Assignment 3

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Contents

Contents **1**

Initial Explanation Regarding the Data **3**

Signal Separation using Singular Value Decomposition (SVD) **9**

Separation of Sources Using ICA **15**
Question 1: Perform ICA and Save Results 15
Question 2: Plot Mixed Signal and Component Vectors 15
Question 3: Plot the Extracted Components 17
Question 4: Plot the Reconstructed Signal 18
Conclusion 20

Comparisons Between ICA and SVD **21**
Question 1: Scatter Plots and Mixing Matrix Directions 21
Question 2: Signal Comparison between SVD and ICA 25
Question 3: Correlation Coefficient and Angle Comparison 26
Question 4: Final Comparison and Analysis of SVD and ICA 27
Question 5: Key Takeaways 27

In this experiment, you will separate the components of the heart signal (ECG) for the mother (Figure 1(ii)), fetus (Figure 1(iv)), and also the noise (Figure 1(iii)) using Blind Source Separation (BSS) techniques from the initial mixed signal (Figure 1(i)). More specifically, you will apply two methods: Principal Component Analysis (PCA) and Independent Component Analysis (ICA), using Singular Value Decomposition (SVD), to achieve this separation.

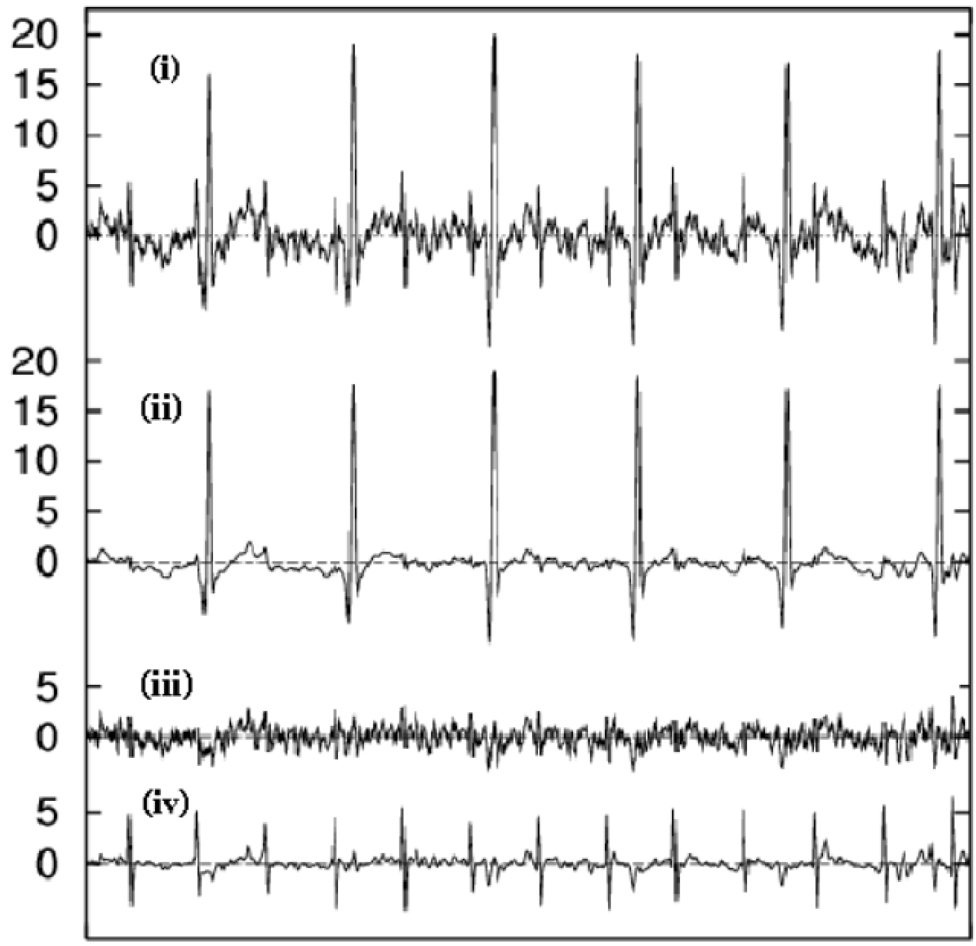

Figure 1: From top to bottom: (i) Mixed ECG signal of the mother and fetus, (ii) Mother ECG signal, (iii) Noise, and (iv) Fetal ECG signal. A 5-second window of these signals has been plotted.

The first method (SVD) only uses second-order statistics, whereas the second method (ICA) utilizes higher-order statistics. In this experiment, you will observe how considering the probabilistic distribution of components will impact the performance quality of these two methods.

In practical applications, one of the serious challenges for evaluating the performance of source separation methods is our lack of knowledge about how the sources are generated and their statistical properties. Therefore, in this experiment, you will use artificially generated ECG artifacts and noise signals, which will allow you to evaluate the quality of the separation in the end.

Initial Explanation Regarding the Data

The ECG signals of the mother and fetus, as well as the noise signal, are provided in the form of the files ‘mceg1.dat‘, ‘fecg1.dat‘, and ‘noise1.dat‘. You can read them in MATLAB using the ‘load‘ command. Each of these files contains a vector named ‘recorded‘ with units of mV and a sampling frequency of 256 Hz.

You will combine these three signals to generate a new signal, which will approximate a real-world signal. Answer the following questions based on the new signal:

1. Plot the time-domain signals of the mother ECG, fetus ECG, noise, and the mixed signal (simulation).

```
% Assume that the loaded data is stored in a variable called 'recorded'
ecg_mother = load('Lab3_data\data\mecg1.dat');  % Mother ECG signal
ecg_fetus = load('Lab3_data\data\fecg1.dat');  % Fetus ECG signal
noise = load('Lab3_data\data\noise1.dat'); % Noise signal

% Combine the signals to create the mixed signal
mixed_signal = ecg_mother + ecg_fetus + noise;

% Smapling Frequency
fs = 256;

% Create the time axis (assuming a sampling frequency of 256 Hz)
t = (0:length(ecg_mother)-1) / fs;

% Plot the mother ECG signal
subplot(4,1,1);
plot(t, ecg_mother);
title('Mother ECG Signal');
xlabel('Time (seconds)');
ylabel('Voltage (mV)');

% Plot the fetus ECG signal
subplot(4,1,2);
plot(t, ecg_fetus);
title('Fetus ECG Signal');
xlabel('Time (seconds)');
ylabel('Voltage (mV)');

% Plot the noise signal
subplot(4,1,3);
plot(t, noise);
title('Noise Signal');
xlabel('Time (seconds)');
ylabel('Voltage (mV)');

% Plot the mixed signal
subplot(4,1,4);
plot(t, mixed_signal);
title('Mixed Signal');
xlabel('Time (seconds)');
ylabel('Voltage (mV)');
```

Source Code 1: EEG - Question 1: Clean and Noisy Signal Plot

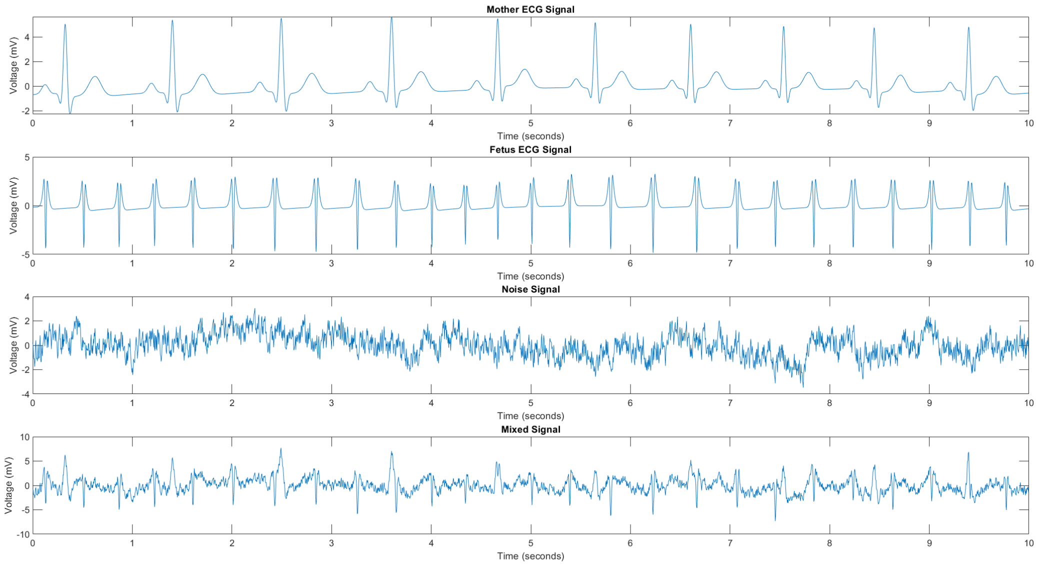

Figure 2: Time-Domain Signals

2. Plot the power spectrum of these signals using the ‘pwelch‘ command. Is the frequency content of the mother and fetal ECG signals comparable? Briefly mention the similarities and differences.

```
% Plot the power spectrum of the mother ECG signal
figure;
subplot(4,1,1);
pwelch(ecg_mother, [], [], [], fs);
title('Power Spectrum of Mother ECG Signal');

% Plot the power spectrum of the fetus ECG signal
subplot(4,1,2);
pwelch(ecg_fetus, [], [], [], fs);
title('Power Spectrum of Fetus ECG Signal');

% Plot the power spectrum of the noise signal
subplot(4,1,3);
pwelch(noise, [], [], [], fs);
title('Power Spectrum of Noise Signal');

% Plot the power spectrum of the mixed signal
subplot(4,1,4);
pwelch(mixed_signal, [], [], [], fs);
title('Power Spectrum of Mixed Signal');
```

Source Code 2: EEG - Question 1: Clean and Noisy Signal Plot

Based on the power spectrum plots of the mother and fetus ECG signals, here are the observations and comparisons of their frequency contents:
Similarities

- **Low-frequency dominance**: Both the mother and fetus ECG signals have significant power concentrated in the lower frequency range, below approximately 30 Hz. This is typical of ECG signals since the heartbeats primarily contribute to lower frequencies.

