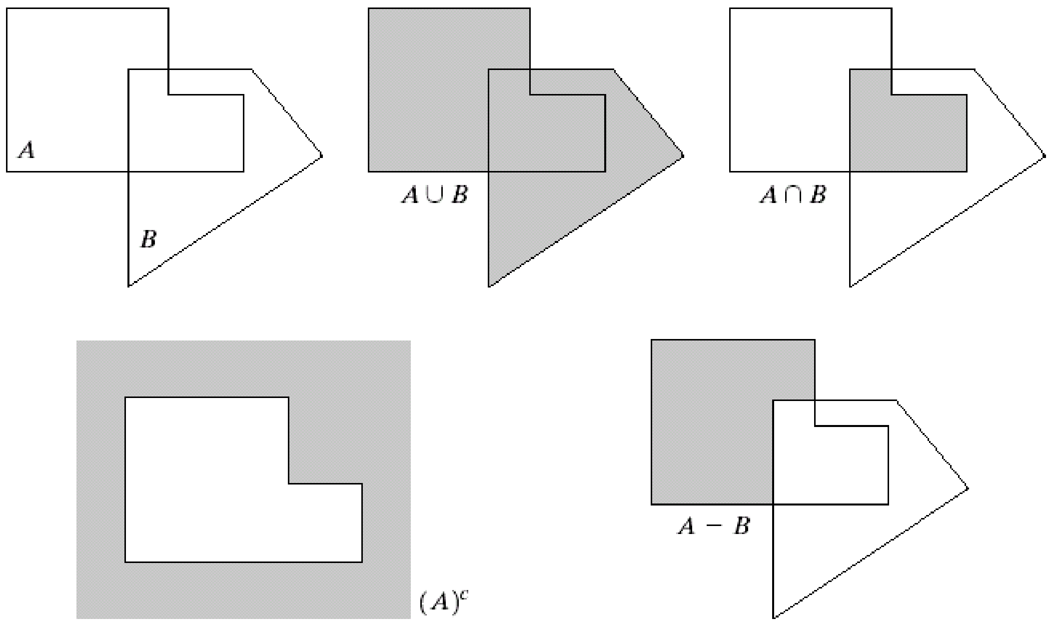
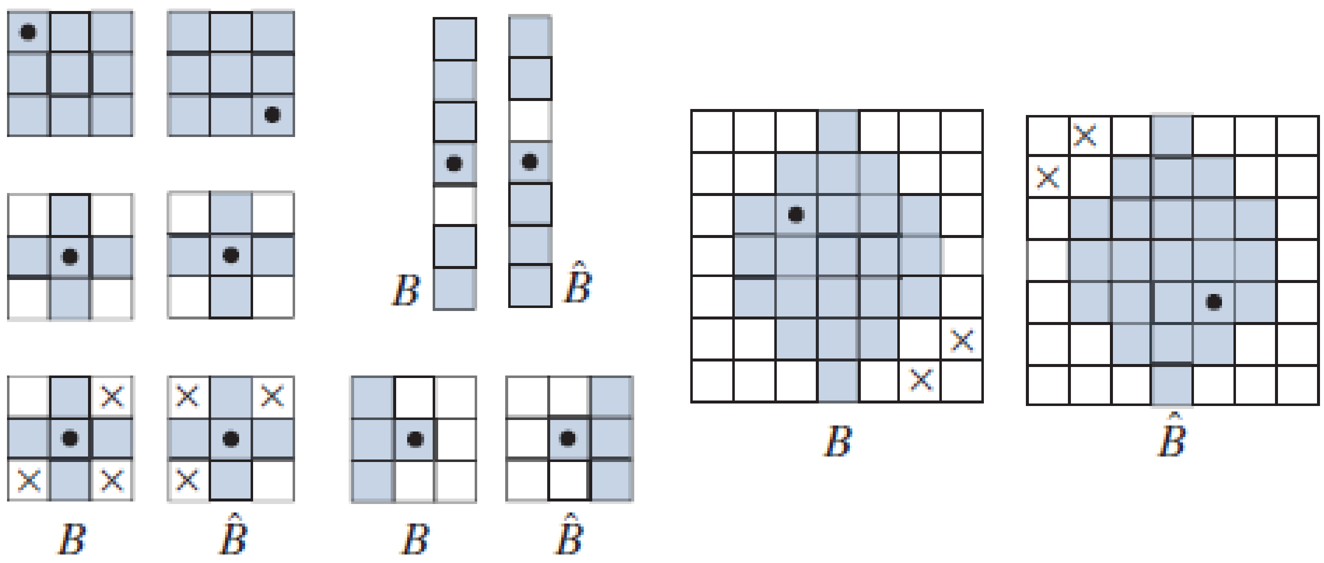
Morphological image processing

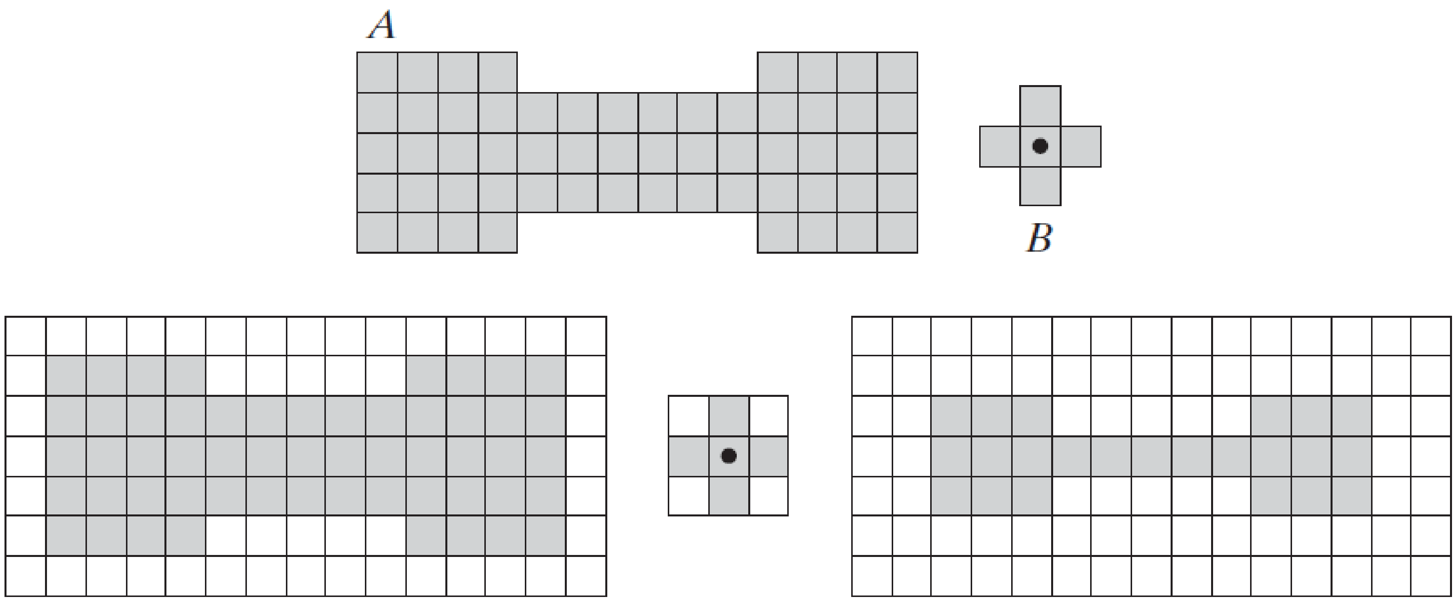
Set operations



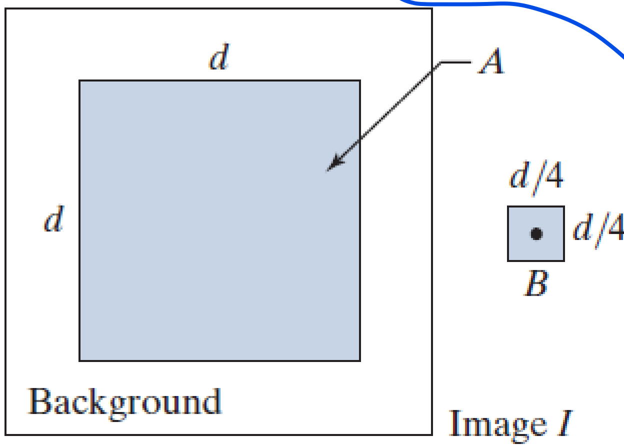
Structuring elements:



Processing a set with a structuring element:



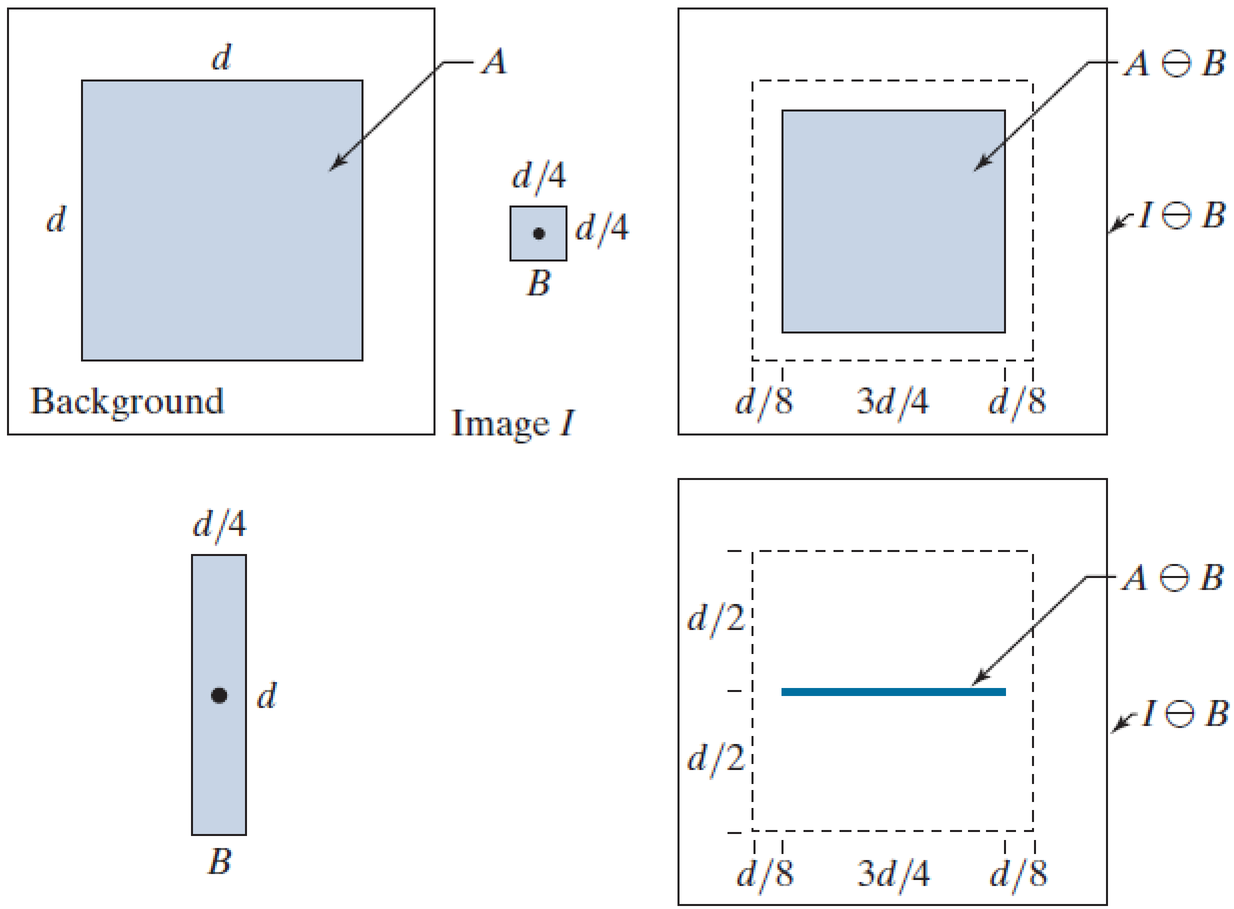
Erosion: $A \ominus B = \{z \mid (B)_z \subseteq A\}$



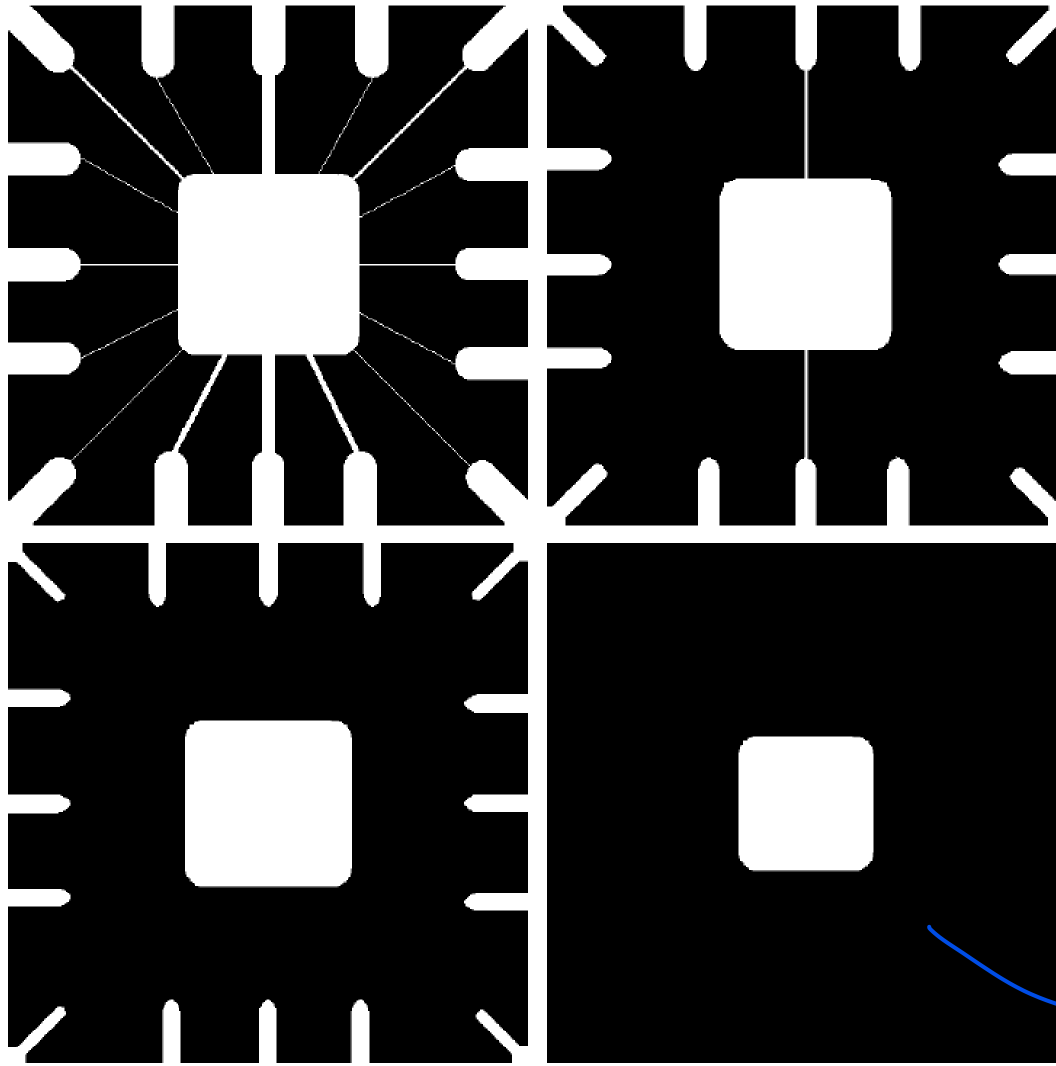
کل B تماماً باید بیافته در A

و فقط مرکزشو نگه داریم

Erosion: $A \ominus B = \{z \mid (B)_z \subseteq A\}$



Erosion
with square
structuring
elements:



11 × 11

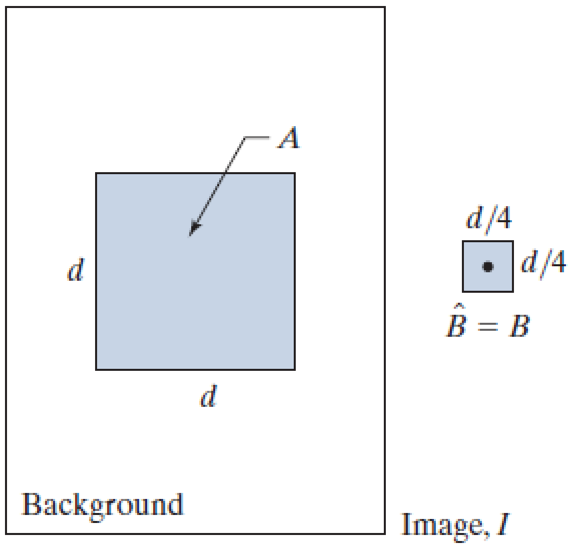
15 × 15

45 × 45

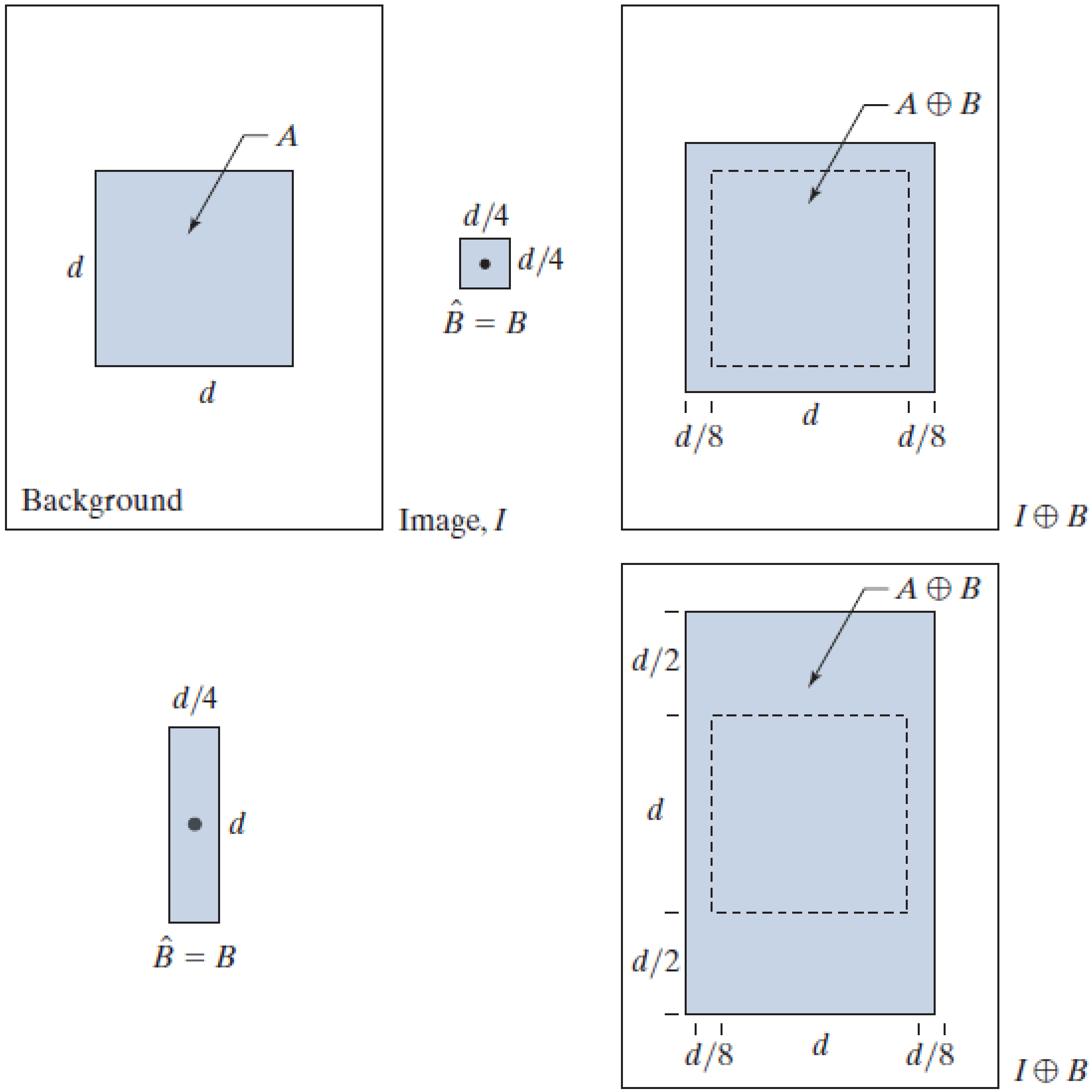
تونستیم مربع رو نگه داریم

برای حذف قسمت هایی که نمی خوایم خوبه

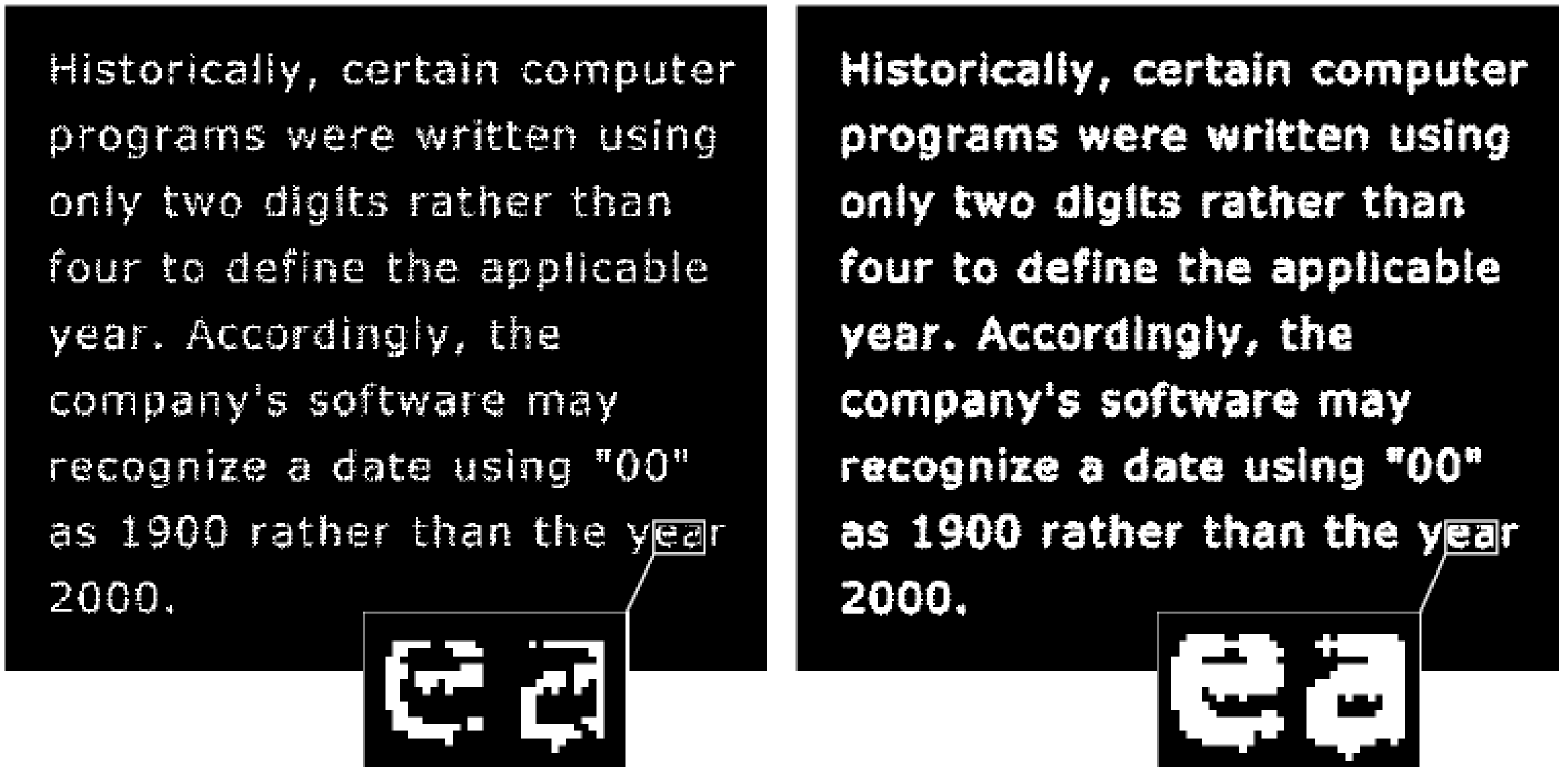
Dilation: $A \oplus B = \{z | (\hat{B})_z \cap A \neq \emptyset\}$



Dilation: $A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \emptyset\}$



Dilation:



0	1	0
1	1	1
0	1	0

Duality with respect to complementation and reflection:

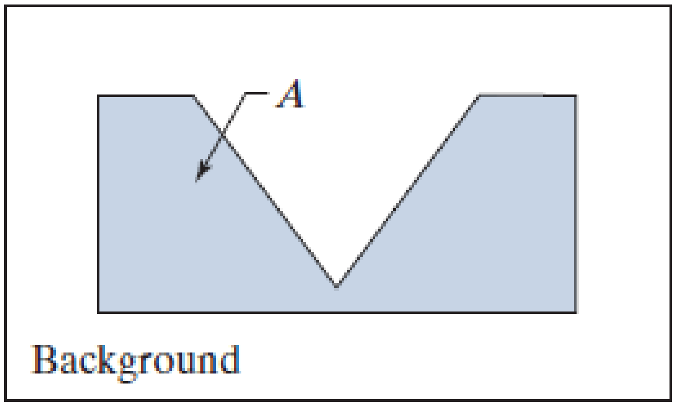
$$(A \ominus B)^c = A^c \oplus \hat{B}$$

$$(A \oplus B)^c = A^c \ominus \hat{B}$$

Opening: $A \circ B = (A \ominus B) \oplus B$

$$A \circ B = \bigcup \{(B)_z \mid (B)_z \subseteq A\}$$

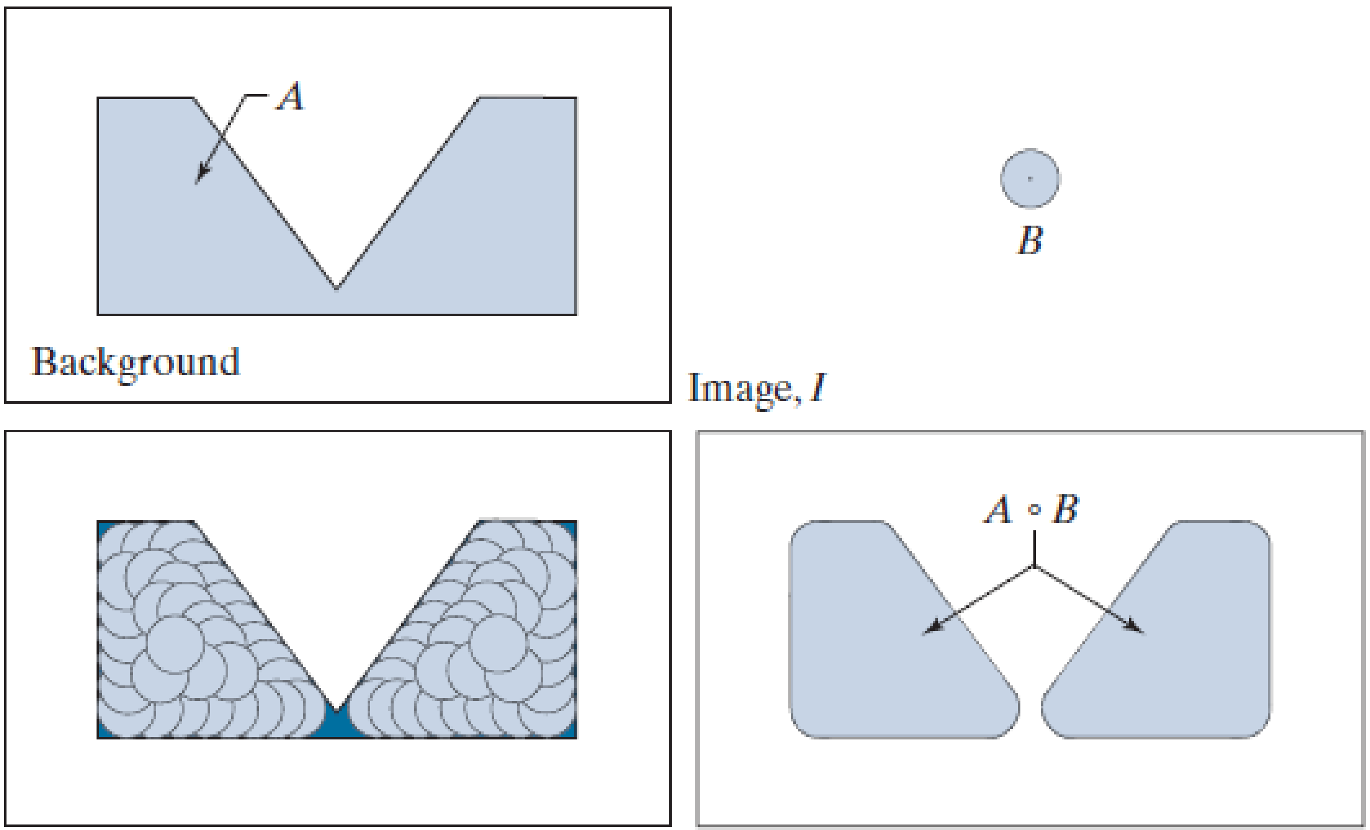
ابعاد تصویر تغییر
نمی‌کنه
فقط ممکنه (بسته به عرض)
لبه‌ها گرد شوند که ابعاد کم باشه



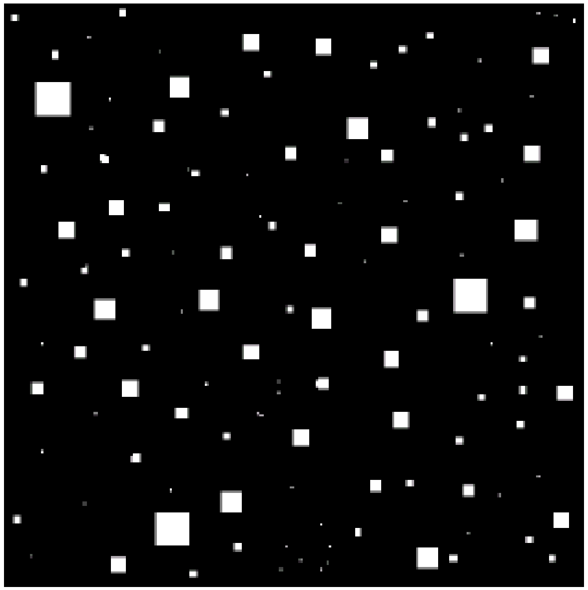
یکی قوانین دو بعد ثو دارد

Opening: $A \circ B = (A \ominus B) \oplus B$

$$A \circ B = \bigcup \{(B)_z \mid (B)_z \subseteq A\}$$

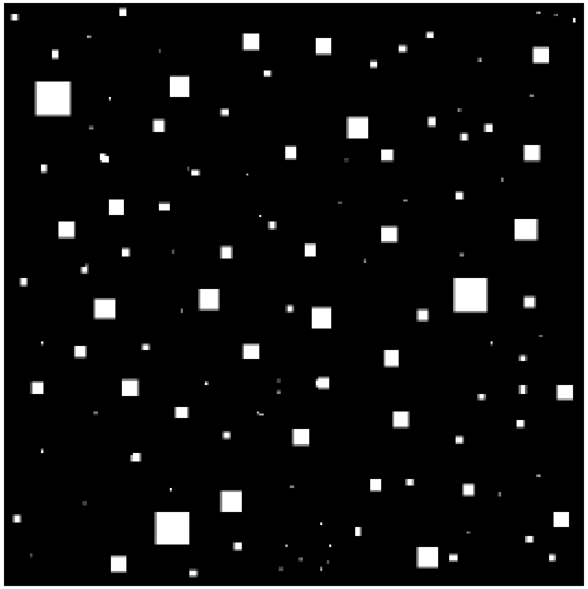


Applying erosion and dilation

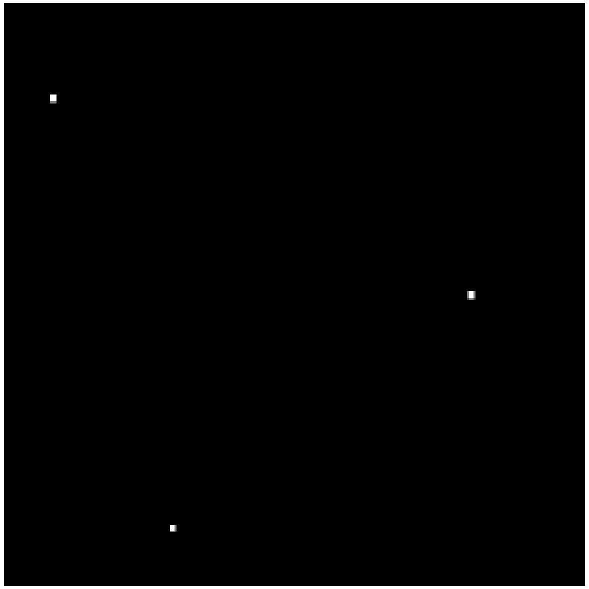


Squares with side sizes:
1, 3, 5, 7, 9, and 15 pixels

Applying erosion and dilation



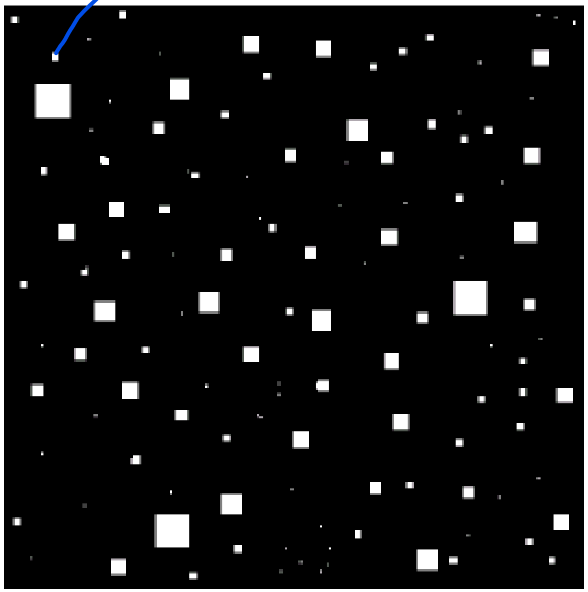
Squares with side sizes:
1, 3, 5, 7, 9, and 15 pixels



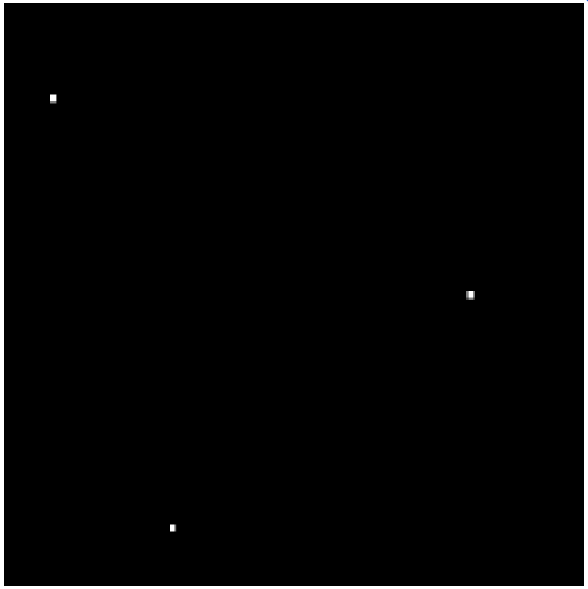
Erosion with square
Structuring element of ones
(side size: 13 pixels)

