Registration

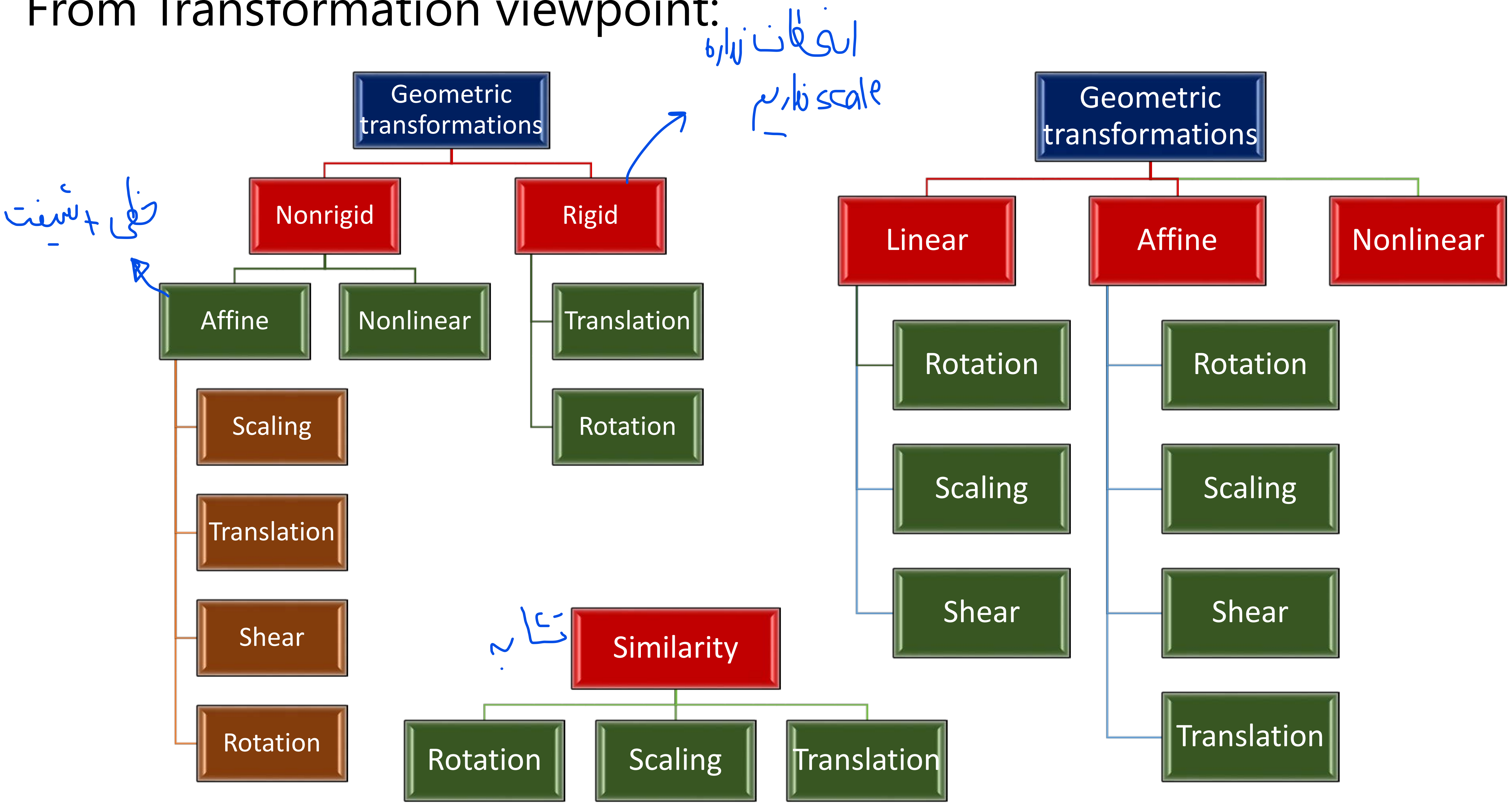
Registration

From applications viewpoint:

1) Moving camera, e.g. panorama
2) Moving objects in the scene
3) Changing the camera/modality (inter-modality), e.g. MR-CT
4) Changing the subject (inter-subject, inter patient), e.g. Atlas.
5) Changing the time of imaging, e.g. medical follow-ups every 6 months for tumor growth monitoring

When modality is not changed: intra-modality
When subject is not changed: intra-subject

From Transformation viewpoint:



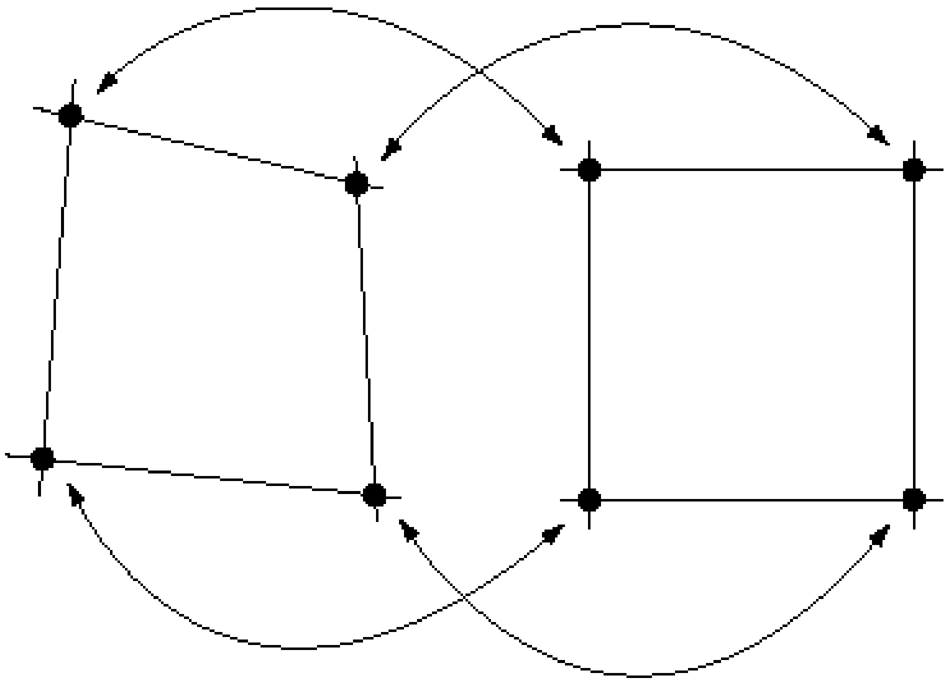
از نقاط استفاده میکنیم

1

Feature-based Registration

خروجی: یک تبدیل هندسی

Tie points

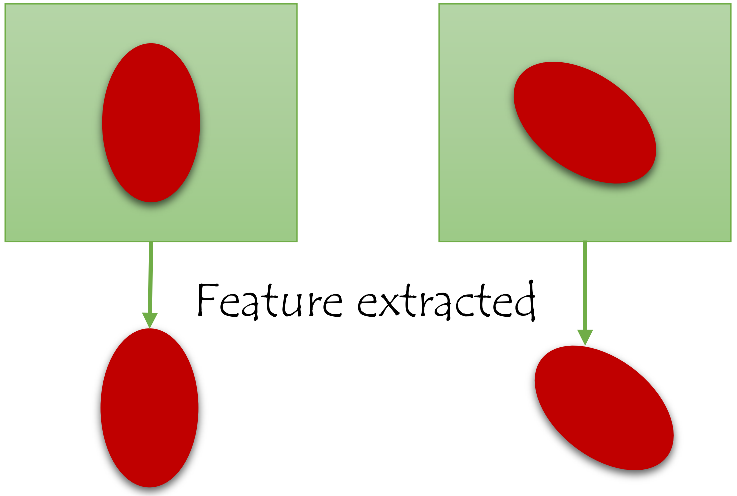


Tie points found by user or automatically.

