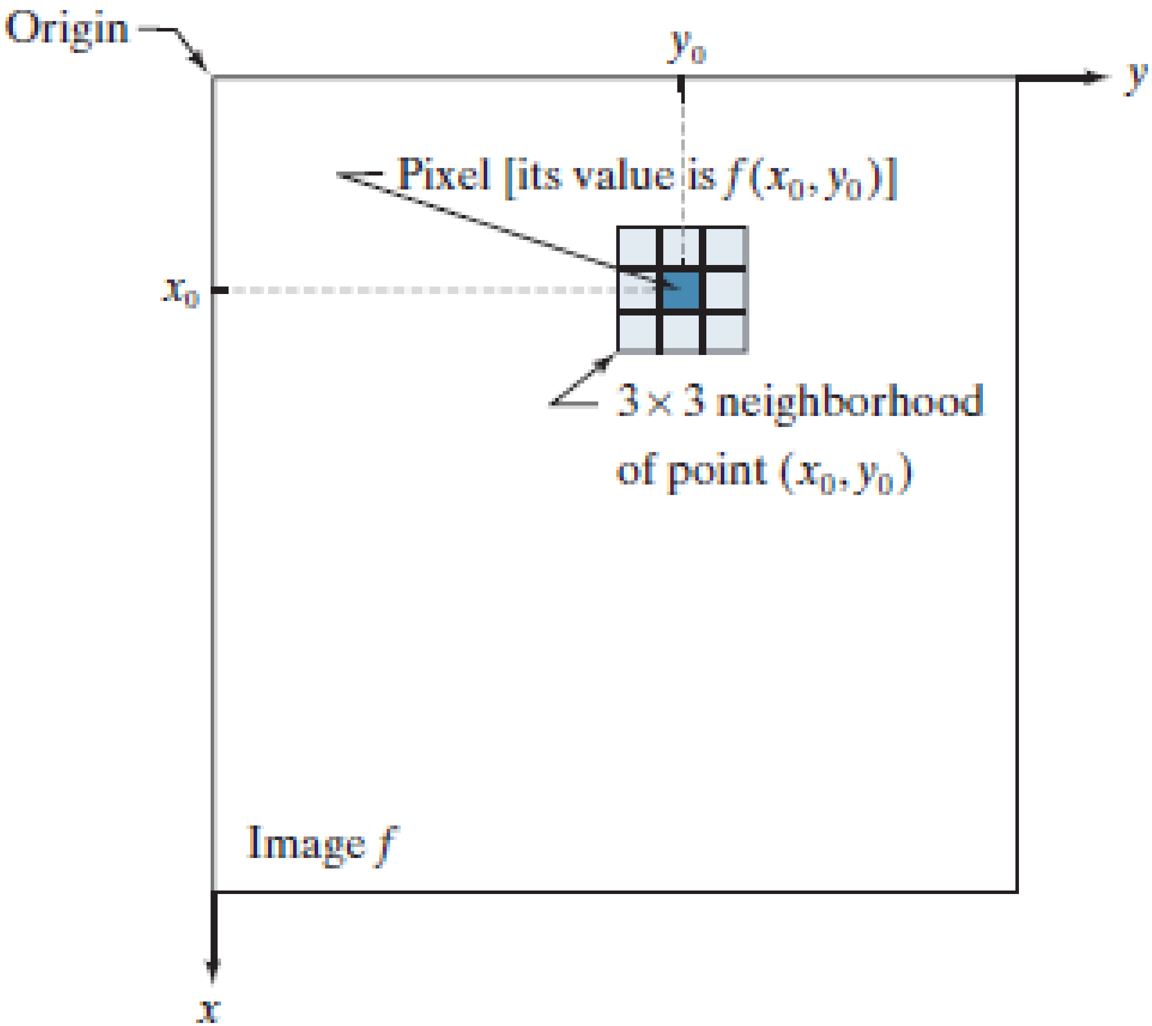
مکانی

Spatial operations

x و y ها مهم هستند.

Spatial operations

$$g(x, y) = T(f(x, y))$$



Fundamentals of spatial filtering

Filtering:

1) Pixel neighborhood, e.g. a small rectangle
2) Operation, e.g., linear or nonlinear

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$

Spatial filters, masks, kernels, templates, windows

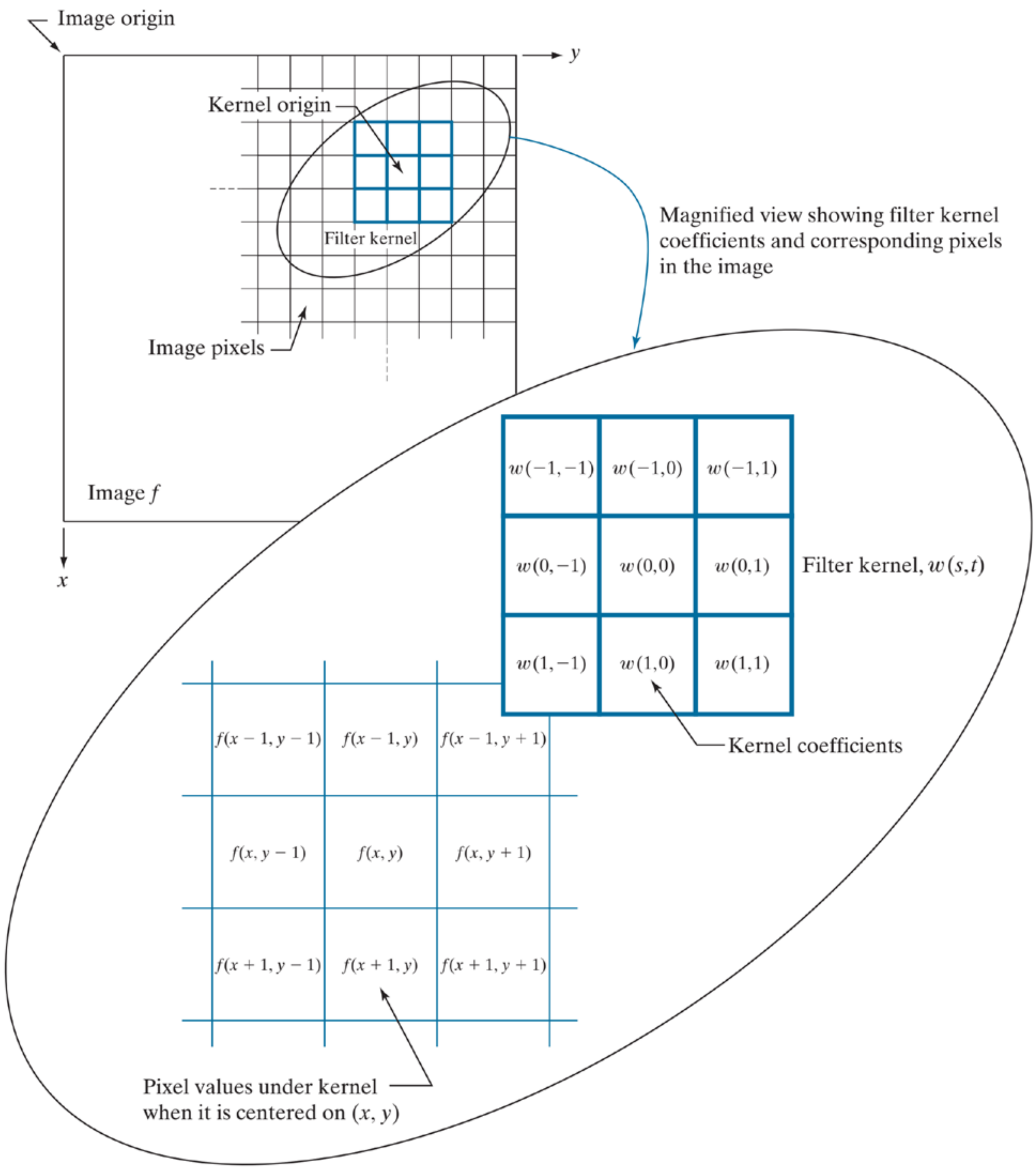


Image origin
y
Kernel origin
Filter kernel
Image pixels
Image f
x
Magnified view showing filter kernel coefficients and corresponding pixels in the image
w(−1,−1)
w(−1,0)
w(−1,1)
w(0,−1)
w(0,0)
w(0,1)
w(1,−1)
w(1,0)
w(1,1)
Filter kernel, w(s,t)
Kernel coefficients
f(x − 1, y − 1)
f(x − 1, y)
f(x − 1, y + 1)
f(x, y − 1)
f(x, y)
f(x, y + 1)
f(x + 1, y − 1)
f(x + 1, y)
f(x + 1, y + 1)
Pixel values under kernel when it is centered on (x, y)

