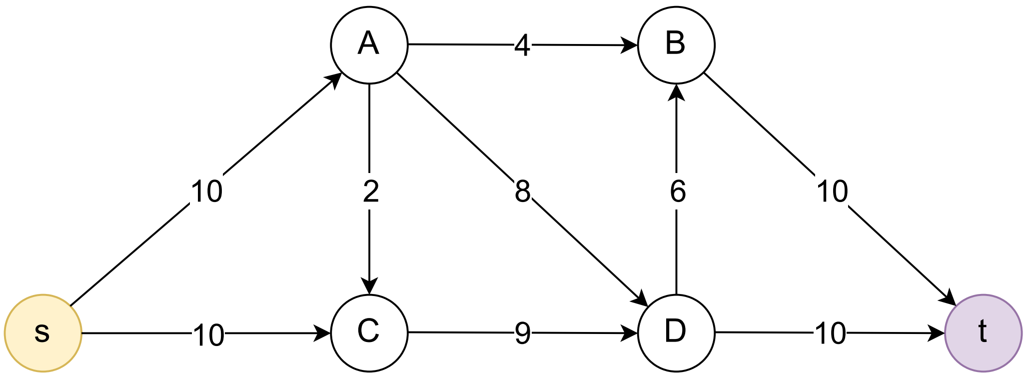
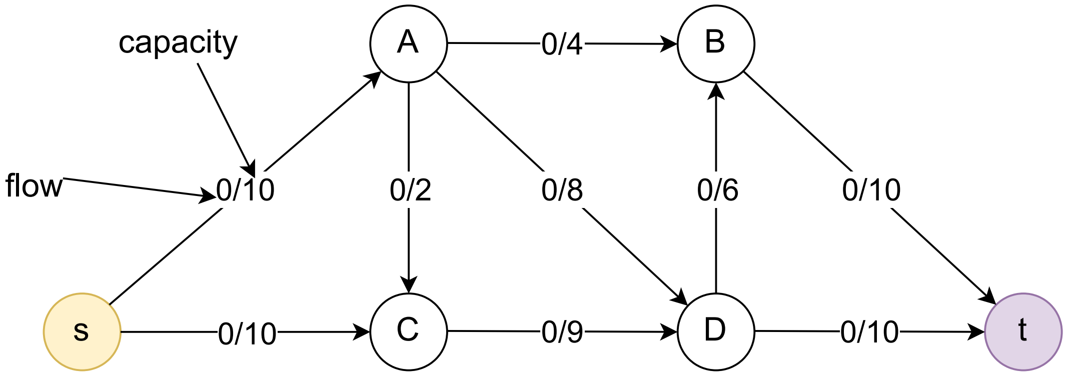
Max flow in Network - Edmonds-karp algorithm(ford fulkerson)



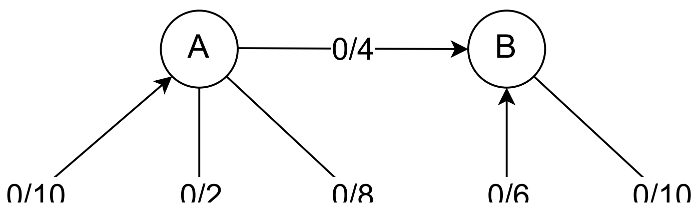
Graph with capacities



Graph with flow 0

Ford–Fulkerson algorithm :

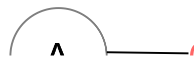
1. Construct the Residual Graph
2. Find a path from the source to the sink with a strictly positive flow.
3. If this path exists, update the flow to include it. Go to Step 1.
4. Else, the flow is maximal.
5. The (s,t)-cut has as S all vertices reachable from the source, and T



as V - S.

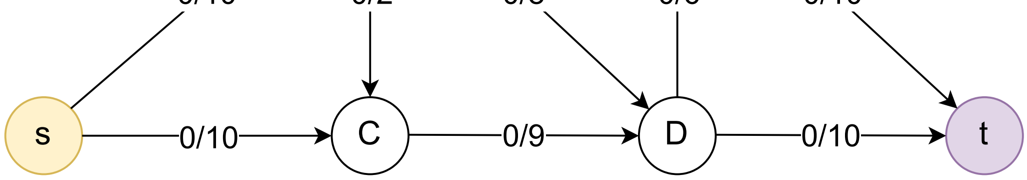
Residual Capacity :

