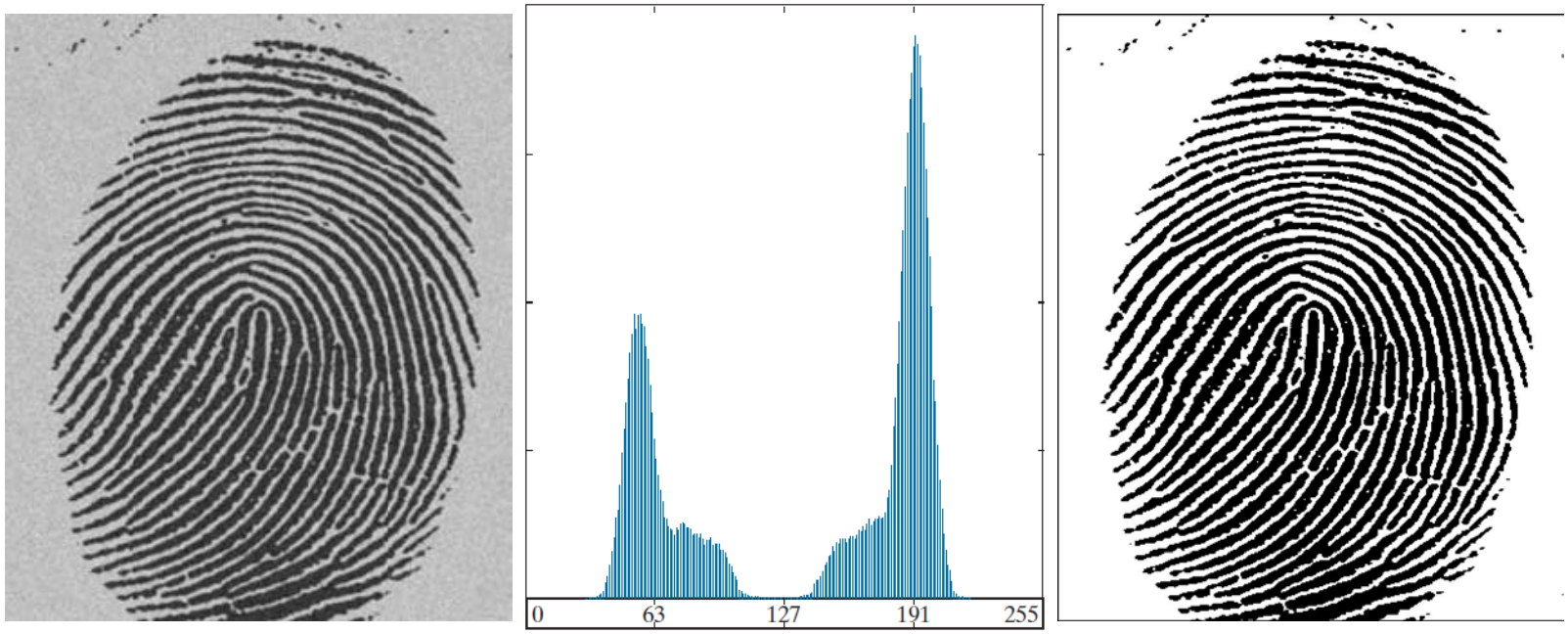
m_{01} T_0 m_{02}

m_{11} $T_1 = \frac{m_{01}+m_{02}}{2}$ m_{12}

$T_2 = \frac{m_{11}+m_{12}}{2}$

$\dots T_{i+1} = \frac{1}{2}(m_{i1} + m_{i2})$

Automatic Global thresholding



Automatic Global thresholding

میانگین قبل ترشولد

Otsu's method

احتمال رخداد شدت های قبل ترشولد

Maximize:

the between-class variance: $\sigma_B^2 = P_1(m_1 - m_G)^2 + P_2(m_2 - m_G)^2$

بالاتر بهتر

میانگین کل تصویر (شدت)

$$P_1(k) = \sum_{i=0}^{k} p_i$$

$$\sigma_B^2(k) = \frac{[m_G P_1(k) - m(k)]^2}{P_1(k)[1 - P_1(k)]}$$

$$P_2(k) = \sum_{i=k+1}^{L-1} p_i = 1 - P_1(k)$$

$$m(k) = \sum_{i=0}^{k} i p_i$$

$$p_i = {}^{n_i}/_{MN}$$

$$m_G = \sum_{i=0}^{L-1} i p_i$$

$$\sigma_B^2(k^*) = \max_{0 \le k \le L-1} \sigma_B^2(k)$$

بهتر

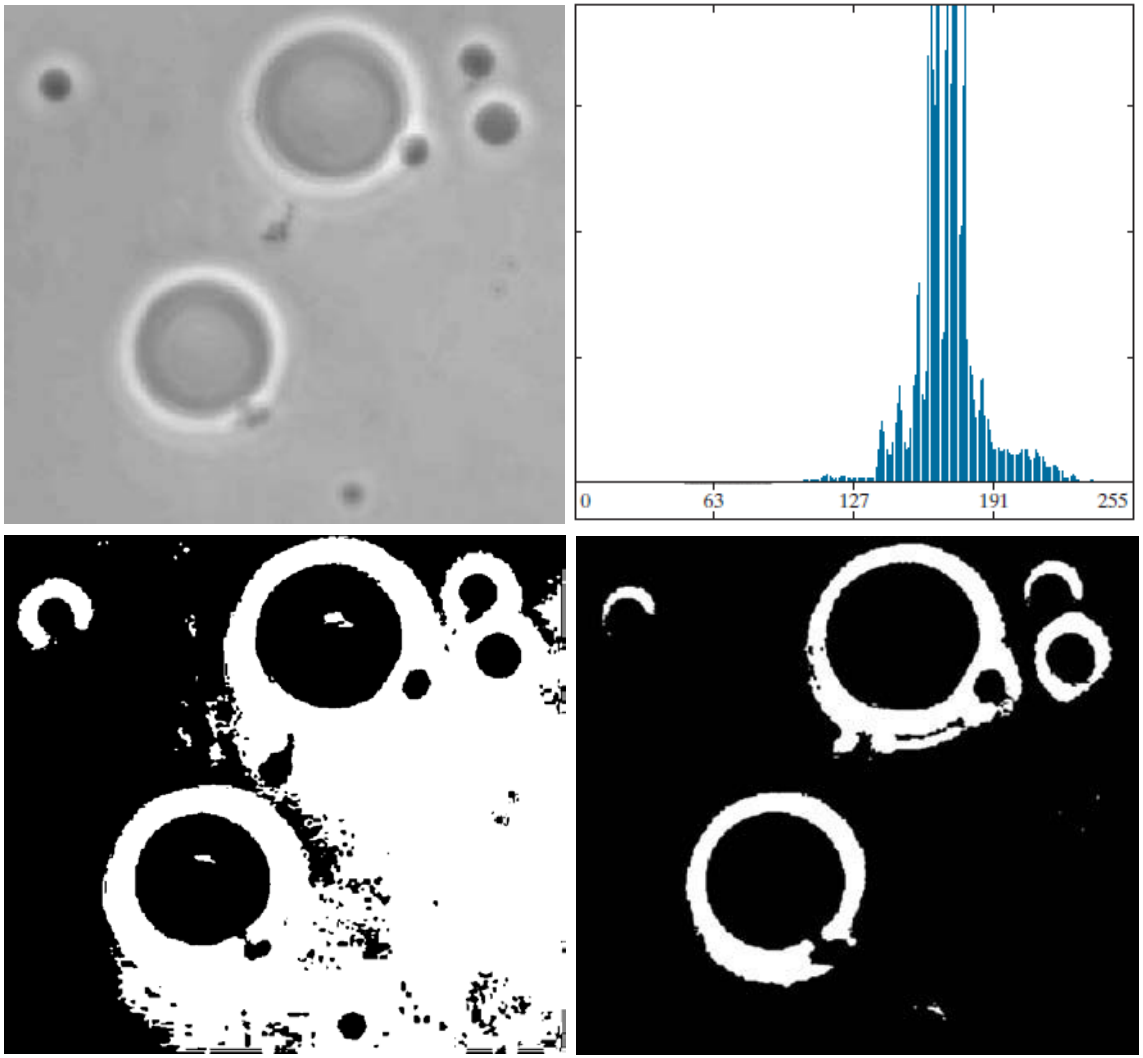
ارزیابی K و ترشولدی که زدیم.

$$\eta(k) = \frac{\sigma_B^2(k)}{\sigma_G^2}$$

بهتر

$$0 \le \eta(k^*) \le 1$$

Automatic Global thresholding
Otsu's method



Basic global Thresholding: 169

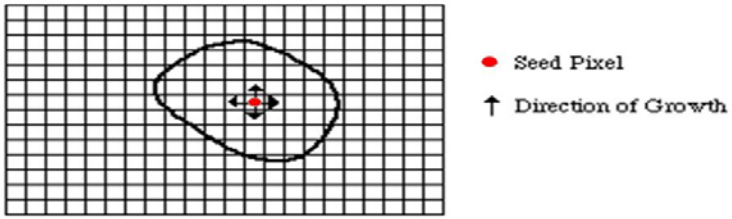
Otsu: 181

Segmentation

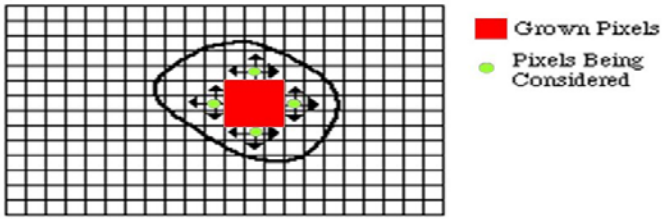
نواحی مشترک

SIMILARITY – BASED SEGMENTATION

Region growing:



(a) Start of Growing a Region



(b) Growing Process After a Few Iterations

Similar pixels join the seed

seed

By user Automatic

$q_2 \rightarrow q_1 \rightarrow p$

transitive

seed p $\underset{?\ \text{similar}}{\overset{\text{neighbor}}{\rightleftarrows}}$ q_1 $\underset{?\ \text{similar}}{\overset{\text{neighbor}}{\rightleftarrows}}$ q_2

(p, q_1) statistics

? similar

Multiple seeds

$p_1, p_2, \ldots, p_n$

statistics

Segmentation

SIMILARITY – BASED SEGMENTATION

measure

Similarity $S(p, q) > T$

Or

Dissimilarity $\mathrm{D}(p, q) < T$

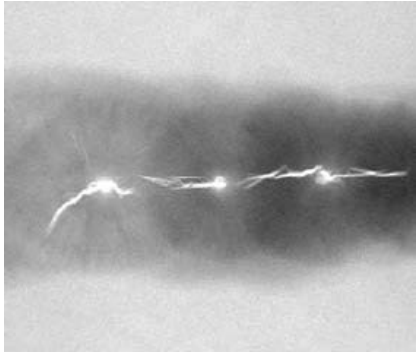
ex: $\left|I_p - I_q\right| < T$

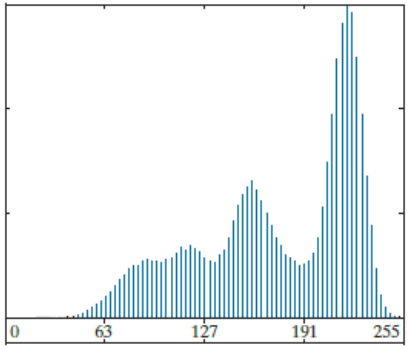
Region growing

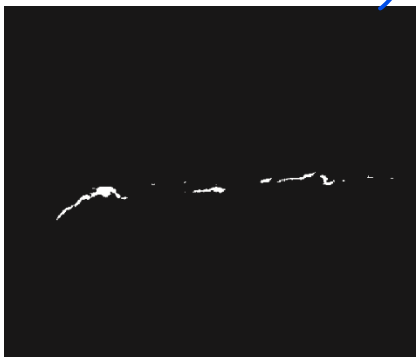
$$Q = \begin{cases} \text{TRUE} & \text{if the absolute difference of the intensities between the seed and the pixel at } (x, y) \text{ is } \leq T \\ \text{FALSE} & \text{otherwise} \end{cases}$$