- **Gradual power decline**: For both signals, as the frequency increases beyond this range (around 30 Hz and above), the power in both spectra gradually decreases, showing less significant content at higher frequencies.

Differences

- **Fetus ECG power distribution**: The fetus ECG signal appears to have more pronounced peaks and variations within the 0 to 30 Hz range, indicating possible stronger contributions from specific frequencies or periodicities compared to the mother ECG.

- **Smoother mother ECG spectrum**: The mother's ECG spectrum shows a smoother and more gradual decline in power, suggesting that its frequency content is more uniform across the low-frequency range compared to the fetus ECG.

- **High-frequency content**: While both signals taper off beyond 30 Hz, the fetus ECG signal shows a bit more activity in the higher frequency ranges (between 30 Hz to 60 Hz) compared to the mother ECG signal, although the power levels are still relatively low.

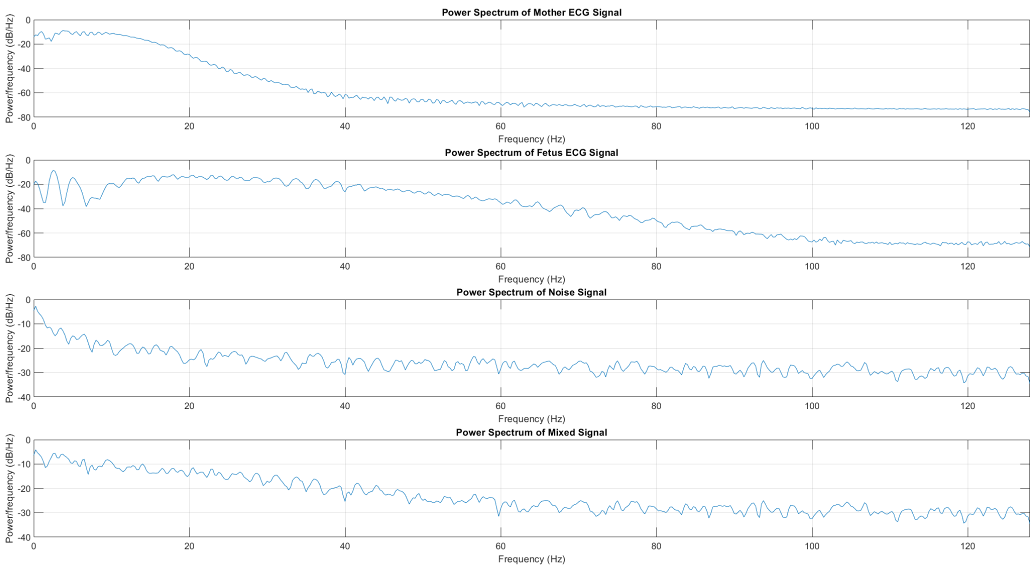

Figure 3: Power Spectrum of the Signals

3. Calculate and report the mean and variance of the three signals. Are they comparable?

Does the variance have any specific relationship with the frequency content of the signals?

```
% Calculate the mean of the signals
mean_mother = mean(ecg_mother);
mean_fetus = mean(ecg_fetus);
mean_noise = mean(noise);
mean_mixed = mean(mixed_signal);

% Calculate the variance of the signals
var_mother = var(ecg_mother);
var_fetus = var(ecg_fetus);
var_noise = var(noise);
var_mixed = var(mixed_signal);

% Display the results without rounding
disp('Mean and Variance of Signals:');
fprintf('Mother ECG - Mean: %g, Variance: %g\n', mean_mother, var_mother);
fprintf('Fetus ECG - Mean: %g, Variance: %g\n', mean_fetus, var_fetus);
fprintf('Noise - Mean: %g, Variance: %g\n', mean_noise, var_noise);
fprintf('Mixed Signal - Mean: %g, Variance: %g\n', mean_mixed, var_mixed);
```

Source Code 3: EEG - Question 1: Clean and Noisy Signal Plot

Signal	**Mean**	**Variance**
Mother ECG	-2.46617e-10	1
Fetus ECG	-4.25004e-10	1
Noise	-4.7691e-10	1
Mixed Signal	-1.14853e-09	2.84744

Table 1: Mean and Variance of Signals

The variances of the mother ECG, fetus ECG, and noise signals are all identical (1.0000). This shows that these signals contribute equally in terms of their energy or spread. However, the mixed signal has a higher variance of 2.8474 due to the combined contributions of the mother, fetus, and noise components.
Variance is a measure of the signal's energy or spread in the time domain, not directly related to the specific frequency content. Signals with higher power across different frequency ranges tend to have higher variance. In this case, the mixed signal's variance is higher because it combines the power of the mother, fetus, and noise signals.
However, variance does not specify where the power is concentrated in the frequency spectrum. To understand the relationship with frequency content, one would need to analyze the power spectral density (PSD) of the signals rather than just the variance.

4. Compute and plot the histogram of these signals using the 'hist' command. In your report, calculate the fourth moment (kurtosis) of the signals using the 'kurtosis' command and report the results. Can you determine the Gaussian nature of their PDFs based on the kurtosis values?

```
% Plot the histograms
figure;
```

```matlab

    % Histogram of the mother ECG signal
    subplot(4,1,1);
    hist(ecg_mother, 50);  % 50 bins for the histogram
    title('Histogram of Mother ECG Signal');
    xlabel('Amplitude');
    ylabel('Frequency');

    % Histogram of the fetus ECG signal
    subplot(4,1,2);
    hist(ecg_fetus, 50);  % 50 bins for the histogram
    title('Histogram of Fetus ECG Signal');
    xlabel('Amplitude');
    ylabel('Frequency');

    % Histogram of the noise signal
    subplot(4,1,3);
    hist(noise, 50);  % 50 bins for the histogram
    title('Histogram of Noise Signal');
    xlabel('Amplitude');
    ylabel('Frequency');

    % Histogram of the mixed signal
    subplot(4,1,4);
    hist(mixed_signal, 50);  % 50 bins for the histogram
    title('Histogram of Mixed Signal');
    xlabel('Amplitude');
    ylabel('Frequency');

    % Calculate the kurtosis (4th moment) of the signals
    kurt_mother = kurtosis(ecg_mother);
    kurt_fetus = kurtosis(ecg_fetus);
    kurt_noise = kurtosis(noise);
    kurt_mixed = kurtosis(mixed_signal);

    % Display the results
    disp('Kurtosis (4th Moment) of Signals:');
    fprintf('Mother ECG: %.4f\n', kurt_mother);
    fprintf('Fetus ECG: %.4f\n', kurt_fetus);
    fprintf('Noise: %.4f\n', kurt_noise);
    fprintf('Mixed Signal: %.4f\n', kurt_mixed);
```

Source Code 4: EEG - Question 1: Clean and Noisy Signal Plot

Signal	**Kurtosis (4th Moment)**	**Gaussian Nature**
Mother ECG	14.0421	Leptokurtic (Heavy Tails)
Fetus ECG	8.9901	Leptokurtic (Heavy Tails)
Noise	2.7662	Near Gaussian (Light Tails)
Mixed Signal	4.6085	Slightly Leptokurtic (Heavy Tails)

Table 2: Kurtosis values and interpretation of Gaussian nature of signals

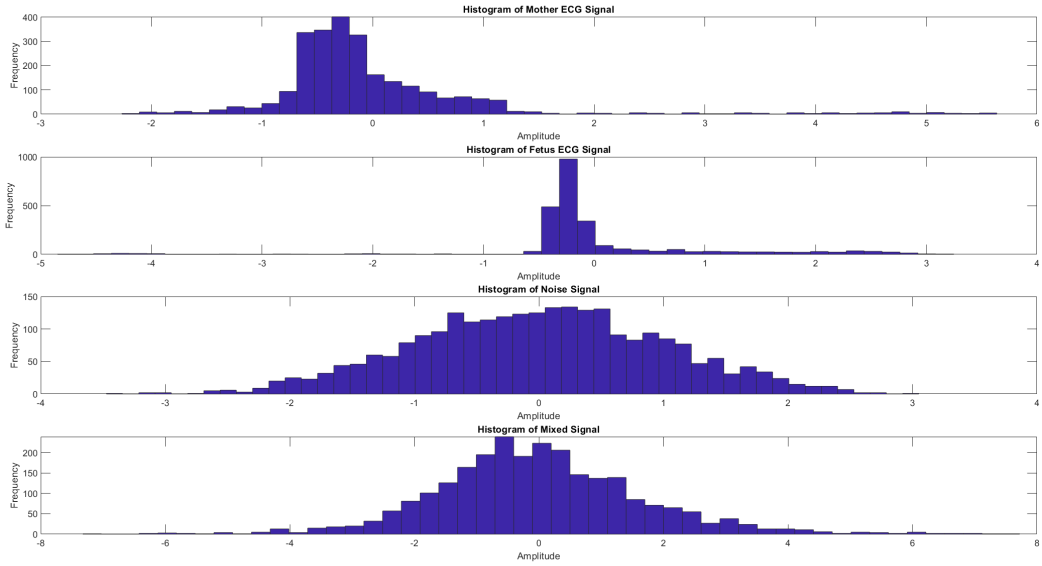

Figure 4: Histograms of the Signals

The kurtosis of a signal is a measure of how heavy or light the tails of the probability distribution function (PDF) are, compared to a Gaussian distribution. A Gaussian distribution has a kurtosis value of 3. Here's how the kurtosis values relate to the Gaussian nature of the signals:

- **Mother ECG**: With a kurtosis of 14.0421, this signal has a much higher value than 3, indicating a leptokurtic distribution. This suggests that the mother ECG signal has heavy tails, meaning more frequent extreme values compared to a Gaussian distribution.
- **Fetus ECG**: The kurtosis value of 8.9901 also indicates a leptokurtic distribution. The fetus ECG signal exhibits heavier tails than a Gaussian distribution but less extreme than the mother ECG signal.
- **Noise**: With a kurtosis of 2.7662, the noise signal is close to the Gaussian reference value of 3, suggesting that it is approximately Gaussian, though slightly platykurtic (lighter tails).
- **Mixed Signal**: A kurtosis of 4.6085 is higher than 3, indicating a slight leptokurtic nature for the mixed signal, meaning its tails are heavier than those of a Gaussian distribution, but not as extreme as the mother or fetus ECG signals.

Signal Separation using Singular Value Decomposition (SVD)

One of the standard and powerful techniques based on second-order statistics for filtering data is Singular Value Decomposition (SVD). In this method, data are projected onto orthogonal axes that correspond to the maximum variance in the data, which can be learned from the data itself (data-driven learning). Initially, the level of variance for each component is determined.

In this section, we will use SVD to analyze the variance content of the components that are separable.

1. Read the data from the file `X.dat` using the `plot3ch` command, which is provided to you. In this file, data from five channels (mother ECG, fetus ECG, noise, and two other channels) is included, but your focus will be on the mixed ECG signal channel recorded over three years. You can access this mixed signal channel, and no further access is required for the other channels. The `plot3ch` command displays the data in two forms. First, the signal is displayed in the time domain. Second, it provides a scatter plot, showing the correlations between the channels. You can use this information to identify specific components like QRS complexes or peak ECG events.

```
% Load the data from dat.X (assuming it's a .mat file)
X = load('Lab3_data\data\X.dat');  % This loads the data into the workspace

% Call the custom plot function 'ch3plot' with the loaded data
plot3ch(X);

% Perform Singular Value Decomposition (SVD)
[U, S, V] = svd(X);
```

Source Code 5: EEG - Question 1: Clean and Noisy Signal Plot

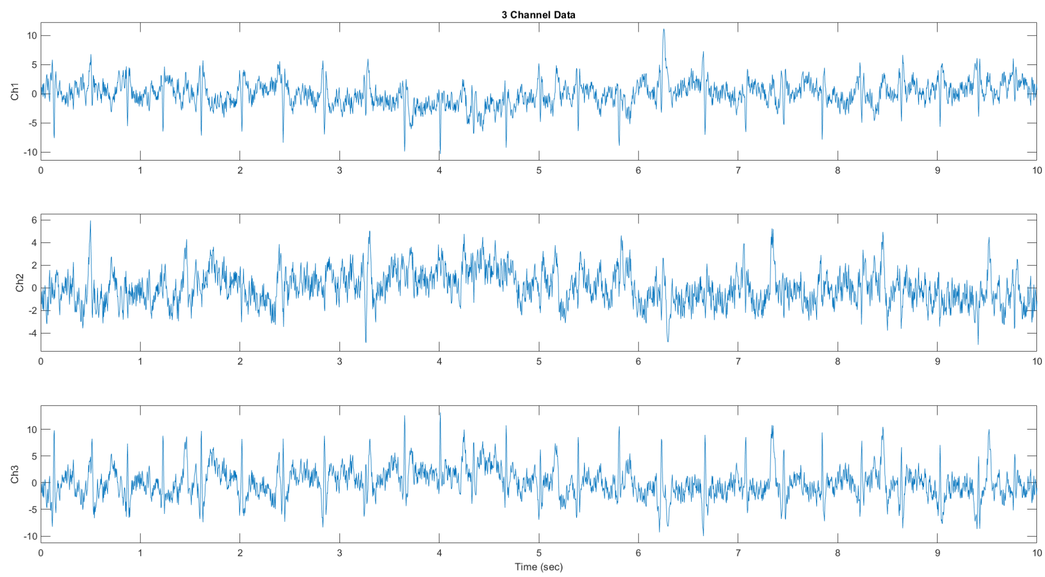

Figure 5: Channels Signals

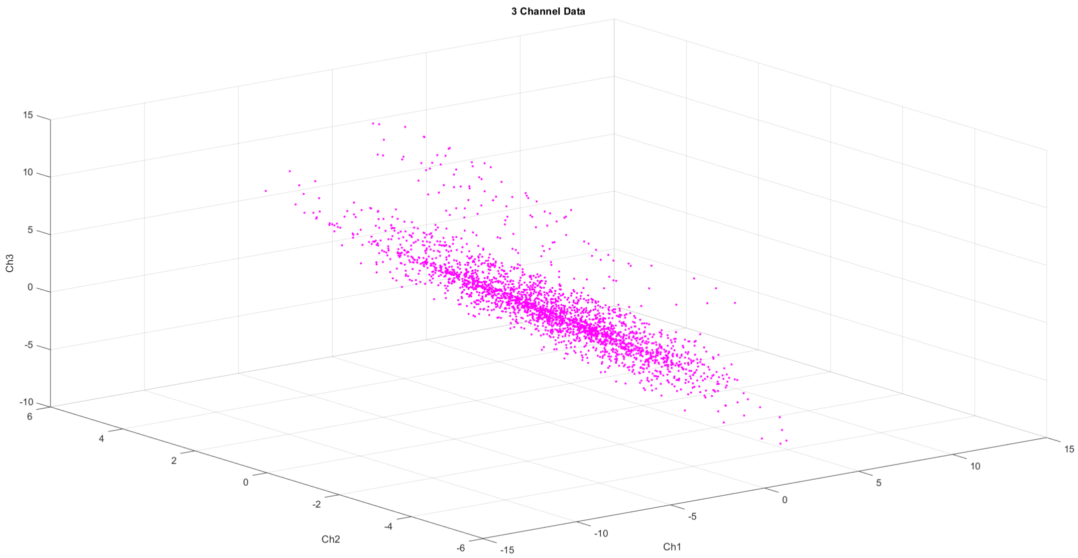

Figure 6: 3D data

2. The goal here is to apply SVD to separate the fetal ECG and mother ECG. The SVD decomposition is represented as:

$$Y = USV^T$$

where Y is the original data matrix $R^{M\times N}$, $U \in R^{M\times M}$, and $V \in R^{N\times N}$ represent the matrices of left and right singular vectors, and S is the diagonal matrix containing singular values. These singular values provide insight into how much each orthogonal component contributes to the data's variance.

Now, using the `plot3dv` command, visualize the singular values (eigenspectrum) to identify the dominant components.

```
% Call the custom plotting function dv3plot
for v = 1:length(V)
    plot3dv(V(:,v), S(:,v));
    hold on
end
```