Averaging

$$
\begin{array}{|c|c|c|c|c|}
\hline
120 & 190 & 140 & 150 & 200 \\ \hline
17 & 21 & 30 & 8 & 27 \\ \hline
89 & 123 & 150 & 73 & 56 \\ \hline
10 & 178 & 140 & 150 & 18 \\ \hline
190 & 14 & 76 & 69 & 87 \\ \hline
\end{array}
\quad \times \quad
\begin{array}{|c|c|c|}
\hline
1/9 & 1/9 & 1/9 \\ \hline
1/9 & 1/9 & 1/9 \\ \hline
1/9 & 1/9 & 1/9 \\ \hline
\end{array}
\quad = \quad
\begin{array}{|c|c|c|c|c|}
\hline
 & & & & \\ \hline
 & 98 & & & \\ \hline
 & & & & \\ \hline
 & & & & \\ \hline
 & & & & \\ \hline
\end{array}
$$

Correlation with impulse

Correlation

Origin f w

(a) 0 0 0 **1** 0 0 0 0 **1 2 4 2 8**

(b) 0 0 0 **1** 0 0 0 0
1 2 4 2 8
Starting position alignment

Zero padding

(c) 0 0 0 0 0 **1** 0 0 0 0 0 0
1 2 4 2 8
Starting position

(d) 0 0 0 0 0 **1** 0 0 0 0 0 0
1 2 4 2 8
Position after 1 shift

(e) 0 0 0 0 0 **1** 0 0 0 0 0 0
1 2 4 2 8
Position after 3 shifts

(f) 0 0 0 0 0 **1** 0 0 0 0 0 0
1 2 4 2 8
Final position

Correlation result

(g) 0 8 2 4 2 1 0 0

Extended (full) correlation result

(h) 0 0 0 8 2 4 2 1 0 0 0 0

Convolution with impulse

Convolution

Origin f w rotated 180°

0 0 0 **1** 0 0 0 0 **8 2 4 2 1** (i)

0 0 0 **1** 0 0 0 0 (j)
8 2 4 2 1
Starting position alignment

Zero padding

0 0 0 0 0 **1** 0 0 0 0 0 0 (k)
8 2 4 2 1
Starting position

0 0 0 0 0 **1** 0 0 0 0 0 0 (l)
8 2 4 2 1
Position after 1 shift

0 0 0 0 0 **1** 0 0 0 0 0 0 (m)
8 2 4 2 1
Position after 3 shifts

0 0 0 0 0 **1** 0 0 0 0 0 0 (n)
8 2 4 2 1
Final position

Convolution result

0 1 2 4 2 8 0 0 (o)

Extended (full) convolution result

0 0 0 1 2 4 2 8 0 0 0 0 (p)

Origin f

$$\begin{matrix}0&0&0&0&0\\0&0&0&0&0\\0&0&1&0&0\\0&0&0&0&0\\0&0&0&0&0\end{matrix}\qquad w\quad \begin{matrix}1&2&3\\4&5&6\\7&8&9\end{matrix}$$

(a)

Padded f

$$\begin{matrix}0&0&0&0&0&0&0\\0&0&0&0&0&0&0\\0&0&0&0&0&0&0\\0&0&0&1&0&0&0\\0&0&0&0&0&0&0\\0&0&0&0&0&0&0\\0&0&0&0&0&0&0\end{matrix}$$

(b)

Correlation with impulse

Initial position for w

$$\begin{matrix}1&2&3&0&0&0&0\\4&5&6&0&0&0&0\\7&8&9&0&0&0&0\\0&0&0&1&0&0&0\\0&0&0&0&0&0&0\\0&0&0&0&0&0&0\\0&0&0&0&0&0&0\end{matrix}$$

(c)

Correlation result

$$\begin{matrix}0&0&0&0&0\\0&9&8&7&0\\0&6&5&4&0\\0&3&2&1&0\\0&0&0&0&0\end{matrix}$$

(d)

Full correlation result

$$\begin{matrix}0&0&0&0&0&0&0\\0&0&0&0&0&0&0\\0&0&9&8&7&0&0\\0&0&6&5&4&0&0\\0&0&3&2&1&0&0\\0&0&0&0&0&0&0\\0&0&0&0&0&0&0\end{matrix}$$

(e)

Convolution with impulse

Rotated w

$$\begin{matrix}9&8&7&0&0&0&0\\6&5&4&0&0&0&0\\3&2&1&0&0&0&0\\0&0&0&1&0&0&0\\0&0&0&0&0&0&0\\0&0&0&0&0&0&0\\0&0&0&0&0&0&0\end{matrix}$$

(f)

Convolution result

$$\begin{matrix}0&0&0&0&0\\0&1&2&3&0\\0&4&5&6&0\\0&7&8&9&0\\0&0&0&0&0\end{matrix}$$

(g)

Full convolution result

$$\begin{matrix}0&0&0&0&0&0&0\\0&0&0&0&0&0&0\\0&0&1&2&3&0&0\\0&0&4&5&6&0&0\\0&0&7&8&9&0&0\\0&0&0&0&0&0&0\\0&0&0&0&0&0&0\end{matrix}$$

(h)

خطی

Spatial correlation and convolution

Correlation:

$$w(x,y) \star f(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$

Convolution:

$$w(x,y) \bigstar f(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x-s, y-t)$$

قرینه کردن

Vector representation of linear filtering

$$R = w_1 z_1 + w_2 z_2 + \ldots + w_{mn} z_{mn}$$

$$= \sum_{k=1}^{mn} w_k z_k$$

$$= w^T z$$

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

$$R = w_1 z_1 + w_2 z_2 + \ldots + w_9 z_9$$

$$= \sum_{k=1}^{9} w_k z_k$$

$$= w^T z$$

Generating spatial filter masks

Averaging:

$$R = \frac{1}{9}\sum_{i=1}^{9} z_i$$

2D Gaussian:

$$h(x,y) = e^{-\frac{x^2+y^2}{2\sigma^2}}$$

3×3 mask:

$$w_1 = h(-1,-1)$$
$$w_2 = h(-1,0)$$
$$\vdots$$
$$w_9 = h(1,1)$$

هرچی میانگین گیری بزرگتر باشه فراز و نشیب‌ها شدت‌ها گرفته می‌شه، تارتر می‌کنه

Smoothing filters: Averaging masks

مثل نمرات درس

$\frac{1}{9} \times$

1	1	1
1	1	1
1	1	1

Averaging
(box filter)