original

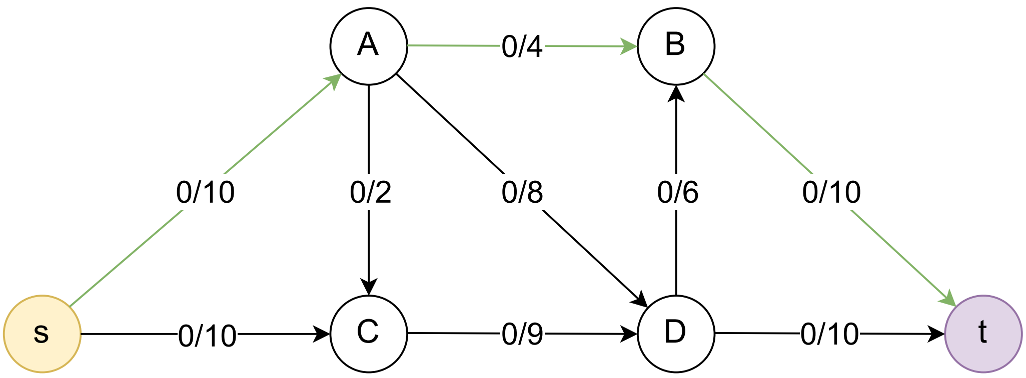


flow network G
6/10
B
residual network Gf
A
4
B

Fin

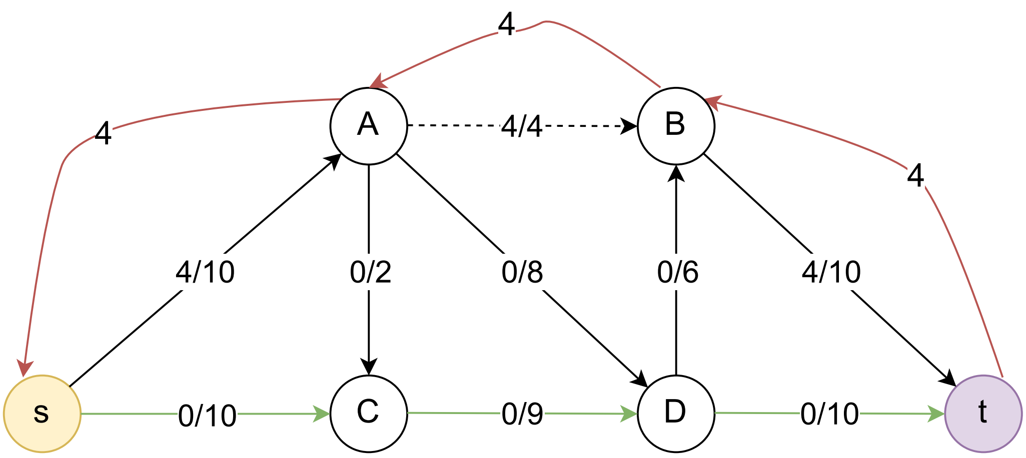


Residual Graph(Gf) for flow 0



Path is : s->
Min flow value along this pa
So we push flow of

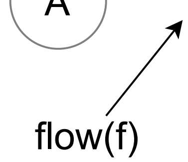
d an $s \leadsto t$ path P in the residual network (Gf), also edges should have +ve residual capacity
choose shortest path(edmonds-karp)



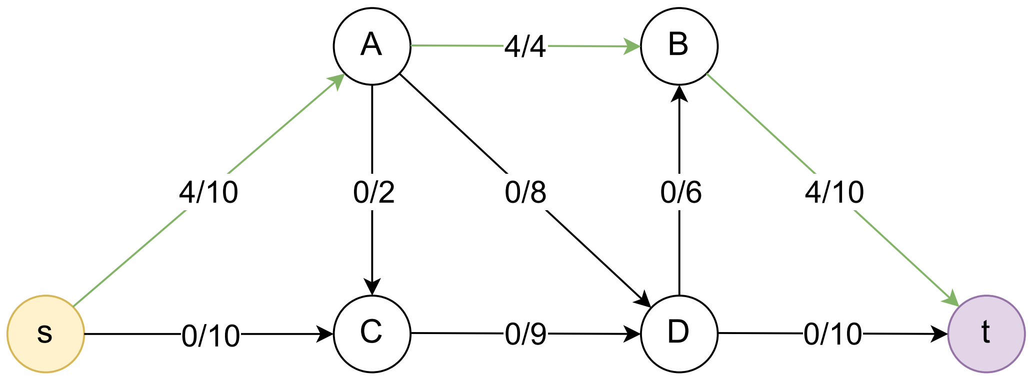
Min flow value
So we pu

Find an $s \leadsto t$ path P in the residual network (Gf), also edges should have +ve residual capacity
choose shortest path(edmonds-karp)

$$c_f(e) = \begin{cases} c(e) - f(e)\ , \text{ if e is forward edge} \\ f(e)\ , \text{ if e is backward edge} \end{cases}$$