pixel threshold

Defective welds

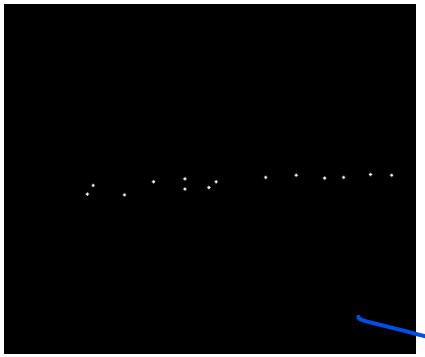


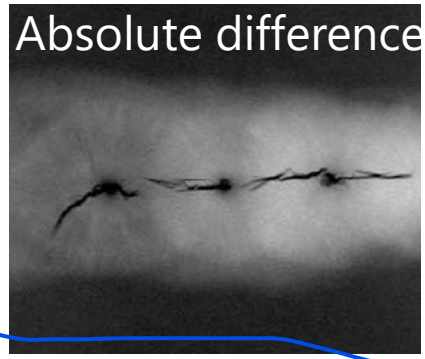


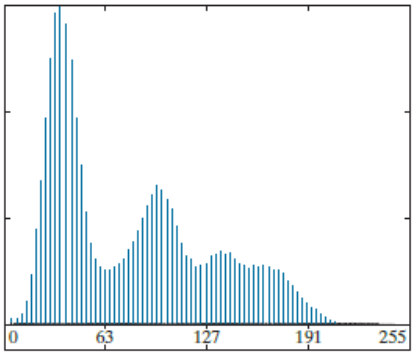


Initial seed image

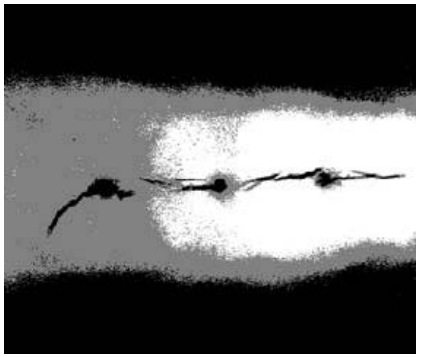
Final (eroded) seed image

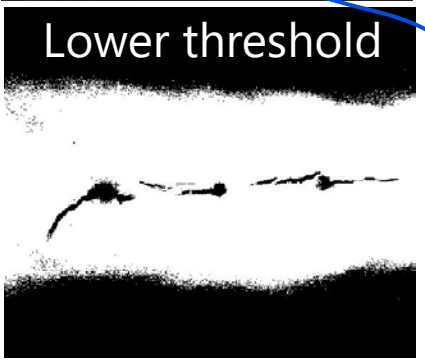


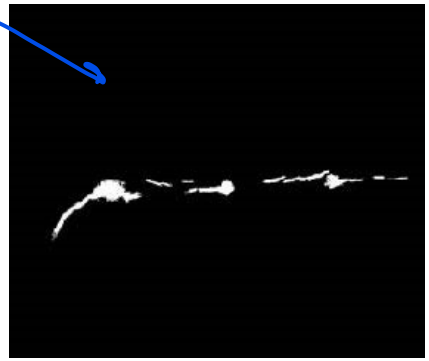




Double threshold

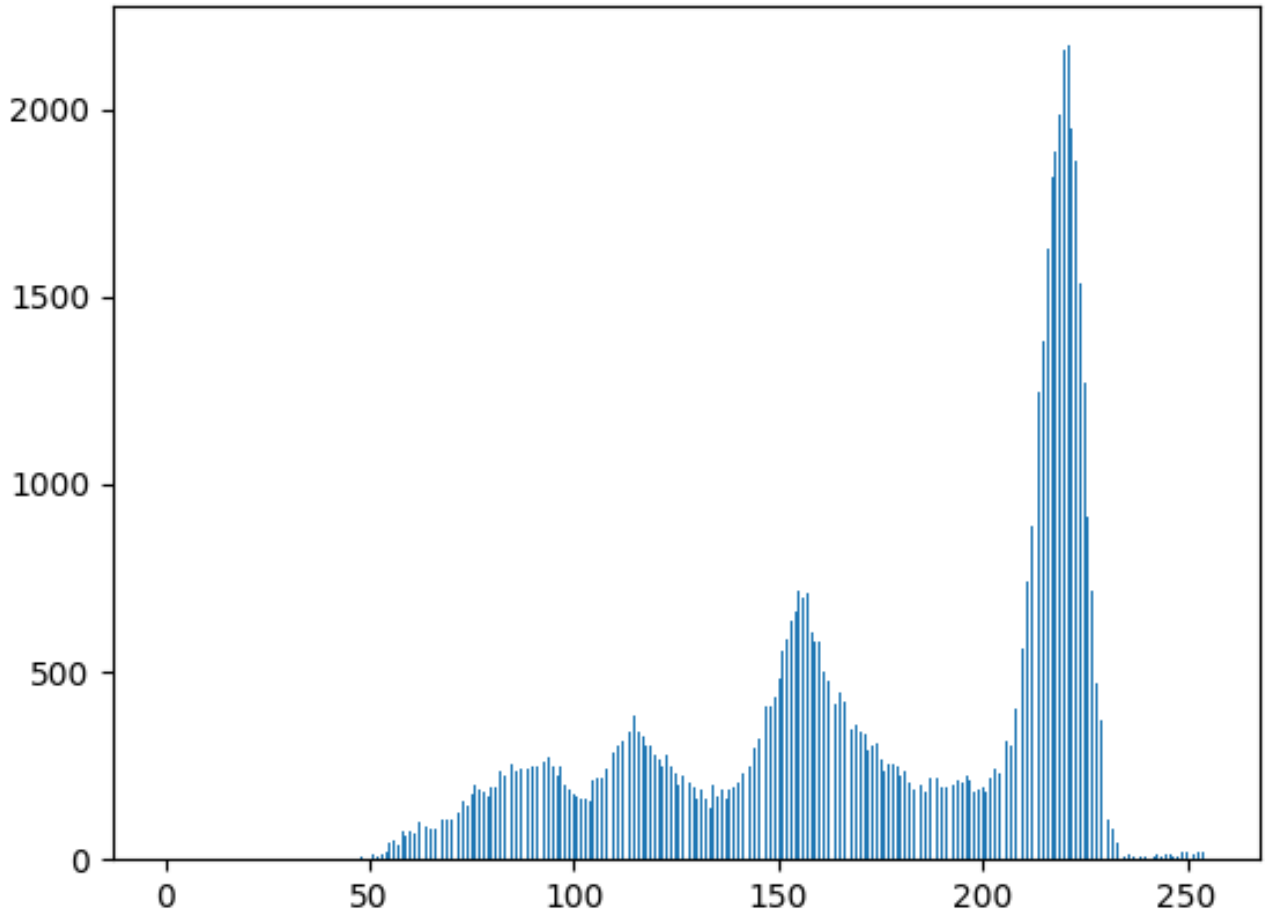






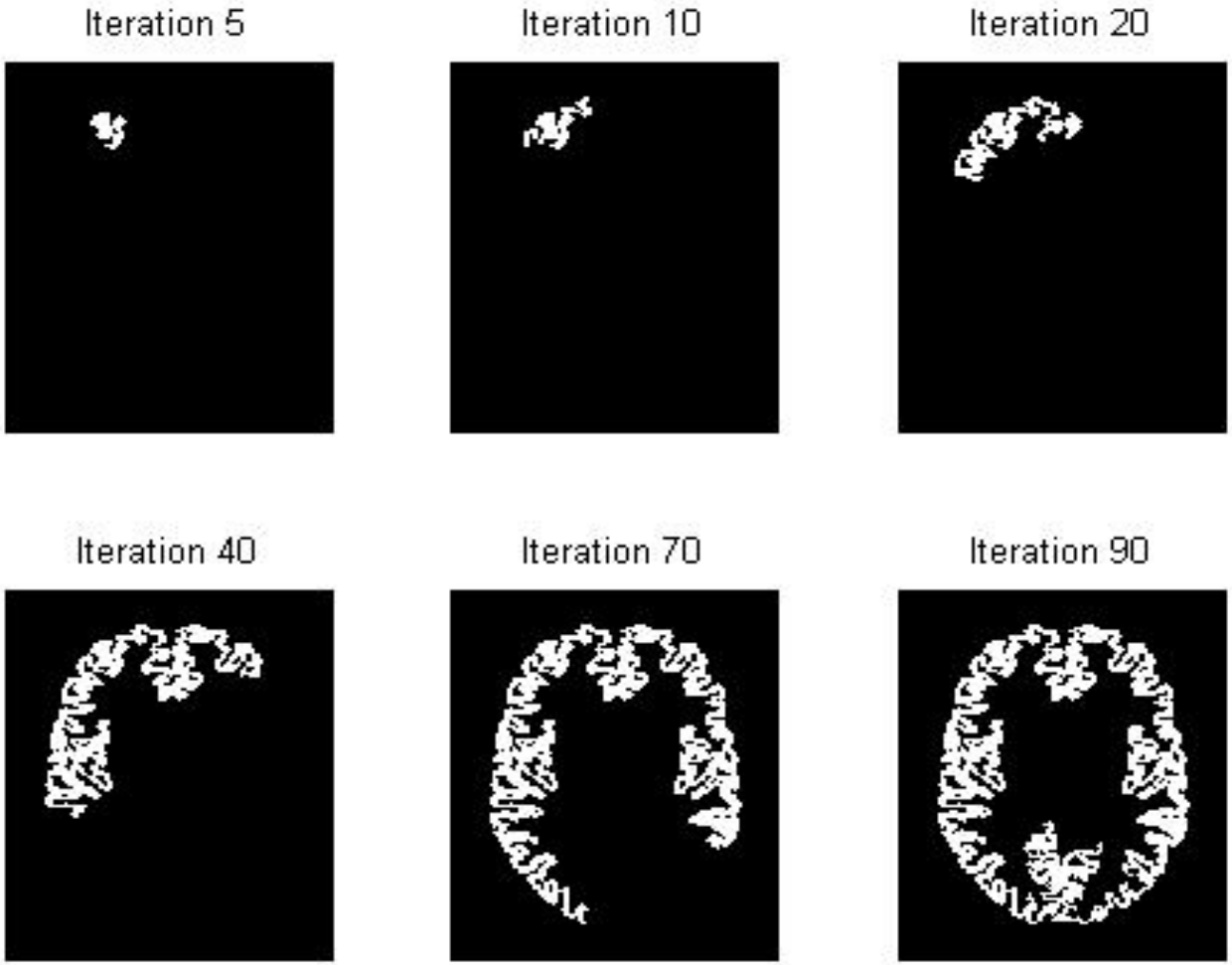
Region growing

Region growing



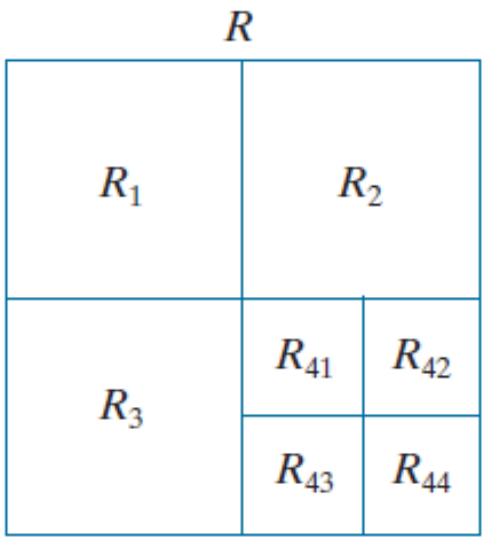
Histogram

Region growing

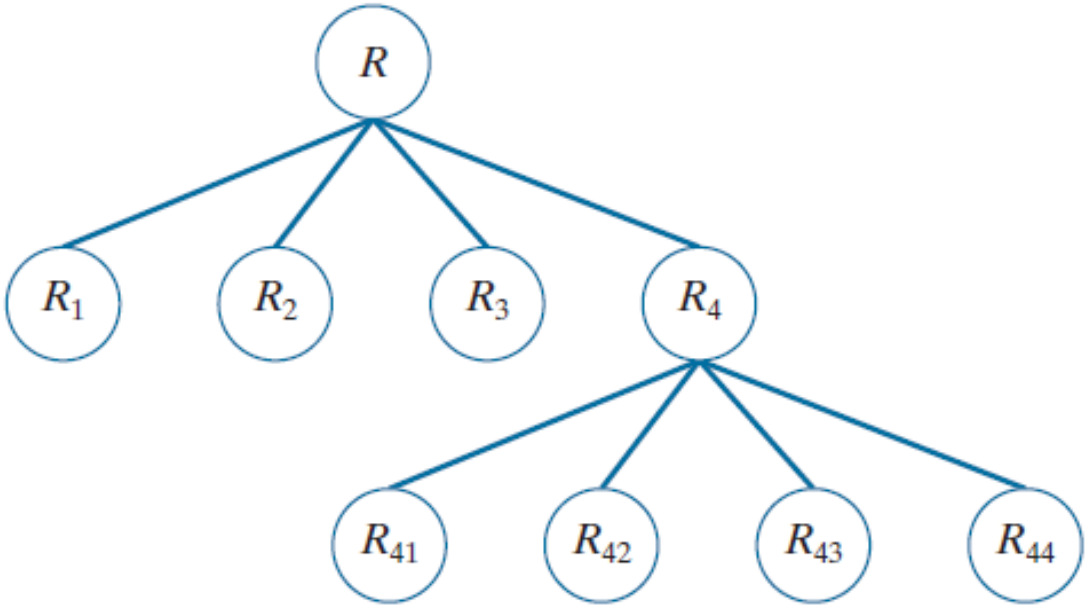


Region splitting and merging

1. Split into four disjoint quadrants any region R_i for which $Q(R_i) = \text{FALSE}$.
2. When no further splitting is possible, merge any adjacent regions R_j and R_k for which $Q(R_j \cup R_k) = \text{TRUE}$.
3. Stop when no further merging is possible.



