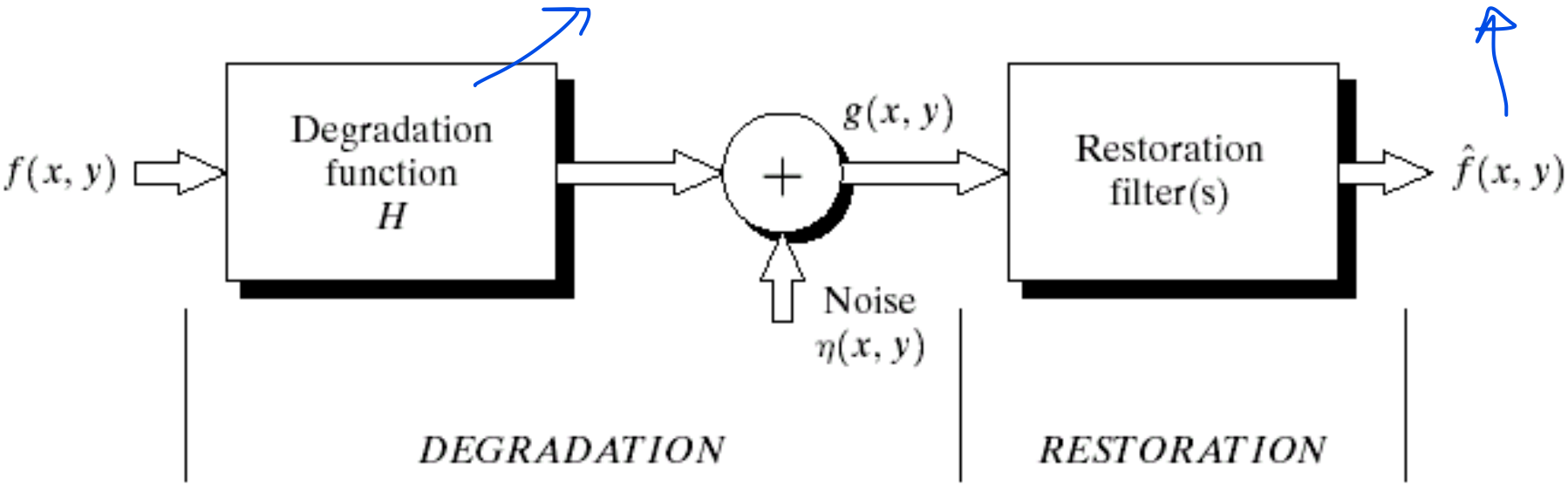
Image restoration

مثلاً مه آلود - لینیت دوربین

نزدیک به f



$$g(x,y) = h(x,y) \star f(x,y) + \eta(x,y)$$

$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

Important Noise PDFs

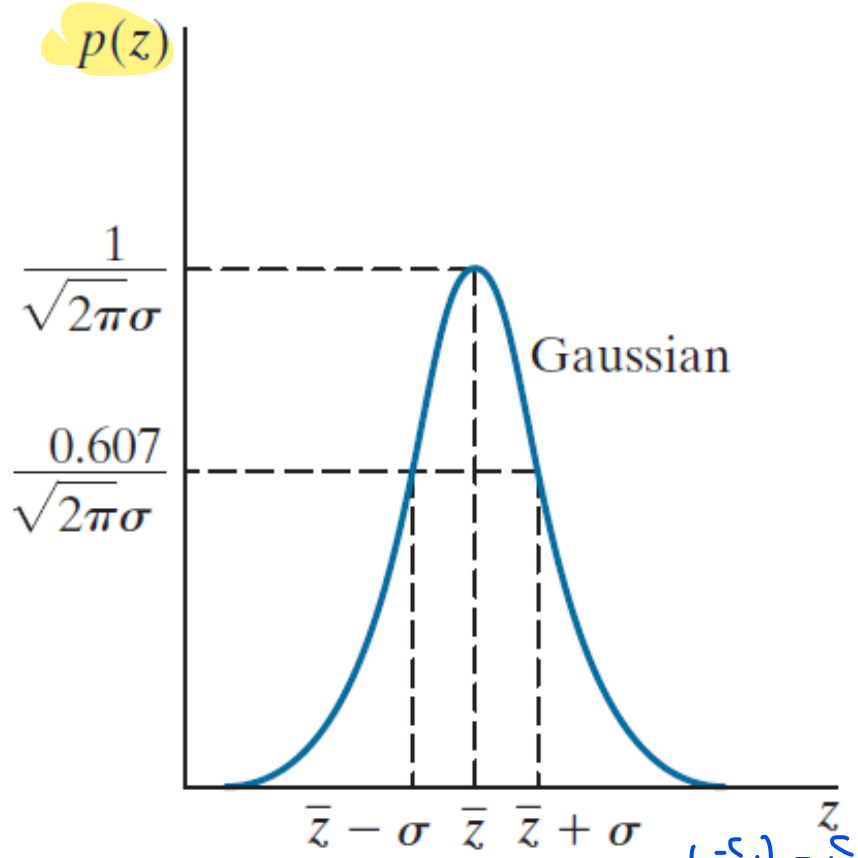
Gaussian Noise:

$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(z-\bar{z})^2/2\sigma^2}$$

$\bar{z}$

σ^2

$$\begin{cases} 70\% & z \in [\bar{z} - \sigma, \bar{z} + \sigma] \\ 95\% & z \in [\bar{z} - 2\sigma, \bar{z} + 2\sigma] \end{cases}$$



شدت (بیشتر)
مکان

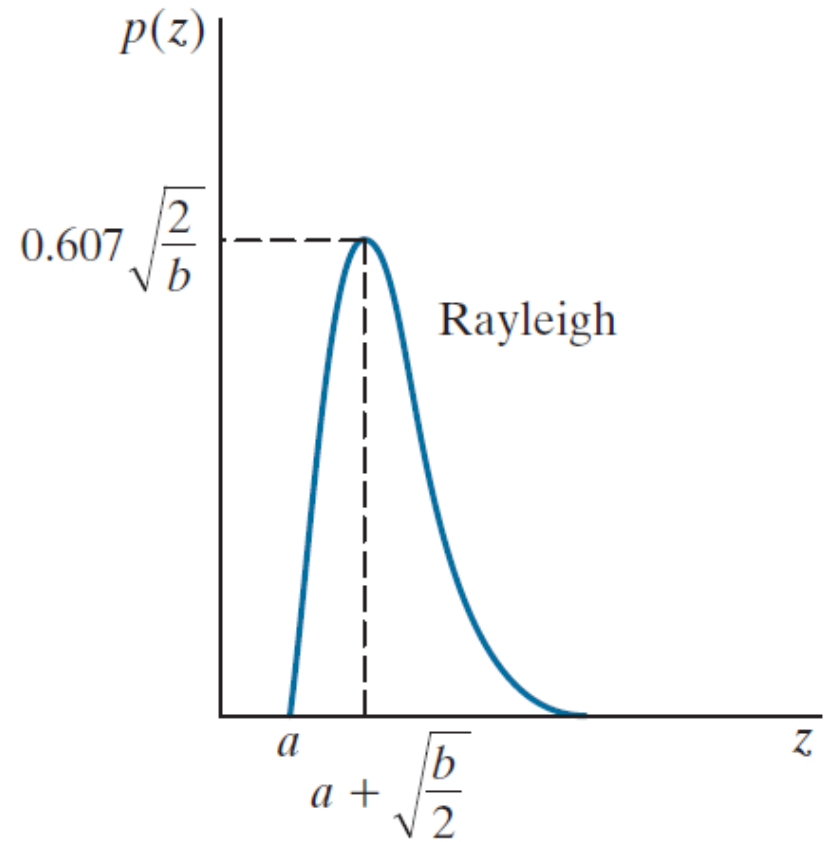
Important Noise PDFs

Rayleigh:

$$p(z) = \begin{cases} \frac{2}{b}(z-a)e^{-(z-a)^2/b} & for\ \ z \geq a \\ 0 & for\ \ z < \cancel{0}\ a \end{cases}$$

$$\bar{z} = a + \sqrt{\frac{\pi b}{4}}$$

$$\sigma^2 = \frac{b(4-\pi)}{4}$$



Important Noise PDFs

Gamma noise

$$p(z) = \begin{cases} \frac{a^b z^{b-1}}{\Gamma(b)} e^{-az} & for\ \ z \geq 0 \\ 0 & for\ \ z < 0 \end{cases}$$

$$\Gamma(b) = \int_0^\infty x^{b-1} e^x dx$$

Erlang noise $\longrightarrow$ b : positive integer

$$p(z) = \begin{cases} \frac{a^b z^{b-1}}{(b-1)!} e^{-az} & for\ \ z \geq 0 \\ 0 & for\ \ z < 0 \end{cases}$$

$$\bar{z} = \frac{b}{a} \quad \sigma^2 = \frac{b}{a^2}$$

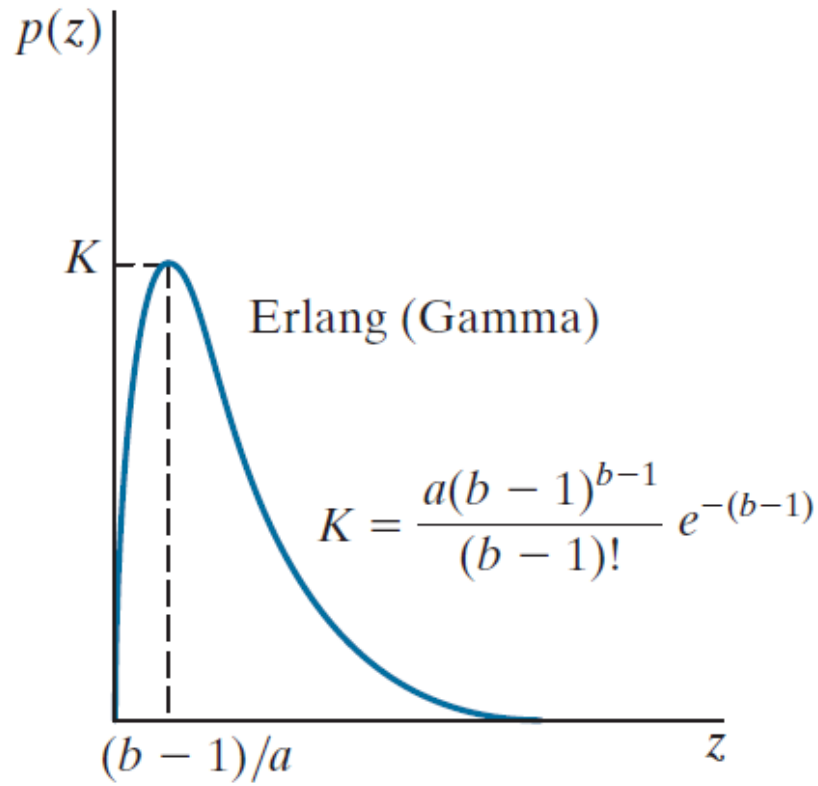
Important Noise PDFs

Erlang:

$$p(z) = \begin{cases} \frac{a^b z^{b-1}}{(b-1)!} e^{-az} & for\ \ z \geq 0 \\ 0 & for\ \ z < 0 \end{cases}$$

$$\bar{z} = \frac{b}{a}$$

$$\sigma^2 = \frac{b}{a^2}$$



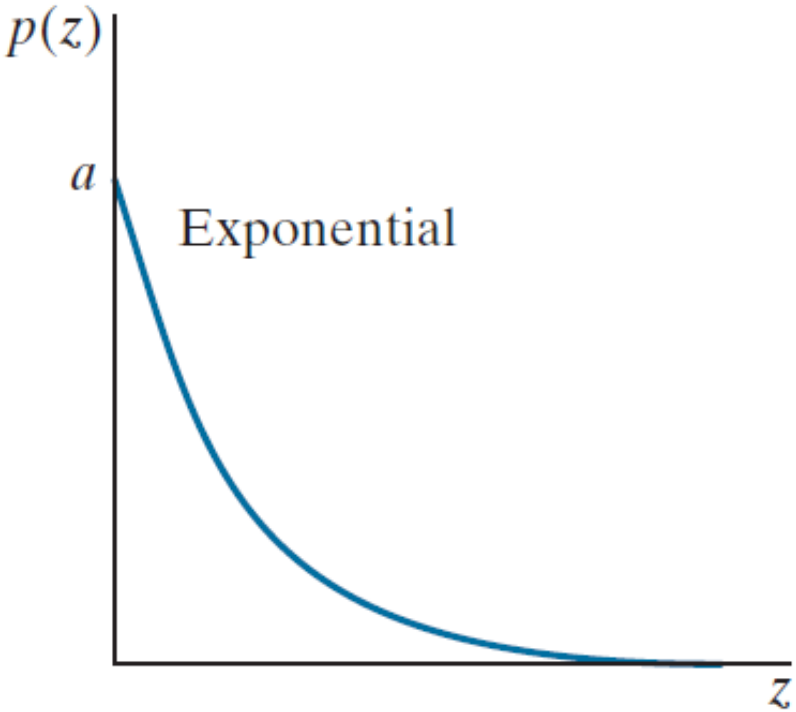
Important Noise PDFs

Exponential:

$$p(z) = \begin{cases} a\,e^{-az} & for\ \ z \geq 0 \\ 0 & for\ \ z < 0 \end{cases}$$

$$\bar{z} = \frac{1}{a}$$

$$\sigma^2 = \frac{1}{a^2}$$



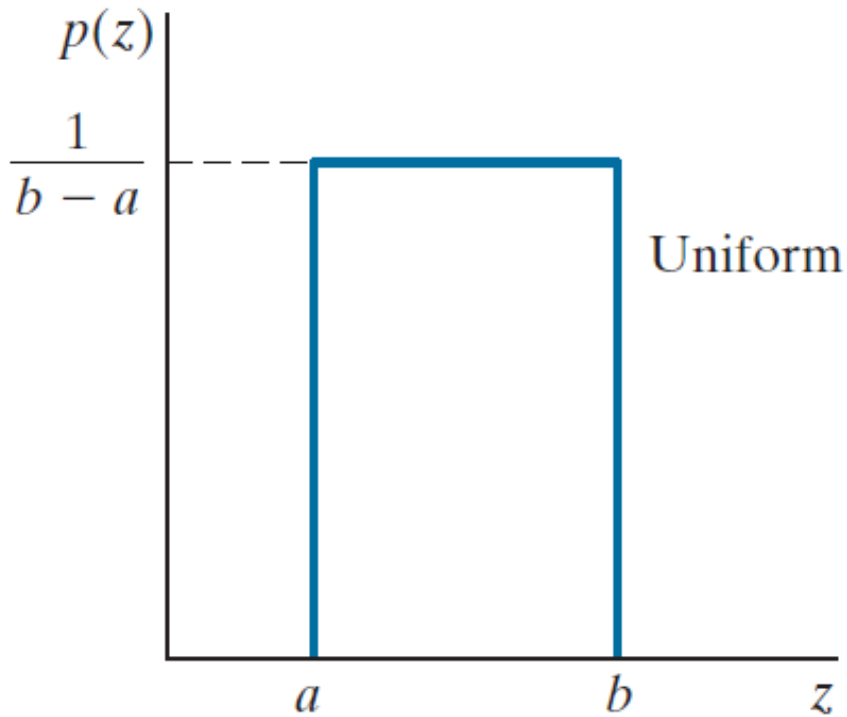
Important Noise PDFs

Uniform:

$$p(z) = \begin{cases} \dfrac{1}{b-a} & if\ a \le z \le b \\ 0 & otherwise \end{cases}$$