$\frac{1}{16} \times$

شدت‌ها
افزایش
نیابند

1	2	1
2	4	2
1	2	1

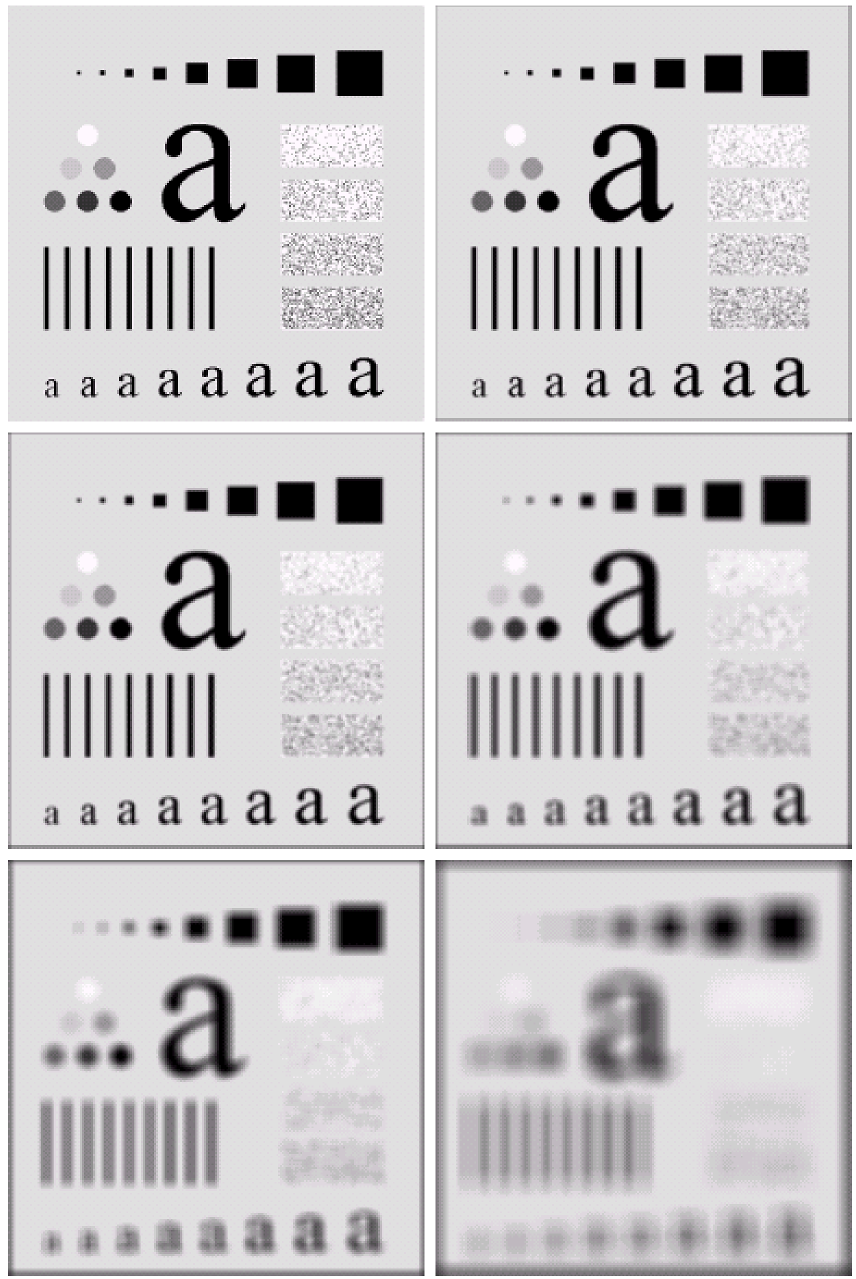
Weighted
averaging

$$g(x, y) = \frac{\sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)}{\sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t)}$$