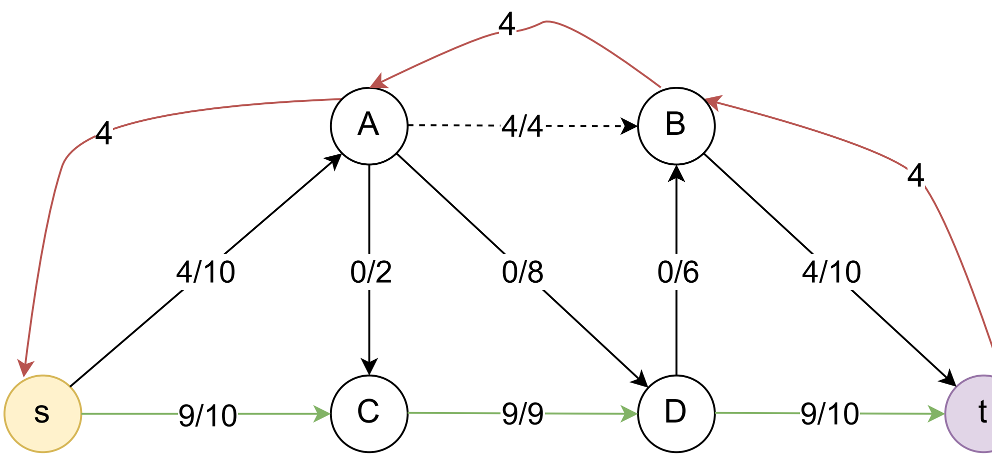


->A->B->t
ath is min{10, 4, 10} = 4
4 along this path

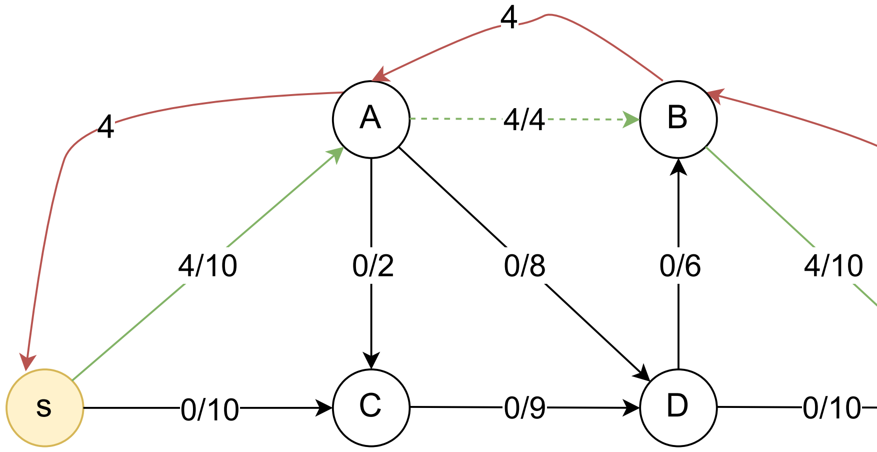


flow at t :

Path is : s->C->D->t
along this path is min{10, 9, 10} = 9
ush a flow of 9 along this path

