IS CODED PROJECT

Business Report

DSBA

Submitted By: Maheep Singh
Batch : PGP-DSBA (PGPDSBA.O.AUG24.A)

Table of Contents

List of Figures 4
List of Tables 5
Problem 1 6
Solution 1 6
1.1 What is the probability that a randomly chosen player would suffer an injury? 6
1.2 What is the probability that a player is a forward or a winger? 6
1.3 What is the probability that a randomly chosen player plays in a striker position and has a foot injury? 6
1.4 What is the probability that a randomly chosen injured player is a striker? 6
Problem 2 7
Solution 2 7
2.1 What proportion of the gunny bags have a breaking strength less than 3.17 kg per sq. cm? 7
2.2 What proportion of the gunny bags have a breaking strength at least 3.6 kg per sq. cm.? 8
2.3 What proportion of the gunny bags have a breaking strength between 5 and 5.5 kg per sq. cm.? 9
2.4 What proportion of the gunny bags have a breaking strength NOT between 3 and 7.5 kg per sq. cm.? 10
Problem 3 11
Solution 3 11
Context 11
Objective 11
Data Description 11
Load the Dataset 11
Data Preparation & Exploratory Data Analysis 12
3.1 Zingaro has reason to believe that the unpolished stones may not be suitable for printing. Do you think Zingaro is justified in thinking so? 14
3.2 Is the mean hardness of the polished and unpolished stones the same? 15
Problem 4 16
Solution 4 16
Context 16
Objective 16
Data Description 16
Load the Dataset 17
Data Preparation & Exploratory Data Analysis 17
4.1 How does the hardness of implants vary depending on dentists? 21
4.2 How does the hardness of implants vary depending on methods? 23

4.3 What is the interaction effect between the dentist and method on the hardness of dental implants for each type of alloy? 26
4.4 How does the hardness of implants vary depending on dentists and methods together? 27

List of Figures

Figure 1: Normal Distribution of Gunny Bags | P(X<3.17) 7
Figure 2: Normal Distribution of Gunny Bags | P(X>=3.6) 8
Figure 3: Normal Distribution of Gunny Bags | P(5<X<5.5) 9
Figure 4: Normal Distribution of Gunny Bags | P(X<3 OR X>7.5) 10
Figure 5: First 5 Rows of the Zingaro Dataset 11
Figure 6: Data-types of columns (Zingaro) 12
Figure 7: Null & Duplicate value Check (Zingaro) 12
Figure 8: Statistical Summary of the Numerical Variables (Zingaro) 12
Figure 9: Univariate Analysis (Zingaro) 13
Figure 10: Outlier Inspection for Unpolished (Zingaro) 13
Figure 11: Outlier Inspection for Polished (Zingaro) 13
Figure 12: Bivariate Analysis – Heatmap (Zingaro) 14
Figure 13: Histogram for Unpolished with Threshold for Printing (Zingaro) 14
Figure 14: P-Value – Problem 3.1 (Zingaro) 15
Figure 15: Histogram – Distribution of Hardness for Polished & Unpolished (Zingaro) 15
Figure 16: P-Value – Problem 3.2 (Zingaro) 16
Figure 17: First 5 Rows of the Dental Implant Dataset 17
Figure 18: Data-types of columns (Dental Implant) 17
Figure 19: Data-types of columns (Dental Implant) – post data-type treatment 17
Figure 20: Null & Duplicate value Check (Dental Implant) 18
Figure 21: Statistical Summary of the Numerical Variables (Dental Implant) 18
Figure 22: Univariate Analysis – Numerical (Dental Implant) 19
Figure 23: Outlier inspection of Response (Dental Implant) 19
Figure 24: Outlier Inspection for Polished (Dental Implant) 19
Figure 25: Univariate Analysis of Categorical Variables (Dental Implant) 20
Figure 26: Bivariate Analysis of Categorical Variables (Dental Implant) 20
Figure 27: Boxplot for Hardness by Dentist for each Alloy (Dental Implant) 21
Figure 28: P-Value – Shapiro-Wilk's Test for both Alloys 22
Figure 29: P-Value – Levene's Test (Dentist) for both Alloys 22
Figure 30: P-Value – One-way ANOVA (Dentist) for both Alloys 22
Figure 31: Boxplot for Hardness by Method for each Alloy (Dental Implant) 23
Figure 32: P-Value – Levene's Test (Method) for both Alloys 24
Figure 33: P-Value – One-way ANOVA (Method) for both Alloys 24
Figure 34: Tukey HSD Test (Method) for both Alloys 25
Figure 35: Interaction Plot between Dentist & Method on Hardness 26
Figure 36: P-Value – Levene's Test (Method X Dentist) for both Alloys 27
Figure 37: 2-way ANOVA for both Alloys 28
Figure 38: Tukey HSD Test (Method X Dentist Interaction) for both Alloys 31

List of Tables

Table 1: Relationship between foot injuries and the positions (Problem 1) 6
Table 2: Statistical Summary of the Numerical Variables (Zingaro) 12
Table 3: Statistical Summary of the Numerical Variables (Dental Implant) 18

Problem 1

A physiotherapist with a male football team is interested in studying the relationship between foot injuries and the positions at which the players play from the data collected.

	Striker	Forward	Attacking Midfielder	Winger	**Total**
Players Injured	45	56	24	20	**145**
Players Not Injured	32	38	11	9	**90**
Total	**77**	**94**	**35**	**29**	**235**

Table 1: Relationship between foot injuries and the positions (Problem 1)

Based on the above data, answer the following questions.

1.1 What is the probability that a randomly chosen player would suffer an injury?
1.2 What is the probability that a player is a forward or a winger?
1.3 What is the probability that a randomly chosen player plays in a striker position and has a foot injury?
1.4 What is the probability that a randomly chosen injured player is a striker?

Solution 1

1.1 What is the probability that a randomly chosen player would suffer an injury?

- We use the following formula for calculating the probability from the given table: -
  P (Injured) = (No. of Players Injured) / (Total no. of Players)
  = 145 / 235
  = 0.617
  = 61.7 %
- The probability that a randomly chosen player would suffer an injury is 61.7%.

1.2 What is the probability that a player is a forward or a winger?

- We use the following formula for calculating the probability from the given table: -
  P (Forward or Winger) = (No. of Forwards + No. of Wingers) / (Total no. of Players)
  = (94 + 29) / 235 = 123 / 235
  = 0.5234
  = 52.34 %
- The probability that a player is a forward or a winger is 52.34%.

1.3 What is the probability that a randomly chosen player plays in a striker position and has a foot injury?

- We use the following formula for calculating the probability from the given table: -
  P (Striker & Injury) = (No. of Injured Strikers) / (Total no. of Players)
  = 45 / 235
  = 0.1915
  = 19.15 %
- The probability that a randomly chosen player plays in a striker position and has a foot injury is 19.15 %.

1.4 What is the probability that a randomly chosen injured player is a striker?

- We use the following formula for calculating the conditional probability from the given table: -
  P (Striker | Injured) = (No. of Injured Strikers) / (Total no. of Injured Players)
  = 45 / 145
  = 0.3103
  = 31.03 %
- The probability that a randomly chosen injured player is a striker is 31.03%

Problem 2

The breaking strength of gunny bags used for packaging cement is normally distributed with a mean of 5 kg per sq. centimetre and a standard deviation of 1.5 kg per sq. centimetre. The quality team of the cement company wants to know the following about the packaging material to better understand wastage or pilferage within the supply chain.
Answer the questions below based on the given information (Provide an appropriate visual representation of your answers, without which marks will be deducted): -

2.1 What proportion of the gunny bags have a breaking strength of less than 3.17 kg per sq. cm?
2.2 What proportion of the gunny bags have a breaking strength of at least 3.6 kg per sq. cm.?
2.3 What proportion of the gunny bags have a breaking strength between 5 and 5.5 kg per sq. cm.?
2.4 What proportion of the gunny bags have a breaking strength NOT between 3 and 7.5 kg per sq. cm.

Solution 2

2.1 What proportion of the gunny bags have a breaking strength less than 3.17 kg per sq. cm?

- It is given that the breaking strength of the gunny bags used for packaging cement is normally distributed with: -
  - Mean = 5 kg per sq. cm.
  - Standard Deviation (SD) = 1.5 kg per sq. cm.
- A normal distribution is a bell-shaped curve that is symmetric around the mean. To answer the question, we will use the properties of normal distribution and the Z-score formula to calculate proportions. The Z-score tells us how far away a particular value is from the mean in terms of standard deviations.
- Computing Z-statistic: -

  Z = (X – Mean) / SD
  = (3.17 – 5) / 1.5
  = -1.22
- We plot the Normal Probability Distribution Function & calculate the area under curve for the required region (highlighted in Red below) to obtain the proportion of breaking strength less than 3.17 kg per sq. cm.

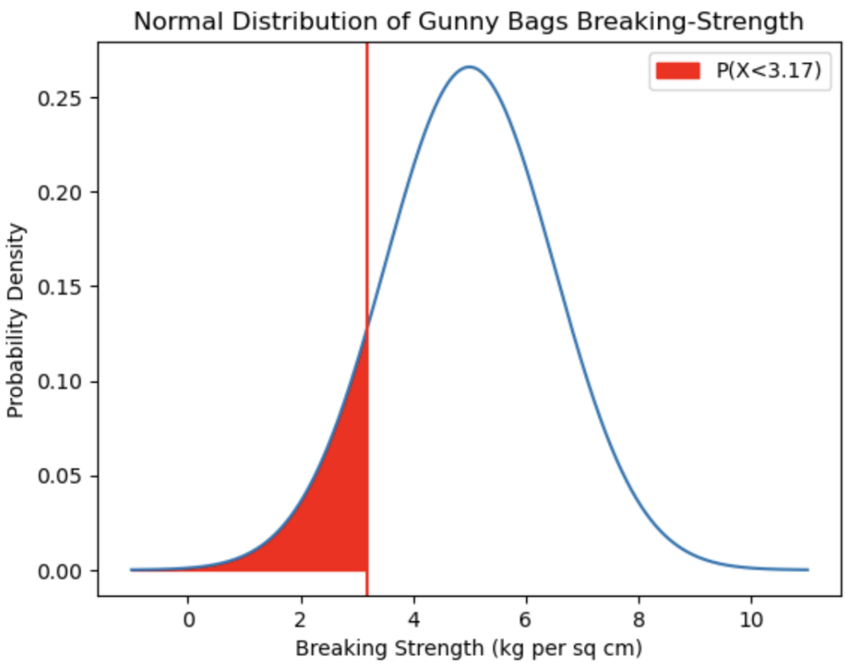


Figure 1: Normal Distribution of Gunny Bags | P(X<3.17)

- With the help of Cumulative Distribution Function of the standard normal distribution, we calculate the area under the curve.
- The area under the curve comes out to be 0.1112, i.e., P(X<3.17) = P(Z<-1.22) = 0.1112.
- **Answer = 11.12% of gunny bags have a breaking strength less than 3.17 kg per sq. cm.**

2.2 What proportion of the gunny bags have a breaking strength at least 3.6 kg per sq. cm.?

- It is given that the breaking strength of the gunny bags used for packaging cement is normally distributed with: -
  - Mean = 5 kg per sq. cm.
  - Standard Deviation (SD) = 1.5 kg per sq. cm.
- A normal distribution is a bell-shaped curve that is symmetric around the mean. To answer the question, we will use the properties of normal distribution and the Z-score formula to calculate proportions. The Z-score tells us how far away a particular value is from the mean in terms of standard deviations.
- Computing Z-statistic: -

  Z = (X – Mean) / SD

  = (3.6 – 5) / 1.5

  = -0.93
- We plot the Normal Probability Distribution Function & calculate the area under curve for the required region (highlighted in Green below) to obtain the proportion of breaking strength at least 3.6 kg per sq. cm.

