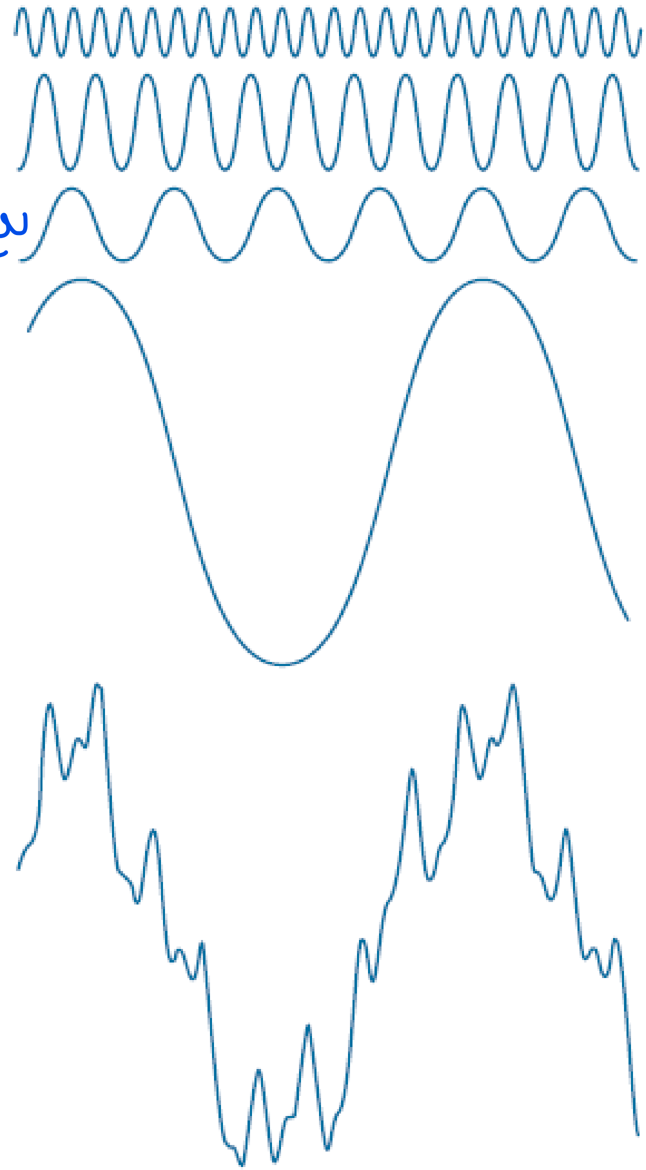
Frequency domain operations

اگر حوزه فرکانس بالا باشد
یعنی شدت‌ها با سرعت بالا از
یک پیکسل به پیکسل دیگر تغییر
می‌کند



Complex numbers

$$C = R + jI$$

$$C^* = R - jI$$

$$C = |C|(\cos\theta + j\sin\theta)$$

$$|C| = \sqrt{R^2 + I^2}$$

$$\theta = \tan^{-1}(I/R)$$

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$C = |C|e^{j\theta}$$

Complex functions

$$F(u) = R(u) + jI(u)$$

$$F^*(u) = R(u) - jI(u)$$

$$|F(u)| = \sqrt{R(u)^2 + I(u)^2}$$

$$\theta(u) = \tan^{-1}\left[{}^{I(u)}\!/_{R(u)}\right]$$

Impulse function

$$\delta(t) = \begin{cases} \infty & if\ t = 0 \\ 0 & if\ \ t \neq 0 \end{cases}$$

$$\delta(x) = \begin{cases} 1 & x = 0 \\ 0 & x \neq 0 \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

$$\sum_{x=-\infty}^{\infty} \delta(x) = 1$$

$$\int_{-\infty}^{\infty} f(t)\delta(t)\, dt = f(0)$$

$$\sum_{x=-\infty}^{\infty} f(x)\delta(x) = f(0)$$

$$\int_{-\infty}^{\infty} f(t)\delta(t - t_0)\, dt = f(t_0)$$

$$\sum_{x=-\infty}^{\infty} f(x)\delta(x - x_0) = f(x_0)$$

Fourier transform

Forward

$$F(\mu) = \int_{-\infty}^{\infty} f(t) e^{-j2\pi\mu t}\, dt$$

Inverse

$$f(t) = \int_{-\infty}^{\infty} F(\mu) e^{j2\pi\mu t}\, d\mu$$

Example

$$f(\mu) = \int_{-\infty}^{\infty} f(t) e^{-j2\pi\mu t} dt = \int_{-W/2}^{W/2} A\, e^{-j2\pi\mu t} dt$$

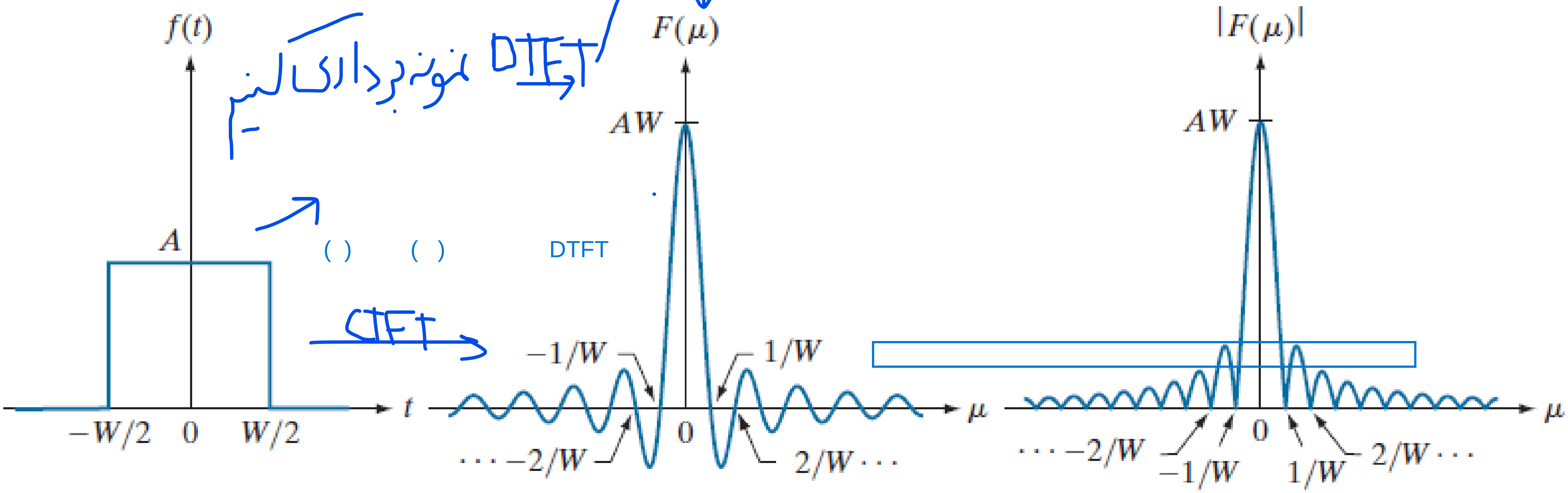
$$= \frac{-A}{j2\pi\mu}\left[e^{-j2\pi\mu t}\right]_{-W/2}^{W/2} = \frac{-A}{j2\pi\mu}\left[e^{-j\pi\mu W} - e^{j\pi\mu W}\right]$$

$$= \frac{A}{j2\pi\mu}\left[e^{j\pi\mu W} - e^{-j\pi\mu W}\right]$$

$$= AW \frac{\sin(\pi\mu W)}{\pi\mu W}$$

$$|F(\mu)| = AT \left|\frac{\sin(\pi\mu W)}{\pi\mu W}\right|$$

متناوب



Signal Transform Spectrum

C++ کاذال شده

با یک قطار ضربه

1D DFT:

سیگنال نمونه برداری شده (گسسته)

DTFT

سیگنال پیوسته

$$\tilde{F}(\mu) = \int_{-\infty}^{\infty} \tilde{f}(t) \exp(-j2\pi\mu t)dt$$

$$= \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} f(t)\delta(t - n\Delta T) \exp(-j2\pi\mu t)dt$$

غربال غربال

$$= \sum_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(t)\delta(t - n\Delta T) \exp(-j2\pi\mu t)dt$$

$$= \sum_{-\infty}^{\infty} f(n\Delta T) \exp(-j2\pi\mu n\Delta T)$$

پیوسته

Sampling : $\tilde{F}$; $\mu = \frac{m}{M\Delta T}$ m = 0, 1, 2, ..., M-1; M samples over $\frac{1}{\Delta T}$

تبدیل فوریه گسسته

خودش هم گسسته است

$$F_m = \sum_{n=0}^{M-1} f_n \exp(-j2\pi mn/M)$$

پریود ثابت

تک

سیگنال

$$F(u) = \sum_{x=0}^{M-1} f(x) \exp(-j2\pi ux/M)$$

2D DFT: $(-1)^{x+y}$ اول

قبل DFT

x شیفت در تصویر

$$F(u,v) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)\exp(-j2\pi(\frac{ux}{M}+\frac{vy}{N}))$$

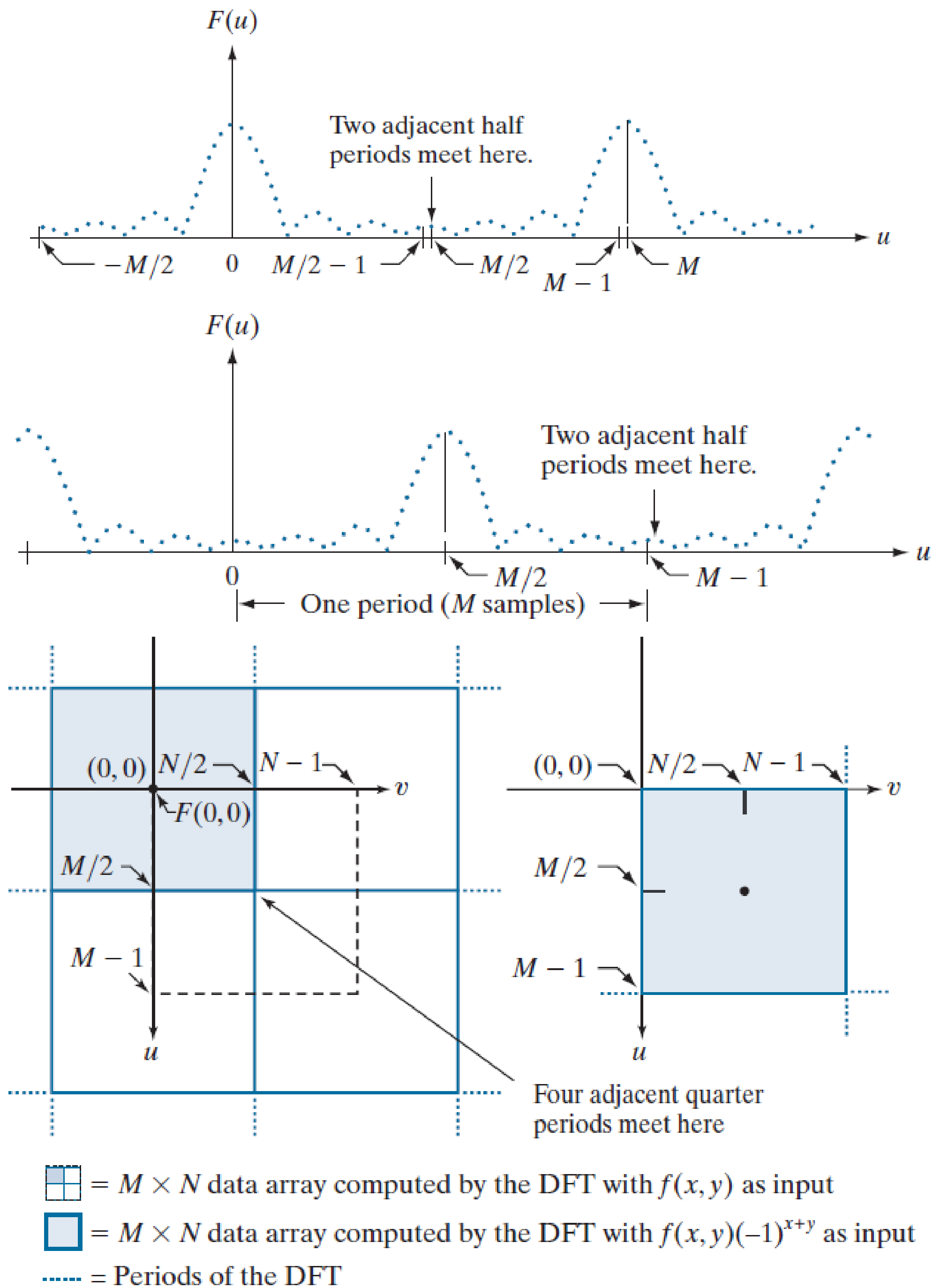
u = 0, 1, 2, ..., M-1 v = 0, 1, 2, ..., N-1

$$f(x,y) = \frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u,v)\exp(j2\pi(\frac{ux}{M}+\frac{vy}{N}))$$

x = 0, 1, 2, ..., M-1 y = 0, 1, 2, ..., N-1

جای DC ← فرکانس صفر ← انرژی بالا

Centering:



F(u)
Two adjacent half periods meet here.
−M/2
0
M/2 − 1
M/2
M − 1
M
u
F(u)
Two adjacent half periods meet here.
0
M/2
M − 1
u
One period (M samples)
(0, 0)
N/2
N − 1
v
F(0, 0)
M/2
M − 1
u
(0, 0)
N/2
N − 1
v
M/2
M − 1
u
Four adjacent quarter periods meet here
= M × N data array computed by the DFT with f(x, y) as input
= M × N data array computed by the DFT with f(x, y)(−1)^{x+y} as input
= Periods of the DFT

$$f(x)\exp\left(j2\pi u_0 \frac{x}{M}\right) \longleftrightarrow F(u - u_0) \quad \& \; u_0 = \frac{M}{2}$$

$$f(x)\exp(j\pi x) \longleftrightarrow F\left(u - \frac{M}{2}\right)$$

$$f(x)\,(-1)^x \longleftrightarrow F\left(u - \frac{M}{2}\right)$$

$$f(x,y)\,(-1)^{x+y} \longleftrightarrow F\left(u - \frac{M}{2}, v - \frac{N}{2}\right)$$

DC را انتقال به مرکز تصویر

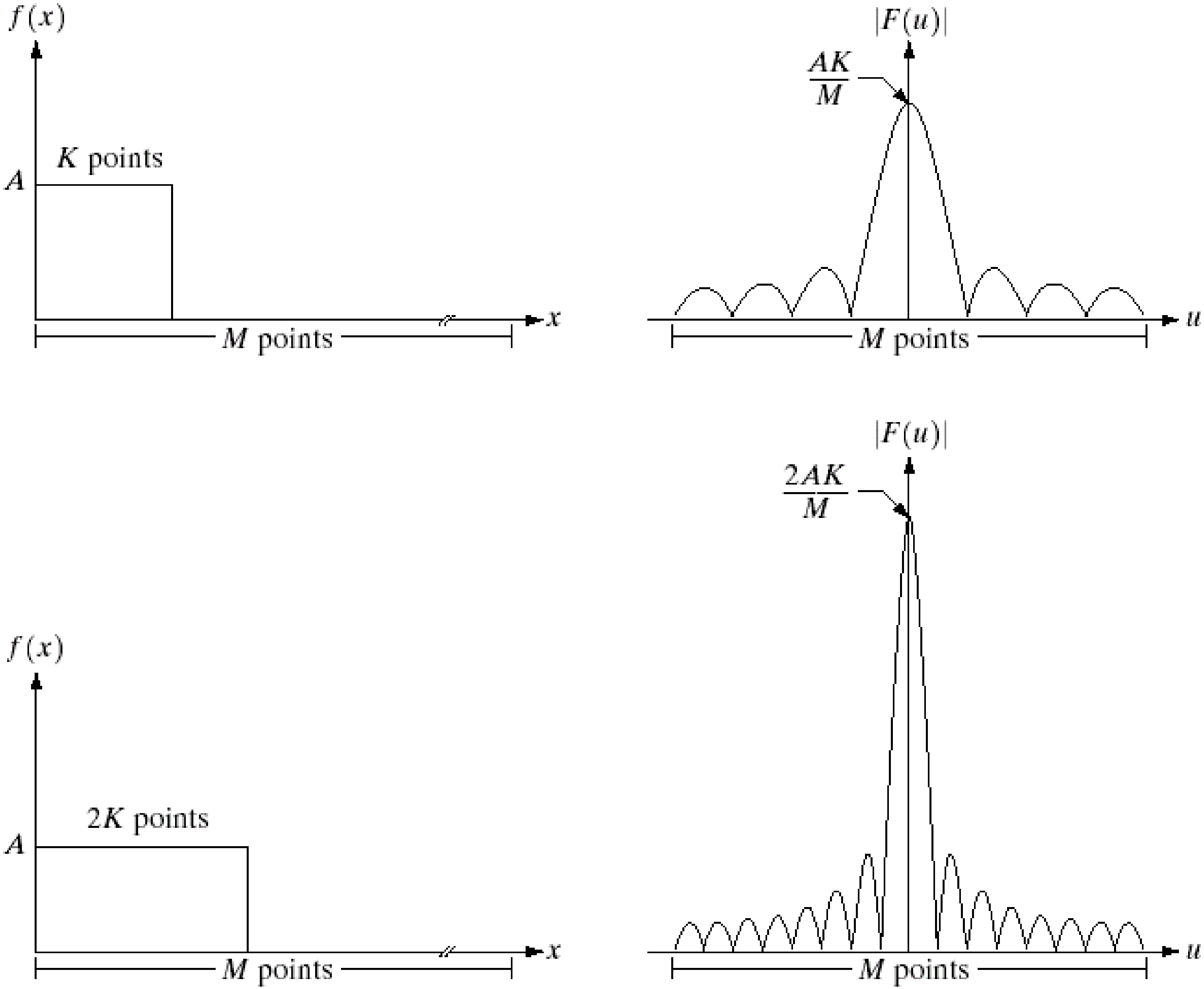
$$F(u,v) = F(u+k_1M, v) = F(u, v+k_2N) = F(u+k_1M, v+k_2N)$$