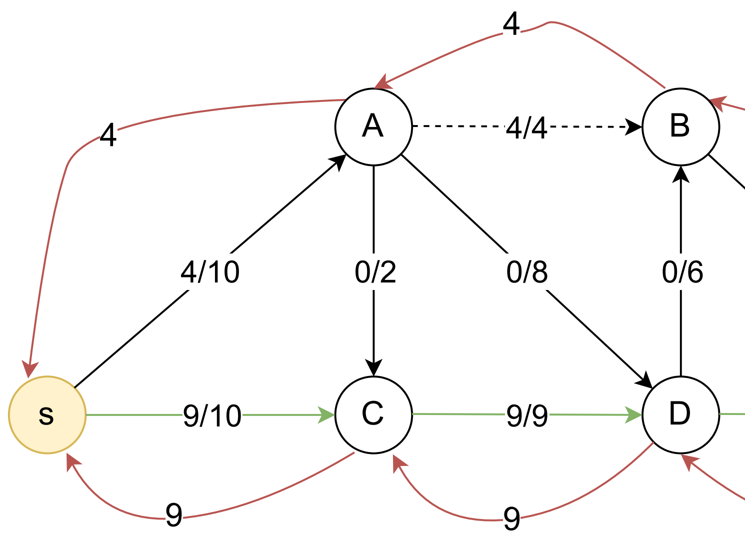


Update residual Graph

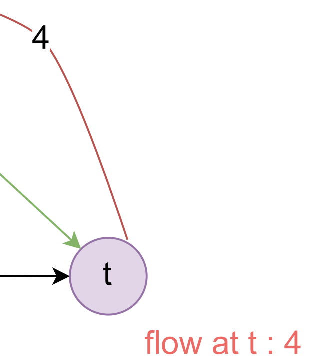


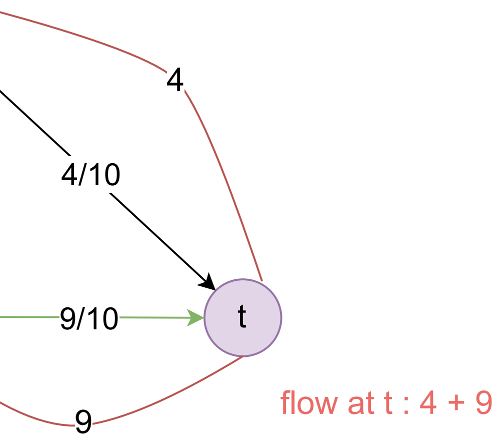
4

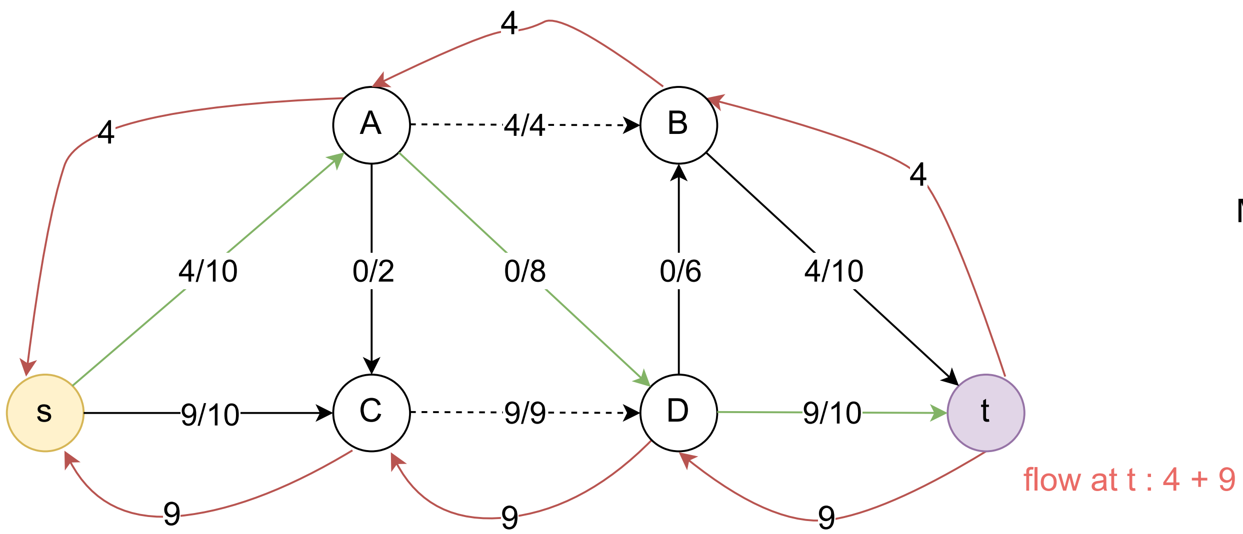
Update residual Graph



ow at t : 4 + 9

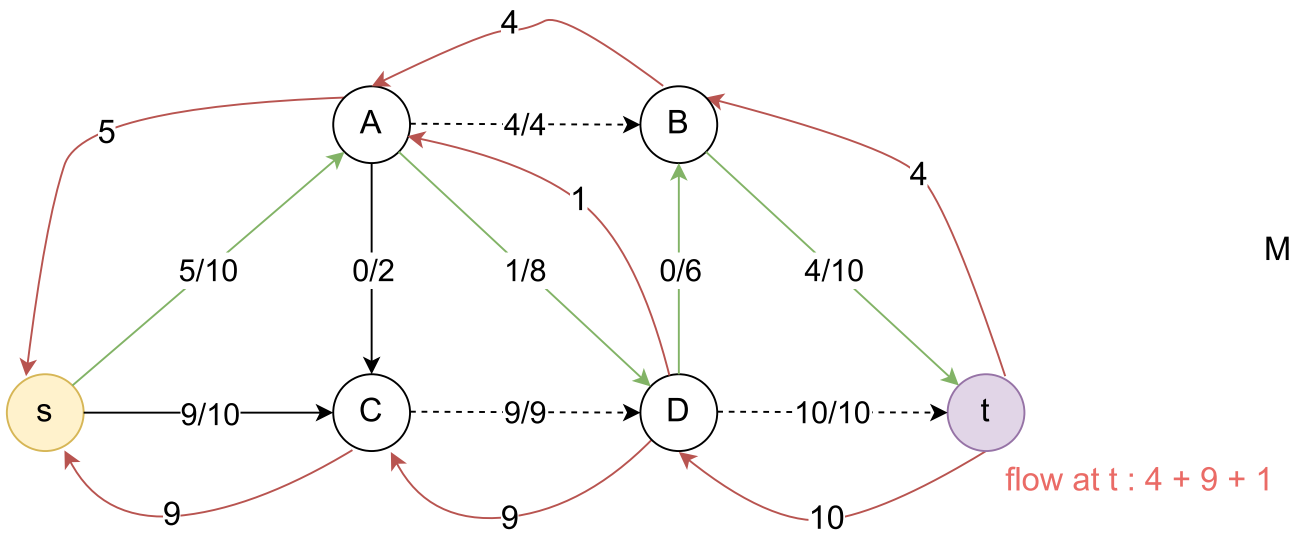

4
t
flow at t : 4


4
4/10
9/10
t
9
flow at t : 4 + 9



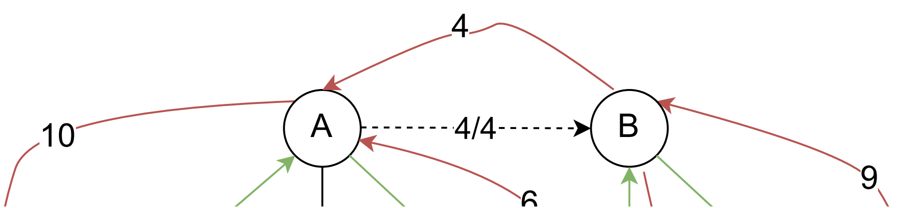