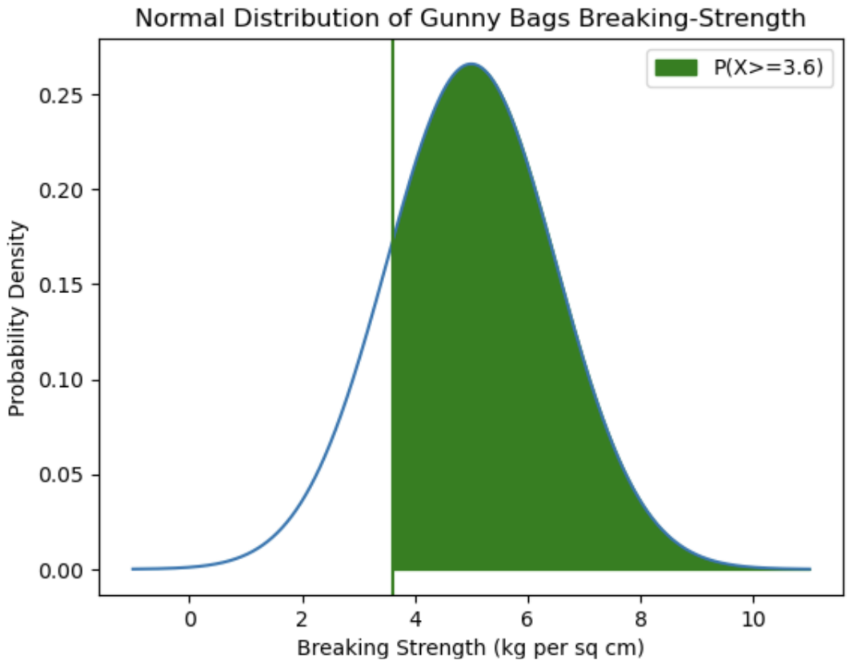


Figure 2: Normal Distribution of Gunny Bags | P(X>=3.6)

- With the help of Cumulative Distribution Function of the standard normal distribution, we calculate the area under the curve.
- The area under the curve comes out to be 0.8247, i.e., P(X>=3.6) = P(Z>=-0.93) = (1 – P(Z<-0.93)) = 0. 0.8247.
- **Answer = 82.47% of gunny bags have a breaking strength at least 3.6 kg per sq. cm.**

2.3 What proportion of the gunny bags have a breaking strength between 5 and 5.5 kg per sq. cm.?

- It is given that the breaking strength of the gunny bags used for packaging cement is normally distributed with: -
  - Mean = 5 kg per sq. cm.
  - Standard Deviation (SD) = 1.5 kg per sq. cm.
- A normal distribution is a bell-shaped curve that is symmetric around the mean. To answer the question, we will use the properties of normal distribution and the Z-score formula to calculate proportions. The Z-score tells us how far away a particular value is from the mean in terms of standard deviations.
- Computing Upper Z-statistic: -

  Z_U = (X – Mean) / SD
  = (5.5 – 5) / 1.5
  = 0.33
- Computing Lower Z-statistic: -

  Z_L = (X – Mean) / SD
  = (5 – 5) / 1.5
  = 0
- We plot the Normal Probability Distribution Function & calculate the area under curve for the required region (highlighted in Orange below) to obtain the proportion of the gunny bags have a breaking strength between 5 and 5.5 kg per sq. cm.

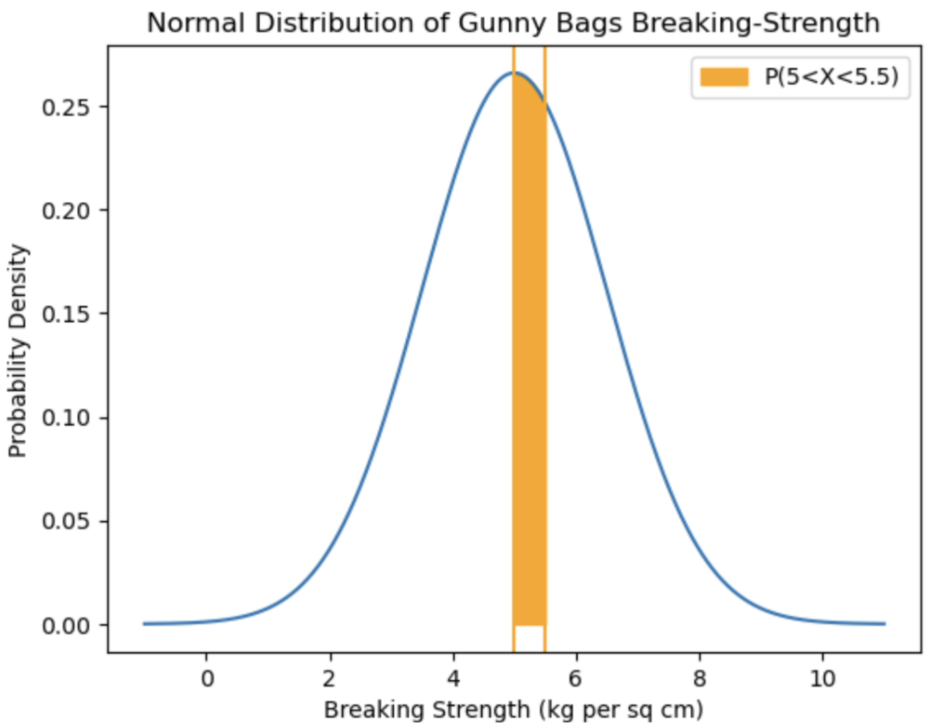


Figure 3: Normal Distribution of Gunny Bags | P(5<X<5.5)

- With the help of Cumulative Distribution Function of the standard normal distribution, we calculate the area under the curve.
- The area under the curve comes out to be 0.1306, i.e., P(X<5.5) - P(X<5) = P(Z<0.33) – P(Z<0) = 0.1306.
- **Answer = 13.06% of gunny bags have a breaking strength between 5 and 5.5 kg per sq. cm.**

2.4 What proportion of the gunny bags have a breaking strength NOT between 3 and 7.5 kg per sq. cm.?

- It is given that the breaking strength of the gunny bags used for packaging cement is normally distributed with: -
  - Mean = 5 kg per sq. cm.
  - Standard Deviation (SD) = 1.5 kg per sq. cm.
- A normal distribution is a bell-shaped curve that is symmetric around the mean. To answer the question, we will use the properties of normal distribution and the Z-score formula to calculate proportions. The Z-score tells us how far away a particular value is from the mean in terms of standard deviations.
- Computing Upper Z-statistic: -
  Z_U = (X – Mean) / SD
  = (7.5 – 5) / 1.5
  = 1.67
- Computing Lower Z-statistic: -
  Z_L = (X – Mean) / SD
  = (3 – 5) / 1.5
  = -1.33
- We plot the Normal Probability Distribution Function & calculate the area under curve for the required region (highlighted in Purple below) to obtain the proportion of the gunny bags have a breaking strength NOT between 3 and 7.5 kg per sq. cm.

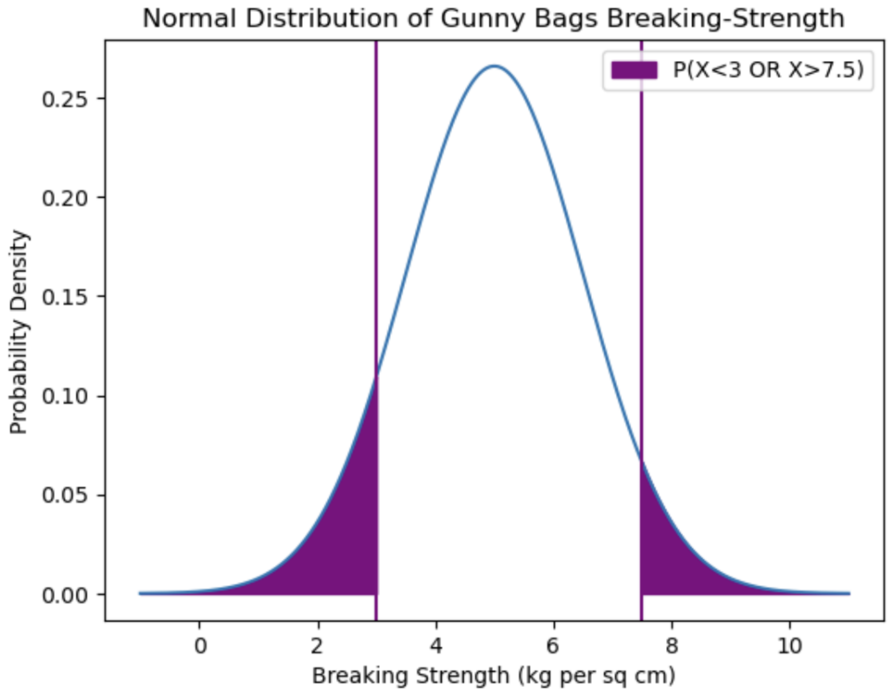


Figure 4: Normal Distribution of Gunny Bags | P(X<3 OR X>7.5)

- With the help of Cumulative Distribution Function of the standard normal distribution, we calculate the area under the curve.
- The area under the curve comes out to be 0.139, i.e., P(X<3) + P(X>7.5) = P(X<3) + (1 - P(X<7.5))
  = P(Z<-1.33) + (1 - P(Z<1.67)) = 0.139.
- **Answer = 13.9% of gunny bags have a breaking strength NOT between 3 and 7.5 kg per sq. cm.**

Problem 3

Zingaro stone printing is a company that specializes in printing images or patterns on polished or unpolished stones. However, for the optimum level of printing of the image, the stone surface has to have a Brinell's hardness index of at least 150. Recently, Zingaro has received a batch of polished and unpolished stones from its clients. Use the data provided to answer the following (assuming a 5% significance level): -
3.1 Zingaro has reason to believe that the unpolished stones may not be suitable for printing. Do you think Zingaro is justified in thinking so?
3.2 Is the mean hardness of the polished and unpolished stones the same?

Solution 3

Context

Zingaro stone printing is a company that specializes in printing images or patterns on polished or unpolished stones. However, for the optimum level of printing of the image, the stone surface has to have a Brinell's hardness index of at least 150. Recently, Zingaro has received a batch of polished and unpolished stones from its clients. Use the data provided to answer the following (assuming a 5% significance level): -

Objective

Suppose you are a Data Scientist at the Zingaro company and the Data Science team has shared some of the key questions that need to be answered. Perform the data analysis to find answers to these questions that will help the company to improve the business.

3.1 Zingaro has reason to believe that the unpolished stones may not be suitable for printing. Do you think Zingaro is justified in thinking so?
3.2 Is the mean hardness of the polished and unpolished stones the same?

Data Description

- **Unpolished**: Brinell's hardness index for unpolished stones
- **Treated and Polished**: Brinell's hardness index for polished & treated stones

Load the Dataset

- Import required libraries to load the dataset, perform data manipulation, analysis & visualization tasks to solve the business problem.
- Below is the view of the first 5 rows of the imported dataset to give a quick overview of the data: -

	Unpolished	Treated and Polished
0	164.481713	133.209393
1	154.307045	138.482771
2	129.861048	159.665201
3	159.096184	145.663528
4	135.256748	136.789227

Figure 5: First 5 Rows of the Zingaro Dataset

Data Preparation & Exploratory Data Analysis

- Checking shape of dataset – 75 rows & 2 columns

- Checking the data types of the columns for the dataset: -

```
Data columns (total 2 columns):
 #   Column                Non-Null Count  Dtype  
---  ------                --------------  -----  
 0   Unpolished            75 non-null     float64
 1   Treated and Polished  75 non-null     float64
```

Figure 6: Data-types of columns (Zingaro)

- Both data types are Float-type.
- Both columns have 75 values.

- Checking for missing/error & duplicate values: -

```
Duplicate Count:  0

Null Values:-
Unpolished              0
Treated and Polished    0
```

Figure 7: Null & Duplicate value Check (Zingaro)

- We have neither duplicate values nor null/error values. Hence, no treatment required.

- Getting the statistical summary of both the numerical fields

	count	mean	std	min	25%	50%	75%	max
Unpolished	75.0	134.110527	33.041804	48.406838	115.329753	135.597121	158.215098	200.161313
Treated and Polished	75.0	147.788117	15.587355	107.524167	138.268300	145.721322	157.373318	192.272856

Figure 8: Statistical Summary of the Numerical Variables (Zingaro)

- Observations & Insights can be summarized below in the table: -

Type	Columns	Observations & Insights
Numerical	Unpolished	✓ Brinell's hardness index for unpolished stones ranges between 48.4 to 200.16. ✓ Mean is 134.11 & Median is 135.60. ✓ Standard Deviation is 33.04
Numerical	Treated and Polished	✓ Brinell's hardness index for treated & polished stones ranges between 107.52 to 192.27. ✓ Mean is 147.79 & Median is 145.72. ✓ Standard Deviation is 15.59

Table 2: Statistical Summary of the Numerical Variables (Zingaro)

- Perform Univariate Analysis – Use Histograms & Boxplots to analyse each numerical variable: -