Source Code 6: EEG - Question 1: Clean and Noisy Signal Plot

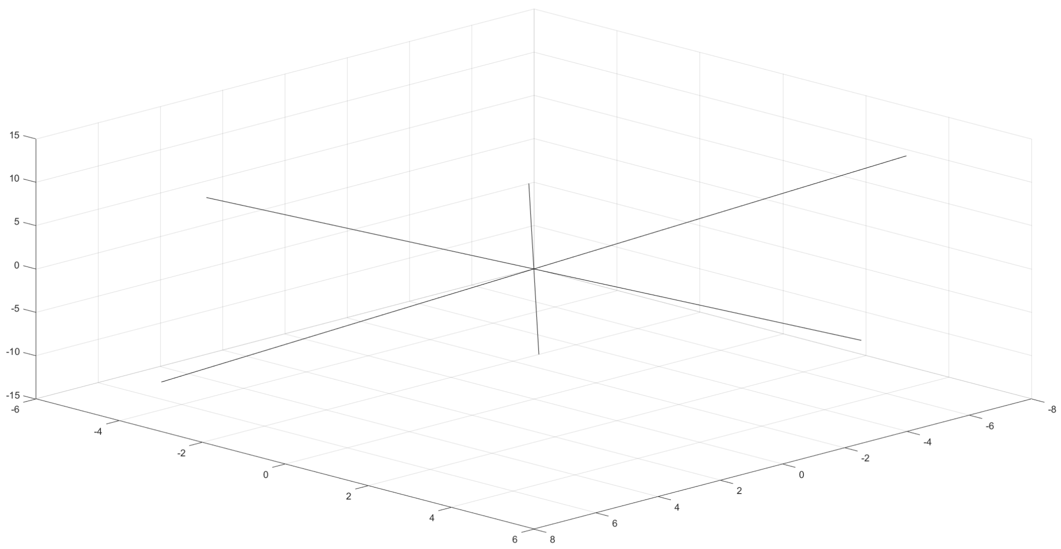

Figure 7: V matrix Columns

3. Plot the three leading columns of the U matrix using the `plot3dv` command, which takes the singular values as input and normalizes the components accordingly.

```
t = (0:length(U)-1) / fs;  % Time vector in seconds

% Plot the first three columns of matrix U
figure;
subplot(3,1,1);
plot(t, U(:,1)); % Plot the first column of U
title('First Column of U');
xlabel('Time (seconds)');
ylabel('Value');

subplot(3,1,2);
plot(t, U(:,2)); % Plot the second column of U
title('Second Column of U');
xlabel('Time (seconds)');
ylabel('Value');

subplot(3,1,3);
plot(t, U(:,3)); % Plot the third column of U
title('Third Column of U');
xlabel('Time (seconds)');
ylabel('Value');

% Extract the singular values from the diagonal of matrix S
singular_values = diag(S);

% Plot the eigenspectrum using the stem function
figure;
stem(singular_values, 'filled');
xlim([0,4]);
title('Eigenspectrum (Singular Values)');
xlabel('Index');
```

```
    ylabel('Singular Value');
```

Source Code 7: EEG - Question 1: Clean and Noisy Signal Plot

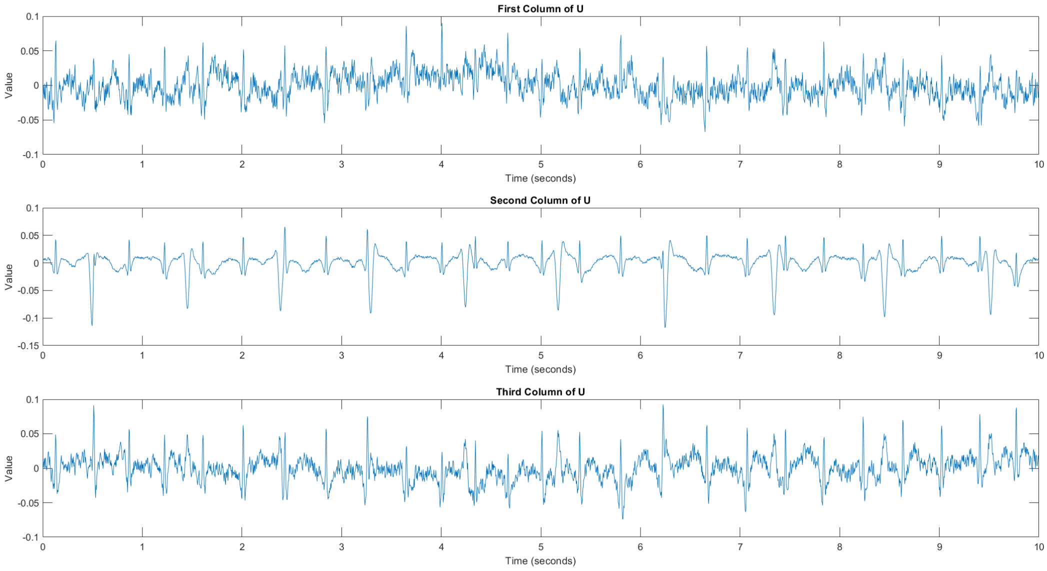

Figure 8: U matrix Columns

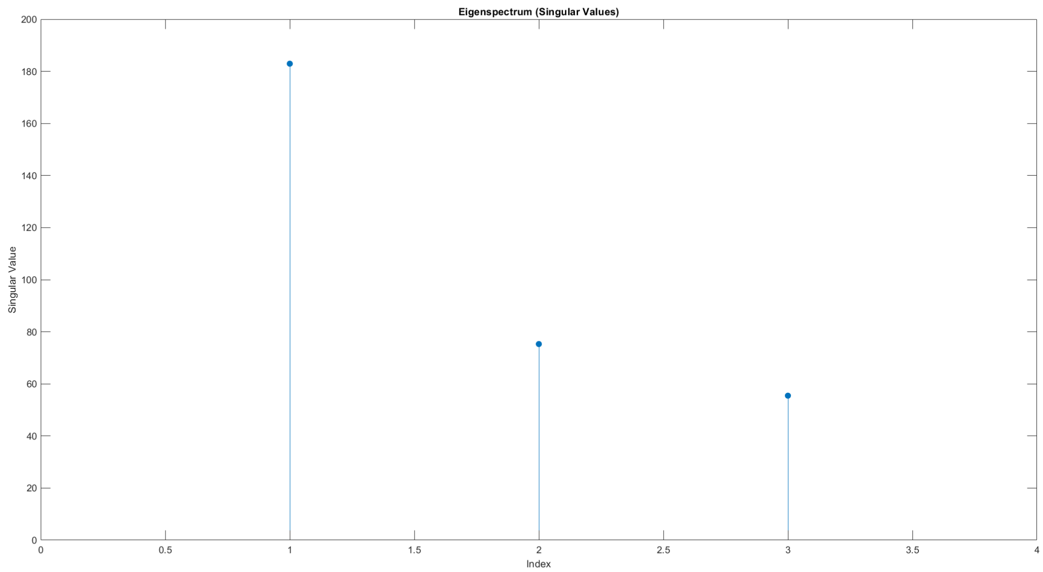

Figure 9: Eigenspectrum

The columns of the U matrix represent orthogonal components in the data space. Each column can be interpreted as capturing distinct features of the signals:

- The **first column of** U is likely to represent the Combination of noise and other signals.
- The **second column of** U likely corresponds to the Fetus ECG signal and mother signals.
- The **third column of** U is likely related to moise and other signals.

So we use second singular value.

4. Identify which components correspond to the mother ECG and fetus ECG signals. These signals should be identified based on their unique features (mother ECG, fetus ECG, and noise). Once identified, modify the matrix S to zero out the singular values corresponding to the mother ECG and noise components, keeping only the singular value related to the fetus ECG. Apply inverse SVD to reconstruct the fetal ECG signal. Finally, compare the result to determine whether the separation was successful.

```
 % Modify the singular values matrix S
 S_mod = zeros(size(S));   % Create a new matrix S_mod initialized to zeros
 S_mod(2,2) = S(2,2);      % Keep only the singular value for the fetus
component (index 3)

 % Reconstruct the matrix using the modified S matrix
 X_Reconstructed = U * S_mod * V';  % Reconstructed matrix with only the
fetal component

 % Plot the original mother ECG signal
 subplot(3,1,1);
 plot(t, X_Reconstructed(:,1));
 title('Reconstructed ECG Signal channel 1');
 xlabel('Index');
 ylabel('Amplitude');

 % Plot the original fetal ECG signal
 subplot(3,1,2);
 plot(t, X_Reconstructed(:,2));
 title('Reconstructed ECG Signal channel 2');
 xlabel('Index');
 ylabel('Amplitude');

 % Plot the reconstructed fetal ECG signal
 subplot(3,1,3);
 plot(t, X_Reconstructed(:,3));
 title('Reconstructed ECG Signal channel 3');
 xlabel('Index');
 ylabel('Amplitude');
```

Source Code 8: EEG - Question 1: Clean and Noisy Signal Plot

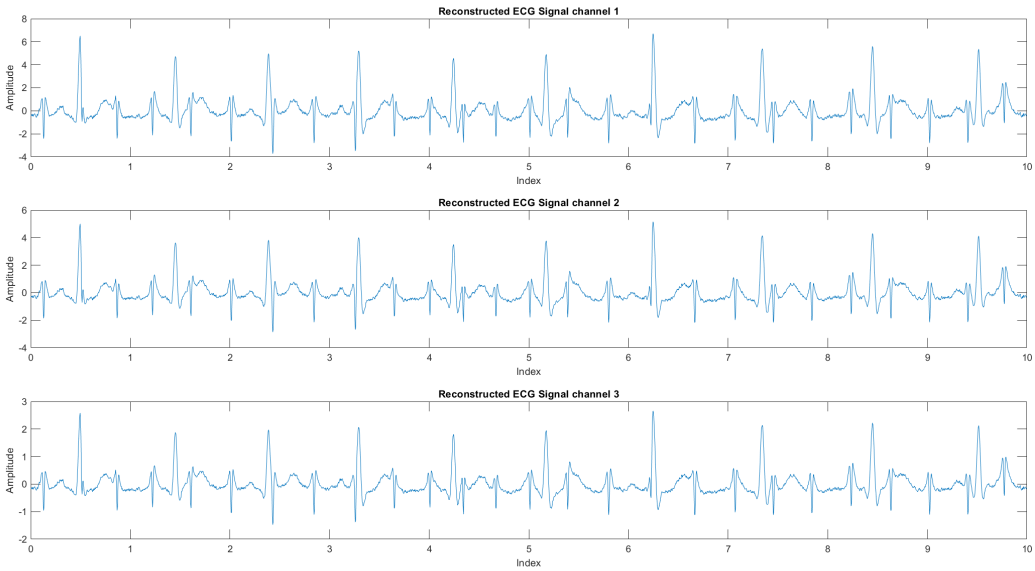

Figure 10: Reconstructed Signals

As we can see, SVD is not a good method for seperataing the mother and fetus ECG signal. However it can remove noise.