Applying erosion and dilation

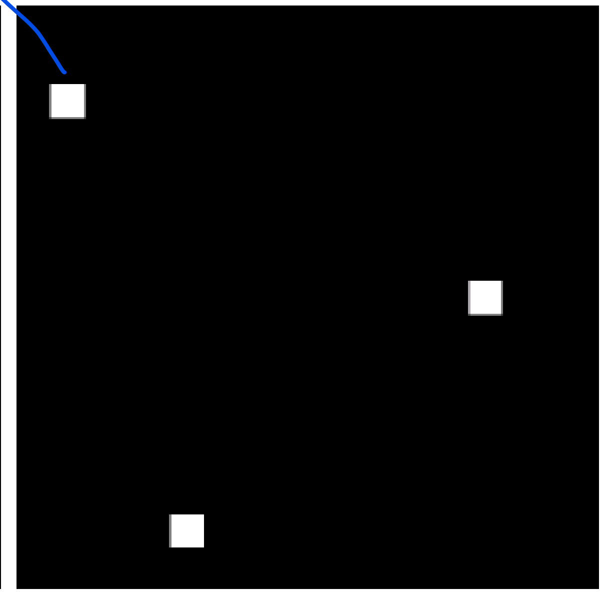
(گوشه‌ها تغییر نکردند (سایزها یکی اند



Squares with side sizes:
1, 3, 5, 7, 9, and 15 pixels

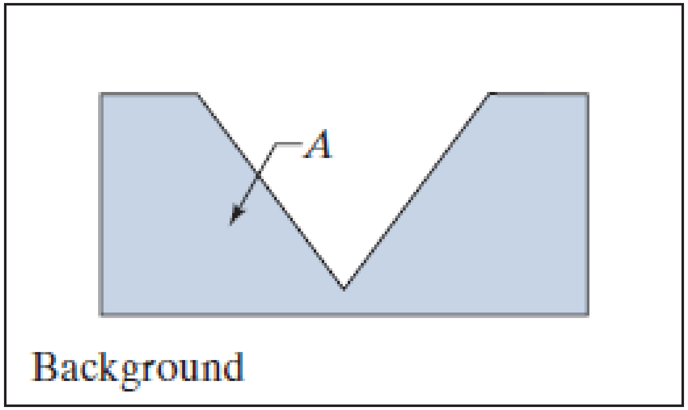


Erosion with square
Structuring element of ones
(side size: 13 pixels)



Dilation with the same
Structuring element

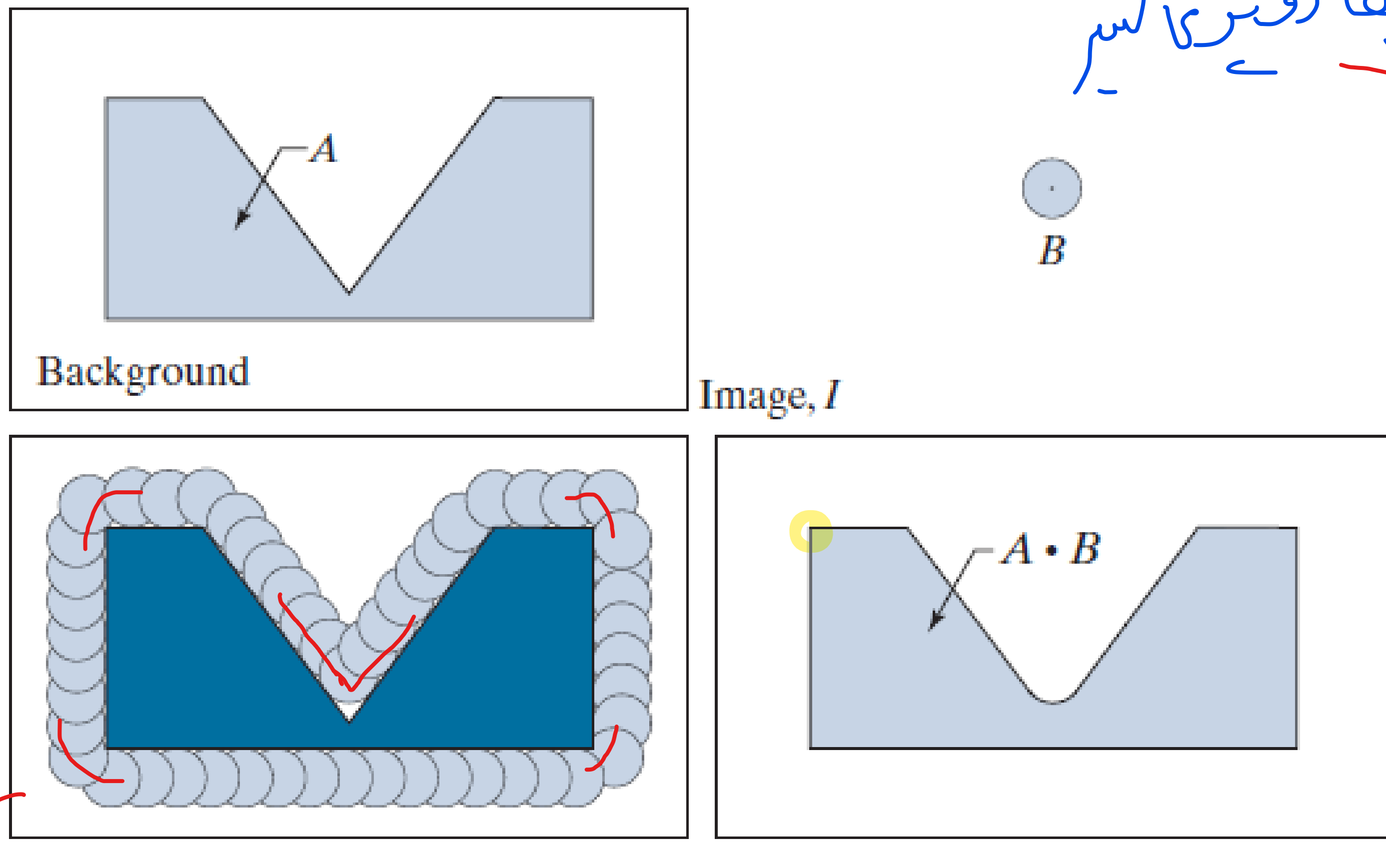
Closing: $A \bullet B = (A \oplus B) \ominus B$



لقه‌ها را پر می‌کند

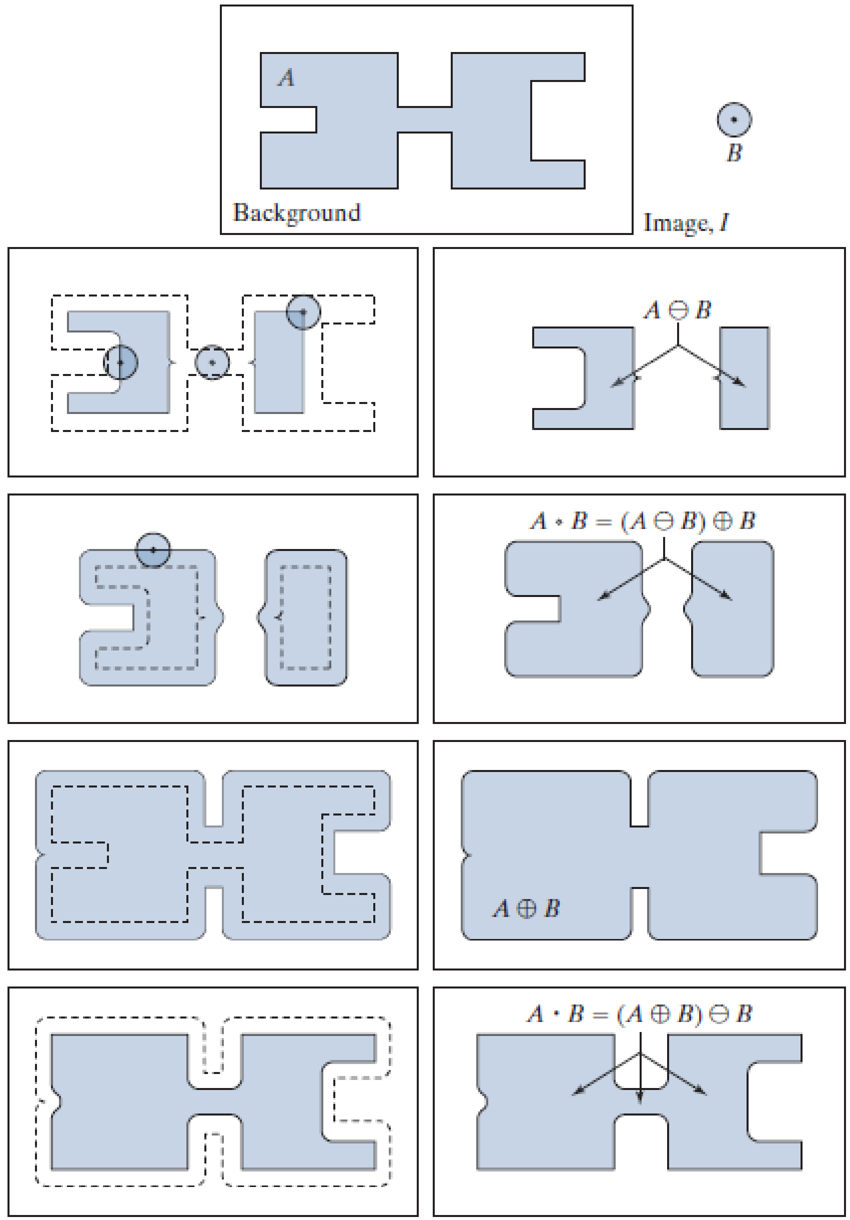
Closing: $A \bullet B = (A \oplus B) \ominus B$

المان رو بیرون شکل حرکت می‌دیم
و لقه‌ها رو پر می‌کنیم



Dilation

Morphological opening and closing



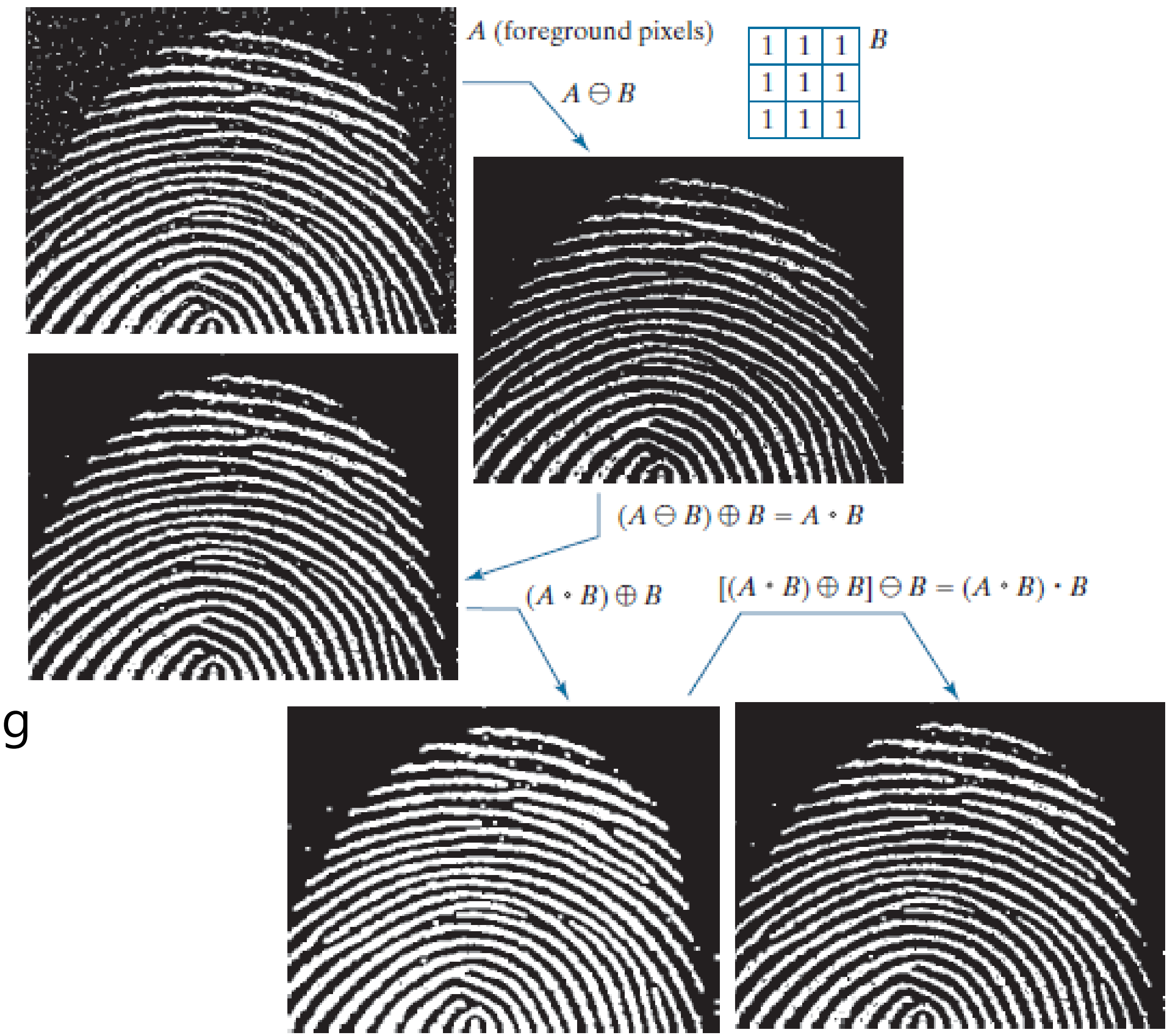
Duality with respect to complementation and reflection:

$$(A \bullet B)^c = (A^c \circ \widehat{B})$$

$$(A \circ B)^c = (A^c \bullet \widehat{B})$$

Applying opening and closing

Noisy



a b c d e f

FIGURE 9.11 (a) Noisy image. (b) Structuring element. (c) Eroded image. (d) Dilation of the erosion (opening of A). (e) Dilation of the opening. (f) Closing of the opening. (Original image courtesy of the National Institute of Standards and Technology.)

Opening

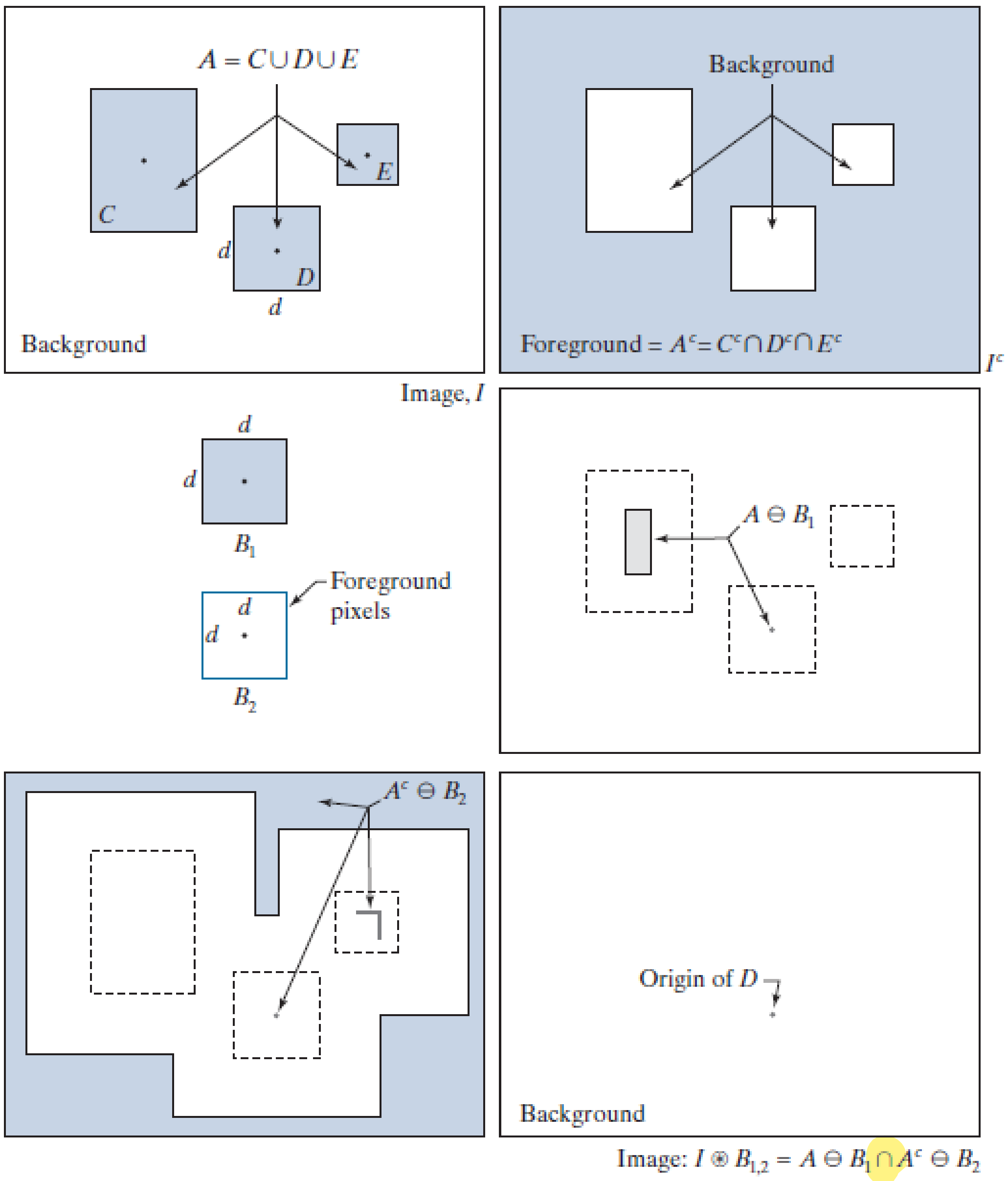
Dilation

Closing

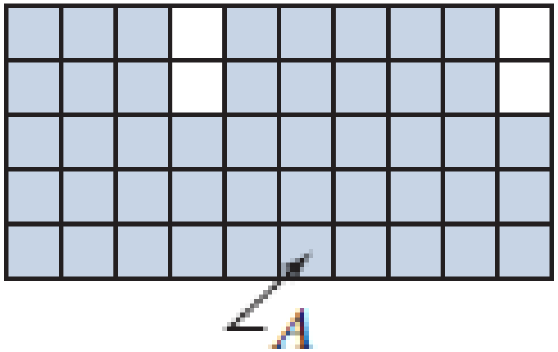
Hit or Miss Transform

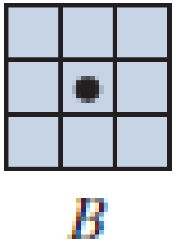
a b
c d
e f

FIGURE 9.12
(a) Image consisting of a foreground (1's) equal to the union, A, of set of objects, and a background of 0's.
(b) Image with its foreground defined as A^c.
(c) Structuring elements designed to detect object D.
(d) Erosion of A by B_1.
(e) Erosion of A^c by B_2.
(f) Intersection of (d) and (e), showing the location of the origin of D, as desired. The dots indicate the origin of their respective components. Each dot is a single pixel.