$$f(x,y) = f(x+k_1M, y) = f(x, y+k_2N) = f(x+k_1M, y+k_2N)$$

$$f(x,y) * h(x,y) = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} f(m,n)h(x-m, y-n)$$

$$f * h \longleftrightarrow F \,.\, H$$
$$f \,.\, h \longleftrightarrow F * H$$

Example

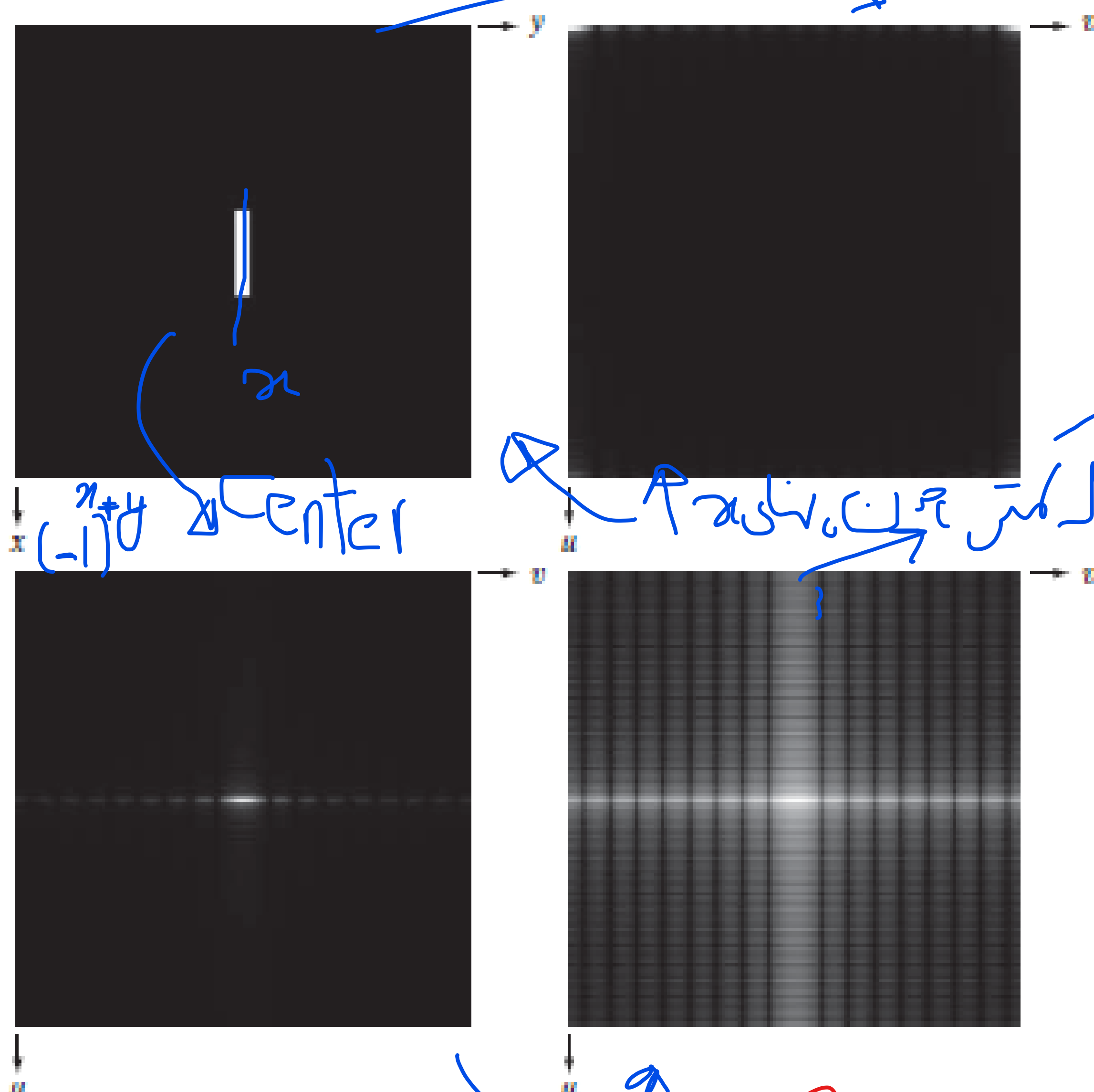


f(x)
K points
A
x
M points
|F(u)|
AK/M
u
M points
f(x)
2K points
A
x
M points
|F(u)|
2AK/M
u
M points

Image Example: Spectrum

a b
c d

FIGURE 4.23
(a) Image.
(b) Spectrum, showing small, bright areas in the four corners (you have to look carefully to see them).
(c) Centered spectrum.
(d) Result after a log transformation.
The zero crossings of the spectrum are closer in the vertical direction because the rectangle in (a) is longer in that direction. The right-handed coordinate convention used in the book places the origin of the spatial and frequency domains at the top left (see Fig. 2.19).



اگر شیفت نزنیم

x

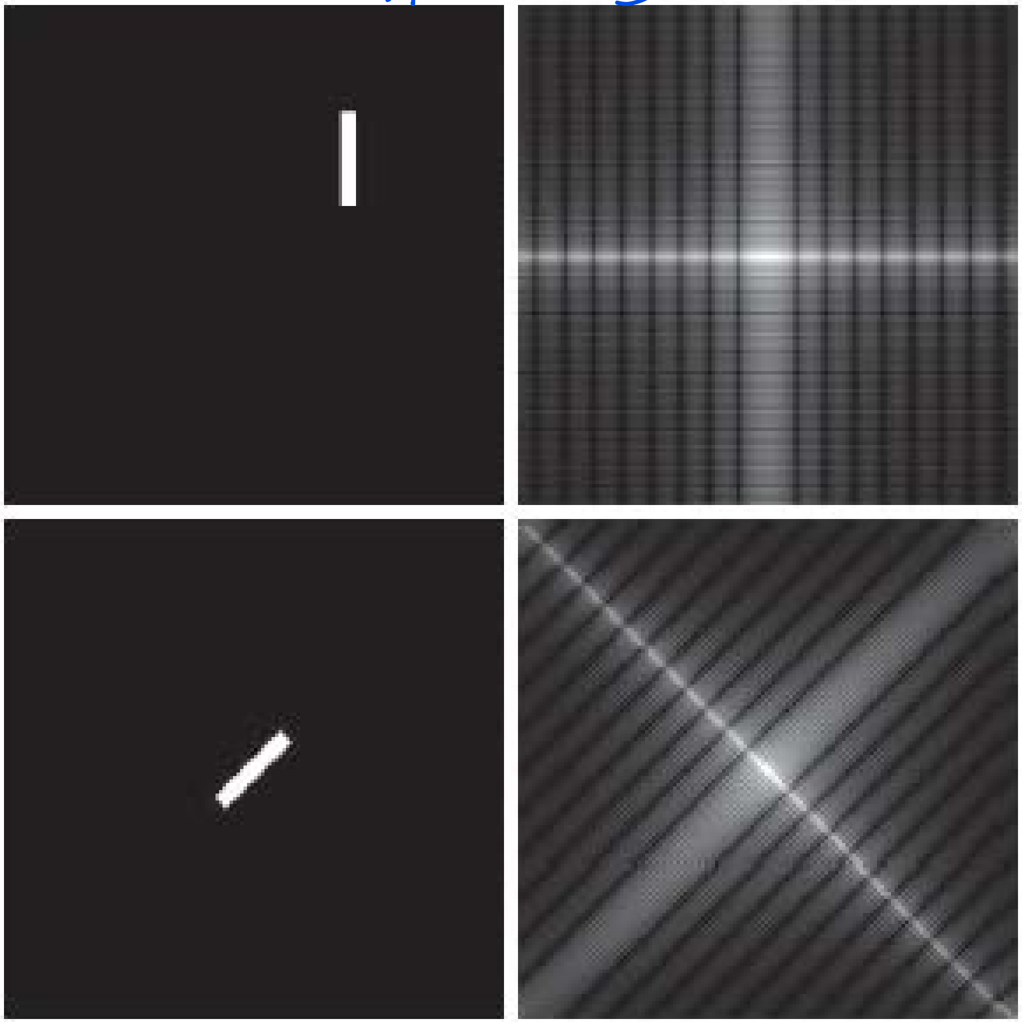
① $\times (-1)^{x+y}$ center

فاصله کمتر چون در راستای x

② Scarm & ③ log

Image Example: Spectrum

جابه جایی در حوزه مکان ایجاد نمی‌کنه چون $|e| = 1$ در spectrum

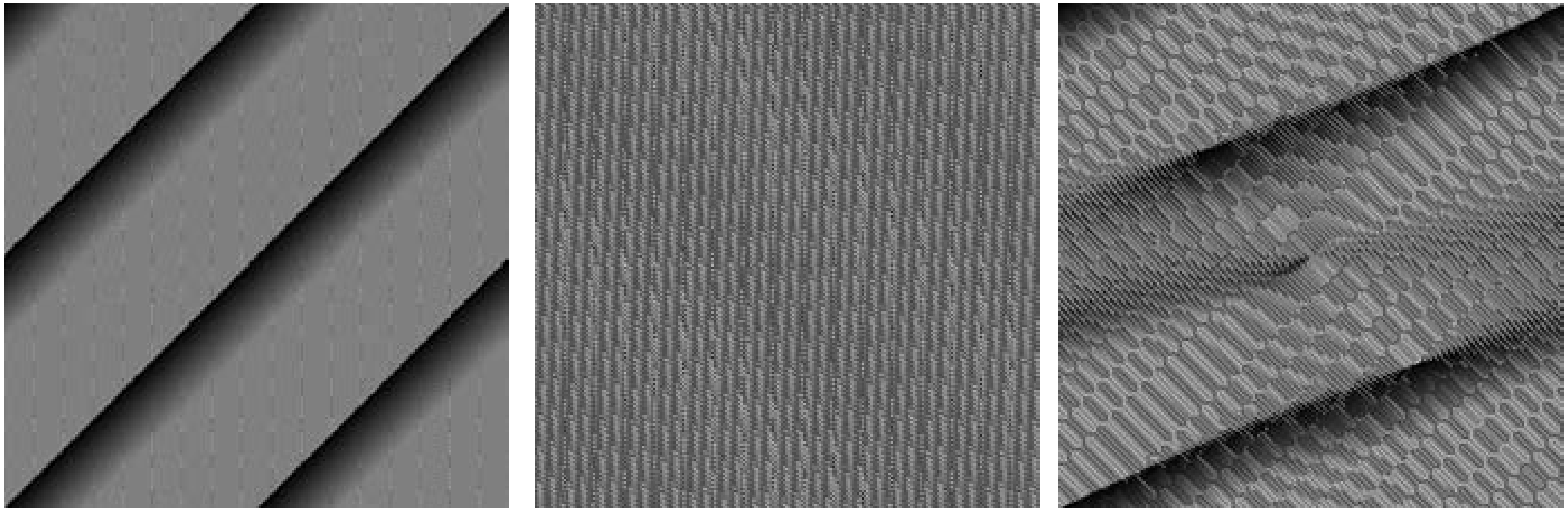


a b
c d

FIGURE 4.24
(a) The rectangle in Fig. 4.23(a) translated.
(b) Corresponding spectrum.
(c) Rotated rectangle.
(d) Corresponding spectrum.
The spectrum of the translated rectangle is identical to the spectrum of the original image in Fig. 4.23(a).

Image Example: Phase

اطلاعات مکان



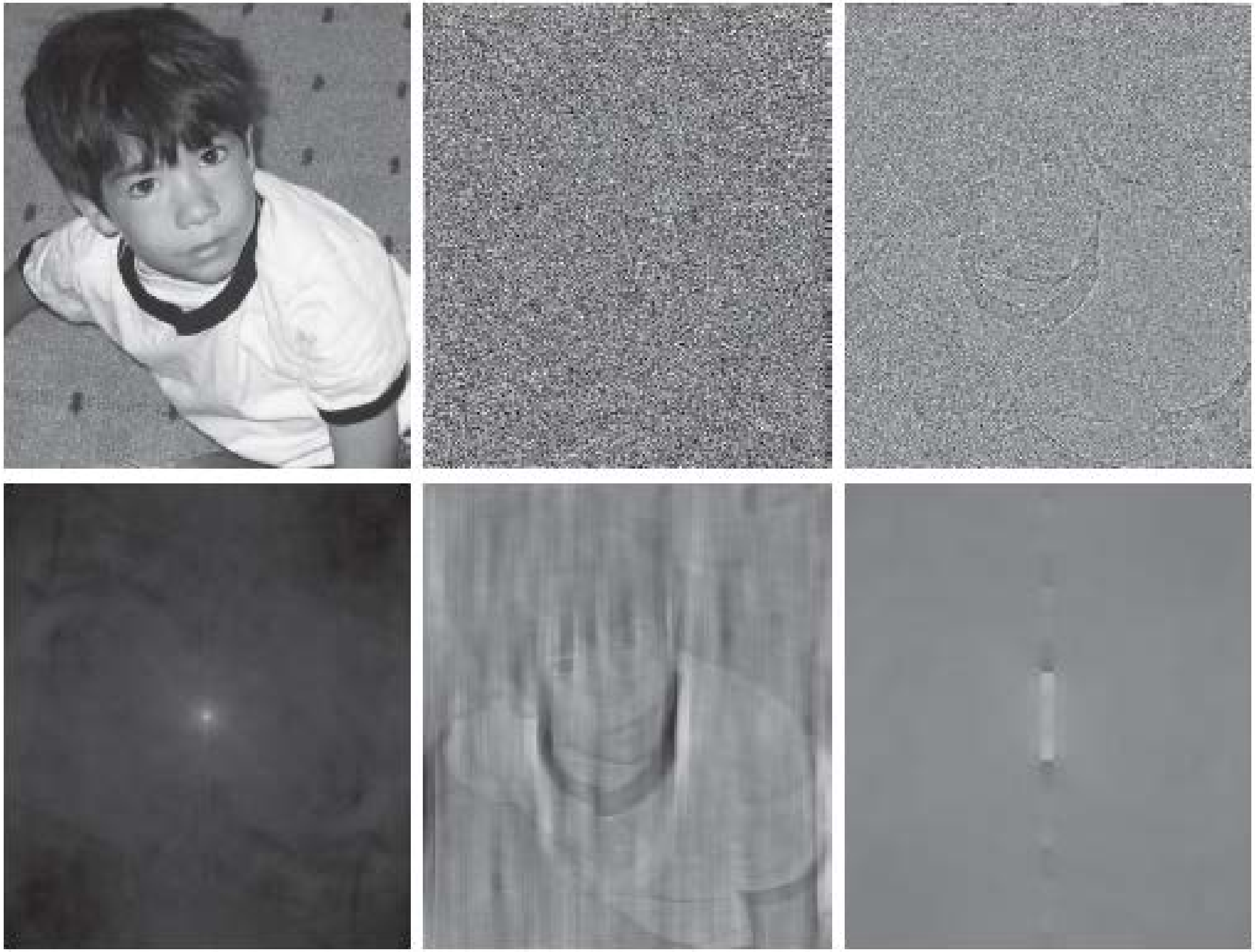
a b c

FIGURE 4.25
Phase angle images of (a) centered, (b) translated, and (c) rotated rectangles.

در فاز اطلاعات شهودی مشخص به ما نمی دهد.

اما اطلاعات اولیه تصویر در فاز است

Image Example: Phase



a b c
d e f

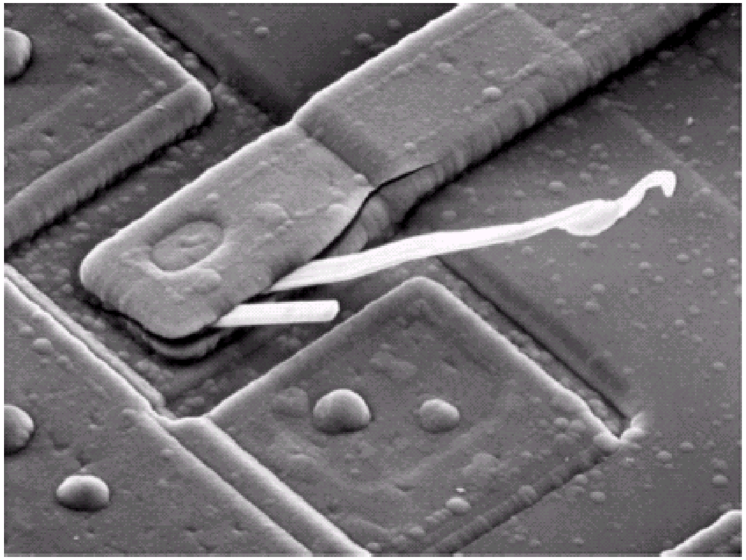
FIGURE 4.26 (a) Boy image. (b) Phase angle. (c) Boy image reconstructed using only its phase angle (all shape features are there, but the intensity information is missing because the spectrum was not used in the reconstruction). (d) Boy image reconstructed using only its spectrum. (e) Boy image reconstructed using its phase angle and the spectrum of the rectangle in Fig. 4.23(a). (f) Rectangle image reconstructed using its phase and the spectrum of the boy's image.