Tie points

Feature based registration

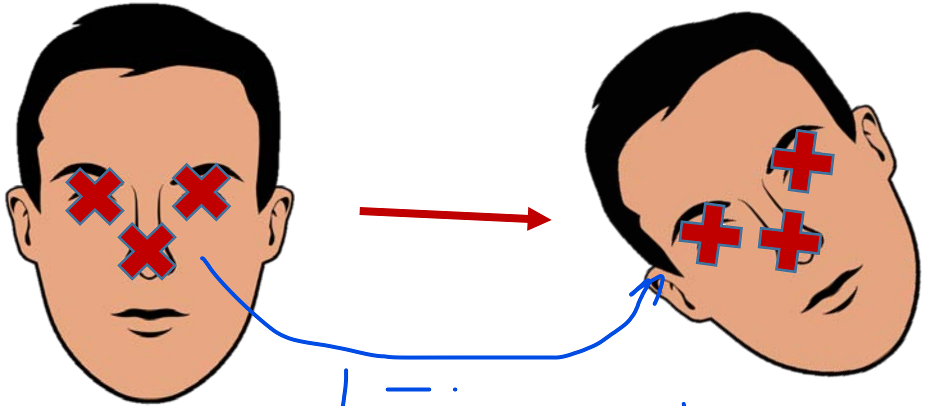
Tie points / contours



Optical / Physical markers

Fiducial markers

Anatomical landmarks



این سه نقطه رو روی دو تصویر داریم

Similarity transform

$p = [x \quad y]^T$ $\qquad$ $p' = [x' \quad y']^T$

Similarity transform

$$p = [x \quad y]^T \qquad\qquad p' = [x' \quad y']^T$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} s\ cos\theta & -s\ sin\theta & t_x \\ s\ sin\theta & s\ cos\theta & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

۴ مجهول نیاز داریم: ۲ نقطه زیرا هر نقطه دو معادله را
به ما میدهد

Affine transform

Affine transform

۶ مجهول

۳ نقطه احتیاج داریم

مشترک دقت بالا چون ممکنه

اون سه نقطه دقیق نباشه

از رگرسیون برای حلش استفاده کنیم

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$p' = Ap$$

Affine transform

$$x'_i = a_{11}x_i + a_{12}y_i + a_{13}$$

$$y'_i = a_{21}x_i + a_{22}y_i + a_{23}$$

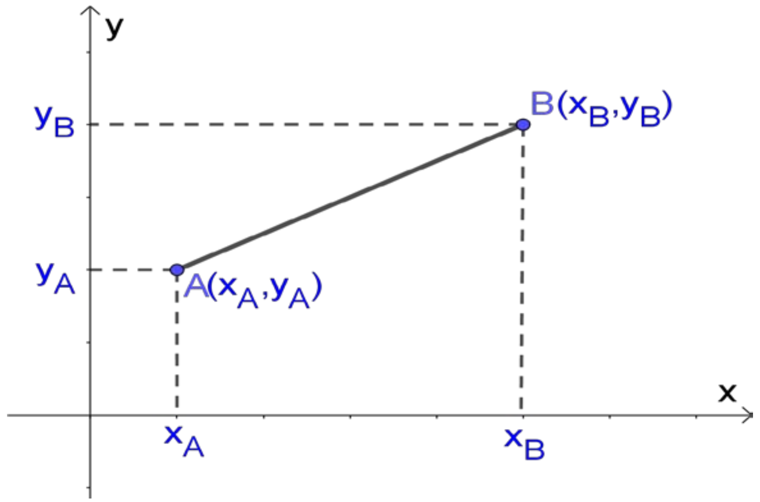
$$\begin{bmatrix} x_1 & y_1 & 1 \\ \vdots & \vdots & \vdots \\ x_n & \overset{y}{\cancel{x}}_n & 1 \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{12} \\ a_{13} \end{bmatrix} = \begin{bmatrix} x'_1 \\ \vdots \\ x'_n \end{bmatrix}$$

$$\begin{bmatrix} x_1 & y_1 & 1 \\ \vdots & \vdots & \vdots \\ x_n & \overset{y}{\cancel{x}}_n & 1 \end{bmatrix} \begin{bmatrix} a_{21} \\ a_{22} \\ a_{23} \end{bmatrix} = \begin{bmatrix} y'_1 \\ \vdots \\ y'_n \end{bmatrix}$$

$$M \qquad \alpha \qquad B$$

Point set registration

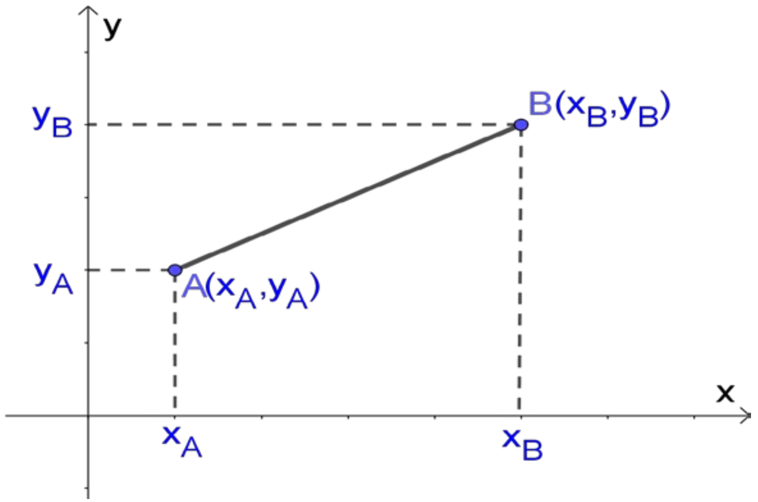
Affine transform



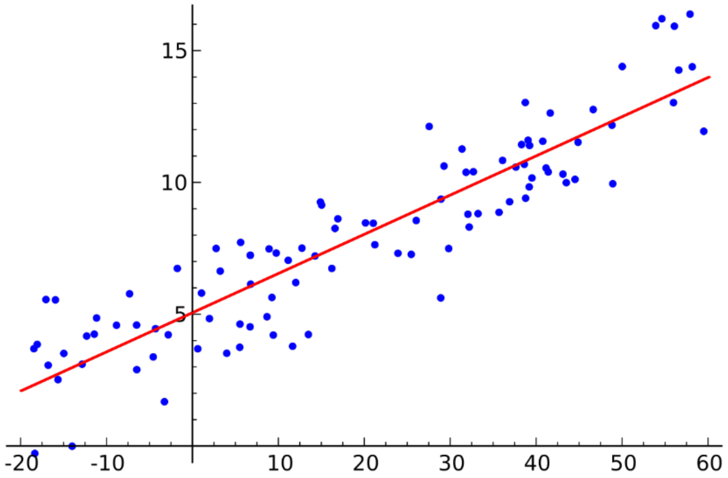
Overdetermined

اگر تعداد معادله‌هامون زیاد باشه

Affine transform



Overdetermined



$$M\alpha = \beta$$

$$\min_{\alpha} ||M\alpha - \beta||^2$$

$$M^T M\alpha = M^T \beta$$

$$\alpha = (M^T M)^{-1} M^T \beta$$

Columns linearly independent

$$(M\alpha - \beta)^T (M\alpha - \beta) = (\alpha^T M^T - \beta^T)(M\alpha - \beta)$$

$$= \alpha^T M^T M\alpha - \alpha^T M^T \beta - \beta^T M\alpha + \beta^T \beta$$

$$= \alpha^T M^T M\alpha - 2\alpha^T M^T \beta + \beta^T \beta$$

$$\frac{\partial}{\partial \alpha} = M^T M\alpha - M^T \beta = 0 \Longrightarrow \alpha = (M^T M)^{-1} M^T \beta$$