Min flow
So v

Find an s⤳t path P in the residual network (Gf), also edges should have +ve residual capacity
choose shortest path(edmonds-karp)

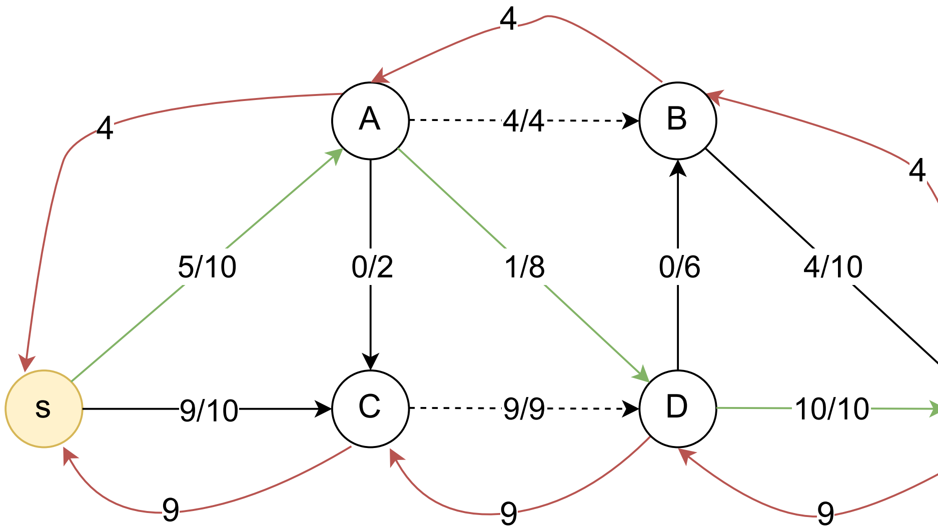


Min flo
S

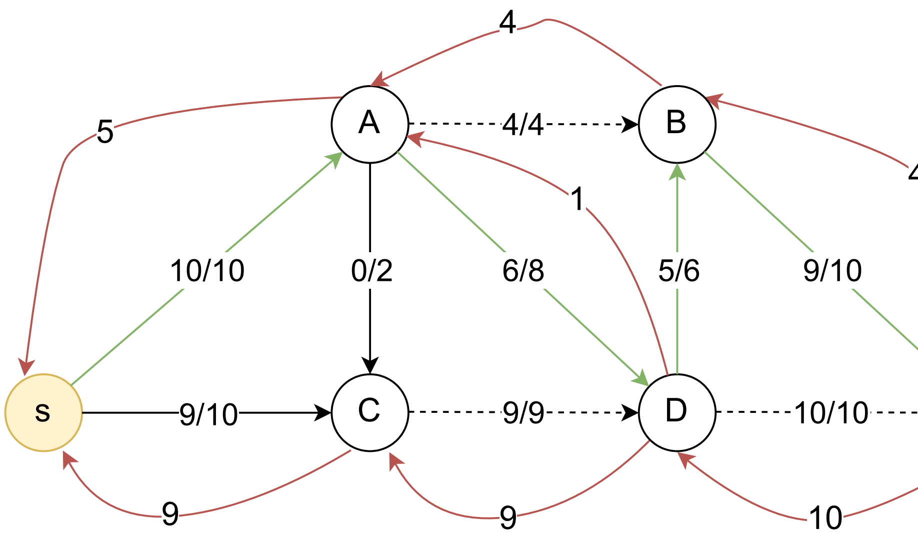
Find an s⤳t path P in the residual network (Gf), also edges should have +ve residual capacity
choose shortest path(edmonds-karp)