$$\bar{z} = \frac{a+b}{2}$$

$$\sigma^2 = \frac{(b-a)^2}{12}$$



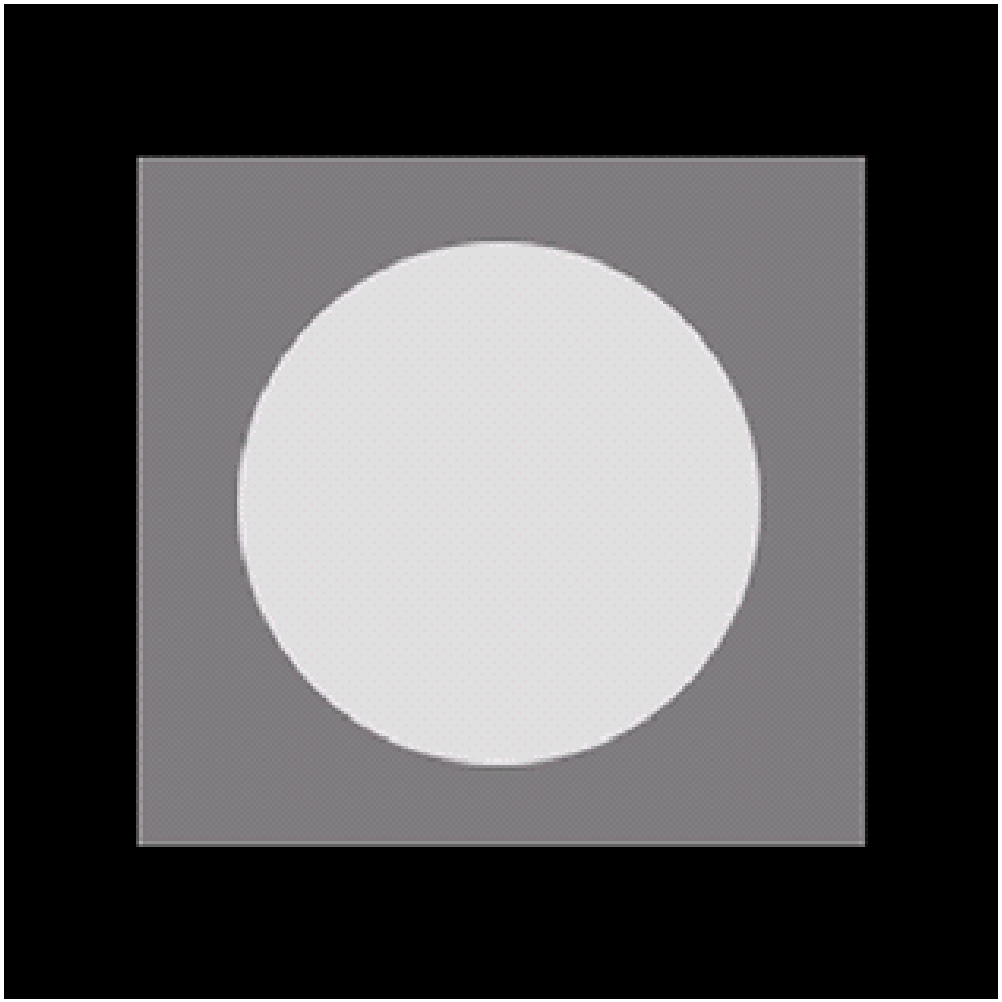
Exponential Noise:

$$p(z) = \begin{cases} a\,e^{-az} & for\ \ z\ \geq 0 \\ 0 & for\ \ z < 0 \end{cases} \qquad \bar{z} = \frac{1}{a} \quad \sigma^2 = \frac{1}{a^2}$$

Uniform Noise:

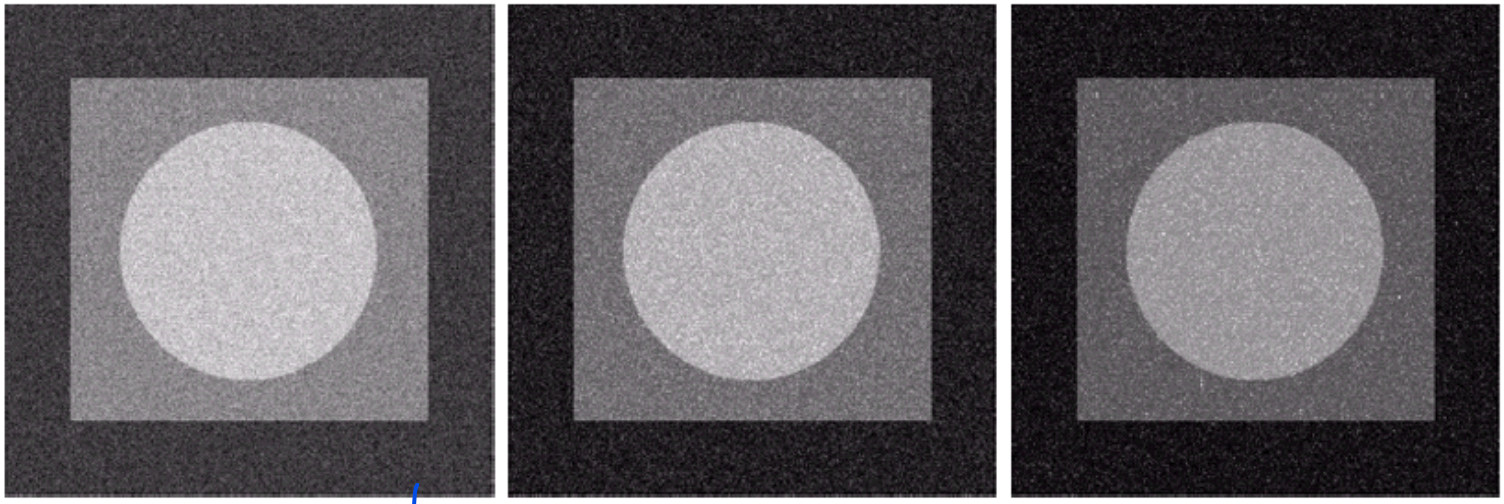
$$p(z) = \begin{cases} \frac{1}{b-a} & for\ \ a \leq z \leq b \\ 0 & for\ \ z < 0 \end{cases} \qquad \bar{z} = \frac{a+b}{z} \quad \sigma^2 = \frac{(b-a)^2}{12}$$

Finding characteristics of PDFs



Test pattern

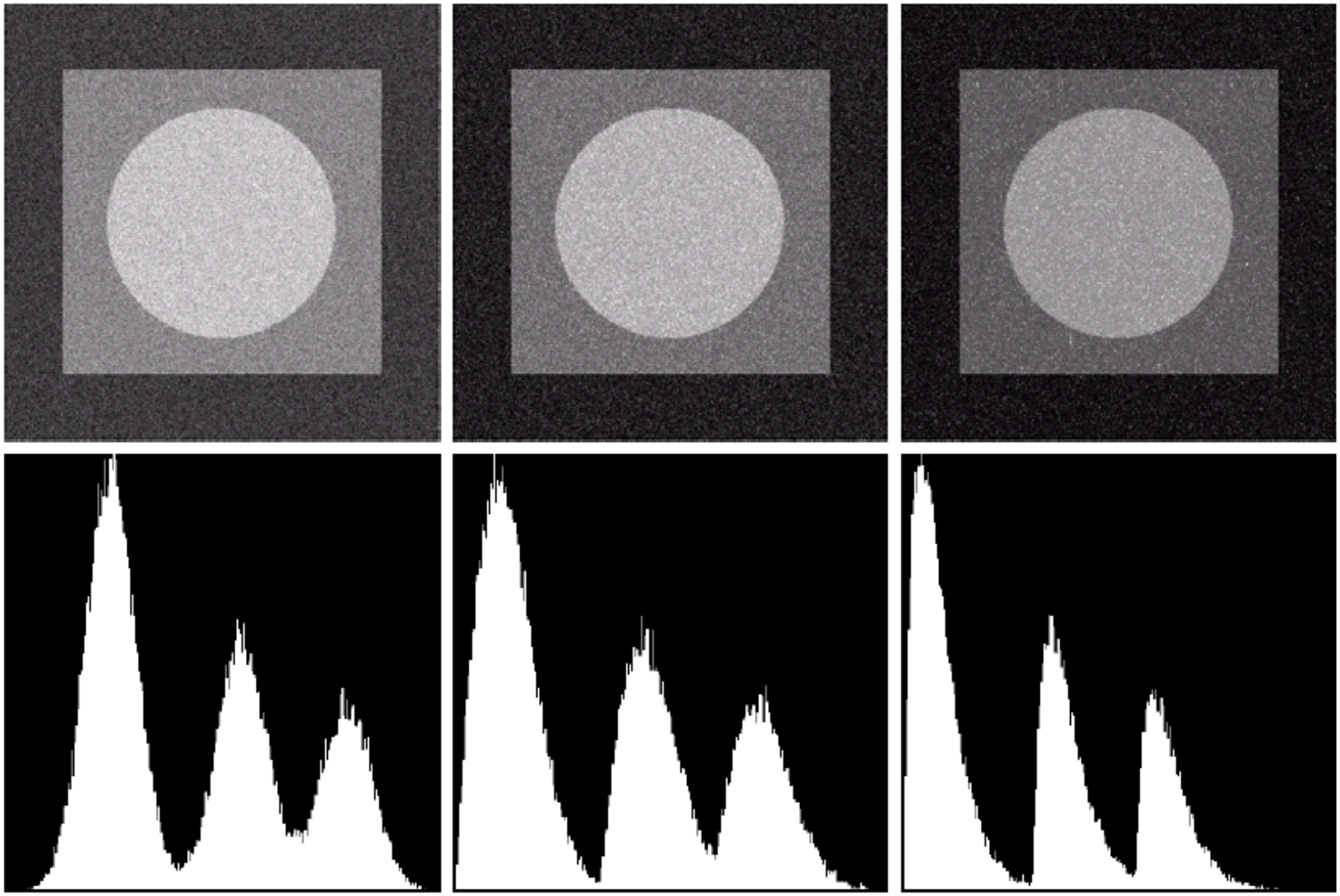
Finding characteristics of PDFs



نویز گوسین بزنیم با میانگین صفر به بعضی از پیکسل‌ها یه عدد مثبت و به بعضی‌ها یه عدد منفی اضافه می‌شه
در نتیجه هیستوگرام از متقارن بودن شدت نزدیک به صفر (۱۲۸) تغییر می‌کنه

Images and histograms after adding various types of noise to the image

Finding characteristics of PDFs

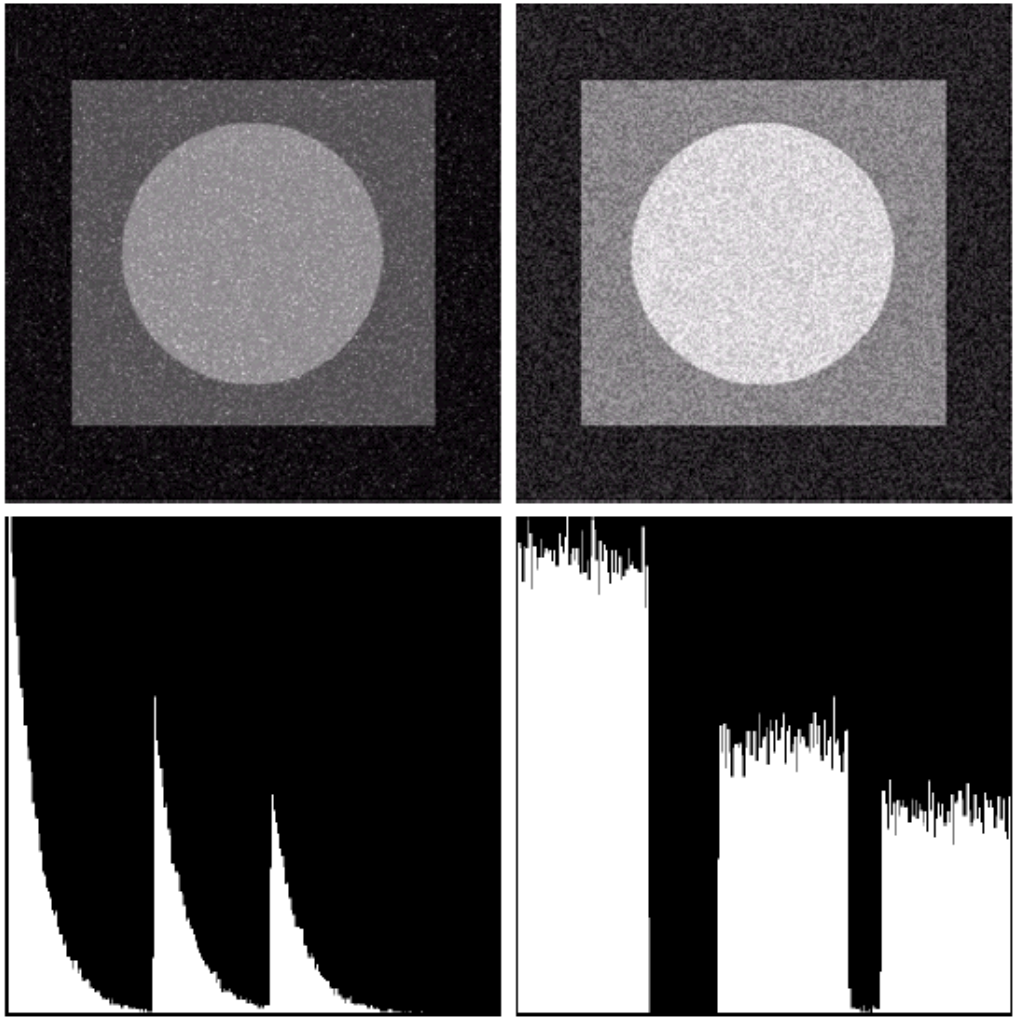


Gaussian Rayleigh Gamma

هر چه σ
بزرگتر باشد،
فاصله دره‌ها بیشتر

Images and histograms after adding various types of noise to the image

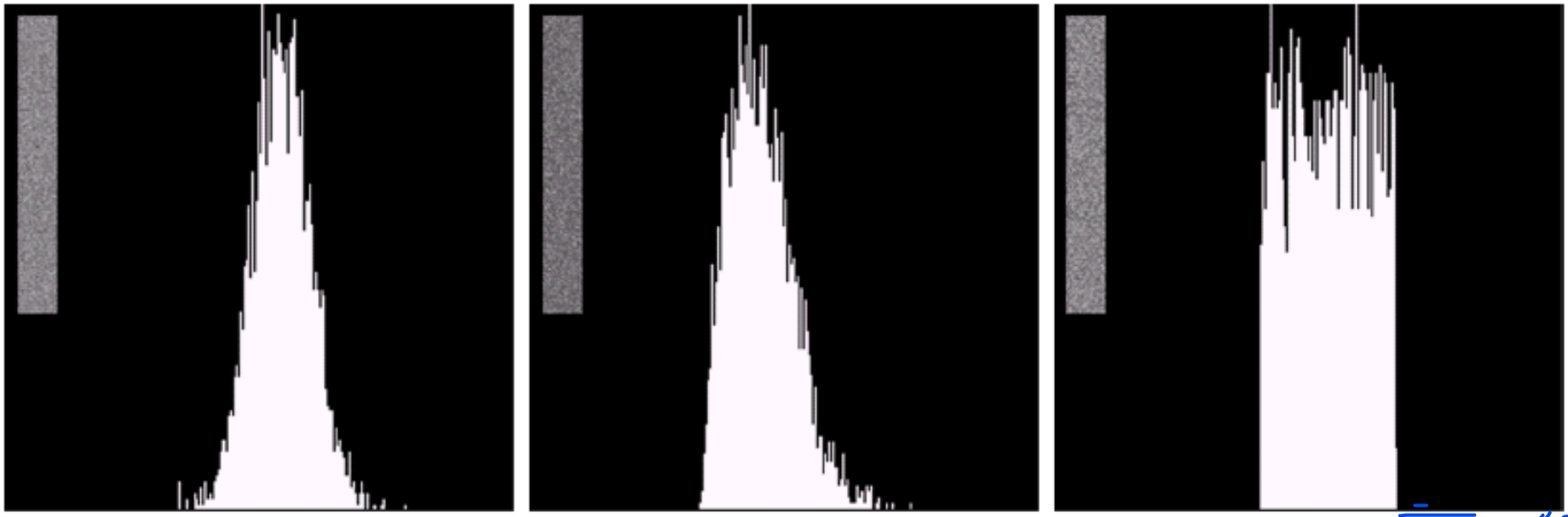
Finding characteristics of PDFs



Exponential Uniform

Images and histograms after adding various types of noise to the image

Using small strips



Gaussian Rayleigh Uniform

شکل به صورت تیز

یه قسمتی از شکل که سفت تقریباً یکسان داره به اندازه یک مستطیل نمونه برداری می کنیم و هیستوگرام