Partitioned

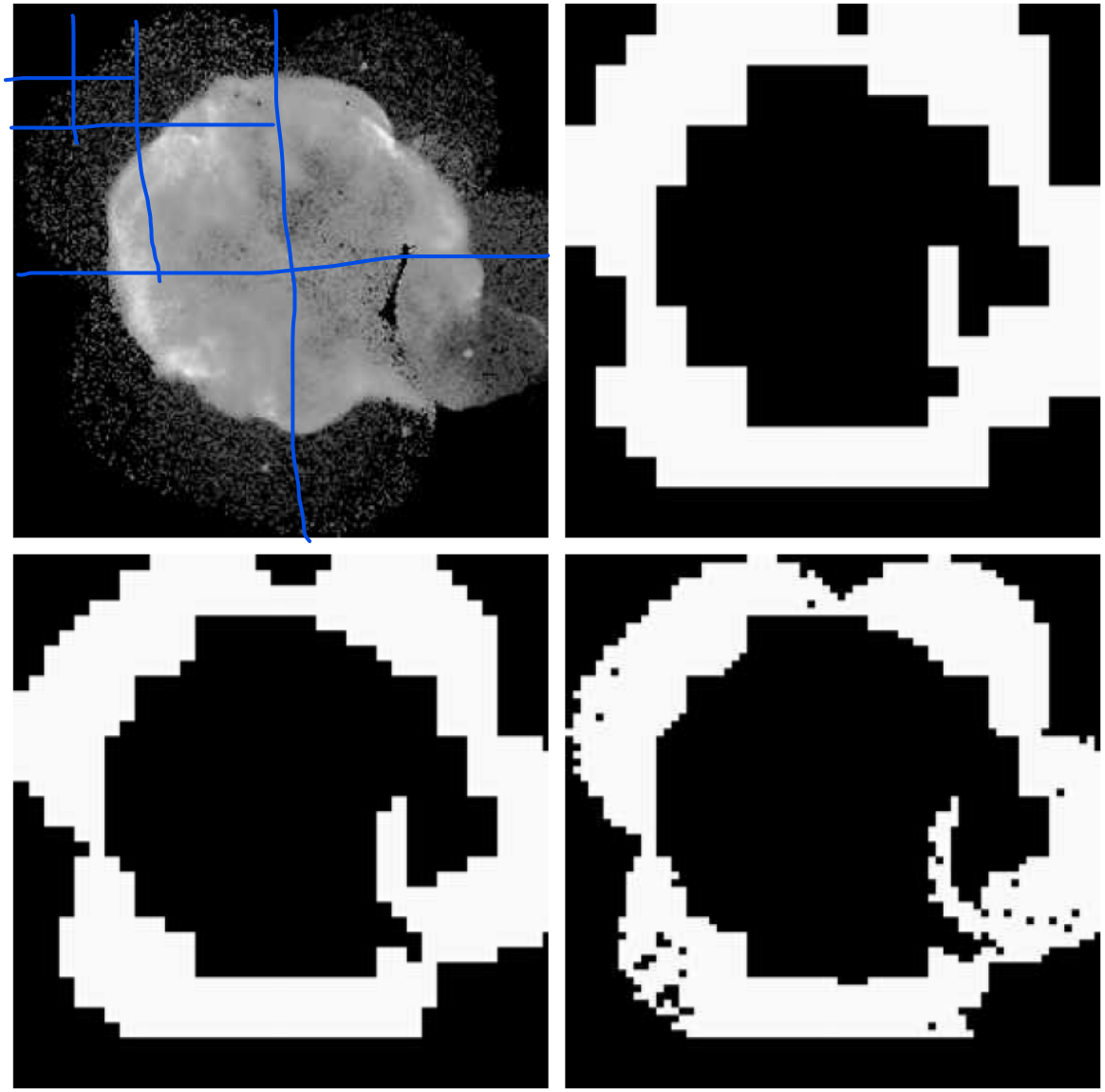


Quadtree

R4 تا خالی داره

Region splitting and merging

Region of interest:
Surround ring



32x32

16x16

8x8

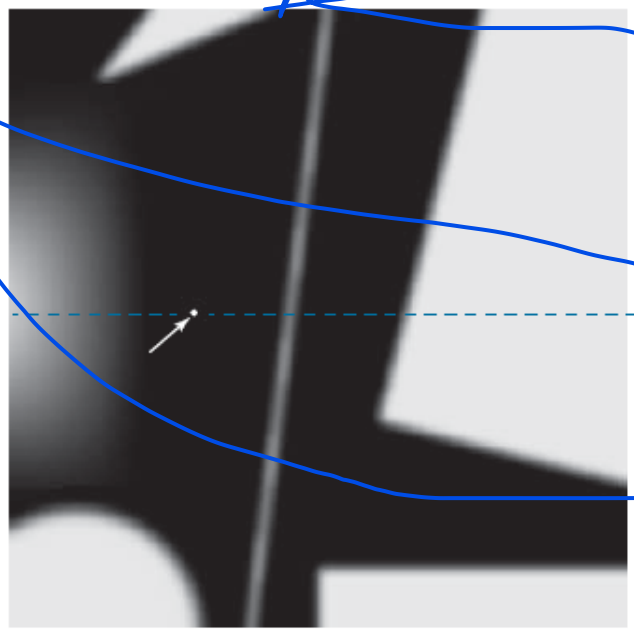
$$Q = \begin{cases} \text{TRUE} & \text{if } \sigma > a \quad \text{AND} \quad 0 < m < b \\ \text{FALSE} & \text{otherwise} \end{cases}$$

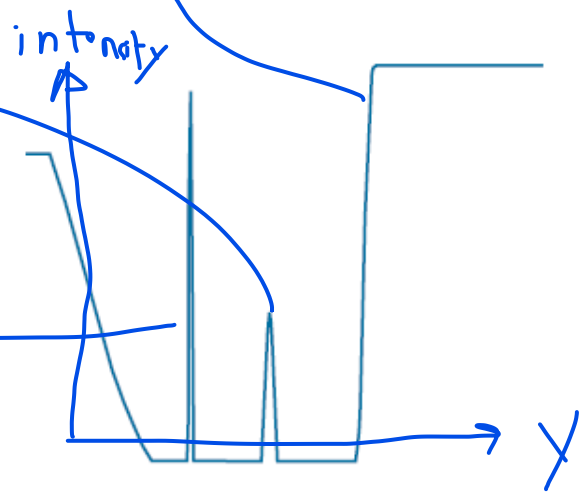
میانگین

Image segmentation based on dissimilarity

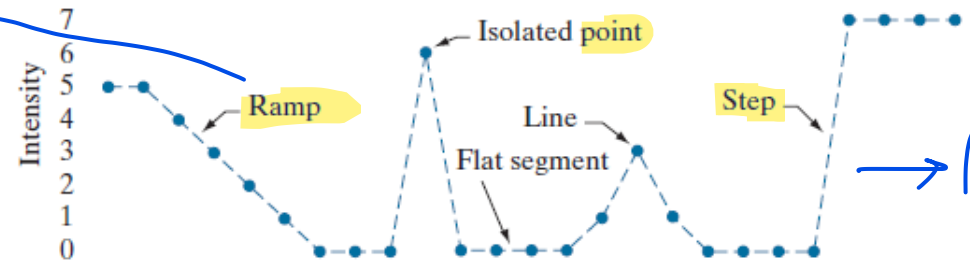
دنبال ریز هستیم (دنبال تفاوت)

Point, line, edge detection





لبه‌یابی داریم



لبه‌یابی نداریم

Intensity values	5	5	4	3	2	1	0	0	0	6	0	0	0	0	1	3	1	0	0	0	0	7	7	7	7	·	·
First derivative		−1	−1	−1	−1	−1	0	0	6	−6	0	0	0	1	2	−2	−1	0	0	0	7	0	0	0			
Second derivative		−1	0	0	0	0	1	0	6	−12	6	0	0	1	1	−4	1	1	0	0	7	−7	0	0			

Image segmentation

Point, line, edge detection

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

3x3 mask

Laplacian

$$\nabla^2 f(x,y) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\frac{\partial^2 f(x,y)}{\partial x^2} = f(x+1,y) + f(x-1,y) - 2f(x,y)$$

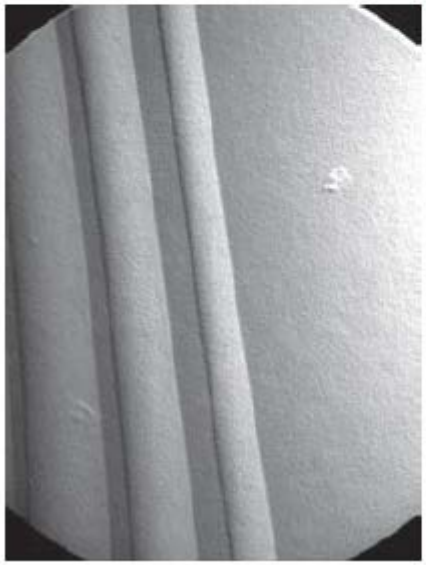
$$\frac{\partial^2 f(x,y)}{\partial y^2} = f(x,y+1) + f(x,y-1) - 2f(x,y)$$

