**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**Solution :**

**import numpy as np**

**import scipy.stats as st**

**mean = 45**

**std = 8**

**time\_limit = 60 - 10**

**p = st.norm.cdf(time\_limit, mean, std)**

**print(1 - p)**

**OutPut : 0.26598552904870054**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Solution :

**Statement A : FALSE**

**A Normal Distribution is Symmetrical around it’s mean , So there are equal numbers of employees above and below the mean . This means that there is are equal number of employees older than 44 and than between 38 and 44**

**Statement B : TRUE**

**The CDF gives the probability that a random variables will be less than are equal to the certain value**

**import numpy as np**

**import scipy.stats as st**

**mean = 38**

**std = 6**

**age\_limit = 30**

**p\_under\_30 = st.norm.cdf(age\_limit, mean, std)**

**Output : 0.147058823529411678**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Solution:**

**If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are independent and iid normal random variables, let's examine the difference between 2 *X*1 and *X*1 + *X*2.**

**1. Distribution of 2X1:**

**Mean : E(2X\_1) = 2E(X\_1) = 2μ**

**Variance : Var(2X\_1) = 4Var(X\_1) = 4σ2**

**(2X1~ N(2μ, 4σ2).**

**2. Distribution of (X1 + X2):**

**Mean : E(X1 + X2) = E(X1) + E(X2) =μ+μ = 2μ**

**Variance :**

**Since X1 and X2 are independent, the variance of the sum is the sum of the variances**

**Var(X1 + X2) = Var(X1) + Var(X2) = 2σ2**

**(X1 + X2~ N(2μ, 2σ2))**

**Both 2X1 and (X1 + X2) have the same mean of (2μ), but (2X) has a larger variance (4σ2) compared to (X1 + X2) (2σ2).The independence of (X1) and (X2) is crucial. If they were not independent, the distributions and properties would be different.**

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**Solution :**

**Given X ~ N(100, 20²)**

**we first convert this to a standard normal distribution by calculating the z-scores**

**z = (X - μ) / σ**

**μ= 100 and σ = 20**

**For the lower bound a:**

**z\_a = (a - 100) / 20**

**For the upper bound b:**

**z\_b = (b - 100) / 20 the probability that X lies between a and b:**

**P(a < X < b) = P(z\_a < Z < z\_b)**

**Since the standard normal distribution is symmetric, z\_a = -z\_b.**

**P(-z\_b < Z < z\_b) = 0.99**

**you can find that the z-score for a cumulative probability of 0.99 is approximately 2.33.**

**z\_b = -2.33**

**z\_b = 2.33**

**Solving for b:**

**(b - 100) / 20 = 2.33**

**b - 100 = 46.6**

**b ≈ 146.6**

**Solving for a:**

**(a - 100) / 20 = -2.33**

**a - 100 = -46.6**

**a ≈ 53.4**

**The values of a and b are approximately 53.4 and 146.6, respectively.**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

**Solution :**

**Given Profit1 ~ N(5, 3²)**

**Profit2 ~ N(7, 4²)**

**assuming $1 = Rs. 45**

**let's convert everything into Rupees.**

**A. To find the Rupee range centered on the mean containing 95% probability:**

**For Profit1:**

**Mean (in Rupees) = 5 million \* 45 = 225 million**

**Standard Deviation (in Rupees) = 3 million \* 45 = 135 million**

**For Profit2:**

**Mean (in Rupees) = 7 million \* 45 = 315 million**

**Standard Deviation (in Rupees) = 4 million \* 45 = 180 million**

**Total Profit = Profit1 + Profit2**

**Total Mean = Mean1 + Mean2 = 225 million + 315 million = 540 million**

**Total Variance = Variance1 + Variance2 = (135 million)² + (180 million)²**

**≈ 18225 million² + 32400 million²**

**≈ 50625 million²**

**Total Standard Deviation (in Rupees) ≈ sqrt(50625 million²) ≈ 225 million**

**We want a range containing 95% probability, which corresponds to roughly 2 standard deviations on each side of the mean in a normal distribution.**

**Range = Total Mean ± (2 \* Total Standard Deviation)**

**= 540 million ± (2 \* 225 million)**

**= 540 million ± 450 million**

**= (90 million, 990 million)**

**So, the Rupee range containing 95% probability for the annual profit of the company is (90 million, 990 million).**

**B. To find the 5th percentile of profit (in Rupees) for the company:**

**The 5th percentile corresponds to a z-score of approximately -1.645**

**Profit = Total Mean + (Z \* Total Standard Deviation)**

**= 540 million + (-1.645 \* 225 million)**

**≈ 143.875 million**

**So, the 5th percentile of profit for the company is approximately 143.875 million Rupees.**

**C. To determine which division has a larger probability of making a loss:**

**For Profit1:**

**Mean = 225 million Rupees**

**Standard Deviation = 135 million Rupees**

**For Profit2:**

**Mean = 315 million Rupees**

**Standard Deviation = 180 million Rupees**

**We need to find the probability that each profit is less than zero:**

**P(Profit1 < 0) and P(Profit2 < 0)**