اطلاعات خاصی به ما
نمیده

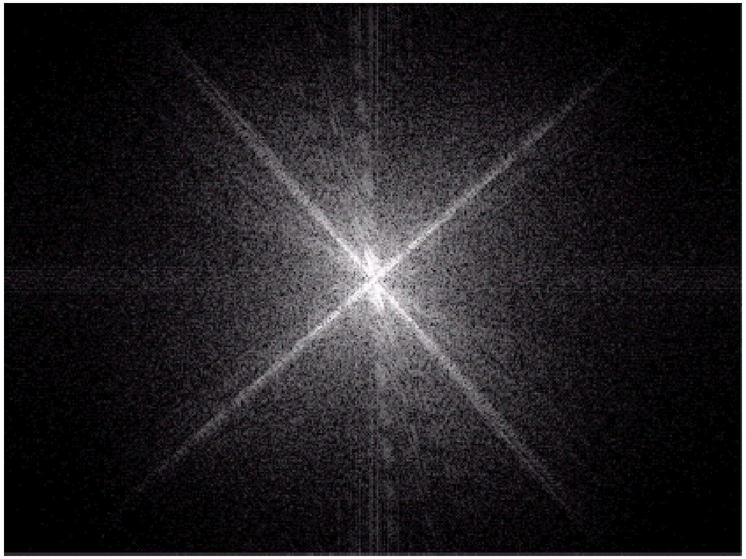
از فاز تبدیل معکوس بگیریم تصویر c که اکثر اطلاعات مشخص است / از spectrum تبدیل معکوس بگیریم

Image Example

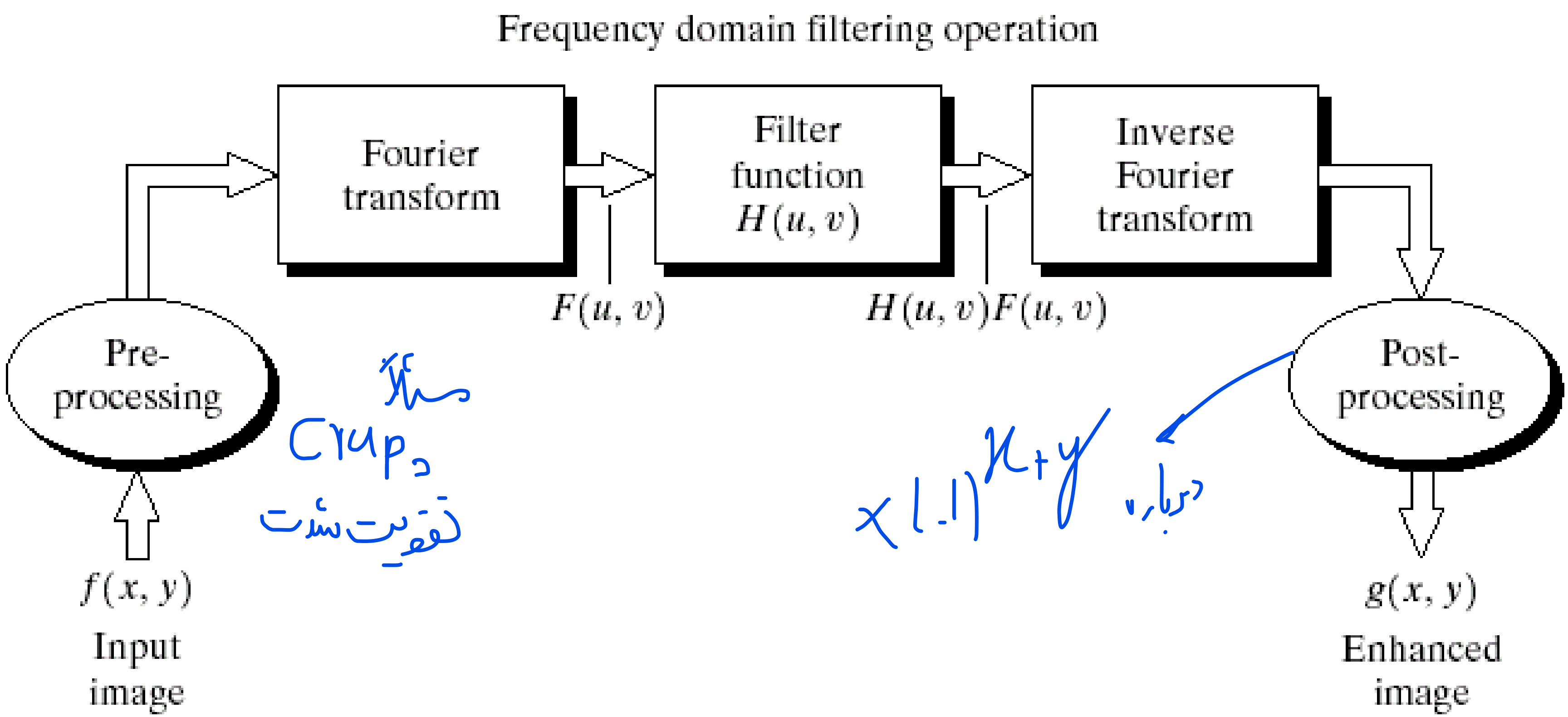
SEM image
of a damaged IC



Fourier spectrum



Steps for filtering in frequency domain



سایز
Crop
تقویت شدت

$\times(-1)^{x+y}$ دوباره

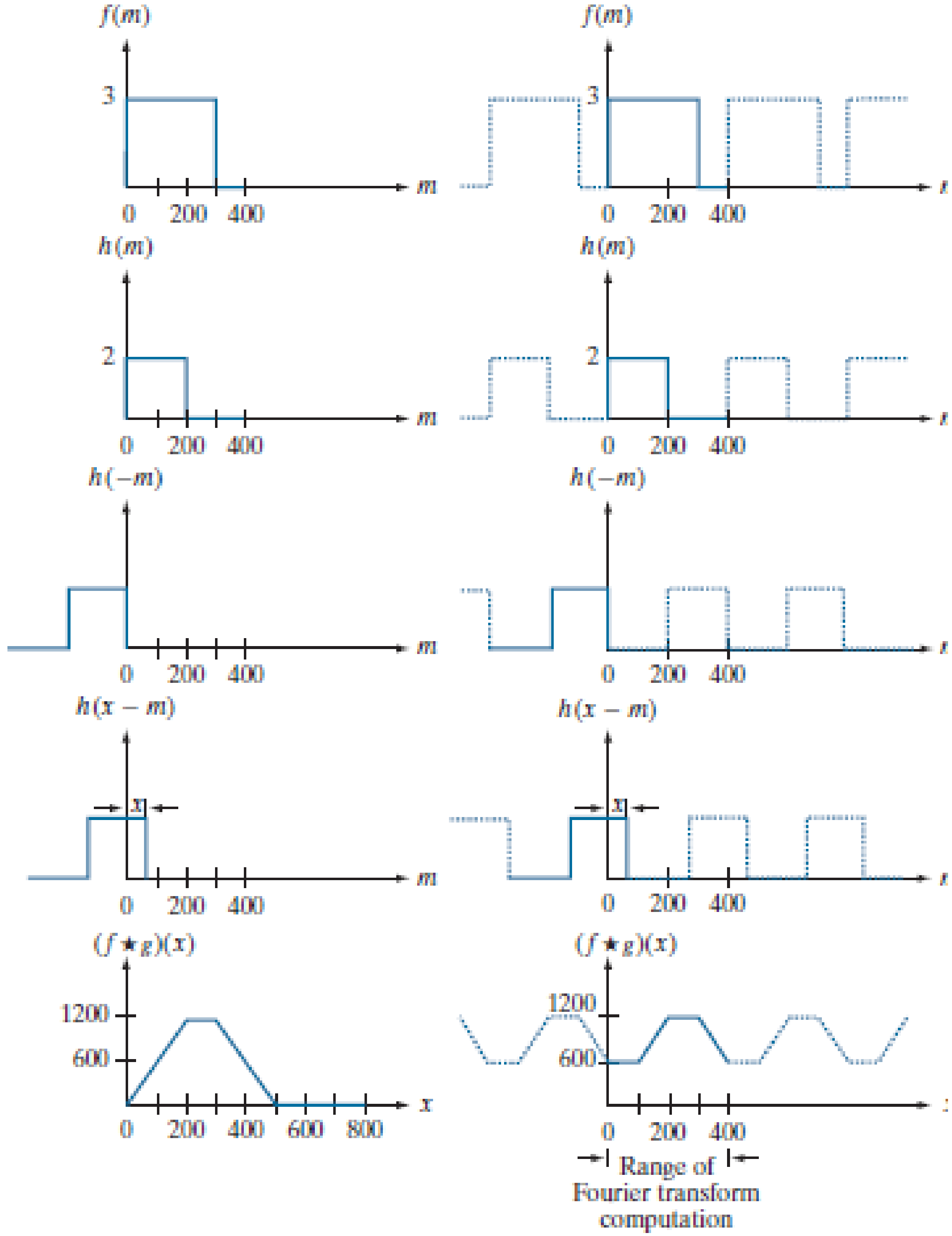
2D Convolution Theorem

$$f(x,y) \bigstar\ h(x,y) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f(m,n)h(x-m,y-n)$$

$$f(x,y) \bigstar h(x,y) \Leftrightarrow F(u,v)H(u,v)$$

Periodicity of DFT

Convolution



Convolution with periodicity

اول این کارو کنیم و بعد بریم تو حوزه فوریه

Padding for periodicity

1D:

$$f_p(x) = \begin{cases} f(x) & 0 \le x \le A-1 \\ 0 & A \le x \le P \end{cases} \qquad h_p(x) = \begin{cases} h(x) & 0 \le x \le B-1 \\ 0 & B \le x \le P \end{cases}$$

$$P \ge A+B-1$$

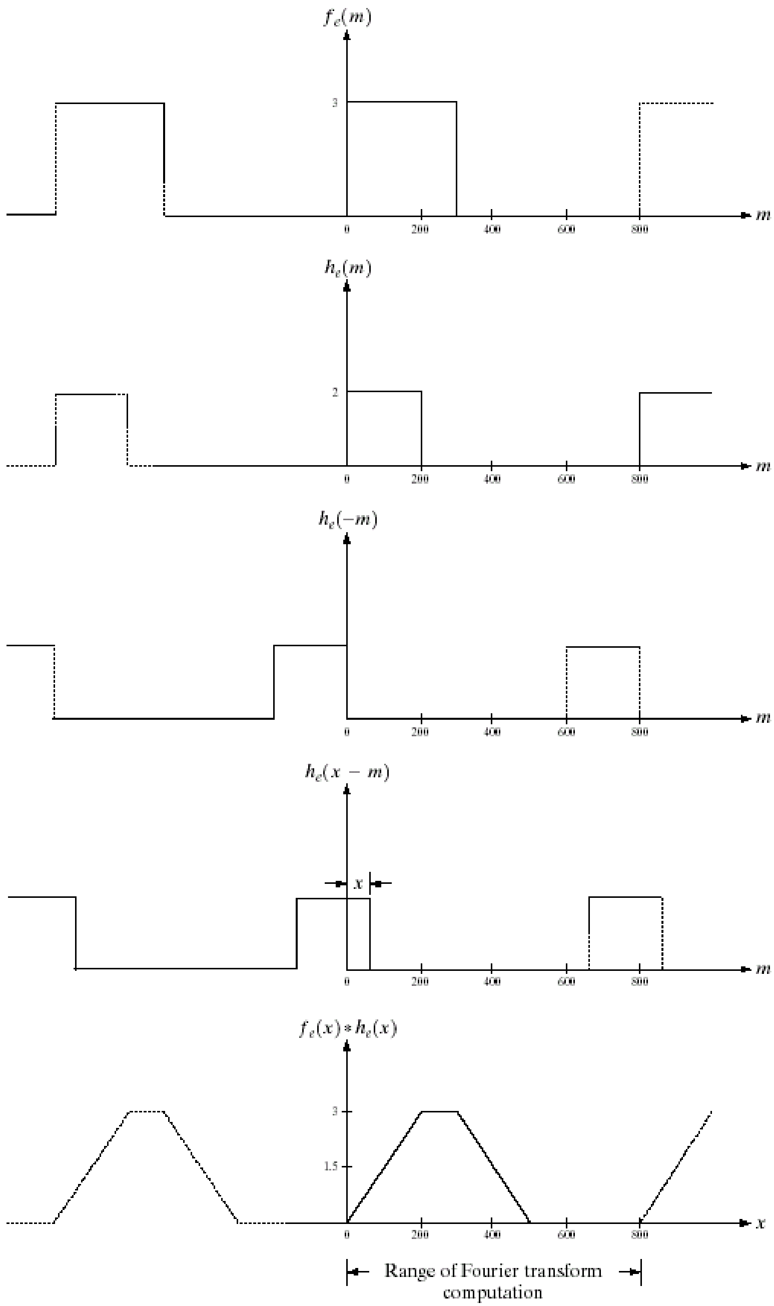
2D :

$f: A \times B \qquad h: C \times D \qquad P \ge A+C-1 \qquad Q \ge B+D-1$

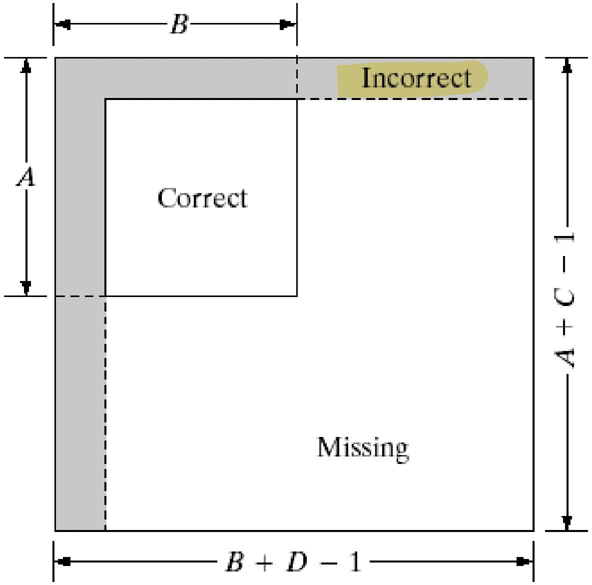
$$f_p(x,y) = \begin{cases} f(x,y) & 0 \le x \le A-1\ ,\ \ 0 \le y \le B-1 \\ 0 & A \le x \le P \quad or \quad B \le y \le Q \end{cases}$$

$$h_p(x,y) = \begin{cases} h(x,y) & 0 \le x \le C-1\ ,\ \ 0 \le y \le D-1 \\ 0 & C \le x \le P \quad or \quad D \le y \le Q \end{cases}$$

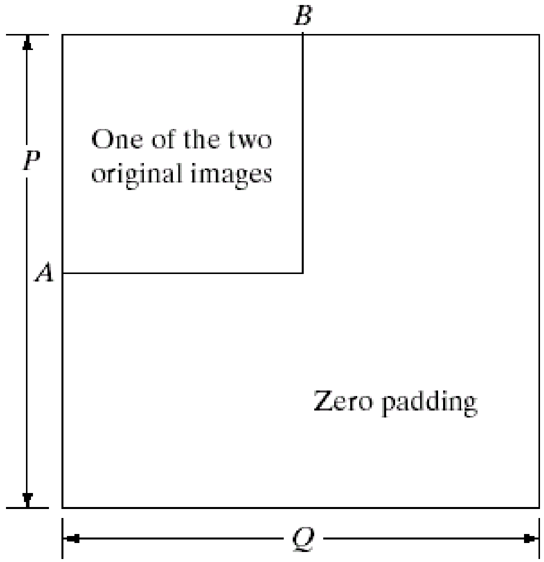
Convolution with extended functions



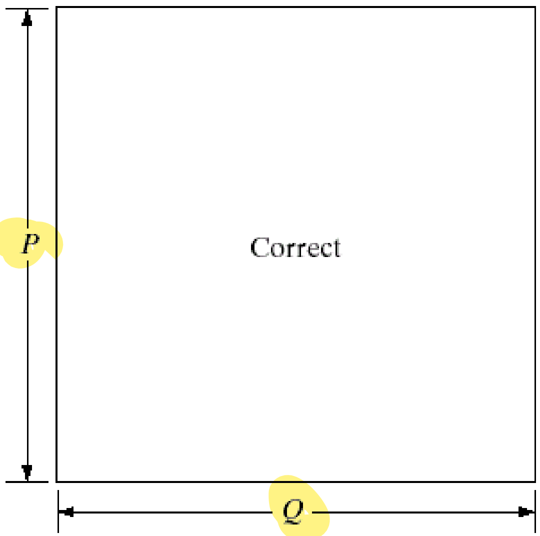
Padding



Result of filtering in the frequency domain without properly padding the input images.



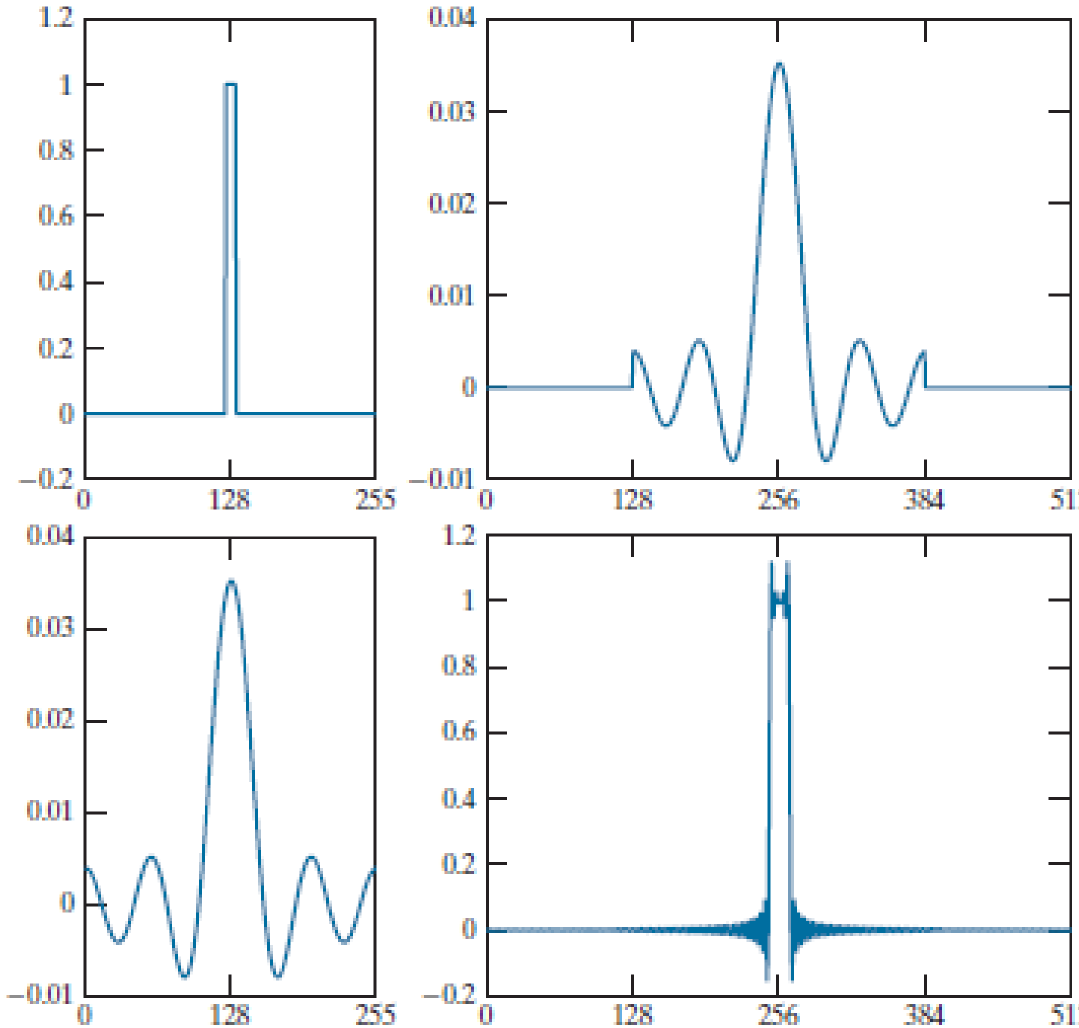
Properly extended (padded) image



$$P = A + C - 1$$
$$Q = B + D - 1$$

Result of filtering in the frequency domain with properly padded input images.