$$\nabla^2 f(x,y) = f(x+1,y) + f(x-1,y) + f(x,y+1)$$
$$+f(x,y-1) - 4f(x,y)$$

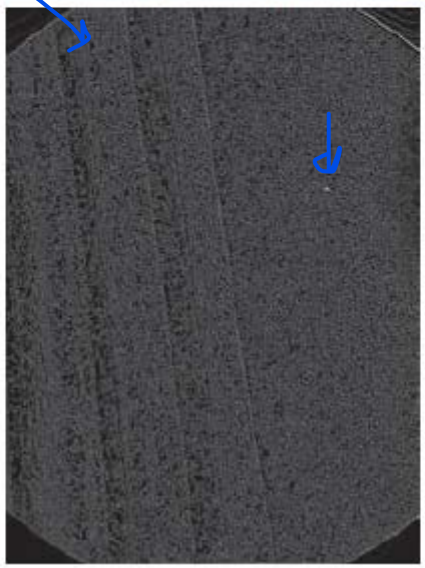
Point detection

کانال

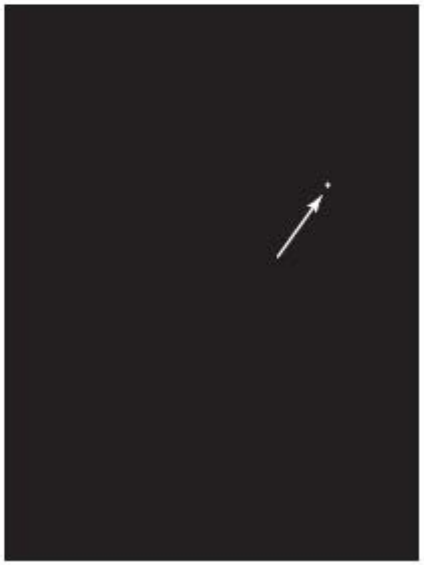
1	1	1
1	−8	1
1	1	1



X-ray image
of a turbine blade

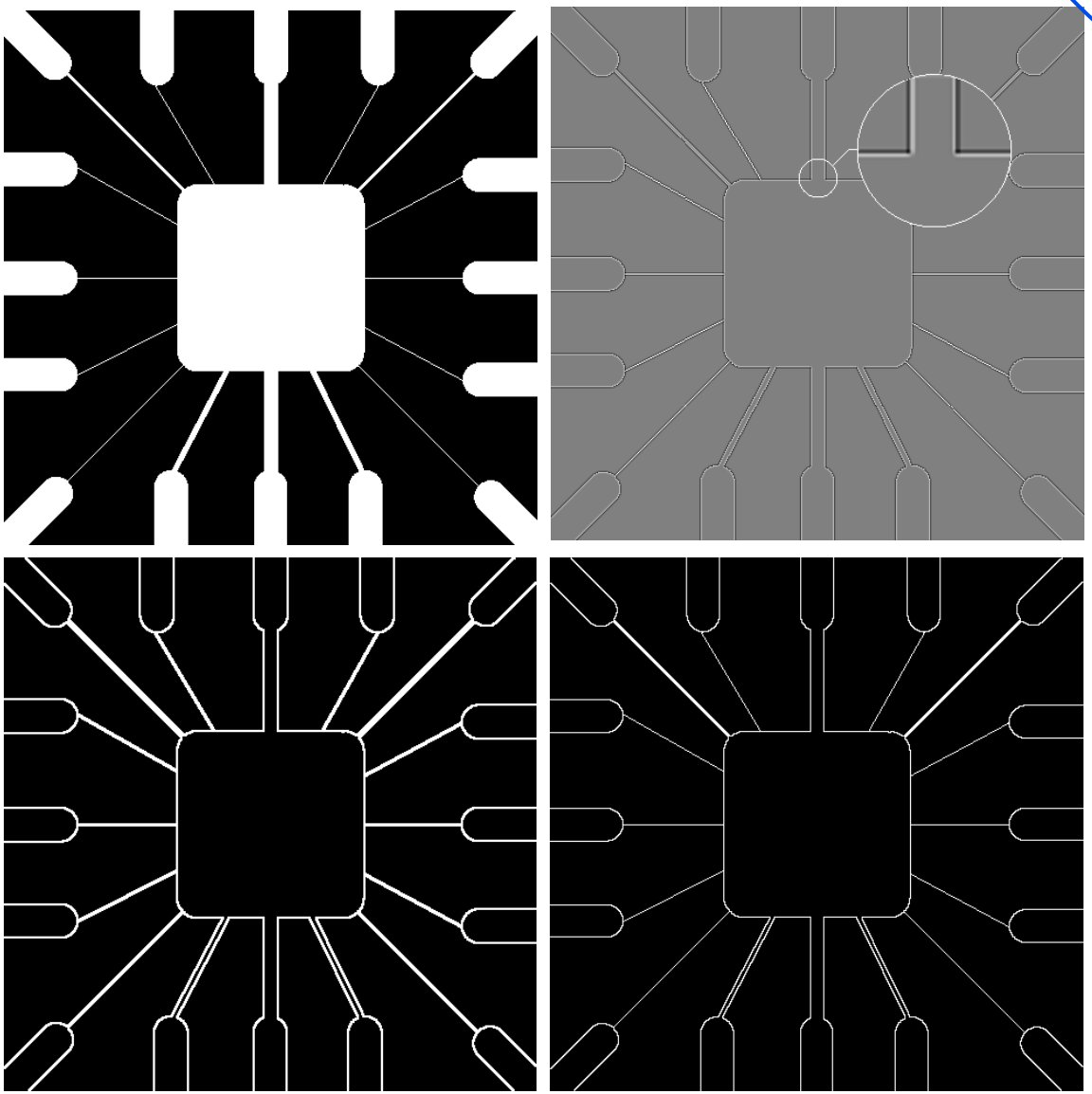


Point
detection



Thresholding

در لبه یه پالس مثبت و یه پالس منفی میشه که در لایه سفید و مشکی نمونه میده



Laplacian positive/negative double line

Absolute

Positive

Line detection

قوی ترین پاسخ به یک لاین خطی است

دو تا بالا در ردیف بالا و پایین هر ضرب یک و تضعیف می‌کند

−1	−1	−1
2	2	2
−1	−1	−1

Horizontal

افقی

2	−1	−1
−1	2	−1
−1	−1	2

+45°

−1	2	−1
−1	2	−1
−1	2	−1

Vertical

−1	−1	2
−1	2	−1
2	−1	−1

−45°

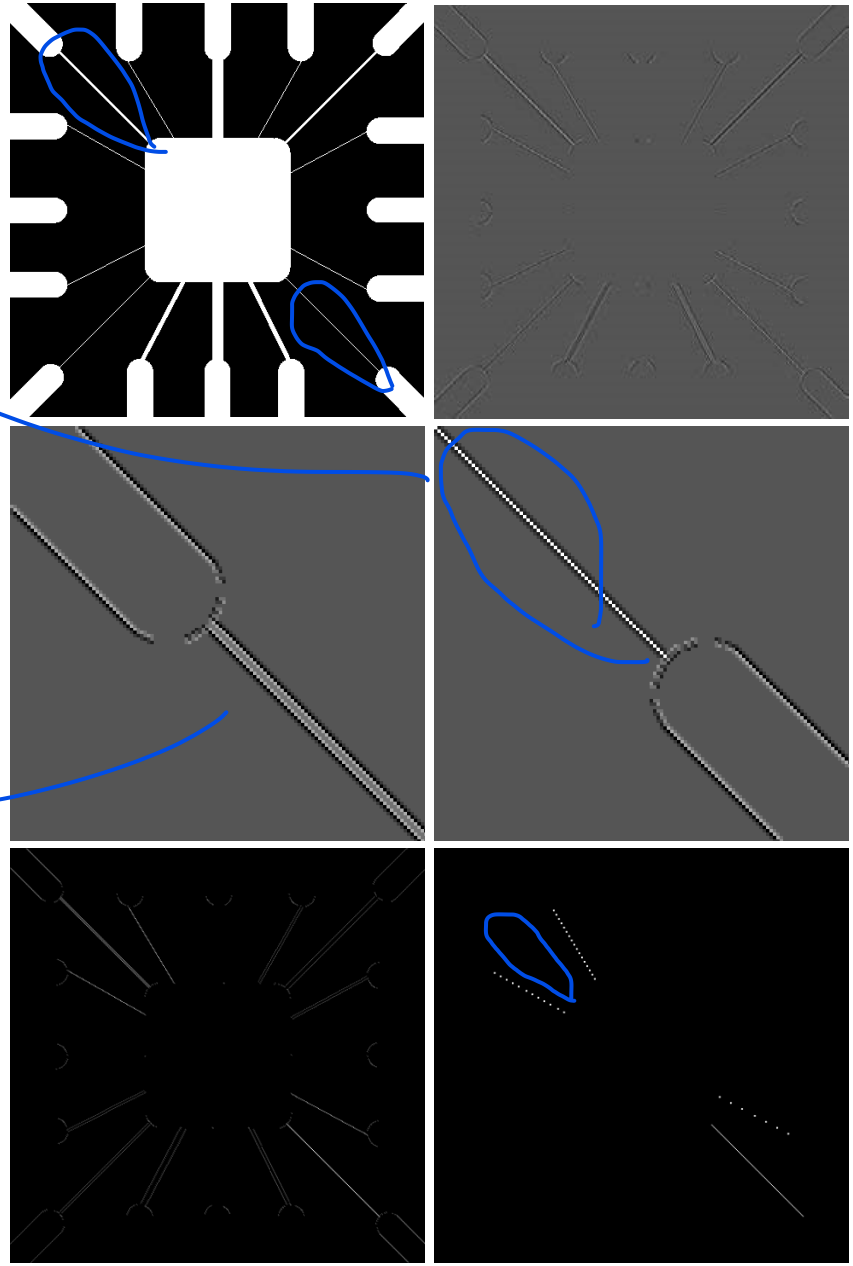
Line detection masks

Line detection

ضخامت ضعیف تر

به

پاسخ بهتری داده



After using
+45 line detector

Top left
zoom

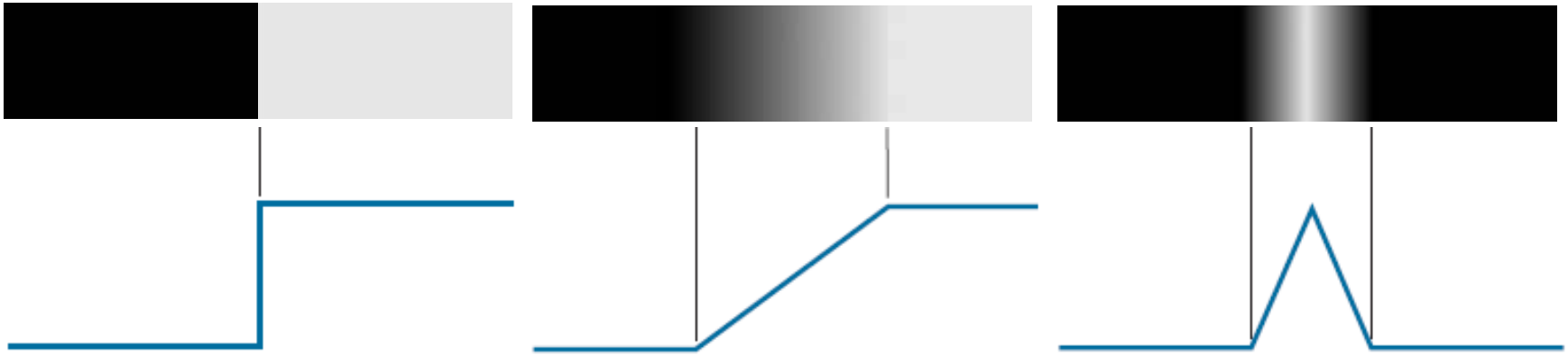
دبل لاین داریم

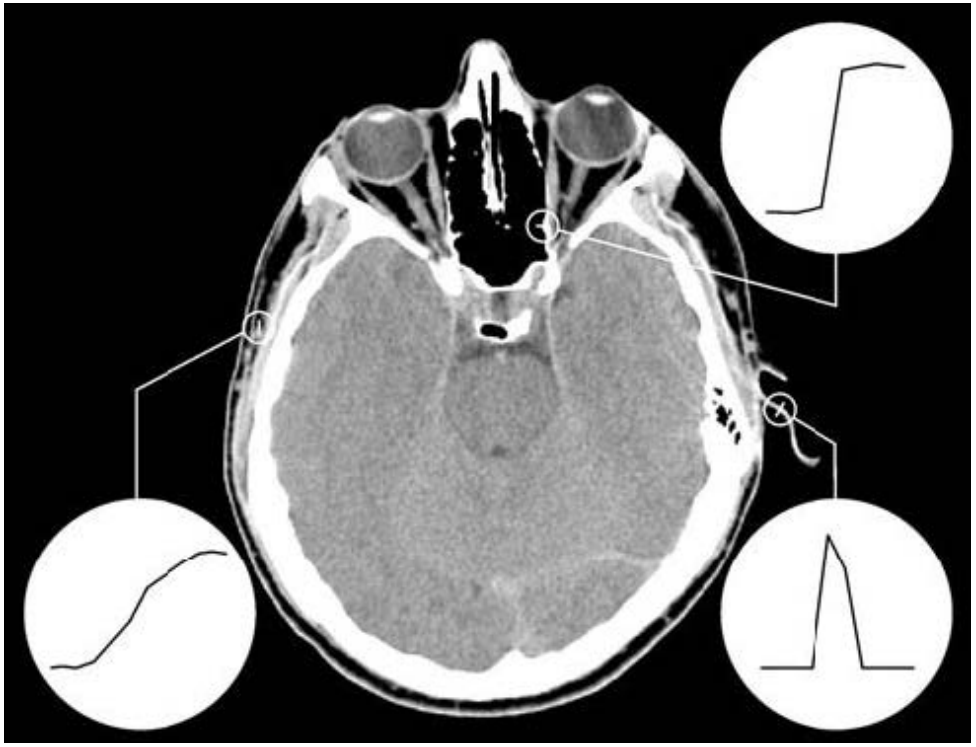
Bottom right
zoom

Negatives
set to zero

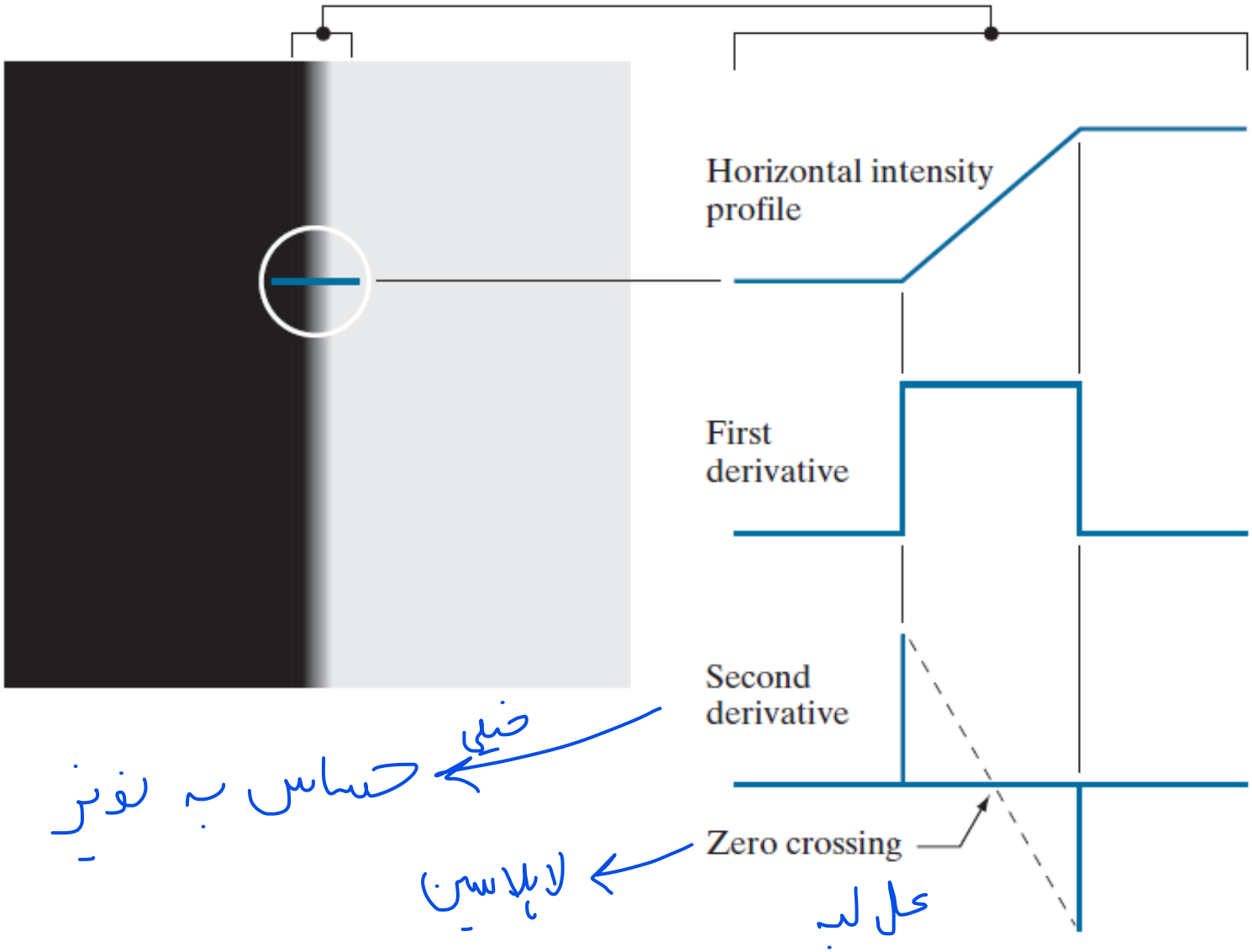
After
thresholding

Edges





Derivatives at the edges



Derivatives at the edges

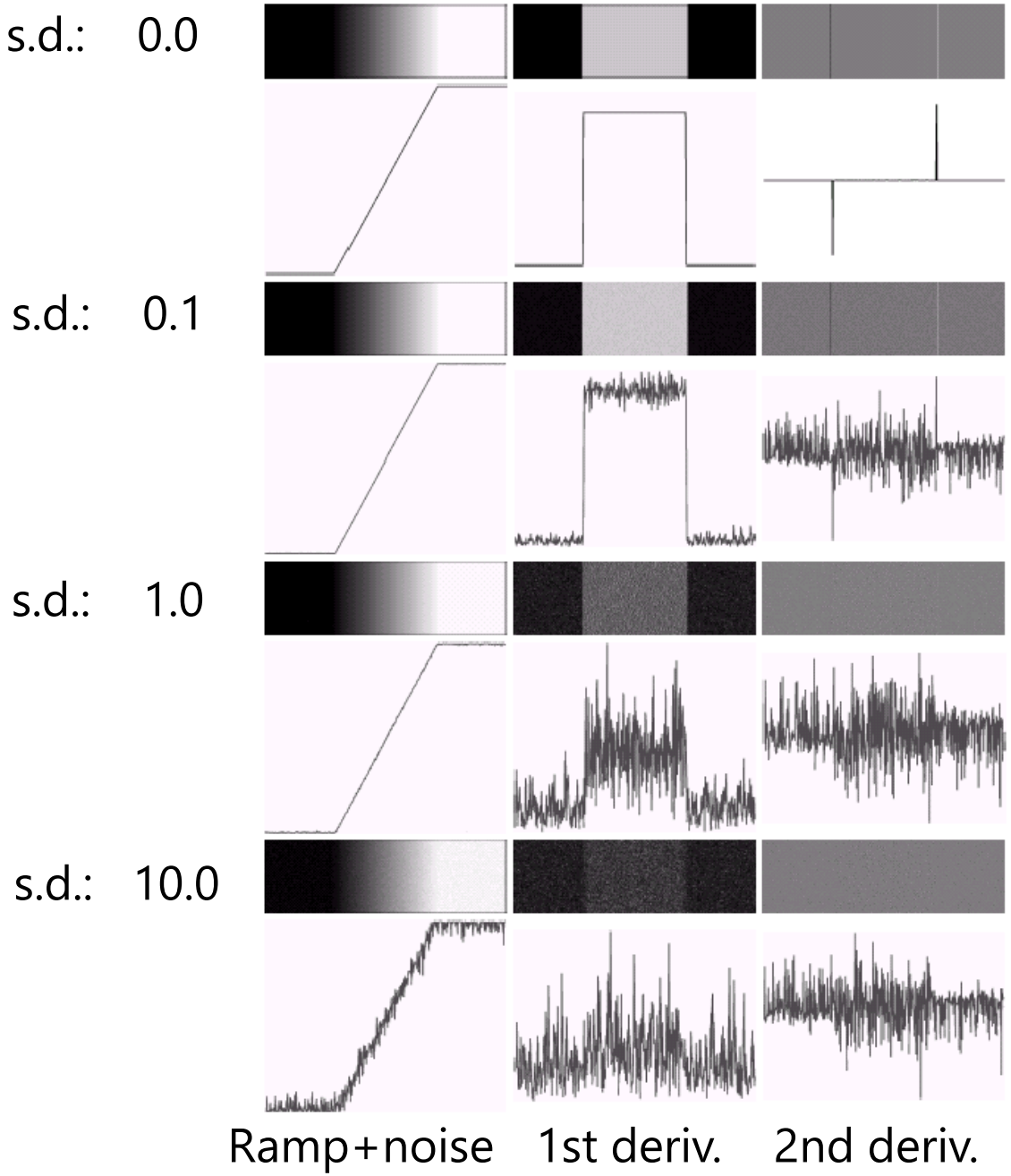


Image gradient

$$\nabla f \equiv \text{grad}(f) \equiv \begin{bmatrix} g_x \\ g_y \end{bmatrix} = \begin{bmatrix} \dfrac{\partial f}{\partial x} \\ \dfrac{\partial f}{\partial y} \end{bmatrix}$$