Finding boundaries

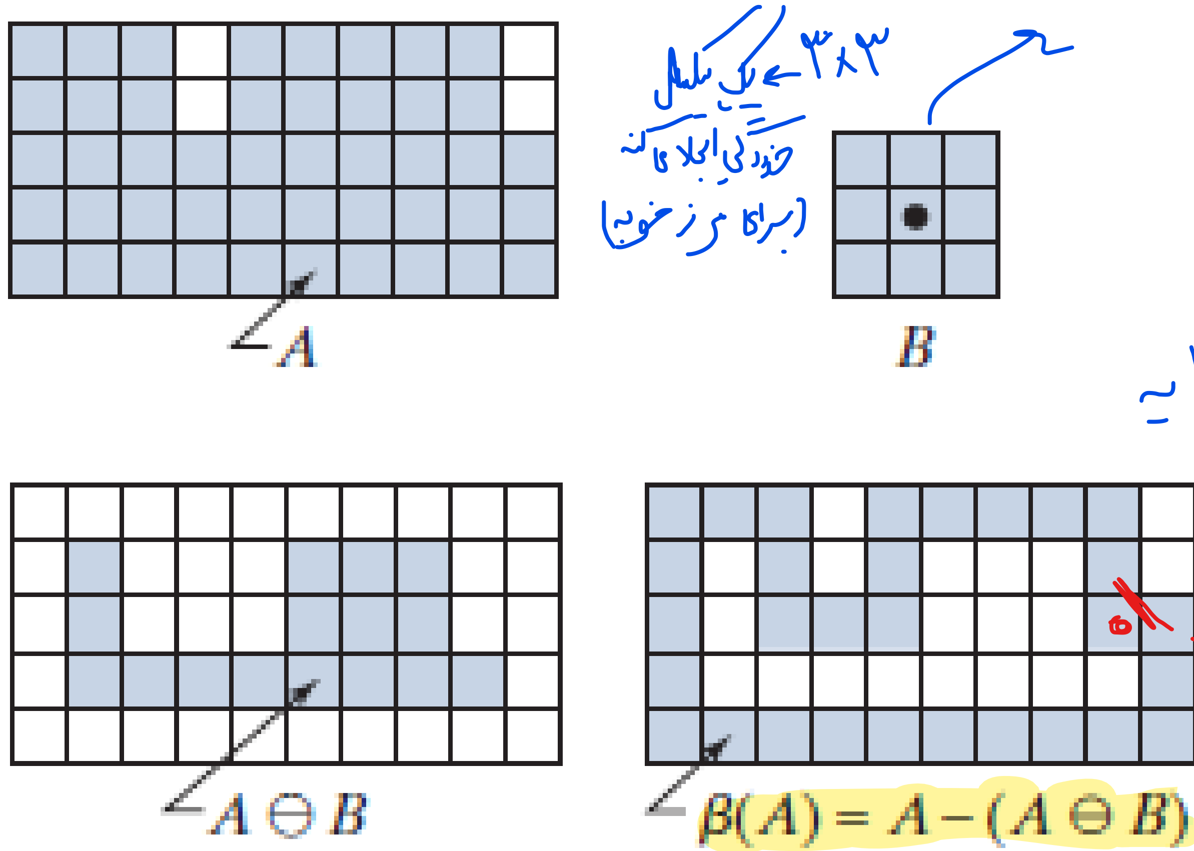




Finding boundaries

برای مرز ۴ همسایه‌ای

۳×۳ ← یک مستطیل
خودش ایجاد کن
(برای مرز خوبه)



Erosion of *A* by *B*

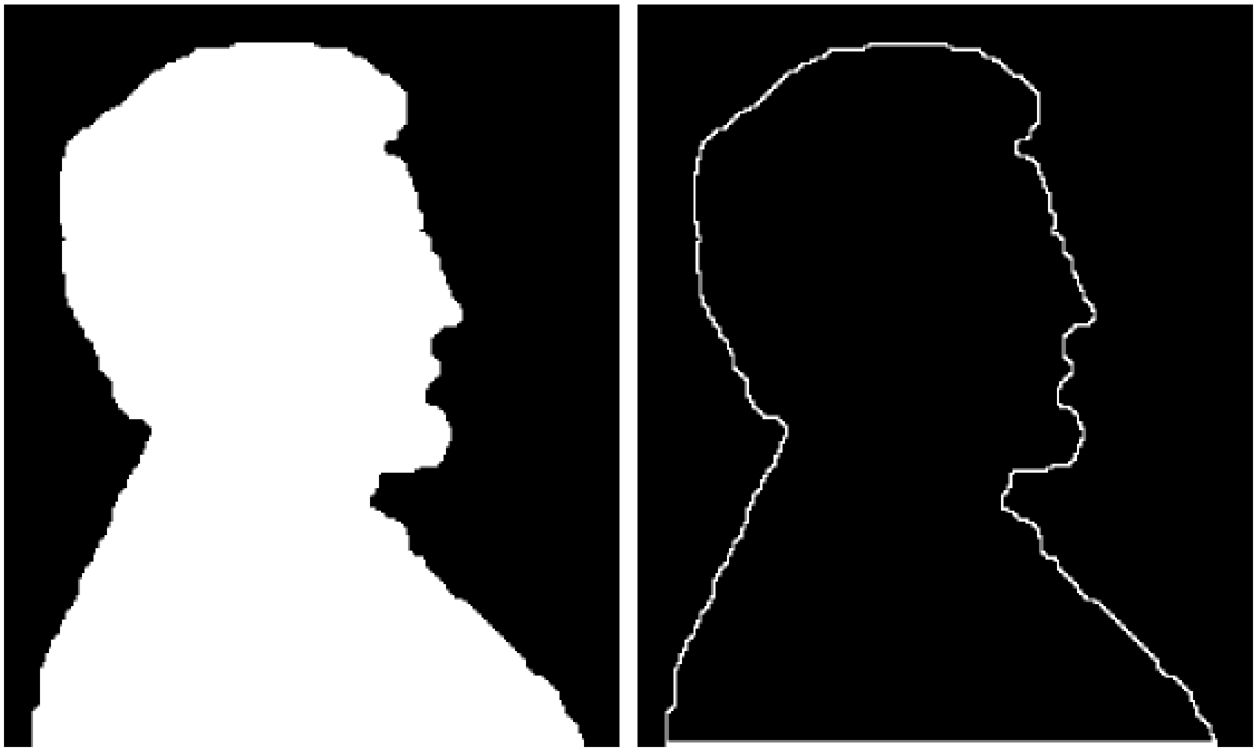
Difference between *A* and its erosion

برای مرز ۸ همسایه

قطری

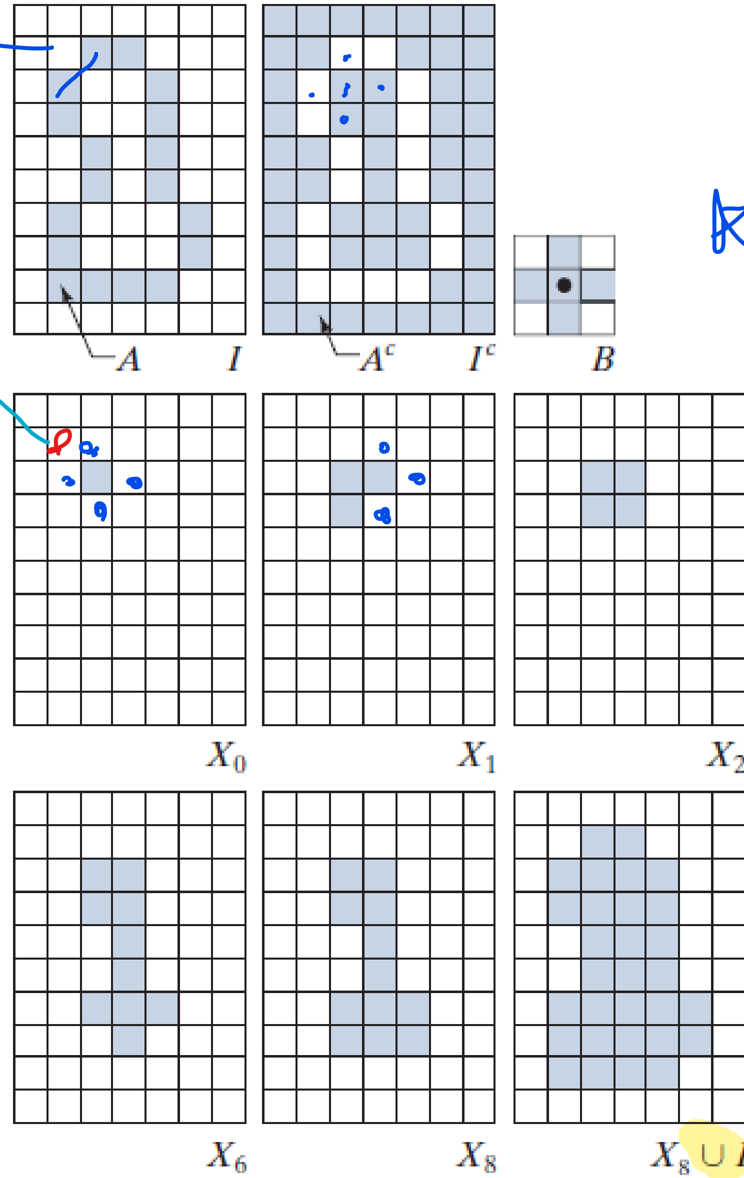
$$\beta(A) = A - (A \ominus B)$$

Finding boundaries



Ones in white

Hole filling



۸ تایی

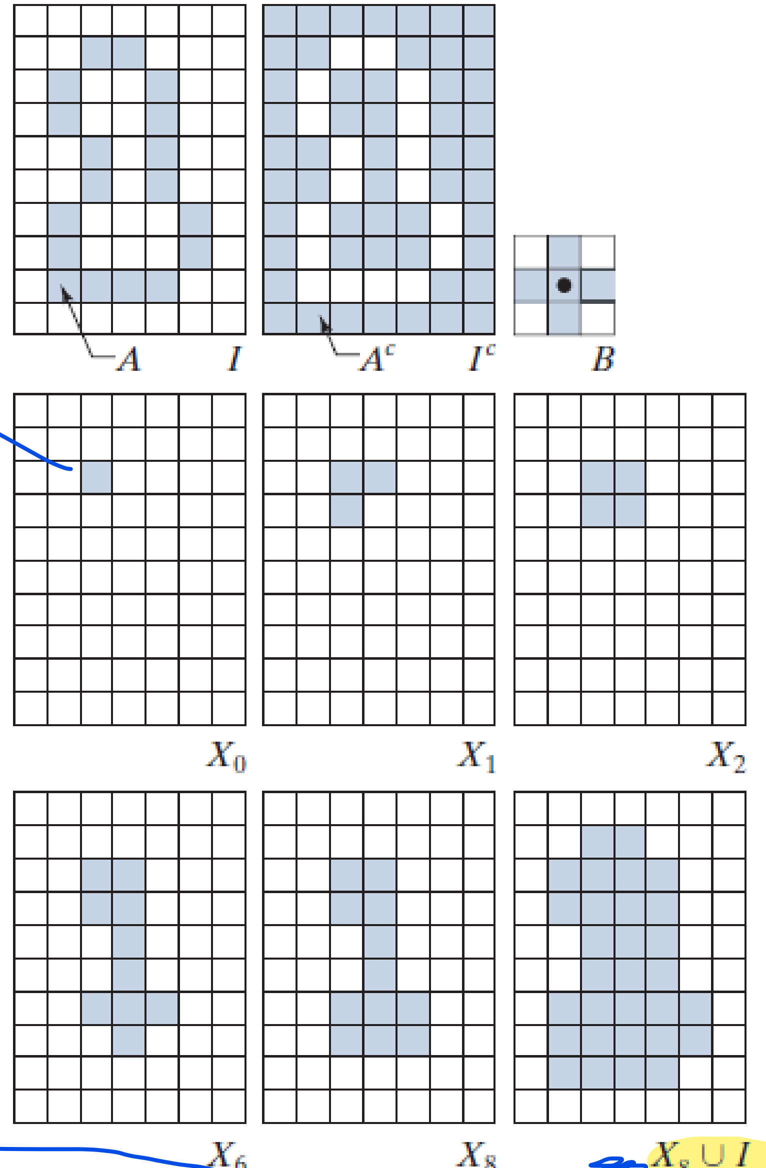
۴ با این مرز جواب نمیداد
چون این نقطه
مرز می شه

اگر این ۸ رو گرفتیم اضافه میشد

Initial point
Inside the boundary

مرزها چهارتایی بودند
یه سلولی بگی درست

Hole filling



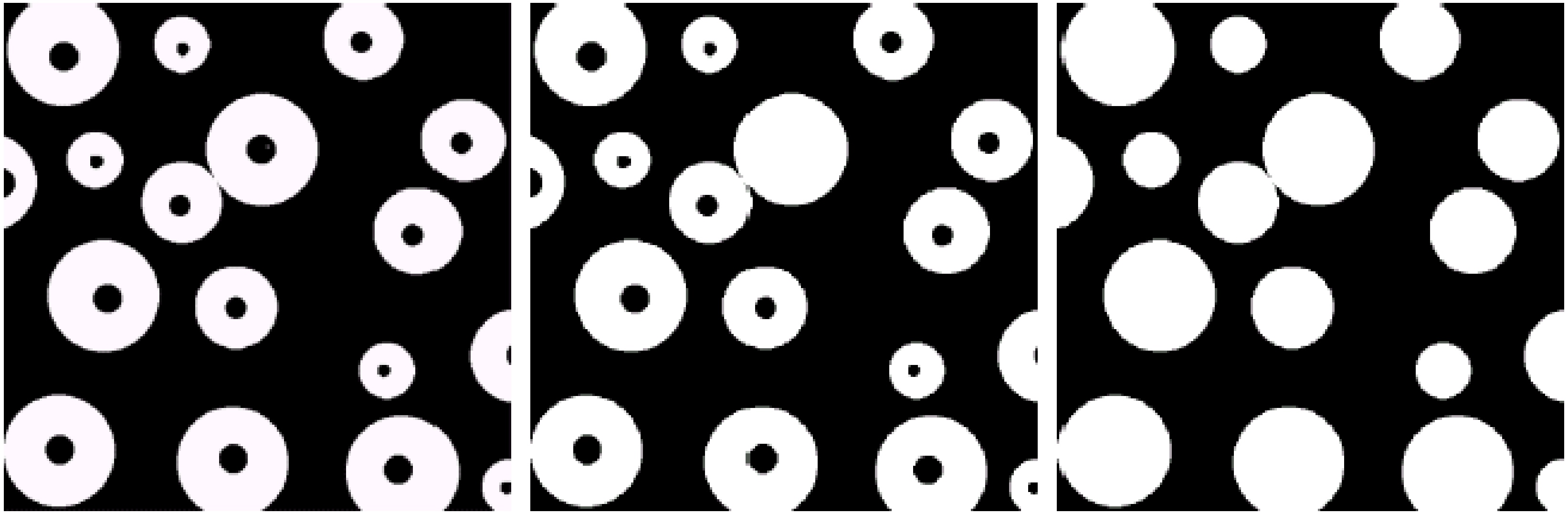
از یه نقطه داخل شکل
شروع می‌کنیم

Initial point
Inside the boundary

اگر $x_K = x_{K-1}$

$$X_k = (X_{k-1} \oplus B) \cap A^c \quad \text{در آخر} \quad k = 1, 2, 3, \ldots$$

Hole filling



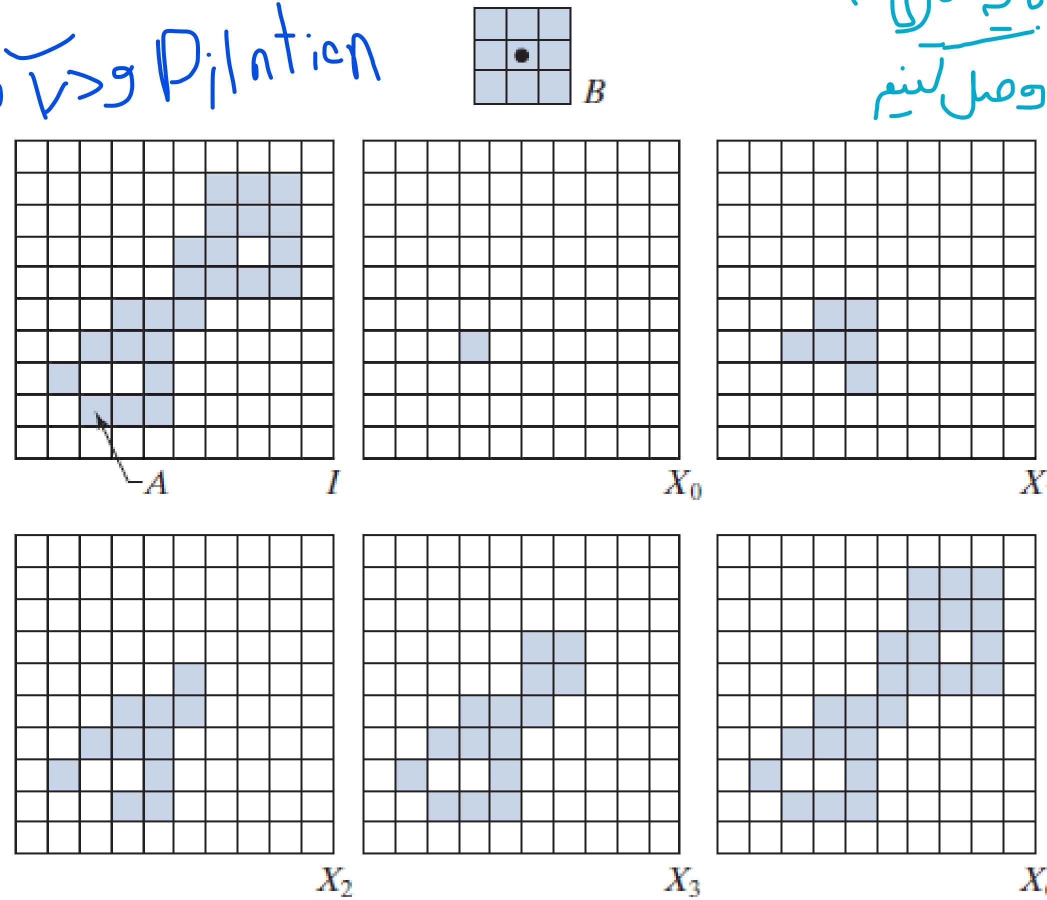
The white dot is the starting point of the region filling algorithm