Padding the filter

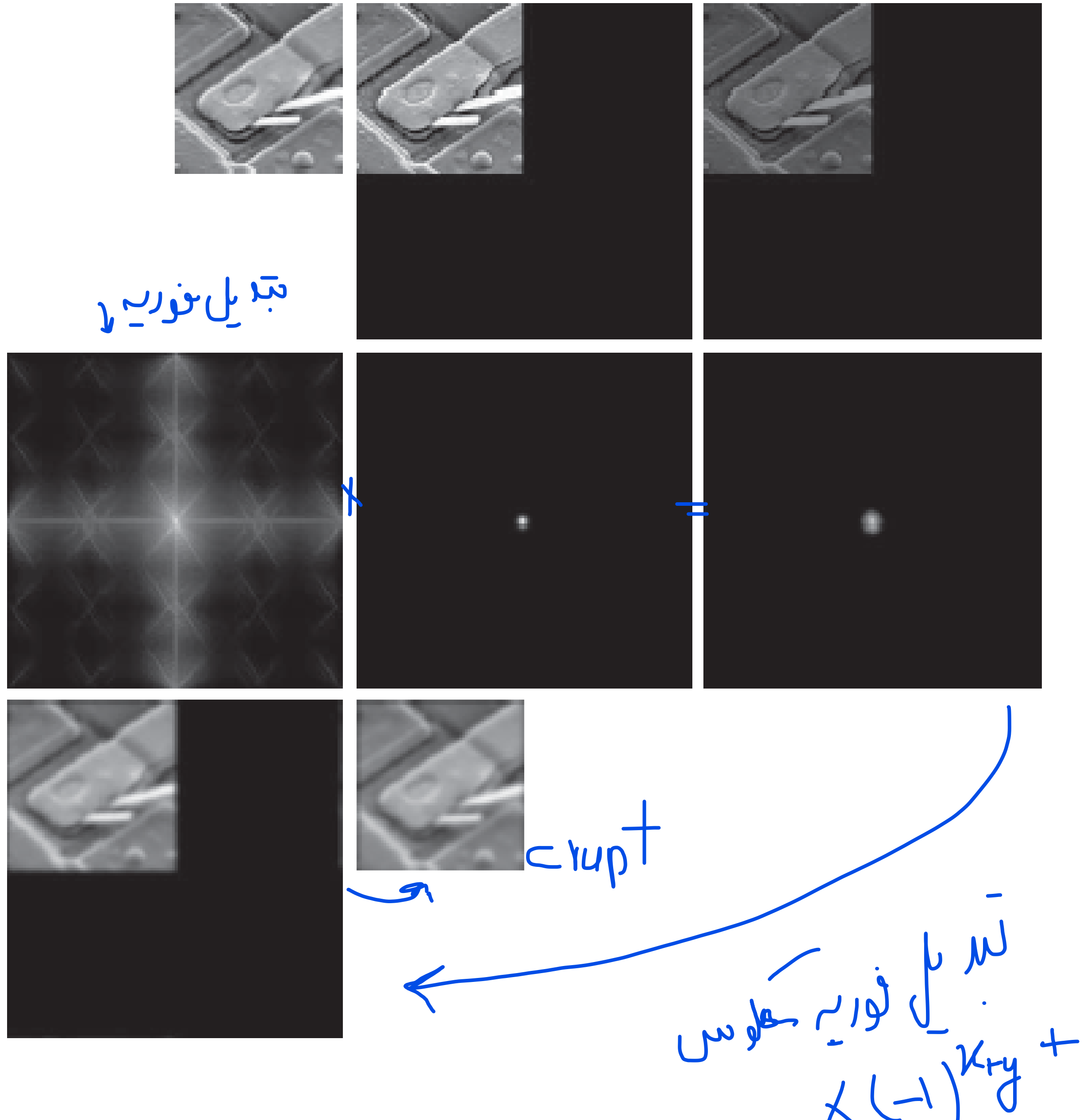


a c
b d

FIGURE 4.33
(a) Filter transfer function specified in the (centered) frequency domain.
(b) Spatial representation (filter kernel) obtained by computing the IDFT of (a).
(c) Result of padding (b) to twice its length (note the discontinuities).
(d) Corresponding filter in the frequency domain obtained by computing the DFT of (c). Note the ringing caused by the discontinuities in (c). Part (b) of the figure is below (a), and (d) is below (c).

Filtering steps

شیفت های منفی رو مثبت کردیم



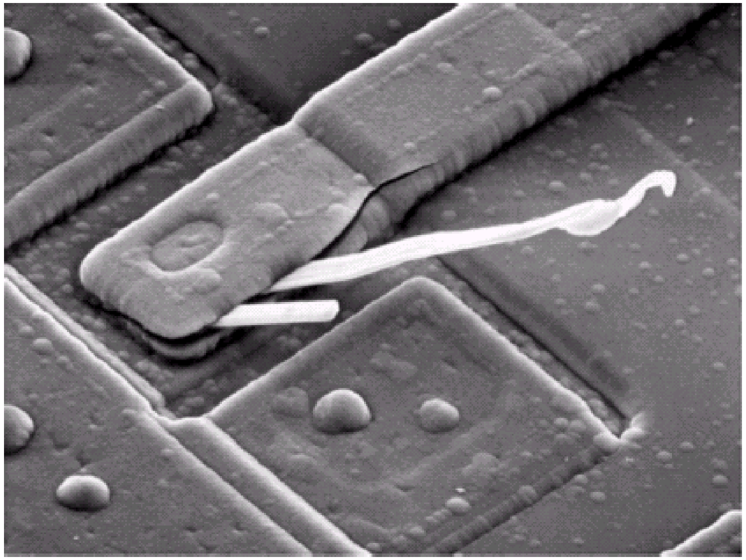
a b c
d e f
g h

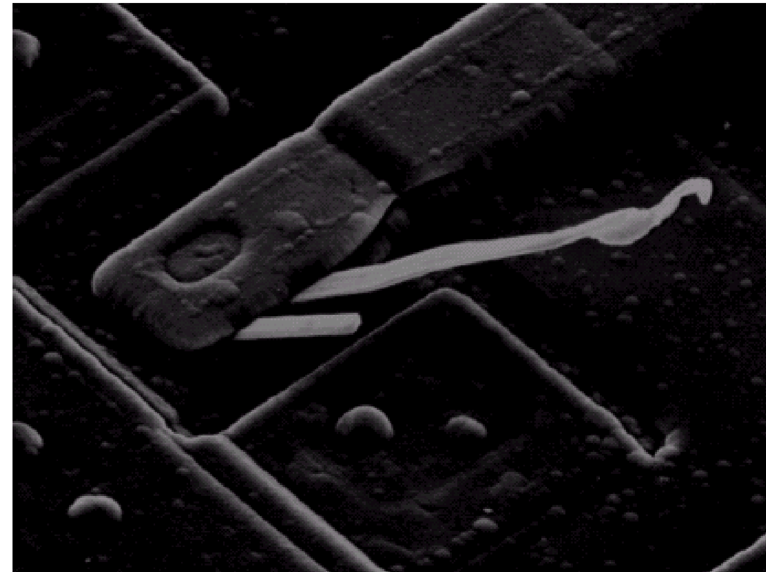
FIGURE 4.35
(a) An $M \times N$ image, f.
(b) Padded image, f_p of size $P \times Q$.
(c) Result of multiplying f_p by $(-1)^{x+y}$.
(d) Spectrum of F.
(e) Centered Gaussian lowpass filter transfer function, H, of size $P \times Q$.
(f) Spectrum of the product HF.
(g) Image g_p, the real part of the IDFT of HF, multiplied by $(-1)^{x+y}$.
(h) Final result, g, obtained by extracting the first M rows and N columns of g_p.

به صورت
real ای گیریم

Application of Notch filters

شدت های منفی رو صفر کردیم





DC رو حذف کنه

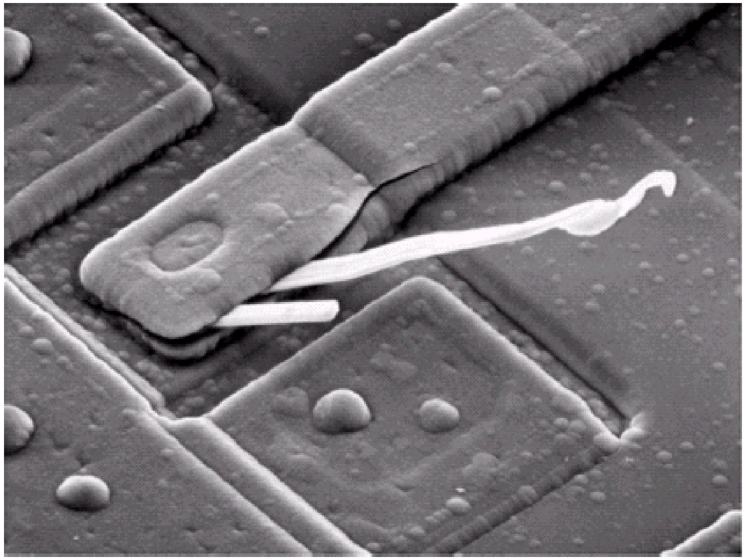
After applying notch filter to set F(0,0) to zero

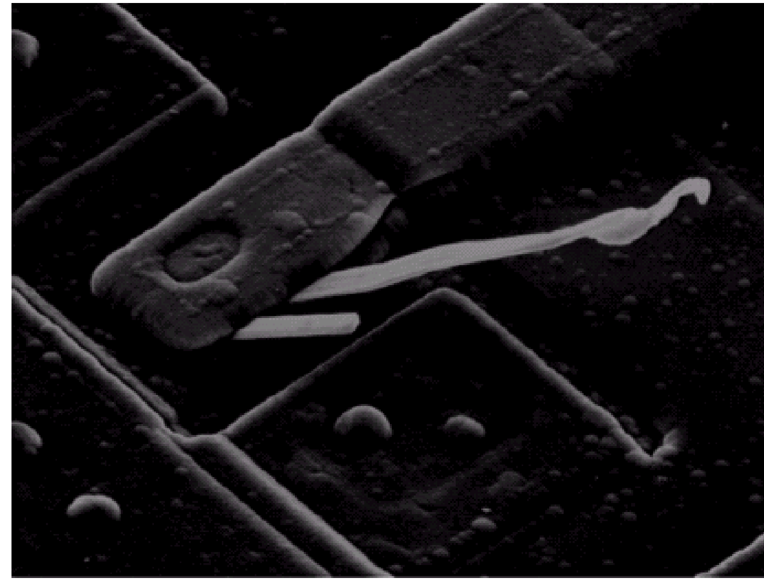
در حوزه زمان میانگین شد آن صفر است

Application of Notch filters

high pass

یک گروه فرکانس رو حذف می‌کنه
خیلی باریک هست



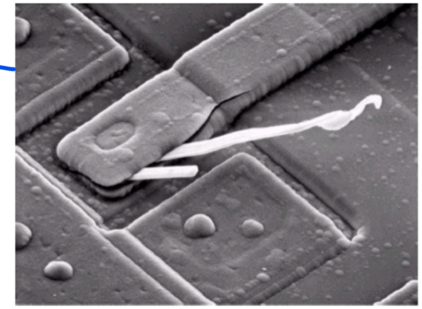


After applying notch filter to set F(0,0) to zero

$$F(0,0) = MN\frac{1}{MN}\sum_{X=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)$$
$$= MN\bar{f}(x,y)$$