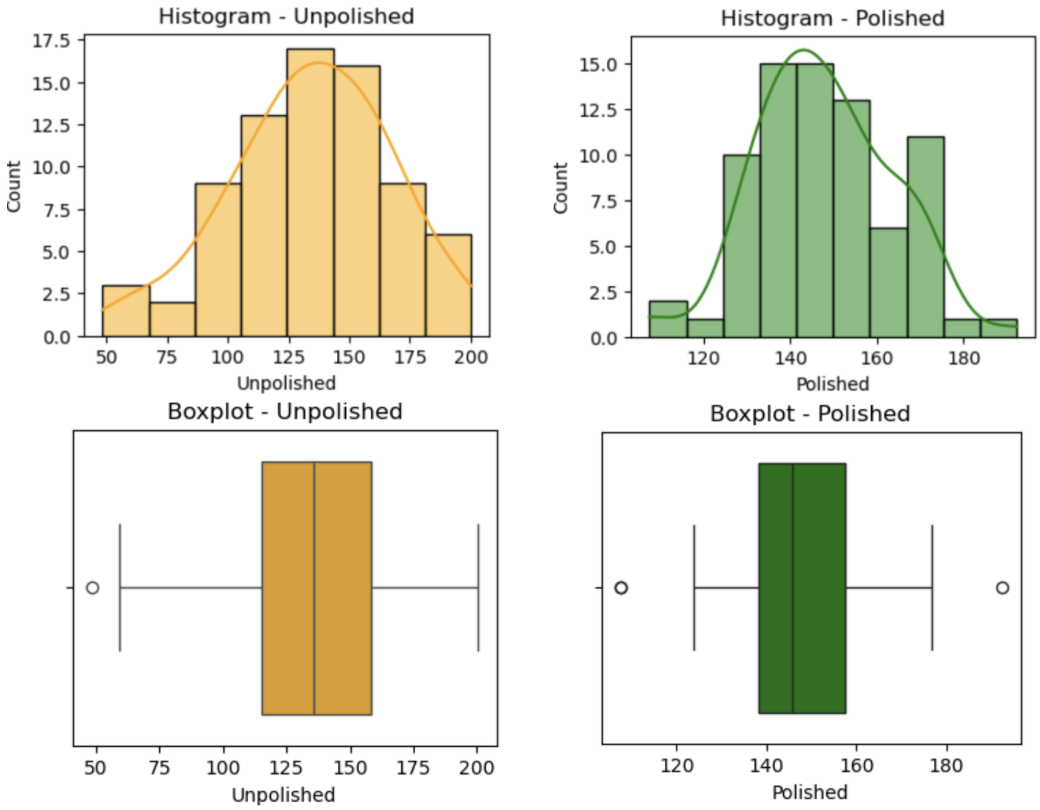


Figure 9: Univariate Analysis (Zingaro)

- Observations & Insights can be summarized below: -
  - Unpolished: -
    - Distribution of Unpolished seems to be symmetric.
    - Unimodal distribution having a single peak at 125
    - Only 1 Outlier observed.
    - Since the outlier count is not significant, we choose to ignore outlier treatment to avoid any loss if information.

```
Lower Wishker at 51.001735124999996 | Upper Whisker at 222.543115725
Lower Whisker Outlier Count = 1
Upper Whisker Outlier Count = 0
Total Outlier Count= 1
```

Figure 10: Outlier Inspection for Unpolished (Zingaro)

  - Polished & Treated: -
    - Distribution of Unpolished seems slightly right skewed.
    - Bimodal distribution having 2 peaks at 140 & 170.
    - Only 2 Outliers observed.
    - Since the outlier count is not significant, we choose to ignore outlier treatment to avoid any loss if information.

```
Lower Wishker at 109.61077220000004 | Upper Whisker at 186.03084539999998
Lower Whisker Outlier Count = 2
Upper Whisker Outlier Count = 1
Total Outlier Count= 3
```

Figure 11: Outlier Inspection for Polished (Zingaro)

- Perform Bivariate Analysis: -

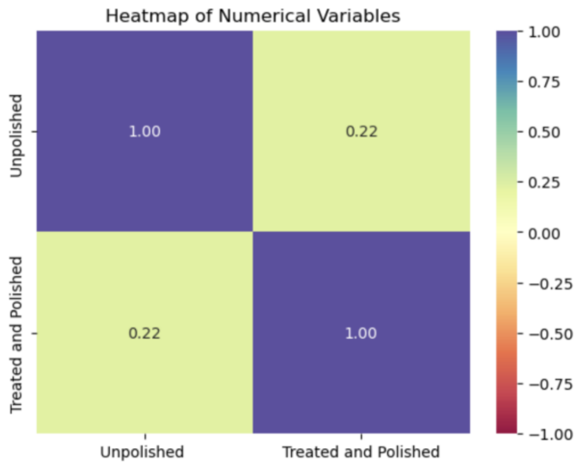


Figure 12: Bivariate Analysis – Heatmap (Zingaro)

- Observations & Insights can be summarized below: -
  - There is no correlation between the hardness of polished & unpolished stones.

3.1 Zingaro has reason to believe that the unpolished stones may not be suitable for printing. Do you think Zingaro is justified in thinking so?

1. Perform Visual Analysis: -

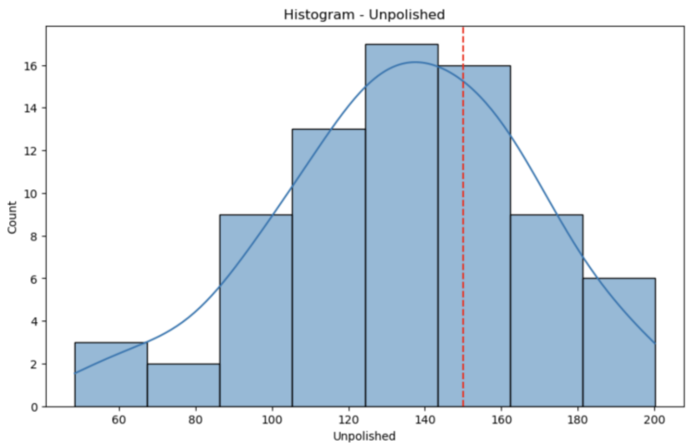


Figure 13: Histogram for Unpolished with Threshold for Printing (Zingaro)

- The histogram above shows the distribution of the Brinell hardness values for unpolished stones. The red dashed line at 150 represents the required hardness threshold for printing.
- From the visual analysis, we can see whether the majority of the unpolished stones meet the required hardness. As evident, most of the distribution lies to the left of the threshold, hence, it supports the conclusion that many unpolished stones are unsuitable for printing.
- To further strengthen the claim for the population, we carry out the hypothesis testing.

2. State the Null and Alternate Hypotheses: -
   - **Null Hypothesis (H_0):** Mean hardness of unpolished stones is greater than or equal to 150 (mean >= 150).
   - **Alternative Hypothesis (H_1):** Mean hardness of unpolished stones is less than 150 (mean < 150).

3. Select Appropriate Test & Decide Significance Level: -
    - To determine if Zingaro's belief is justified, we can perform **one-sample t-test** to see if the mean hardness of unpolished stones is less than 150.
    - It is given in the question to consider significance level as 5%.

4. Compute the P-value: -
    - We conduct 1 sample t-test (left-tailed) with Unpolished hardness data and population mean 150. Below is the Python output: -

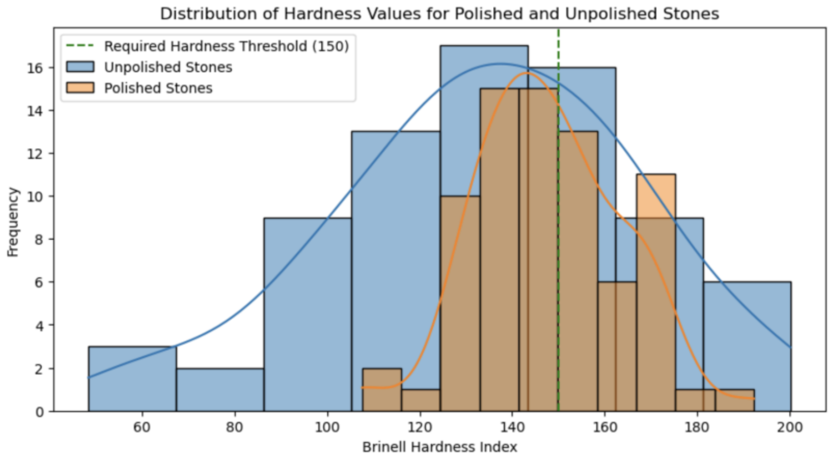
```
The p-value is  4.171286997419652e-05
As the p-value 4.171286997419652e-05 is less than the level of significance, we reject the null hypothesis.
```

Figure 14: P-Value – Problem 3.1 (Zingaro)

5. Conclusion from the Test Result: -
    - Since the p-value is much smaller than the significance level of 0.05, we reject the null hypothesis.
    - This suggests there is statistical evidence that the mean hardness of unpolished stones is significantly less than 150. Therefore, **Zingaro is justified in thinking that unpolished stones may not be suitable for printing.**

3.2 Is the mean hardness of the polished and unpolished stones the same?

1. Perform Visual Analysis: -




```
The mean hardness of polished stones is 147.79
The mean hardness of unpolished stones is 134.11
The standard deviation of polished stones is 15.59
The standard deviation of unpolished stones is 33.04
```

Figure 15: Histogram – Distribution of Hardness for Polished & Unpolished (Zingaro)

- The chart above shows the distribution of the Brinell hardness values for both polished (in Red) and unpolished stones (in Blue). The green dashed line at 150 represents the required hardness threshold for printing.
- From the visual analysis, we can observe that the polished stones generally have a higher hardness than the unpolished stones, and more polished stones are above the threshold of 150. This supports the conclusion that the mean hardness of polished and unpolished stones is significantly different.
- To further strengthen the claim for the population, we carry out the hypothesis testing.

2. State the Null and Alternate Hypotheses: -
    - **Null Hypothesis (H_0):** The mean hardness of polished stones is equal to the mean hardness of unpolished stones (mean_polished = mean_unpolished)
    - **Alternative Hypothesis (H_1):** The mean hardness of polished stones is different from the mean hardness of unpolished stones (mean_polished ≠ mean_unpolished)

3. Select Appropriate Test & Decide Significance Level: -
   - To determine if there is a difference in the mean hardness of polished and unpolished stones, we can use a **two-sample t-test (two-tailed)** for the means of two independent groups.
   - Also, since the **standard deviations are different for both samples**, we have to keep that in mind while performing the test.
   - It is given in the question to consider significance level as 5%.

4. Compute the P-value: -
   - We conduct 2 sample t-test (two-tailed) with Unpolished & Polished hardness data, considering unequal variances for both samples. Below is the Python output: -

```
The p-value is  0.001588379295584306
As the p-value 0.001588379295584306 is less than the level of significance, we reject the null hypothesis.
```

Figure 16: P-Value – Problem 3.2 (Zingaro)

5. Conclusion from the Test Result: -
   - Since the p-value is smaller than the significance level of 0.05, we reject the null hypothesis.
   - This suggests **there is statistical evidence that the mean hardness of polished stones is significantly different from the mean hardness of unpolished stones.**

Problem 4

Dental implant data: The hardness of metal implants in dental cavities depends on multiple factors, such as the method of implant, the temperature at which the metal is treated, the alloy used as well as the dentists, who may favour one method above another, and may work better in his/her favourite method. The response is the variable of interest.

4.1 How does the hardness of implants vary depending on dentists?
4.2 How does the hardness of implants vary depending on methods?
4.3 What is the interaction effect between the dentist and method on the hardness of dental implants for each type of alloy?
4.4 How does the hardness of implants vary depending on dentists and methods together?

Solution 4

Context

Dental implant data: The hardness of metal implants in dental cavities depends on multiple factors, such as (1) the method of implant (2) the temperature at which the metal is treated (3) the alloy used, as well as, (4) the dentists, who may favour one method above another, and may work better in his/her favourite method. The response is the variable of interest.

Objective

Suppose you are a Data Scientist and the Data Science team has shared some of the key questions that need to be answered. Perform the data analysis to find answers to these questions.

4.1 How does the hardness of implants vary depending on dentists?
4.2 How does the hardness of implants vary depending on methods?
4.3 What is the interaction effect between the dentist and method on the hardness of dental implants for each type of alloy?
4.4 How does the hardness of implants vary depending on dentists and methods together?

Data Description

- **Dentist**: Categories of Dentists from 1 to 5
- **Method**: Categories of Method deployed for the Implant from 1 to 3
- **Alloy**: Type of the Alloy used for the Implant. We have 2 types in the dataset
- **Temp**: Temperature at which the metal is treated (3 values)
- **Response**: Hardness of the Implants

Load the Dataset

- Import required libraries to load the dataset, perform data manipulation, analysis & visualization tasks to solve the business problem.
- Below is the view of the first 5 rows of the imported dataset to give a quick overview of the data: -

	Dentist	Method	Alloy	Temp	Response
0	1	1	1	1500	813
1	1	1	1	1600	792
2	1	1	1	1700	792
3	1	1	2	1500	907
4	1	1	2	1600	792

Figure 17: First 5 Rows of the Dental Implant Dataset

Data Preparation & Exploratory Data Analysis

- Checking shape of dataset – 90 rows & 5 columns

- Checking the data types of the columns for the dataset: -

```
Data columns (total 5 columns):
 #   Column    Non-Null Count  Dtype
---  ------    --------------  -----
 0   Dentist   90 non-null     int64
 1   Method    90 non-null     int64
 2   Alloy     90 non-null     int64
 3   Temp      90 non-null     int64
 4   Response  90 non-null     int64
dtypes: int64(5)
```

Figure 18: Data-types of columns (Dental Implant)

- All 5 columns have 90 values.
- All 5 variables are of Int64 data-type. Since, Dentist, Method & Alloy are categorical variables, we must change them to categorical data-type. Post the change, below is the updated data-types for all the variables: -

```
Data columns (total 5 columns):
 #   Column    Non-Null Count  Dtype
---  ------    --------------  -----
 0   Dentist   90 non-null     category
 1   Method    90 non-null     category
 2   Alloy     90 non-null     category
 3   Temp      90 non-null     int64
 4   Response  90 non-null     int64
dtypes: category(3), int64(2)
```

Figure 19: Data-types of columns (Dental Implant) – post data-type treatment

- Checking for missing/error & duplicate values: -

```
Duplicate Count:  0

Null Values:-
Dentist     0
Method      0
Alloy       0
Temp        0
Response    0
```

Figure 20: Null & Duplicate value Check (Dental Implant)

- We have neither duplicate values nor null/error values. Hence, no treatment required.