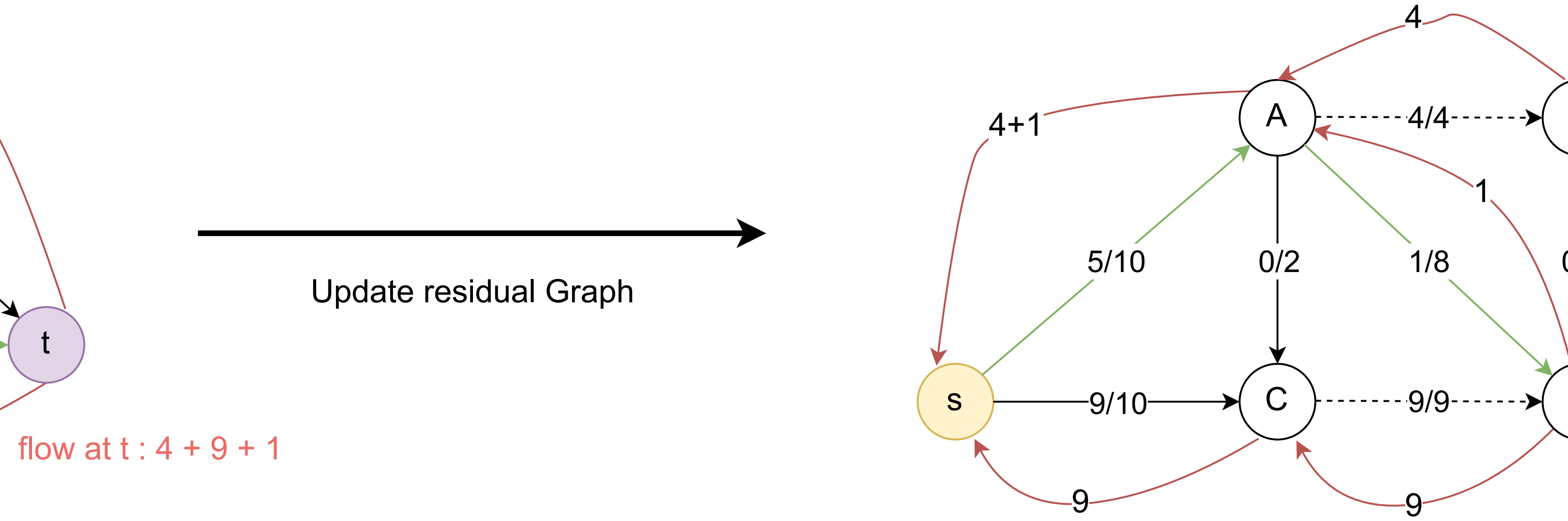
Path is : s->A->D->t
value along this path is min{4, 8, 1} = 1
we push a flow of 1 along this path