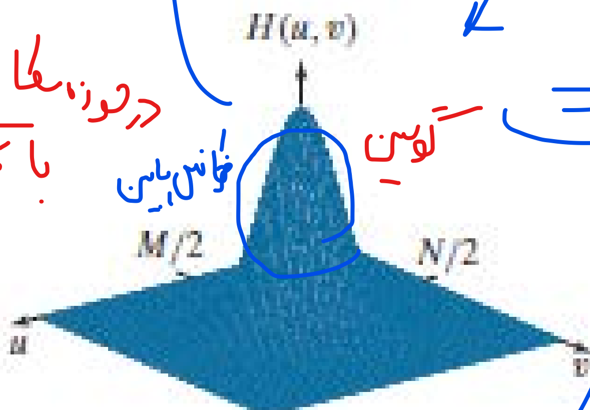
Application of LP and HP filters

مثل اینکه در حوزه مکان میانگین گیری کنیم



Original

مثل در حوزه مکان
با ماسک گوسین
کانولوشن

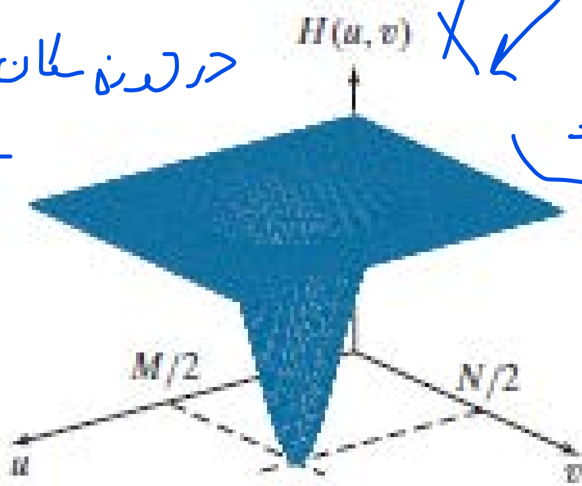


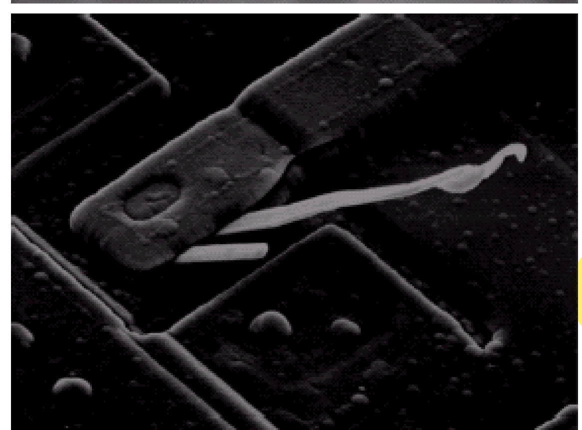
فرکانس های بالا رو وزن کم داده

LP filter applied

در حوزه مکان مثل مشتق گیری

sharpening و



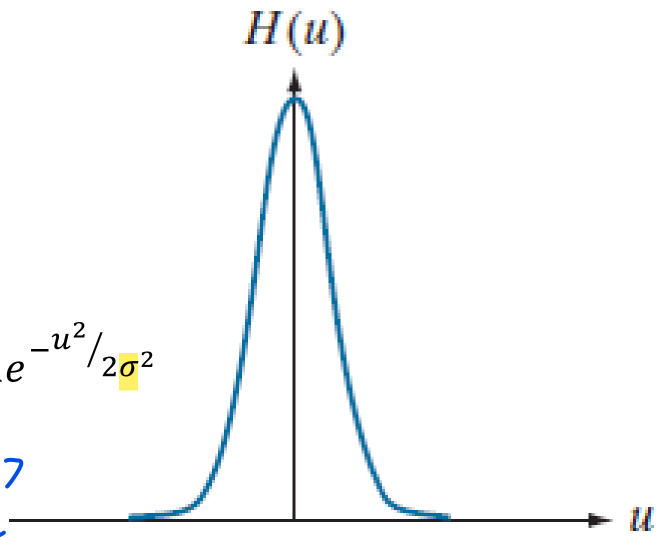


فیلتری که فرکانس های بالا رو عبور میده

HP filter applied

sharp

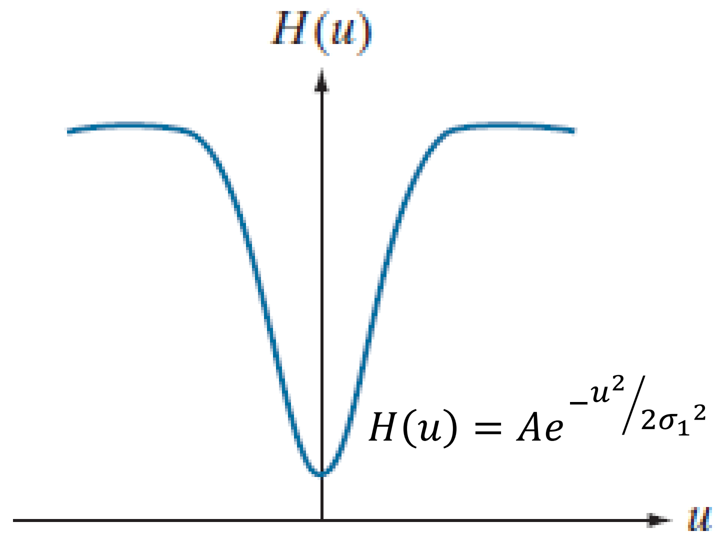
Gaussian LP and HP filters



$H(u) = Ae^{-u^2/2\sigma^2}$

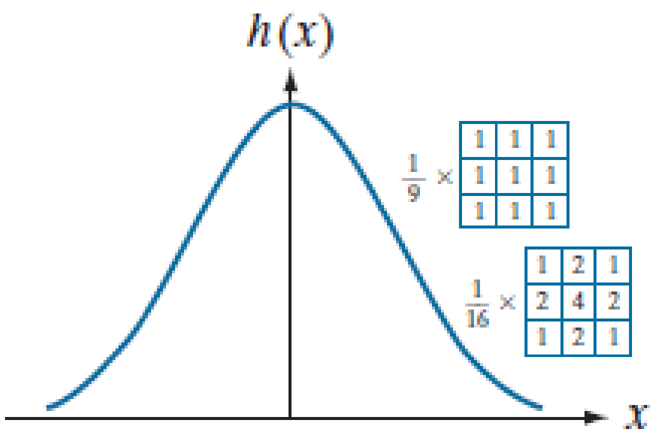
در هر دو حوزه ها
شکل گوسین
دارن

Gaussian frequency domain LPF



$H(u) = Ae^{-u^2/2\sigma_1^2} - Be^{-u^2/2\sigma_2^2}$

Gaussian frequency domain HPF



$\frac{1}{9} \times$

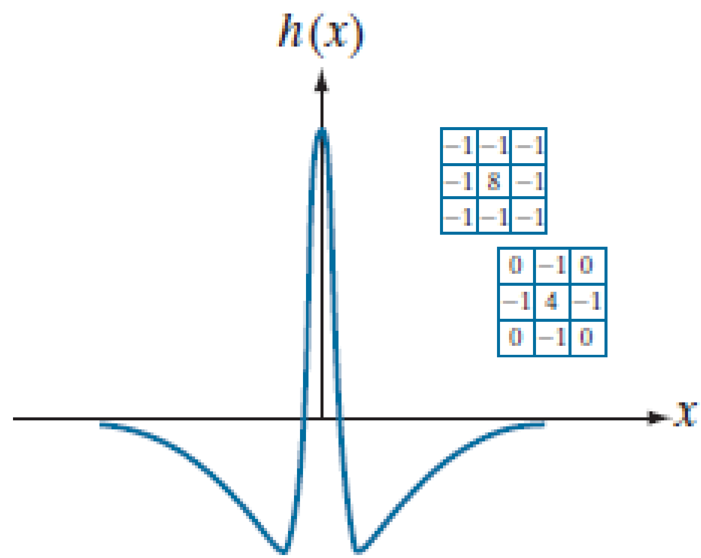
1	1	1
1	1	1
1	1	1

$\frac{1}{16} \times$

1	2	1
2	4	2
1	2	1

Gaussian spatial LPF

$h(x) = \sqrt{2\pi}\sigma Ae^{-2\pi^2\sigma^2x^2}$

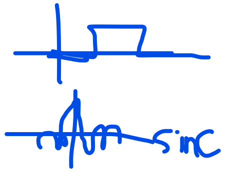


−1	−1	−1
−1	8	−1
−1	−1	−1

0	−1	0
−1	4	−1
0	−1	0

Gaussian spatial HPF

$h(x) = \sqrt{2\pi}\sigma Ae^{-2\pi^2\sigma_1^2x^2} - \sqrt{2\pi}\sigma Be^{-2\pi^2\sigma_2^2x^2}$



در حوزه فرکانس

در حوزه مکان

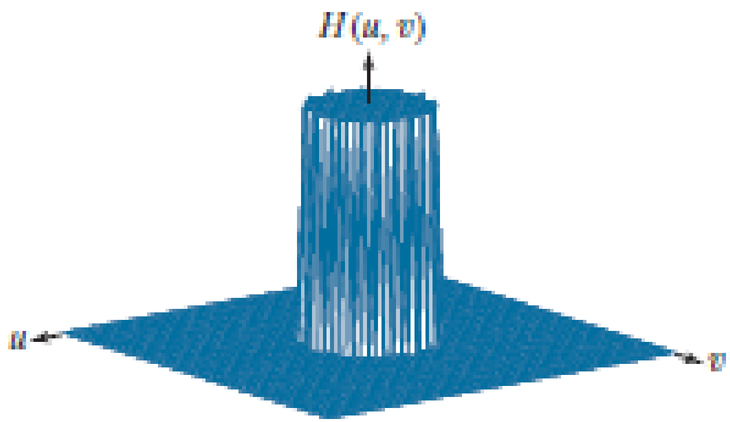
ایده‌آل Ideal LPF

low

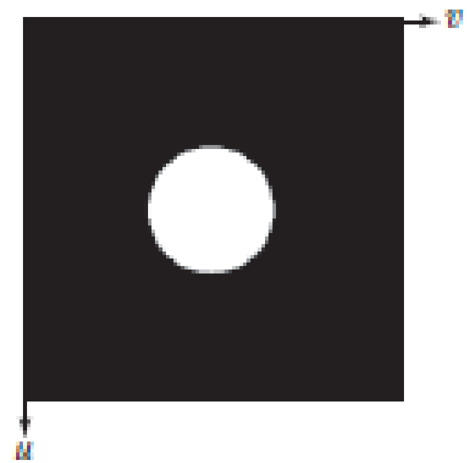
Filter

باعث تولید حلقه

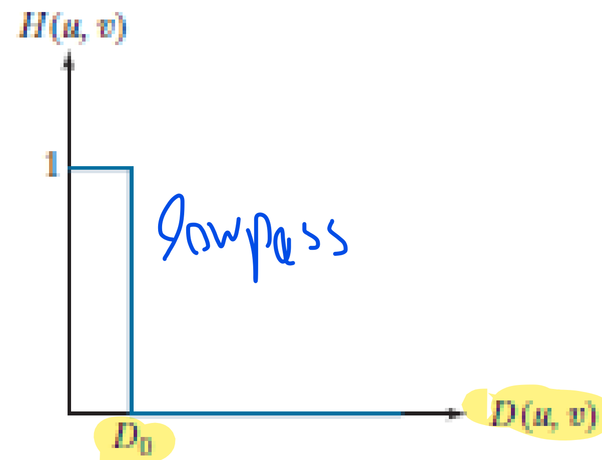
تبدیل به ضربه ای



Ideal LPF
transfer function



Filter image

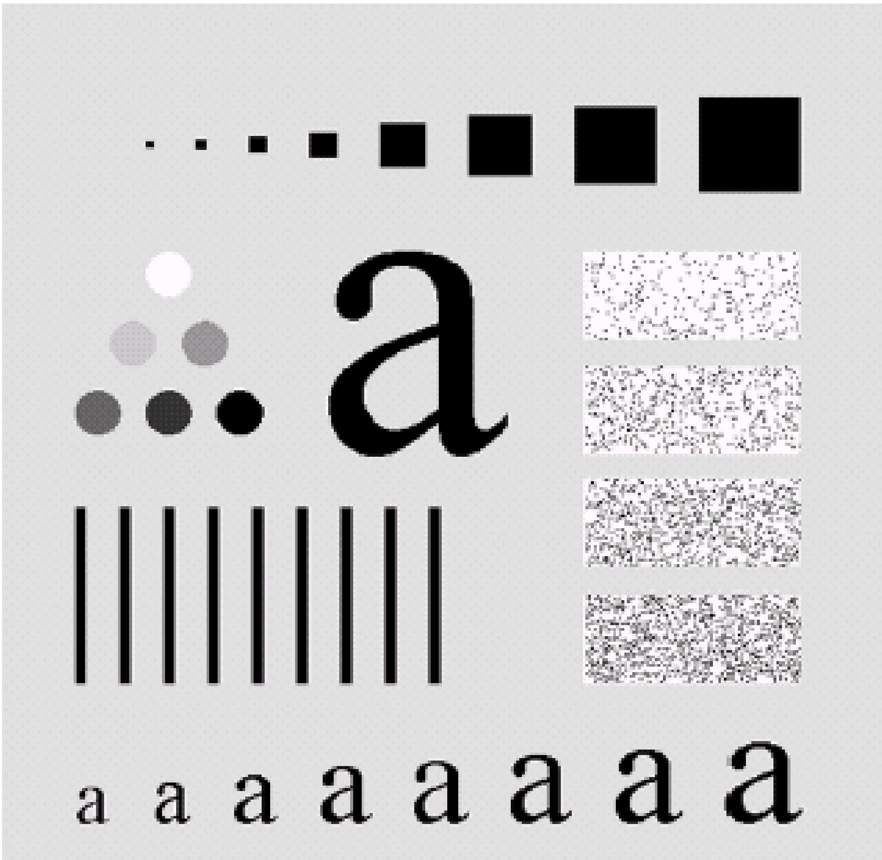


Filter radial
cross section

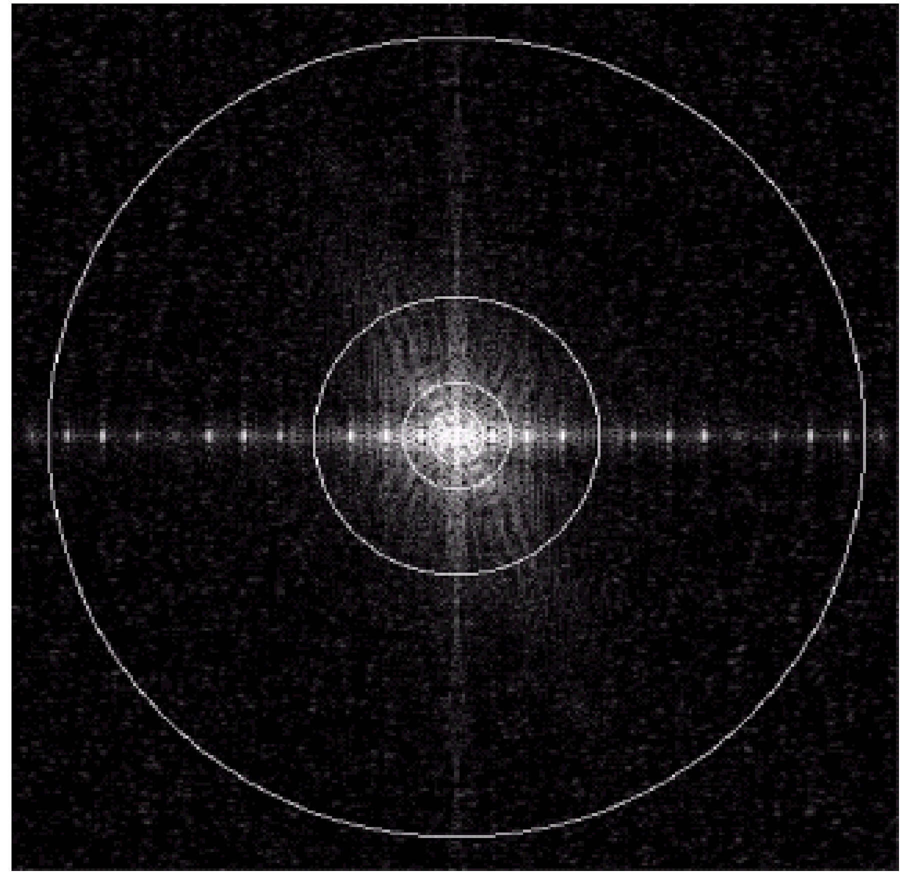
$$H(u,v) = \begin{cases} 1 & if\, D(u,v) \le D_0 \\ 0 & if\ D(u,v) > D_0 \end{cases}$$

$$D(u,v) = \left[(u - {}^{P}/_{2})^2 + (v - {}^{Q}/_{2})^2\right]^{1/2}$$

Choosing cutoff for LPF



500x500 image



Fourier spectrum
Circle radii: 5, 15, 30, 80, and 230
Power enclosed: 92, 94.6, 95.4, 98, and 99.5