- Getting the statistical summary of both the numerical fields

	count	unique	top	freq	mean	std	min	25%	50%	75%	max
Dentist	90.0	5.0	1.0	18.0	NaN	NaN	NaN	NaN	NaN	NaN	NaN
Method	90.0	3.0	1.0	30.0	NaN	NaN	NaN	NaN	NaN	NaN	NaN
Alloy	90.0	2.0	1.0	45.0	NaN	NaN	NaN	NaN	NaN	NaN	NaN
Temp	90.0	NaN	NaN	NaN	1600.000000	82.107083	1500.0	1500.0	1600.0	1700.0	1700.0
Response	90.0	NaN	NaN	NaN	741.777778	145.767845	289.0	698.0	767.0	824.0	1115.0

Figure 21: Statistical Summary of the Numerical Variables (Dental Implant)

- Observations & Insights can be summarized below in the table: -

Type	Columns	Observations & Insights
Numerical	Response	✓ Implant hardness ranges between 289 to 1115. ✓ Mean is 741.77 & Median is 767. ✓ Standard Deviation is 145.77.
Numerical	Temp	✓ Temperature ranges between 1500 to 1700. ✓ Mean & Mean are 1600. ✓ Standard Deviation is 82.12.
Categorical	Alloy	✓ Alloy types have 2 values. ✓ Each alloy has 45 values each.
Categorical	Method	✓ Method types have 3 values. ✓ Each Method has 30 values each.
Categorical	Dentist	✓ Dentist types have 5 values. ✓ Each Dentist has 18 values each.

Table 3: Statistical Summary of the Numerical Variables (Dental Implant)

- Perform Univariate Analysis: –
    - Use Histograms & Boxplots to analyse each numerical variable: -

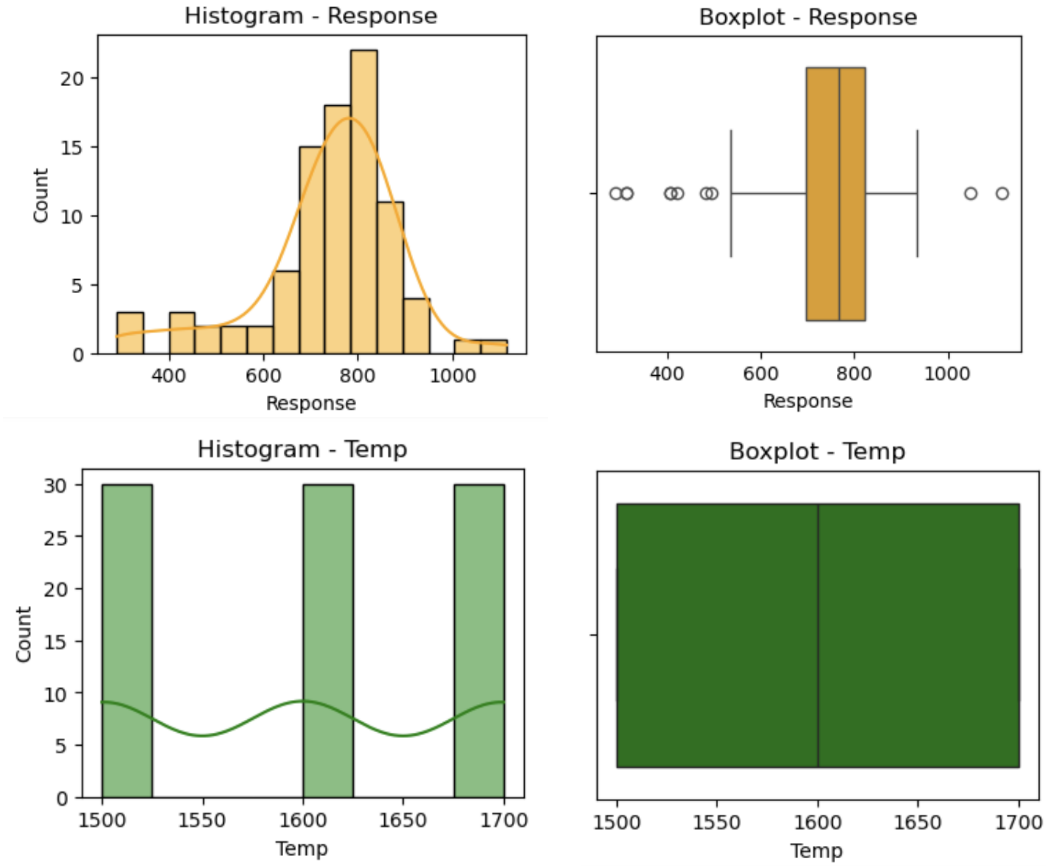


Figure 22: Univariate Analysis – Numerical (Dental Implant)

- Observations & Insights can be summarized below: -
    - ➢ Response: -
        - ✓ Distribution of Unpolished seems to be symmetric.
        - ✓ Unimodal distribution having a single peak at 800.
        - ✓ Total 8 Outliers observed.
        - ✓ We choose to ignore outlier treatment to avoid any loss of information. This is to account for any error by chance on part of a Dentist using a particular method, leading to an unusual hardness of the implant.

```
Lower Wishker at 509.0 | Upper Whisker at 1013.0
Lower Whisker Outlier Count = 8
Upper Whisker Outlier Count = 2
Total Outlier Count= 10
```

Figure 23: Outlier inspection of Response (Dental Implant)

- ➢ Temp: -
    - ✓ Distribution of Temp seems to be uniform.
    - ✓ 3 values observed at 1500, 1600 & 1700.
    - ✓ No Outliers observed.
    - ✓ No Outlier treatment required

```
Lower Wishker at 1200.0 | Upper Whisker at 2000.0
Lower Whisker Outlier Count = 0
Upper Whisker Outlier Count = 0
Total Outlier Count= 0
```

Figure 24: Outlier Inspection for Polished (Dental Implant)

- Use Countplots to analyse each categorical variable.
    - Each variable is uniformly distributed, implying each categorical value has equal no of values.

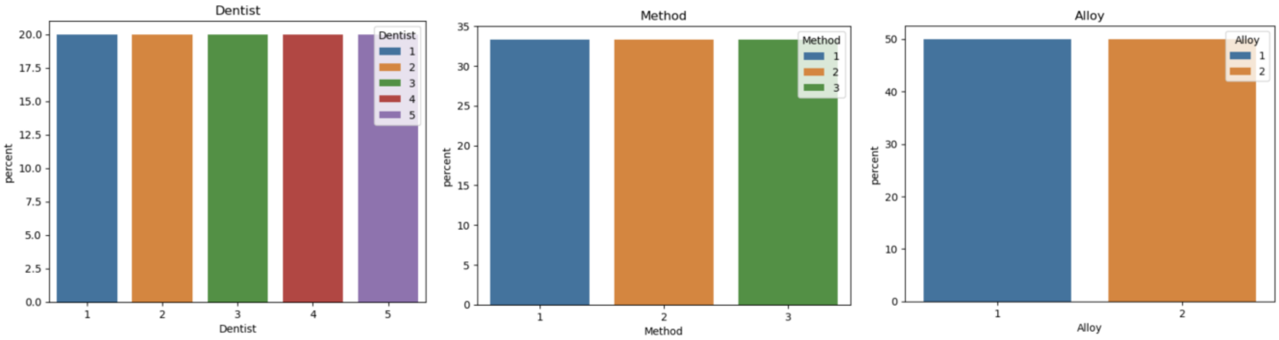


Figure 25: Univariate Analysis of Categorical Variables (Dental Implant)

- Perform Bivariate Analysis: –
    - Use Heatmap to analyse the correlation between numerical variables.
        - There is no correlation between Response & Temp

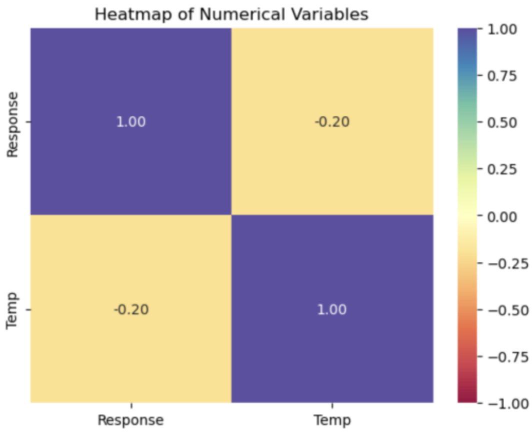


Figure 26: Bivariate Analysis of Categorical Variables (Dental Implant)

- Bivariate analysis of Categorical & Categorical vs Numerical shall be done under 'Perform Visual Analysis' section in context with the question asked.

4.1 How does the hardness of implants vary depending on dentists?

1. Perform Visual Analysis: -

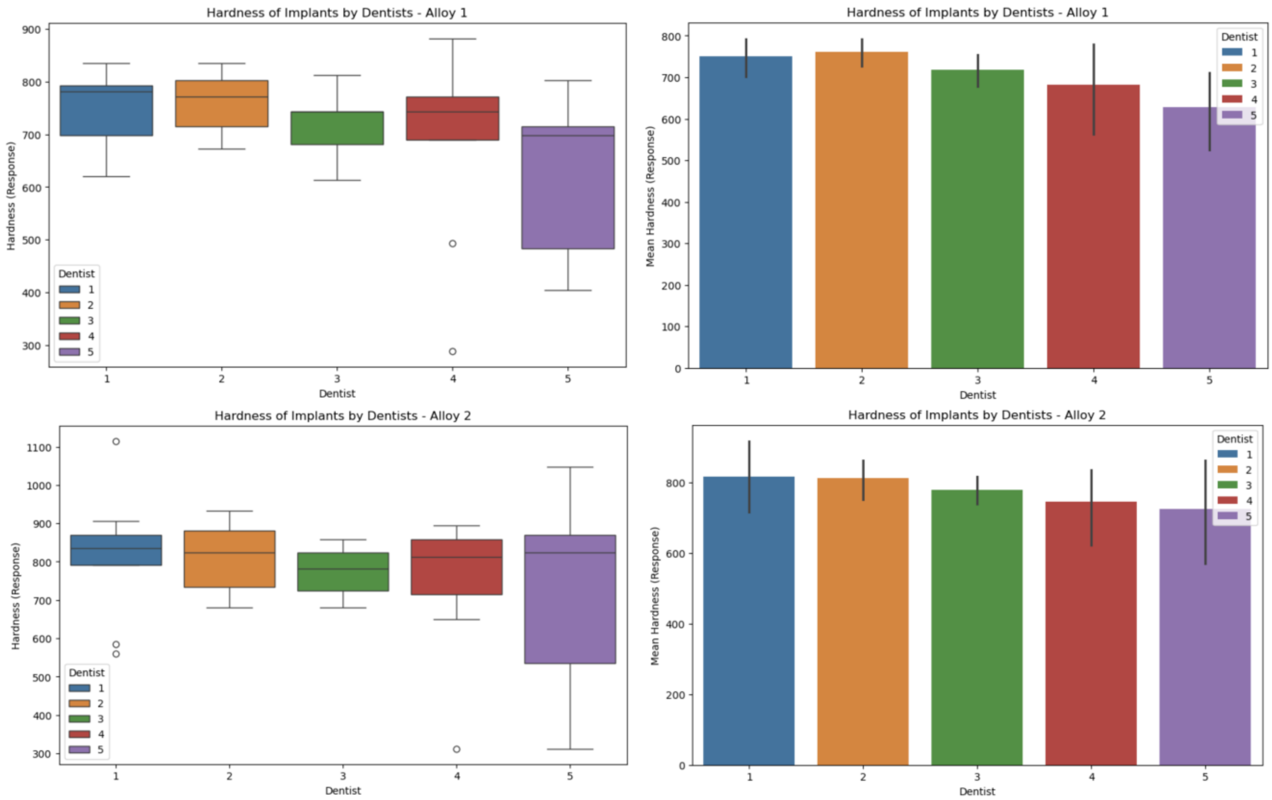


Figure 27: Boxplot for Hardness by Dentist for each Alloy (Dental Implant)

- **Alloy 1**: The distribution of implant hardness for each dentist shows trivial variation, but for it to be statistically significant we will have to carry out one-way ANOVA.
- **Alloy 2**: The implant hardness distribution appears to be relatively consistent across dentists. However, for us to statistically establish the result we will have to carry out one-way ANOVA.