Separation of Sources Using ICA

In this section, we use a technique based on higher-order statistics called Independent Component Analysis (ICA) to separate the maternal, fetal, and noise signals. We aim to separate the mixed signals, which are composed of "sources $\times$ mixed matrix = observations", into their components. Mathematically, this relationship can be written as:

$$X^T = AZ^T$$

It should be noted that for visualizing the observation matrix, it is represented as `channels × time samples`, where N is the number of channels and M is the number of time samples ($X \in \mathbb{R}^{M\times N}$).

Thus, we can estimate the sources Z in the following way:

$$\hat{Z}^T = WX^T$$

Where matrix W is the separating matrix, and W^{-1} is the estimate of matrix A, which represents the mixing matrix.

Question 1: Perform ICA and Save Results

Apply the ICA technique to the observation matrix X^T using the ICA function. This will give you the estimated matrices W and $\hat{Z}$. Save these results and also calculate and store the matrix W^{-1}, which estimates the mixed matrix A.

```
% Load data
load('mecg1.dat');
load('fecg1.dat');
load('noise1.dat');
load('X.dat');

Fs = 256;
t = (0:length(X)-1)./Fs;

% Perform ICA on X'T
[W, ZHAT] = ica(X');

% Pseudo-inverse of W
Winv = pinv(W);

% Save the results
save('ICA_output','W','ZHAT','Winv');
```

Source Code 9: Perform ICA on X^T

Question 2: Plot Mixed Signal and Component Vectors

Plot the initial data (scatter plot) using the function `plot3ch`, and plot the three component vectors using the function `plot3dv`. It is worth mentioning that `plot3dv` plots the column vectors of matrix W^{-1} as inputs. The results can be saved as a figure file (`fig` format).

```
% Scatter plot
plot_title = "Mixed signal";
plot3ch(X, Fs, plot_title);

figure;
col = ["blue", "green", "red"];

```

```
% Plot the columns of Winv
for i = 1:size(Winv, 2)
    plot3dv(Winv(:, i), 1, col(i));
    title('Component vectors');
    xlabel('x'); ylabel('y'); zlabel('z');
end

% Save the figure
savefig('component_vectors.fig');
```

Source Code 10: Scatter Plot and 3D Plot of Component Vectors

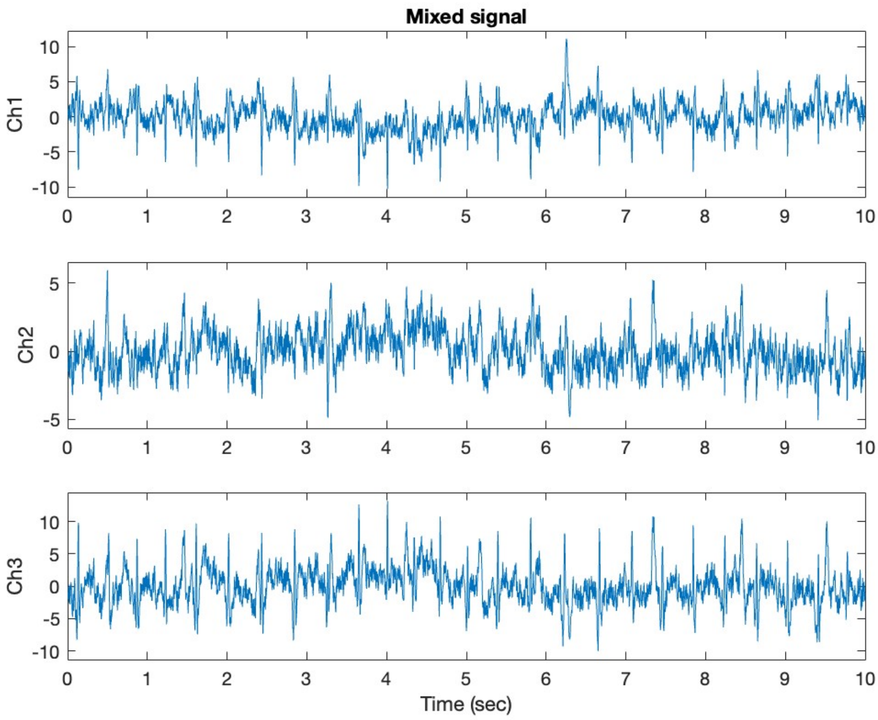

Figure 11: Time plot of the mixed signal

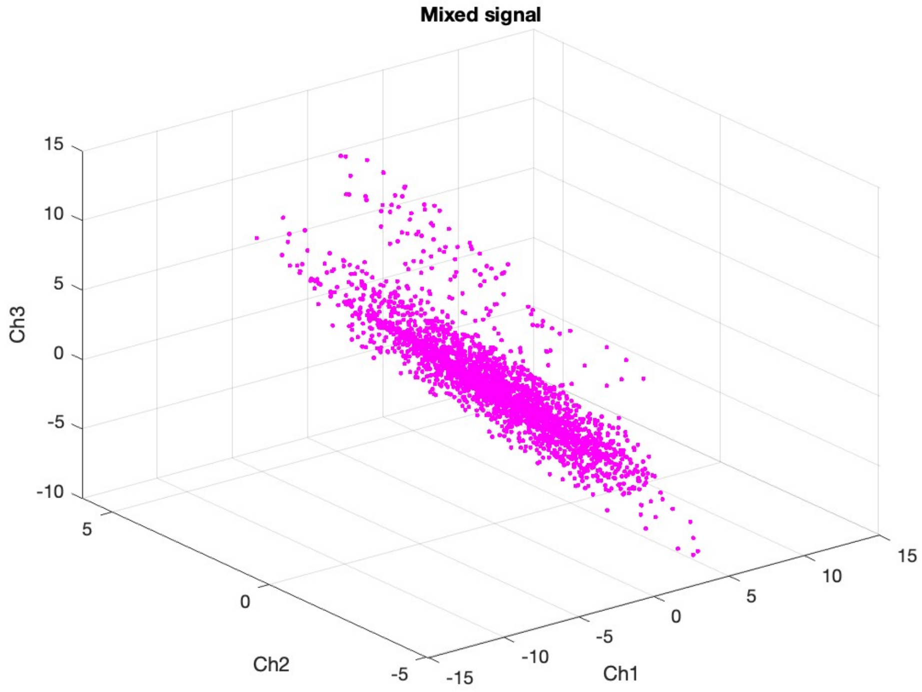

Figure 12: Scatter plot of the mixed signal

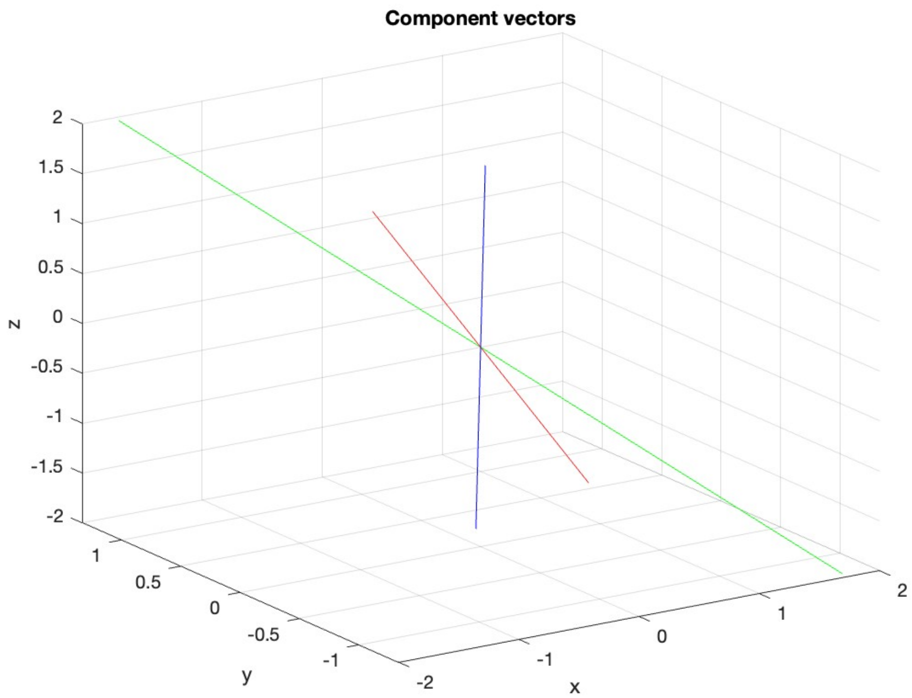

Figure 13: 3D plot of the component vectors from W^{-1}

Question 3: Plot the Extracted Components

Plot the three rows (components) of matrix $\hat{Z}^T$ over time. Also, reconstruct the signal by using one component (e.g., component 3) from matrix $\hat{Z}$ and matrix W^{-1} to transform it back to the sensor space (observation space).

```
% Three rows components of ZHAT over time
figure;
for i = 1:size(ZHAT,1)
    hold on;
    plot(t, i*5 + ZHAT(i,:))
    xlabel('Time (s)')
end

% Label each component
yticks([5 10 15]);
yticklabels({"Component 1", "Component 2", "Component 3"});
ytickangle(30);
title('Components');

% Reconstruct the signal using only component 3
n = 3;
X_reconstructed = Winv(:,n) * ZHAT(n,:);

% Save the reconstructed signal
save('X_reconstructed_ICA','X_reconstructed');
```