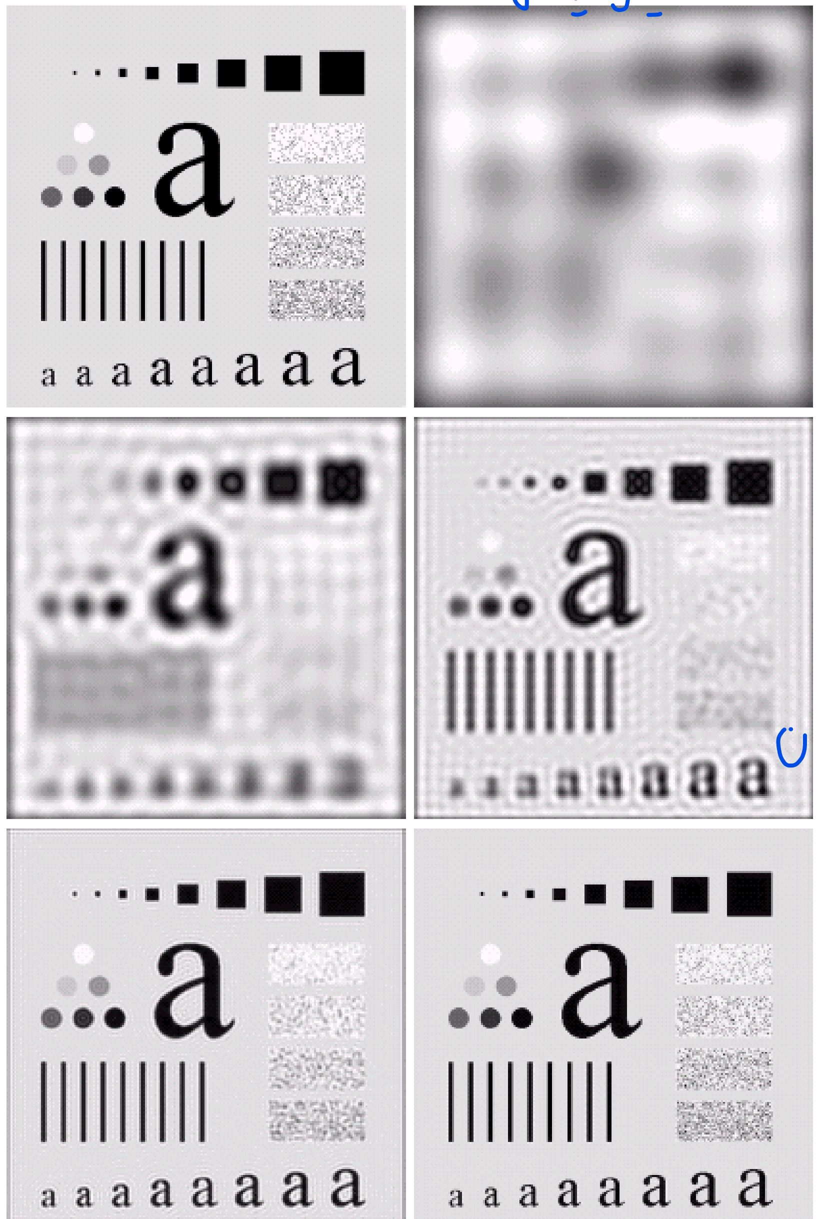
Choosing cutoff for ILPF

فرکانس‌های بالا حذف شوند

فیلتر ایده‌آل

Original image

cutoff=5



cutoff=15

cutoff=30

معایب ← ایجاد شدن موج

یک sinc با تصویر کانوالو شده در حوزه مکان

و تبدیل یک ضربه ای

cutoff=80

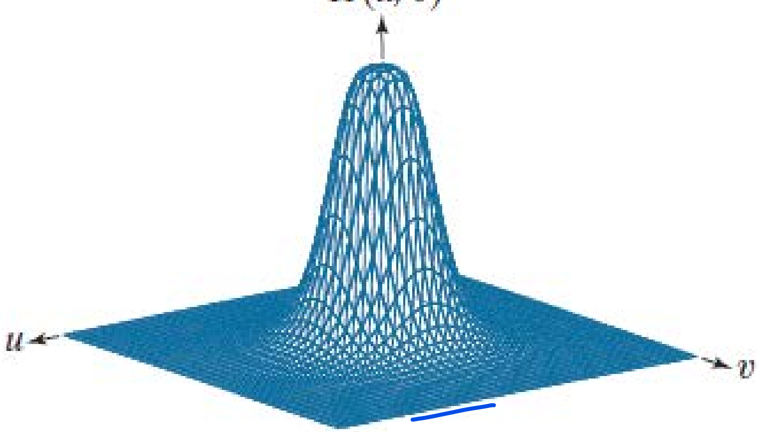
cutoff=230

ILPF عیب خلش

Butterworth LPF

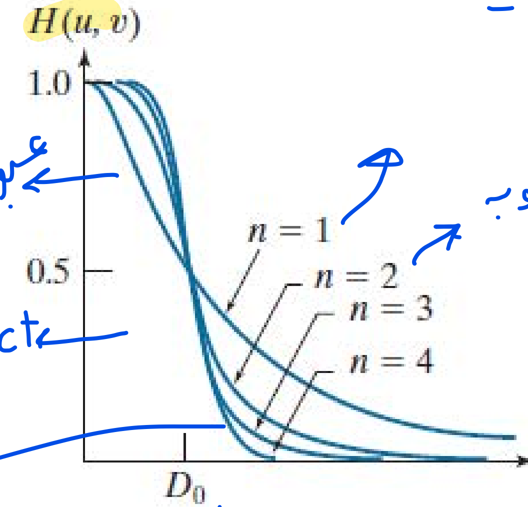
در حوزه فرکانس

همیشه + ← عبور-reject



خوب نیست

تمایز عبور و reject



عبور کامل

reject

خوب

باند عبور

دوباره مثل

(امواجی) دارم ringing

$$H(u,v) = \frac{1}{1 + \left[D(u,v) / D_0 \right]^{2n}}$$

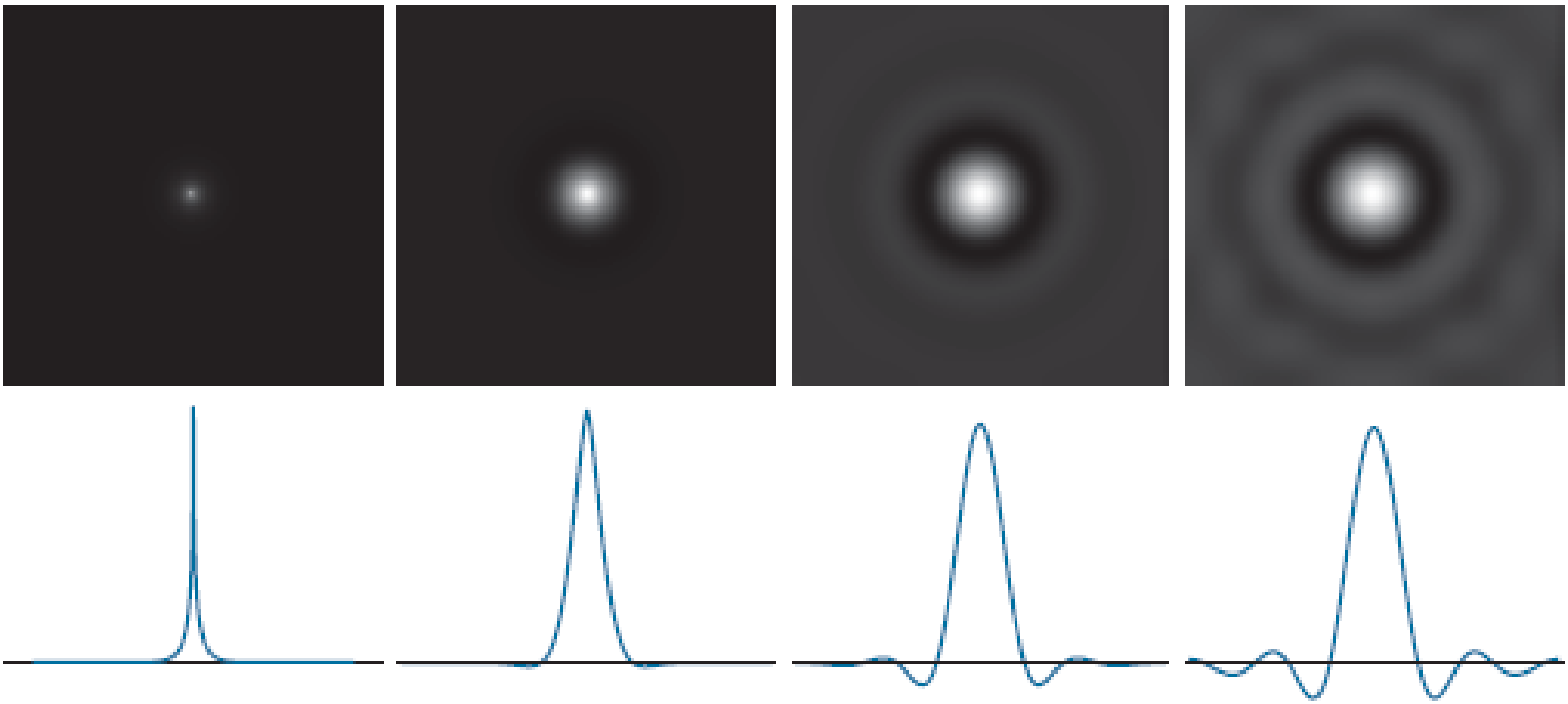
هرچه n بیشتر باشه

شبیه فیلتر ILPF

میشه

Spatial representation of BLPF

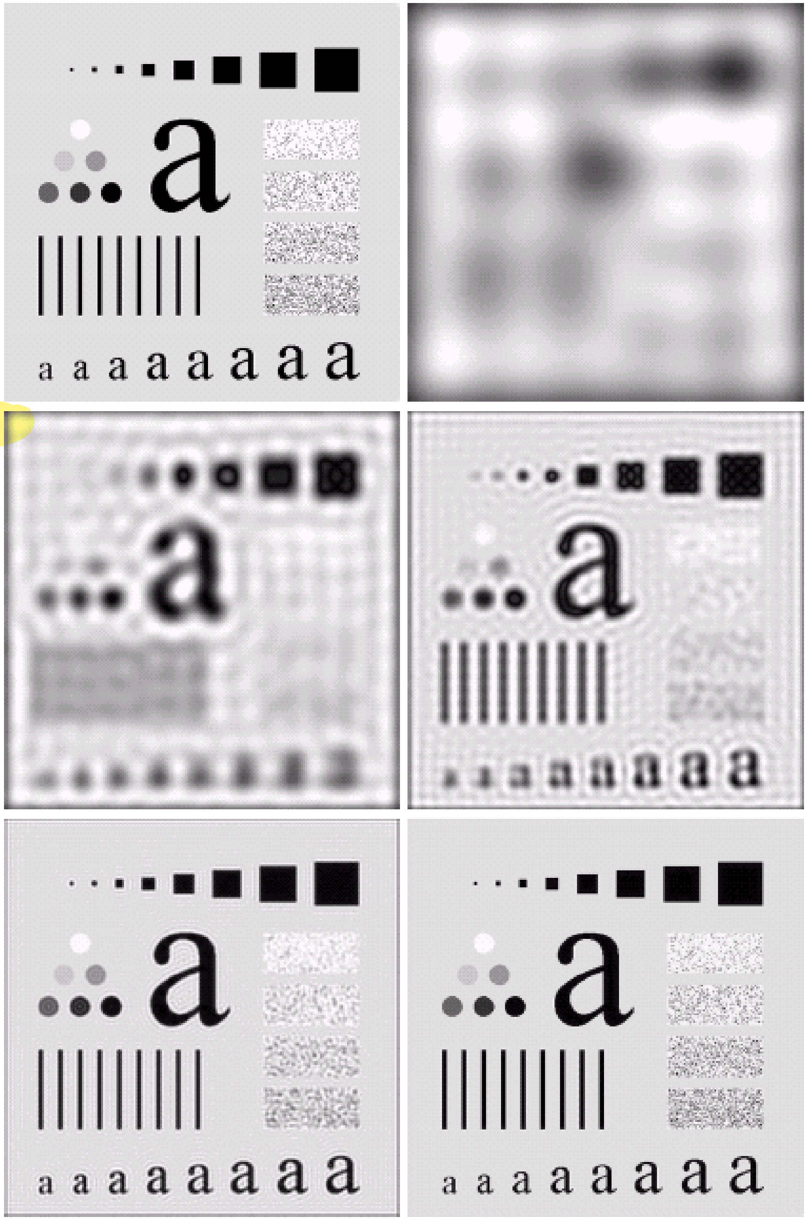
درحوزه مکان



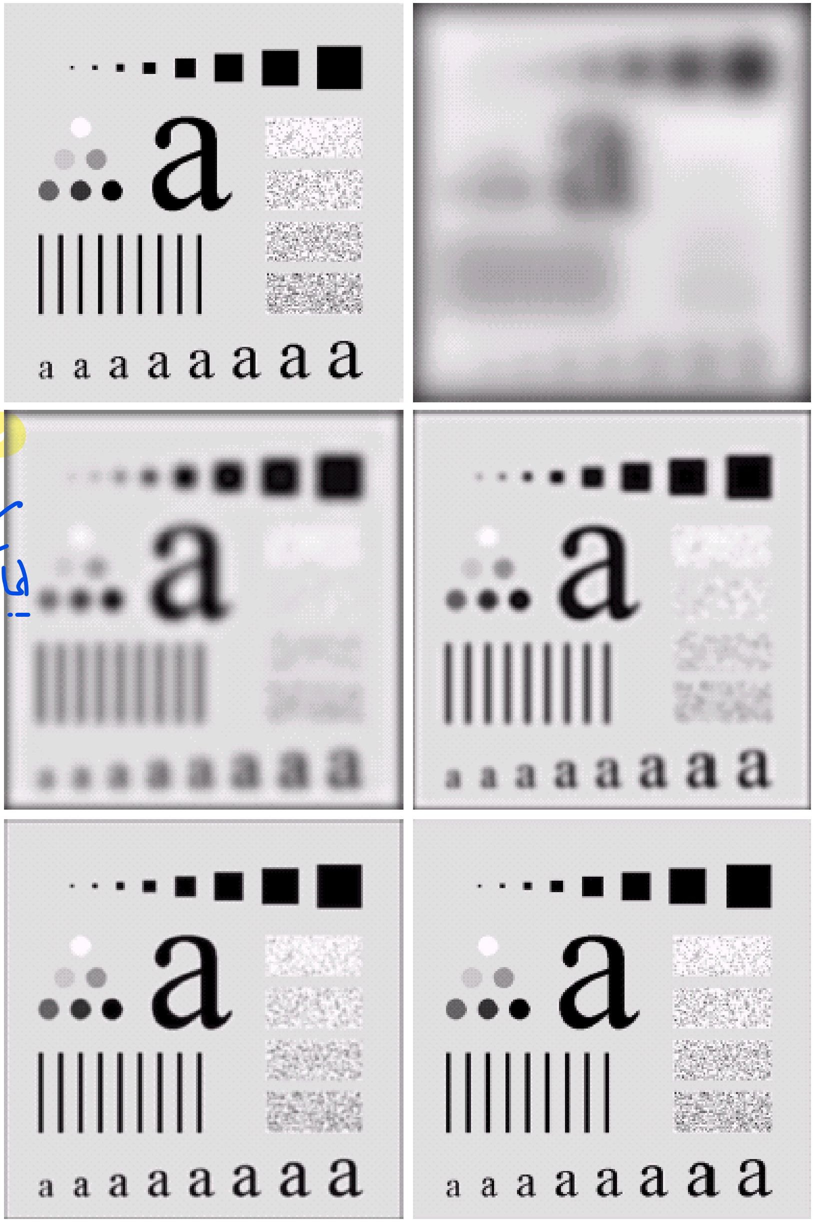
BLPF order 1 | BLPF order 2 | BLPF order 5 | BLPF order 20

شبیه ILPF | شبیه sinc داره است)