$\leftarrow 9$ $\qquad$ $16 \rightarrow$

Choosing the size of the mask

Original



$n=3$

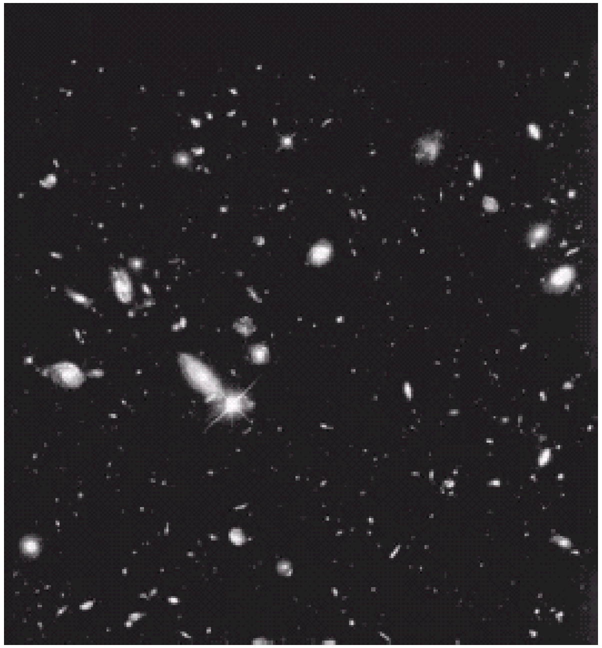
$n=5$

$n=9$

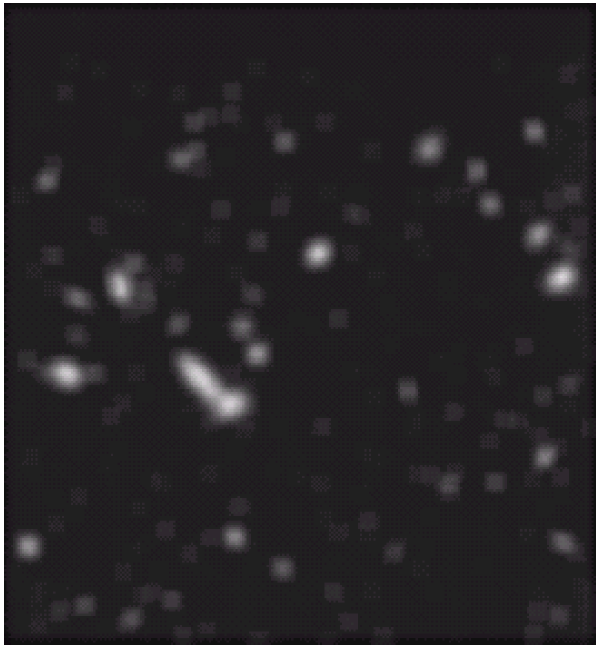
$n=15$

$n=35$

Choosing the size of the mask

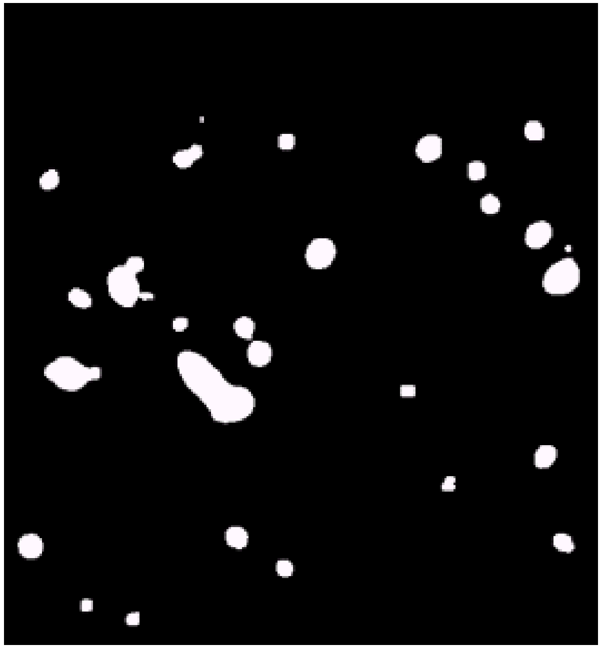


Original:
Hubble telescope image



Filtered by:
15x15 aver. mask

object های کوچک بخاطر اطرافشون
حذف میشوند



Thresholding

Nonlinear filters: Order statistics filters

0^{th} percentile: minimum

the darkest pixels

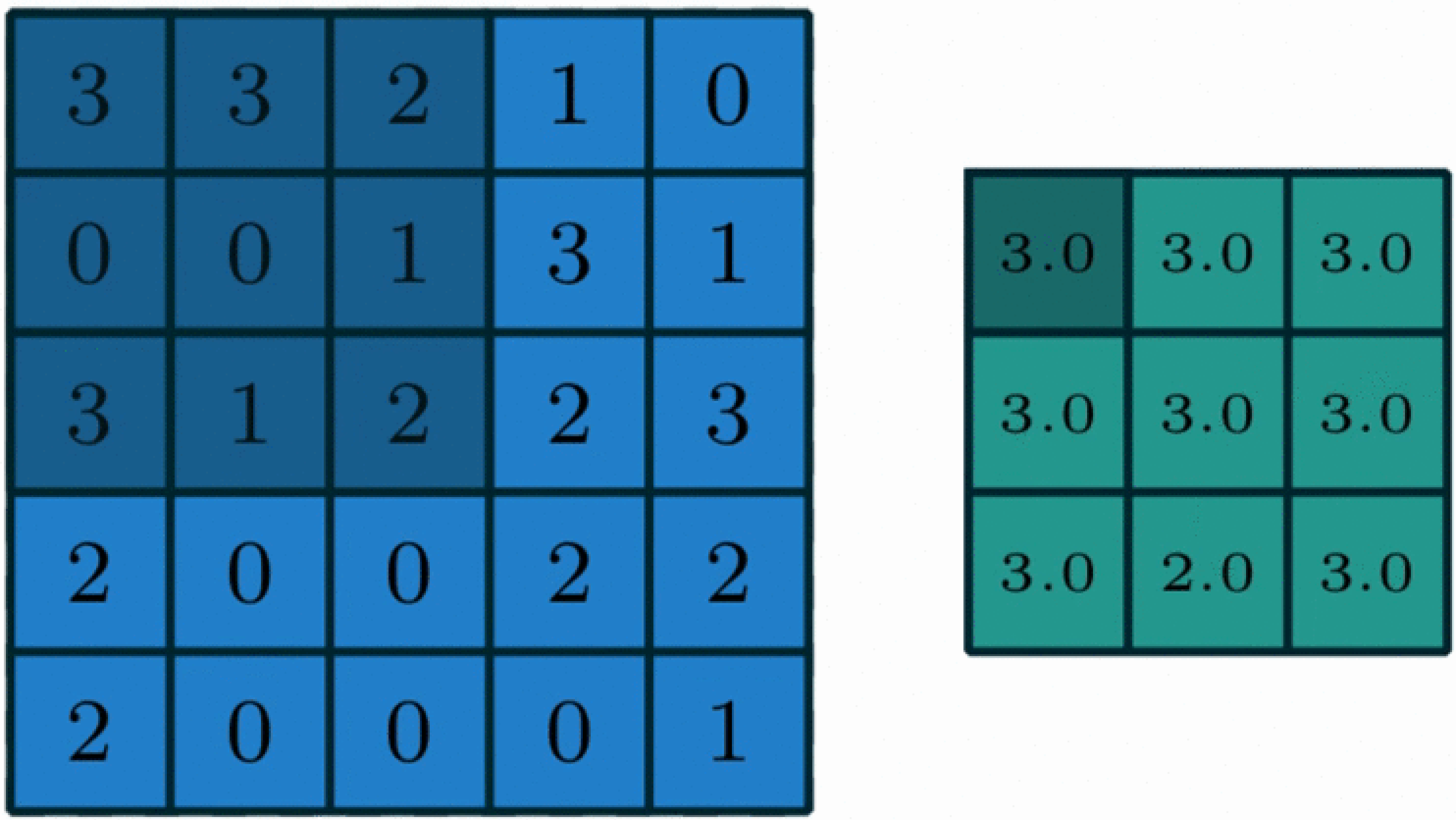
50^{th} percentile: median

eliminate impulse noise

100^{th} percentile: maximum

the brightest pixels

Nonlinear filters: Max filter



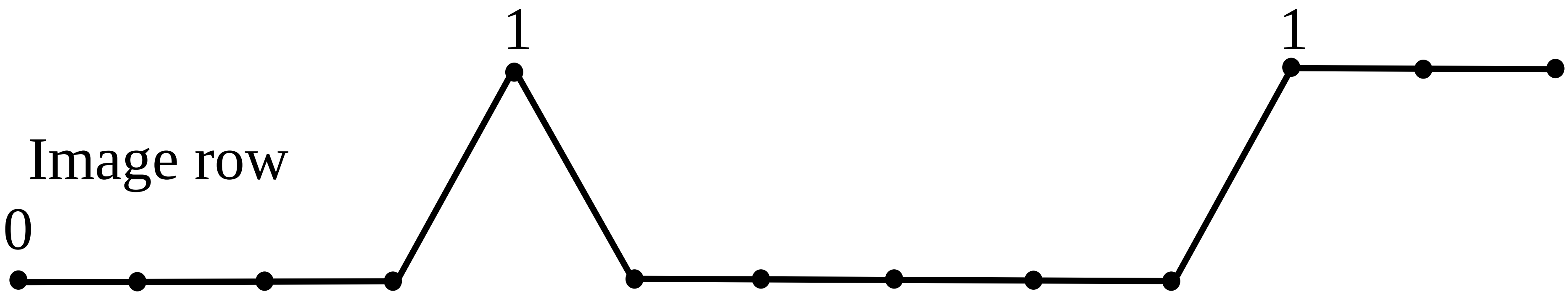
Nonlinear filters: Median filter

Input Image					median	Filtered Img
7	8	4	5	5	5 5 7 8 8 9 9 9 9	8
5	9	4	3	8		
5	2	7	2	2		
6	1	9	2	4		
3	2	6	9	4		

In this example:

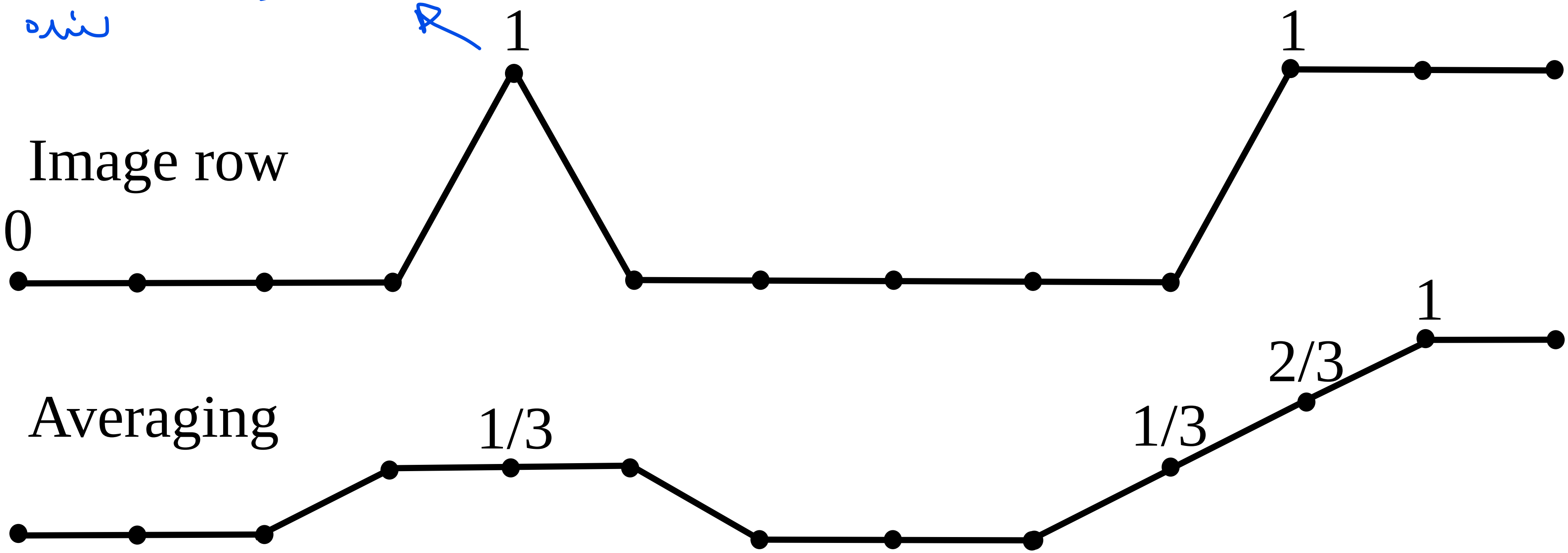
On the image borders, the median is found after mirroring the section under the window with respect to the central element