Extraction of connected components

مجموعه یکپارچه

هر دو نقطه رو بتونیم با یه مسیری که تماما در مجموعه باشه وصل کنیم

اشتراک A با فرآیند Dilation رو بگیریم



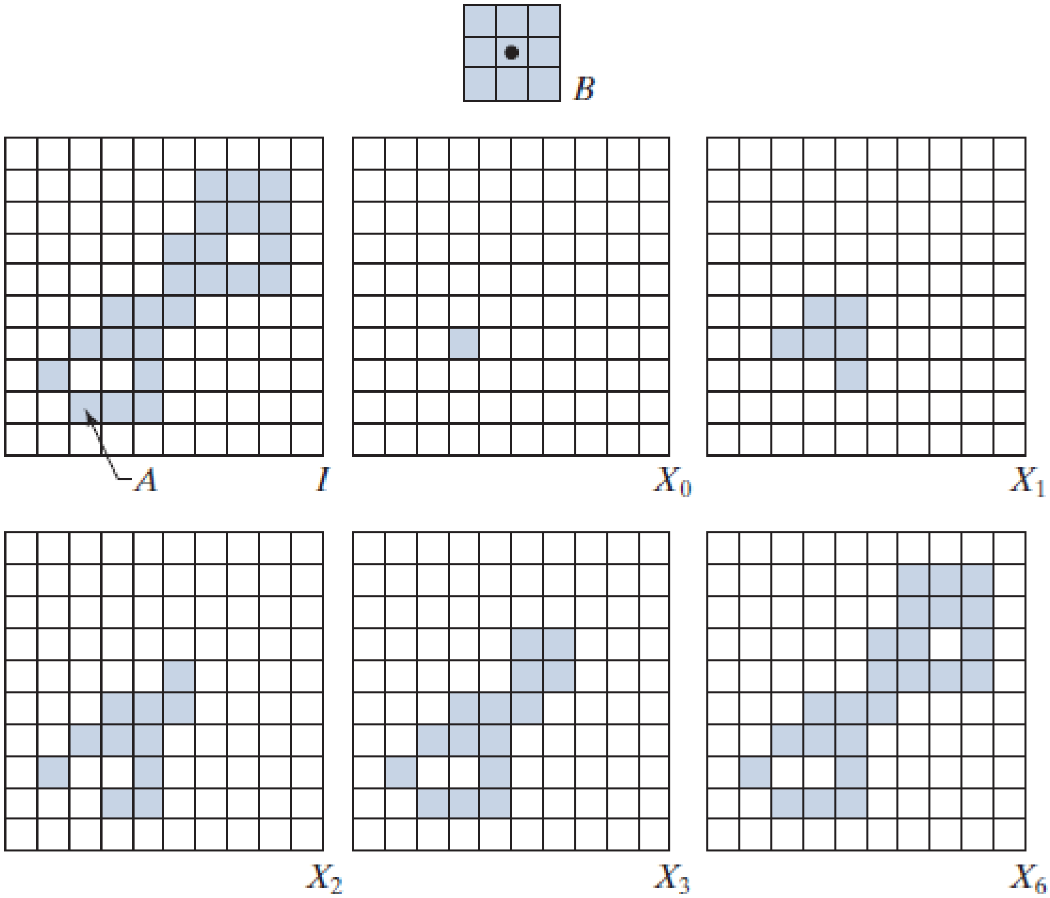
After first step

After second step

Final result

تعداد نقاطی که بهم وصل اند رو پیدا کنیم

Extraction of connected components



After first step

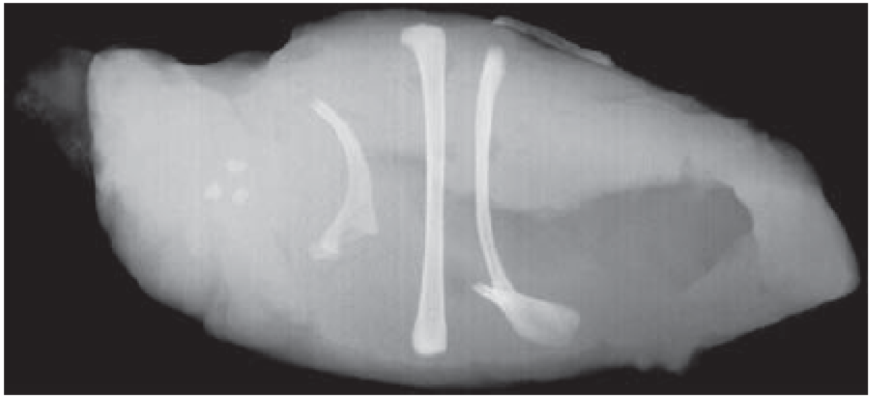
After second step

Final result

$$X_k = (X_{k-1} \oplus B) \cap A \qquad k = 1, 2, 3, \ldots$$

Extraction of connected components

X-ray image



Thresholded

Eroded with a 5x5 struct. el. of ones

# of pixels in the connected components

Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

Convex hull

hit or miss

Struct. el.

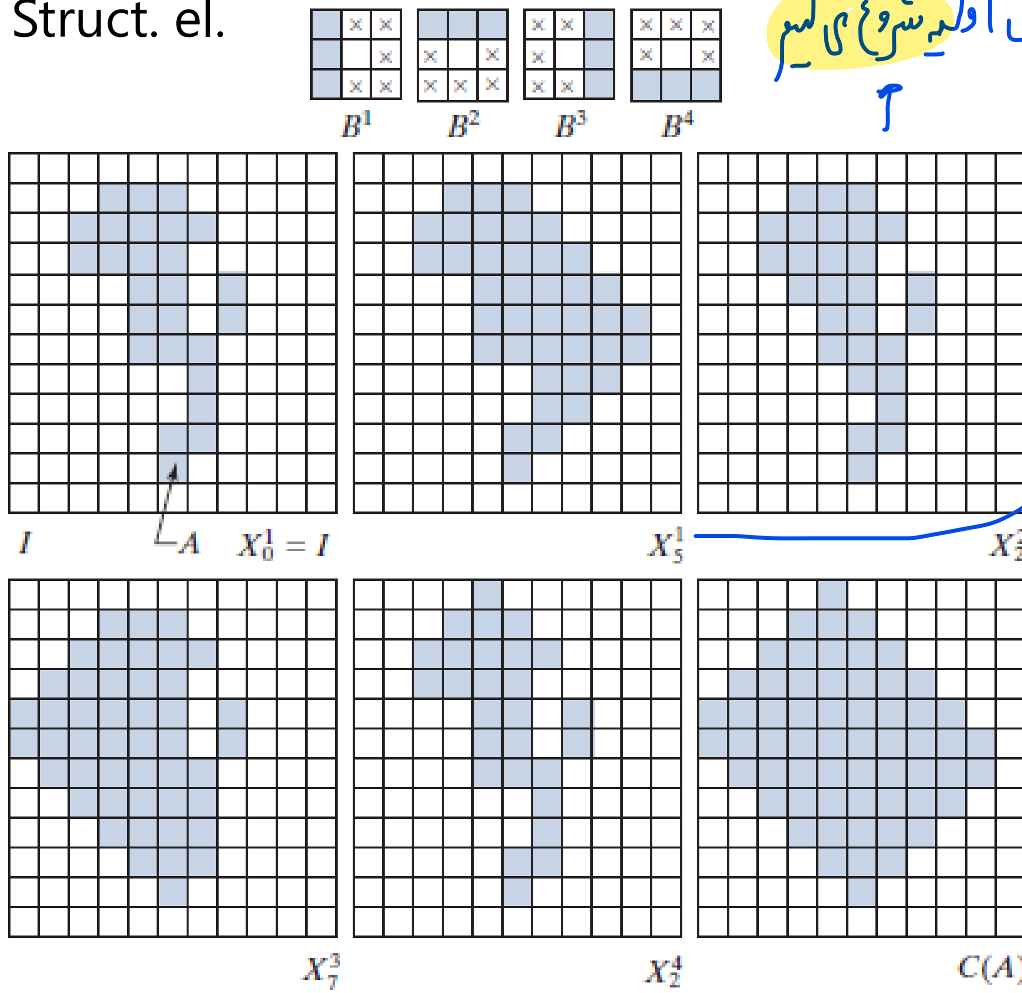
هر بار با B_i جلو رفتیم

از شکل اولیه شروع می‌کنیم

یه بار با B_1

یه بار با B_2

در آخر اجتماع می‌گیریم



مینیمم‌ترین حالت نیست

Convex hull

اجتماع حالت‌های دیگه

match با شکل

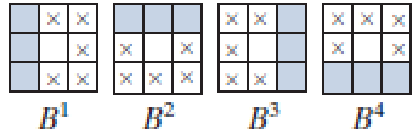
قسمت‌های خالی رو پر می‌کنیم

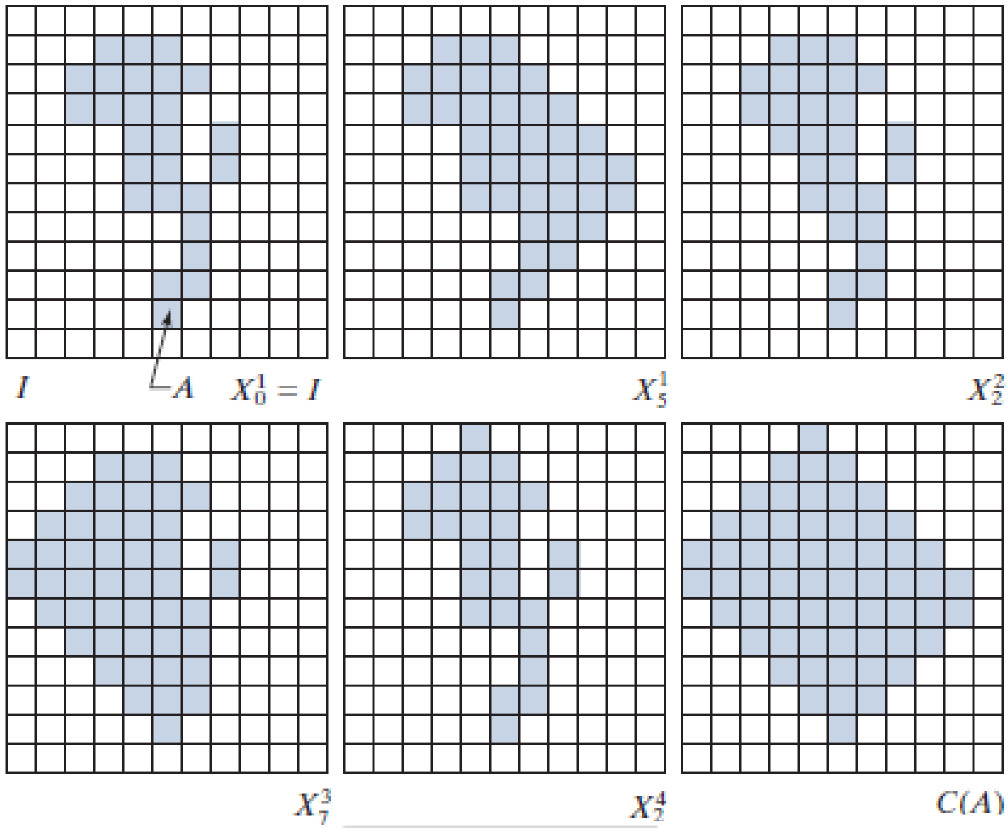
$$X_k^i = (X_{k-1} \circledast B^i) \cup A \qquad i = 1,2,3,4 \;\; and \;\; k = 1,2,3,\ldots$$

$X_0^i = A$ until: $X_k^i = X_{k-1}^i$ $\qquad D^i = X_k^i$ $\qquad C(A) = \bigcup_{i=1}^{4} D^i$

Convex hull

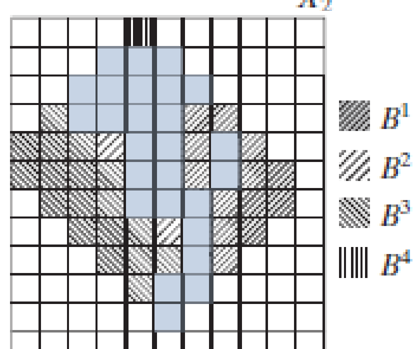
Struct. el.

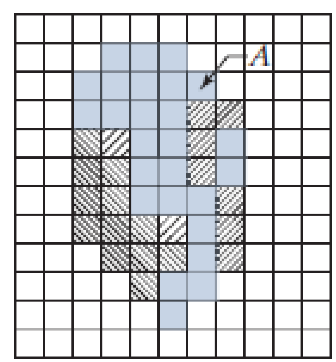




Convex hull

Showing contributions



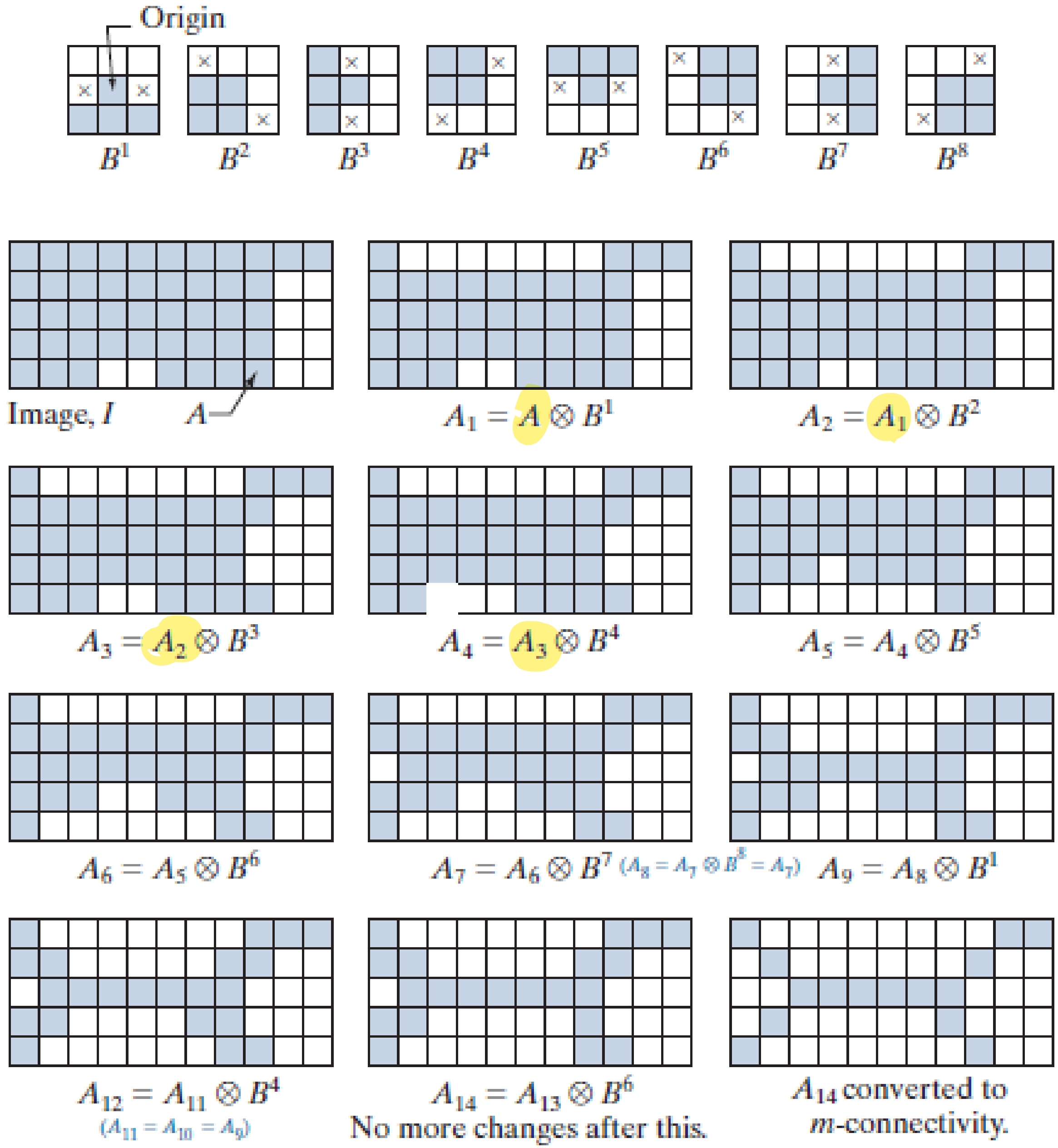


Limiting growth
In the convex hull
algorithm

Thinning

match شد حذفش می‌کنیم

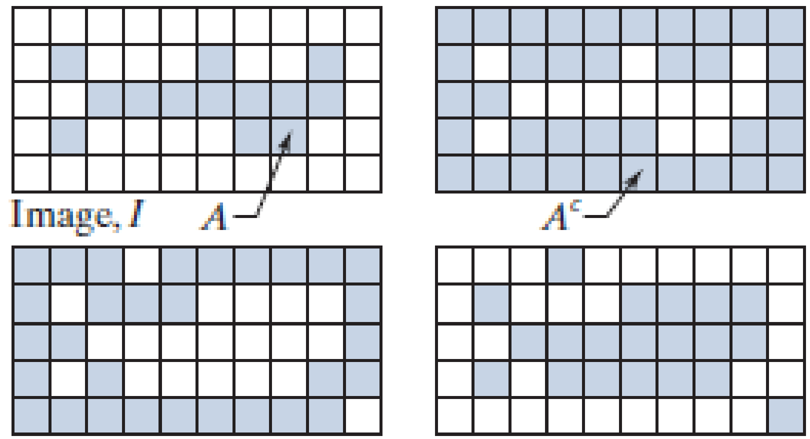
$$A \otimes B = A - (A \circledast B)$$
$$= A \cap (A \circledast B)^c$$



$$\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$$

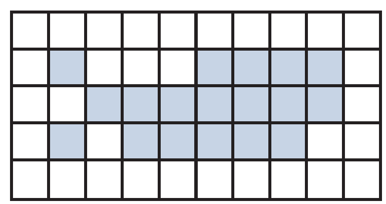
$$A\otimes\{B\} = ((\ldots((A\otimes B^1)\otimes B^2)\ldots)\otimes B^n)$$