$$g_x = \frac{\partial f(x,y)}{\partial x} = f(x+1,y) - f(x,y)$$

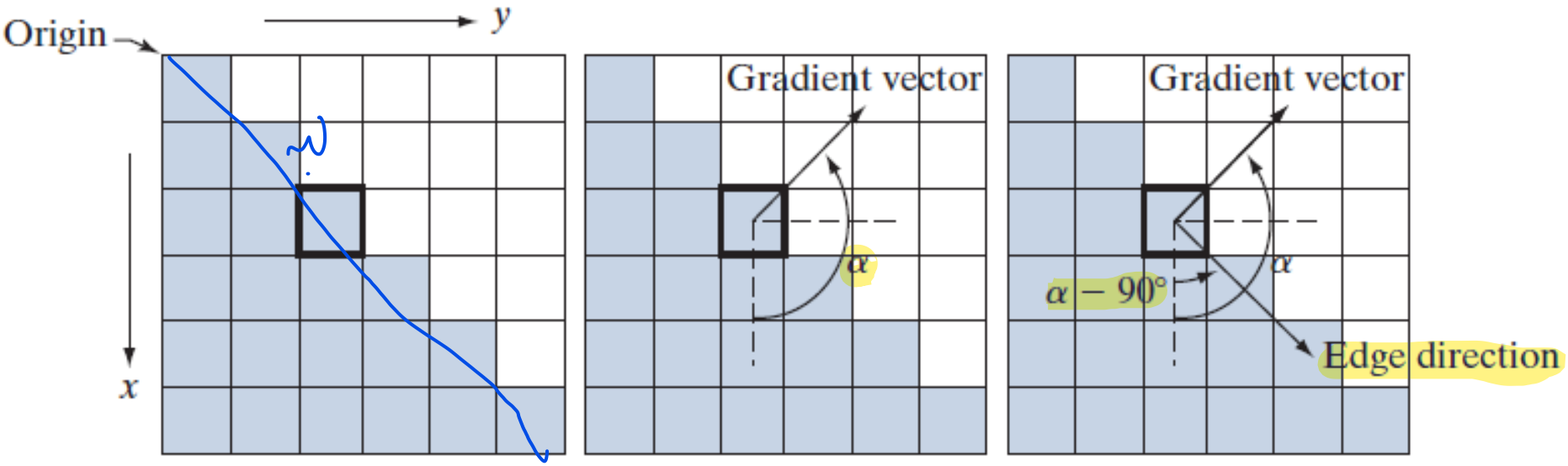
$$g_y = \frac{\partial f(x,y)}{\partial y} = f(x,y+1) - f(x,y)$$

$$M(x,y) = mag(\nabla f) = \sqrt{g_x^2 + g_y^2}$$

$$\alpha(x,y) = \tan^{-1}\left[\frac{g_y}{g_x}\right]$$

تمرد برلیه

Image gradient



$$\nabla f \equiv \text{grad}(f) \equiv \begin{bmatrix} g_x \\ g_y \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

هرچه بزرگتر

احتمال وجود لبه بیشتر

$$M(x, y) = 2\sqrt{2}$$

Masks to compute gradients

−1	
1	

−1	1

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

−1	0
0	1

0	−1
1	0

Roberts

$$g_x = \frac{\partial f}{\partial x} = (z_9 - z_5)$$

$$g_y = \frac{\partial f}{\partial y} = (z_8 - z_6)$$

mask ها
برای
بدست آوردن
g_x و g_y

−1	−1	−1
0	0	0
1	1	1

−1	0	1
−1	0	1
−1	0	1

Prewitt

$$g_x = \frac{\partial f}{\partial x} = (z_7 + z_8 + z_9) - (z_1 + z_2 + z_3)$$

$$g_y = \frac{\partial f}{\partial y} = (z_3 + z_6 + z_9) - (z_1 + z_4 + z_7)$$

−1	−2	−1
0	0	0
1	2	1

−1	0	1
−2	0	2
−1	0	1

Sobel

$$g_x = \frac{\partial f}{\partial x} = (z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)$$

$$g_y = \frac{\partial f}{\partial y} = (z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)$$

Masks to compute gradients

لبه های قطری رو پیدا می کنند

0	1	1
−1	0	1
−1	−1	0

−1	−1	0
−1	0	1
0	1	1

Prewitt

0	1	2
−1	0	1
−2	−1	0

−2	−1	0
−1	0	1
0	1	2

Sobel

To detect diagonal edges

Edge directions

$|G_x|$

برای لبه های
افقی بهتر کار کرده
(لبه ی)

$|G_y|$

$|G_x|+|G_y|$

Edge directions

$|G_x|$

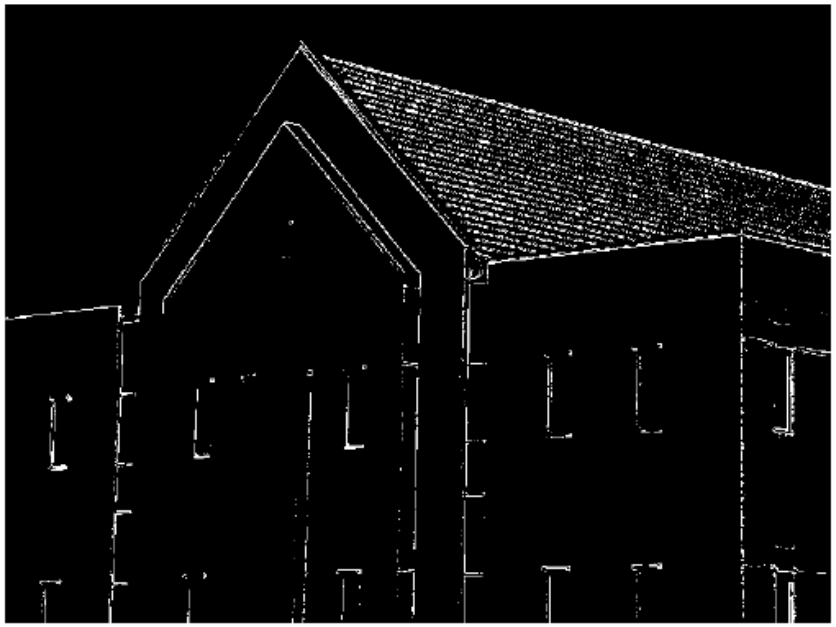
$|G_y|$

$|G_x|+|G_y|$

نویزها رو دیوار رو
شدند

Results after smoothing by a 5x5 averaging filter

Thresholded



Without prior smoothing

With prior smoothing

Edge directions

چرا؟ ببین

Diagonal edge detection

Marr-Hildreth edge detector

روش تشخیص مرز یابی (۲ روش)

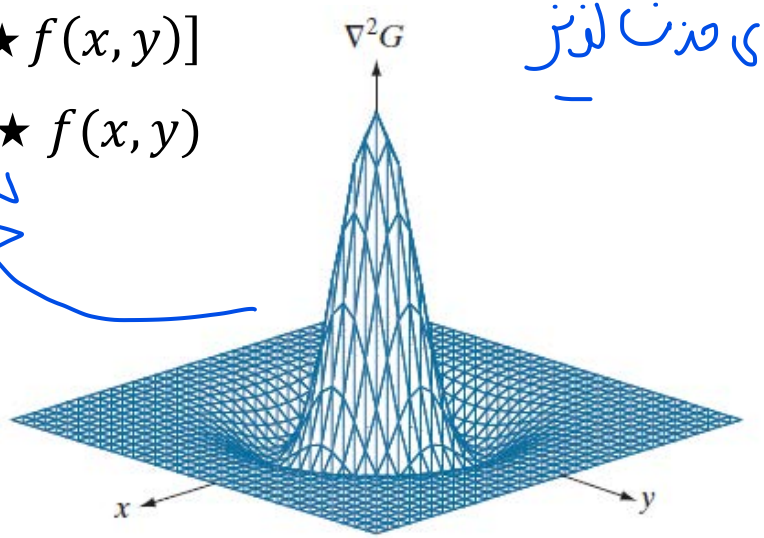
Using Laplacian of Gaussian (LoG)

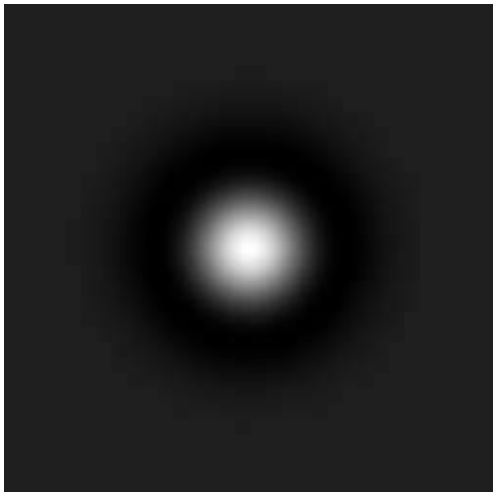
مشتق دوم

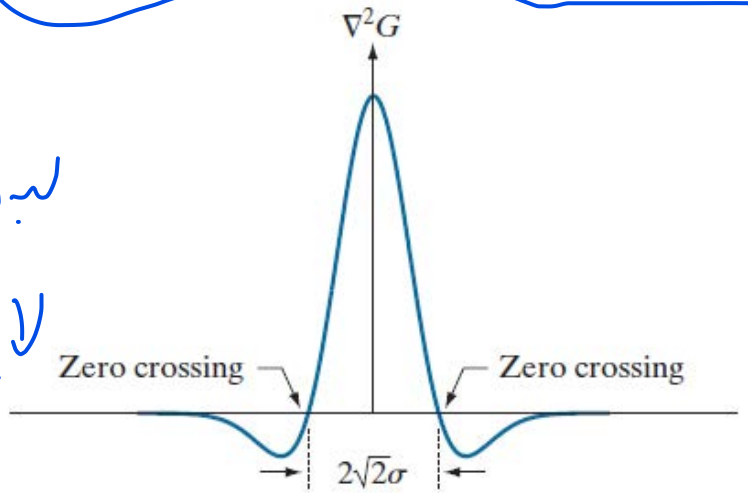
برای حذف نویز

$$g(x,y) = \nabla^2[G(x,y) \star f(x,y)]$$

$$g(x,y) = \nabla^2[G(x,y)] \star f(x,y)$$







0	0	−1	0	0
0	−1	−2	−1	0
−1	−2	16	−2	−1
0	−1	−2	−1	0
0	0	−1	0	0

لبه ها، zero crossing های

لاپلاسین هست

Marr-Hildreth edge detector Using Laplacian of Gaussian (LoG)

$$G(x,y) = \mathrm{e}^{-\frac{x^2+y^2}{2\sigma^2}}$$

$n \times n$ and $n \geq 6\sigma$

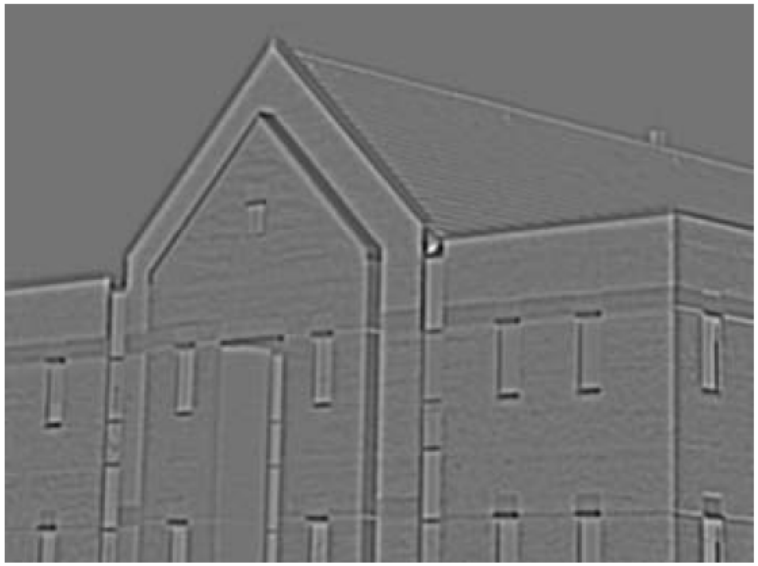
$$\nabla^2 G(x,y) = \frac{\partial^2 G(x,y)}{\partial x^2} + \frac{\partial^2 G(x,y)}{\partial y^2}$$

$$= \frac{\partial}{\partial x}\left[\frac{-x}{\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}\right] + \frac{\partial}{\partial y}\left[\frac{-y}{\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}\right]$$

$$= \left[\frac{x^2}{\sigma^4} - \frac{1}{\sigma^2}\right] e^{-\frac{x^2+y^2}{2\sigma^2}} + \left[\frac{y^2}{\sigma^4} - \frac{1}{\sigma^2}\right] e^{-\frac{x^2+y^2}{2\sigma^2}}$$

$$\nabla^2 G(x,y) = [\frac{x^2+y^2-2\sigma^2}{\sigma^4}] e^{-\frac{x^2+y^2}{2\sigma^2}}$$

Laplacian of Gaussian (LoG)