2. State the Null and Alternate Hypotheses: -

- Although the test would be done separately for Alloy 1 & Alloy 2, Null & Alternate Hypothesis statements would remain the same.
- **Null Hypothesis (H_0):** There is no significant difference in the hardness of implants depending on the Dentist for each Alloy (to be tested each for Alloy 1 and Alloy 2).
- **Alternative Hypothesis (H_1):** There is a significant difference in the hardness of implants depending on the Dentist for each Alloy (to be tested each for Alloy 1 and Alloy 2).

3. Select Appropriate Test & Decide Significance Level: -

- In a one-way ANOVA test, we compare the means from several populations to test if there is any significant difference between them. In this question, we will compare Hardness for each of the Dentist groups, hence, one-way ANOVA is the appropriate test.
- It is given in the question to consider significance level as 5%.

4. Check for Assumptions: -

- **Independence**: The observations within each group and between groups must be independent of each other. This assumption ensures that there is no systematic relationship among observations, which might otherwise affect the validity of the ANOVA results.
  - ➢ We are assuming in our question that the observations from each group & between the groups are independent of each other as nowhere in the question it is stated otherwise.

- **Normality:** Response variable follows a normal distribution
  - We will apply the Shapiro-Wilk's test on the Response variable.
  - ✓ Null Hypothesis (H_0): The Implant Hardness follow a normal distribution for each alloy.
  - ✓ Alternative Hypothesis (H_1): The Implant Hardness do not follow a normal distribution for each alloy.
  - We carry out the test on the Response variable for each of Alloy 1 & Alloy 2.
  - Below is the Python output for both alloys: -

```
Shapiro | Alloy 1 - As the p-value 1.1945308699072215e-05 is less than the level of significance, we reject the null hypothesis i.e. Implant Hardness for
Alloy 1 do not follow a normal distribution
Shapiro | Alloy 2 - As the p-value 0.00040293129942514585 is less than the level of significance, we reject the null hypothesis i.e. Implant Hardness for
Alloy 2 do not follow a normal distribution
```

Figure 28: P-Value – Shapiro-Wilk's Test for both Alloys

  - As the P-value < 0.05 for both Alloys, we reject the null hypothesis, implying the Normality condition is not met.
  - However, we continue with ANOVA as it is mentioned in the question to proceed even if the assumptions are not met.

- **Homogeneity of Variances:** The variances of the response variable should be approximately equal across all groups/levels.
  - We will apply the Levene's test on the Response variable for various groups/levels of Dentists.
    - ✓ Null Hypothesis (H_0): All the Dentist population variances are equal for each alloy type.
    - ✓ Alternative Hypothesis (H_1): At least one Dentist population variance is different from the rest for each alloy type.
  - We carry out the test on the Response variable among various Dentist groups/levels for each of Alloy 1 & Alloy 2.
  - Below is the python output for both alloys: -

```
Levene | Alloy 1 - As the p-value 0.2565537418543795 is greater than the level of significance, we fail to reject the null hypothesis i.e. All the Dentis
t population variances are equal for Alloy 1
Levene | Alloy 2 - As the p-value 0.23686777576324952 is greater than the level of significance, we fail to reject the null hypothesis i.e. All the Denti
st population variances are equal for Alloy 2
```

Figure 29: P-Value – Levene's Test (Dentist) for both Alloys

  - As the p-value > 0.05 for both Alloys, we fail to reject the null hypothesis, implying all the Dentist population variances are equal for both the Alloys.
  - The Homogeneity of Variances condition is Met.

5. Compute the P-value: -
   - We conduct 1-way ANOVA with Response variable for all Dentist Levels. Below is the Python output: -

```
F-OneWay | Alloy 1 - As the p-value 0.11656712140267628 is greater than the level of significance, we fail to reject the null hypothesis i.e. there is no
significant difference in the Implant Hardness between dentists for Alloy 1.
F-OneWay | Alloy 2 - As the p-value 0.7180309510793431 is greater than the level of significance, we fail to reject the null hypothesis i.e. there is no
significant difference in the Implant Hardness between dentists for Alloy 2.
```

Figure 30: P-Value – One-way ANOVA (Dentist) for both Alloys

6. Conclusion from the Test Result: -
   - **Alloy 1**: As the P-value (0.117) > 0.05, we **fail to reject the null hypothesis**. Thus, there is **no significant difference in the hardness** of implants between Dentist Levels for Alloy 1.
   - **Alloy 2**: As the P-value (0.718) > 0.05, we **fail to reject the null hypothesis**. Thus, there is **no significant difference in the hardness** of implants between Dentist Levels for Alloy 2.

4.2 How does the hardness of implants vary depending on methods?

1. Perform Visual Analysis: -

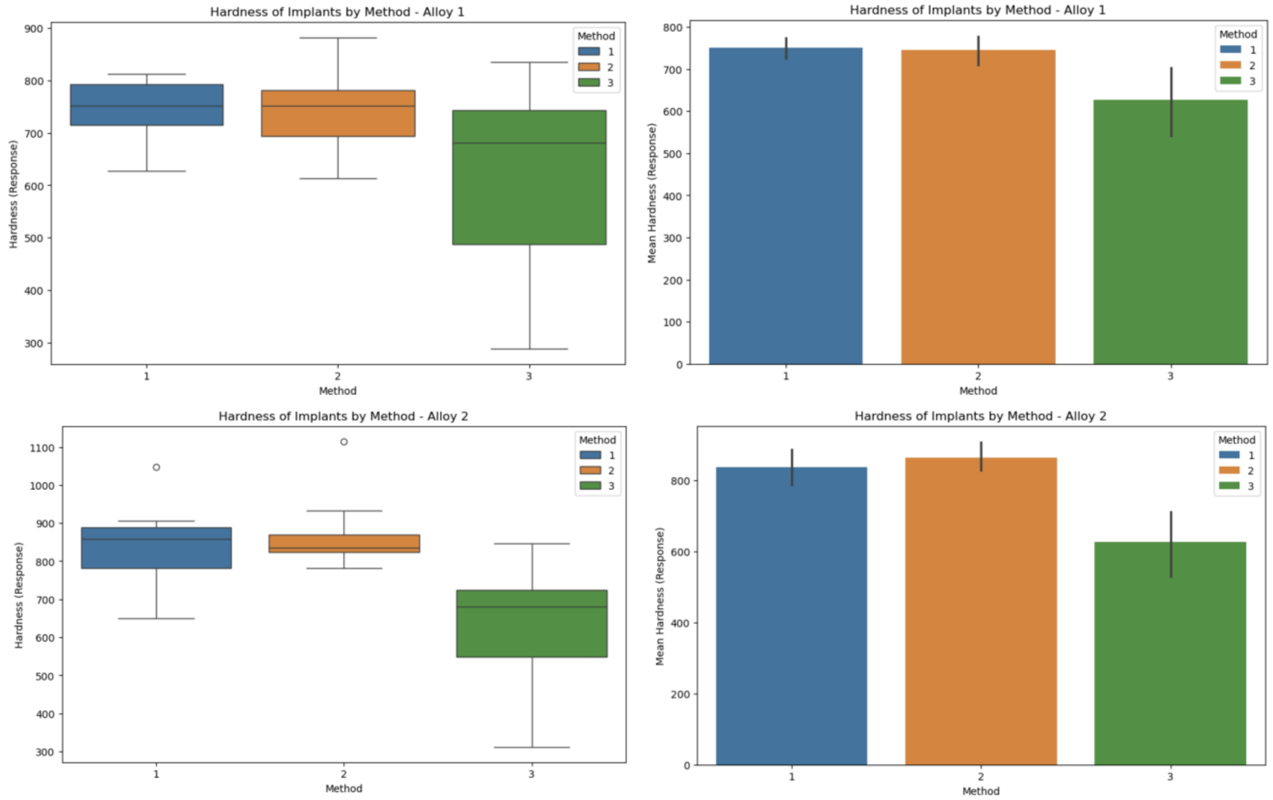


Figure 31: Boxplot for Hardness by Method for each Alloy (Dental Implant)

- **Alloy 1**: The distribution of implant hardness for the 3rd Method shows variation, but for it to be statistically significant we will have to carry out one-way ANOVA.
- **Alloy 2**: The distribution of implant hardness for the 3rd Method shows variation, but for it to be statistically significant we will have to carry out one-way ANOVA.

2. State the Null and Alternate Hypotheses: -
   - Although the test would be done separately for Alloy 1 & Alloy 2, Null & Alternate Hypothesis statements would remain the same.
   - **Null Hypothesis (H_0):** There is no significant difference in the hardness of implants depending on the Method for each Alloy (to be tested each for Alloy 1 and Alloy 2).
   - **Alternative Hypothesis (H_1):** There is a significant difference in the hardness of implants depending on the Method for each Alloy (to be tested each for Alloy 1 and Alloy 2).

3. Select Appropriate Test & Decide Significance Level: -
   - In a one-way ANOVA test, we compare the means from several populations to test if there is any significant difference between them. In this question, we will compare Hardness for each of the Method groups, hence, one-way ANOVA is the appropriate test.
   - It is given in the question to consider significance level as 5%.

4. Check for Assumptions: -
   - **Independence**: The observations within each group and between groups must be independent of each other. This assumption ensures that there is no systematic relationship among observations, which might otherwise affect the validity of the ANOVA results.
     - We are assuming in our question that the observations from each group & between the groups are independent of each other as nowhere in the question it is stated otherwise.

- **Normality:** Response variable follows a normal distribution
  - We will apply the Shapiro-Wilk's test on the Response variable.
    - ✓ Null Hypothesis (H_0): The Implant Hardness follow a normal distribution for each alloy.
    - ✓ Alternative Hypothesis (H_1): The Implant Hardness do not follow a normal distribution for each alloy.
  - We carry out the test on the Response variable for each of Alloy 1 & Alloy 2.
  - Please note that this step has been done already in the previous question. Python output is same as computed in Question 4.1 (refer, Figure 28)
  - As the P-value < 0.05 for both Alloys, we reject the null hypothesis, implying the Normality condition is not met.
  - However, we continue with ANOVA as it is mentioned in the question to proceed even if the assumptions are not met.

- **Homogeneity of Variances:** The variances of the response variable should be approximately equal across all groups.
  - We will apply the Levene's test on the Response variable for various groups of Methods.
    - ✓ Null Hypothesis (H_0): All the Method population variances are equal for each alloy type.
    - ✓ Alternative Hypothesis (H_1): At least one Method population variance is different from the rest for each alloy type.
  - We carry out the test on the Response variable among various Method groups for each of Alloy 1 & Alloy 2.
  - Below is the python output for both alloys: -

```
Levene | Alloy 1 - As the p-value 0.0034160381460233975 is less than the level of significance, we reject the null hypothesis i.e. At least one Method va
riance is different from the rest for Alloy 1
Levene | Alloy 2 - As the p-value 0.04469269939158668 is less than the level of significance, we reject the null hypothesis i.e. At least one Method vari
ance is different from the rest for Alloy 2
```

Figure 32: P-Value – Levene's Test (Method) for both Alloys

  - As the P-value < 0.05 for both Alloys, we reject the null hypothesis, implying the Homogeneity of Variances condition is Not Met.
  - However, we continue with ANOVA as it is mentioned in the question to proceed even if the assumptions are not met.

5. Compute the P-value: -
   - We conduct 1-way ANOVA with Response variable for all Method Levels. Below is the Python output: -

```
F-OneWay | Alloy 1 - As the p-value 0.004163412167505543 is less than the level of significance, we reject the null hypothesis i.e. there is significant
difference in the Implant Hardness between Methods for Alloy 1
F-OneWay | Alloy 2 - As the p-value 5.415871051443187e-06 is less than the level of significance, we reject the null hypothesis i.e. there is significant
difference in the Implant Hardness between dentists for Alloy 2
```

Figure 33: P-Value – One-way ANOVA (Method) for both Alloys

6. Conclusion from the Test Result: -
   - **Alloy 1**: As the P-value (0.004) < 0.05, we **reject the null hypothesis**. Thus, there is **significant difference in the hardness** of implants between Method Levels for Alloy 1.
   - **Alloy 2**: As the P-value (5.42e-06) < 0.05, we **reject the null hypothesis**. Thus, there is **significant difference in the hardness** of implants between Method Levels for Alloy 2.

7. Identify Method Pairs with Significant Difference in Hardness: -
   - Perform **Multiple Comparison Test (Tukey HSD)** to identify which Method Level Mean Hardness is different from other Level.
   - **Null Hypothesis (H_0):** There is no significant difference in the Mean Implant Hardness between the two Method-levels being compared for each Alloy.
   - **Alternative Hypothesis (H_1):** There is a significant difference in the Mean Implant Hardness between the two Method-levels being compared for each Alloy.