Thickening



Thinning the complement of the set

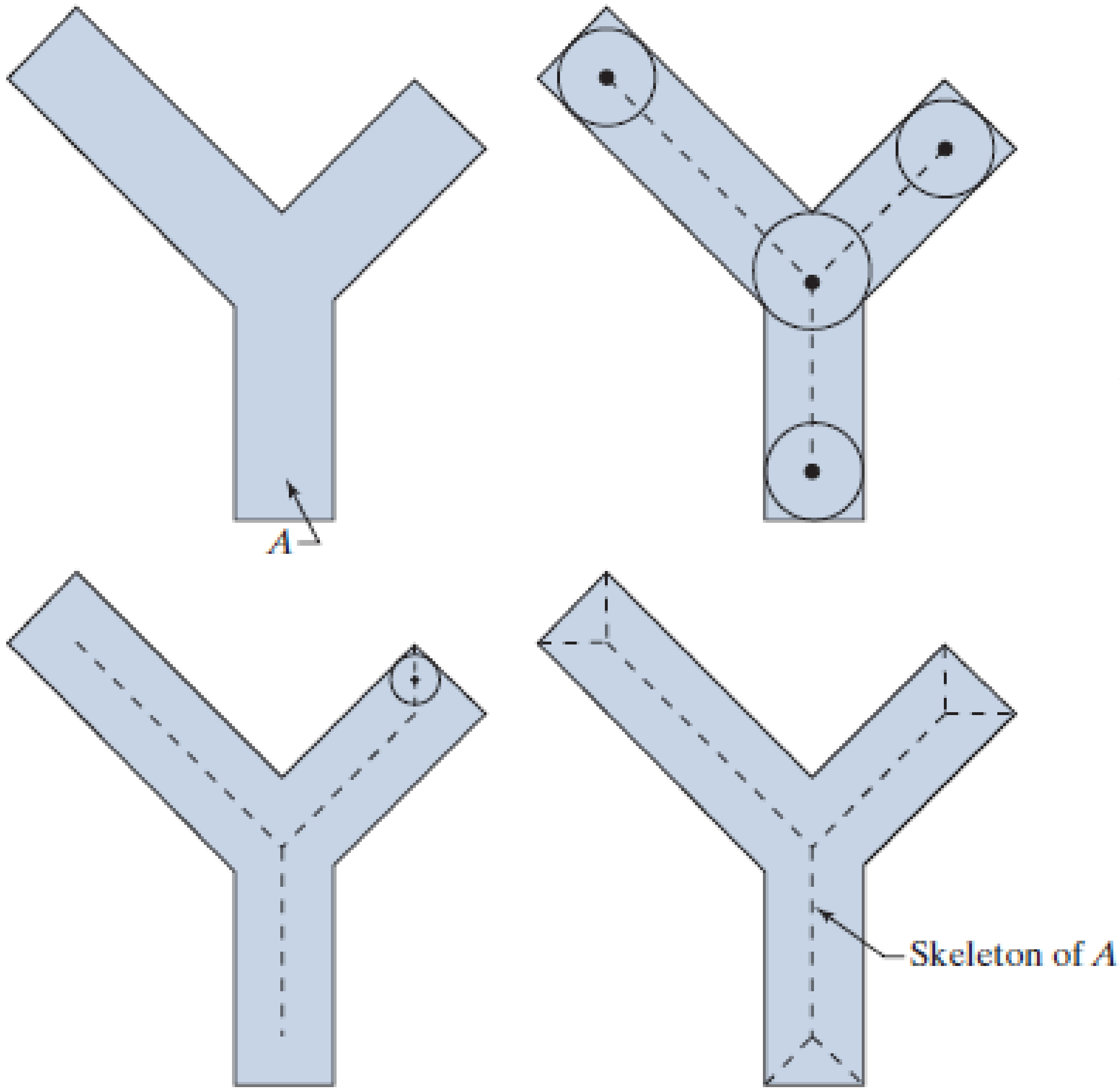
Eliminating the disconnected points



$$A \odot B = A \cup (A \circledast B)$$

$$A \odot \{B\} = ((\dots((A \odot B^1) \odot B^2) \dots) \odot B^n)$$

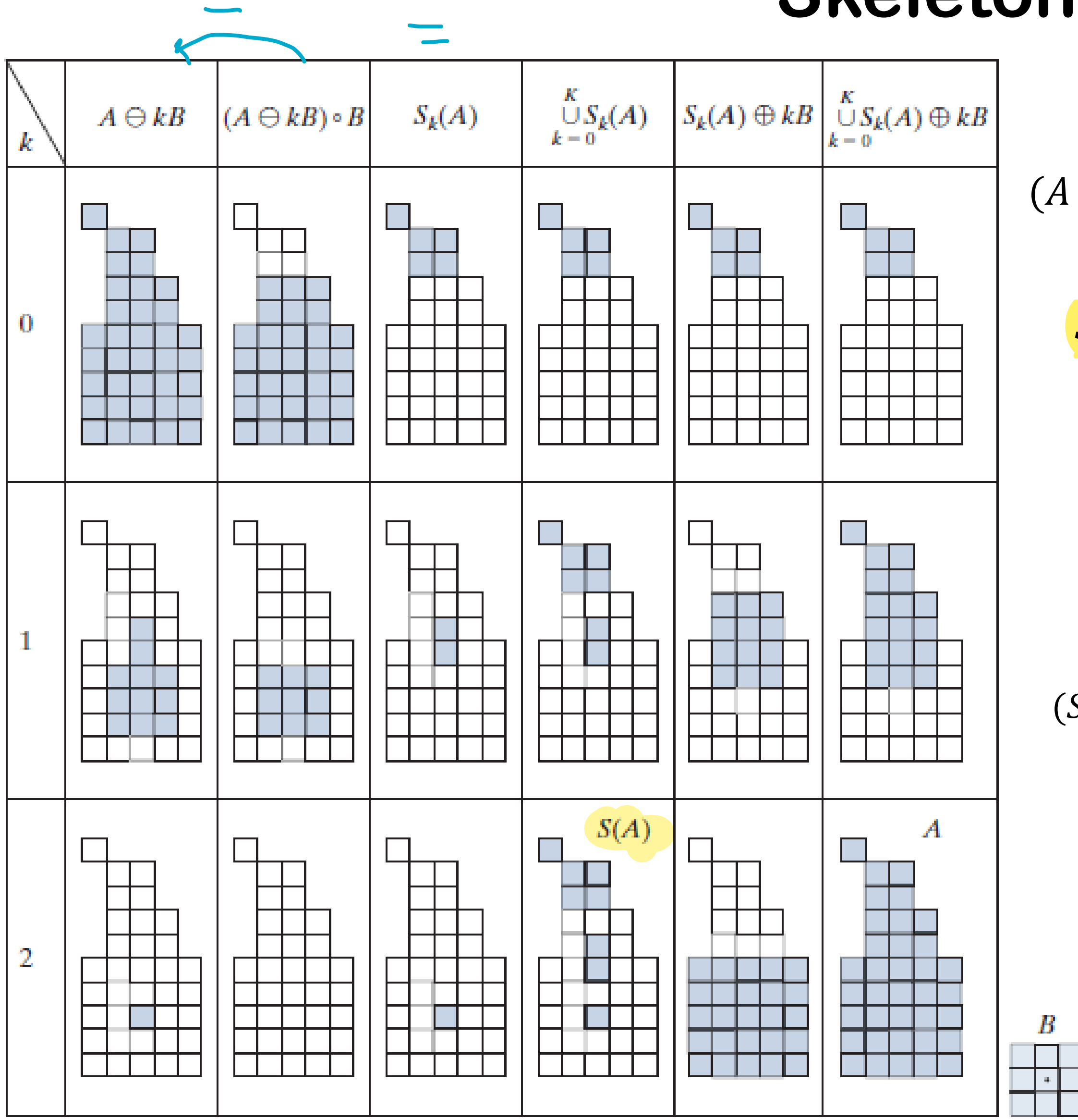
Skeleton



Maximum disks with centers on the skeleton

Complete skeleton

Skeleton



Morphological skeleton

Reconstructed

$$S(A) = \bigcup_{k=0}^{K} S_k(A)$$

$$(A - kB) = ((\ldots((A \ominus B) \ominus B) \ominus \cdots) \ominus B)$$

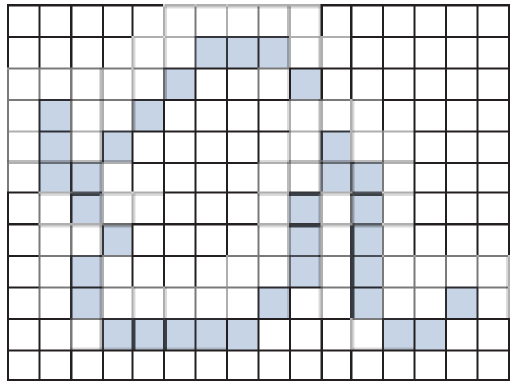
$$S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$$

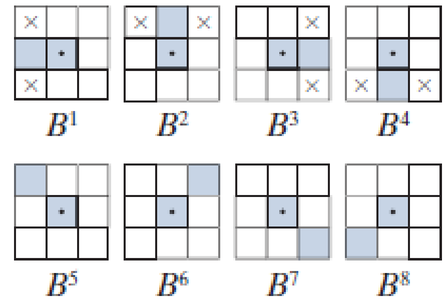
$$K = \max\{k \mid (A \ominus kB) \neq \emptyset\}$$

$$A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$$

$$(S_k(A) \oplus kB) = ((\ldots((S_k(A) \oplus B) \oplus B) \oplus \ldots) \oplus B)$$

Pruning

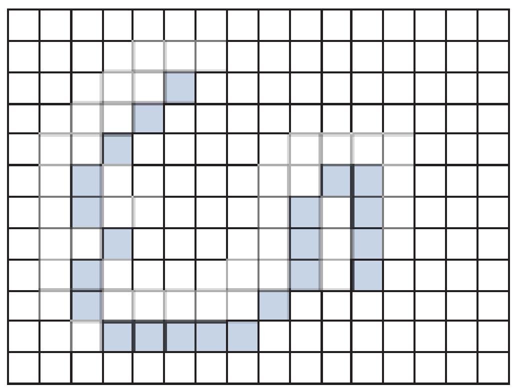


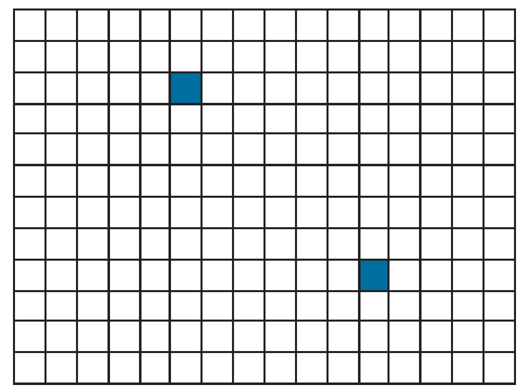


Struct. el.
Used for deleting
end points

After three cycles
Of thinning

$$X_1 = A \otimes \{B\}$$



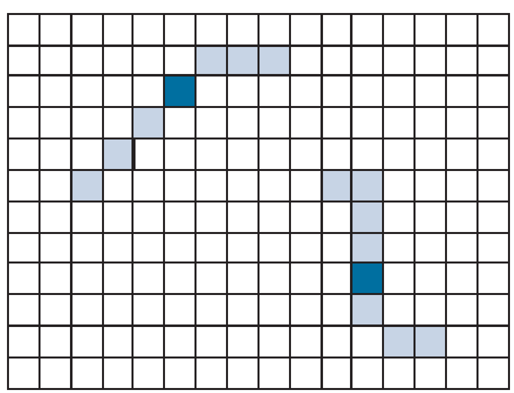


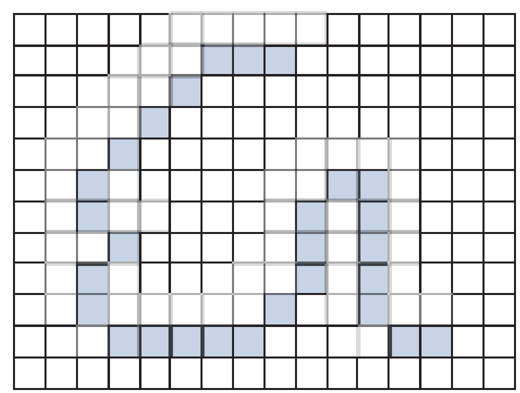
End points

$$X_2 = \bigcup_{k=1}^{8} (X_1 \circledast B^k)$$

Dilation of
end points

$$X_3 = (X_2 \oplus H) \cap A$$

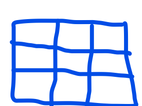




Pruned

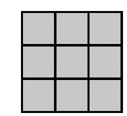
$$X_4 = X_1 \cup X_3$$

H →



Morphological operations and their properties

Translation	$(A)_Z = \{w \mid w = a + z, \quad for\ a \in A\}$	Translates the origin of $\boldsymbol{A}$ to point z.
Reflection	$\hat{B} = \{w \mid w = -b, \quad for\ b \epsilon B\}$	Reflects all elements of $\boldsymbol{B}$ about the origin of this set.
Complement	$A^c = \{w \mid w \notin A\}$	Set of points not in $\boldsymbol{A}$.
Difference	$A - B = \{w \mid w \in A, w \notin B\} = A \cap B^C$	Set of points that belong to $\boldsymbol{A}$ but not to $\boldsymbol{B}$.
Dilation	$A \oplus B = \left\{z \mid \left(\hat{B}\right)_z \cap A \neq \emptyset\right\}$	"Expands" the boundary of $\boldsymbol{A}$. (I)
Erosion	$A \ominus B = \{z \mid (B)_z \subseteq A\}$	"Contracts" the boundary of $\boldsymbol{A}$. (I)
Opening	$A \circ B = (A \ominus B) \oplus B$	Smoothes contours, breaks narrow isthmuses, and eliminates small islands and sharp peaks. (I)
Closing	$A \bullet B = (A \oplus B) \ominus B$	Smoothes contours, fuses narrow breaks and long thin gulfs, and eliminates small holes. (I)

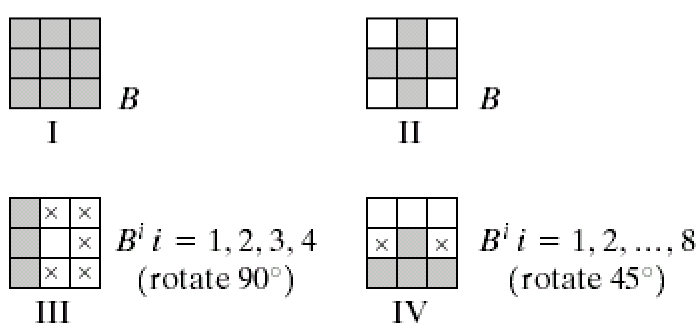


B

I

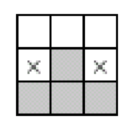
Morphological operations and their properties

Hit-or-miss transform	$A \circledast B = (A \ominus B_1) \cap (A^C \ominus B_2)$ $= (A \ominus B_1) - (A \oplus \hat{B}_2)$	The set of points (coordinates) at which, simultaneously, B_1 found a match ("hit") in A and B_2 found a match in A^c.
Boundary extraction	$\beta(A) = A - (A \ominus B)$	Set of points on the boundary of set A. (I)
Region filling	$X_k = (X_{k-1} \oplus B) \cap A^c;\ X_0 = p\ and$ k=1,2,3,…	Fills a region in A, given a point p in the region. (II)
Connected components	$X_k = (X_{k-1} \oplus B) \cap A;\ X_0 = p\ and$ k=1,2,3,…	Finds a connected component Y in A, given a point p in Y. (I)
Convex hull	$X_k^i = \left(X_{k-1}^i \circledast B^i\right) \cup A;\ I = 1,2,3,4;$ k=1,2,3,…; $X_0^I = A; and$ $D^i = X_{\text{conv}}^i.$	Finds the convex hull $C(A)$ of set A, where "conv" indicates convergence in the sense that $X_k^i = X_{k-1}^i$. (III)



Morphological operations and their properties

Thinning	$A \otimes B = A - (A \circledast B)$ $= A \cap (A \circledast B)^c$ $A \otimes \{B\} =$ $\left(\left(\ldots\left((A \otimes B^1) \otimes B^2\right)\ldots\right) \otimes B^n\right)$ $\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$	Thins set A. The first two equations give the basic definition of thinning. The last two equations denote thinning by a sequence of structuring elements. This method is normally used in practice. (IV)
Thickening	$A \odot B = A \cup (A \circledast B)$ $A \odot \{B\} =$ $((\ldots(A \odot B^1) \odot B^2 \ldots) \odot B^n)$	Thickens set A. (See preceding comments on sequences of structuring elements.) Uses IV with 0's and 1's reversed.