Comparison of averaging and median filters



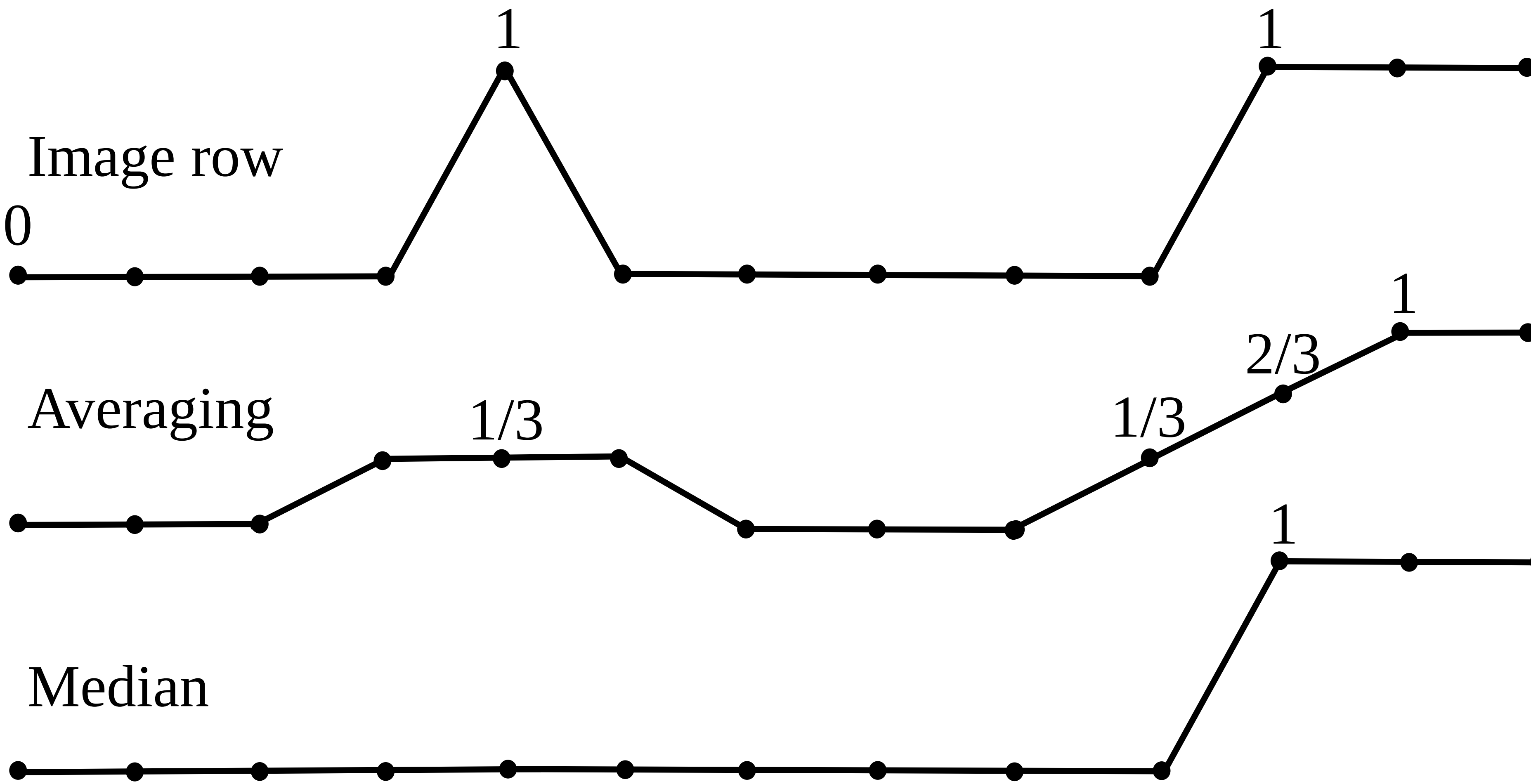
Comparison of averaging and median filters

اگه نویز باشه تضعیف شده

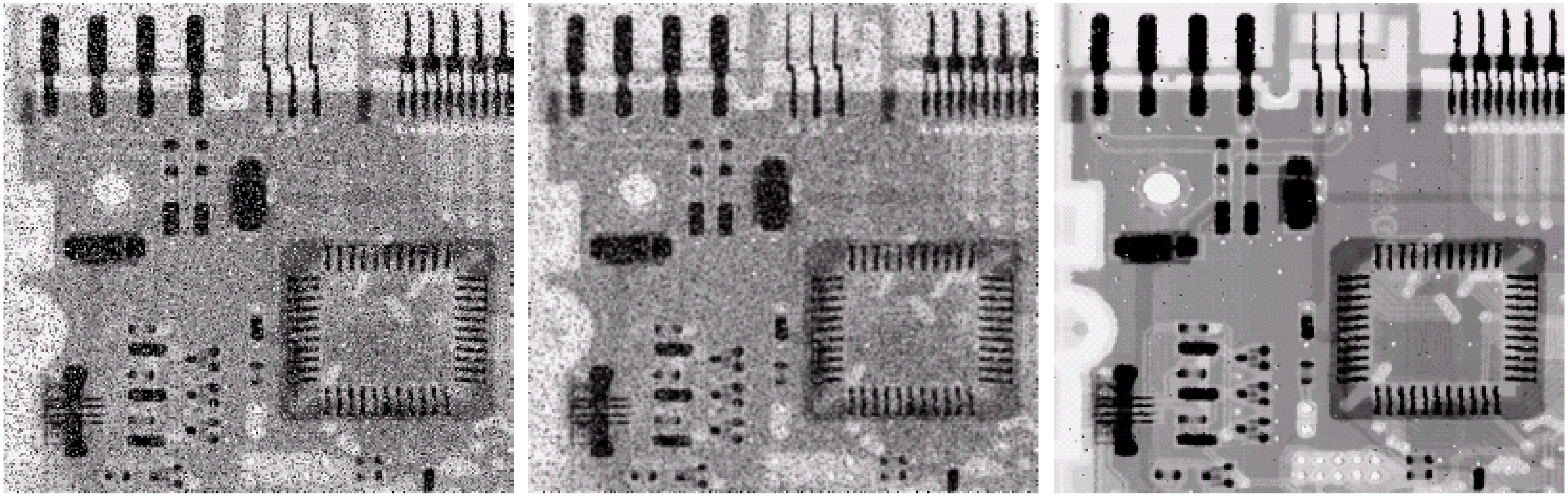


فیلترهای میانه ← نویزهای impulsi رو حذف می‌کنن

Comparison of averaging and median filters



Comparison of averaging and median filters



Original:
X-ray image
of a circuit board
+
Salt and pepper
noise

Filtered by:
3x3 aver. mask

Filtered by:
3x3 median filter

Sharpening filters

Smoothing: averaging → integration

Sharpening → differentiation

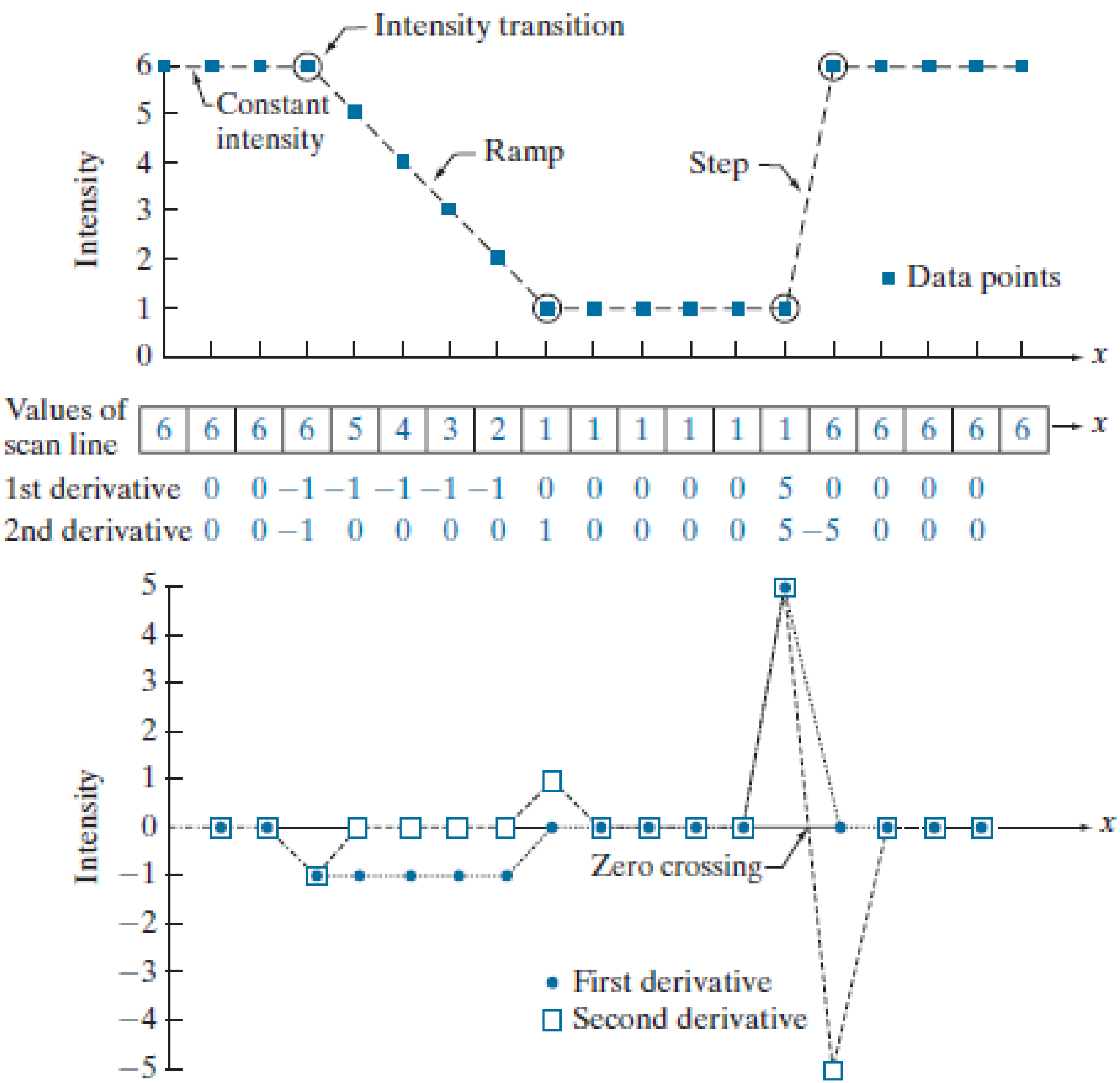
-1st derivative:

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

-2nd derivative:

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$

Sharpening filters



Implementation of second-order derivative

Laplacian: Simplest isotropic filter

مشتق دوم

$$\Delta f = \nabla^2 f = \nabla \cdot \nabla f = \sum_{i=1}^{n} \frac{\partial^2 f}{\partial x_i^2}$$

Implementation of second-order derivative

$$\frac{\partial^2 f}{\partial x^2} = f(x+1,y) + f(x-1,y) - 2f(x,y)$$

$$\frac{\partial^2 f}{\partial y^2} = f(x,y+1) + f(x,y-1) - 2f(x,y)$$

$$\nabla^2 f(x,y) = f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)$$

Laplacian masks

0	1	0
1	−4	1
0	1	0

1	1	1
1	−8	1
1	1	1

Negative difference equations used →

0	−1	0
−1	4	−1
0	−1	0

−1	−1	−1
−1	8	−1
−1	−1	−1

Isotropic for 90-degree increments

Isotropic for 45-degree increments

Sharpening: $g(x,y) = f(x,y) + c[\nabla^2 f(x,y)]$

$c < 0$

Image sharpening

Blurred →

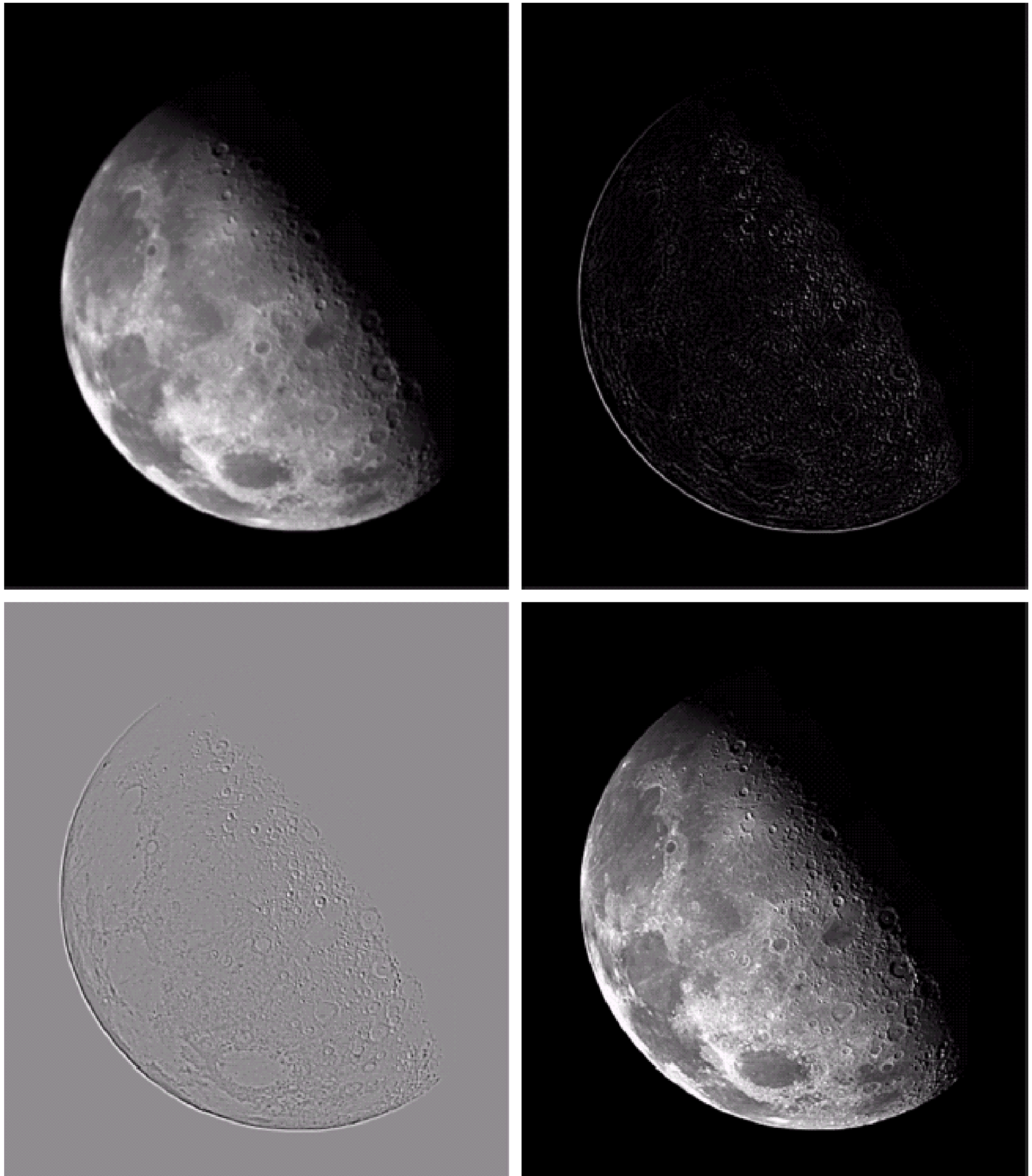
← Laplacian

فقط برای

این

عکس

Scaled Laplacian →



Sharpened Image ($c=-1$) ←

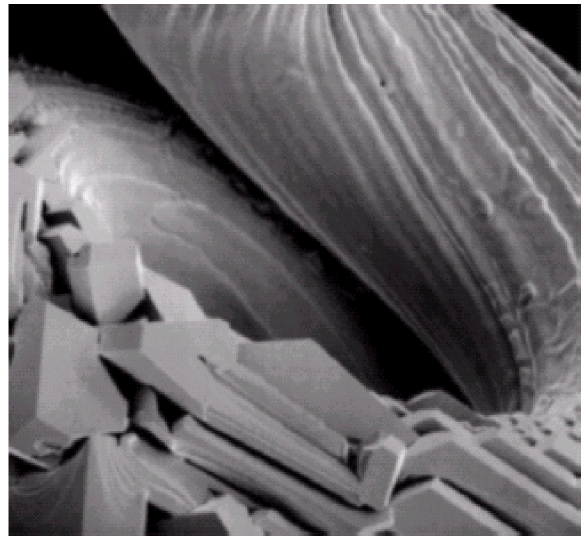
Simplification

$$g(x,y) = f(x,y) - \nabla^2 f(x,y)$$
$$= f(x,y) - [f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1)] + 4f(x,y)$$
$$= 5f(x,y) - [f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1)]$$