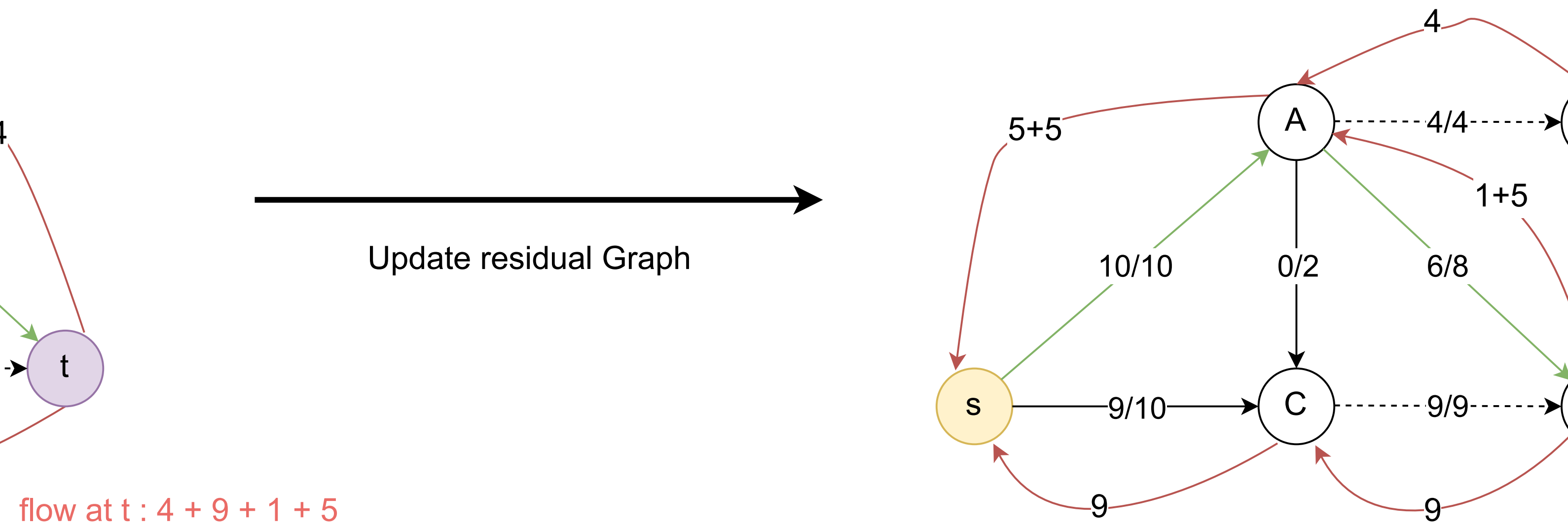


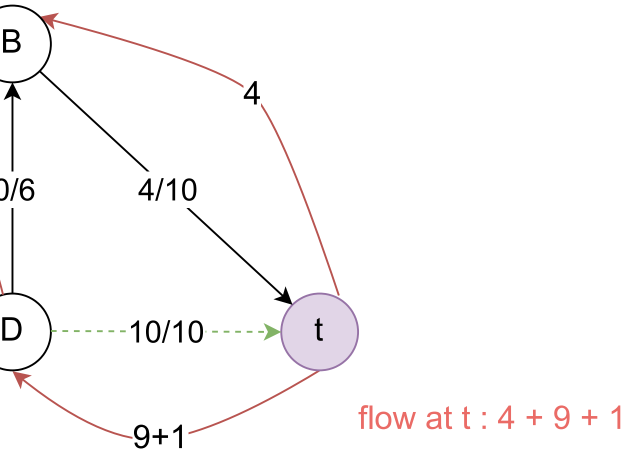
Path is : s->A->D->B->t
w value along this path is min{5, 7, 6, 6} = 5
So we push a flow of 5 along this path

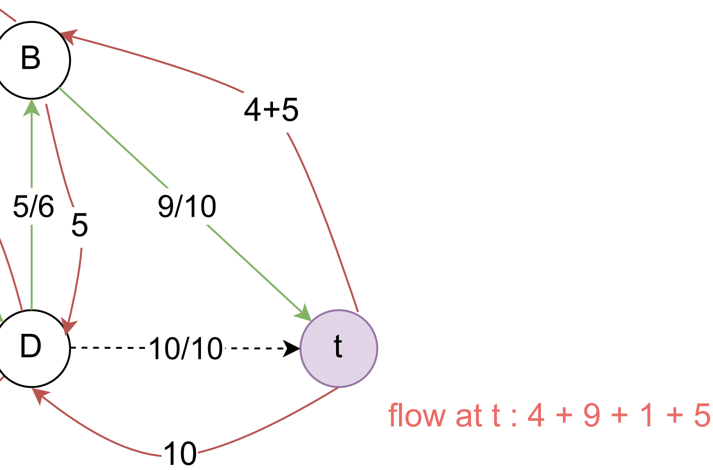


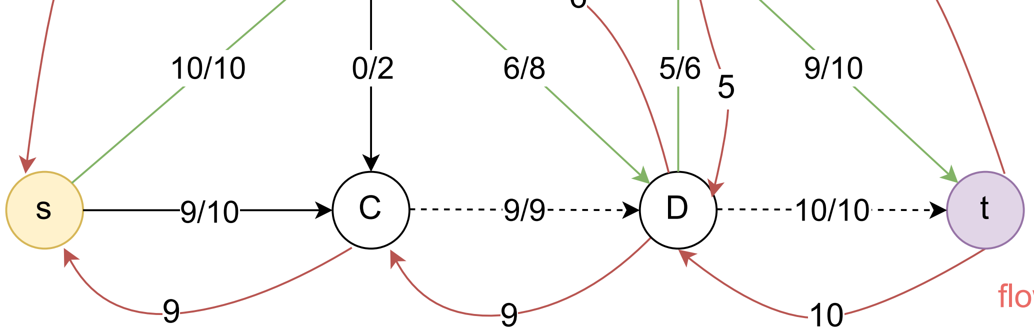
No such path is found so we stop


t
flow at t : 4 + 9 + 1
Update residual Graph
4
4+1
A
4/4
1
5/10
0/2
1/8
s
9/10
C
9/9
9
9


t
flow at t : 4 + 9 + 1 + 5
Update residual Graph
4
5+5
A
4/4
1+5
10/10
0/2
6/8
s
9/10
C
9/9
9
9