BLPF vs. ILPF



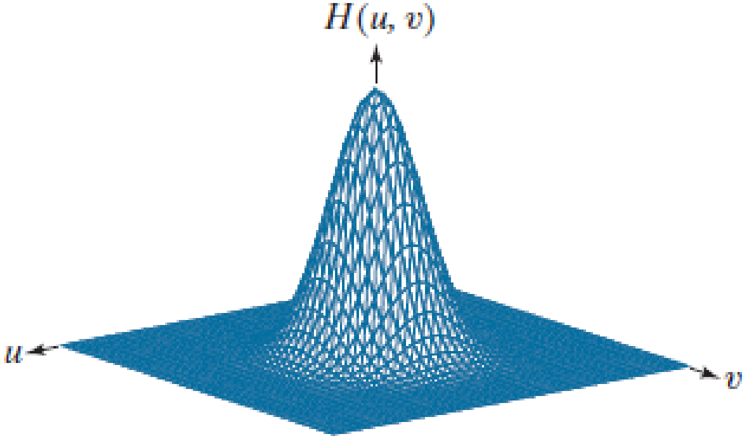
رینگ
ایجاد نکرده

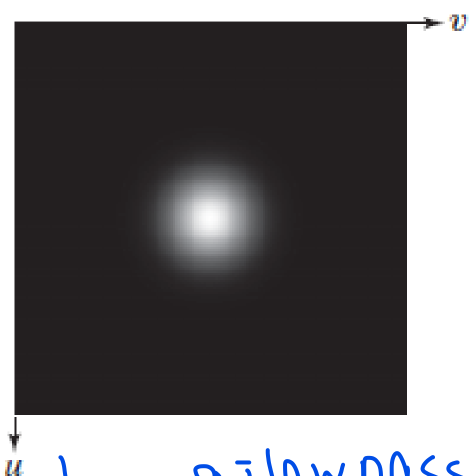


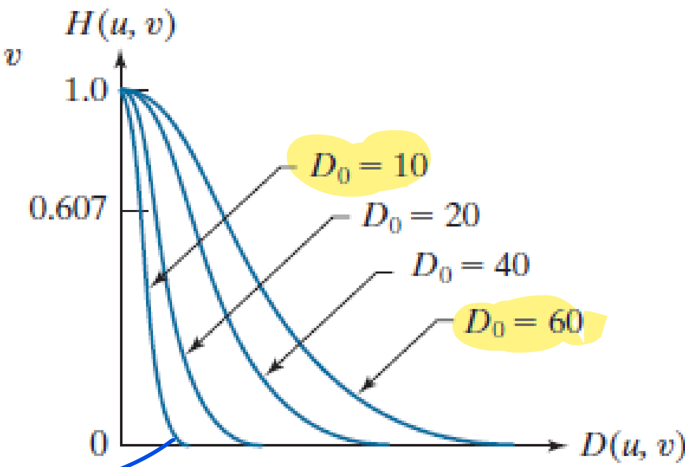
ILPF

BLPF order 2

Gaussian LPF





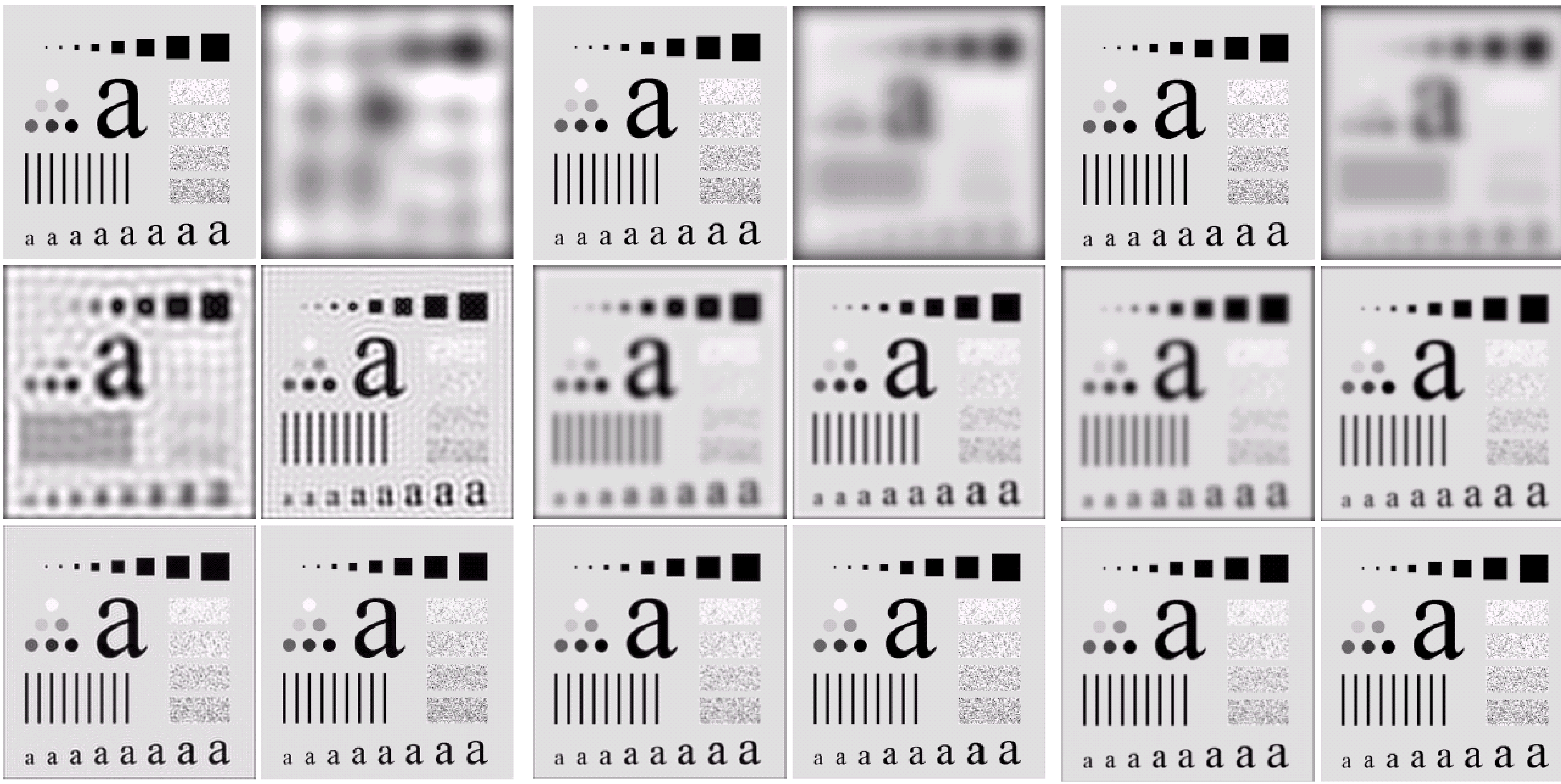


lowpass تر هست | با این ترها رو عبور می‌دیم تارتر می‌شه

$$H(u,v) = e^{-D^2(u,v)/2D_0^2}$$

cutoff

GLPF vs BLPF vs ILPF



ILPF

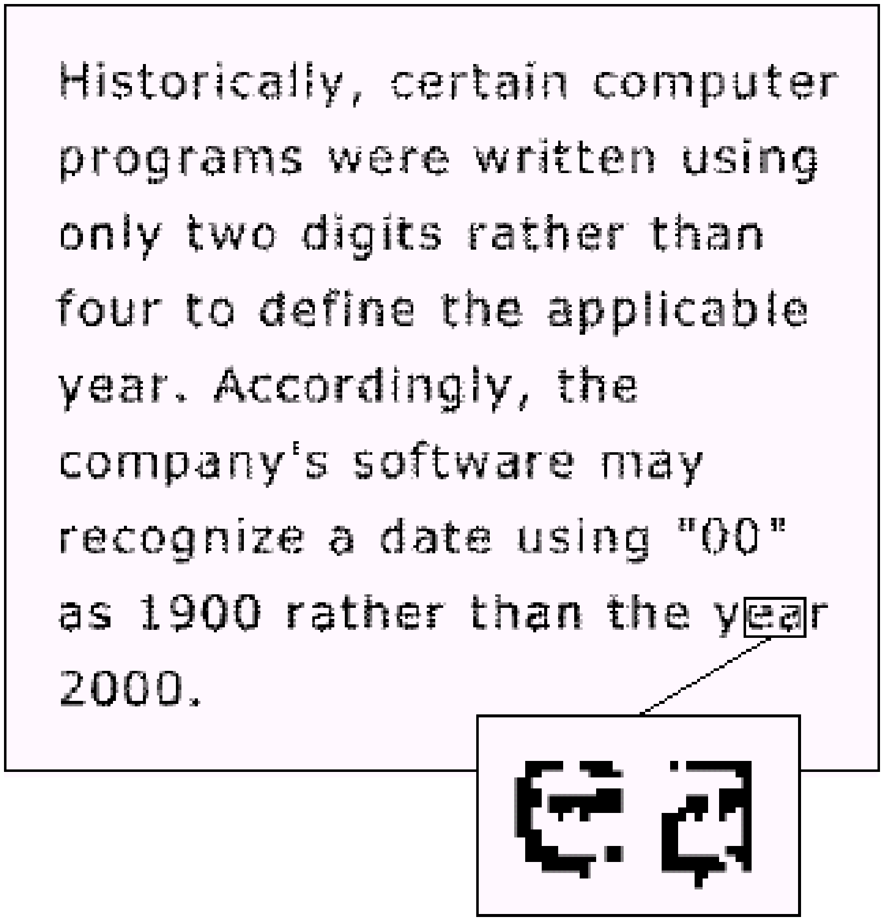
BLPF order 2

GLPF

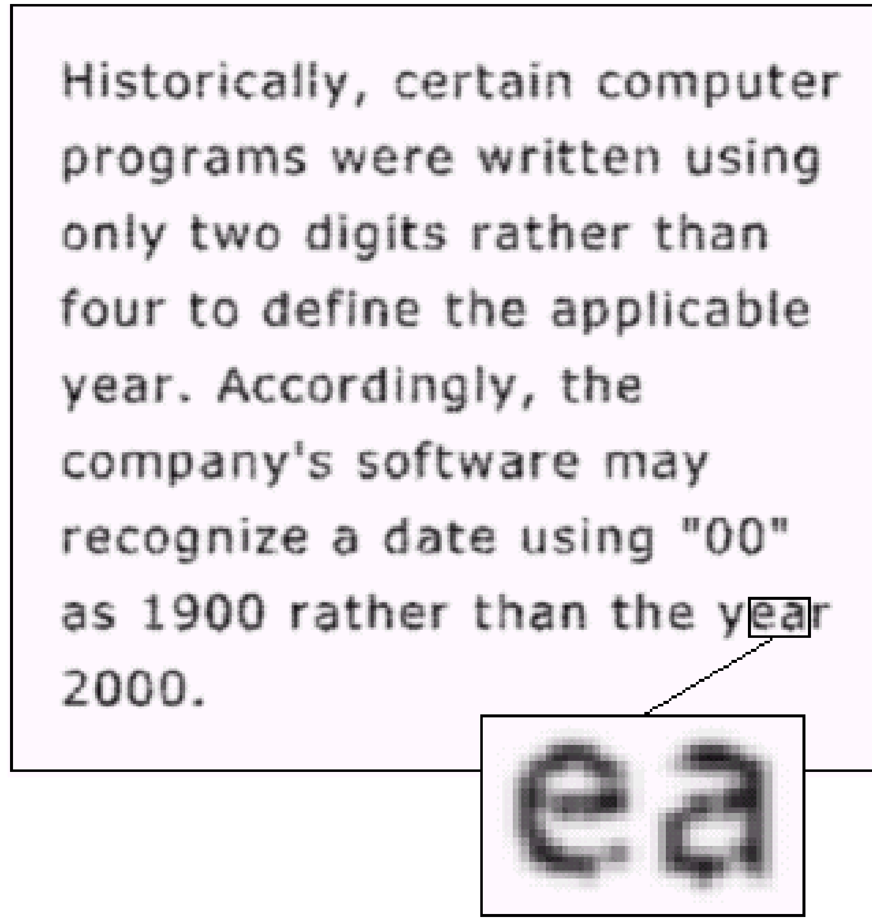
بدون ringing است

GLPF application

معادل سازی در حوزه مکان


Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.

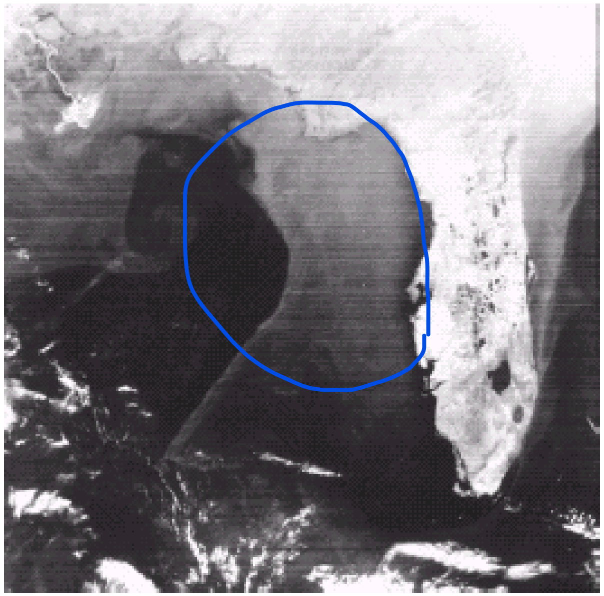
Broken letters


Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.

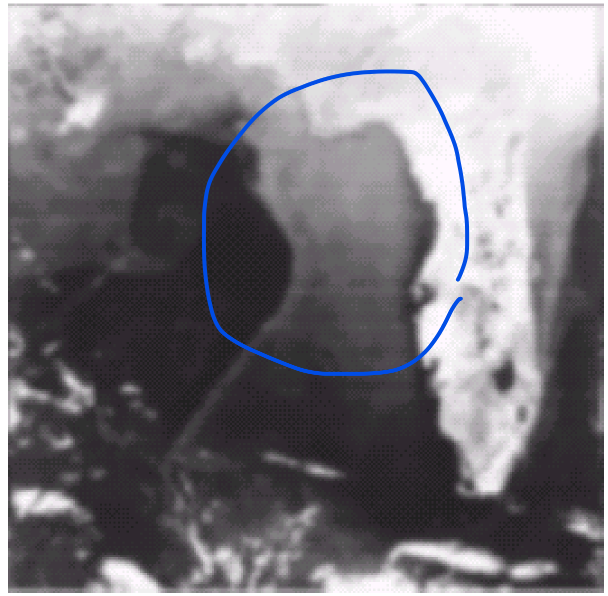
Result of using GLPF

GLPF application

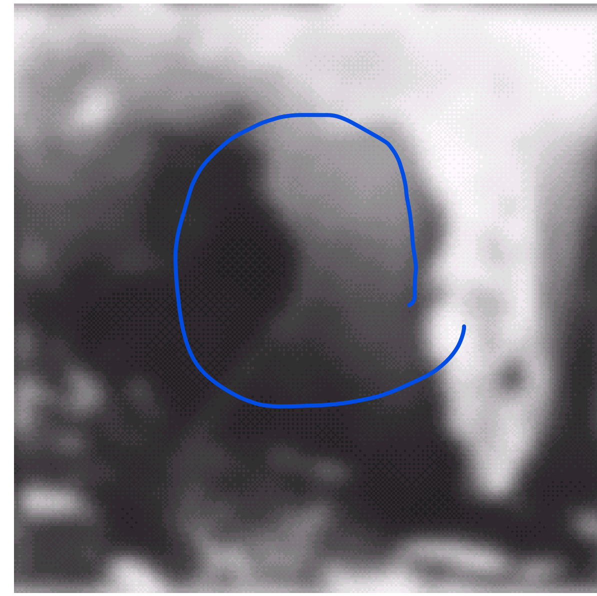
خط‌های افقی حذف شده است



Original



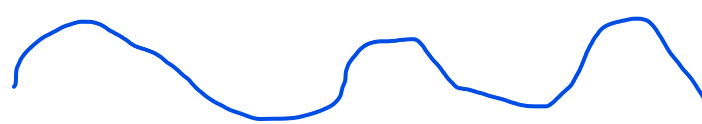
Result of using GLPF (D_0 =30)



Result of using GLPF (D_0 =10)

تارتر شده

نگه داشت ← (فرکانس های پایین ر)



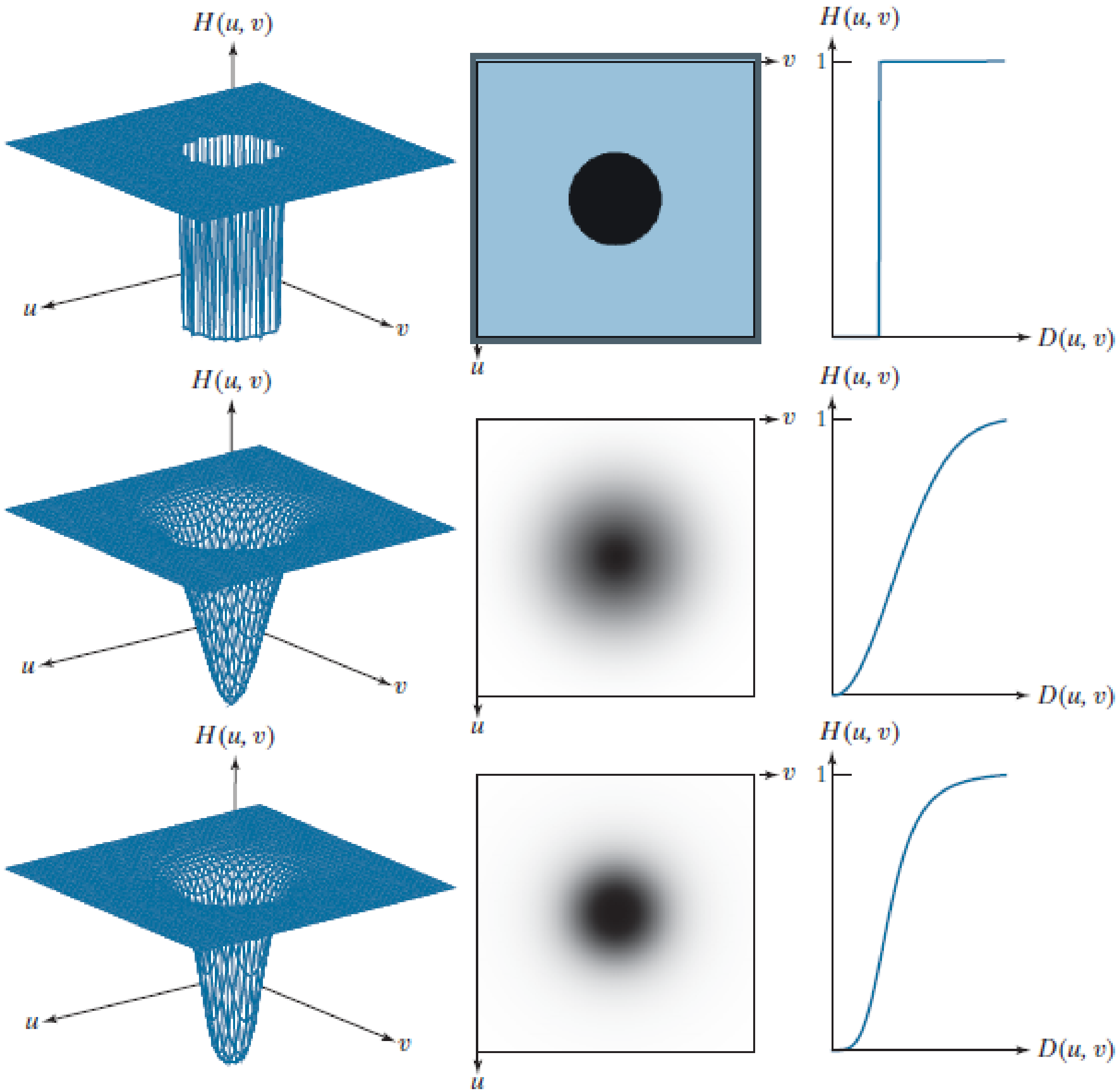
High-pass filters

$$H_{HP}(u,v) = 1 - H_{LP}(u,v)$$

Ideal

Butterworth

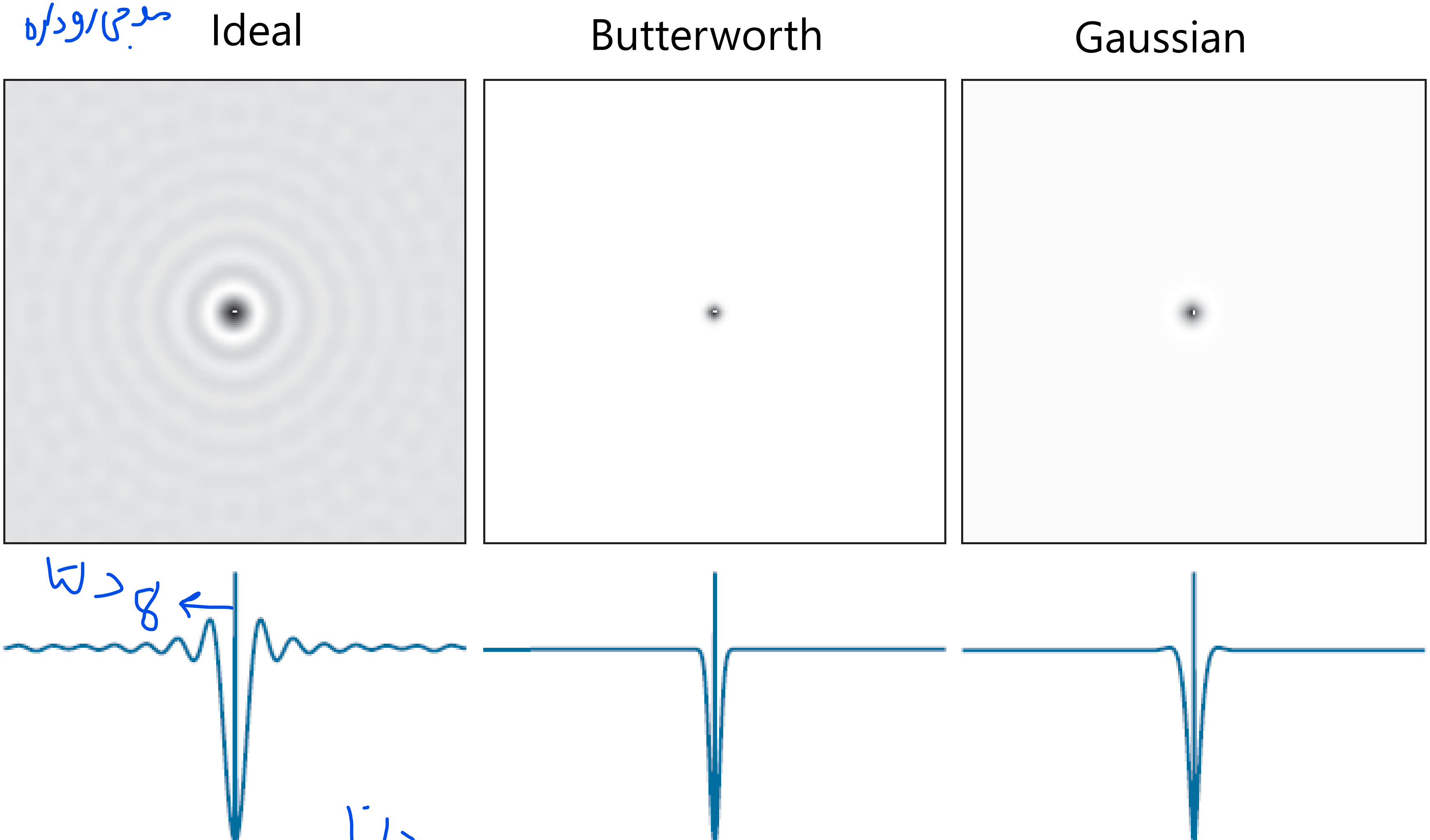
Gaussian



تیزسازی

Spatial representation of high-pass filters

موج داره



$\omega \searrow \ \delta$

$\omega \searrow$

$1 - H_{LP} \longleftrightarrow \delta - \text{sinc}$

Application of ideal high-pass filters

DC حذف ها

$D_0 = 15$ $D_0 = 30$ $D_0 = 80$

برای لبه خوبه

از 80 به بالا عبوری‌اند

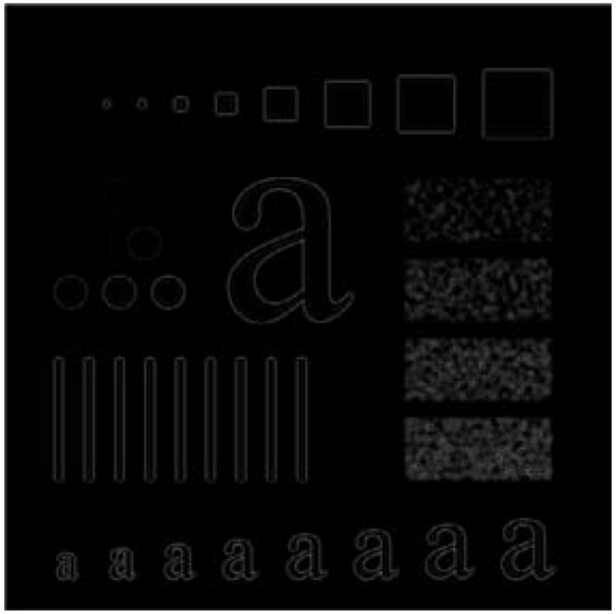
بیش‌ترین حذف فرکانس های پایین

ringing داره

Application of Butterworth high-pass filters

$D_0 = 15$

$D_0 = 30$



$D_0 = 80$

ringing نداریم

BHPF of order 2
Smoother results in
comparison with IHPF

Application of Gaussian high-pass filters

$D_0 = 15$

$D_0 = 30$

$D_0 = 80$

Laplacian in frequency domain

Laplacian in frequency domain

ضرب می‌کنیم در F

$$H(u,v) = -4\pi^2(u^2 + v^2)$$

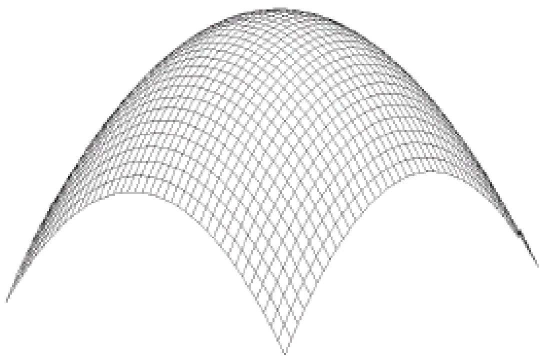
$$H(u,v) = -4\pi^2\left[(u - {}^{P}\!/_{2})^2 + (v - {}^{Q}\!/_{2})^2\right]$$
$$= -4\pi^2 D^2(u,v)$$