Source Code 11: Plot of Extracted Components

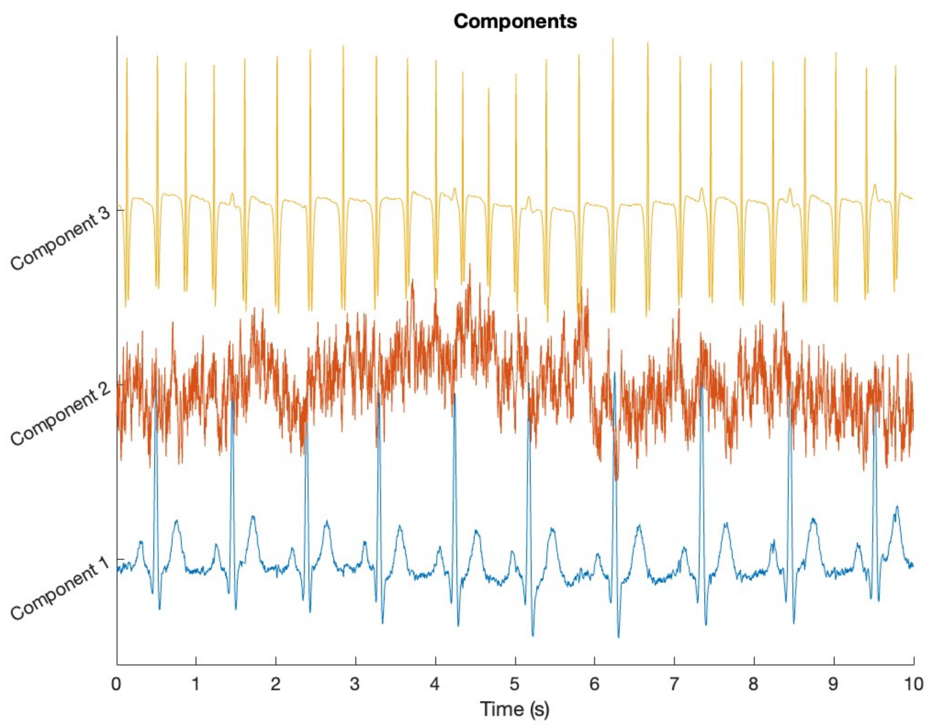

Figure 14: Extracted components from ICA

Question 4: Plot the Reconstructed Signal

Plot and save the reconstructed signal using ICA. In your opinion, has the desired component been successfully recovered?

```
% Plot the reconstructed signals
figure;
hold on;
for i = 1:3
    plot(t, i*6 + X_reconstructed(i,:));
    xlabel("Time (s)");
    ylabel("Amplitude");
end

yticks([1 2 3].*6);
yticklabels({"Channel 1", "Channel 2", "Channel 3"});
ytickangle(30);
title('X reconstructed');

hold off;
```

Source Code 12: Plot of Reconstructed Signals

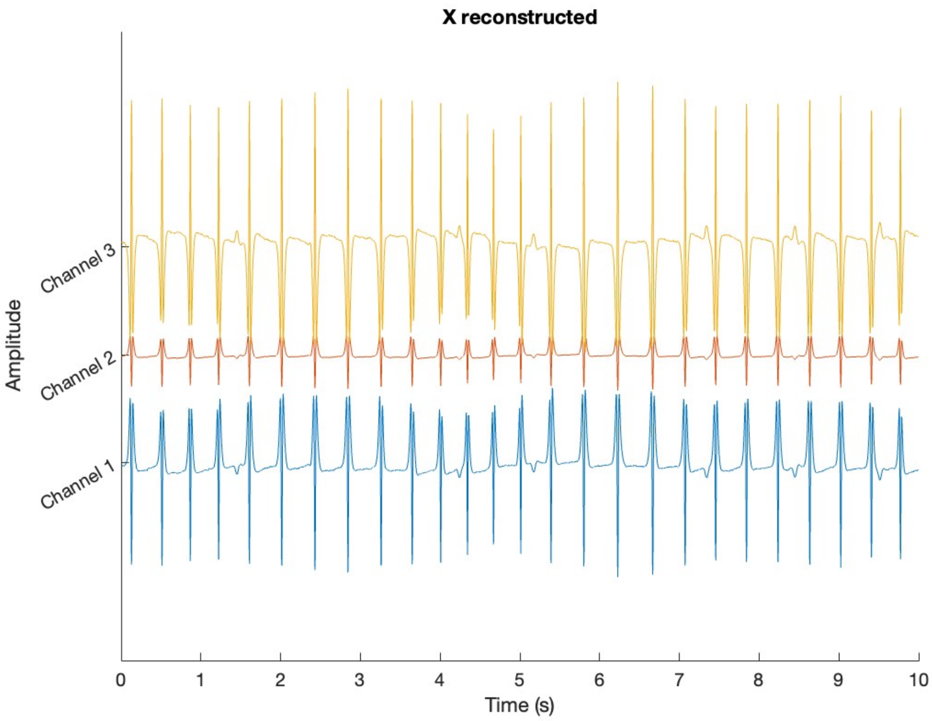

Figure 15: Reconstructed signals from ICA

Soloution

The signal was reconstructed successfully using Independent Component Analysis (ICA). By selecting the appropriate independent components from the output of ICA, we were able to reconstruct the desired signal from the mixed sources.
The reconstructed signal closely resembles the original clean signals, as can be seen in the plotted results. This demonstrates the effectiveness of ICA in separating the fetal ECG (fECG) from the maternal ECG (mECG) and noise. Given that the signal was heavily mixed, the ability of ICA to isolate and reconstruct the clean signal shows the strength of this statistical method in source separation tasks.

Conclusion

In this section, we applied ICA to mixed ECG signals to separate and reconstruct individual components. The ICA method was effective in extracting independent components, and the signal reconstruction demonstrated the method's ability to isolate and clean up the noisy mixed signals.

Comparisons Between ICA and SVD

In this section, we compare the performance of Independent Component Analysis (ICA) and Singular Value Decomposition (SVD) in denoising the signals. The comparison is made based on the directions of the mixing matrices, the angles between them, and the quality of signal reconstruction.

Question 1: Scatter Plots and Mixing Matrix Directions

Plot the following in a 3D space (corresponding to three channels): - Scatter plot of the observation matrix X - Scatter plot of the reconstructed observation matrix using SVD - Scatter plot of the reconstructed observation matrix using ICA - Direction of the columns of matrix V - Direction of the columns of matrix W^{-1}

```
%% Q4.1
Fs = 256;
load('X.dat')
plot_title = "Mixed signal";
plot3ch(X,Fs,plot_title)

load('X_reconstructed_SVD')
plot_title = "Mixed signal SVD";
plot3ch(X_reconstructed,Fs,plot_title)

load('X_reconstructed_ICA')
plot_title = "Mixed signal ICA";
plot3ch(X_reconstructed',Fs,plot_title)

figure;
plot3dv(A(:,1), [], 'r');
hold on;
plot3dv(A(:,2), [], 'g');
plot3dv(A(:,3), [], 'b');
title('Directions of matrix A columns (ICA mixing matrix)');
xlabel('Ch1'); ylabel('Ch2'); zlabel('Ch3');
hold off;

figure;
plot3dv(V(:,1), [], 'r');
hold on;
plot3dv(V(:,2), [], 'g');
plot3dv(V(:,3), [], 'b');
title('Directions of matrix V columns (SVD right singular vectors)');
xlabel('Ch1'); ylabel('Ch2'); zlabel('Ch3');
hold off;
```

Source Code 13: Scatter Plot and Direction of Mixing Matrices

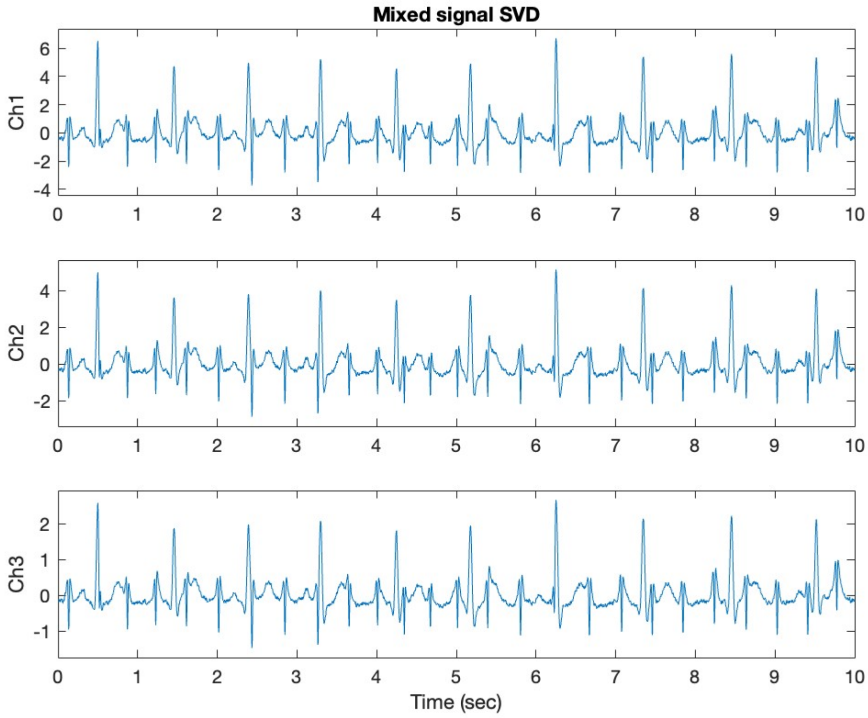

Figure 16: Timming plot of the reconstructed signal (SVD)