Registration

Landmark

Image A

Image B

Mapped A

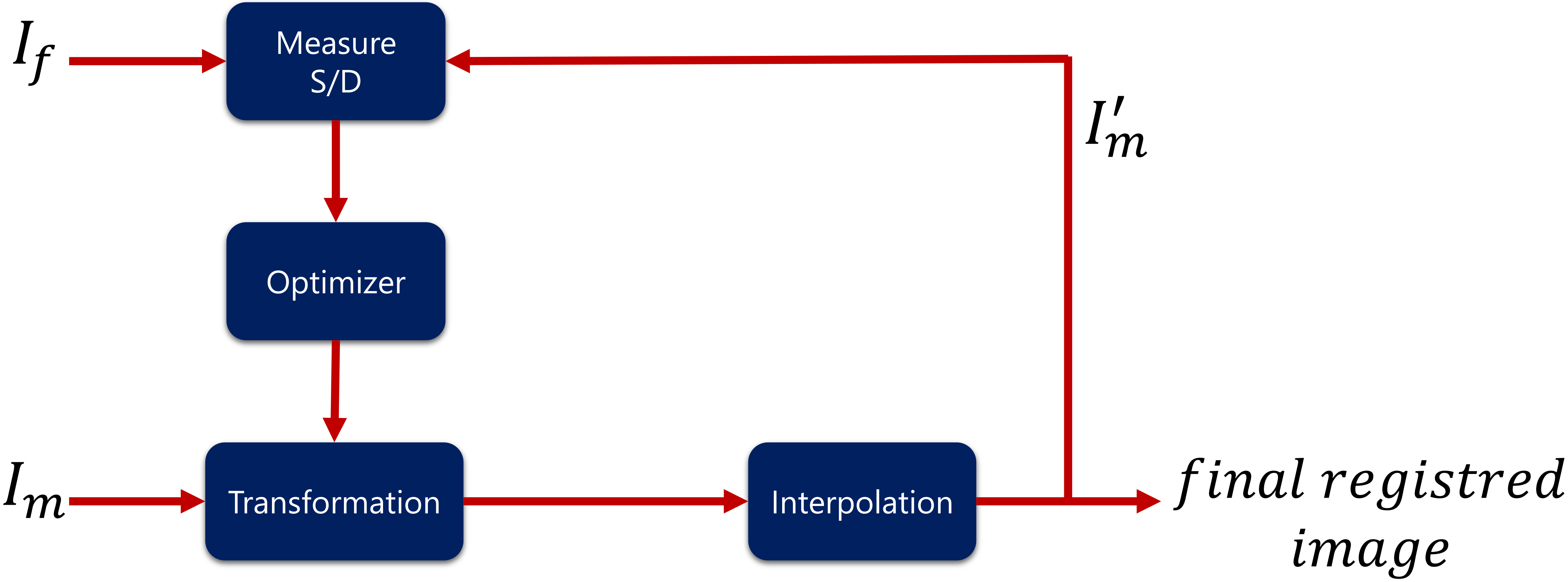
Original B

به صورت دقیق تناظر مشخص
نداریم

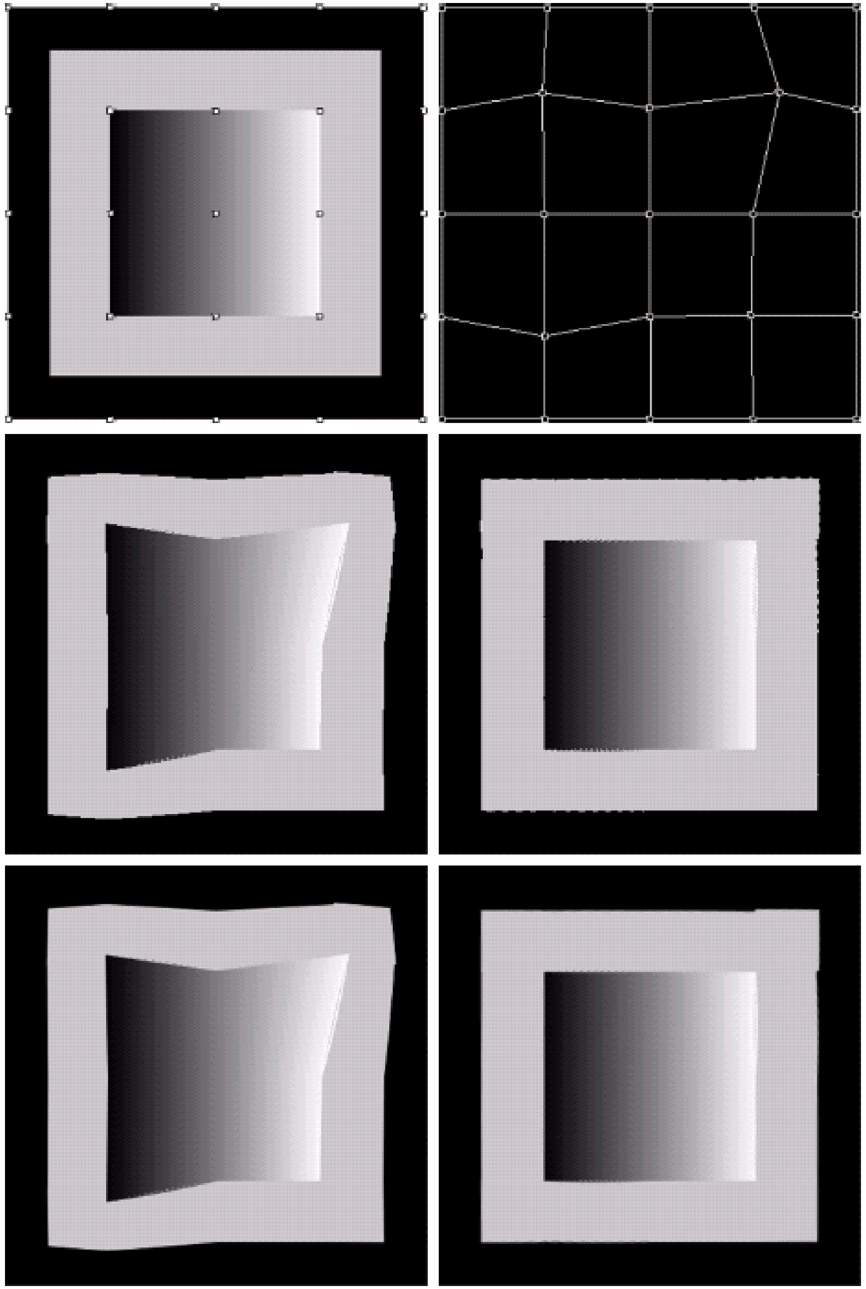
Intensity-based Registration

از پترن های شدت استفاده کنیم

Intensity-based registration



Transformation



Tie points
in the image

Tie points after
geometric
distortion

Distorted using
nearest neighbor
interpolation

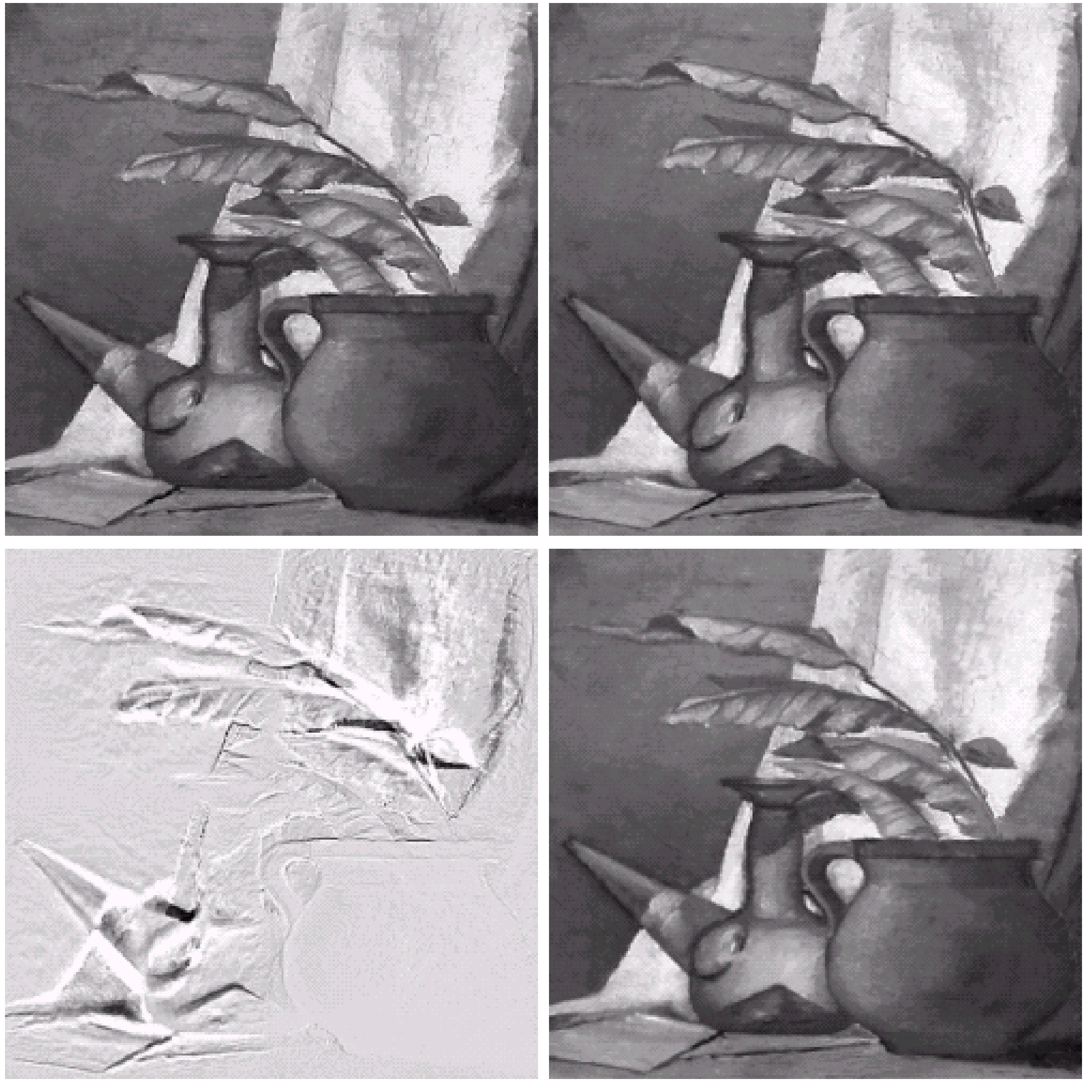
Restored

Distorted using
Bilinear interpolation

Restored

Before geometric distortion

Distorted using bilinear interpolation



Difference

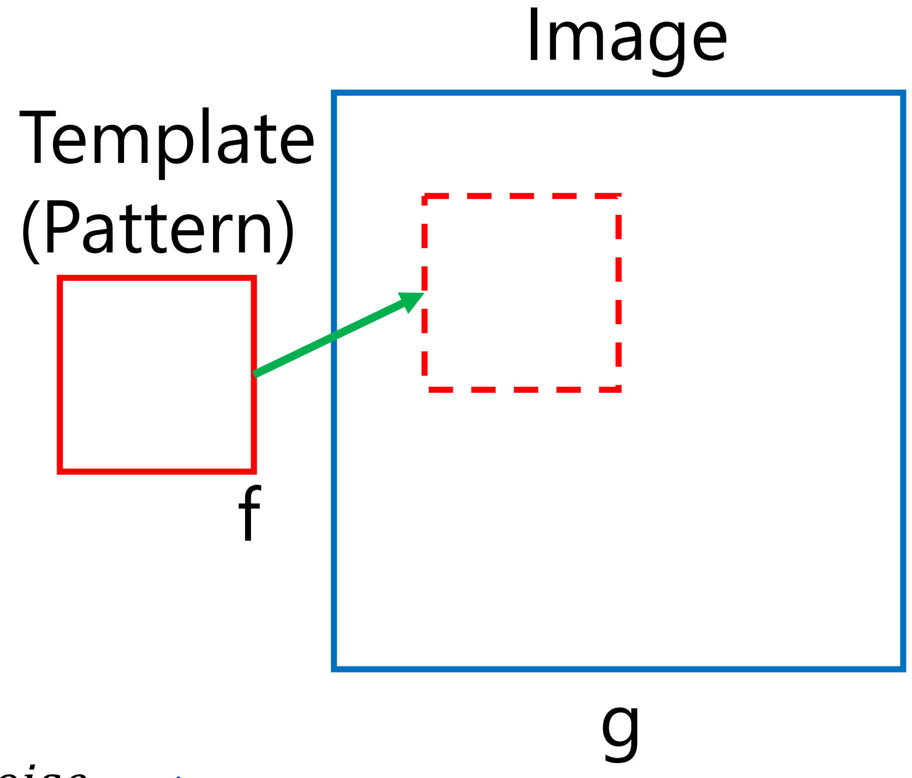
Restored

Match (Similarity) Measure or Mismatch (Dissimilarity) Measure

For intramodality:

Sum of absolute differences (SAD): $Min \iint_A |f-g|$

$$\rightarrow \sum_{i,j \in A} \sum |f(i,j) - g(i+u, j+\text{v})|$$



Sum of squared differences (SSD): $Min \iint_A [f-g]^2$

$$\rightarrow \sum_{i,j \in A} \sum [f(i,j) - g(i+u, j+v)]^2 \rightarrow Difference\ only\ in\ noise.$$