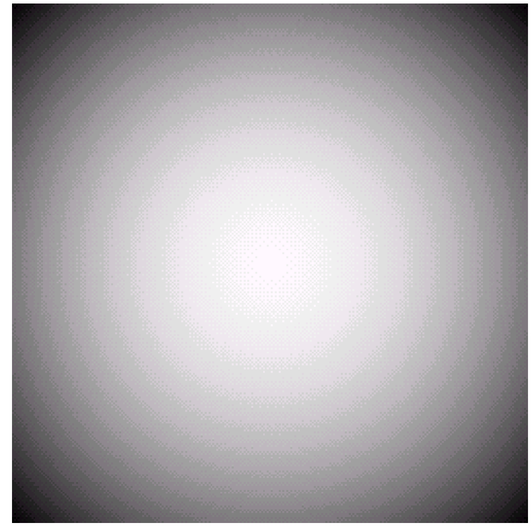
DC بی انته مرکز

Laplacian

حوزه فرکانس

Freq. domain

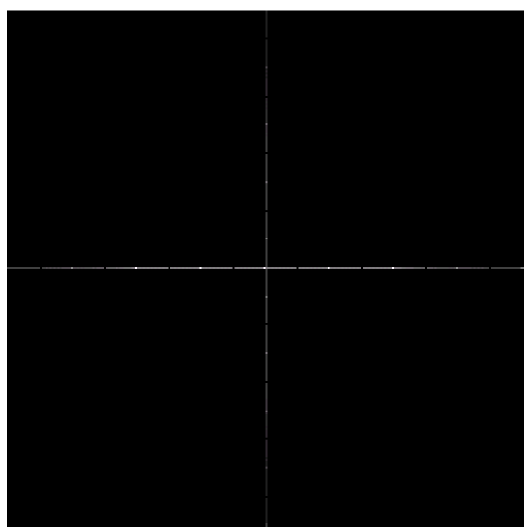


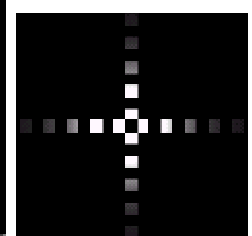


Image

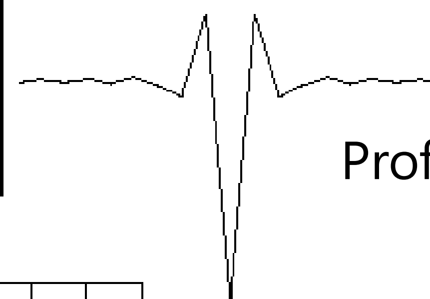
Spat. domain

حوزه مکان





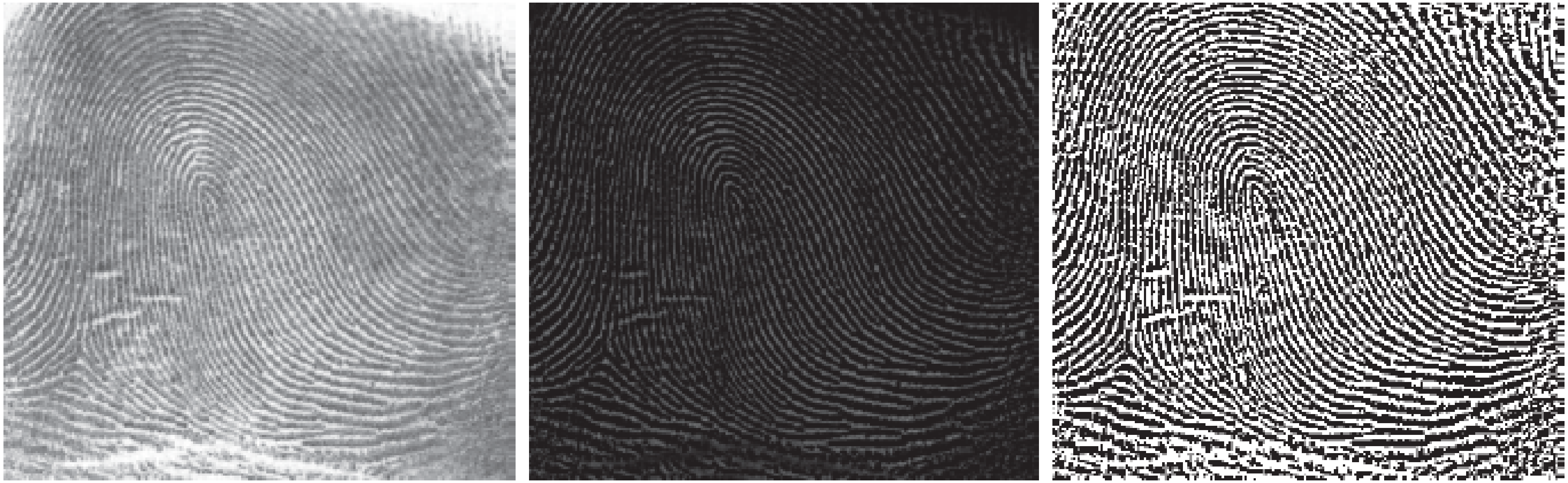
Zoom



Profile

0	1	0
1	−4	1
0	1	0

Mask



a b c

FIGURE 4.55 (a) Smudged thumbprint. (b) Result of highpass filtering (a). (c) Result of thresholding (b). (Original image courtesy of the U.S. National Institute of Standards and Technology.)

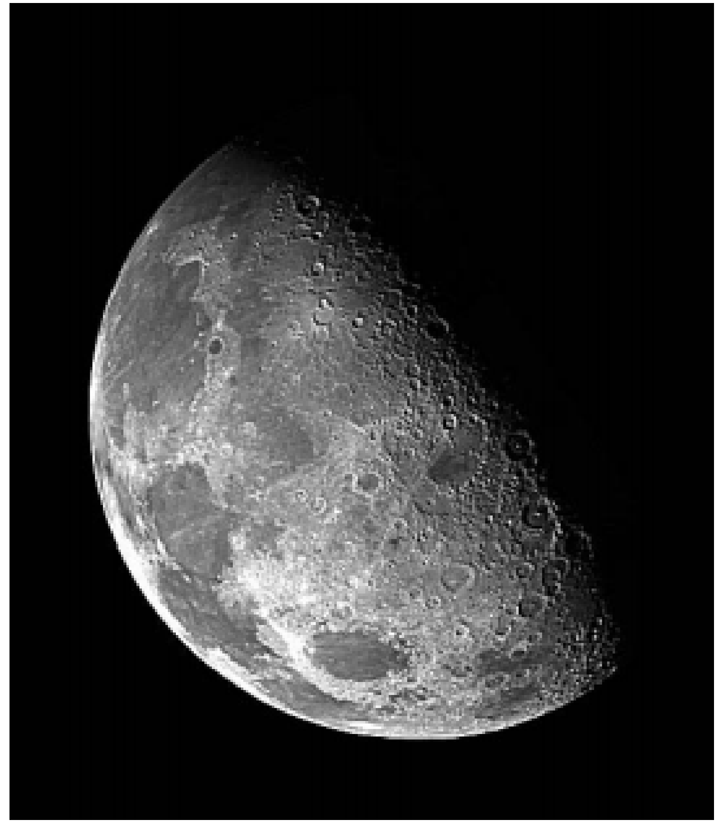
Laplacian for sharpening

$$\nabla^2 f(x,y) = \Im^{-1}\{H(u,v)F(u,v)\}$$

$$g(x,y) = f(x,y) + c\nabla^2 f(x,y)$$

$$\begin{aligned} g(x,y) &= \Im^{-1}\{F(u,v) - H(u,v)F(u,v)\} \\ &= \Im^{-1}\{[1 - H(u,v)]F(u,v)\} \\ &= \Im^{-1}\{[1 + 4\pi^2 D^2(u,v)]F(u,v)\} \end{aligned}$$

Laplacian for sharpening



Unsharp masking, high boost filtering, and high-frequency emphasis filtering

$$g_{mask}(x,y) = f(x,y) - f_{LP}(x,y)$$

→ blurr

$$f_{LP}(x,y) = \mathfrak{I}^{-1}[H_{LP}(u,v)F(u,v)]$$

$$g(x,y) = f(x,y) + k * g_{mask}(x,y)$$

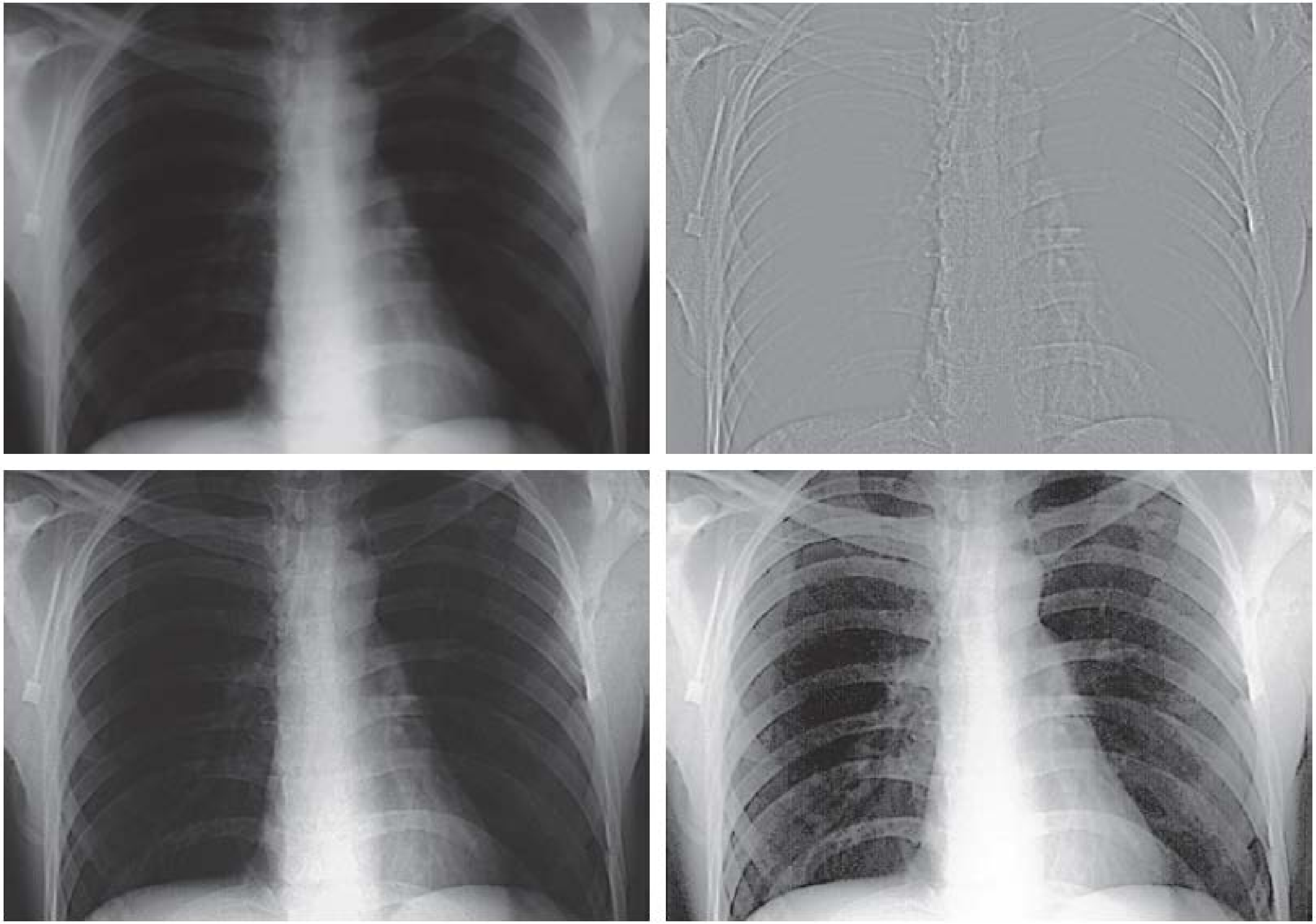
$$g(x,y) = \mathfrak{I}^{-1}\{\big[1 + k * [1 - H_{LP}(u,v)]\big]F(u,v)\}$$

$$g(x,y) = \mathfrak{I}^{-1}\{\underbrace{[1 + k * H_{HP}(u,v)]}_{\text{High-frequency emphasis filter}}F(u,v)\}$$

$$g(x,y) = \mathfrak{I}^{-1}\{[k_1 + k_2 * H_{HP}(u,v)]F(u,v)\}$$

Chest X-ray

Result of Butterworth high-frequency filtering



Result of Butterworth high-frequency emphasis filtering

Result of histogram equalization

Selective filtering

Bandreject filters:

Ideal (IBRF)	Gaussian (GBRF)	Butterworth (BBRF)
$H(u,v)=\begin{cases}0 & \text{if } C_0-\frac{W}{2}\le D(u,v)\le C_0+\frac{W}{2}\\ 1 & \text{otherwise}\end{cases}$	$H(u,v)=1-e^{-\left[\frac{D^2(u,v)-C_0^2}{D(u,v)W}\right]^2}$	$H(u,v)=\dfrac{1}{1+\left[\dfrac{D(u,v)W}{D^2(u,v)-C_0^2}\right]^{2n}}$

Bandpass filters:

$$H_{BP}(u,v)=1-H_{BR}(u,v)$$

میان گذر ـ میان نگذر

Selective filtering

Notchreject filters:

$$H_{NR}(u,v) = \prod_{k=1}^{Q} H_k(u,v) H_{-k}(u,v)$$

$$D_k(u,v) = \left[(u - {}^{M}/_{2} - u_k)^2 + (v - {}^{N}/_{2} - v_k)^2 \right]^{1/2}$$

$$D_{-k}(u,v) = \left[(u - {}^{M}/_{2} + u_k)^2 + (v - {}^{N}/_{2} + v_k)^2 \right]^{1/2}$$

Example: Butterworth notchreject filters:

$$H_{NR}(u,v) = \prod_{k=1}^{3} \left[\frac{1}{1 + \left[{}^{D_{0k}}/_{D_k(u,v)} \right]^{2n}} \right] \left[\frac{1}{1 + \left[{}^{D_{0k}}/_{D_{-k}(u,v)} \right]^{2n}} \right]$$

Notchpass filters:

$$H_{NP}(u,v) = 1 - H_{NR}(u,v)$$

Computation of 2D Fourier transform as series of 1D transforms

Separability of the 2D DFT:

$$F(u,v) = \sum_{x=0}^{M-1} e^{-j2\pi ux/M} \sum_{y=0}^{N-1} f(x,y) e^{-j2\pi vy/N}$$

هر سطر

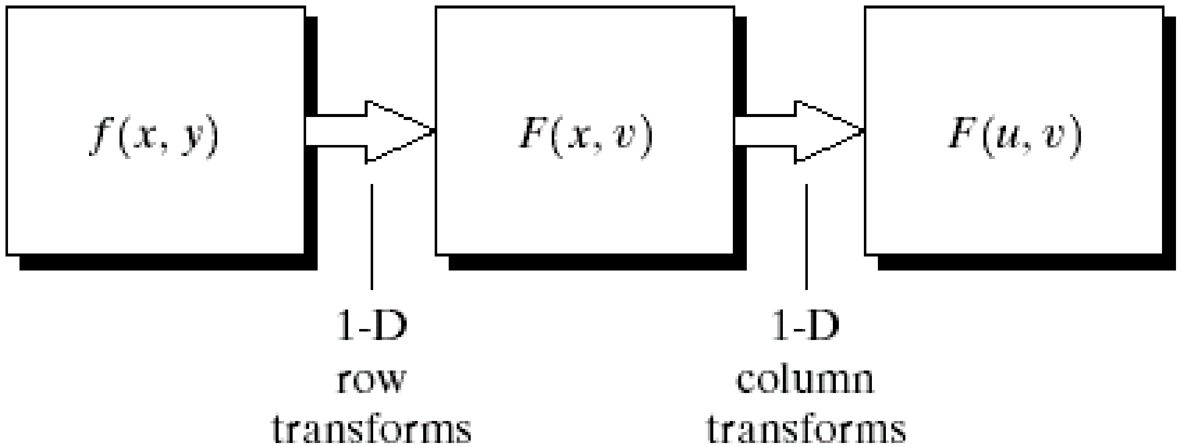
صریم بر اساس

ستون

$$= \sum_{x=0}^{M-1} F(x,v) e^{-j2\pi ux/M}$$

$$F(x,v) = \sum_{y=0}^{N-1} f(x,y) e^{-j2\pi vy/N}$$

Computation of 2D Fourier transform as series of 1D transforms



1D FFT

1D FFT

دو سری از تبدیل فوریه یک بعدی استفاده کنیم

IDFT from DFT

$$MNf^*(x,y) = \sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F^*(u,v)e^{-j2\pi\left({}^{ux}/_{M}+{}^{vy}/_{N}\right)}$$