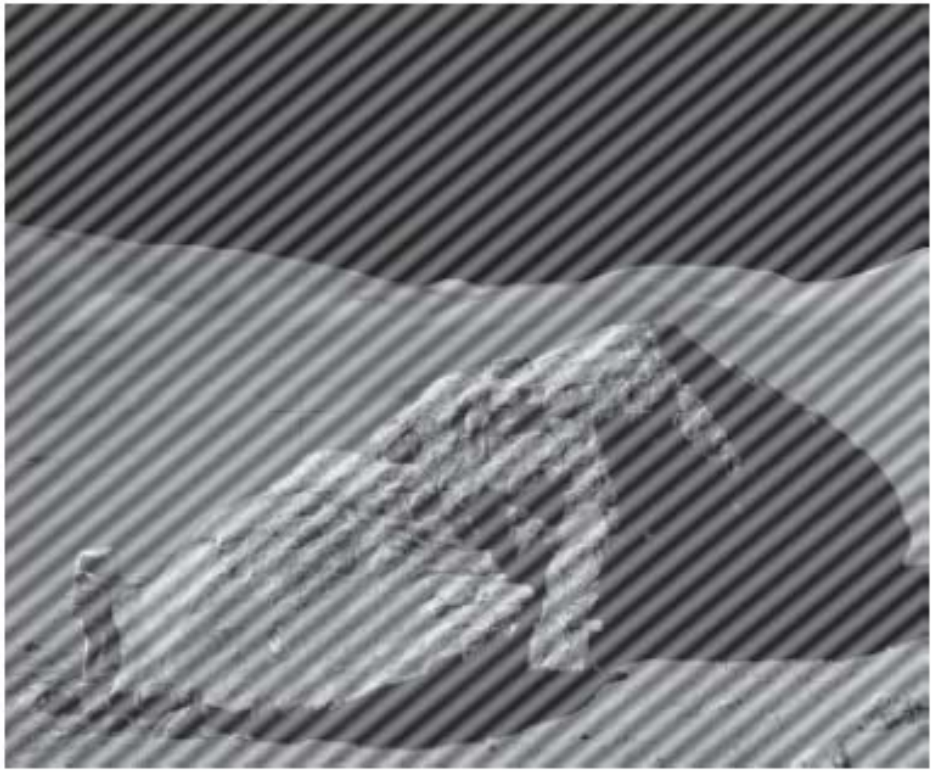
آن را رسم می کنیم، از شکل بدست آمده می توانیم بفهمیم چه نویزی اضافه شده است و

رویدیت

بیاریم

Image corrupted by sinusoidal noise

همینجوری یه عدد بهش اضافه کنی (بسته به مکانش اضافه کنی)



نویز به مکان وابسته است

می‌بریم تا حوزه فرکانس

و بعد نویز رو تشخیص میدیم

$\sin \leftrightarrow \delta(V+s) - \delta(V-s)$

یه ناچ فیلتر بزنیم که بیاد این impals رو حذف کنه

جایی که تبدیل فوریه و حوزه فرکانس بهتر از حوزه مکان هست

Image corrupted by sinusoidal noise

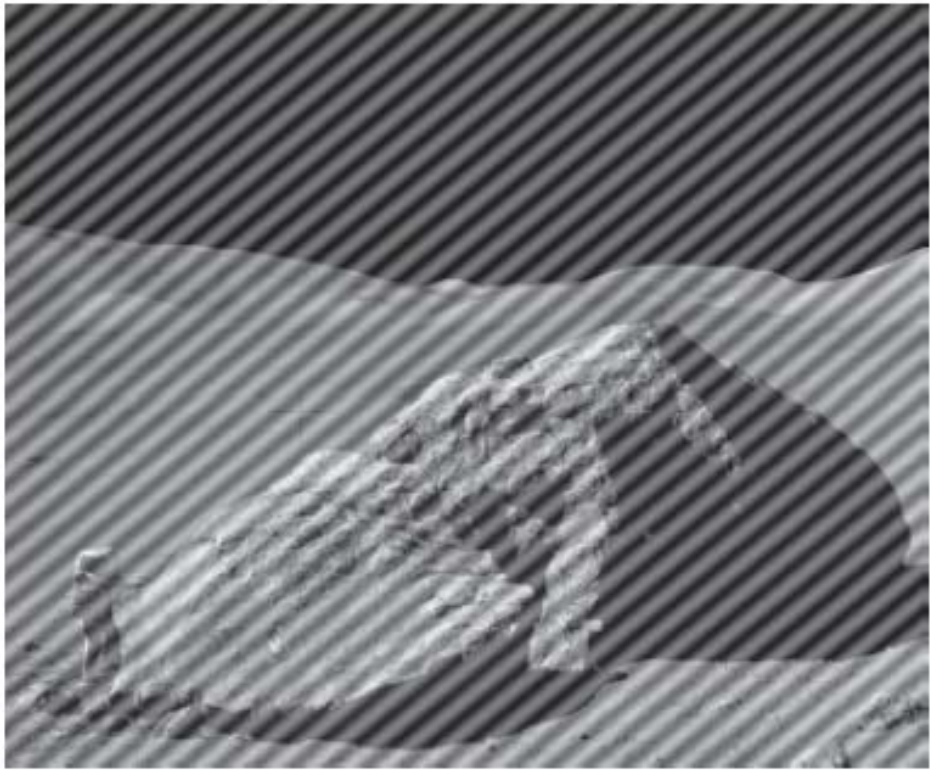


Image corrupted by sinusoidal noise

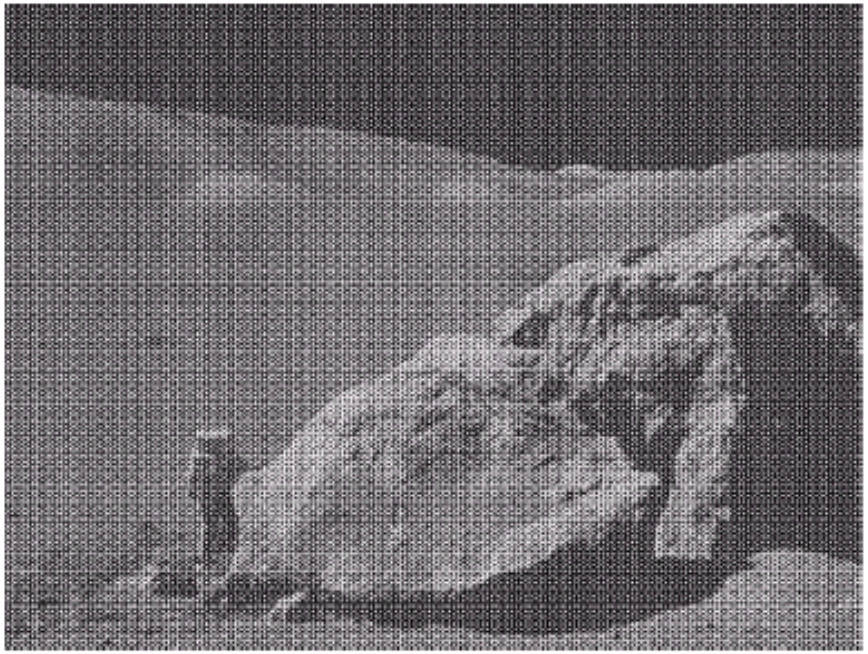
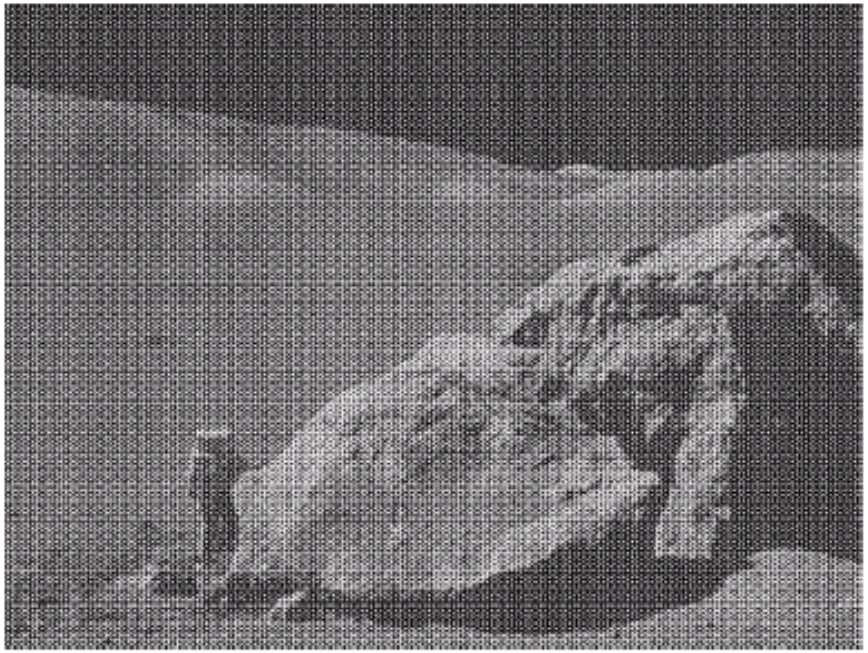
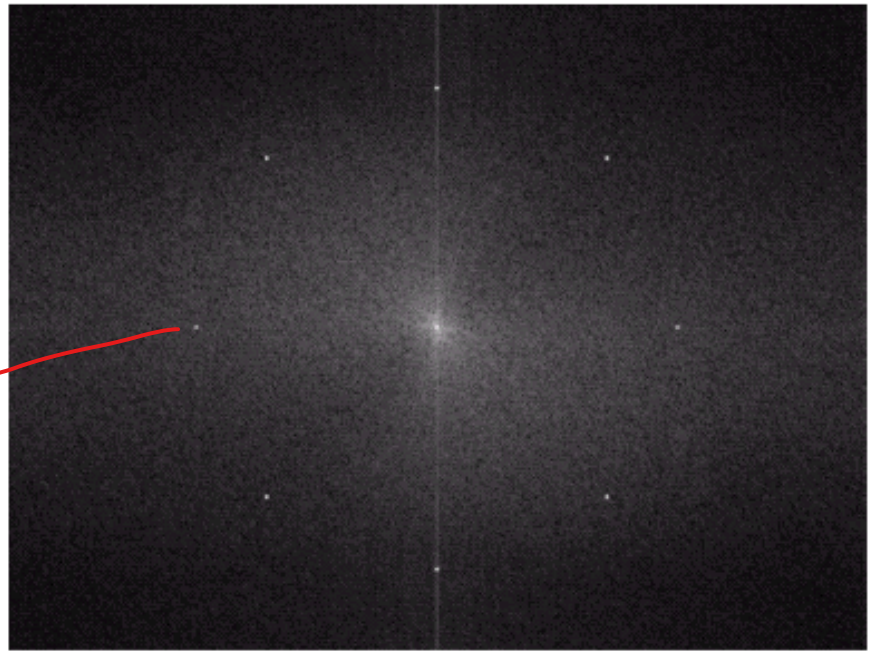


Image corrupted by sinusoidal noise





عدد فرکانس

۴ زوج یک

نویز

Restoration using spatial filters

حذف می‌کند، تضعیف می‌کند

Mean filters

Arithmetic :

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} g(s,t)$$

سایز پنجره

Geometric :

$$\hat{f}(x,y) = \left[\prod_{(s,t)\in S_{xy}} g(s,t) \right]^{\frac{1}{mn}}$$

مناسب برای یک پیکسل منفرد

نویز Pepper

Harmonic :

$$\hat{f}(x,y) = \frac{mn}{\sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}}$$

تصاویر سرد سیاه

Contraharmonic :

$$\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

Restoration using spatial filters

Mean filters

Contraharmonic: $$\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

Restoration using spatial filters

Mean filters

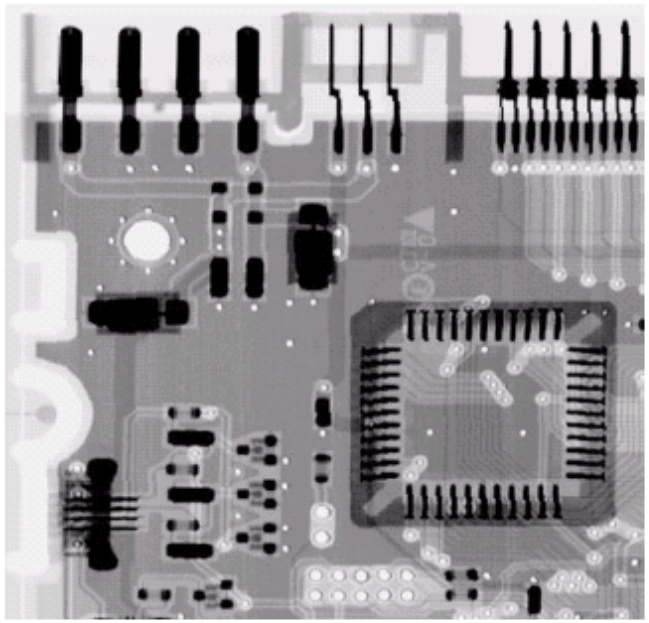
تغییر می‌کند mask کرنل

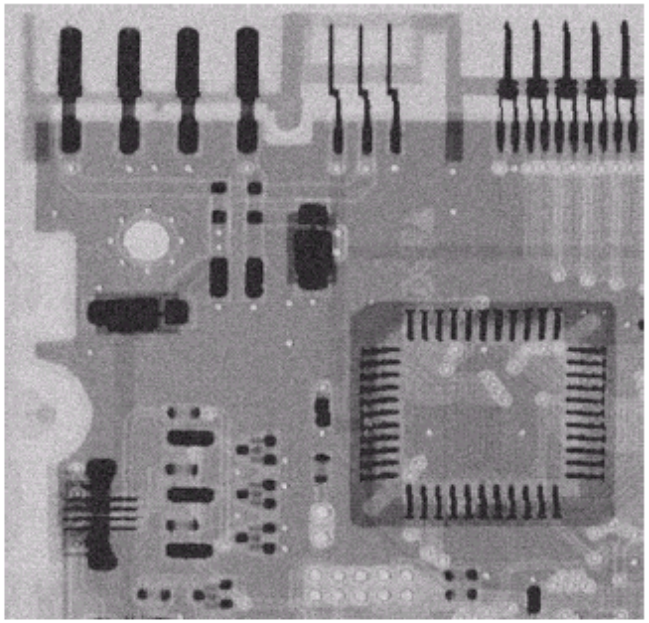
Contraharmonic: $$\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

$$\begin{cases} Q > 0 & \textit{Eliminate pepper noise} \\ Q < 0 & \textit{Eliminate salt noise} \\ Q = 0 & \textit{Arithmetic} \\ Q = -1 & \textit{Harmonic} \end{cases}$$