به اختلافات حساسیت بیشتری دارد
مشتق گرفتن راحت‌تر

Amirkabir University of Technology (Tehran Polytechnic) Biomedical Engineering Department

خوب $f=g$

$$Min \iint_A [f-g]^2 = Min\left(\iint_A f^2 + \iint_A g^2 - 2\iint_A fg\right) \overset{g \text{ constant energy}}{\equiv} Max \iint_A fg$$

$\rightarrow$ *Drawback when g has varying energy.*

بازهم با NCC یکی نیست
چون ساس اول
جزیی از f نیست

خوب نیست
چرا؟

نرم نیست
نرم هم باشه بازهم
معادل نیست

$$Cauchy - Schwartz: \iint_A f.g \le \sqrt{\iint_A f^2 . \iint_A g^2} \quad , Equality: g = cf$$

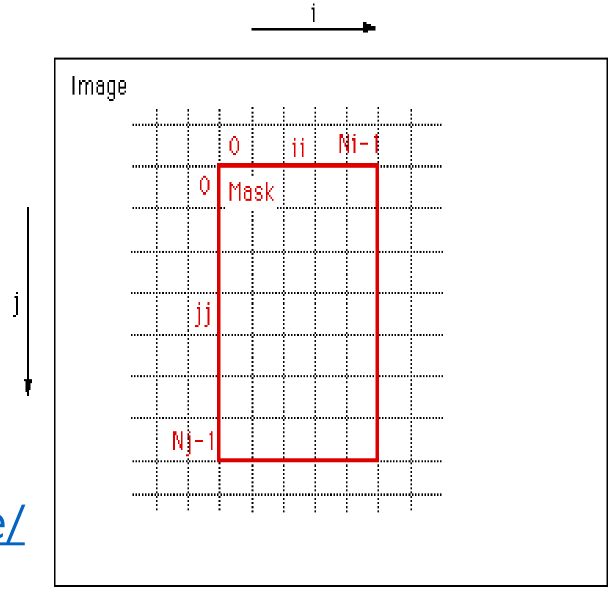
$$\sum_{i,j\in A}\sum f(i,j).g(i,j) \le \sqrt{\sum_{i,j\in A}\sum f^2(i,j) . \sum_{i,j\in A}\sum g^2(i,j)}$$

$$Equality: g(i,j) = cf(i,j)$$

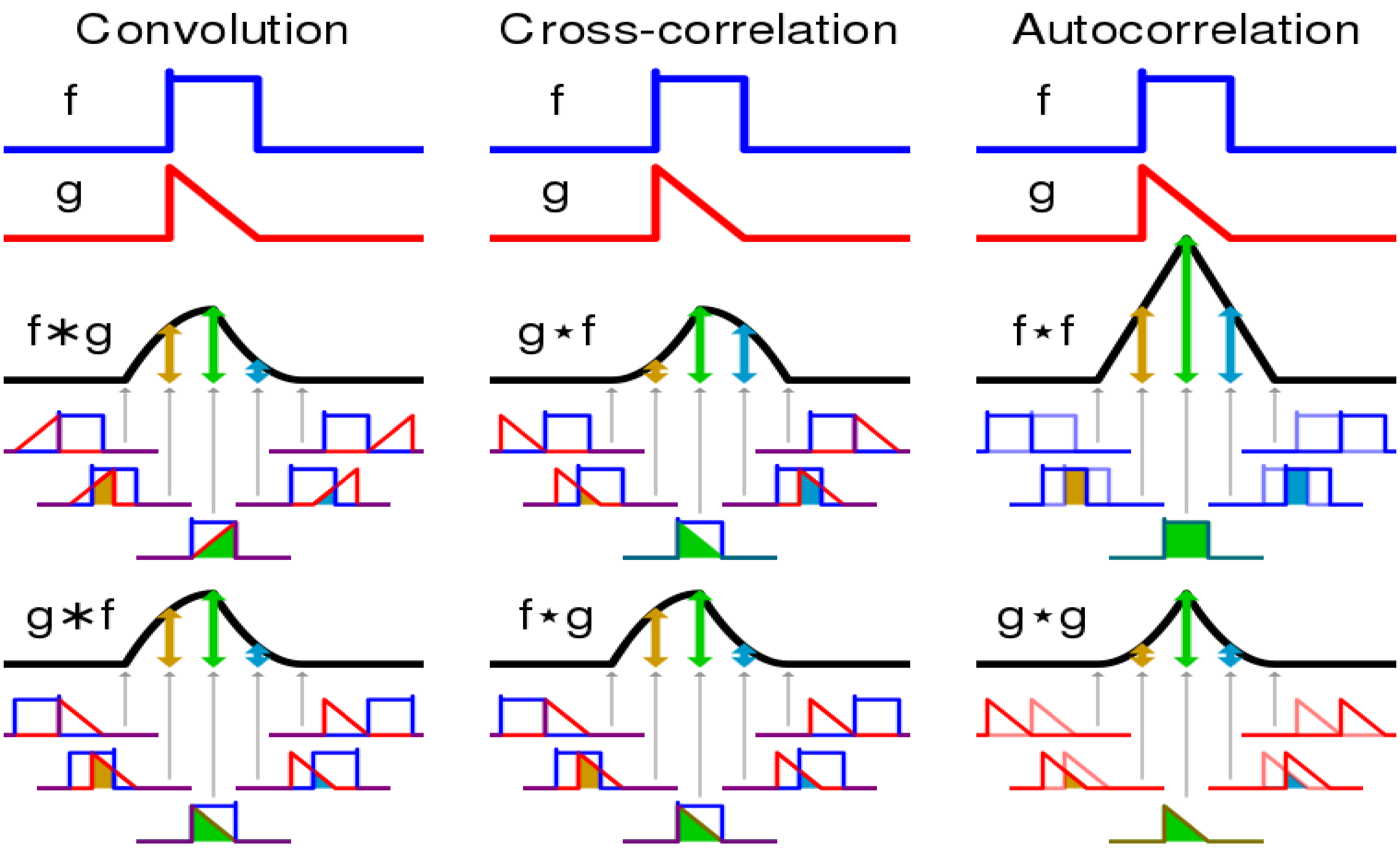
$$\iint_A f(x,y).\,g\underbrace{(x+u,y+v)}_{Shift:u,v}\,dxdy \leq \sqrt{\iint_A f^2(x,y)dxdy\,.\iint_A g^2(x+u,y+v)dxdy}$$

$$\iint_{-\infty}^{+\infty} f(x,y).\,g(x+u,y+v)dxdy \rightarrow Cross\ Correlation\ of\ f\ \&\ g$$