- Perform multiple pairwise comparison (Tukey HSD). Below is the Python output: -

```
Tukey HSD | Alloy 1
   Multiple Comparison of Means - Tukey HSD, FWER=0.05
=========================================================
group1 group2  meandiff p-adj   lower     upper   reject
---------------------------------------------------------
     1      2   -6.1333  0.987 -102.714  90.4473  False
     1      3    -124.8 0.0085 -221.3807 -28.2193  True
     2      3 -118.6667 0.0128 -215.2473  -22.086  True
---------------------------------------------------------
```

```
Tukey HSD | Alloy 2
   Multiple Comparison of Means - Tukey HSD, FWER=0.05
=========================================================
group1 group2 meandiff p-adj    lower     upper   reject
---------------------------------------------------------
     1      2     27.0 0.8212  -82.4546 136.4546  False
     1      3   -208.8 0.0001 -318.2546  -99.3454  True
     2      3   -235.8    0.0 -345.2546 -126.3454  True
---------------------------------------------------------
```

Figure 34: Tukey HSD Test (Method) for both Alloys

- Inference: -
  - **Alloy 1**: -
    - Comparison between **Method 1 and Method 2**:
      - ✓ Mean Difference: -6.13
      - ✓ P-value: 0.987
      - ✓ P-value > 0.05: **No significant difference** (Fail to reject null hypothesis)
    - Comparison between **Method 1 and Method 3**:
      - ✓ Mean Difference: -124.8
      - ✓ P-value: 0.0085
      - ✓ P-value < 0.05: **Significant difference** (Reject null hypothesis)
    - Comparison between **Method 2 and Method 3**:
      - ✓ Mean Difference: -118.67
      - ✓ P-value: 0.0128
      - ✓ P-value < 0.05: **Significant difference** (Reject null hypothesis)
  - **Alloy 2**: -
    - Comparison between **Method 1 and Method 2**:
      - ✓ Mean Difference: 27
      - ✓ P-value: 0.8212
      - ✓ P-value > 0.05: **No significant difference** (Fail to reject null hypothesis)
    - Comparison between **Method 1 and Method 3**:
      - ✓ Mean Difference: -208.8
      - ✓ P-value: 0.0001
      - ✓ P-value < 0.05: **Significant difference** (Reject null hypothesis)
    - Comparison between **Method 2 and Method 3**:
      - ✓ Mean Difference: -235.8
      - ✓ P-value: 0.0000
      - ✓ P-value < 0.05: **Significant difference** (Reject null hypothesis)
- **Summary**: -
  - ➢ Alloy 1: Method 3 significantly differs from both Methods 1 and 2.
  - ➢ Alloy 2: Method 3 significantly differs from both Methods 1 and 2.

4.3 What is the interaction effect between the dentist and method on the hardness of dental implants for each type of alloy?

1. Create Interaction Plot: -

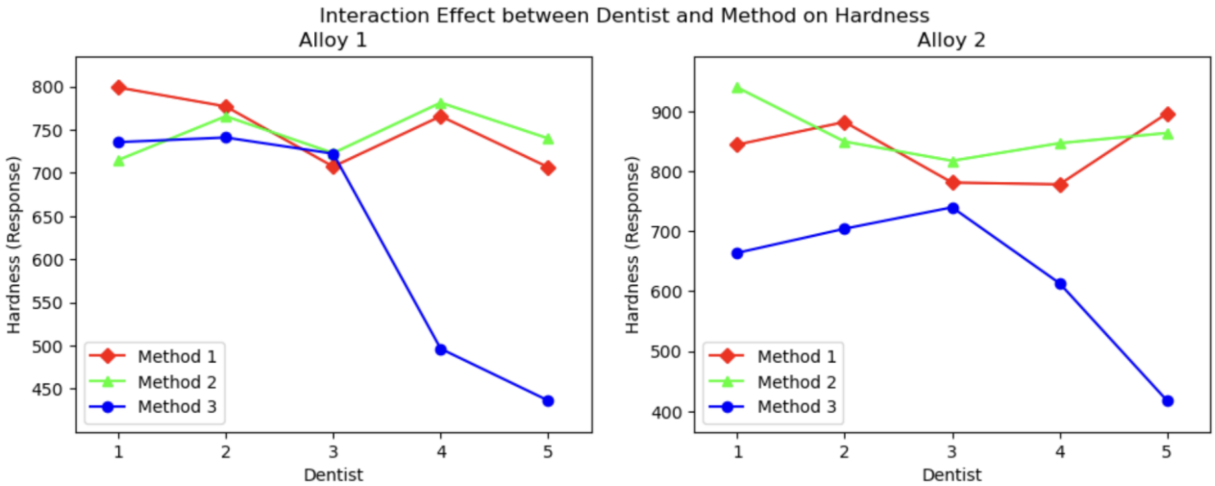


Figure 35: Interaction Plot between Dentist & Method on Hardness

2. Draw Inferences from the Interaction Plot: -

- Alloy 1: -
  - Non-Parallel Lines:
    - ✓ The lines for different methods are not entirely parallel, indicating there is an interaction effect between Dentist and Method on the Hardness of implants.
    - ✓ This means the effect of the Method on Hardness is not consistent across different Dentists.
  - Difference in Hardness by Method:
    - ✓ Method 3 generally shows a lower hardness compared to Method 1 and Method 2 for most Dentists.
    - ✓ There is some variation in how each Dentist performs each Method, which suggests a difference in proficiency or preference.
- Alloy 2: -
  - Strong Interaction Effect:
    - ✓ There is a more pronounced difference between the Methods across different Dentists, as evident in the above chart having non-parallel lines.
    - ✓ This indicates that the Method's effect on hardness varies substantially depending on the dentist.
  - Variation between Method 2 and Method 3:
    - ✓ Method 2 & Method 3 have more fluctuating results, and the differences between these methods (specially Method 3) vary significantly across different dentists.
    - ✓ For some Dentists, Method 3 performs much worse compared to the others.
- Summary: -
  - There is a significant **interaction effect** between **Dentist** and **Method** for both **Alloy 1** and **Alloy 2**. This implies that the choice of method and the specific dentist performing it, both, influence the hardness of dental implants.
  - **Alloy 2** shows a more pronounced interaction, indicating that different dentists' outcomes vary substantially depending on which method they use, more so than **Alloy 1**.
  - **Method 3** often shows poorer performance compared to **Method 1 & Method 2**, suggesting that it may be less reliable or less preferred for achieving high hardness values.
- Next Steps: -
  - To statistically establish the interaction inference, we must carry out 2-way ANOVA.

4.4 How does the hardness of implants vary depending on dentists and methods together?

1. Perform Visual Analysis: -
   - Please refer Question 4.3 – Interaction Plot (Figure 35), for the visual plot & analysis on the same.

2. State the Null and Alternate Hypotheses: -
   - Although the test would be done separately for Alloy 1 & Alloy 2, Null & Alternate Hypothesis statements would remain the same.
   - **Null Hypothesis (H_0):** There is no interaction effect between Dentists and Methods on Implant Hardness for each Alloy (to be tested each for Alloy 1 and Alloy 2).
   - **Alternative Hypothesis (H_1):** There is an interaction effect between Dentists and Methods on Implant Hardness for each Alloy (to be tested each for Alloy 1 and Alloy 2).

3. Select Appropriate Test & Decide Significance Level: -
   - This problem concerns the effect of two independent variables on a dependent variable. Two-way ANOVA test is the appropriate test here.
   - It is given in the question to consider significance level as 5%.

4. Check for Assumptions: -
   - **Independence**: The observations within each group and between groups must be independent of each other. This assumption ensures that there is no systematic relationship among observations, which might otherwise affect the validity of the ANOVA results.
     - ➢ We are assuming in our question that the observations from each group & between the groups are independent of each other as nowhere in the question it is stated otherwise.
   - **Normality:** Response variable follows a normal distribution
     - ➢ We will apply the Shapiro-Wilk's test on the Response variable.
       - ✓ Null Hypothesis (H_0): The Implant Hardness follow a normal distribution for each alloy.
       - ✓ Alternative Hypothesis (H_1): The Implant Hardness do not follow a normal distribution for each alloy.
     - ➢ We carry out the test on the Response variable for each of Alloy 1 & Alloy 2.
     - ➢ Please note that this step has been done already in the previous question. Python output is same as computed in Question 4.1 (refer, Figure 28)
     - ➢ As the P-value < 0.05 for both Alloys, we reject the null hypothesis, implying the Normality condition is not met.
     - ➢ However, we continue with ANOVA as it is mentioned in the question to proceed even if the assumptions are not met.

   - **Homogeneity of Variances:** The variances of the response variable should be approximately equal across all groups.
     - ➢ We will apply the Levene's test on the Response variable for various groups of Methods & Dentists.
       - ✓ Null Hypothesis (H_0): All the Dentist **X** Method population variances are equal for each alloy type.
       - ✓ Alternative Hypothesis (H_1): At least one Dentist **X** Method population variance is different from the rest for each alloy type.
     - ➢ We carry out the test on the Response variable among various Dentist & Method groups for each of Alloy 1 & Alloy 2.
     - ➢ Levene Test's | Response on Dentist – Please refer Figure 29 (already done in previous section) | Condition Met
     - ➢ Levene Test's | Response on Method – Please refer Figure 32 (already done in previous section) | Condition Not Met
     - ➢ Levene Test's | Response on Dentist X Method. Below is the python output for both alloys: -

```
Levene | Alloy 1 - As the p-value 0.3128166652989495 is greater than the level of significance, we fail to reject the null hypothesis i.e. All the popula
tion variances are equal for Alloy 1
Levene | Alloy 2 - As the p-value 0.7831735515657826 is greater than the level of significance, we fail to reject the null hypothesis i.e. All the popula
tion variances are equal for Alloy 2
```

Figure 36: P-Value – Levene's Test (Method X Dentist) for both Alloys

- ➢ As the P-value > 0.05 for both Alloys, we fail reject the null hypothesis, implying the Homogeneity of Variances condition is Met.

5. Compute the P-value: -
   - We conduct 2-way ANOVA with Response variable with the below formula: -
   - Formula = 'Response ~ C(Dentist) + C(Method) + C(Dentist):C(Method)'

     where,

     ✓ Response – Dependent variable

     ✓ Dentist – Independent categorical variable

     ✓ Method – Independent categorical variable

     ✓ Dentist X Method – Interaction categorical variable between Dentist & Method
   - Below is the Python output: -

------------------------2-Way ANOVA - Alloy 1------------------------

	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist)	4.0	106683.688889	26670.922222	3.899638	0.011484
C(Method)	2.0	148472.177778	74236.088889	10.854287	0.000284
C(Dentist):C(Method)	8.0	185941.377778	23242.672222	3.398383	0.006793
Residual	30.0	205180.000000	6839.333333	NaN	NaN

------------------------2-Way ANOVA - Alloy 2------------------------

	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist)	4.0	56797.911111	14199.477778	1.106152	0.371833
C(Method)	2.0	499640.400000	249820.200000	19.461218	0.000004
C(Dentist):C(Method)	8.0	197459.822222	24682.477778	1.922787	0.093234
Residual	30.0	385104.666667	12836.822222	NaN	NaN

Figure 37: 2-way ANOVA for both Alloys

6. Conclusion from the Test Result: -
   - **Alloy 1**: -
     - **Dentist**:

       ✓ F-statistic: 3.90 (i.e. between-group variability **greater than** within-group variability)

       ✓ P-value: 0.011

       ✓ P-value < 0.05: There is a significant effect of Dentist on Hardness (Reject null hypothesis)
     - **Method**:

       ✓ F-statistic: 10.85 (i.e. between-group variability **greater than** within-group variability)

       ✓ P-value: 0.0003

       ✓ P-value < 0.05: There is a significant effect of Method on Hardness (Reject null hypothesis)
     - **Interaction (Dentist X Method)**:

       ✓ F-statistic: 3.4 (i.e. between-group variability **greater than** within-group variability)

       ✓ P-value: 0.0068

       ✓ P-value < 0.05: There is a significant effect of Interaction between Dentist & Method on Hardness (Reject null hypothesis). This means that the effect of the method on hardness depends on a specific dentist.
   - **Alloy 2**: -
     - **Dentist**:

       ✓ F-statistic: 1.11 (i.e. between-group variability **nearly equal** to within-group variability)

       ✓ P-value: 0.372

       ✓ P-value > 0.05: There is no significant effect of Dentist on Hardness (Fail to reject null hypothesis)
     - **Method**:

       ✓ F-statistic: 19.46 (i.e. between-group variability **greater than** within-group variability)

       ✓ P-value: 0.000004

       ✓ P-value < 0.05: There is a significant effect of Method on Hardness (Reject null hypothesis)
     - **Interaction (Dentist X Method)**:

       ✓ F-statistic: 1.92 (i.e. between-group variability **nearly equal** to within-group variability)

       ✓ P-value: 0.093

       ✓ P-value > 0.05: There is no significant effect of Interaction between Dentist & Method on Hardness (Fail to reject null hypothesis). This means that the effect of the method is consistent across dentists.
   - **Summary**:
     - **Alloy 1**: Both **Dentist** and **Method** have significant effects on **Hardness**, and there is a significant Interaction effect between the two. This indicates that the effectiveness of a **Method** varies depending on the specific **Dentist**.
     - **Alloy 2**: The **Method** significantly affects **Hardness**, but the **Dentist** does not. There is also no significant **Interaction** effect, suggesting that the effect of the **Method** is consistent across different **Dentists**.