Restoration using spatial filters

X-ray

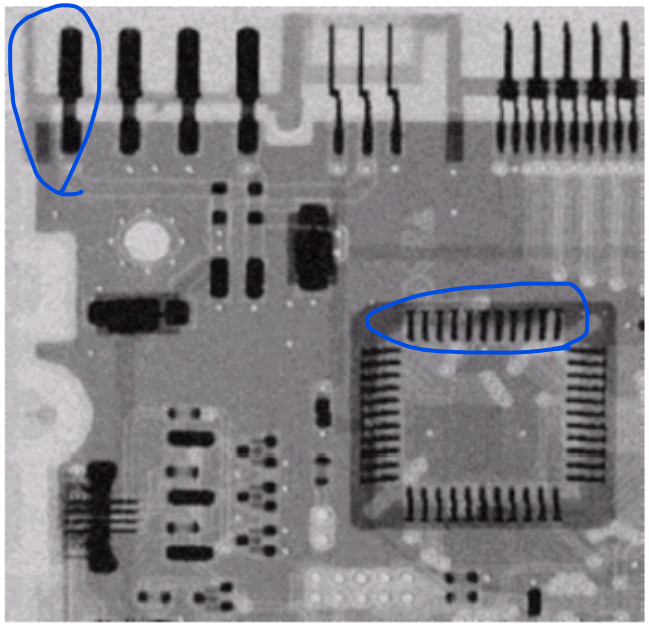


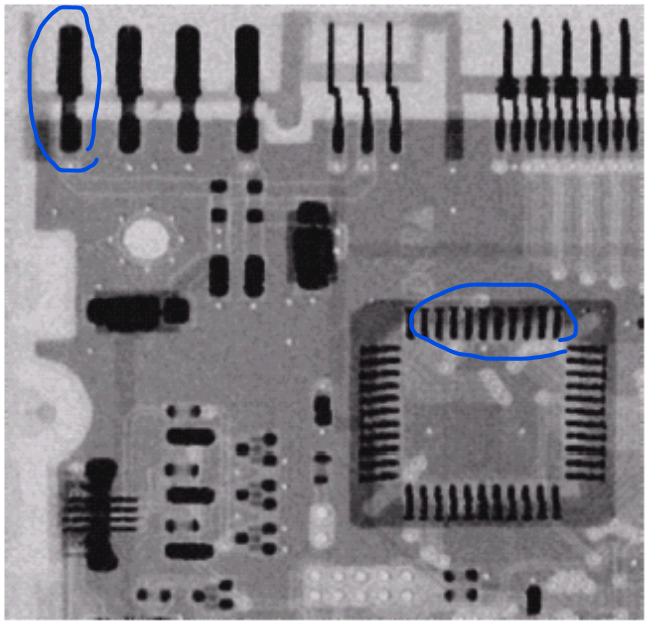


+Gaussian noise

Arithmetic mean 3x3

blur کنه



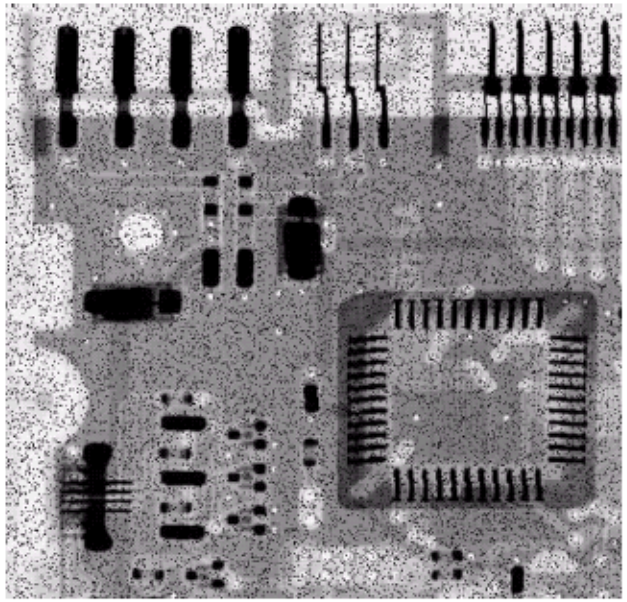


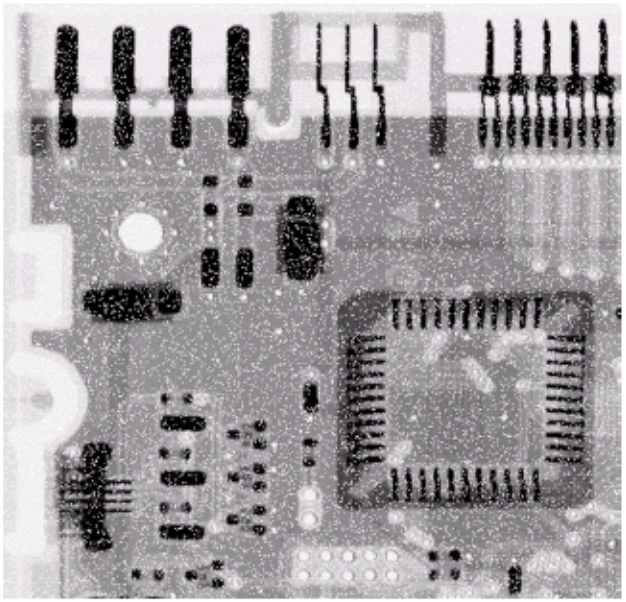
Geometric mean 3x3

تیزتر از حسابی

sharp تر

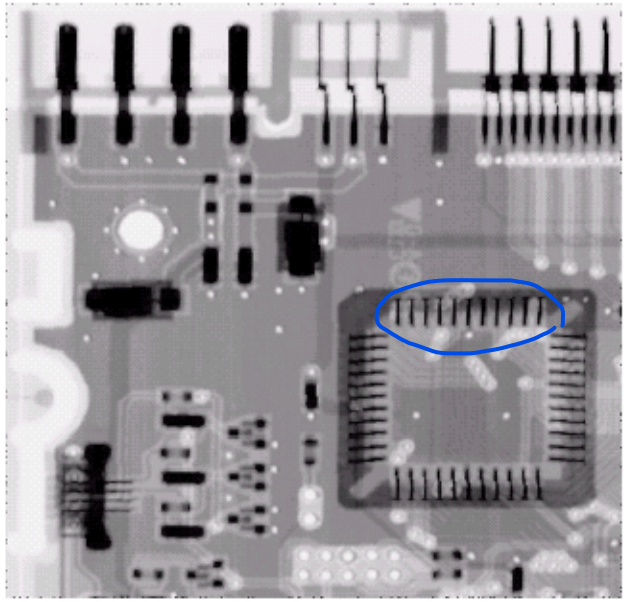
Corrupted by pepper noise (P=0.1)

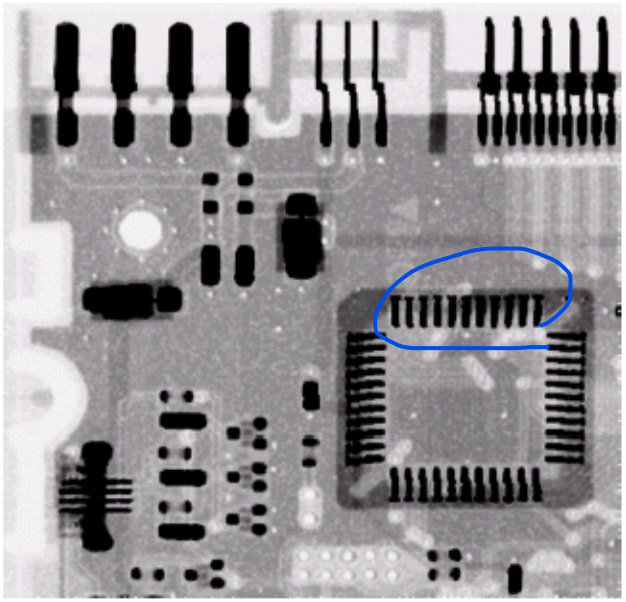




Corrupted by salt noise (P=0.1)

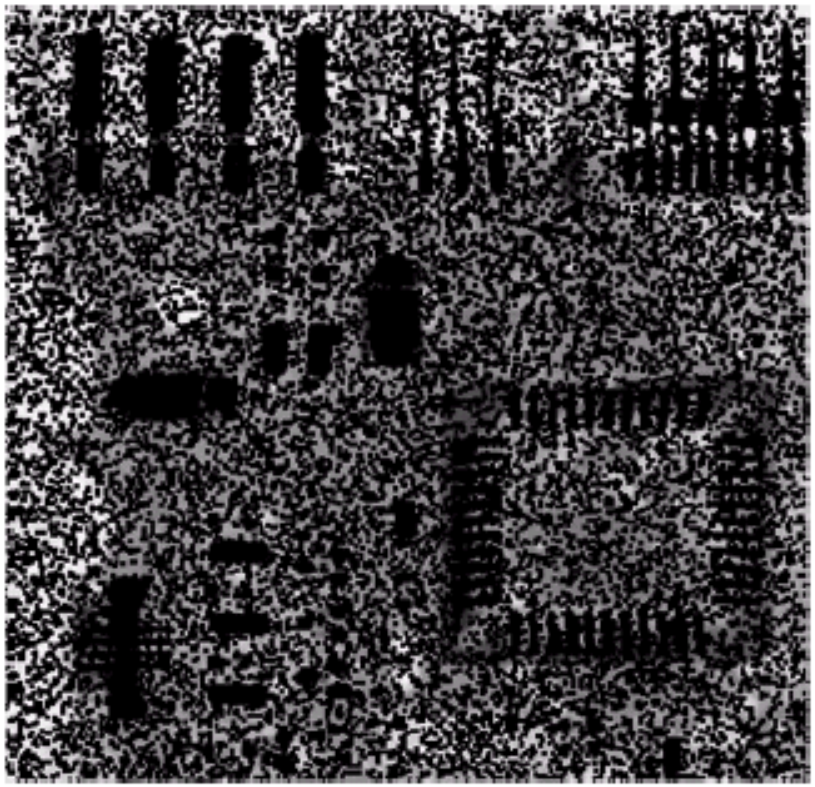
With contraharmonic filtering of order 1.5



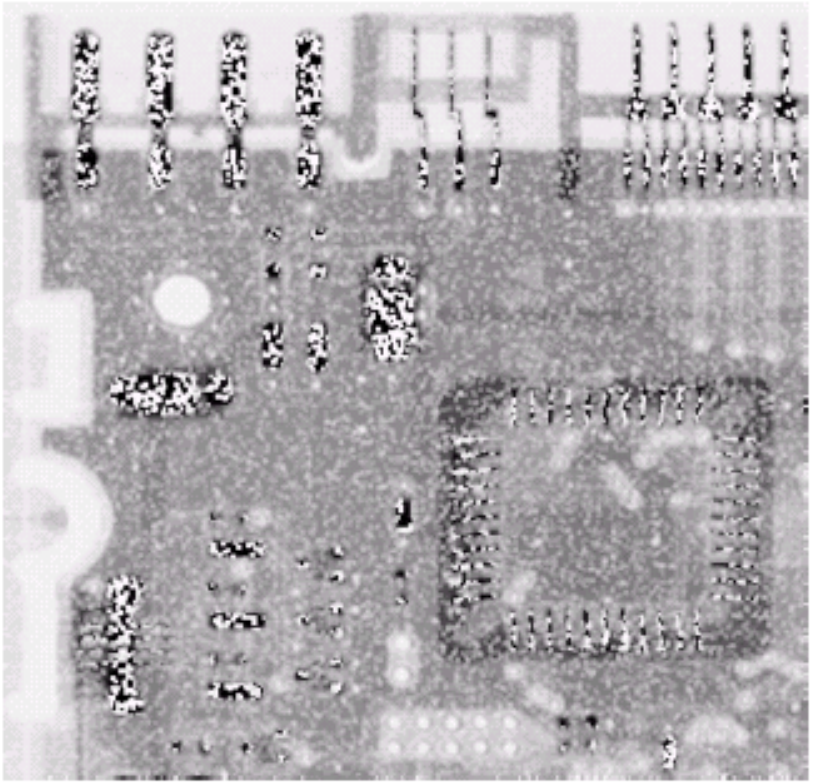


With *Q=-1.5*

Selecting the wrong sign



Q=-1.5 for image with pepper noise



Q=1.5 for image with salt noise

اول مرتب کنه
شدت هارو

Order statistic filters

برای impalse خوبه

$$\text{Median } \tilde{f}(x,y) = \underset{(s,t)\epsilon S_{xy}}{median}\{g(s,t)\}$$

برای salt بده

$$\text{Max } \tilde{f}(x,y) = \underset{(s,t)\epsilon S_{xy}}{max}\{g(s,t)\}$$

برای pepper بده

$$\text{Min } \tilde{f}(x,y) = \underset{(s,t)\epsilon S_{xy}}{min}\{g(s,t)\}$$

$$\text{Midpoint } \tilde{f}(x,y) = \frac{1}{2}\left[\underset{(s,t)\epsilon S_{xy}}{min}\{g(s,t)\} + \underset{(s,t)\epsilon S_{xy}}{max}\{g(s,t)\}\right]$$

Order statistics averaging

اسمش

Alpha-trimmed mean filter :

d/2 از بالا، d/2 از پایین میریزیم دور بعد از مرتب کردن

و بعد از باقی مانده، میانگین می‌گیریم.

$$\tilde{f}(x,y) = \frac{1}{mn-d} \sum_{(s,t)\in S_{xy}} g_r(s,t)$$

remain

0 → میانگین‌گیری

mn-1 → میانه‌گیری

~~1~~ 2 3 4 5 6 ~~8~~

d=2

Order statistics averaging

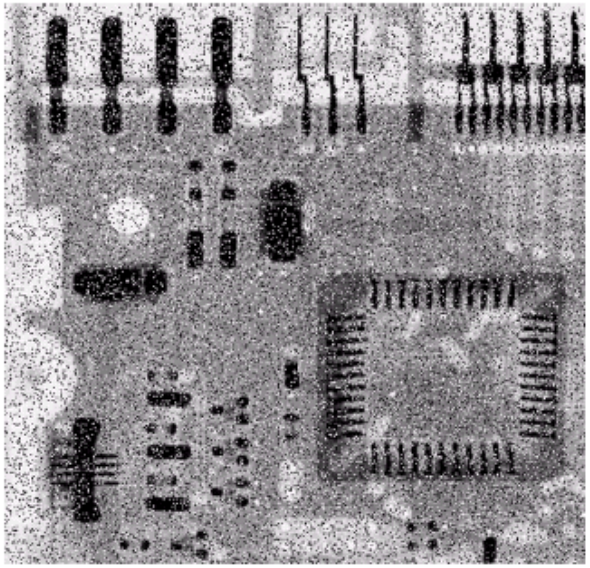
Alpha-trimmed mean filter :

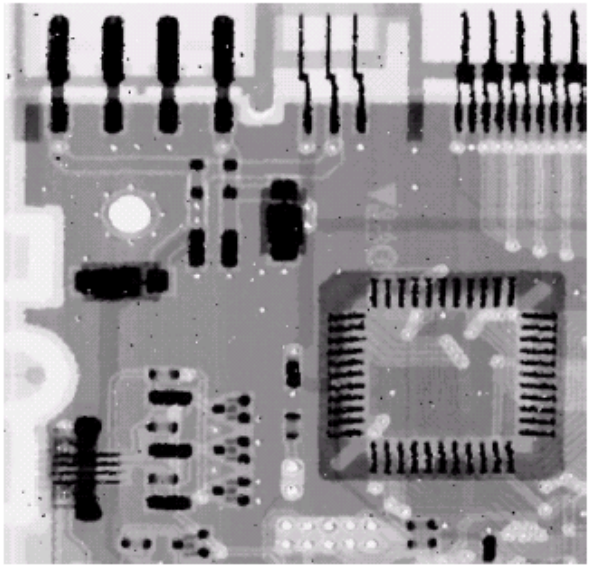
$$\tilde{f}(x,y) = \frac{1}{mn-d} \sum_{(s,t)\in s_{xy}} g_r(s,t)$$