LoG
s.d.=4

حل
انحدبه

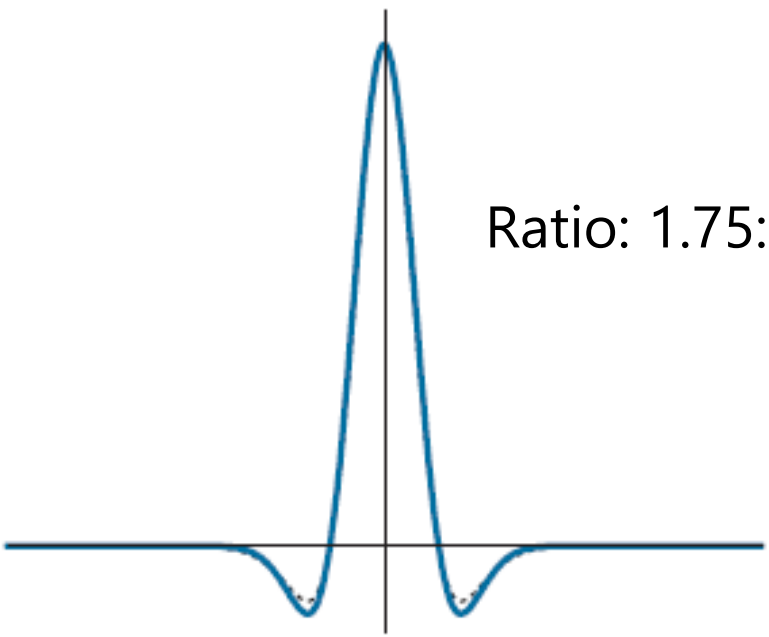
Threshold: 0
closed loop edges

Threshold: 4% of max of LoG

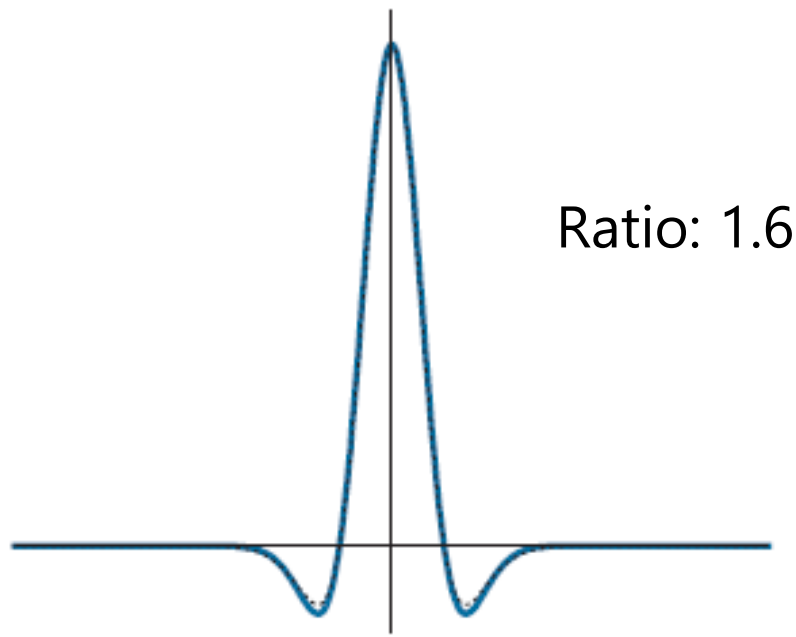
LoG estimation with DoG

$$DoG(x,y) = \frac{1}{2\pi\sigma_1^2} e^{-\frac{x^2+y^2}{2\sigma^2}} - \frac{1}{2\pi\sigma_2^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

$$\sigma^2 = \frac{\sigma_1^2\sigma_2^2}{\sigma_1^2 - \sigma_2^2} - \ln[\frac{\sigma_1^2}{\sigma_2^2}]$$



Ratio: 1.75:1



Ratio: 1.6:1

مشتق اول ←

Canny edge detector

لبه با فاصله متاکزر از یک نداشته باشیم

$$G(x,y) = e^{-\frac{x^2+y^2}{2\sigma^2}}$$

1) Smooth image:

$$f_S(x,y) = G(x,y) \star f(x,y)$$

2) Gradient: (f_S)

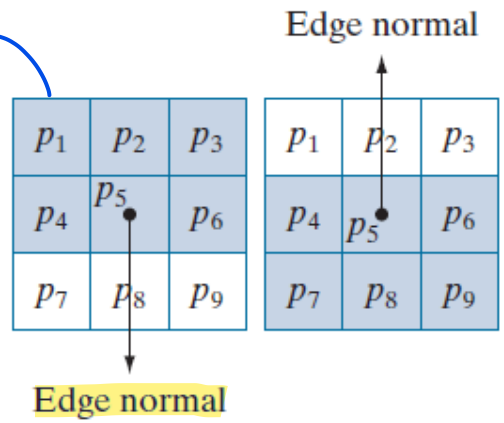
$$M(x,y) = \sqrt{g_x^2 + g_y^2}$$

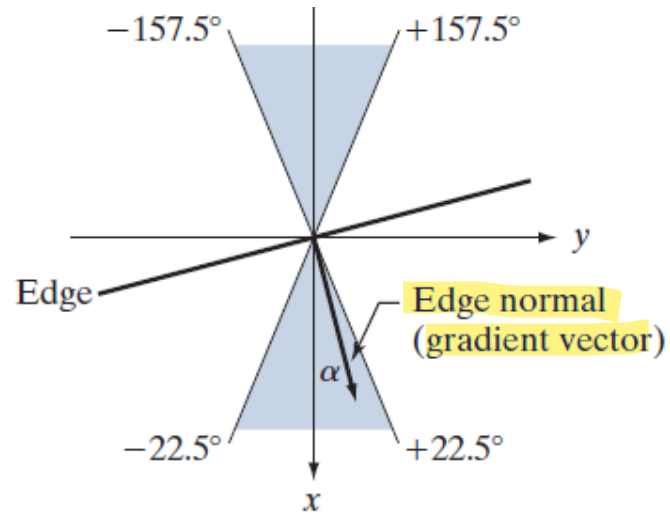
$$\alpha(x,y) = \tan^{-1}\left[\frac{g_y}{g_x}\right]$$

مرحله اول

3) Form the non-maxima suppressed image: $g_N(x, y)$

ماتریس M (صفحه قبل)



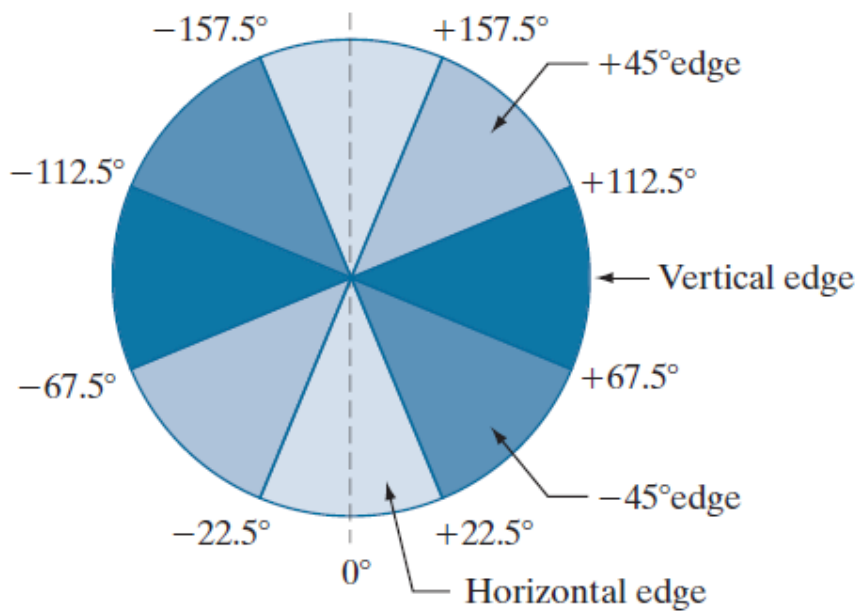


If not maximum along the edge normal:

$$g_N(x, y) = 0$$

If maximum:

$$g_N(x, y) = M(x, y)$$



مرحله دوم

لبه کاذب

4) Double thresholding to reduce false edges and linking:

لبه های قوی

$$g_{NH}(x,y) = g_N(x,y) \geq T_H$$

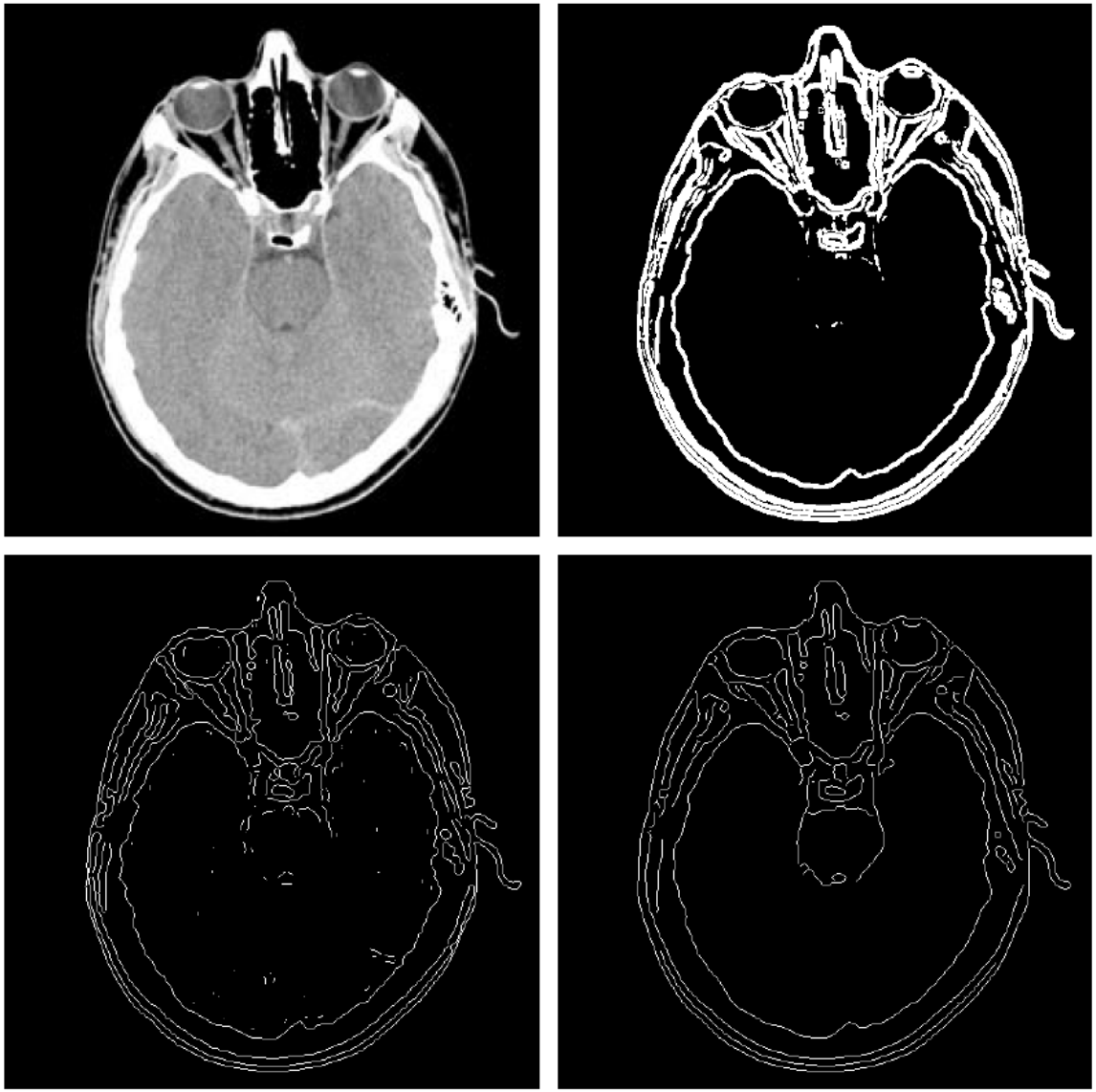
تمایز بین لبه های قوی تر از ضعیف تر

$$g_{NL}(x,y) = g_N(x,y) \geq T_L$$

لبه های ضعیف

$$g_{NL}(x,y) = g_{NL}(x,y) - g_{NH}(x,y)$$

(a) Locate the next unvisited edge pixel, p, in $g_{NH}(x, y)$.
(b) Mark as valid edge pixels all the weak pixels in $g_{NL}(x, y)$ that are connected to p using, say, 8-connectivity.
(c) If all nonzero pixels in $g_{NH}(x, y)$ have been visited go to Step d. Else, return to Step a.
(d) Set to zero all pixels in $g_{NL}(x, y)$ that were not marked as valid edge pixels.



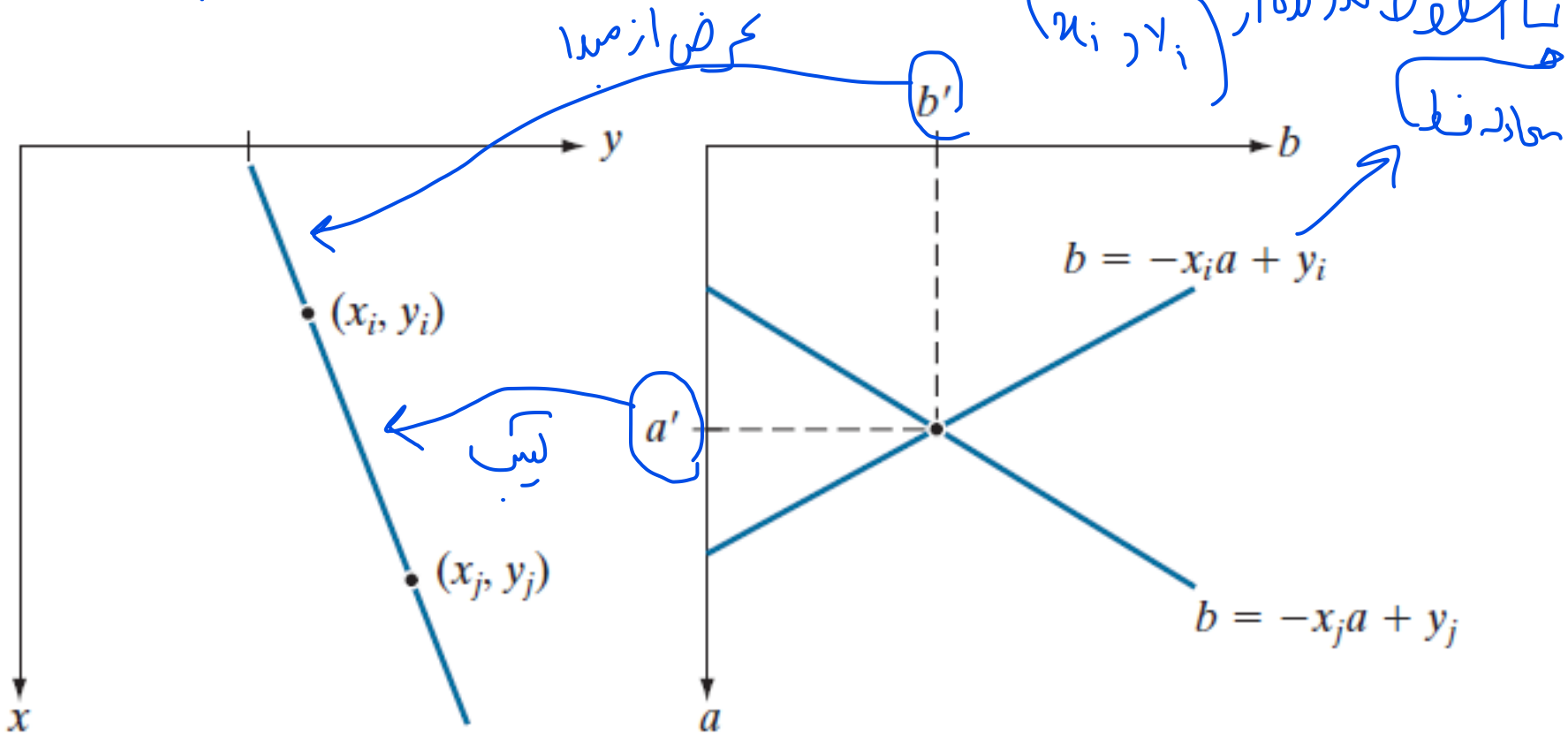
Thresholded gradient of smoothed image

(ناپیوستگی دارد) Marr-Hildreth

Canny (link)

