Applications of a Directed Graph (Network-flow) to Find the Maximum Flow

Course: Graph Theory (CPTS 553)

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Objective of the Study

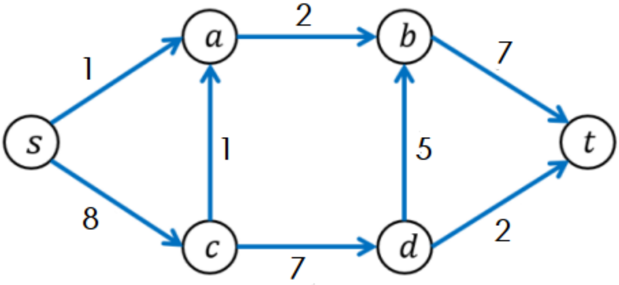
- The primary objective is to show the application of a directed graph, network flow
- Network flow is one of the vastly used approaches for solving problems
- This work talks about the concepts of flow-network their approaches in solving real-world problems

Network Flow

- In graph theory, network flow or flow-network is a directed graph with numerical capacities on its edges, and in this graph, the objective is to construct a flow
- There are different network flow problems, such as maximum flow problems, minimum-cost flow problems, nowhere-zero, etc.
- There are different algorithms available for constructing the flow-network. The common algorithms are "Dinic's algorithm", "Edmonds-Karp algorithm", "Ford-Fulkerson algoithm"

An Example of Network flow graph

This random network flow graph has source (s), sink(t), nodes (a, b, c, d), and directed edges with different capacities.



Ford-Fulkerson Algorithm

Ford-Fulkerson Algorithm is used to find the maximum flow of the network. The Ford-Fulkerson method continues finding augmenting paths and augments the flow until no more augmenting paths from source (s) to sink (t) exist. Here is the algorithm;

- Start with initial flow as 0.
- If there is a augmenting path from source to sink,add this path-flow to flow
- Retund the flow

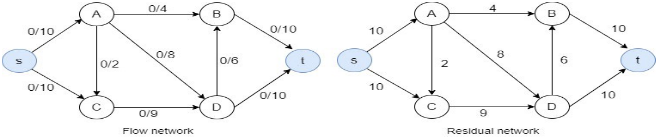
Maximum Flow Algorithm

To find the maximum flow (and min-cut as a by-product), the Ford-Fulkerson method can be used and this finds the augmenting paths through the residual graph and augments the flow until no more augmenting paths can be found, thus obtained the maximum flow

Application 1: Finding the Max Flow

In this application, we tried to find the maximum fow using the network flow by Ford-Fulkerson Algorithm

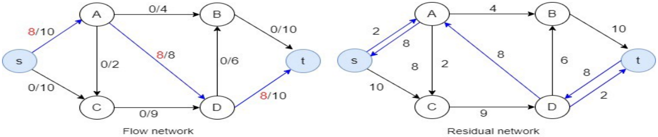
Figure: Network flow with zero initial input



Application 1: Finding the Max Flow

Now, we select another augmenting path, say, $s \to A \to D \to t$,and the bottleneck value is 8.

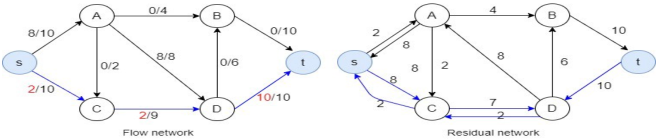
Figure: Network flow with zero initial input



Application 1: Finding the Max Flow

Now, we select another augmenting path, say, $s \to C \to D \to t$. And the bottleneck value is 2.

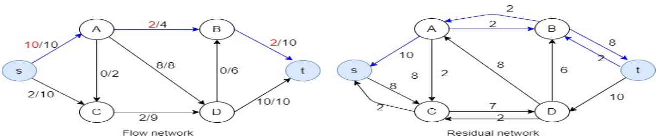
Figure: Network flow with zero initial input



Application 1: Finding the Max Flow

Still we have paths for augmentation in the flow-network. Let select the path $s \rightarrow A \rightarrow B \rightarrow t$ and the bottleneck value is 2.

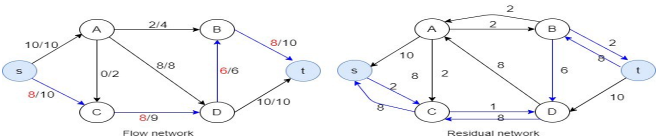
Figure: Network flow with zero initial input



Application 1: Finding the Max Flow

We have to augment further, and select the path $s \to C \to D \to B \to t$, and the bottleneck value is 6.

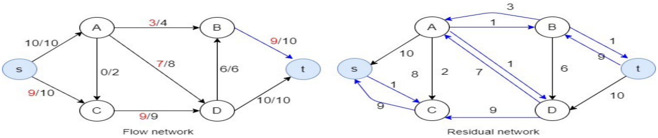
Figure: Network flow with zero initial input



Application 1: Finding the Max Flow

Now, we have more path to augment, $s \to C \to D \to A \to B \to t$, and the bottleneck value is 1.

Figure: Network flow with zero initial input



Now, there are no paths left from the source (s) to the sink (t) in the residual graph to update. Hence, this attained the maximum flow through the edged following the capacity constraints. Thus, the Ford-Fulkerson completed the update of flow and we can get the maximum flow from the graph now.

Application 1: Maximum Flow Value

We know that the maximum flow is equal to the flow coming out of the source, hence, the maximum flow for our case is $10+9 = 19$.

Application 2: Finding the Max Flow value by linear optimization

This showed an approach of finding the maximum flow using the concept of linear programming. This has designed the problem and then formulated the objective functions and state the constraints to solve the LP problem to find the Max Flow vlaue.

Colclusion

- Directed graphs are good way to solve many real world problems
- If the problem can be formulated into a directed graph properly then the application of an appropriate method can find the solution for the problem