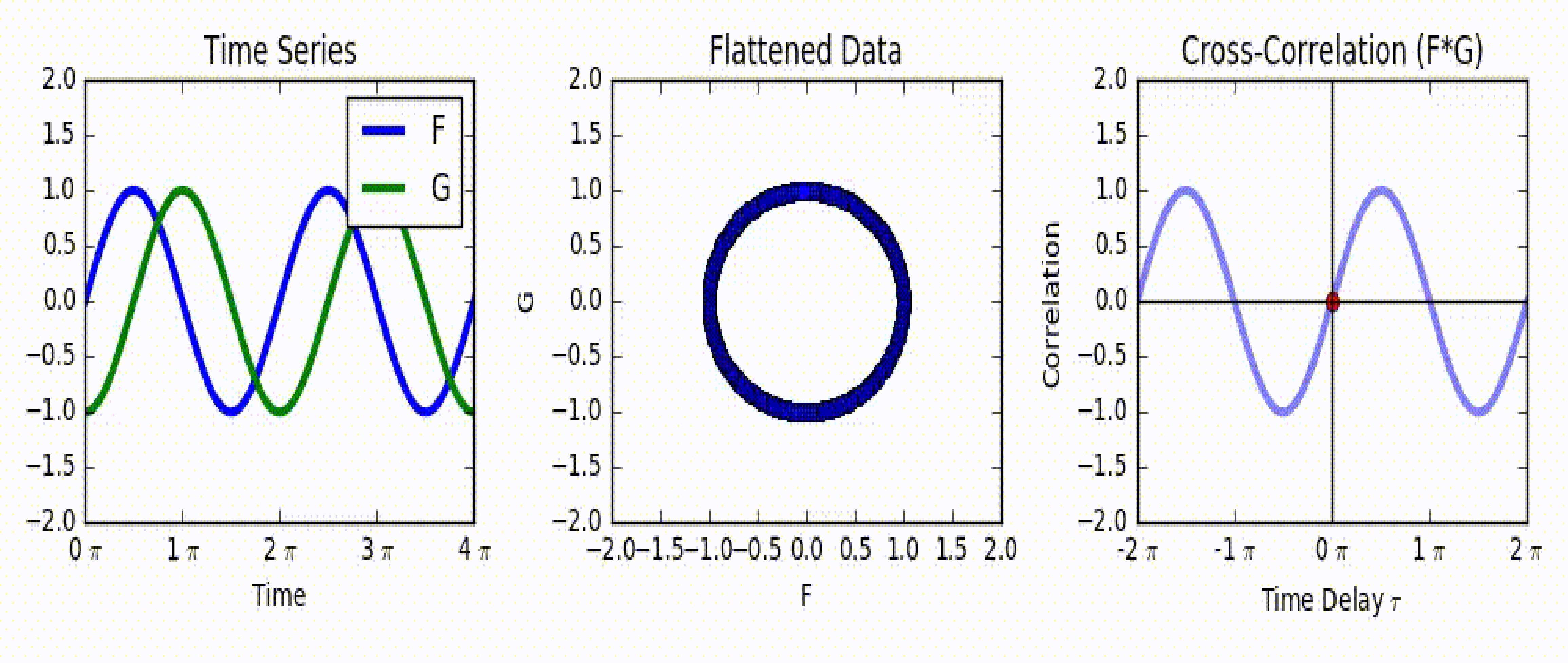
$$\rightarrow Cannot\ directly\ be\ used\ as\ similarity\ measure.$$



http://paulbourke.net/miscellaneous/correlate/



https://en.wikipedia.org/wiki/File:Comparison_convolution_correlation.svg



https://en.wikipedia.org/wiki/File:Cross_correlation_animation.gif

Similatry

$$C_{fg} = \iint_{-\infty}^{+\infty} f(x,y).\, g(x+u, y+v)\,dxdy$$

$$Normalized\ Cross\ Correlation\ (NCC) = \frac{C_{fg}}{\sqrt{\iint f^2 . \iint g^2}}$$

مقدارشو max کنیم

ثابته حذف می‌کنیم

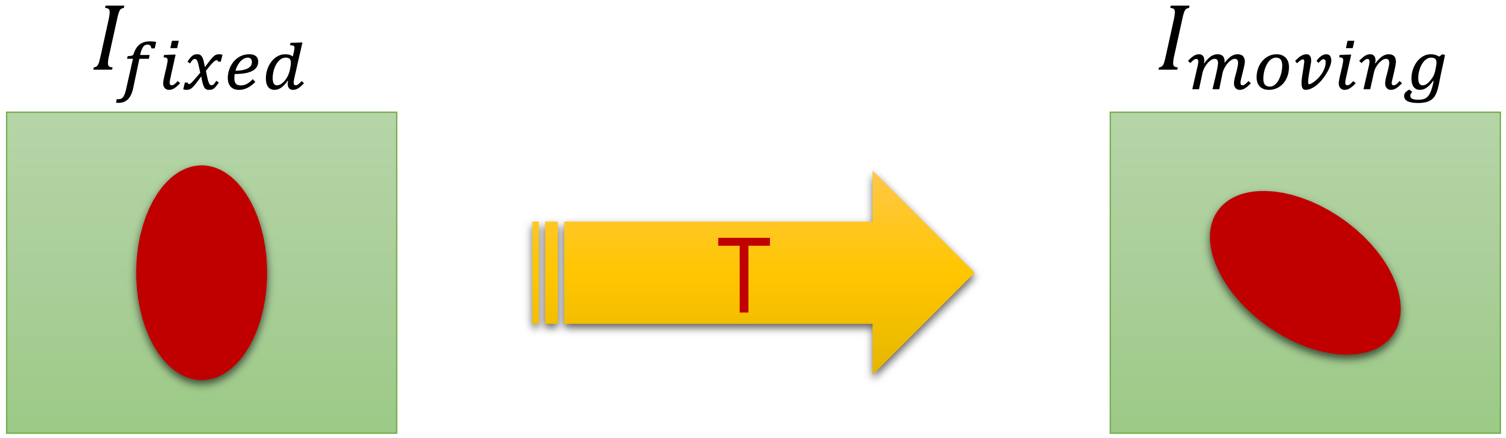
$$NCC = \frac{C_{fg}}{\sqrt{\iint_A g^2(x+u, y+v)\,dxdy}}$$

$$Maximum\ when: g = cf$$

برای اینکه یه عکس روز گرفته باشه و هم یه عکس شب ولی به چون g ضریبی از f هست

Registration

Intensity based registration



SDD مثل

معیار تفاوت:

$$\hat{T} = \arg\min_{T} \; D(I_f, I_m, T)$$

NCC مثل

معیار شباهت:

$$\hat{T} = \arg\max_{T} \; S(I_f, I_m, T)$$

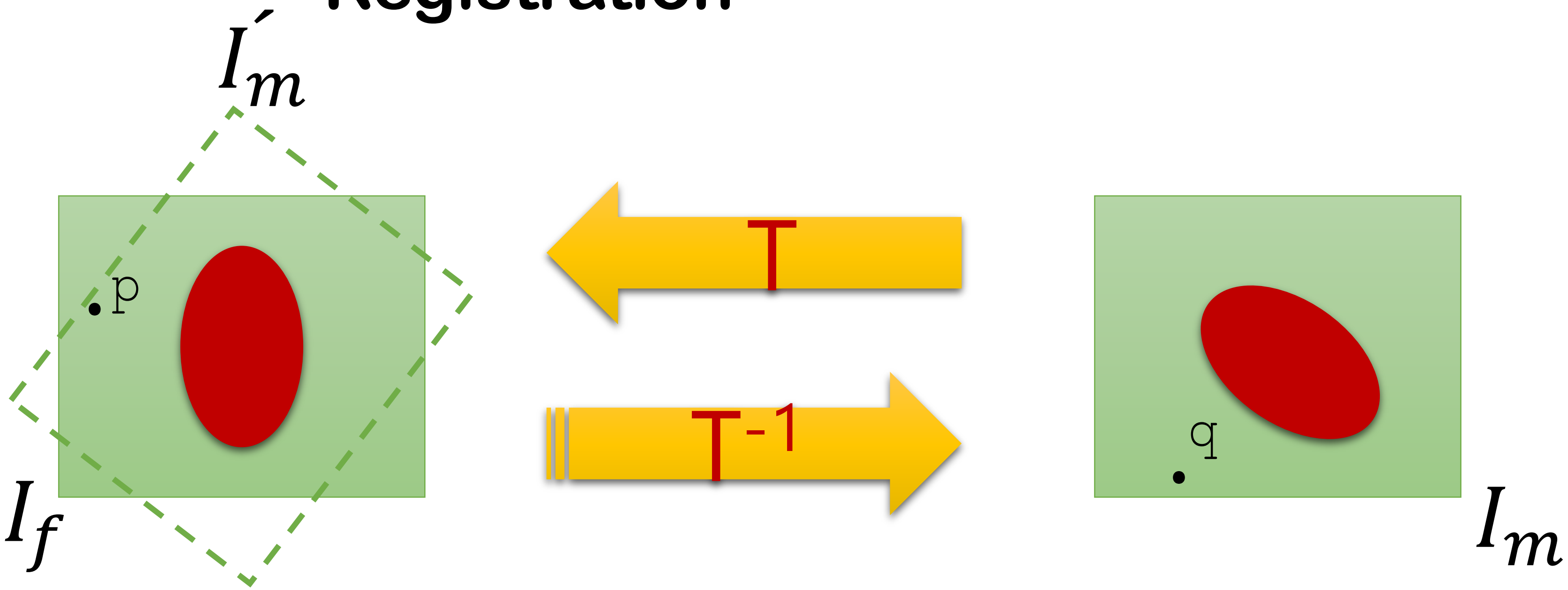
Similarty

Registration

$$p = \begin{bmatrix} x_p \\ y_p \end{bmatrix} \qquad p = T(q)$$

$$q = \begin{bmatrix} x_q \\ y_q \end{bmatrix} \qquad q = T^{-1}(p)$$

$$\begin{bmatrix} x_p \\ y_p \end{bmatrix} = T\left(\begin{bmatrix} x_q \\ y_q \end{bmatrix}\right)$$



$$I'_m(\mathrm{p}) = I_m(q) = I_m\left(T^{-1}(p)\right) \longrightarrow \sum_{p\in\Omega}\left[I_f(p) - I'_m(\mathrm{p})\right]^2 = \sum_{p\in\Omega}\left[I_f(p) - I_m\left(T^{-1}(p;\theta)\right)\right]^2$$

$$\min_T E = \min_T E(\theta) \longrightarrow E(\theta) = \sum_{p\in\Omega}\left[I_f(p) - I_m\left(T^{-1}(p;\theta)\right)\right]^2$$

Registration

optizimation

Gradient Descent: $\theta = [\theta_1, \theta_2, \ldots, \theta_k]^T$

$$\theta_{t+1} = \theta_t - \eta \nabla E(\theta_t) \qquad \nabla E(\theta_t) = \frac{\partial E}{\partial \theta}(\theta_t)$$

Numerical Computation: $\theta = [\theta_1, \theta_2, \ldots, \theta_k]^T$

$$\frac{\partial E}{\partial \theta_i} = \frac{E(\theta_1, \ldots, \theta_i + \Delta\theta_i, \ldots, \theta_k) - E(\theta_1, \ldots, \theta_i, \ldots, \theta_k)}{\Delta \theta_i}$$

Registration

$Continuous\ (Chain\ rule)$:

$$\frac{\partial E}{\partial \theta} = \sum_{p \in \Omega} -2\left[I_f(p) - I_m\left(T'(p;\theta)\right)\right] \frac{\partial I_m}{\partial T'} \frac{\partial T'}{\partial \theta}$$

$$T' = T^{-1}$$

$$\frac{\partial I_m}{\partial T'}\left(T'\ (p)\right) = \left[I_{mx}\left(T'(p;\theta)\right) \qquad I_{my}\left(T'(p;\theta)\right)\right]$$

$$I_{mx}\left(T'(p;\theta)\right) = \frac{I_m(x+\Delta x, y) - I_m(x,y)}{\Delta x} \qquad I_{my}\left(T'(p;\theta)\right) = \frac{I_m(x, y+\Delta y) - I_m(x,y)}{\Delta y}$$

$$T'(p;\theta) = \begin{bmatrix} ax_p - by_p + t_x \\ bx_p + ay_p + t_y \end{bmatrix}$$

Registration

$Continuous\ (Chain\ rule)$:

$$\frac{\partial E}{\partial \theta} = \sum_{p \in \Omega} -2\left[I_f(p) - I_m\left(T'(p;\theta)\right)\right] \frac{\partial I_m}{\partial T'} \frac{\partial T'}{\partial \theta}$$

$$T' = T^{-1}$$

$$\frac{\partial I_m}{\partial T'}\left(T'\ (p)\right) = \left[I_{mx}\left(T'(p;\theta)\right) \qquad I_{my}\left(T'(p;\theta)\right)\right]$$

$$I_{mx}\left(T'(p;\theta)\right) = \frac{I_m(x+\Delta x, y) - I_m(x, y)}{\Delta x} \qquad I_{my}\left(T'(p;\theta)\right) = \frac{I_m(x, y+\Delta y) - I_m(x, y)}{\Delta y}$$

$$T'(p;\theta) = \begin{bmatrix} ax_p - by_p + t_x \\ bx_p + ay_p + t_y \end{bmatrix}$$

$$\theta = [a \quad b \quad t_x \quad t_y]^T$$

$$p = [x_p \quad y_p]$$

$$\frac{\partial T'}{\partial \theta} =$$

Registration

۱×۴

$Continuous\ (Chain\ rule):$

$$\frac{\partial E}{\partial \theta} = \sum_{p \in \Omega} -2\underbrace{\left[I_f(p) - I_m\left(T'(p;\theta)\right)\right]}_{\text{اسکالر}} \underbrace{\frac{\partial I_m}{\partial T'}}_{۱×۲} \underbrace{\frac{\partial T'}{\partial \theta}}_{۲×۴} \qquad T' = T^{-1}$$

جزیی تصویر

$$\frac{\partial I_m}{\partial T'}\left(T'(p)\right) = \left[I_{mx}\left(T'(p;\theta)\right) \quad I_{my}\left(T'(p;\theta)\right)\right]$$

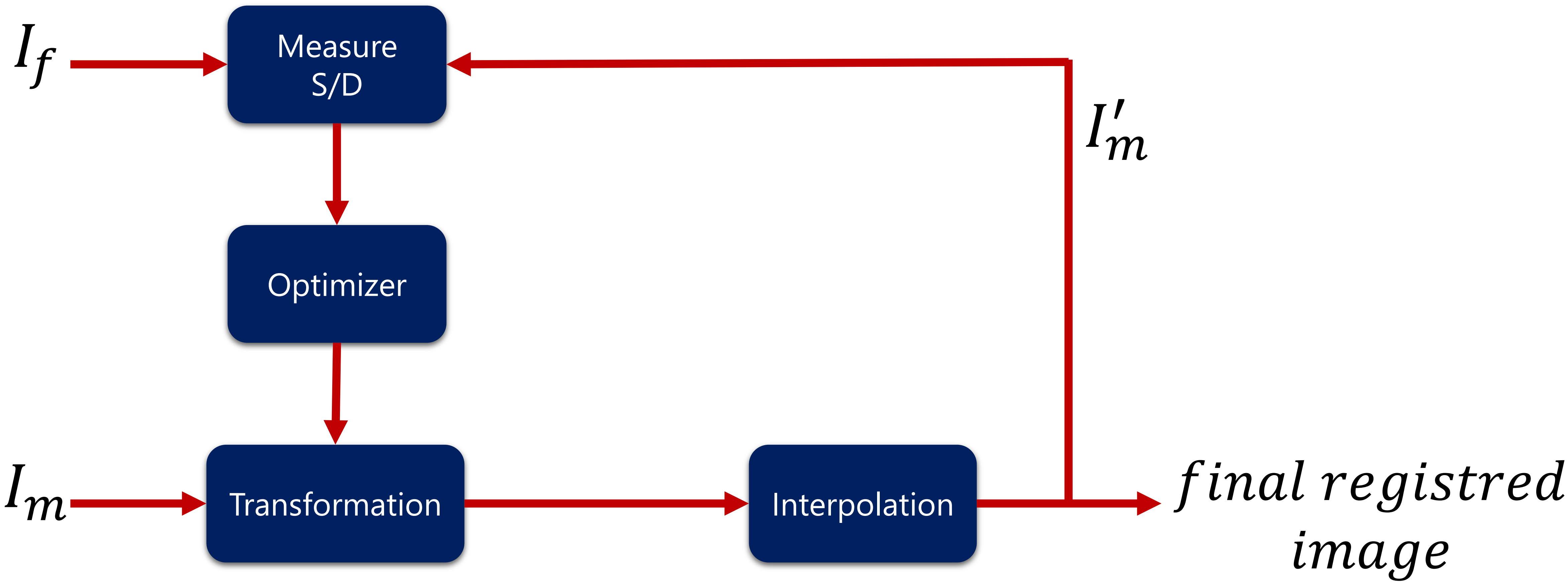
$$I_{mx}\left(T'(p;\theta)\right) = \frac{I_m(x+\Delta x, y) - I_m(x,y)}{\Delta x} \qquad I_{my}\left(T'(p;\theta)\right) = \frac{I_m(x, y+\Delta y) - I_m(x,y)}{\Delta y}$$