$g_r(s,t)$: remaining after $d/2$ lowest & $d/2$ highest are removed

$$\begin{cases} d = 0 & arithmetic\ mean \\ d = mn - 1 & median \end{cases}$$

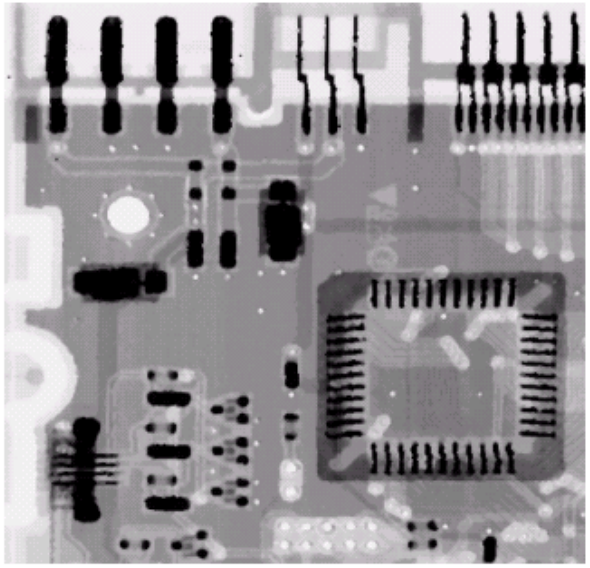
Corrupted by salt and pepper noise
$P_a = P_b = 0.1$

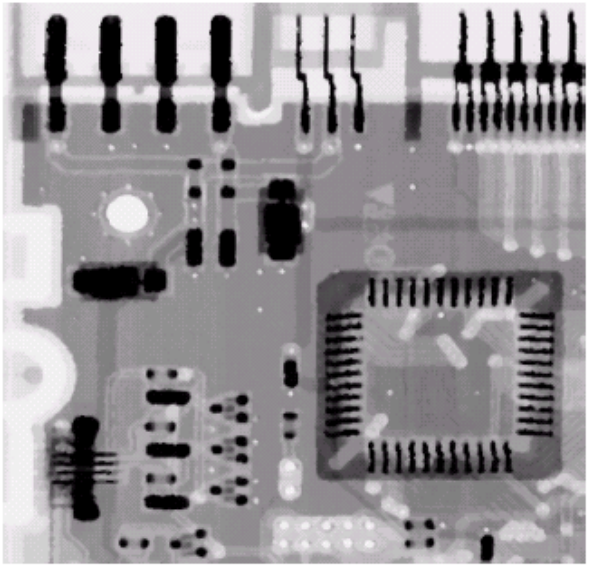




Median filtering (3x3)

2nd time median filtering (3x3)

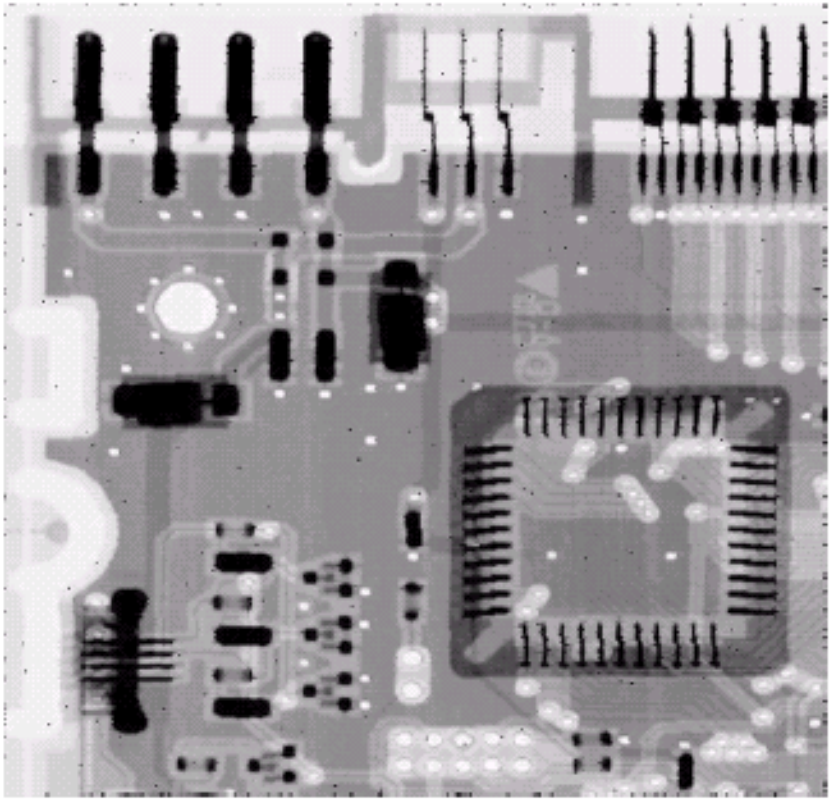




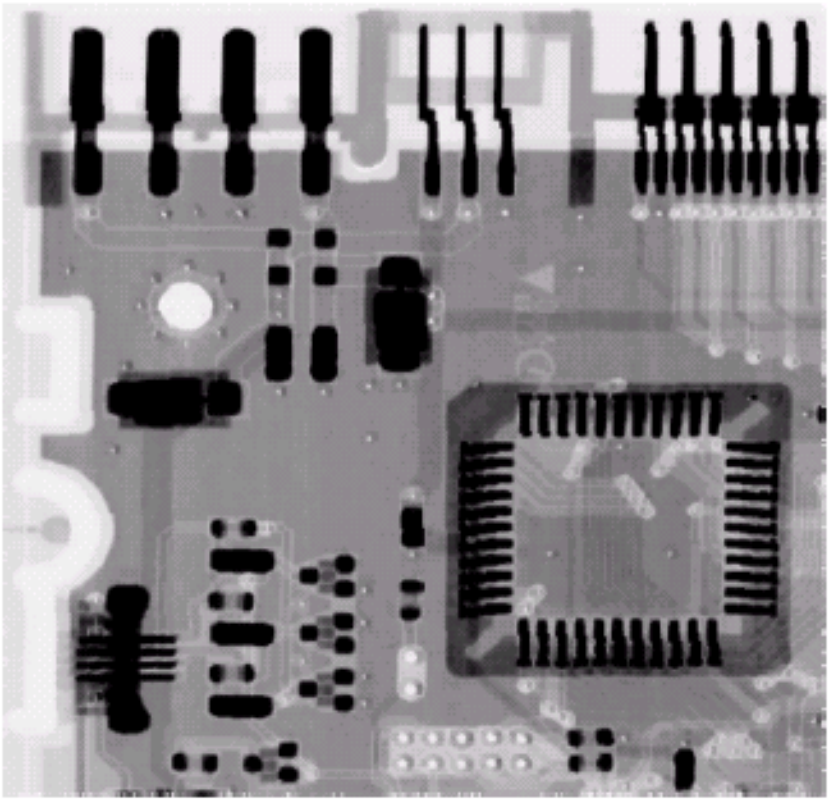
3rd time median filtering (3x3)

قسمت های سفیدتر نازک تر می‌شوند

قسمت های تیره تصویر، نازک تر می‌شوند



Max filtering (3x3)
on image with pepper noise



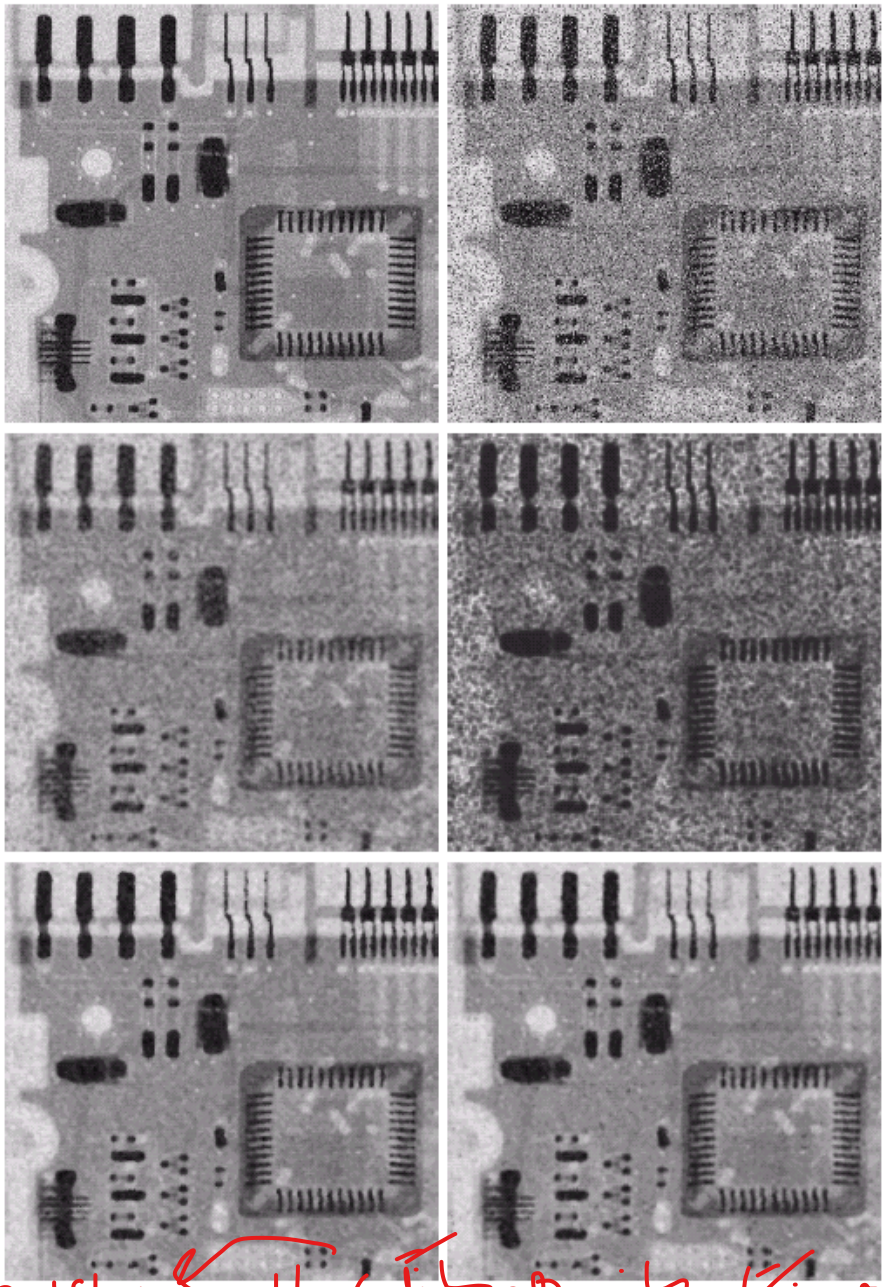
Min filtering (3x3)
on image with salt noise

۲۲ بهمن

میانگین گیری خوبه

+ uniform noise

یکنواخت



+ salt and pepper noise

Filtering with arithmetic mean filter (5x5)

Filtering with geometric mean filter (5x5)

Filtering with median filter (5x5)

Filtering with alpha-trimmed mean filter

قدرت یکنواخت رو با میانگین گیری

تست impulse با میانه گیری

هر نگاه میانه و هم میانگین دار

برای میانگین صفری خوبه!

Adaptive filtering

Adaptive local noise reduction filter:

$g(x, y)$: Noise image

σ_η^2 : Noise variance

m_L : Local mean of pixels in S_{xy}

σ_L^2 : Local variance of pixels in S_{xy}

Adaptive filtering

Adaptive local noise reduction filter:

اگر نداشتیم مثل قبل ما یه تیله

که دست نخورده رو برمی‌گردونیم و کار رو بهت میاریم

Expectations:

1) $\sigma_\eta^2 = 0 \longrightarrow g(x,y)$

2) $\sigma_L^2 \gg \sigma_\eta^2 \longrightarrow g(x,y)$

3) $\sigma_L^2 = \sigma_\eta^2 \longrightarrow m_L$

میانگین گیری رو انجام

بدهم

تصویر sharp

و contrast خوب داره

Adaptive filtering

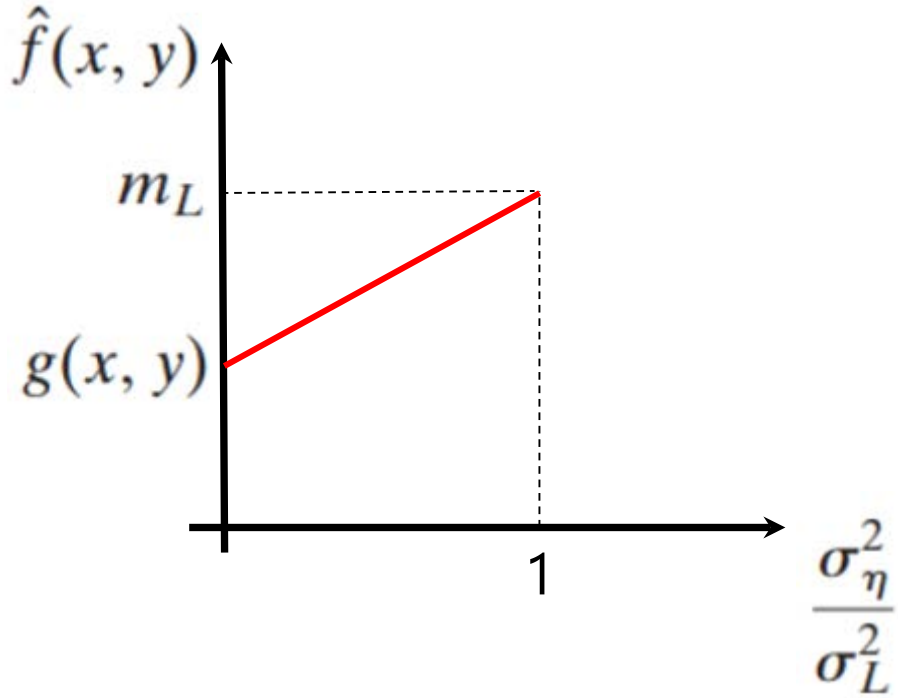
Adaptive local noise reduction filter:

Expectations:

1) $\sigma_\eta^2 = 0 \longrightarrow g(x,y)$

2) $\sigma_L^2 \gg \sigma_\eta^2 \longrightarrow g(x,y)$

3) $\sigma_L^2 = \sigma_\eta^2 \longrightarrow m_L$



Adaptive filtering

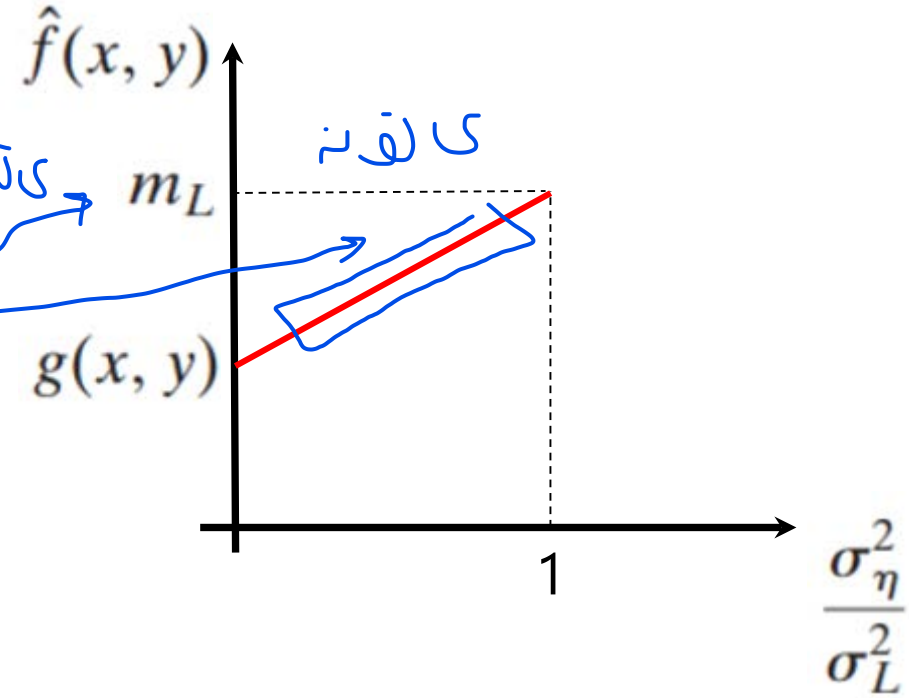
Adaptive local noise reduction filter:

Expectations:

1) $\sigma_\eta^2 = 0 \longrightarrow g(x,y)$

2) $\sigma_L^2 \gg \sigma_\eta^2 \longrightarrow g(x,y)$

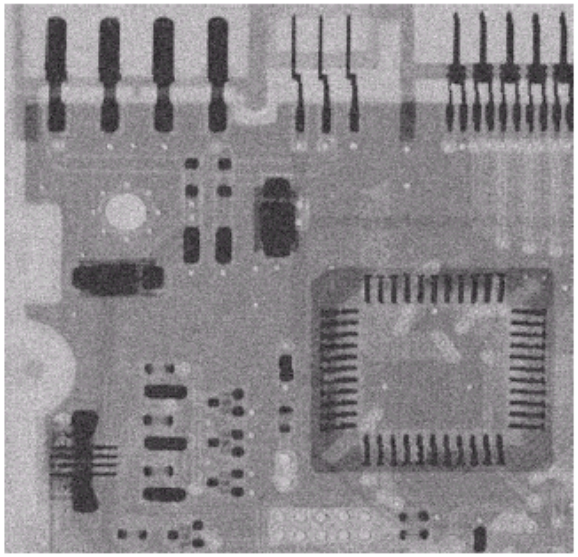
3) $\sigma_L^2 = \sigma_\eta^2 \longrightarrow m_L$

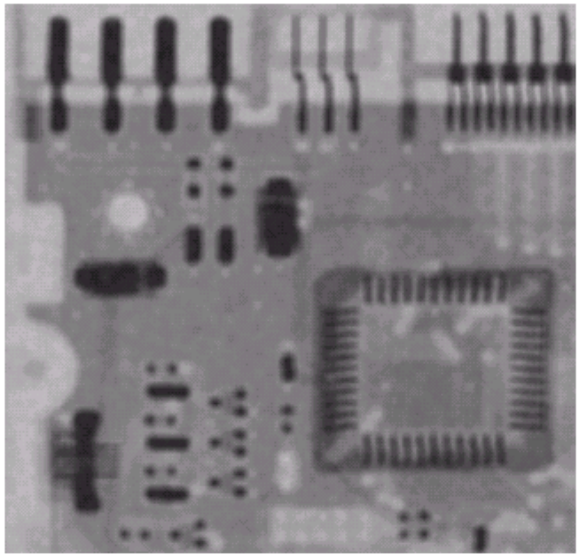


Adaptive expression:

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma_\eta^2}{\sigma_L^2}[g(x,y) - m_L]$$

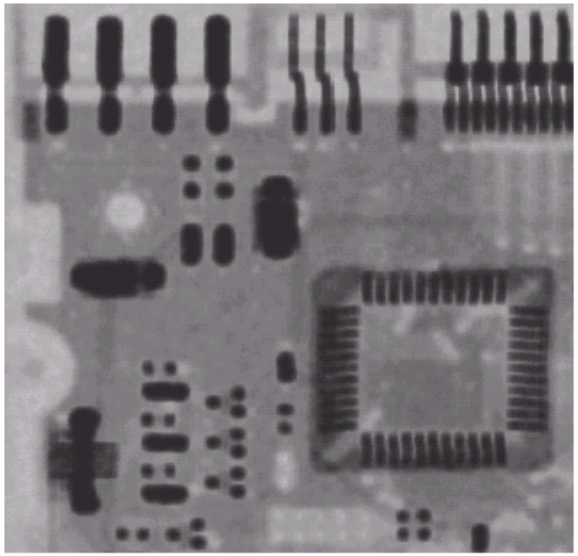
+Gaussian with zero mean variance 1000

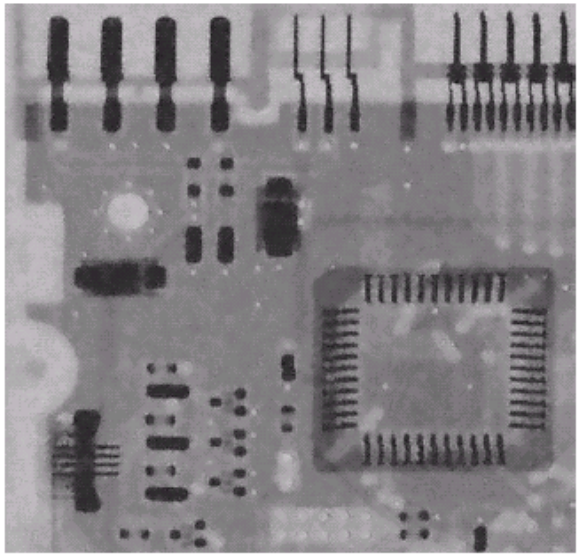




Arithmetic mean filtering (7x7)

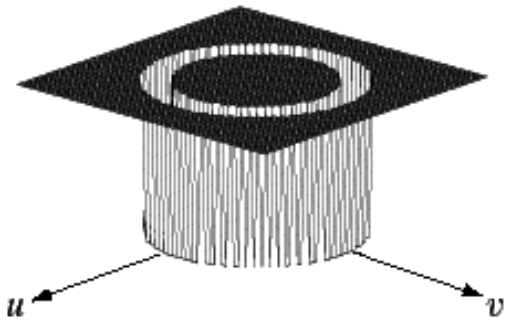
Geometric mean filtering (7x7)



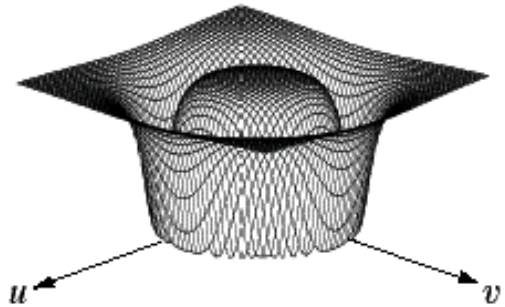


Adaptive noise reduction filtering (7x7)

Bandreject filters



Ideal

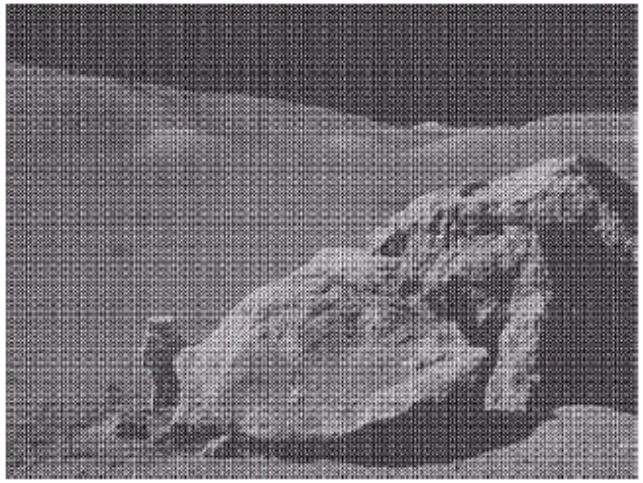


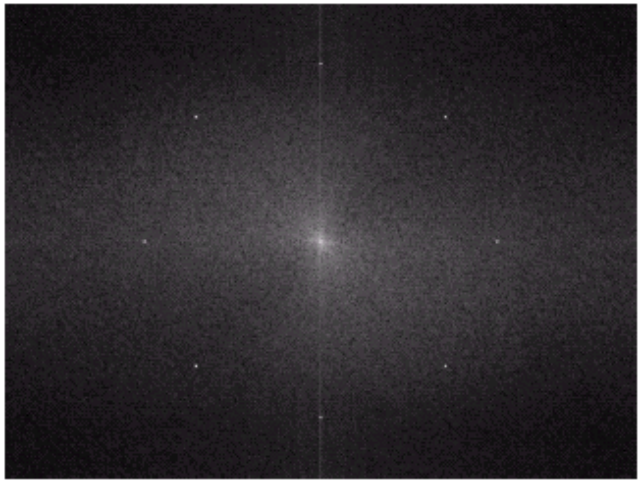
Butterworth
(order 1)

Gaussian

Applying bandreject filters

Corrupted by Sinusoidal noise

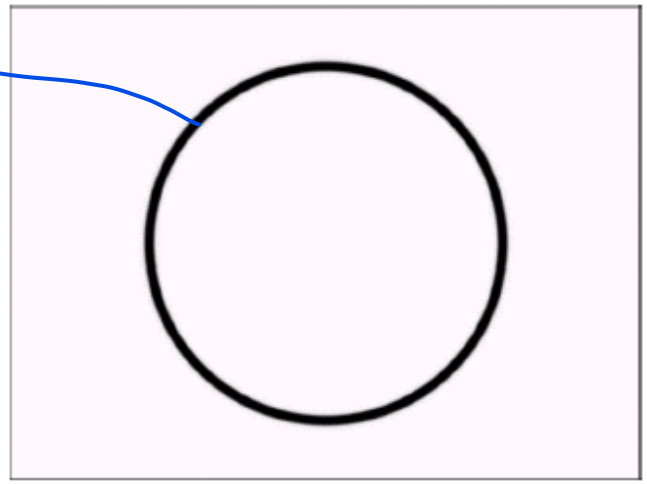




Spectrum

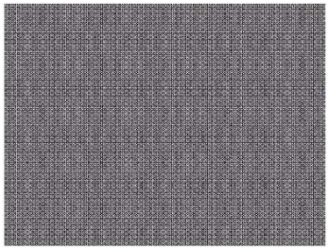
reject

Butterworth bandreject filter

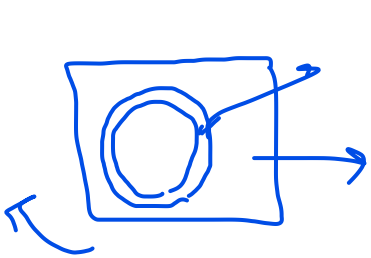


Result of filtering

Noise pattern obtained by bandpass filtering



تکمیل نویز

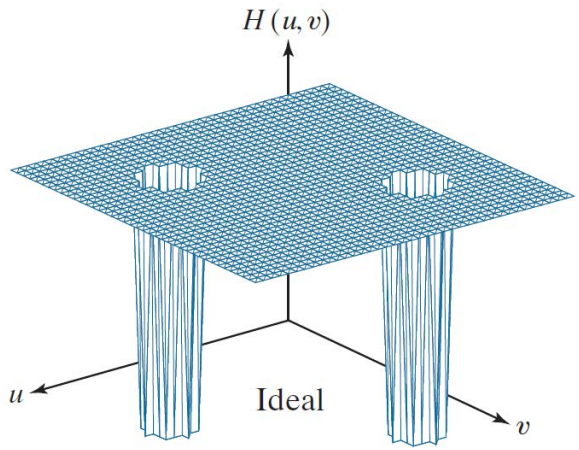


سفید

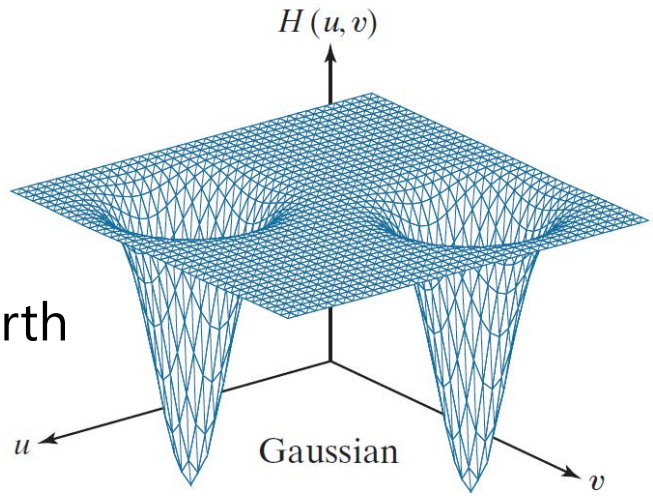
مشکی

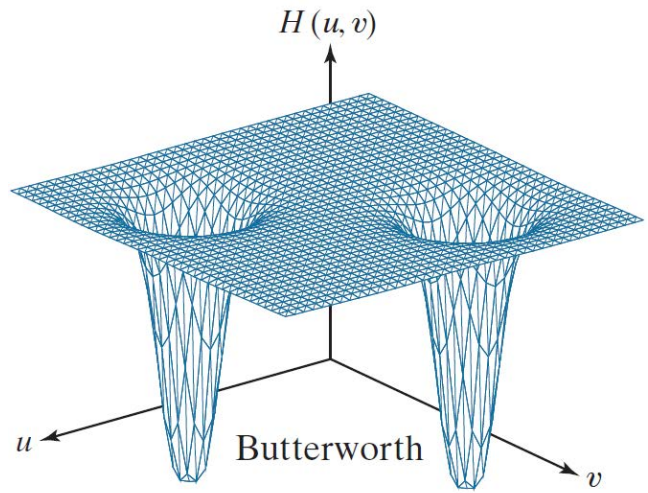
Notch filters

Ideal



Butterworth

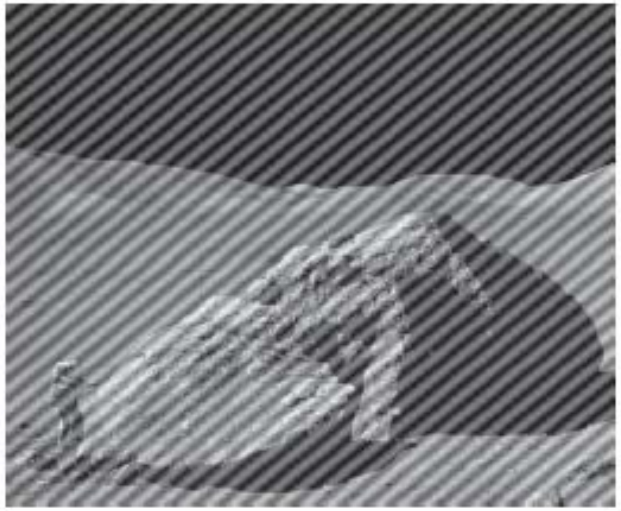




Gaussian

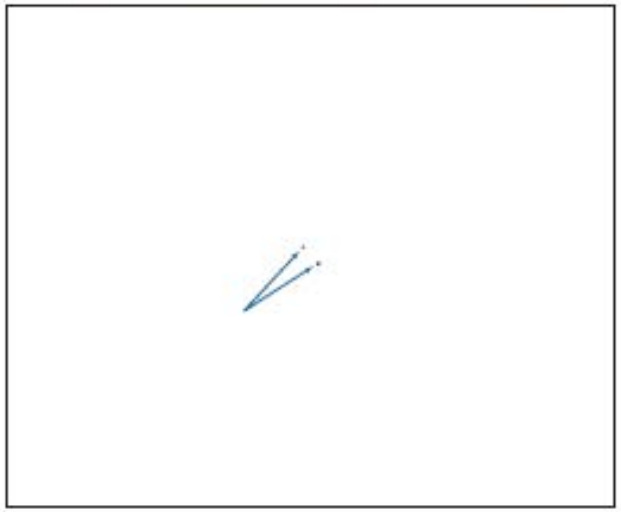
Applying bandreject filters

Corrupted by Sinusoidal noise



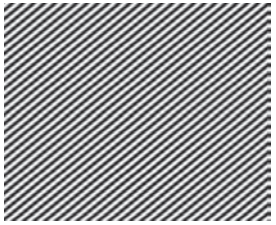
Spectrum

Notch filter



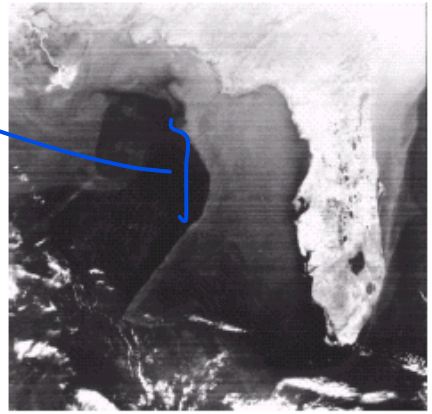
Result of filtering

Noise pattern obtained by Notch pass filtering



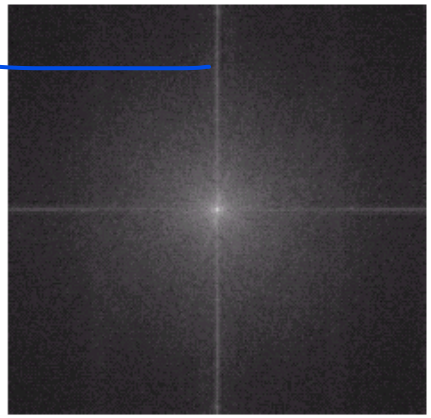
این خطوط افقی رو میشه در
حوزه فرکانس دید.