7. Identify Method Pairs with Significant Difference in Hardness: -
   - Perform **Multiple Comparison Test (Tukey HSD)** to identify which Method-Dentist Combination Mean Hardness is different from other combinations.
   - **Null Hypothesis (H_0):** There is no significant difference in the Mean Implant Hardness between the two Method-Dentist Combination being compared for each Alloy.
   - **Alternative Hypothesis (H_1):** There is significant difference in the Mean Implant Hardness between the two Method-Dentist Combination being compared for each Alloy.
   - Perform multiple pairwise comparison (Tukey HSD). Below is the Python output: -

Tukey HSD | Alloy 1

Multiple Comparison of Means - Tukey HSD, FWER=0.05

group1	group2	meandiff	p-adj	lower	upper	reject
Method1-Dentist1	Method1-Dentist2	-22.0	1.0	-270.8283	226.8283	False
Method1-Dentist1	Method1-Dentist3	-91.6667	0.9853	-340.495	157.1617	False
Method1-Dentist1	Method1-Dentist4	-33.3333	1.0	-282.1617	215.495	False
Method1-Dentist1	Method1-Dentist5	-92.3333	0.9844	-341.1617	156.495	False
Method1-Dentist1	Method2-Dentist1	-84.0	0.9933	-332.8283	164.8283	False
Method1-Dentist1	Method2-Dentist2	-33.3333	1.0	-282.1617	215.495	False
Method1-Dentist1	Method2-Dentist3	-76.0	0.9975	-324.8283	172.8283	False
Method1-Dentist1	Method2-Dentist4	-17.6667	1.0	-266.495	231.1617	False
Method1-Dentist1	Method2-Dentist5	-59.0	0.9998	-307.8283	189.8283	False
Method1-Dentist1	Method3-Dentist1	-63.3333	0.9996	-312.1617	185.495	False
Method1-Dentist1	Method3-Dentist2	-58.0	0.9999	-306.8283	190.8283	False
Method1-Dentist1	Method3-Dentist3	-76.6667	0.9972	-325.495	172.1617	False
Method1-Dentist1	Method3-Dentist4	-302.6667	0.007	-551.495	-53.8383	True
Method1-Dentist1	Method3-Dentist5	-362.6667	0.0007	-611.495	-113.8383	True
Method1-Dentist2	Method1-Dentist3	-69.6667	0.999	-318.495	179.1617	False
Method1-Dentist2	Method1-Dentist4	-11.3333	1.0	-260.1617	237.495	False
Method1-Dentist2	Method1-Dentist5	-70.3333	0.9989	-319.1617	178.495	False
Method1-Dentist2	Method2-Dentist1	-62.0	0.9997	-310.8283	186.8283	False
Method1-Dentist2	Method2-Dentist2	-11.3333	1.0	-260.1617	237.495	False
Method1-Dentist2	Method2-Dentist3	-54.0	0.9999	-302.8283	194.8283	False
Method1-Dentist2	Method2-Dentist4	4.3333	1.0	-244.495	253.1617	False
Method1-Dentist2	Method2-Dentist5	-37.0	1.0	-285.8283	211.8283	False
Method1-Dentist2	Method3-Dentist1	-41.3333	1.0	-290.1617	207.495	False
Method1-Dentist2	Method3-Dentist2	-36.0	1.0	-284.8283	212.8283	False
Method1-Dentist2	Method3-Dentist3	-54.6667	0.9999	-303.495	194.1617	False
Method1-Dentist2	Method3-Dentist4	-280.6667	0.016	-529.495	-31.8383	True
Method1-Dentist2	Method3-Dentist5	-340.6667	0.0016	-589.495	-91.8383	True
Method1-Dentist3	Method1-Dentist4	58.3333	0.9999	-190.495	307.1617	False
Method1-Dentist3	Method1-Dentist5	-0.6667	1.0	-249.495	248.1617	False
Method1-Dentist3	Method2-Dentist1	7.6667	1.0	-241.1617	256.495	False
Method1-Dentist3	Method2-Dentist2	58.3333	0.9999	-190.495	307.1617	False
Method1-Dentist3	Method2-Dentist3	15.6667	1.0	-233.1617	264.495	False
Method1-Dentist3	Method2-Dentist4	74.0	0.9981	-174.8283	322.8283	False
Method1-Dentist3	Method2-Dentist5	32.6667	1.0	-216.1617	281.495	False
Method1-Dentist3	Method3-Dentist1	28.3333	1.0	-220.495	277.1617	False
Method1-Dentist3	Method3-Dentist2	33.6667	1.0	-215.1617	282.495	False
Method1-Dentist3	Method3-Dentist3	15.0	1.0	-233.8283	263.8283	False
Method1-Dentist3	Method3-Dentist4	-211.0	0.166	-459.8283	37.8283	False
Method1-Dentist3	Method3-Dentist5	-271.0	0.0229	-519.8283	-22.1717	True
Method1-Dentist4	Method1-Dentist5	-59.0	0.9998	-307.8283	189.8283	False
Method1-Dentist4	Method2-Dentist1	-50.6667	1.0	-299.495	198.1617	False
Method1-Dentist4	Method2-Dentist2	0.0	1.0	-248.8283	248.8283	False
Method1-Dentist4	Method2-Dentist3	-42.6667	1.0	-291.495	206.1617	False
Method1-Dentist4	Method2-Dentist4	15.6667	1.0	-233.1617	264.495	False
Method1-Dentist4	Method2-Dentist5	-25.6667	1.0	-274.495	223.1617	False
Method1-Dentist4	Method3-Dentist1	-30.0	1.0	-278.8283	218.8283	False
Method1-Dentist4	Method3-Dentist2	-24.6667	1.0	-273.495	224.1617	False
Method1-Dentist4	Method3-Dentist3	-43.3333	1.0	-292.1617	205.495	False
Method1-Dentist4	Method3-Dentist4	-269.3333	0.0243	-518.1617	-20.505	True
Method1-Dentist4	Method3-Dentist5	-329.3333	0.0025	-578.1617	-80.505	True
Method1-Dentist5	Method2-Dentist1	8.3333	1.0	-240.495	257.1617	False
Method1-Dentist5	Method2-Dentist2	59.0	0.9998	-189.8283	307.8283	False
Method1-Dentist5	Method2-Dentist3	16.3333	1.0	-232.495	265.1617	False
Method1-Dentist5	Method2-Dentist4	74.6667	0.9979	-174.1617	323.495	False
Method1-Dentist5	Method2-Dentist5	33.3333	1.0	-215.495	282.1617	False
Method1-Dentist5	Method3-Dentist1	29.0	1.0	-219.8283	277.8283	False
Method1-Dentist5	Method3-Dentist2	34.3333	1.0	-214.495	283.1617	False
Method1-Dentist5	Method3-Dentist3	15.6667	1.0	-233.1617	264.495	False
Method1-Dentist5	Method3-Dentist4	-210.3333	0.1692	-459.1617	38.495	False
Method1-Dentist5	Method3-Dentist5	-270.3333	0.0234	-519.1617	-21.505	True
Method2-Dentist1	Method2-Dentist2	50.6667	1.0	-198.1617	299.495	False
Method2-Dentist1	Method2-Dentist3	8.0	1.0	-240.8283	256.8283	False
Method2-Dentist1	Method2-Dentist4	66.3333	0.9994	-182.495	315.1617	False
Method2-Dentist1	Method2-Dentist5	25.0	1.0	-223.8283	273.8283	False
Method2-Dentist1	Method3-Dentist1	20.6667	1.0	-228.1617	269.495	False

Tukey HSD | Alloy 2

Multiple Comparison of Means - Tukey HSD, FWER=0.05

group1	group2	meandiff	p-adj	lower	upper	reject
Method1-Dentist1	Method1-Dentist2	37.6667	1.0	-303.2292	378.5625	False
Method1-Dentist1	Method1-Dentist3	-63.3333	1.0	-404.2292	277.5625	False
Method1-Dentist1	Method1-Dentist4	-66.3333	1.0	-407.2292	274.5625	False
Method1-Dentist1	Method1-Dentist5	52.0	1.0	-288.8958	392.8958	False
Method1-Dentist1	Method2-Dentist1	95.3333	0.999	-245.5625	436.2292	False
Method1-Dentist1	Method2-Dentist2	5.0	1.0	-335.8958	345.8958	False
Method1-Dentist1	Method2-Dentist3	-27.0	1.0	-367.8958	313.8958	False
Method1-Dentist1	Method2-Dentist4	2.3333	1.0	-338.5625	343.2292	False
Method1-Dentist1	Method2-Dentist5	19.3333	1.0	-321.5625	360.2292	False
Method1-Dentist1	Method3-Dentist1	-180.6667	0.8085	-521.5625	160.2292	False
Method1-Dentist1	Method3-Dentist2	-140.3333	0.9635	-481.2292	200.5625	False
Method1-Dentist1	Method3-Dentist3	-104.6667	0.9973	-445.5625	236.2292	False
Method1-Dentist1	Method3-Dentist4	-231.3333	0.4686	-572.2292	109.5625	False
Method1-Dentist1	Method3-Dentist5	-427.0	0.0049	-767.8958	-86.1042	True
Method1-Dentist2	Method1-Dentist3	-101.0	0.9981	-441.8958	239.8958	False
Method1-Dentist2	Method1-Dentist4	-104.0	0.9975	-444.8958	236.8958	False
Method1-Dentist2	Method1-Dentist5	14.3333	1.0	-326.5625	355.2292	False
Method1-Dentist2	Method2-Dentist1	57.6667	1.0	-283.2292	398.5625	False
Method1-Dentist2	Method2-Dentist2	-32.6667	1.0	-373.5625	308.2292	False
Method1-Dentist2	Method2-Dentist3	-64.6667	1.0	-405.5625	276.2292	False
Method1-Dentist2	Method2-Dentist4	-35.3333	1.0	-376.2292	305.5625	False
Method1-Dentist2	Method2-Dentist5	-18.3333	1.0	-359.2292	322.5625	False
Method1-Dentist2	Method3-Dentist1	-218.3333	0.5587	-559.2292	122.5625	False
Method1-Dentist2	Method3-Dentist2	-178.0	0.8234	-518.8958	162.8958	False
Method1-Dentist2	Method3-Dentist3	-142.3333	0.9594	-483.2292	198.5625	False
Method1-Dentist2	Method3-Dentist4	-269.0	0.2485	-609.8958	71.8958	False
Method1-Dentist2	Method3-Dentist5	-464.6667	0.0017	-805.5625	-123.7708	True
Method1-Dentist3	Method1-Dentist4	-3.0	1.0	-343.8958	337.8958	False
Method1-Dentist3	Method1-Dentist5	115.3333	0.9932	-225.5625	456.2292	False
Method1-Dentist3	Method2-Dentist1	158.6667	0.912	-182.2292	499.5625	False
Method1-Dentist3	Method2-Dentist2	68.3333	1.0	-272.5625	409.2292	False
Method1-Dentist3	Method2-Dentist3	36.3333	1.0	-304.5625	377.2292	False
Method1-Dentist3	Method2-Dentist4	65.6667	1.0	-275.2292	406.5625	False
Method1-Dentist3	Method2-Dentist5	82.6667	0.9998	-258.2292	423.5625	False
Method1-Dentist3	Method3-Dentist1	-117.3333	0.992	-458.2292	223.5625	False
Method1-Dentist3	Method3-Dentist2	-77.0	0.9999	-417.8958	263.8958	False
Method1-Dentist3	Method3-Dentist3	-41.3333	1.0	-382.2292	299.5625	False
Method1-Dentist3	Method3-Dentist4	-168.0	0.8735	-508.8958	172.8958	False
Method1-Dentist3	Method3-Dentist5	-363.6667	0.0279	-704.5625	-22.7708	True
Method1-Dentist4	Method1-Dentist5	118.3333	0.9914	-222.5625	459.2292	False
Method1-Dentist4	Method2-Dentist1	161.6667	0.9005	-179.2292	502.5625	False
Method1-Dentist4	Method2-Dentist2	71.3333	1.0	-269.5625	412.2292	False
Method1-Dentist4	Method2-Dentist3	39.3333	1.0	-301.5625	380.2292	False
Method1-Dentist4	Method2-Dentist4	68.6667	1.0	-272.2292	409.5625	False
Method1-Dentist4	Method2-Dentist5	85.6667	0.9997	-255.2292	426.5625	False
Method1-Dentist4	Method3-Dentist1	-114.3333	0.9937	-455.2292	226.5625	False
Method1-Dentist4	Method3-Dentist2	-74.0	0.9999	-414.8958	266.8958	False
Method1-Dentist4	Method3-Dentist3	-38.3333	1.0	-379.2292	302.5625	False
Method1-Dentist4	Method3-Dentist4	-165.0	0.8868	-505.8958	175.8958	False
Method1-Dentist4	Method3-Dentist5	-360.6667	0.0302	-701.5625	-19.7708	True
Method1-Dentist5	Method2-Dentist1	43.3333	1.0	-297.5625	384.2292	False
Method1-Dentist5	Method2-Dentist2	-47.0	1.0	-387.8958	293.8958	False
Method1-Dentist5	Method2-Dentist3	-79.0	0.9999	-419.8958	261.8958	False
Method1-Dentist5	Method2-Dentist4	-49.6667	1.0	-390.5625	291.2292	False
Method1-Dentist5	Method2-Dentist5	-32.6667	1.0	-373.5625	308.2292	False
Method1-Dentist5	Method3-Dentist1	-232.6667	0.4596	-573.5625	108.2292	False
Method1-Dentist5	Method3-Dentist2	-192.3333	0.7376	-533.2292	148.5625	False
Method1-Dentist5	Method3-Dentist3	-156.6667	0.9191	-497.5625	184.2292	False
Method1-Dentist5	Method3-Dentist4	-283.3333	0.1871	-624.2292	57.5625	False
Method1-Dentist5	Method3-Dentist5	-479.0	0.0011	-819.8958	-138.1042	True
Method2-Dentist1	Method2-Dentist2	-90.3333	0.9994	-431.2292	250.5625	False
Method2-Dentist1	Method2-Dentist3	-122.3333	0.9884	-463.2292	218.5625	False
Method2-Dentist1	Method2-Dentist4	-93.0	0.9992	-433.8958	247.8958	False
Method2-Dentist1	Method2-Dentist5	-76.0	0.9999	-416.8958	264.8958	False
Method2-Dentist1	Method3-Dentist1	-276.0	0.2169	-616.8958	64.8958	False
Method2-Dentist1	Method3-Dentist2	-235.6667	0.4396	-576.5625	105.2292	False

