Algorithm 2 Merge incremental matrix with decomposition results, $(U,\ \Sigma,\ V) = mix(M_{n-1},\ C_{n-1},\ U_m,\ \Sigma_m,\ V_m)$.

Input:

Initial matrix M_{n-1} and incremental matrix C_{n-1}.

Decomposition results $U_m,\ \Sigma_m,\ V_m$ of matrix M.

Output:

New decomposition results $U,\ \Sigma,\ V$.

1: Project C_{n-1} on the orthogonal space spanned by U_m, $L = U_m^{\mathrm{T}} \times C_{n-1}$.

2: Compute H which is orthogonal to U_m, $H = C_{n-1} - U_m \times L$.

3: Obtain the unitary orthogonal basis J from matrix H.

4: Compute the coordinates of matrix H, $K = J^{\mathrm{T}} \times H$.

5: Execute SVD on the new matrix $[U\ J]$, $[U',\ \Sigma',\ V'] = svd([U\ J])$.

6: Obtain new decomposition results, $([U\ \ J],\ \ U') \rightarrow U,\ \Sigma' \rightarrow \Sigma,\ V' \rightarrow V$.

7: **return** $U,\ S,\ V$.