$$T'(p;\theta) = \begin{bmatrix} ax_p - by_p + t_x \\ bx_p + ay_p + t_y \end{bmatrix} \qquad \theta = [a \quad b \quad t_x \quad t_y]^T \qquad p = [x_p \quad y_p]$$

$$\frac{\partial T'}{\partial \theta} = \begin{bmatrix} x_p & -y_p & 1 & 0 \\ y_p & x_p & 0 & 1 \end{bmatrix}$$

Jacobian

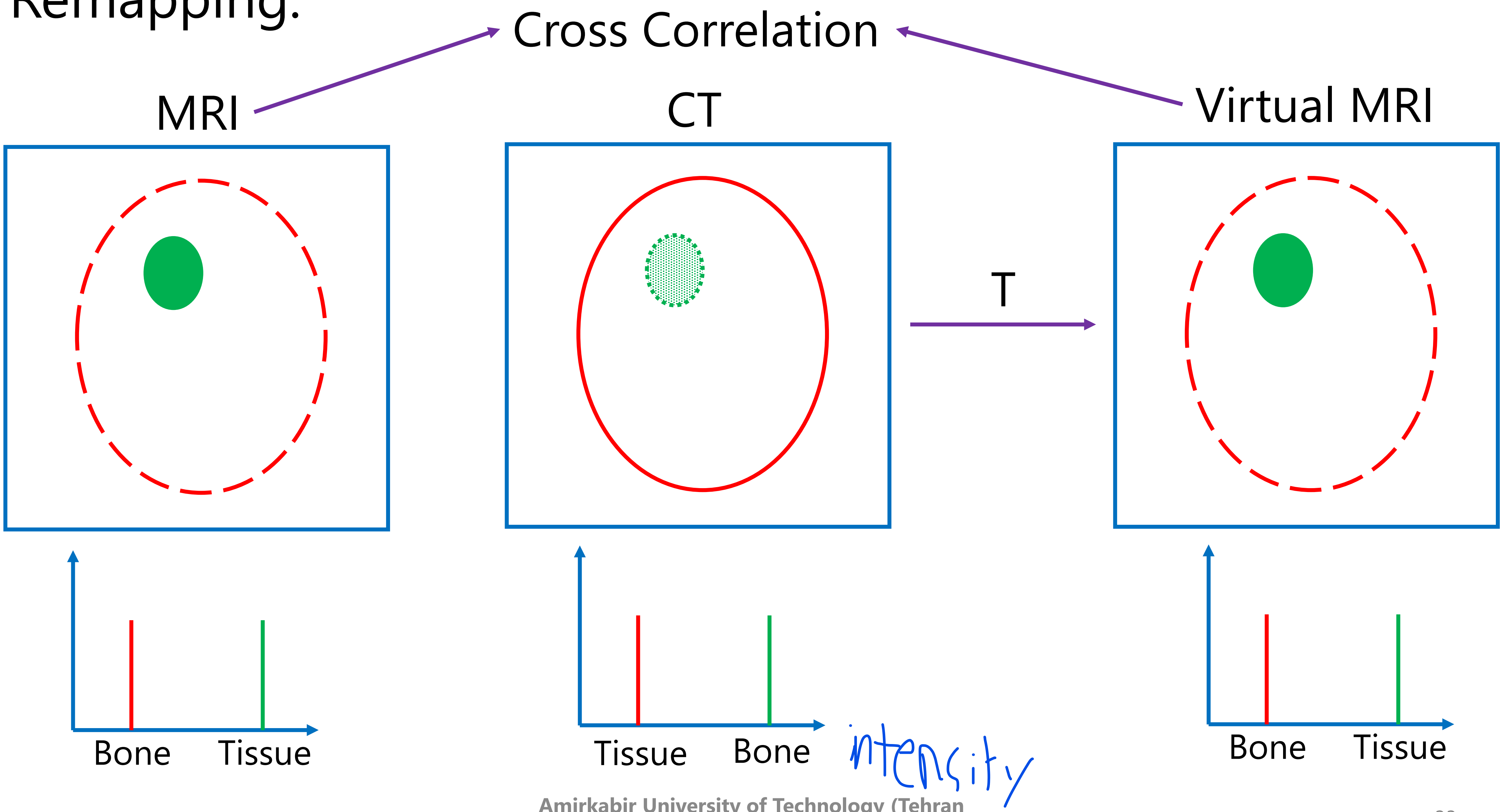
Intensity-based registration



For intermodality:

1. Feature based registration
2. Remapping
3. Mutual information as similarity measure

Remapping:

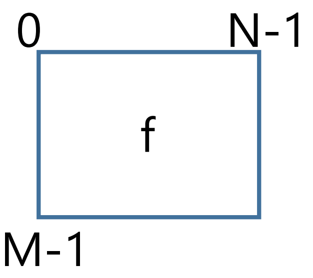

Cross Correlation
MRI
CT
Virtual MRI
T
Bone
Tissue
Tissue
Bone
intensity
Bone
Tissue

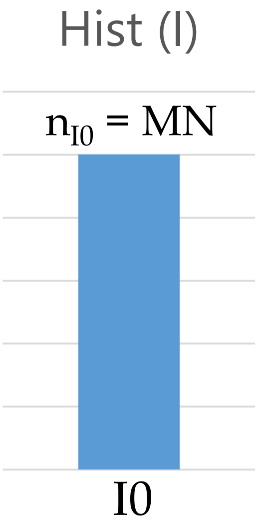
Mutual information:

$Entropy$:

$$H(I) = -\sum_{i} P_i \log P_i$$

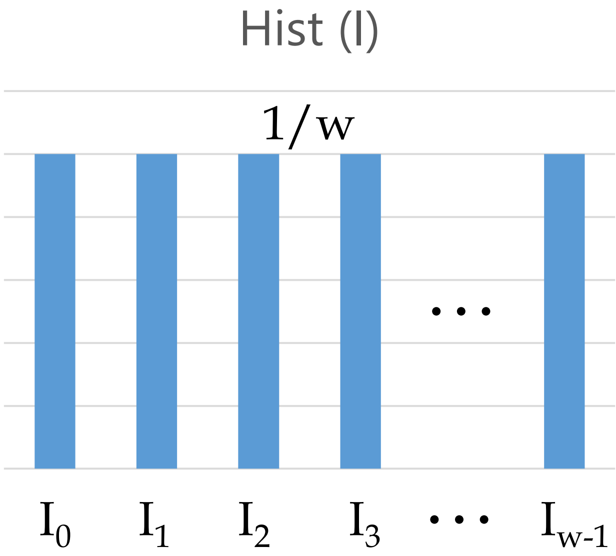
$$P_i = \frac{n_i}{MN}$$





$$P(I) = 1 \Longrightarrow$$

$$H(I) = -\sum_{i \in \{I_0\}} 1 \times \log 1 = 0$$



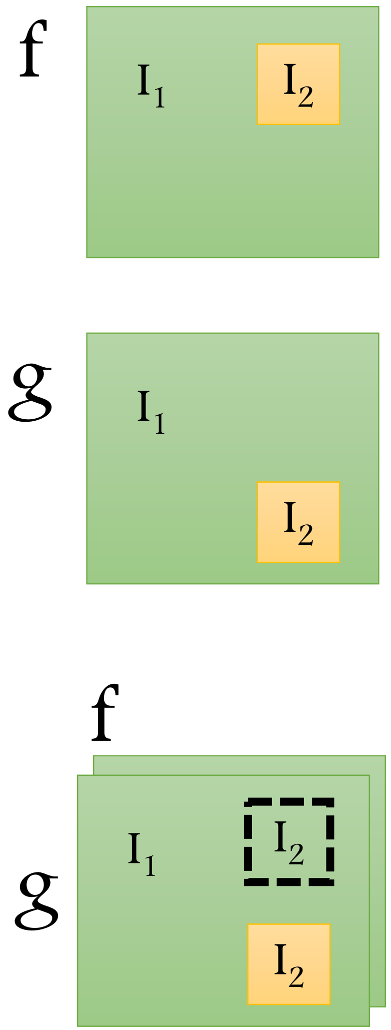
$$P_i = \frac{1}{w} \Longrightarrow$$

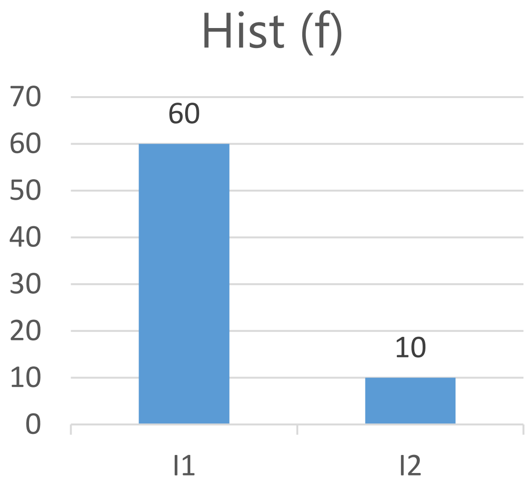
$$H = -\sum_{i \in \{I_0, \dots, I_{w-1}\}} \frac{1}{w} \times \log \frac{1}{w}$$

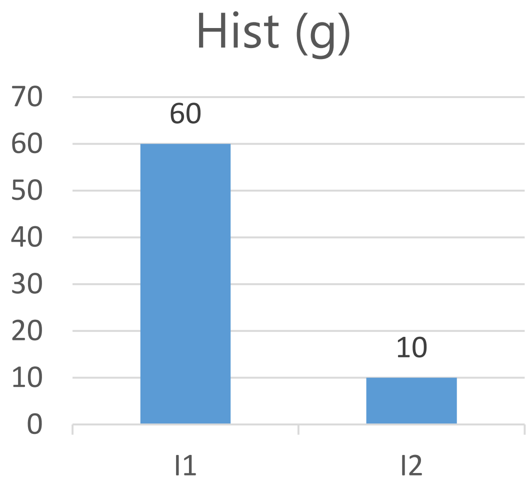
$$= -w \times \frac{1}{w} \log \frac{1}{w} = \log w$$

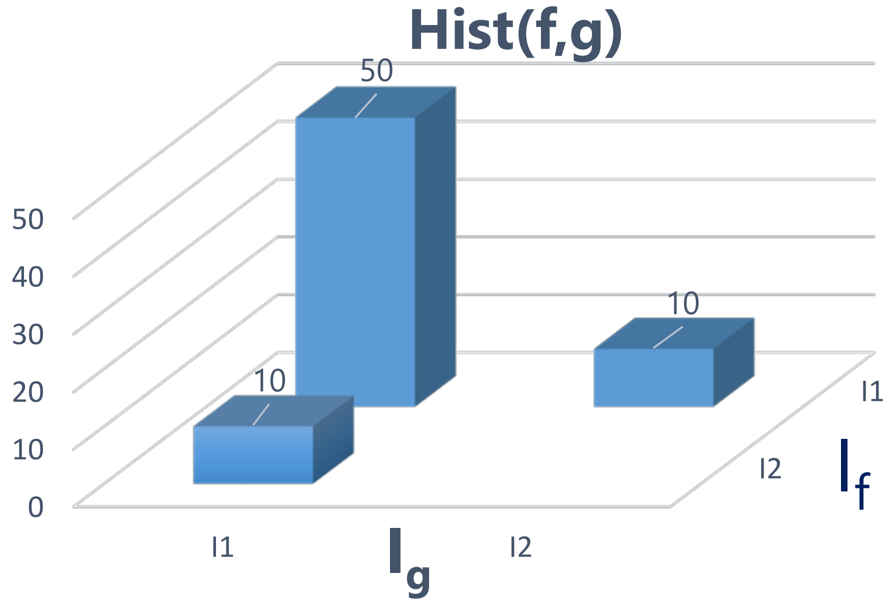
Similarity measure: Mutual information

$Joint\ histogram$:









Similarity measure: Mutual information

$Joint\ Entropy$:

$$H(I,J) = -\sum_{i,j} P_{ij} \log P_{ij} \xrightarrow{\text{مستقل } j \text{ و } i}$$

$$= -\sum_{i}\sum_{j} P_i P_j \log\left(P_i P_j\right)$$

$$= -\sum_{i}\sum_{j} P_i P_j \left(\log P_i + \log P_j\right)$$

$$= -\sum_{i}\sum_{j} P_i P_j \log P_i - \sum_{i}\sum_{j} P_i P_j \log P_j$$

$$= -\sum_{i} P_i \log P_i - \sum_{j} P_j \log P_j = H(I) + H(J)$$

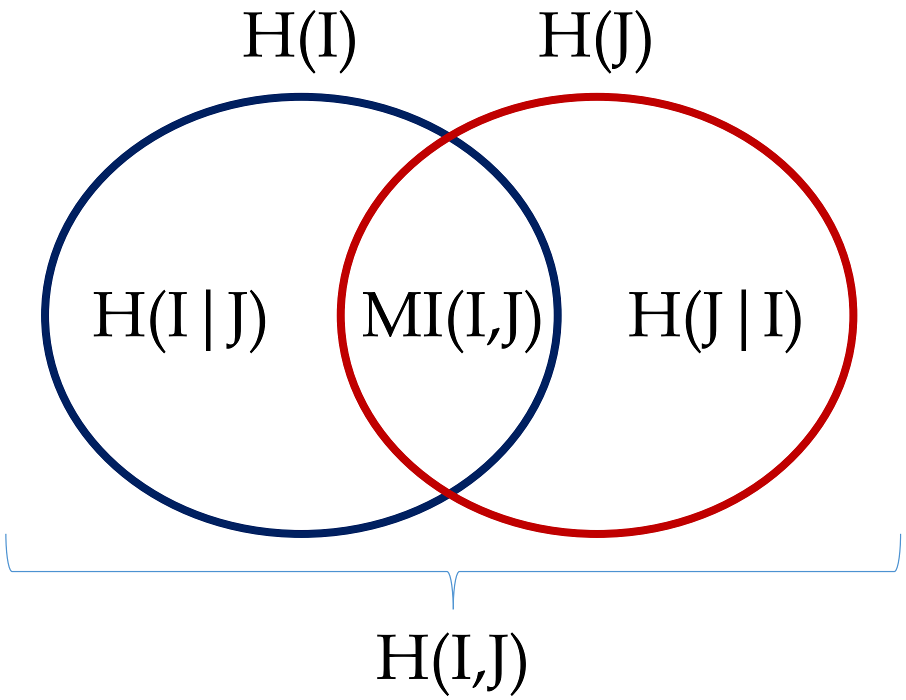
Similarity measure: Mutual information

Conditional Entropy: $H(I \mid J) = E_J\{H(I \mid J = j)\} \longrightarrow Entropy\ of\ I\ conditioned\ on\ J = j$

$$H(I \mid J) = -\sum_j P_j \sum_i P_{i|j} \log\left(P_{i|j}\right)$$

$$= -\sum_j \sum_i P_j \frac{P_{ij}}{P_j} \log\left(\frac{P_{ij}}{P_j}\right) = -\sum_{i,j} P_{ij} \log \frac{P_{ij}}{P_j}$$

$$\Longrightarrow H(I|J) = H(I,J) - H(J)$$



Similarity measure: Mutual information

$Mutual\ information$:

$$MI(I,J) = \sum_i \sum_j P_{ij} \log\left(\frac{P_{ij}}{P_i P_j}\right) = H(I) - H(I|J) = H(I) + H(J) - H(I,J)$$

$$\begin{cases} MI = 0 & \text{مستقل } i\,,j \\ MI = H(I) = H(J) & \text{یکسان} \end{cases}$$

$$MI(I,J) = \sum_i \sum_j P_{i|j} P_j \log\left(\frac{P_{i|j} P_j}{P_i P_j}\right) = \sum_i \sum_j P_{i|j} P_j \left(\log\left(P_{i|j}\right) - \log P_j\right)$$

$$= \sum_i P_{i|j} \log P_{i|j} - \sum_i P_i \log P_i = \sum_{i,j} P_{i,j} \log \frac{P_{ij}}{P_j} - \sum_i P_i \log P_i = H(I) - H(I|J)$$