IV

B^i $i = 1, 2, \ldots, 8$
(rotate 45°)

Morphological operations and their properties

Skeletons

$$S(A) = \bigcup_{k=0} S_k(A)$$

$$S_k(A) = \bigcup_{k=0} \{(A \ominus kB) - [(A \ominus kB) \circ B]\}$$

Reconstruction of A:

$$A = \bigcup_{k=0}^{k} (S_k(A) \oplus kB)$$

Finds the skeleton $S(A)$ of set A. The last equation indicates that A can be reconstructed from its skeleton subsets $S_k(A)$. In all three equations, K is the value of the iterative step after which the set A erodes to the empty set. The notation $(A \ominus kB)$ denotes the kth iteration of successive erosion of A by B. (I)

Pruning

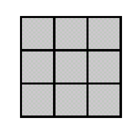
$$X_1 = A \otimes \{B\}$$

$$X_2 = \bigcup_{k=1}^{8} (X_1 \circledast B^k)$$

$$X_3 = (X_2 \oplus H) \cap A$$

$$X_4 = X_1 \cup X_3$$

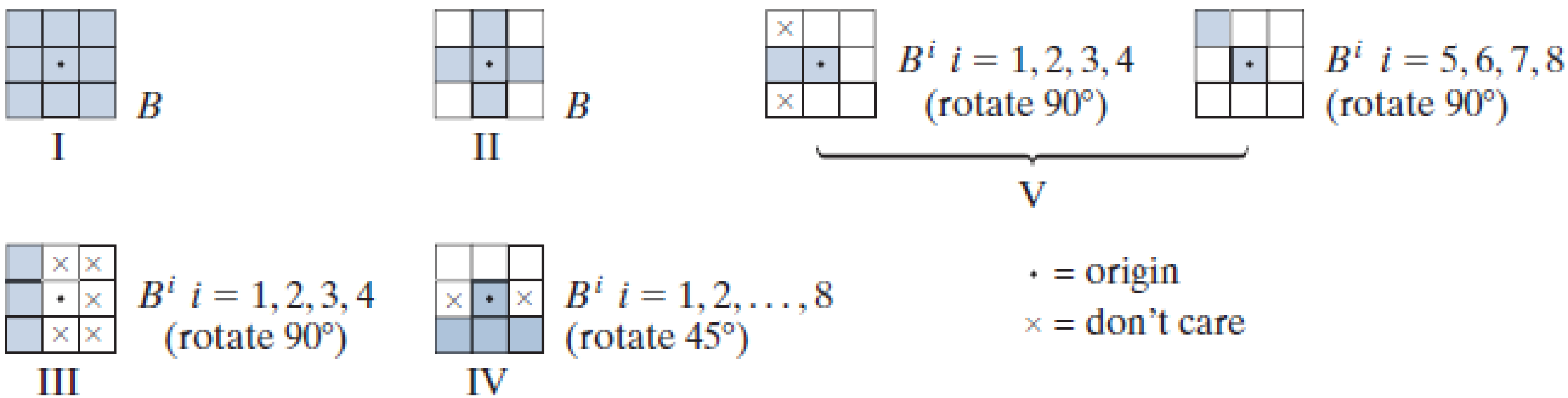
X_4 is the result of pruning set A. The number of times that the first equation is applied to obtain X_1 must be specified. Structuring elements V are used for the first two equations. In the third equation H denotes structuring element I.



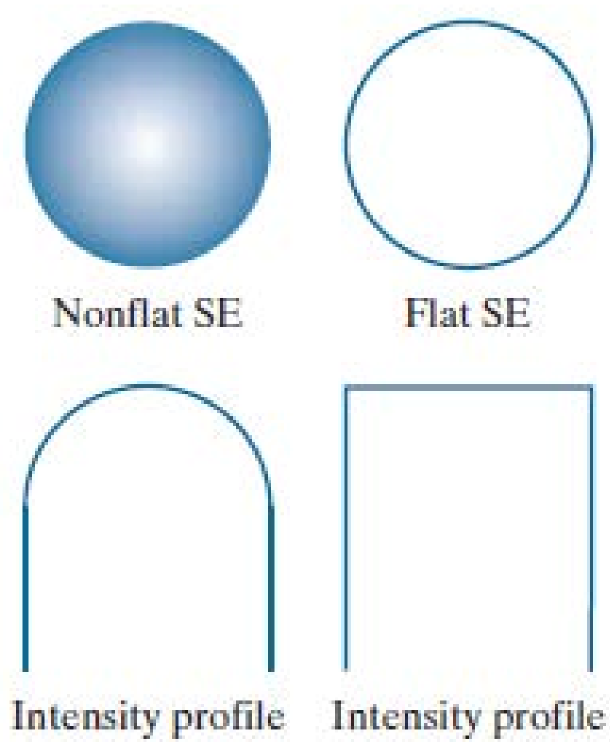
B

I

Frequently used structuring elements



Grayscale morphological operations



ثبات در تغییر سطح

Grayscale morphological operations

Erosion with non-flat SE
$$[f \ominus b_N](x,y) = \min_{(s,t)\in b_N}\{f(x+s,y+t) - b_N(s,t)\}$$

Dilation with non-flat SE
$$[f \oplus b_N](x,y) = \max_{(s,t)\in b_N}\{f(x-s,y-t) + b_N(s,t)\}$$

Erosion with flat SE
$$[f \ominus b](x,y) = \min_{(s,t)\in b_N}\{f(x+s,y+t)\}$$

Dilation with flat SE
$$[f \oplus b](x,y) = \max_{(s,t)\in b_N}\{f(x-s,y-t)\}$$

Duality with respect to complementation and reflection:

$$(f \ominus b)^c = (f^c \oplus \hat{b})$$

$$(f \oplus b)^c = (f^c \ominus \hat{b})$$

Flat erosion　　　Flat dilation

خوردگی رو شئ‌ها

min

max

Grayscale morphological operations

برای حذف salt

تا حدودی میشه گفت کنتراست کم میشه

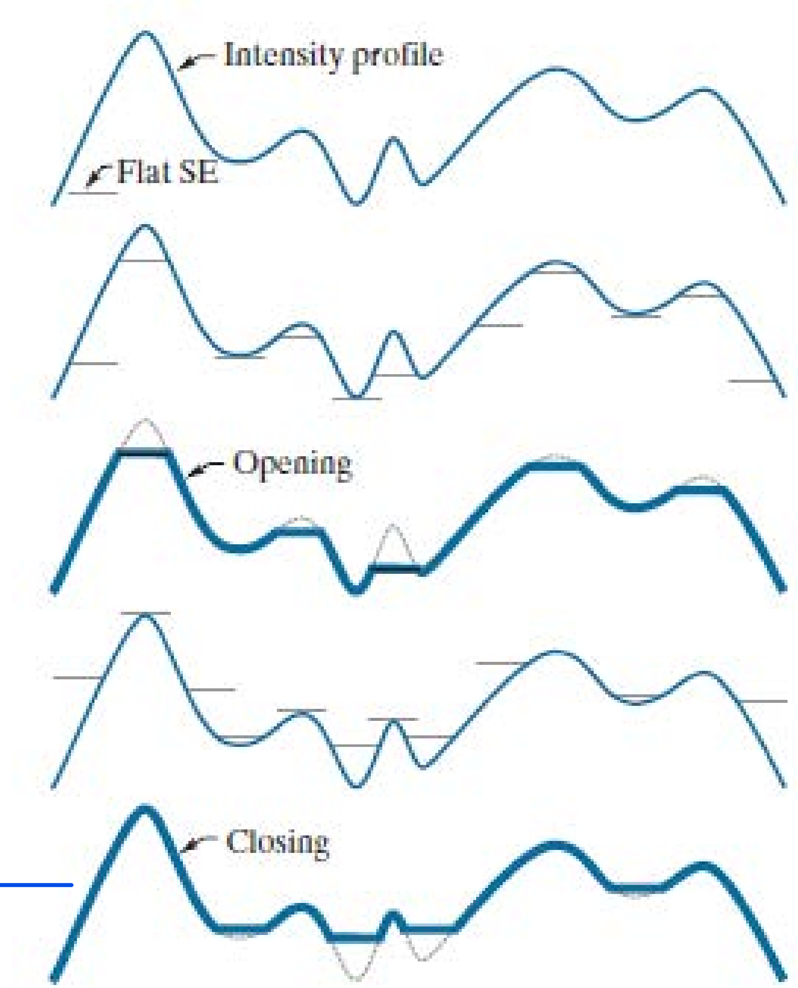
Opening $f \circ b = (f \ominus b) \oplus b$

Closing $f \bullet b = (f \oplus b) \ominus b$

Duality with respect to complementation and reflection:

$$(f \bullet b)^c = f^c \circ \hat{b}$$

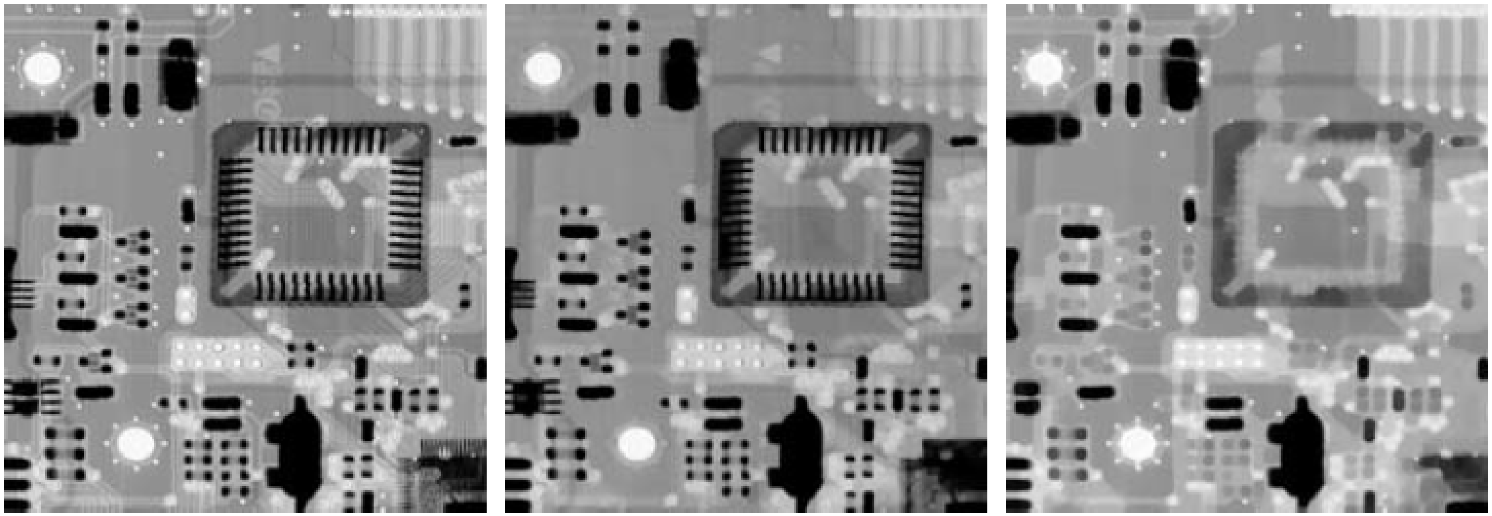
$$(f \circ b)^c = f^c \bullet \hat{b}$$



Pepper برای حذف

تا حدودی میشه گفت کنتراست کم میشه

Grayscale morphological operations

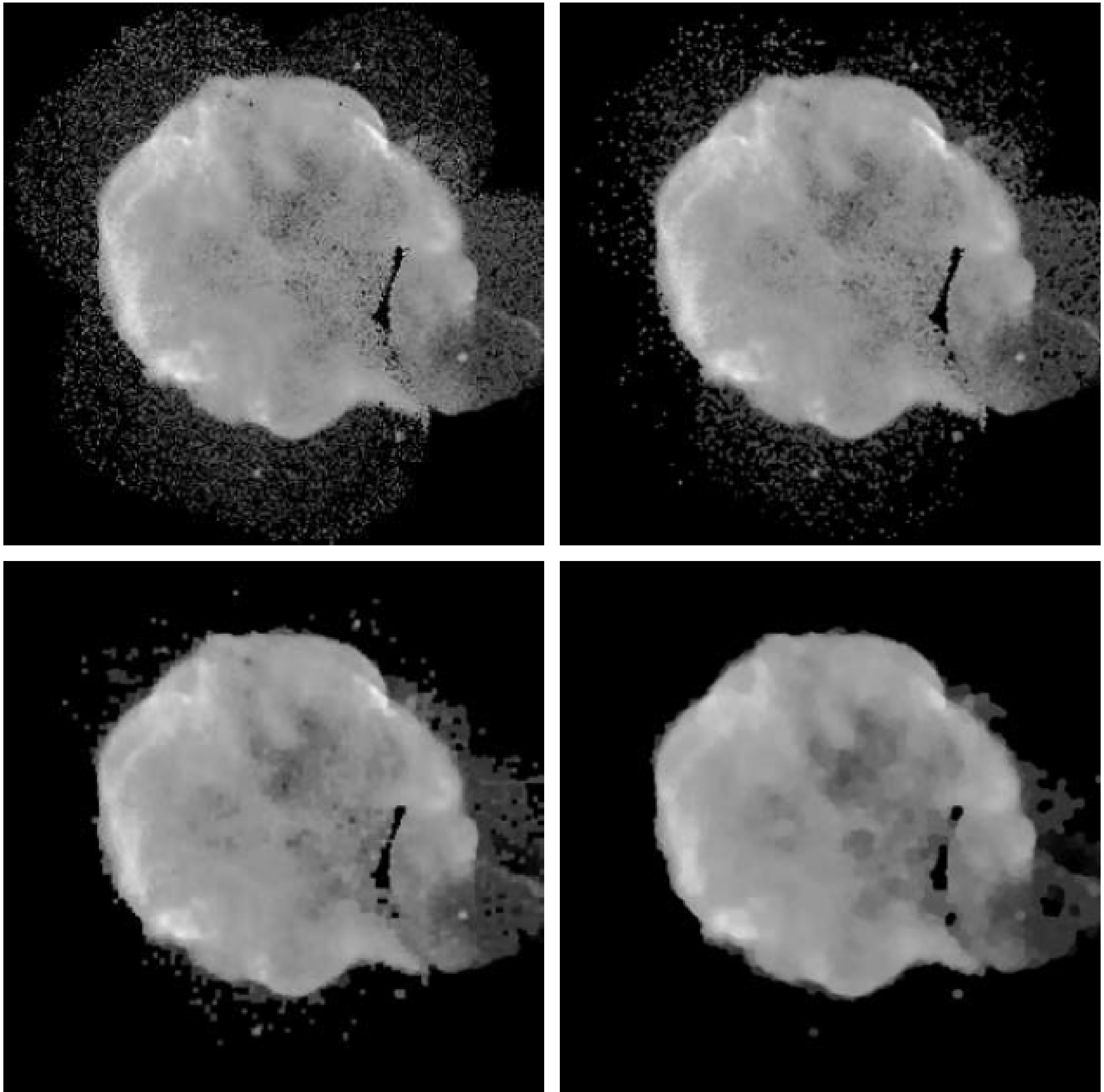


Opening Closing

کلیت شدت‌ها یک در هر دو حفظ شده

تصویر روشن یا تاریک‌تر نمی‌شه

Opening and closing sequences



Size: 1

Size: 3

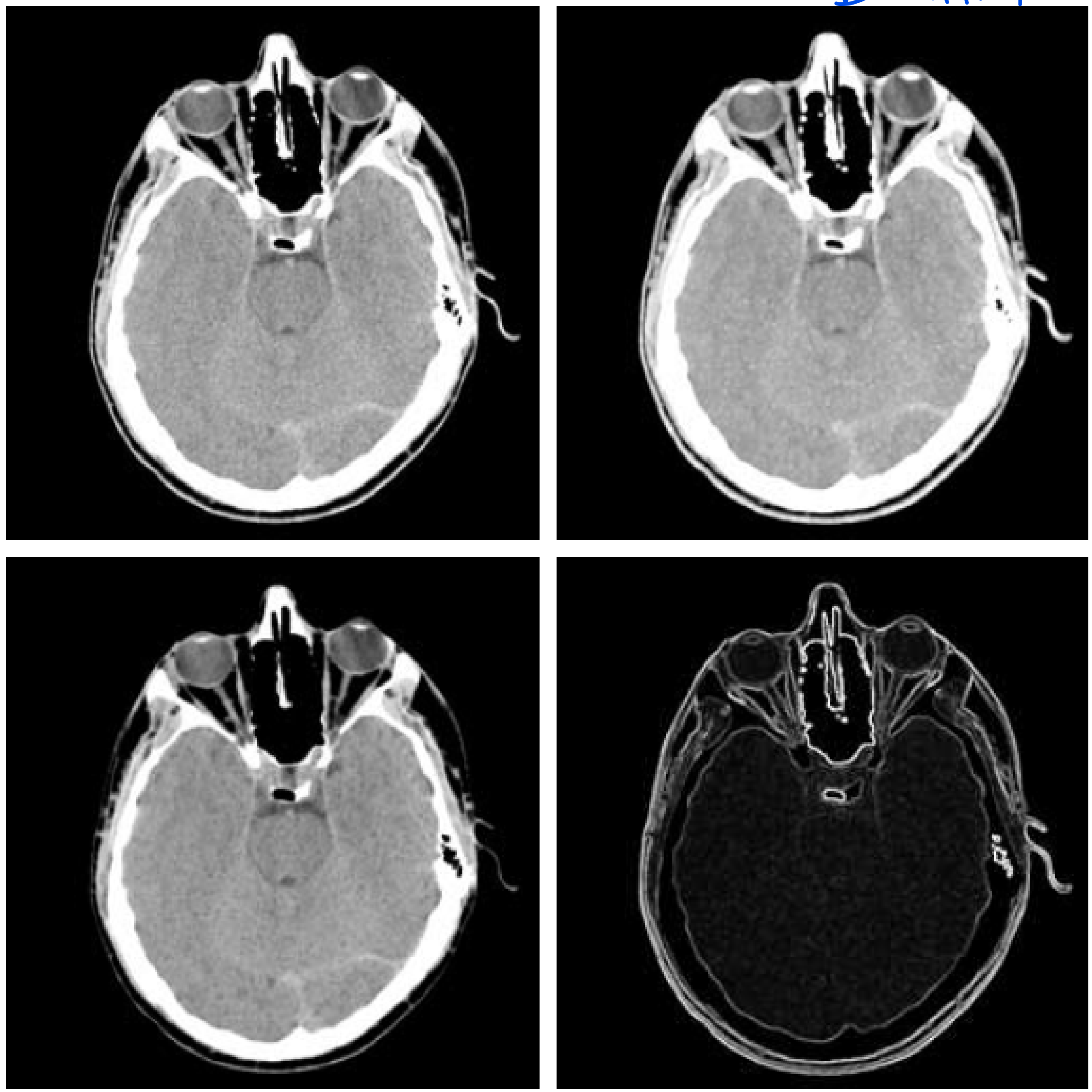
Size: 5

Morphological Gradient

$$g = (f \oplus b) - (f \ominus b)$$

مثل مشتق عمل می‌کنه

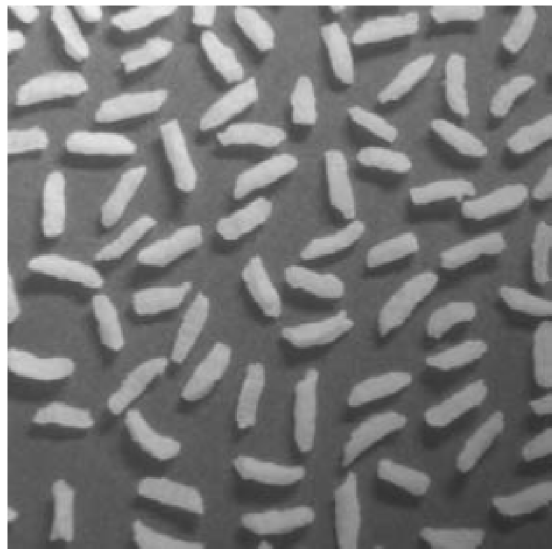
Dilation

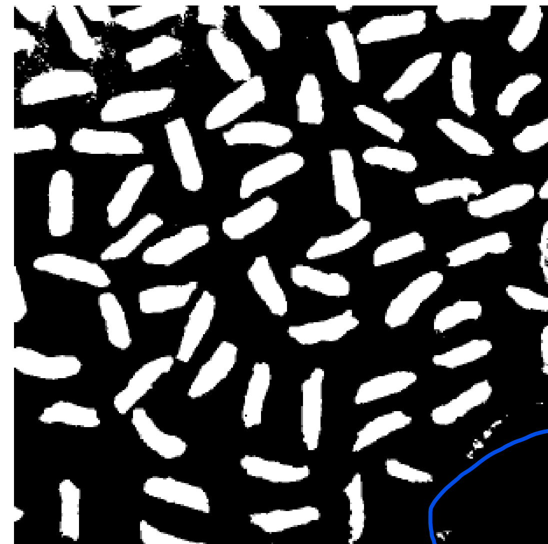


erosion

Top-hat, Bottom-hat Transformation

نورپردازی بیشتر





نورپردازی کمتر

اسکرین شات

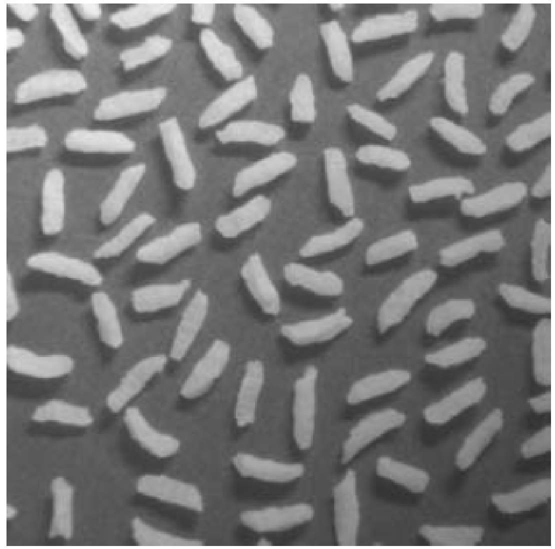
روش اول :

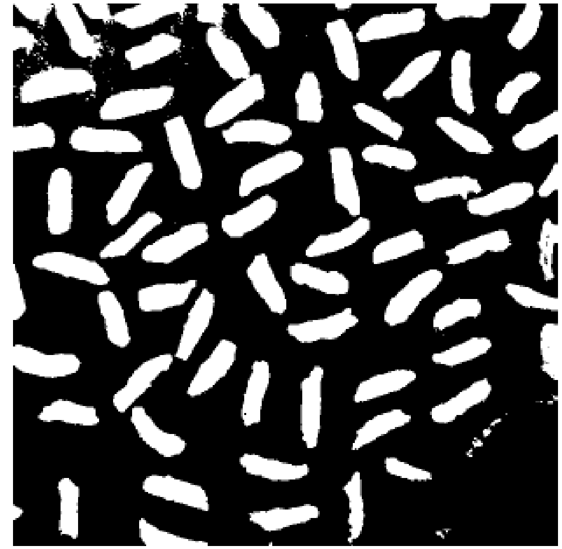
ترشولد مشکل اول

Top-hat, Bottom-hat Transformation

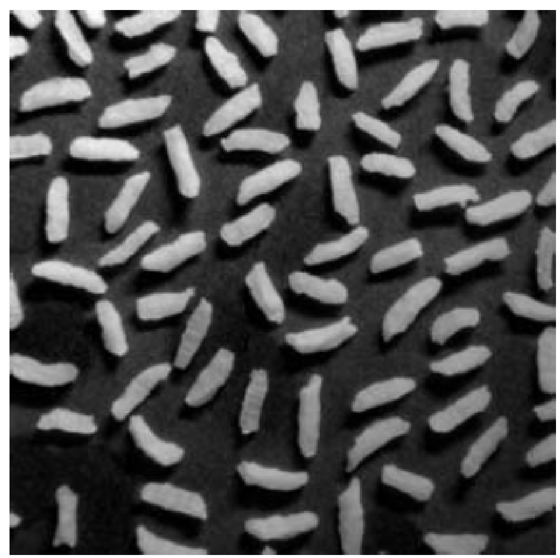
$$T_{hat}(f) = f - (f \circ b)$$

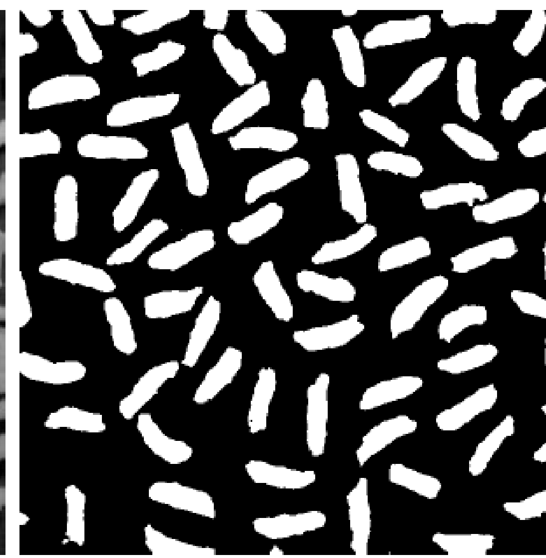
$$B_{hat}(f) = (f \bullet b) - f$$





پس زمینه





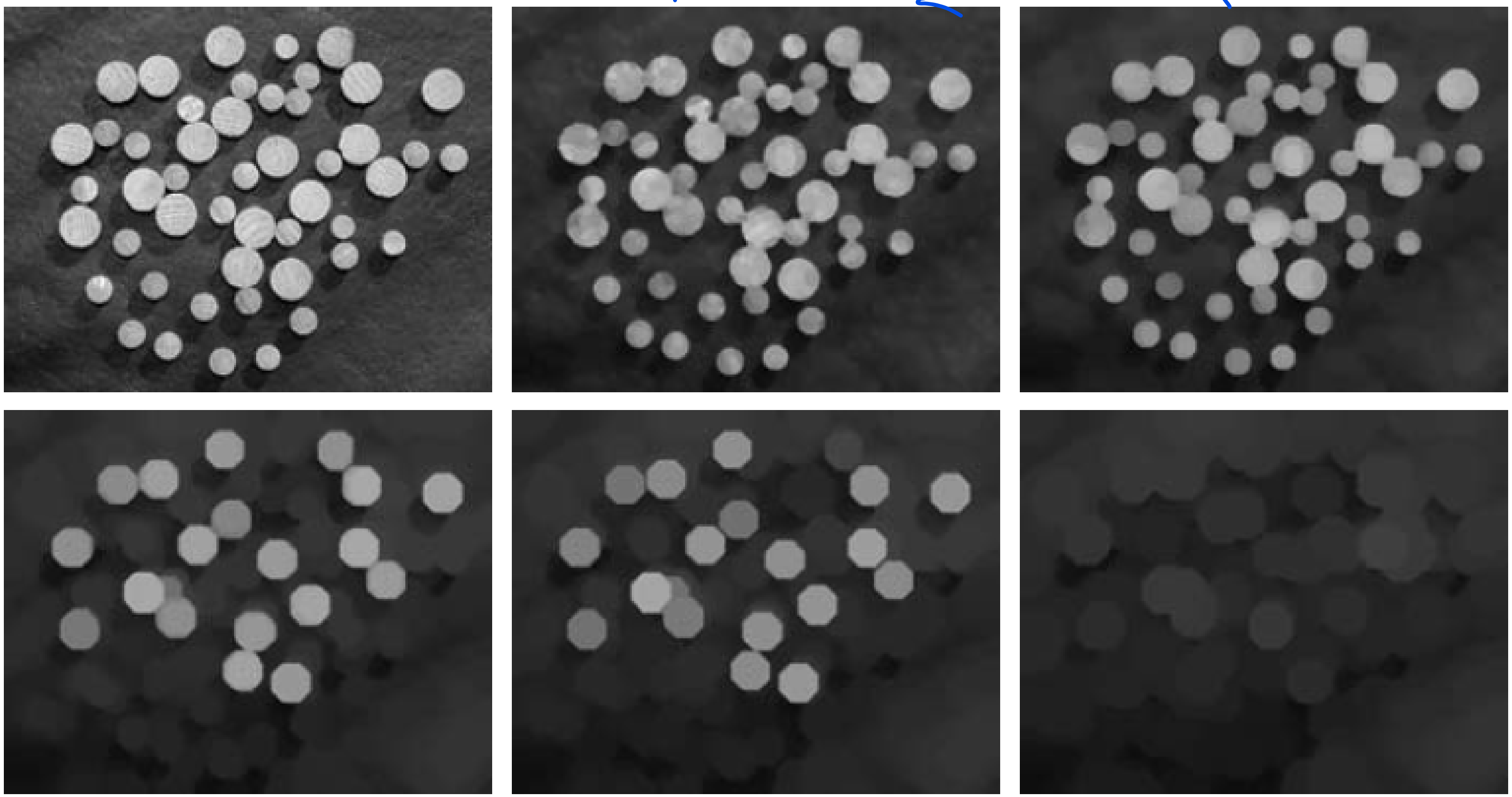
Granulometry

مساحت = جمع شدت ها

اصطلاحاً دانه ها رو بشماریم با ابعاد متفاوت

⊖

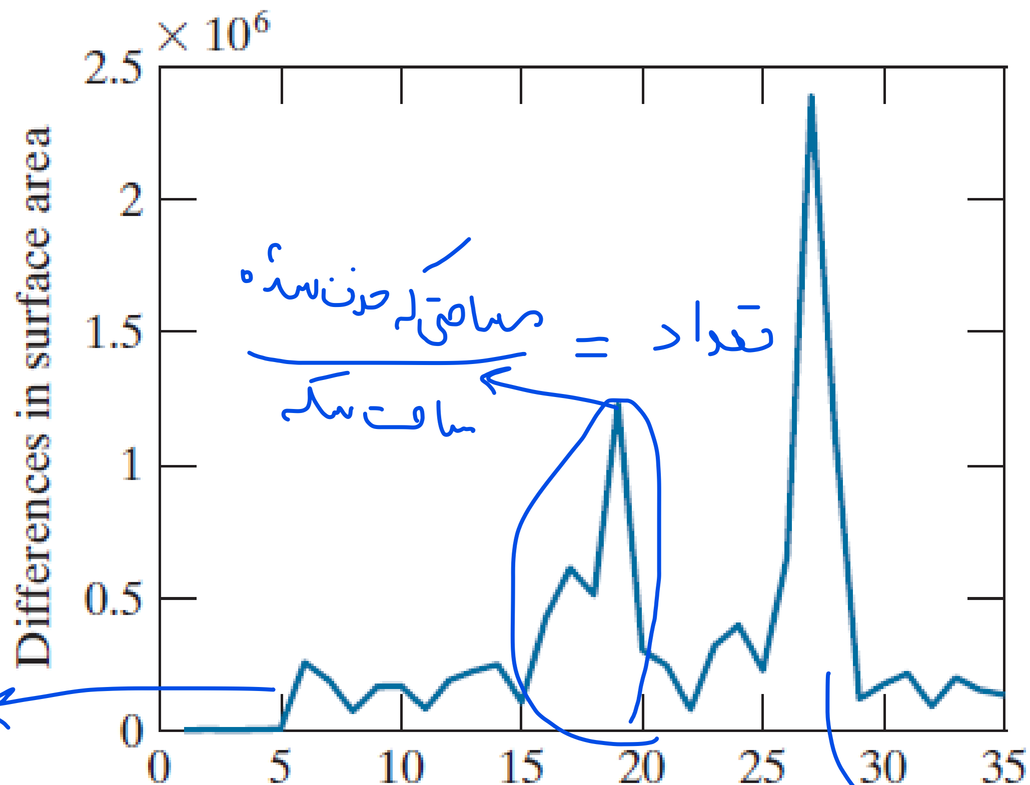
open



آن وقت اختلافات اینا یعنی این تیکه ها تو اون شعاع حذف شدند

شکل

Granulometry



تعداد = مساحت حذف شده / مساحت سکه

بیشتر ها بالاست
چون هنوز چیز خاصی
حذف نشده

شعاع

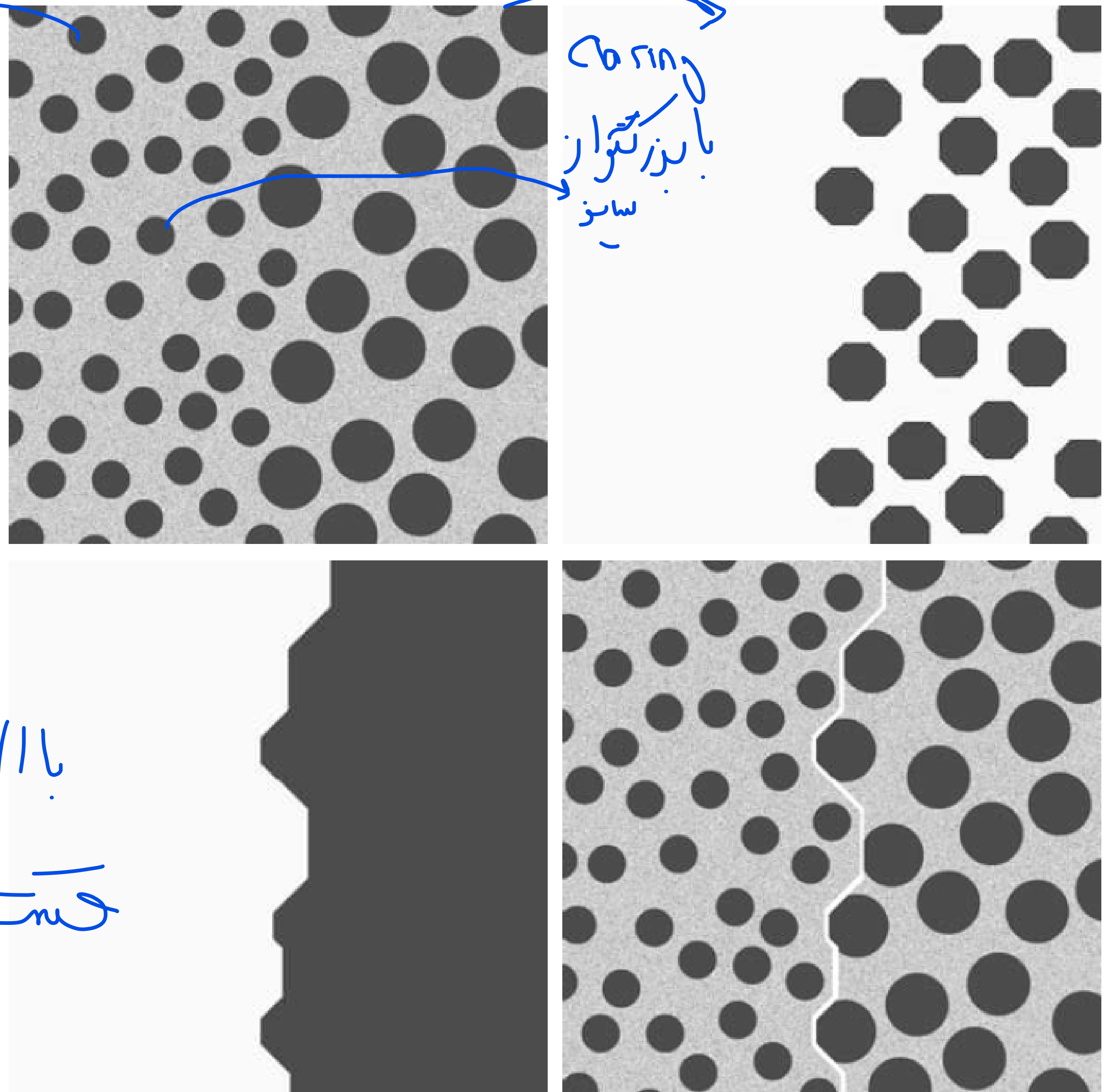
مربوط به سکه بزرگ‌ها

Textural segmentation

می خواهیم مرز رو پیدا کنیم

مثل دره

closing
با بزرگتر از سایز



opening
با المان بزرگتر از
حفره سفید بین
دایره‌ها

gradient
بزنیم

Boundary between regions of different texture