Edge linking: Hough transform

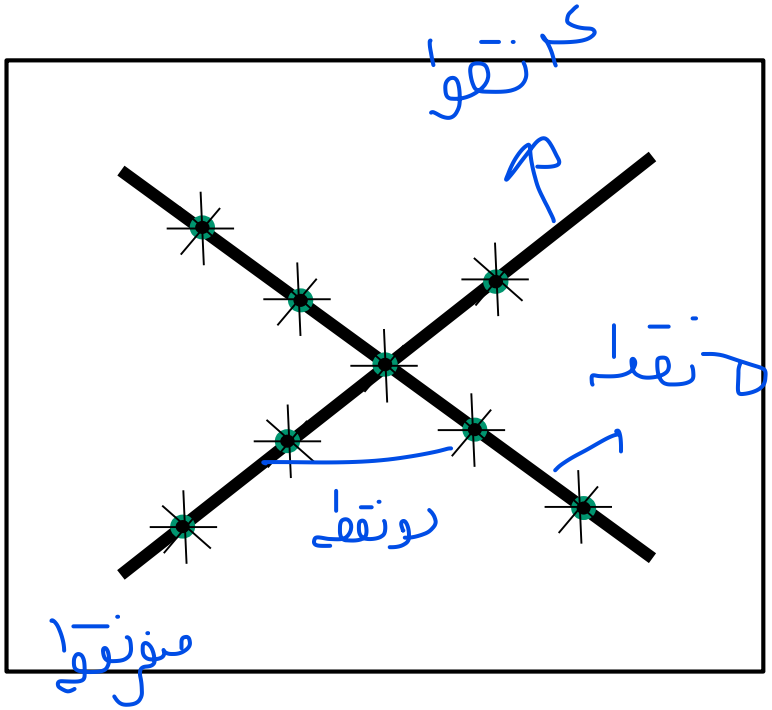
برای لبه‌های مستقیم



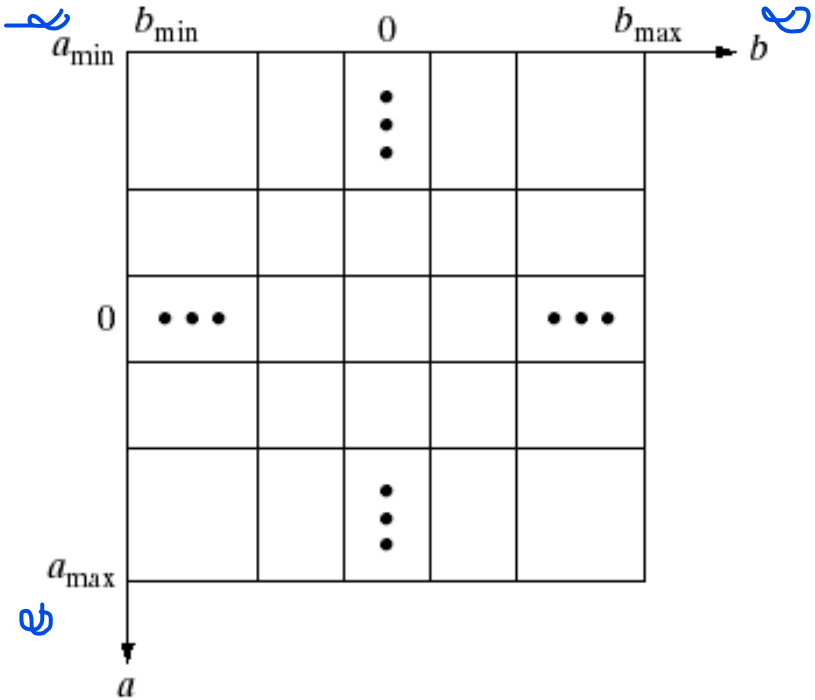
xy-plane

Parameter plane

Hough transform



From edge detection



Subdivision of the parameter plane

ماکسیمم خانه‌ها

threshold نقاط را رو نقاط بعد از

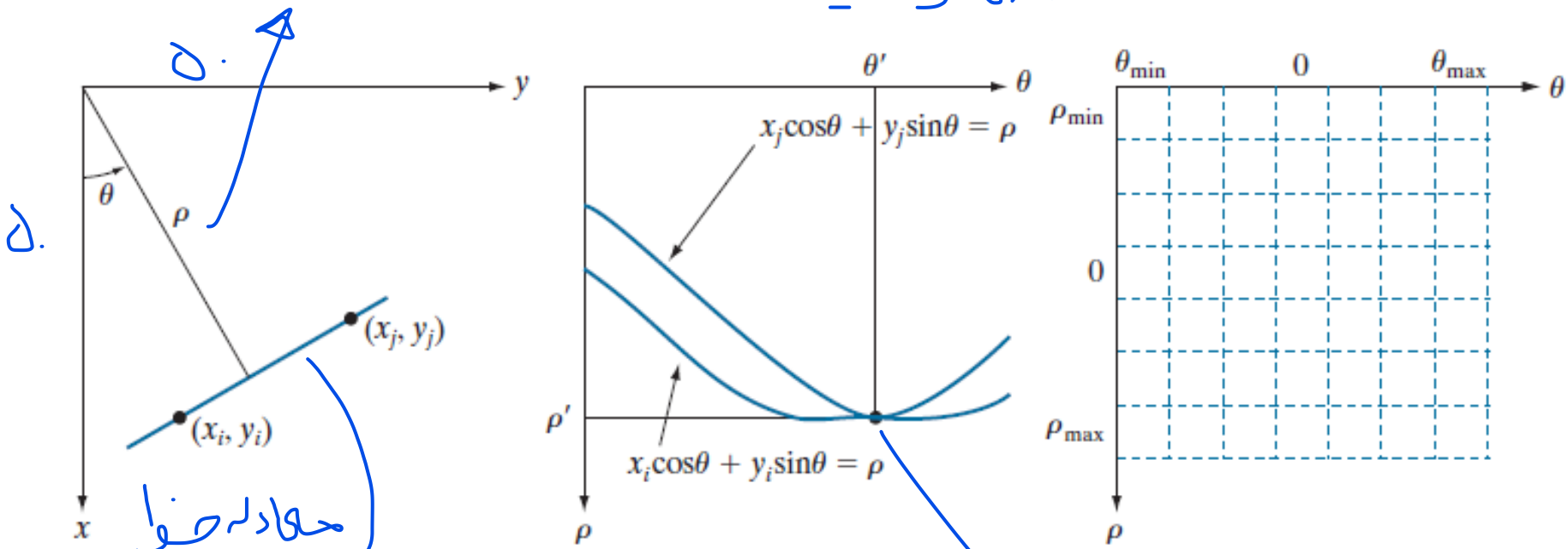
داریم

۵ و ۶ خط گذرنده از دنیا طرصیدا

حداکثر خطی تصویر ۵.۴

Hough transform

به ازای هر نقطه یک sin



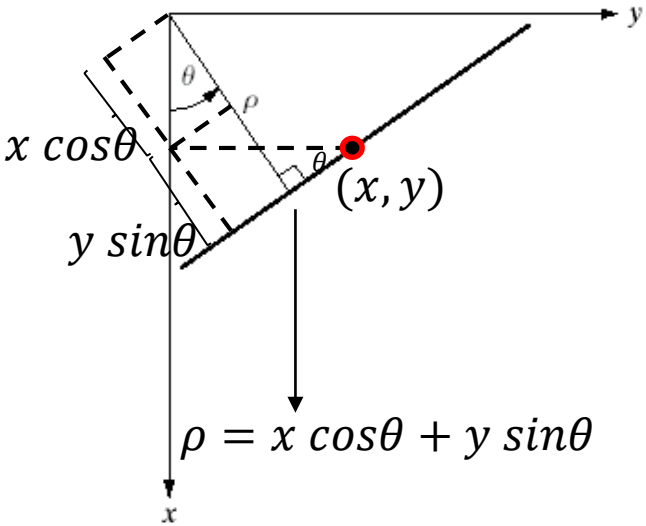
معادله خط

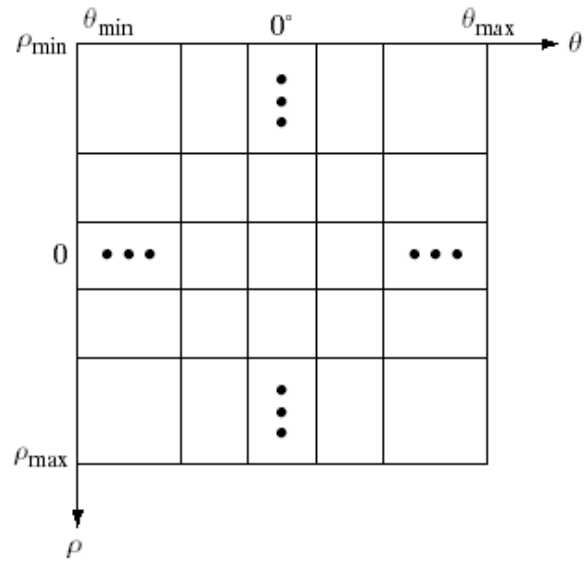
$$x\cos\theta + y\sin\theta = \rho$$

ρ و θ خطی که از نقاط گذرنده

In polar coordinates

Hough transform





In polar coordinates

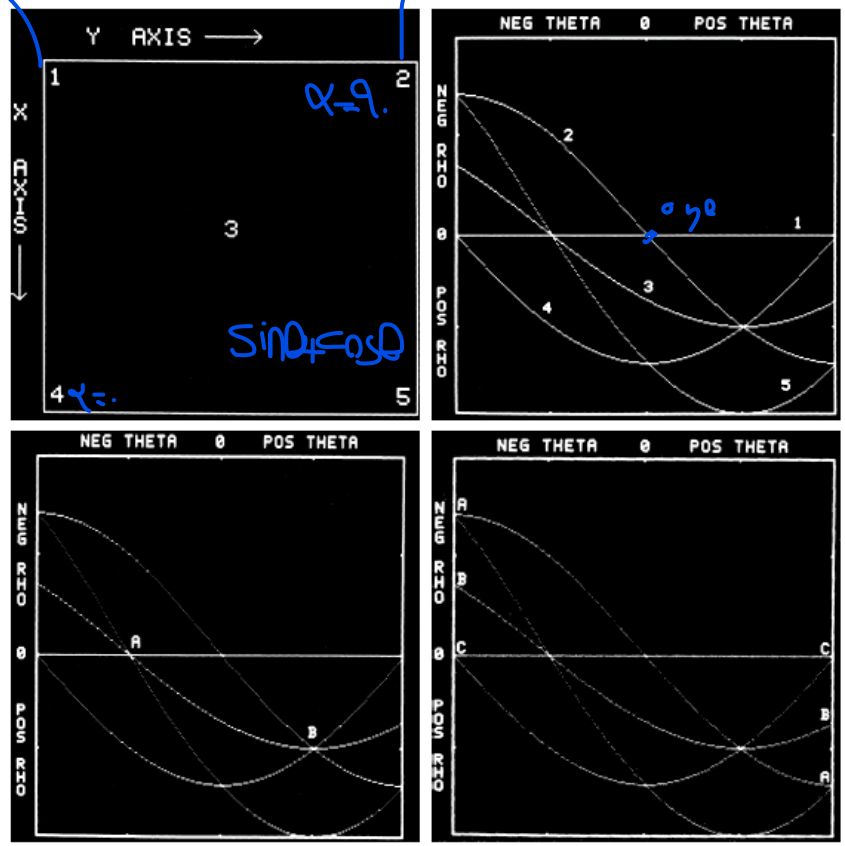
برای پیدا کردن اندازه پارامترها از min و max x و y نظر روی فضا تقسیم

Hough transform

معادله خطی تمام خطوط گذرنده از نقطه یک

sin θ

Image with 5 points



Parameter space

$$x\,cos\theta + y\,sin\theta = \rho$$

$$\sqrt{x^2+y^2}\,cos\alpha\,cos\theta + sin\alpha\,sin\theta = \rho$$

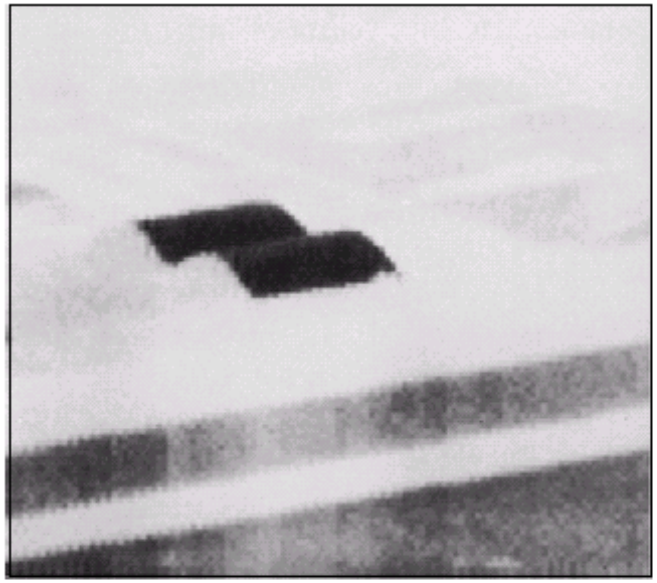
$$\sqrt{x^2+y^2}\cos(\theta-\alpha) = \rho$$