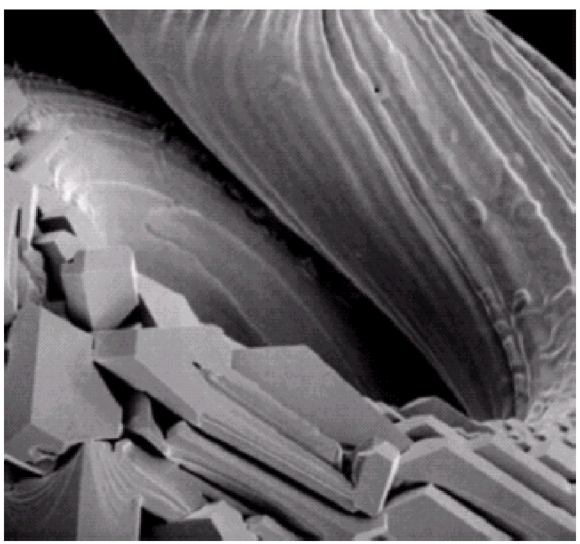
0	−1	0
−1	5	−1
0	−1	0

−1	−1	−1
−1	9	−1
−1	−1	−1

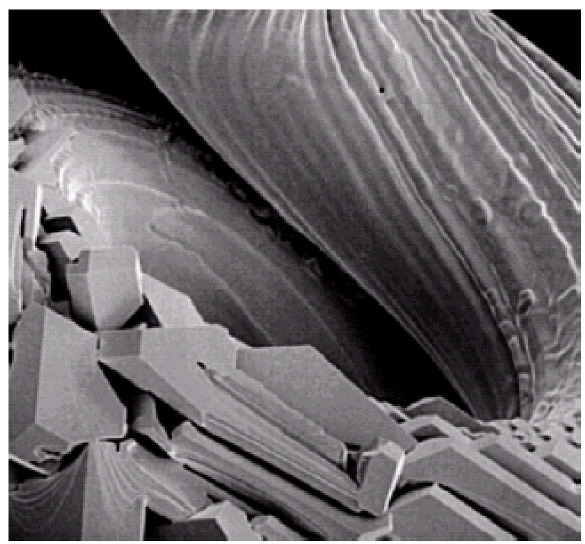
لبه‌ها رو بیشتر sharp
میکنه
۵ درجه



Scanning Electron Image ←



Without Diagonal Elements →



With Diagonal Elements ←

Unsharp masking and high-boost filtering

روش دیگر

$$g_{mask}(x,y) = f(x,y) - \bar{f}(x,y)$$

Blurred

Sharpened

$$\rightarrow \quad g(x,y) = f(x,y) + k * g_{mask}(x,y)$$

از مشتقات جاهاشون
[illegible]

$k > 0$

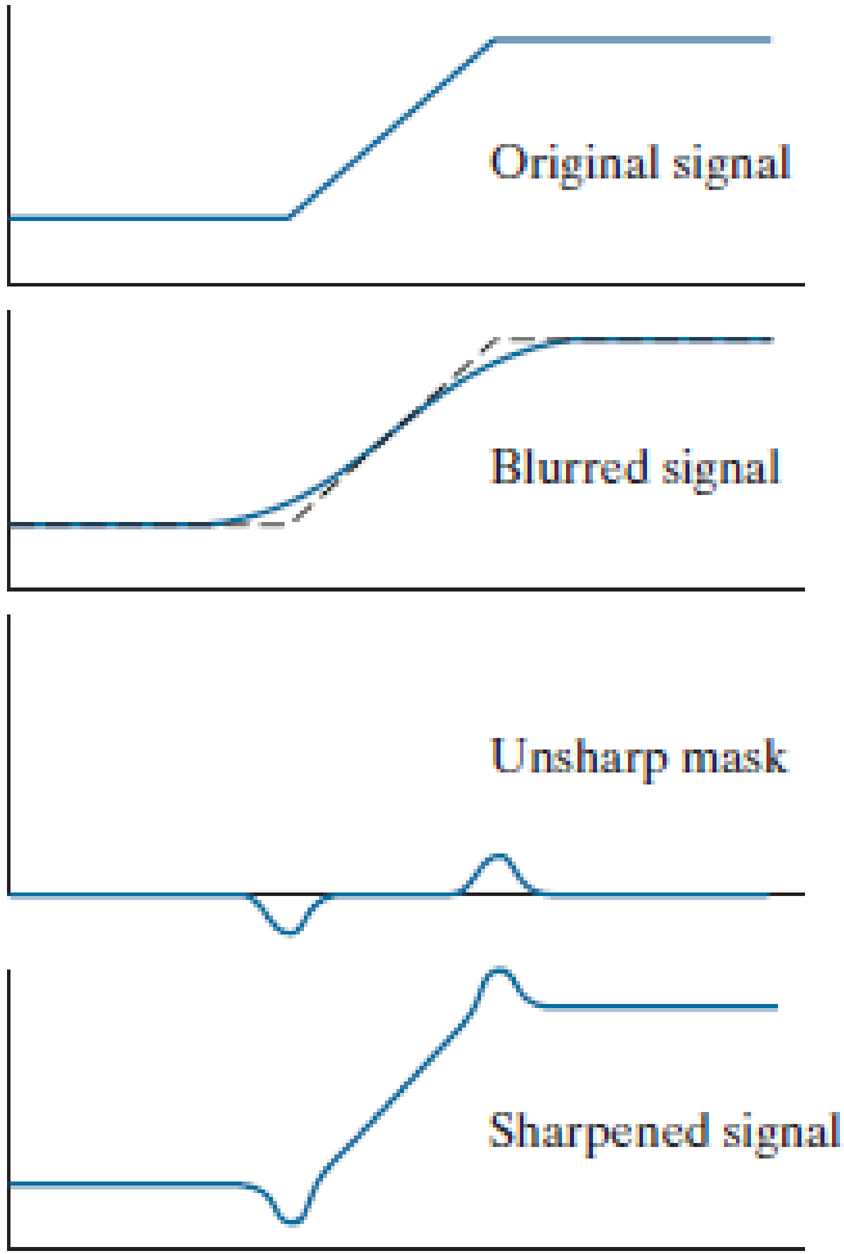
$k = 1 \leftarrow$ Unsharp masking

$k > 1 \leftarrow$ High-boost filtering

$k < 1 \leftarrow$ Deemphasize mask

Unsharp masking

$\bar{f}$



First-order derivatives

مشتق اول به نویز
حساسیت کمتری دارد

Gradient: $$\nabla f = \begin{bmatrix} g_x \\ g_y \end{bmatrix} = \begin{bmatrix} \dfrac{\partial f}{\partial x} \\ \dfrac{\partial f}{\partial y} \end{bmatrix}$$

Gradient image: $$M(x,y) = mag(\nabla f) = \sqrt{g_x^2 + g_y^2}$$

$$M(x,y) \approx |g_x| + |g_y|$$

Gradient masks

Roberts cross-gradient operators:

$$g_x = z_9 - z_5$$
$$g_y = z_8 - z_6$$

$$M(x,y) = [(z_9 - z_5)^2 + (z_8 - z_6)^2]^{1/2}$$

$$M(x,y) \approx |z_9 - z_5| + |z_8 - z_6|$$

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

−1	0
0	1

0	−1
1	0

مشتق اول

Gradient masks

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

Sobel operators:

$$g_x = \frac{\partial f}{\partial x} = (z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)$$

$$g_y = \frac{\partial f}{\partial y} = (z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)$$

$$M(x, y) \approx |(z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)| + |(z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)|$$