Method2-Dentist1	Method3-Dentist2	26.0	1.0	-222.8283	274.8283	False
Method2-Dentist1	Method3-Dentist3	7.3333	1.0	-241.495	256.1617	False
Method2-Dentist1	Method3-Dentist4	-218.6667	0.1324	-467.495	30.1617	False
Method2-Dentist1	Method3-Dentist5	-278.6667	0.0173	-527.495	-29.8383	True
Method2-Dentist2	Method2-Dentist3	-42.6667	1.0	-291.495	206.1617	False
Method2-Dentist2	Method2-Dentist4	15.6667	1.0	-233.1617	264.495	False
Method2-Dentist2	Method2-Dentist5	-25.6667	1.0	-274.495	223.1617	False
Method2-Dentist2	Method3-Dentist1	-30.0	1.0	-278.8283	218.8283	False
Method2-Dentist2	Method3-Dentist2	-24.6667	1.0	-273.495	224.1617	False
Method2-Dentist2	Method3-Dentist3	-43.3333	1.0	-292.1617	205.495	False
Method2-Dentist2	Method3-Dentist4	-269.3333	0.0243	-518.1617	-20.505	True
Method2-Dentist2	Method3-Dentist5	-329.3333	0.0025	-578.1617	-80.505	True
Method2-Dentist3	Method2-Dentist4	58.3333	0.9999	-190.495	307.1617	False
Method2-Dentist3	Method2-Dentist5	17.0	1.0	-231.8283	265.8283	False
Method2-Dentist3	Method3-Dentist1	12.6667	1.0	-236.1617	261.495	False
Method2-Dentist3	Method3-Dentist2	18.0	1.0	-230.8283	266.8283	False
Method2-Dentist3	Method3-Dentist3	-0.6667	1.0	-249.495	248.1617	False
Method2-Dentist3	Method3-Dentist4	-226.6667	0.1035	-475.495	22.1617	False
Method2-Dentist3	Method3-Dentist5	-286.6667	0.0128	-535.495	-37.8383	True
Method2-Dentist4	Method2-Dentist5	-41.3333	1.0	-290.1617	207.495	False
Method2-Dentist4	Method3-Dentist1	-45.6667	1.0	-294.495	203.1617	False
Method2-Dentist4	Method3-Dentist2	-40.3333	1.0	-289.1617	208.495	False
Method2-Dentist4	Method3-Dentist3	-59.0	0.9998	-307.8283	189.8283	False
Method2-Dentist4	Method3-Dentist4	-285.0	0.0137	-533.8283	-36.1717	True
Method2-Dentist4	Method3-Dentist5	-345.0	0.0013	-593.8283	-96.1717	True
Method2-Dentist5	Method3-Dentist1	-4.3333	1.0	-253.1617	244.495	False
Method2-Dentist5	Method3-Dentist2	1.0	1.0	-247.8283	249.8283	False
Method2-Dentist5	Method3-Dentist3	-17.6667	1.0	-266.495	231.1617	False
Method2-Dentist5	Method3-Dentist4	-243.6667	0.0596	-492.495	5.1617	False
Method2-Dentist5	Method3-Dentist5	-303.6667	0.0067	-552.495	-54.8383	True
Method3-Dentist1	Method3-Dentist2	5.3333	1.0	-243.495	254.1617	False
Method3-Dentist1	Method3-Dentist3	-13.3333	1.0	-262.1617	235.495	False
Method3-Dentist1	Method3-Dentist4	-239.3333	0.0688	-488.1617	9.495	False
Method3-Dentist1	Method3-Dentist5	-299.3333	0.0079	-548.1617	-50.505	True
Method3-Dentist2	Method3-Dentist3	-18.6667	1.0	-267.495	230.1617	False
Method3-Dentist2	Method3-Dentist4	-244.6667	0.0576	-493.495	4.1617	False
Method3-Dentist2	Method3-Dentist5	-304.6667	0.0065	-553.495	-55.8383	True
Method3-Dentist3	Method3-Dentist4	-226.0	0.1057	-474.8283	22.8283	False
Method3-Dentist3	Method3-Dentist5	-286.0	0.0131	-534.8283	-37.1717	True
Method3-Dentist4	Method3-Dentist5	-60.0	0.9998	-308.8283	188.8283	False

Method2-Dentist1	Method3-Dentist3	-200.0	0.6868	-540.8958	140.8958	False
Method2-Dentist1	Method3-Dentist4	-326.6667	0.0709	-667.5625	14.2292	False
Method2-Dentist1	Method3-Dentist5	-522.3333	0.0003	-863.2292	-181.4375	True
Method2-Dentist2	Method2-Dentist3	-32.0	1.0	-372.8958	308.8958	False
Method2-Dentist2	Method2-Dentist4	-2.6667	1.0	-343.5625	338.2292	False
Method2-Dentist2	Method2-Dentist5	14.3333	1.0	-326.5625	355.2292	False
Method2-Dentist2	Method3-Dentist1	-185.6667	0.7793	-526.5625	155.2292	False
Method2-Dentist2	Method3-Dentist2	-145.3333	0.9525	-486.2292	195.5625	False
Method2-Dentist2	Method3-Dentist3	-109.6667	0.9958	-450.5625	231.2292	False
Method2-Dentist2	Method3-Dentist4	-236.3333	0.4352	-577.2292	104.5625	False
Method2-Dentist2	Method3-Dentist5	-432.0	0.0043	-772.8958	-91.1042	True
Method2-Dentist3	Method2-Dentist4	29.3333	1.0	-311.5625	370.2292	False
Method2-Dentist3	Method2-Dentist5	46.3333	1.0	-294.5625	387.2292	False
Method2-Dentist3	Method3-Dentist1	-153.6667	0.9291	-494.5625	187.2292	False
Method2-Dentist3	Method3-Dentist2	-113.3333	0.9942	-454.2292	227.5625	False
Method2-Dentist3	Method3-Dentist3	-77.6667	0.9999	-418.5625	263.2292	False
Method2-Dentist3	Method3-Dentist4	-204.3333	0.657	-545.2292	136.5625	False
Method2-Dentist3	Method3-Dentist5	-400.0	0.0105	-740.8958	-59.1042	True
Method2-Dentist4	Method2-Dentist5	17.0	1.0	-323.8958	357.8958	False
Method2-Dentist4	Method3-Dentist1	-183.0	0.7951	-523.8958	157.8958	False
Method2-Dentist4	Method3-Dentist2	-142.6667	0.9586	-483.5625	198.2292	False
Method2-Dentist4	Method3-Dentist3	-107.0	0.9967	-447.8958	233.8958	False
Method2-Dentist4	Method3-Dentist4	-233.6667	0.4529	-574.5625	107.2292	False
Method2-Dentist4	Method3-Dentist5	-429.3333	0.0046	-770.2292	-88.4375	True
Method2-Dentist5	Method3-Dentist1	-200.0	0.6868	-540.8958	140.8958	False
Method2-Dentist5	Method3-Dentist2	-159.6667	0.9083	-500.5625	181.2292	False
Method2-Dentist5	Method3-Dentist3	-124.0	0.9869	-464.8958	216.8958	False
Method2-Dentist5	Method3-Dentist4	-250.6667	0.3458	-591.5625	90.2292	False
Method2-Dentist5	Method3-Dentist5	-446.3333	0.0028	-787.2292	-105.4375	True
Method3-Dentist1	Method3-Dentist2	40.3333	1.0	-300.5625	381.2292	False
Method3-Dentist1	Method3-Dentist3	76.0	0.9999	-264.8958	416.8958	False
Method3-Dentist1	Method3-Dentist4	-50.6667	1.0	-391.5625	290.2292	False
Method3-Dentist1	Method3-Dentist5	-246.3333	0.3717	-587.2292	94.5625	False
Method3-Dentist2	Method3-Dentist3	35.6667	1.0	-305.2292	376.5625	False
Method3-Dentist2	Method3-Dentist4	-91.0	0.9994	-431.8958	249.8958	False
Method3-Dentist2	Method3-Dentist5	-286.6667	0.1746	-627.5625	54.2292	False
Method3-Dentist3	Method3-Dentist4	-126.6667	0.9842	-467.5625	214.2292	False
Method3-Dentist3	Method3-Dentist5	-322.3333	0.0786	-663.2292	18.5625	False
Method3-Dentist4	Method3-Dentist5	-195.6667	0.7158	-536.5625	145.2292	False

Figure 38: Tukey HSD Test (Method X Dentist Interaction) for both Alloys

- Inference: -
  - **Alloy 1**: -
    - ✓ Tukey HSD test revealed several significant differences between specific combinations of Dentist and Method, highlighting that **Method 3 is particularly challenging in terms of achieving consistent hardness outcomes**.
    - ✓ **Method 3** appears to be **highly variable**, with significant differences observed between **Dentist 1** and **Dentist 5** as well as within **Dentist 5** for different methods.
    - ✓ The **lack of consistency** across different dentists and methods implies that **Method 3** is **more sensitive to individual differences** in skill and requires a higher degree of **procedural control**.
    - ✓ **Method 1 and Method 2** provide more **consistent outcomes**, suggesting that they are easier to perform consistently across different dentists.
    - ✓ **Recommendations: -**
      - ➢ **Standardize Method 3**: Improve the procedural standardization of Method 3 to reduce the observed variability across dentists.
      - ➢ **Training for Method 3**: Provide targeted training to dentists, particularly those (e.g., **Dentist 5**) who showed inconsistent results with **Method 3**. Comparison between **Method 1 and Method 3**:
  - **Alloy 2**: -
    - ✓ Tukey HSD test revealed several significant differences were observed between combinations involving **Method 3**, further indicating that **Method 3** is difficult to standardize across dentists.
    - ✓ **Method 3** consistently produces significantly different hardness outcomes compared to other methods (**Method 1 & Method 2**), especially when performed by **Dentist 5**.
    - ✓ The inconsistency of **Method 3** indicates that it may be more **complex** or require more precise procedural guidelines compared to the other methods.
    - ✓ **Method 1 & Method 2** appear to be more robust and consistent, producing similar outcomes across different **Dentists.**
    - ✓ **Recommendations: -**
      - ➢ **Improve Procedural Guidelines for Method 3:** Create a more standardized procedure for Method 3 to help dentists achieve consistent outcomes.
      - ➢ **Training Focused on Method 3:** Provide additional training to dentists, particularly Dentist 5, on Method 3 to improve the consistency of outcomes.

- **Consider Using Method 1 and 2**: When consistency in implant hardness is crucial, Method 1 and 2 should be preferred over Method 3.

- **Summary: -**
  - Across both **Alloy 1** and **Alloy 2**, **Method 3** stands out as the most inconsistent method, leading to significant differences in implant hardness across different **dentists** and even for the **same dentist** using different methods.
  - **Method 3** is more challenging and seems to depend heavily on **individual dentist skill** or **proficiency**, leading to variability in hardness outcomes. Improved **procedural standardization** and **targeted training** for **Method 3** are needed to achieve more uniform results.
  - In contrast, **Method 1 and Method 2** are generally more **consistent** across different dentists, suggesting that they may be easier to standardize and less sensitive to individual variability.