Original
with horizontal
sensor scan lines

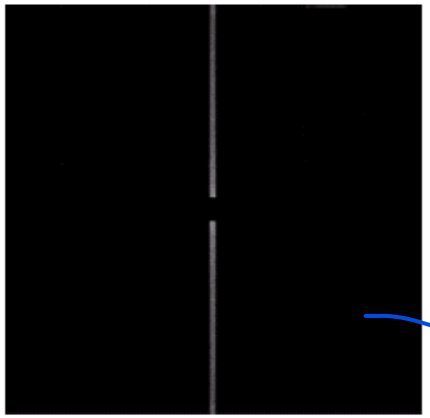


خطوط افقی هستند.

Spectrum



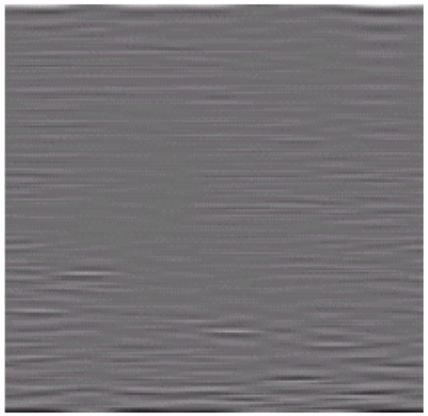
Notch pass filter



سیاه ← سفید
سفید ← سیاه

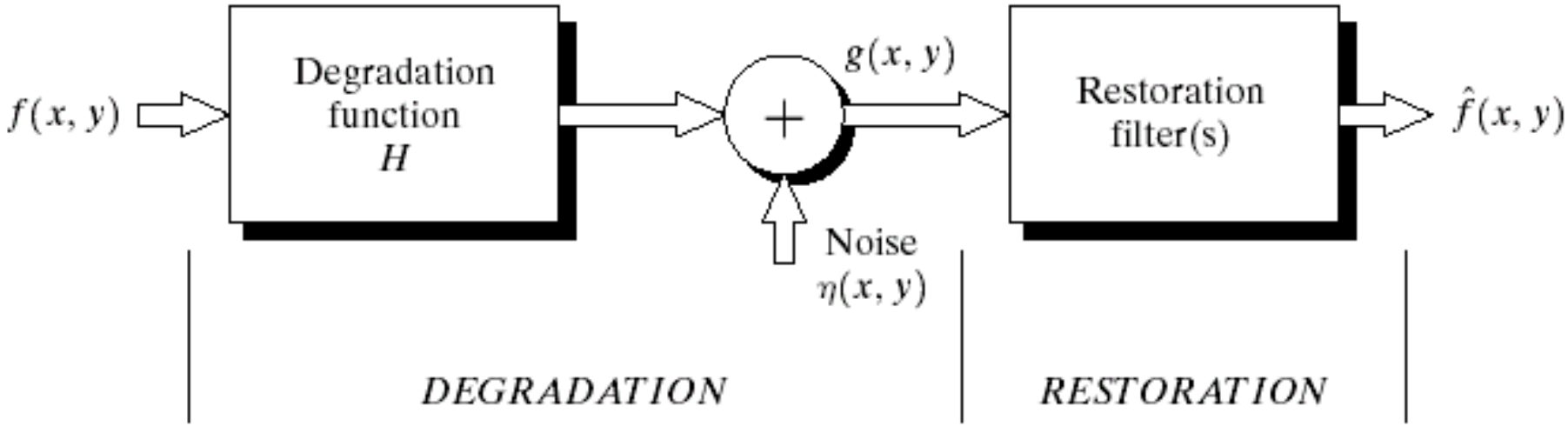
خط های
افقی رو
بهمون داده

IFT of
notch pass
filtered
image



Result of
notch reject
filtering

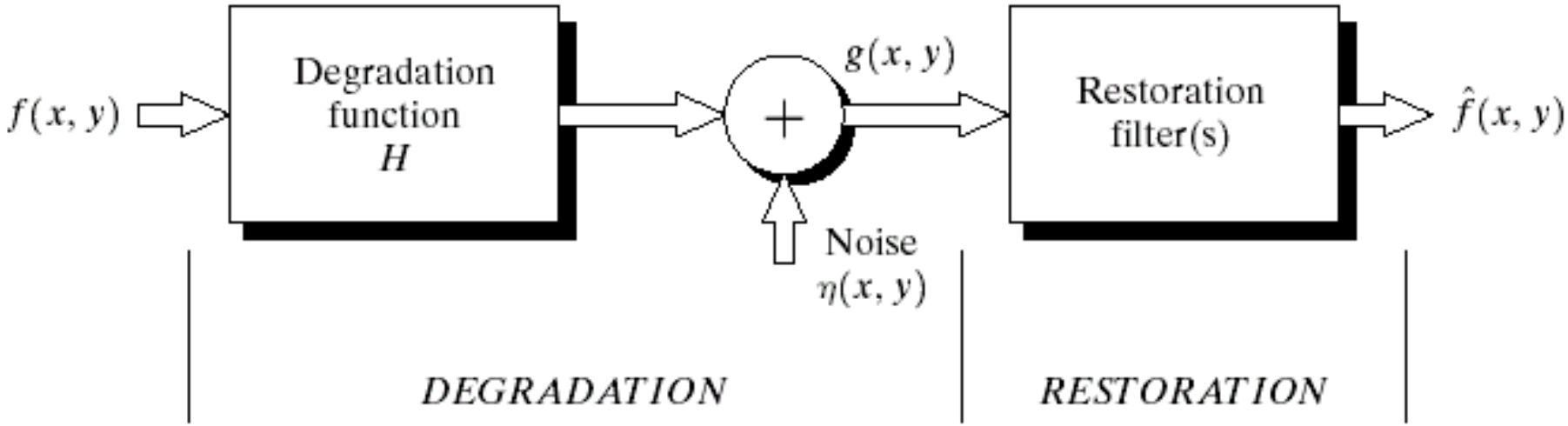
Linear, Position-Invariant Degradations



$$g(x, y) = H[f(x, y)] + \eta(x, y)$$

Linear (additivity and homogeneity):

Linear, Position-Invariant Degradations



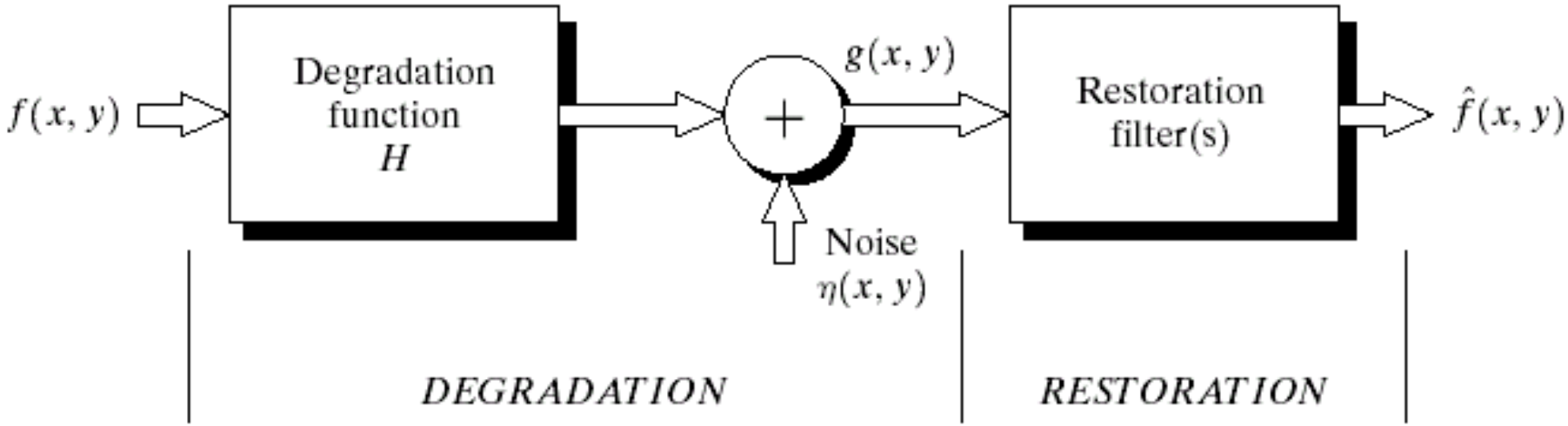
$$g(x,y) = H[f(x,y)] + \eta(x,y)$$

Linear (additivity and homogeneity):

$$H[af_1(x,y) + bf_2(x,y)] = aH[f_1(x,y)] + bH[f_2(x,y)]$$

Position invariant:

Linear, Position-Invariant Degradations



$$g(x,y) = H[f(x,y)] + \eta(x,y)$$

Linear (additivity and homogeneity):

$$H[af_1(x,y) + bf_2(x,y)] = aH[f_1(x,y)] + bH[f_2(x,y)]$$

Position invariant:

$$H[f(x-\alpha, y-\beta)] = g(x-\alpha, y-\beta)$$

$$f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)\delta(x-\alpha, y-\beta)d\alpha d\beta$$

$$g(x,y) = H[f(x,y)] = H\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)\delta(x-\alpha, y-\beta)d\alpha d\beta\right]$$

Additivity:

$$g(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} H[f(\alpha,\beta)\delta(x-\alpha, y-\beta)]d\alpha d\beta$$

Homogeneity:

$$g(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)H[\delta(x-\alpha, y-\beta)]d\alpha d\beta$$

$h(x,\alpha,y,\beta) = H[\delta(x-\alpha, y-\beta)] \longrightarrow$ Point Spread Function (PSF)

$$g(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)h(x,\alpha,y,\beta)\, d\alpha d\beta$$

PI:

$$H[\delta(x-\alpha, y-\beta)] = h(x-\alpha, y-\beta)$$

$$g(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)h(x-\alpha, y-\beta)d\alpha d\beta$$

In presence of additive noise:

$$g(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)h(x,\alpha,y,\beta)d\alpha d\beta + \eta(x,y)$$

PI:

$$g(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)h(x-\alpha, y-\beta)d\alpha d\beta + \eta(x,y)$$

Estimating the degradation function

1) By image observation:

Processing, e.g., sharpening, a subimage $g_s(x, y)$ with high contrast $\hat{f}_s(x, y)$

Estimating the degradation function

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Processing, e.g., sharpening, a subimage $g_s(x, y)$ with high contrast $\hat{f}_s(x, y)$

$$H_s(u, v) = \frac{G_s(u, v)}{\hat{F}_s(u, v)}$$

2) By experimentation:

Obtaining the impulse response, e.g., when the imaging device is available:

از یه جای نورانی عکس می‌گیریم و impulse رو بدست
میاریم که می‌شه پاسخ ضربه، بعد اینو کانوالو می‌کنیم با تصویر
و می‌تونیم بفهمیم اگر این تصویر اینو آل رو با این دوربین بگیریم چی بمون میده

تقریب Estimating the degradation function

1) By image observation:

Processing, e.g., sharpening, a subimage $g_s(x, y)$ with high contrast $\hat{f}_s(x, y)$

$$H_s(u, v) = \frac{G_s(u, v)}{\hat{F}_s(u, v)}$$

2) By experimentation:

Obtaining the impulse response, e.g., when the imaging device is available:

$$H(u, v) = \frac{G(u, v)}{A}$$

دامنه نقطه نور

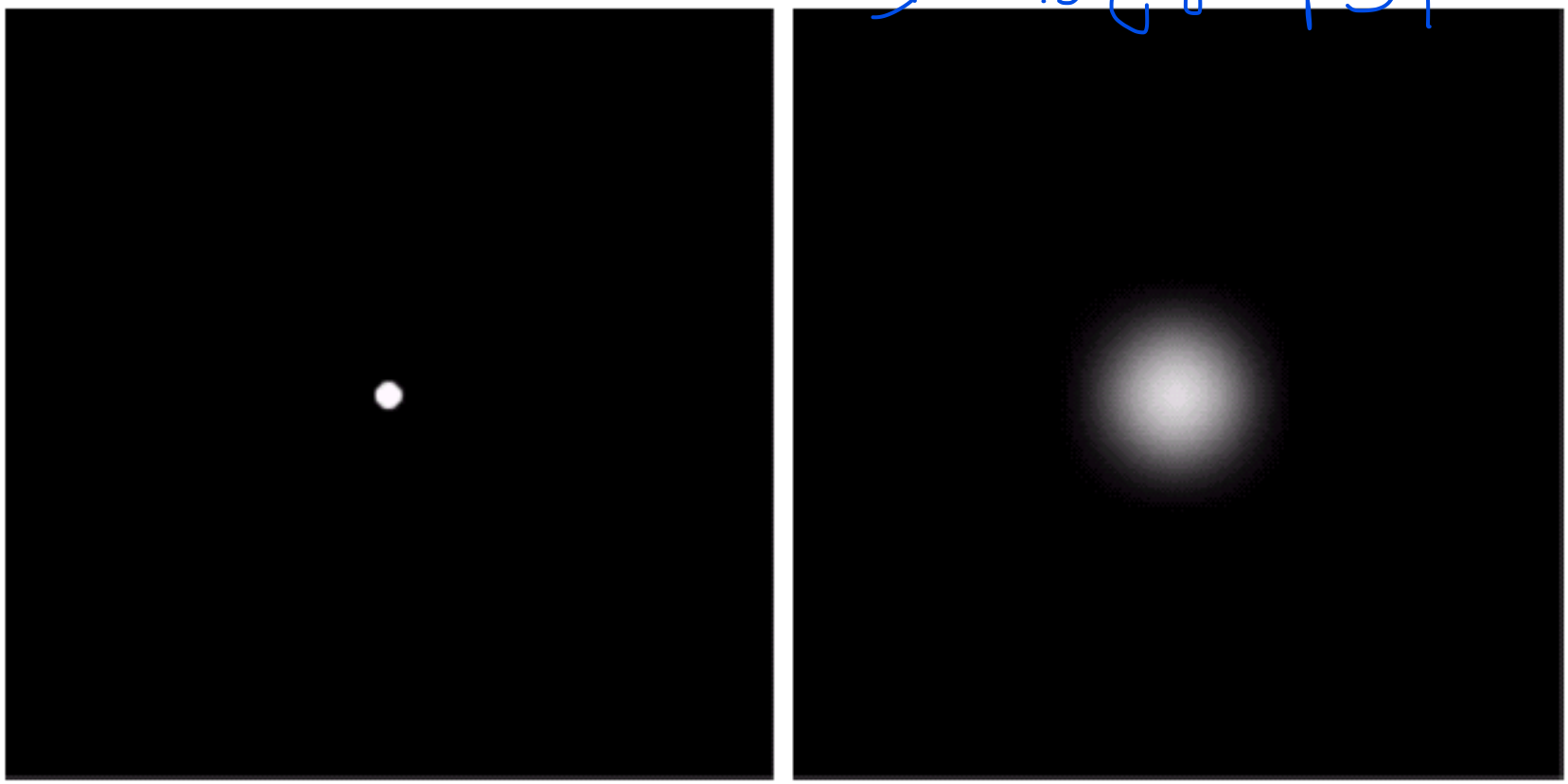
3) By modelling:

Atmospheric turbulence:

$$H(u, v) = e^{-k(u^2+v^2)^{5/6}}$$

کاربهای تجربیات

Estimating the degradation function



Impulse

Degraded impulse

By experimentation

Estimating the degradation function

Atmospheric turbulence

Negligible

Severe
k=0.0025

Mild
k=0.001

Low
k=0.00025

By modelling

...3) By modelling:

Motion blur:

$$g(x,y) = \int_0^T f[x - x_0(t), y - y_0(t)]\, dt$$

T is the duration of the exposure

$$G(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y) e^{-j2\pi(ux+vy)}\, dxdy$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left[\int_0^T f[x - x_0(t), y - y_0(t)] dt\right] e^{-j2\pi(ux+vy)} dxdy$$

$$G(u,v) = \int_0^T \left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f[x - x_0(t), y - y_0(t)] e^{-j2\pi(ux+vy)} dxdy\right] dt$$

$$G(u,v) = \int_0^T F(u,v) e^{-j2\pi[ux_0(t)+vy_0(t)]} dt$$

$$= F(u,v) \int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]} dt$$

$$H(u,v) = \int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]}\, dt$$

Example: $x_0(t) = {}^{at}/_{T}$

$$H(u,v) = \int_0^T e^{-j2\pi u x_0(t)} dt$$

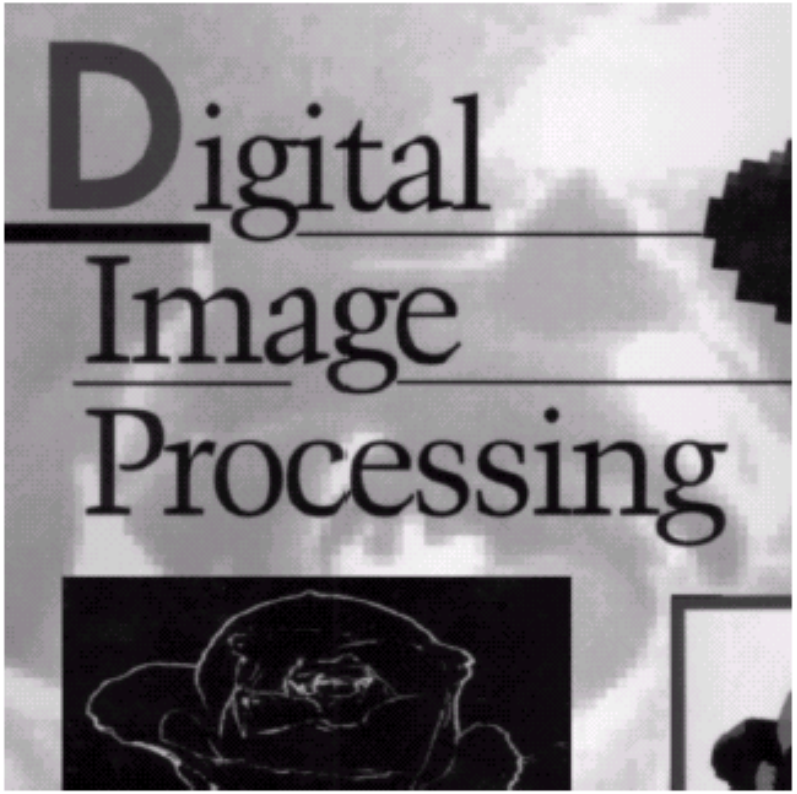
$$= \int_0^T e^{-j2\pi u a t/T} dt$$

$$= \frac{T}{\pi u a} \sin(\pi u a) e^{-j\pi u a}$$

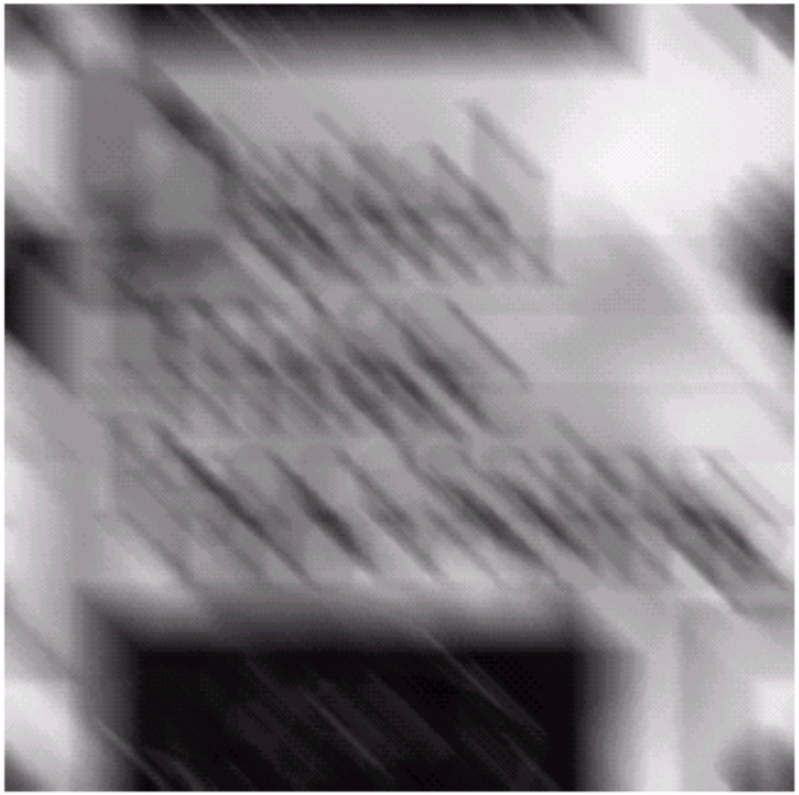
Example: $x_0(t) = {}^{at}/_{T}$ $\quad y_0 = {}^{bt}/_{T}$

$$H(u,v) = \frac{T}{\pi(ua+vb)} \sin[\pi(ua+vb)] e^{-j\pi(ua+vb)}$$

Motion blur



Original



$a=b=1$ and $T=1$

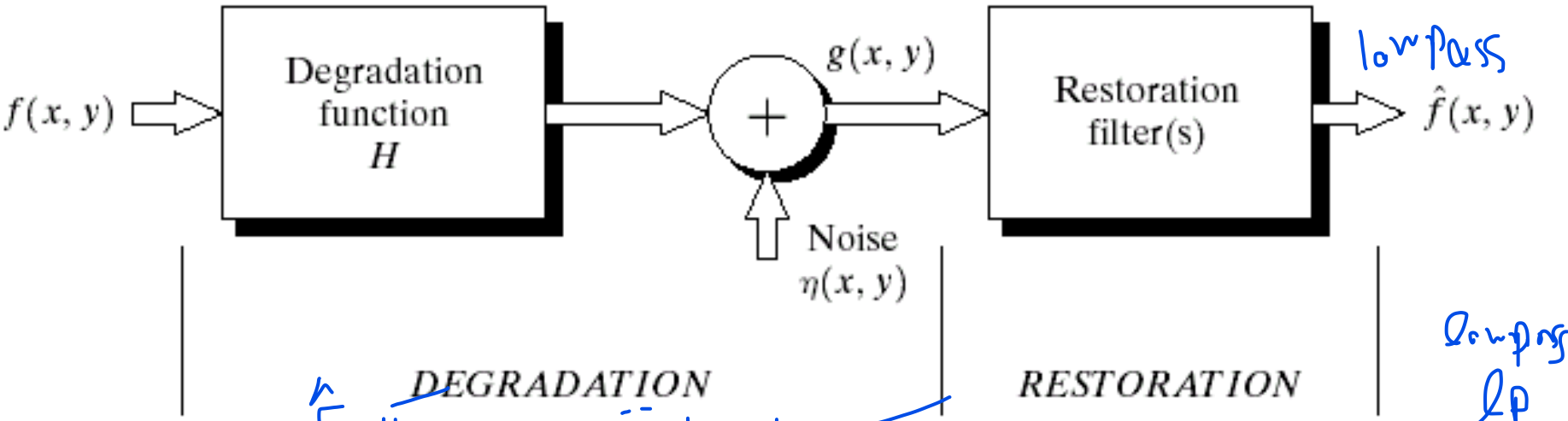
Inverse filtering

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)}$$

در شرایط نویز بنا به

Inverse filtering

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)}$$



$\hat{F}$ را به $1/H$ تقسیم کنیم. بعد از فرکانس بالا رو حذف کنیم.

$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

$$\hat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

Possible solution:
Limiting the frequency range

$\frac{0}{0} = \infty$

نویز تقویت می‌شه

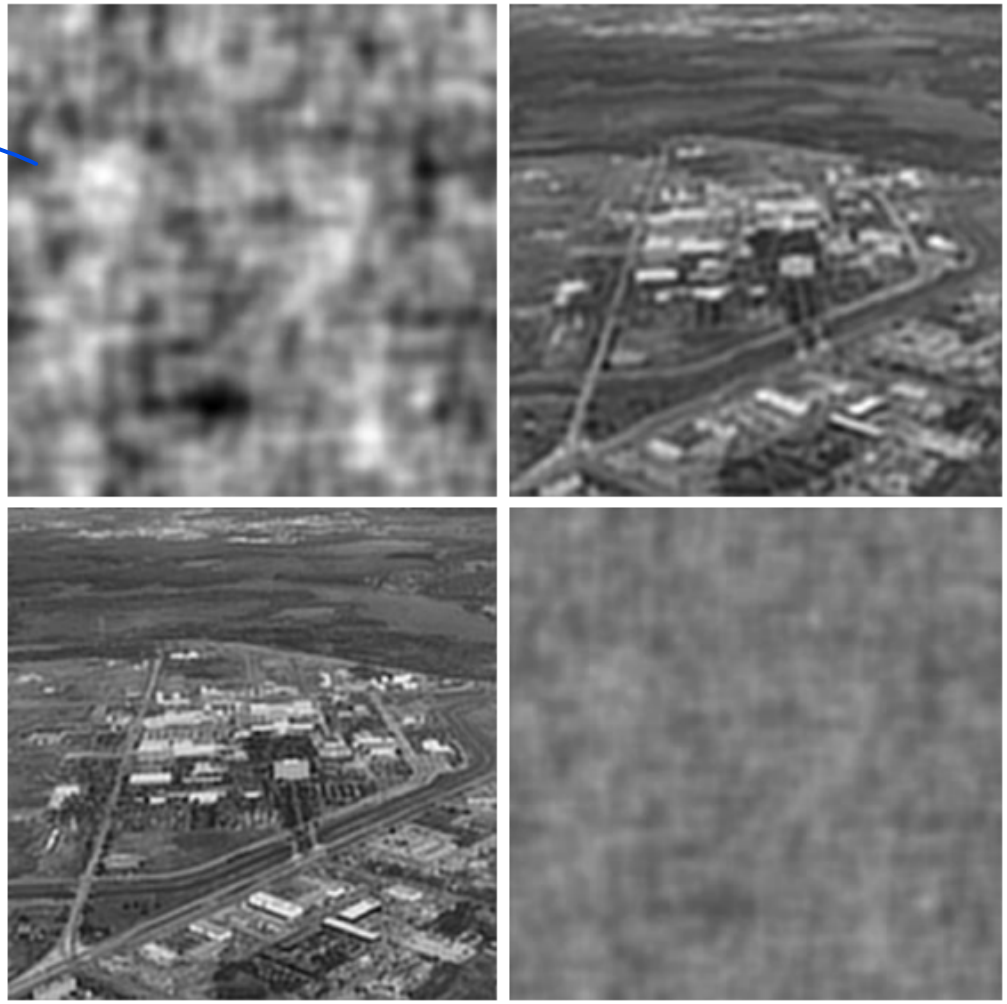
اگر مراتع ها هست پس تو

Inverse filtering

$$H(u,v) = e^{-k[(u-M/2)^2+(v-N/2)^2]^{5/6}}$$

$\frac{1}{H}$ بزنیم بجای زهم

Using the full filter



Using a BLP of order 10

Cut-off outside the radius of 40 (degraded)

low pass

Using a BLP of order 10

Cut-off outside the radius of 70

tradeoff

Using a BLP of order 10

Cut-off outside the radius of 85 (noisy)

hipass

Minimum Mean Square Error (MMSE) or Wiener Filtering

$$e^2 = E\{(f - \hat{f})^2\}$$

$$\hat{F}(u,v) = \overbrace{\left[\frac{H^*(u,v)S_f(u,v)}{S_f(u,v)|H(u,v)|^2 + S_\eta(u,v)}\right]}^{H_w} G(u,v)$$

$$= \left[\frac{H^*(u,v)}{|H(u,v)|^2 + {S_\eta(u,v)}/{S_f(u,v)}}\right] G(u,v)$$

$$= \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + {S_\eta(u,v)}/{S_f(u,v)}}\right] G(u,v)$$

$H(u,v)$ = degradation function

$H^*(u,v)$ = complex conjugate of $H(u,v)$

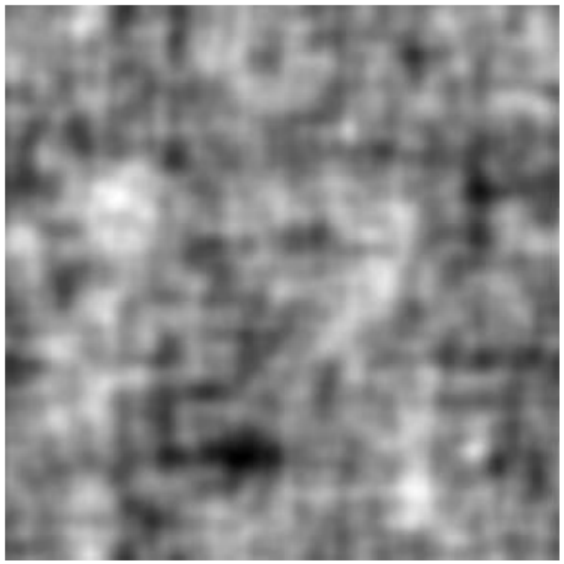
$|H(u,v)|^2 = H^*(u,v)H(u,v)$

$S_\eta(u,v)$= Power spectrum of the noise

$S_f(u,v)$= Power spectrum of the undegraded image

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + K}\right] G(u,v)$$

Minimum Mean Square Error (MMSE) or Wiener Filtering



Inverse filtering

Radially limited inverse filtering

Wiener filtering

بهترین

Corrupted by motion blur and additive noise

Inverse filtering

Wiener filtering

noise variance one order of magnitude smaller →

noise variance five orders of magnitude smaller than original →