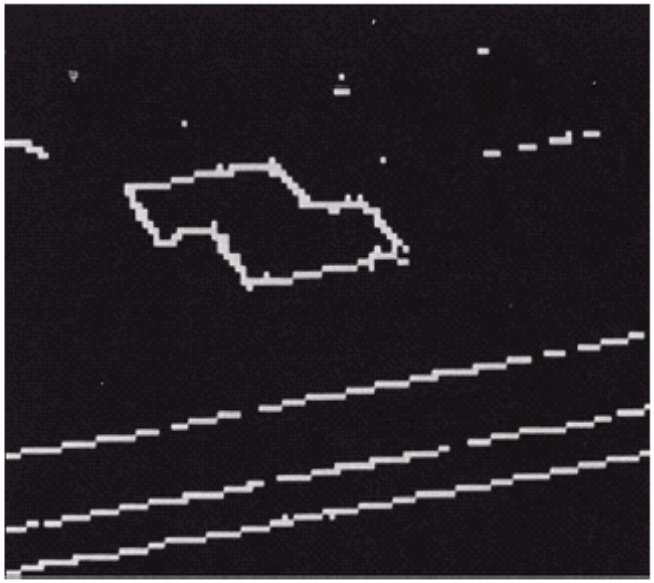
$$0 \le \alpha \le 90$$

$$\alpha - 90 \le \theta \le \alpha + 90$$

Hough transform

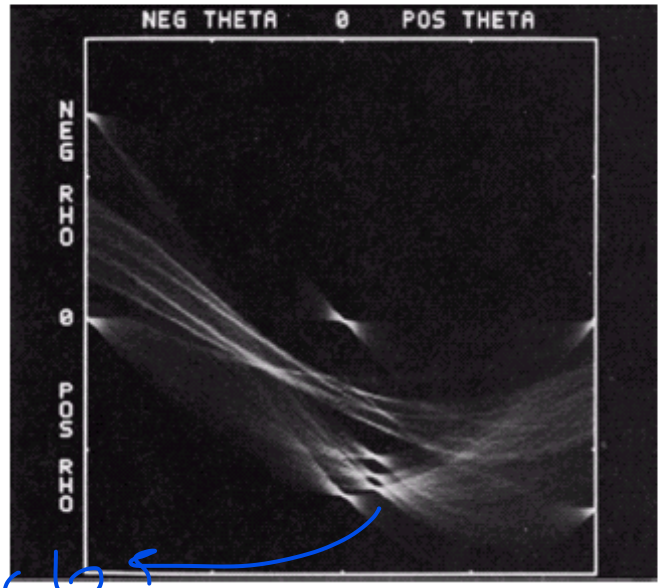
Infrared image

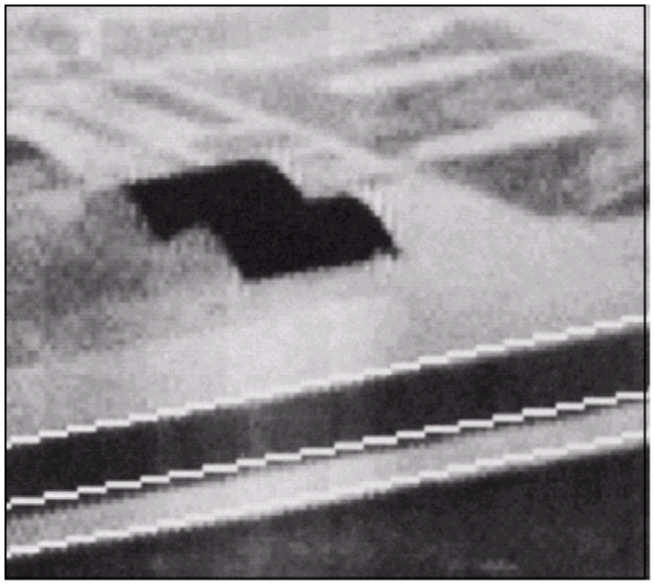




Thresholded gradient image

Hough transform



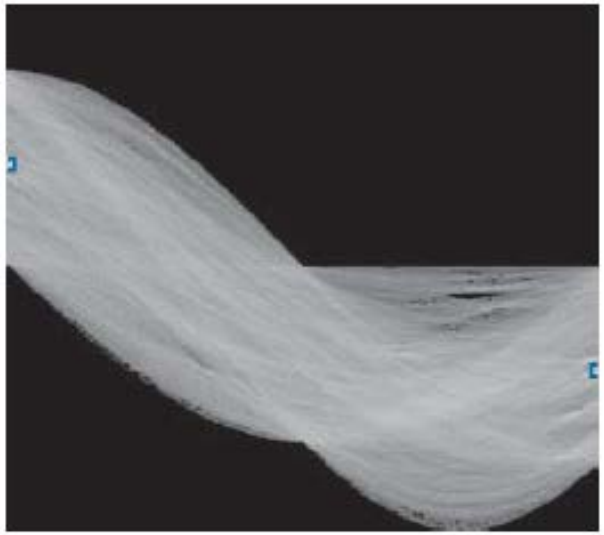


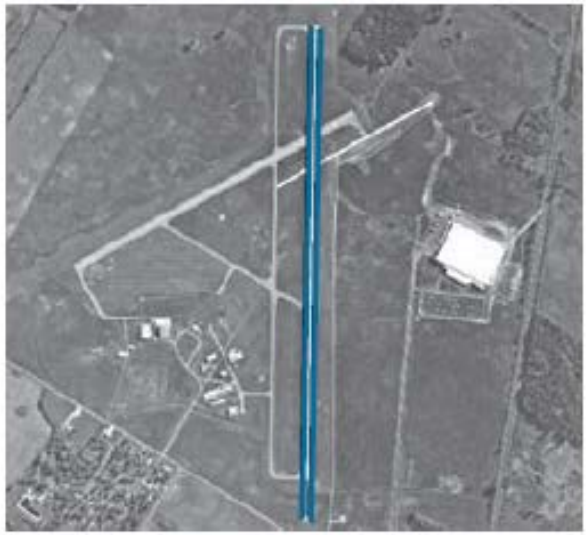
Linked pixel

حاوی بیشترین نقطه

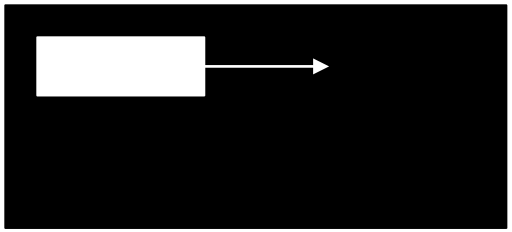
Hough transform

threshold





Use of motion in segmentation

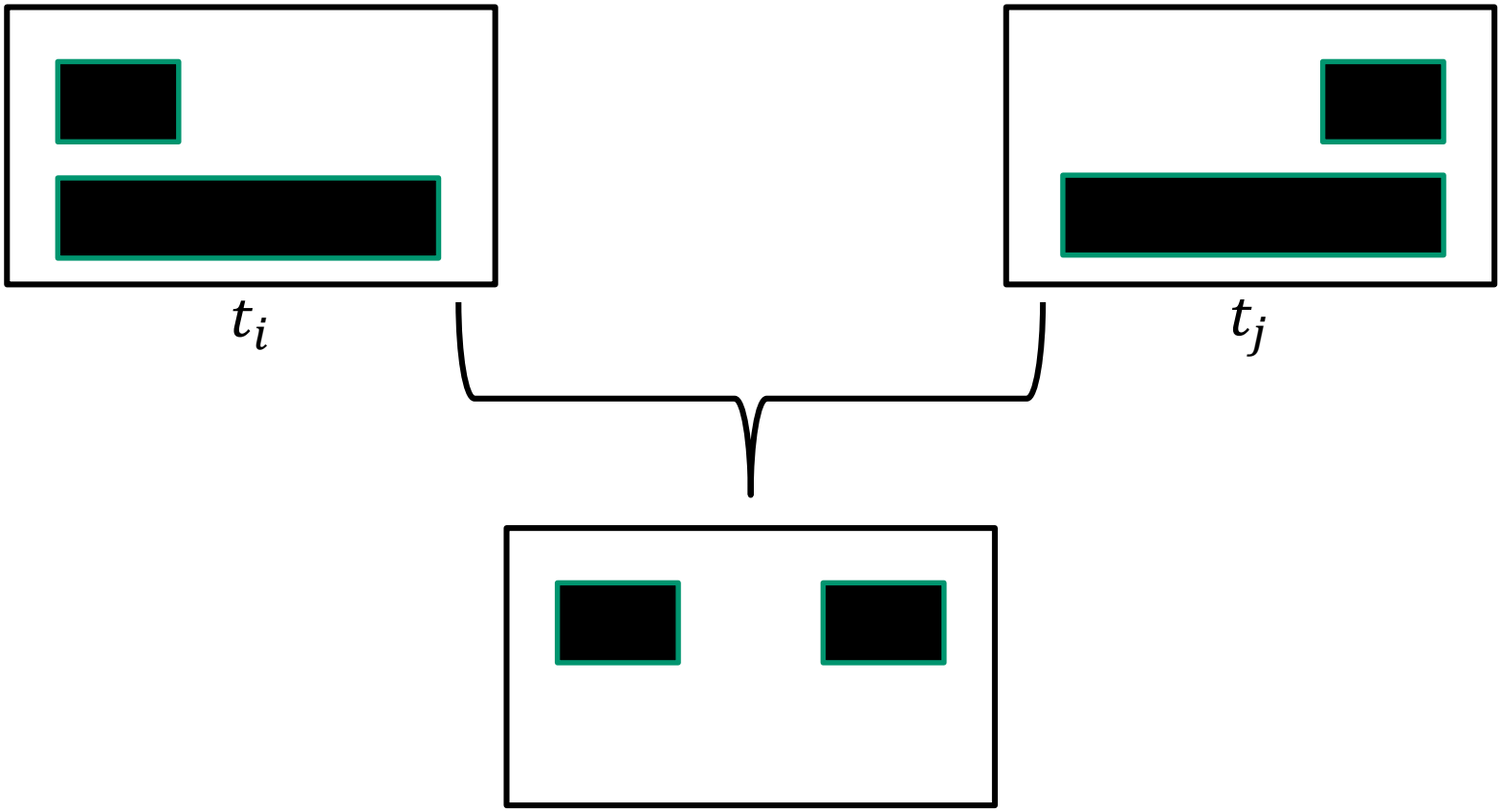


Use of motion in segmentation

t_i

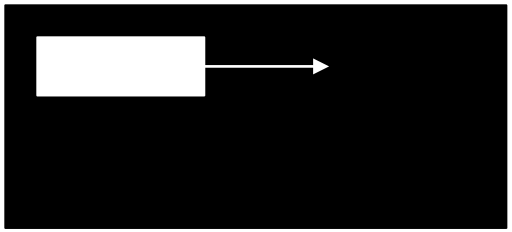
t_j

Use of motion in segmentation



$$d_{ij}(x,y) = \begin{cases} 1 & if\ \left|f(x,y,t_i) - f\left(x,y,t_j\right)\right| > T \\ 0 & Otherwise \end{cases}$$

Use of motion in segmentation



$$d_{ij}(x,y) = \begin{cases} 1 & if\ \left|f(x,y,t_i) - f\left(x,y,t_j\right)\right| > T \\ 0 & Otherwise \end{cases}$$

Accumulative difference images (ADIs):

تصویر اولیه

Absolute:

$$A_k(x,y) = \begin{cases} A_{k-1}(x,y) + 1 & if\ |R(x,y) - f(x,y,k)| > T \\ A_{k-1}(x,y) & Otherwise \end{cases}$$

فریم k ام

Positive:

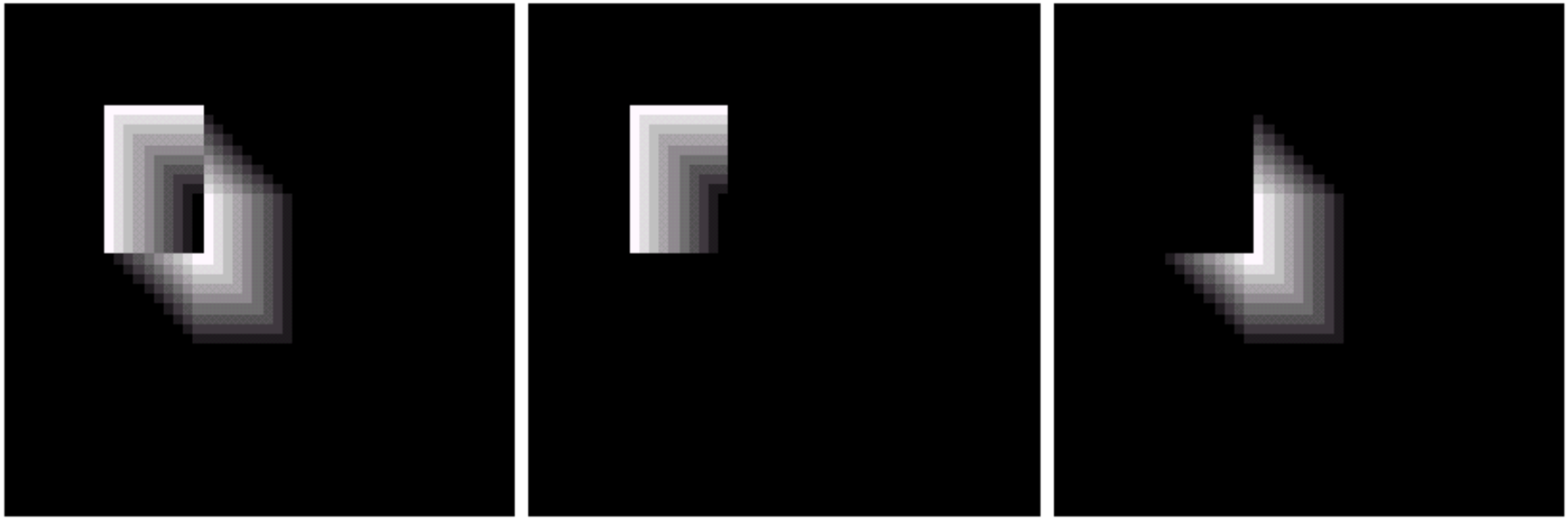
$$P_k(x,y) = \begin{cases} P_{k-1}(x,y) + 1 & if\ [R(x,y) - f(x,y,k)] > T \\ P_{k-1}(x,y) & Otherwise \end{cases}$$

Negative:

$$N_k(x,y) = \begin{cases} N_{k-1}(x,y) + 1 & if\ [R(x,y) - f(x,y,k)] < -T \\ N_{k-1}(x,y) & Otherwise \end{cases}$$

Use of motion in segmentation

Accumulative difference images (ADI)



Absolute ADI = Positive ADI ∪ Negative ADI

کجا بوده و کجا داره میره

کجا بوده

مسیر نشون میده

Establishing a reference image

2 frames in a sequence

Eastbound auto subtracted and background restored

Positive گیریم و پیکسل از تصویر دوم بگذاریم توی اول

جای اولی رو میده