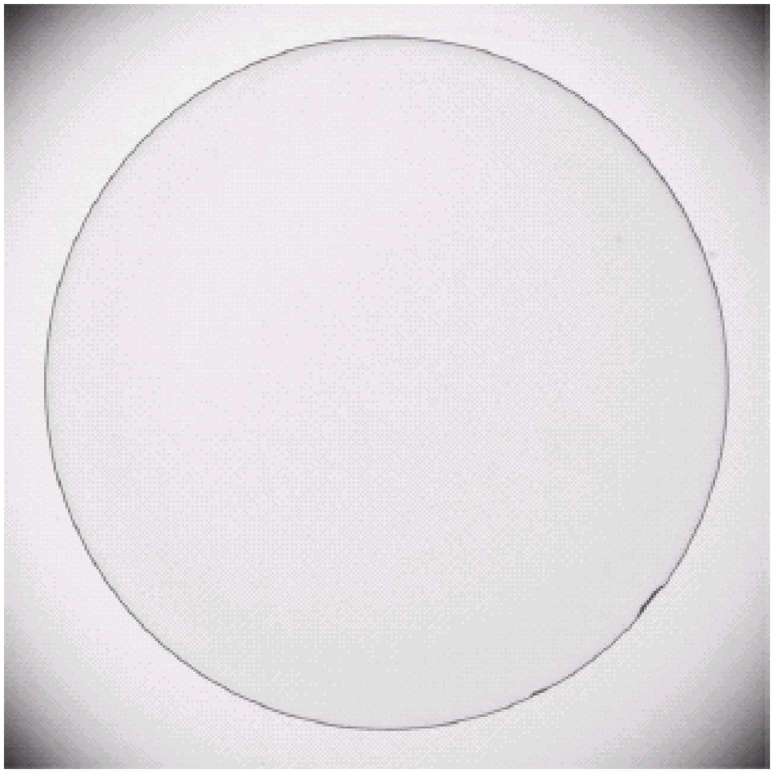
در راستای x

−1	−2	−1
0	0	0
1	2	1

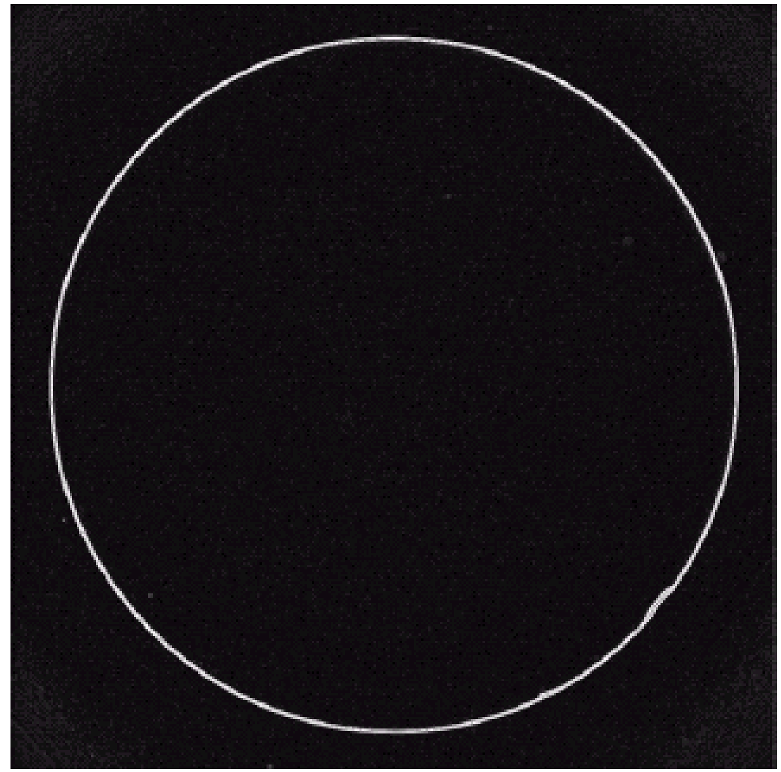
−1	0	1
−2	0	2
−1	0	1

Gradient application

Industrial inspection:

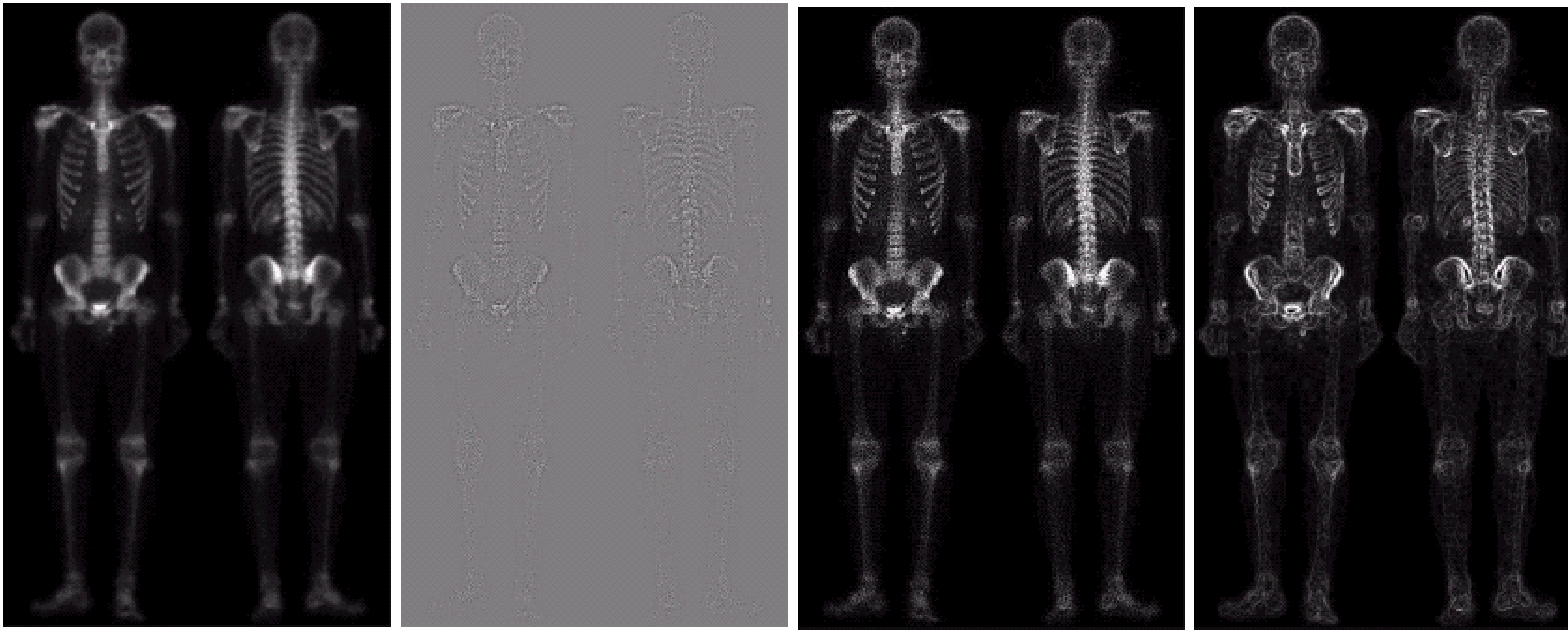


Contact lens



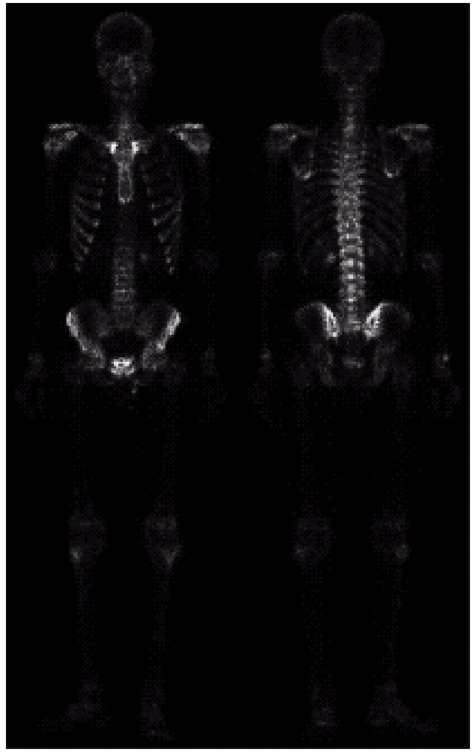
Sobel operators used to highlight defects

threshold



(a) Bone scan (b) Laplacian (c)Sharpened (d) Sobel of (a)

(e) Smoothing of (d) by 5x5 averaging



(f) Product of (c) and (e)

(g) Sum of (a) and (f)

(g) power-law transformation of (g)