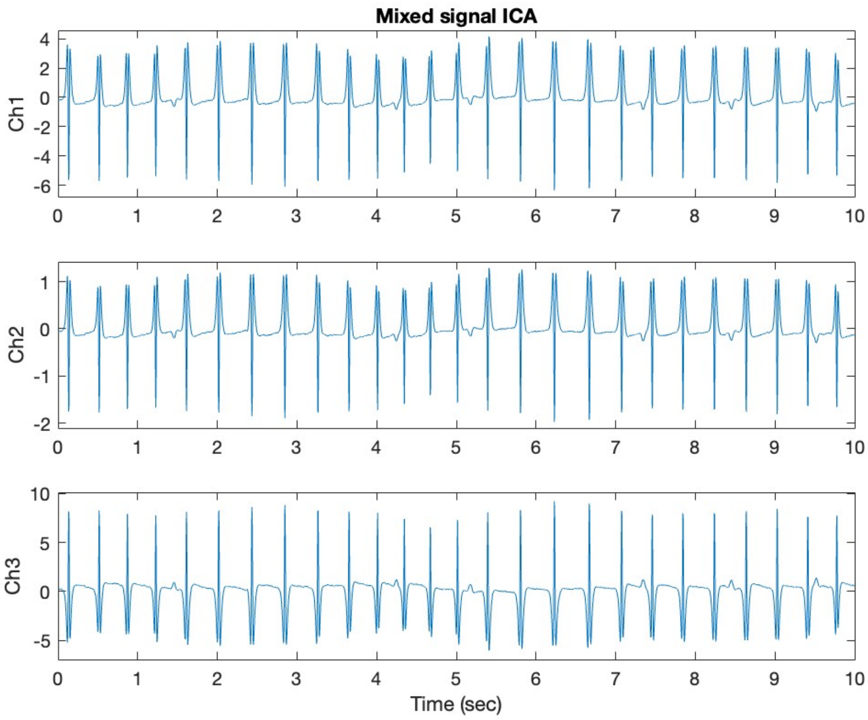

Figure 17: Timming plot of the reconstructed signal (ICA)

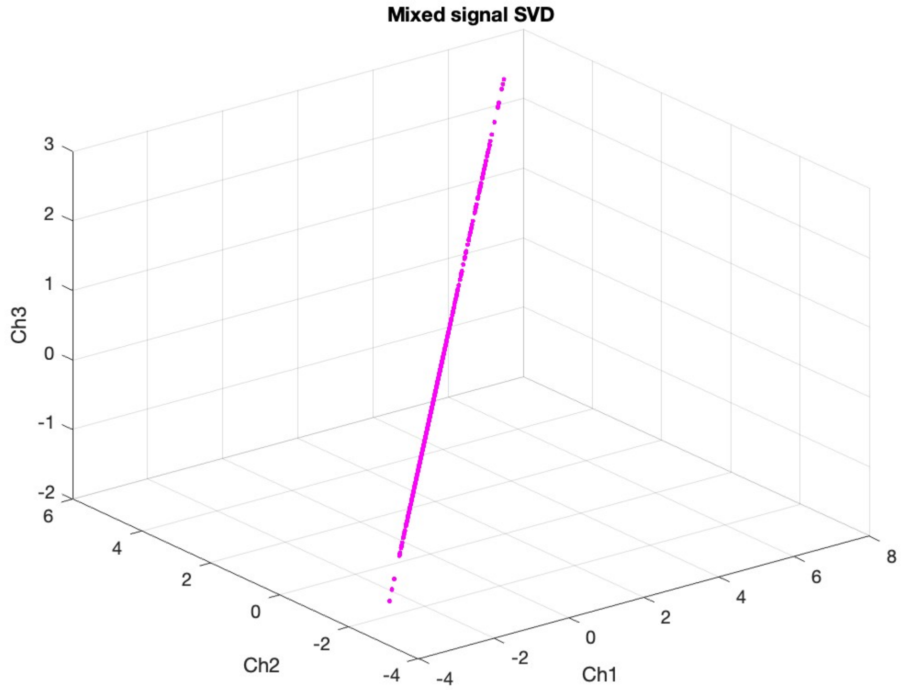

Figure 18: Scatter plot of the reconstructed signal (SVD)

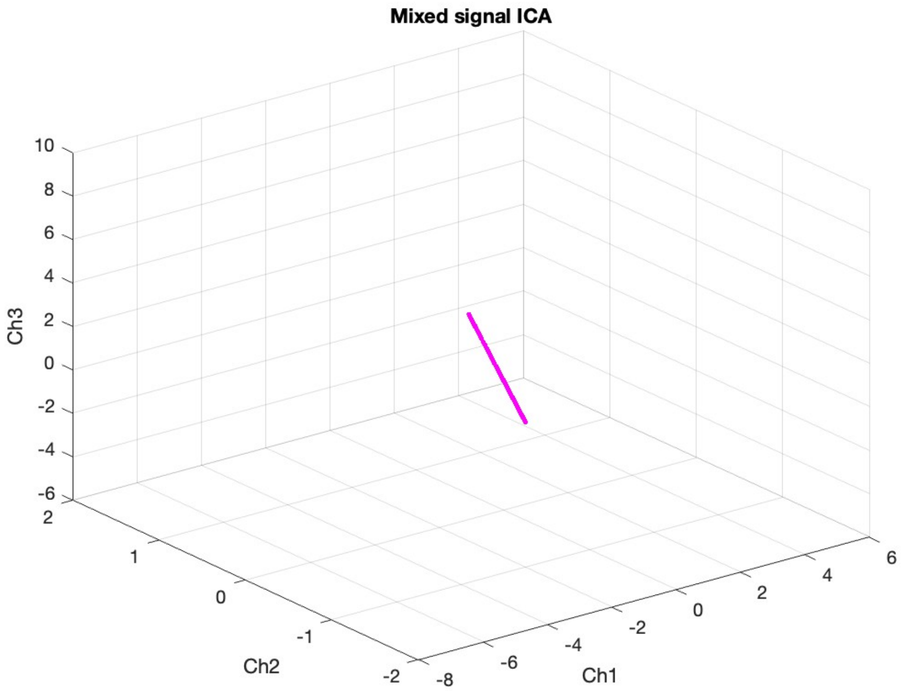

Figure 19: Scatter plot of the reconstructed signal (ICA)

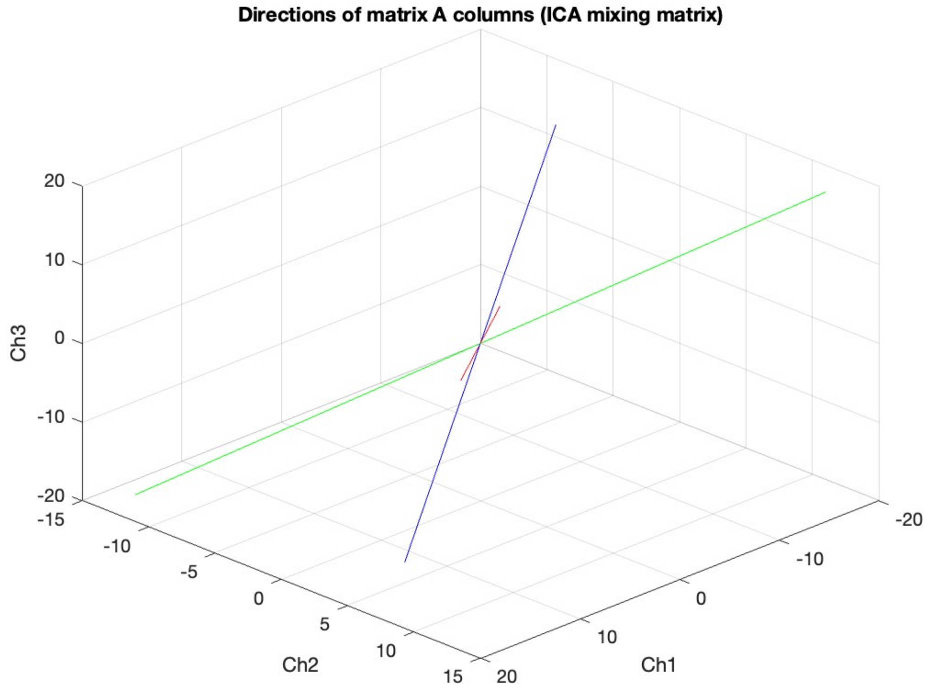

Figure 20: Directions of matrix A (ICA mixing matrix)

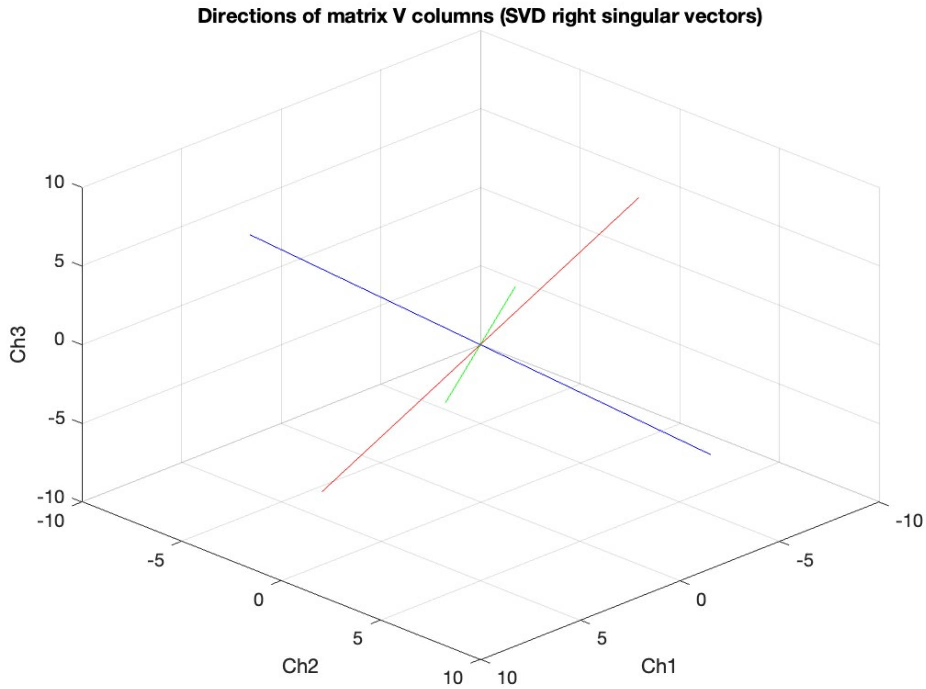

Figure 21: Directions of matrix V (SVD right singular vectors)

Table 3: Angles Between Columns of Matrix V (SVD) and Matrix A (ICA)

Angle	**SVD (V)**	**ICA (A)**
Angle 1 and 2	6.6613×10^{-16}	2.2343
Angle 1 and 3	2.2204×10^{-16}	1.2074
Angle 3 and 2	-2.7756×10^{-16}	5.3267

Question 2: Signal Comparison between SVD and ICA

Plot and compare the results from SVD and ICA methods with the ideal fetal ECG signal $fECG$. Identify which method provides better performance.

```
% Load the signals
load('fecg2.dat');
load('SVD_Output');
load('ICA_Output');

% Plot the signals for comparison
fs = 256;
t = 0 : 1/fs : 2560/fs - 1/fs;

figure;
% The ideal signal
subplot(3,1,1)
plot(t, fecg2);
title('Ideal Signal (fecg2)');
grid minor;
xlabel('Time (s)');
ylabel('Amplitude');
% The SVD-denoised signal
load('X_reconstructed_SVD')
subplot(3,1,2)
plot(t, X_reconstructed(:,1));
title('Denoised by SVD');
grid minor;
xlabel('Time (s)');
ylabel('Amplitude');
% The ICA-denoised signal
load('X_reconstructed_ICA')
subplot(3,1,3)
plot(t, X_reconstructed(2,:));
title('Denoised by ICA');
grid minor;
xlabel('Time (s)');
ylabel('Amplitude');

% Save the figure
savefig('Signal_Comparison_SVD_ICA.fig');
saveas(gcf, 'Signal_Comparison_SVD_ICA.png');
```

Source Code 14: Signal Comparison Between SVD and ICA

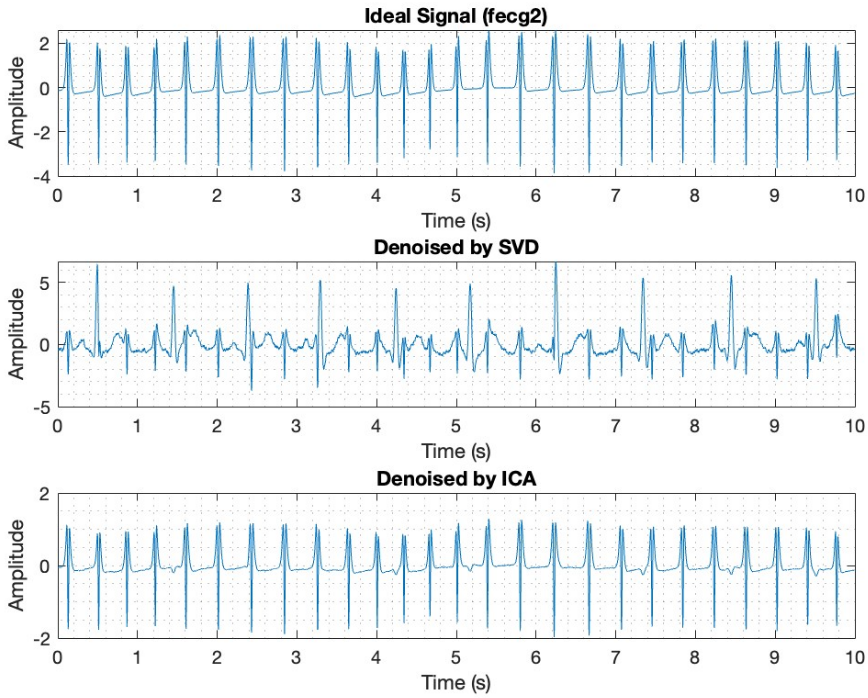

Figure 22: Comparison of Signals: Ideal, SVD-denoised, ICA-denoised

Soloution

In this comparison, we analyze the ideal $fECG$ signal against the signals denoised by both SVD and ICA. From the visual plots: - The SVD-denoised signal retains much of the original structure but appears to have more noise compared to the ICA-denoised signal. - The ICA-denoised signal closely follows the shape of the ideal $fECG$, showing a higher correlation and smoother reconstruction.
Based on visual inspection, the ICA method shows better performance in removing noise and reconstructing the original signal more accurately.

Question 3: Correlation Coefficient and Angle Comparison

Compute and compare the correlation coefficient between the original $fECG$ signal and the reconstructed signals (from SVD and ICA) for one of the channels using the `corrcoef` function.

```
% Load the outputs
load('SVD_Output');
load('ICA_Output');

% Correlation coefficient
load('fecg2.dat')
load('X_reconstructed_SVD')
r_SVD = corrcoef(fecg2,X_reconstructed(:,1));

load('X_reconstructed_ICA')
r_ICA = corrcoef(fecg2,X_reconstructed(2,:));

% Display correlation coefficients
disp('Correlation coefficient for SVD:');
disp(r_SVD);
```

```

    disp('Correlation coefficient for ICA:');
    disp(r_ICA);
```

Source Code 15: Correlation Coefficient Comparison

Soloution

The correlation coefficients between the reconstructed signals and the ideal $fECG$ signal are as follows:

$$r_{SVD} = \begin{pmatrix} 1.0000 & 0.4955 \\ 0.4955 & 1.0000 \end{pmatrix}$$

$$r_{ICA} = \begin{pmatrix} 1.0000 & 0.9978 \\ 0.9978 & 1.0000 \end{pmatrix}$$

From these results, the ICA method shows a much higher correlation (0.9978) with the ideal signal compared to the SVD method (0.4955), indicating that ICA provides a more accurate reconstruction of the original $fECG$ signal.

Question 4: Final Comparison and Analysis of SVD and ICA

Perform the final comparison between SVD and ICA, outlining the advantages and disadvantages of both methods.

Soloution

Both ICA and SVD have their respective strengths and weaknesses:
- **SVD**: This method is based on matrix factorization and focuses on orthogonal components. It works well in noise reduction when the noise is distributed across multiple components. However, it tends to underperform when dealing with non-Gaussian noise or when statistical independence is required, as seen in the low correlation with the ideal $fECG$ signal.
- **ICA**: This method excels in separating independent components, especially when dealing with mixed signals like ECG data. It showed a high correlation with the ideal $fECG$, indicating better performance in reconstructing the clean signal. However, ICA requires careful selection of the components, and improper selection can lead to inaccurate results.

In summary, ICA outperforms SVD in the context of this task due to its ability to isolate statistically independent sources, making it more effective for denoising the $fECG$ signal.

Question 5: Key Takeaways

Discuss the most important points you have learned from this experiment.

Soloution

Key takeaways from this experiment are:
- ICA is highly effective in separating independent components from mixed signals, particularly when dealing with non-Gaussian sources like ECG data.
- SVD, while effective in reducing noise, is not as robust when statistical independence is needed, making it less suitable for applications like fetal ECG extraction.
- The importance of selecting appropriate components in ICA is crucial for obtaining accurate signal reconstructions.
- Correlation analysis is a powerful tool to quantify the accuracy of signal reconstruction and should be used alongside visual inspection for more comprehensive evaluations.