B
D
t
0/6
4/10
4
10/10
9+1
flow at t : 4 + 9 + 1

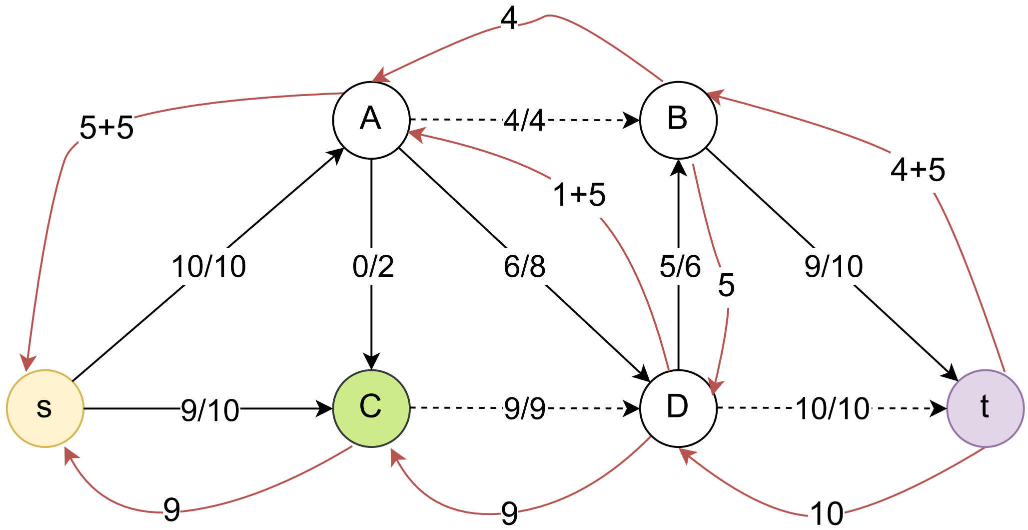

B
D
t
5/6
5
9/10
4+5
10/10
10
flow at t : 4 + 9 + 1 + 5



flow at t : 4 + 9 + 1 + 5

Find an s⤳t path P in the residual network (Gf), also edges should have +ve residual capacity
choose shortest path(edmonds-karp)

Min-cut



So {S, C}

So V - {S, C}

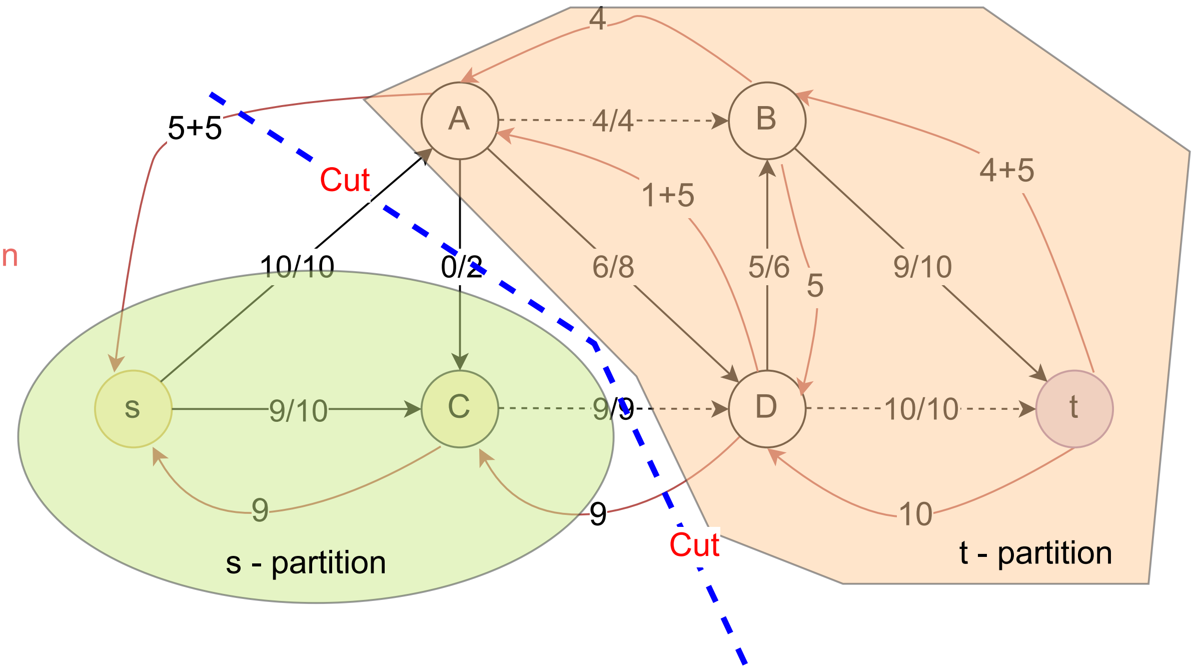
In the residual graph, find all nodes reachable from s (with +ve residual capacity)

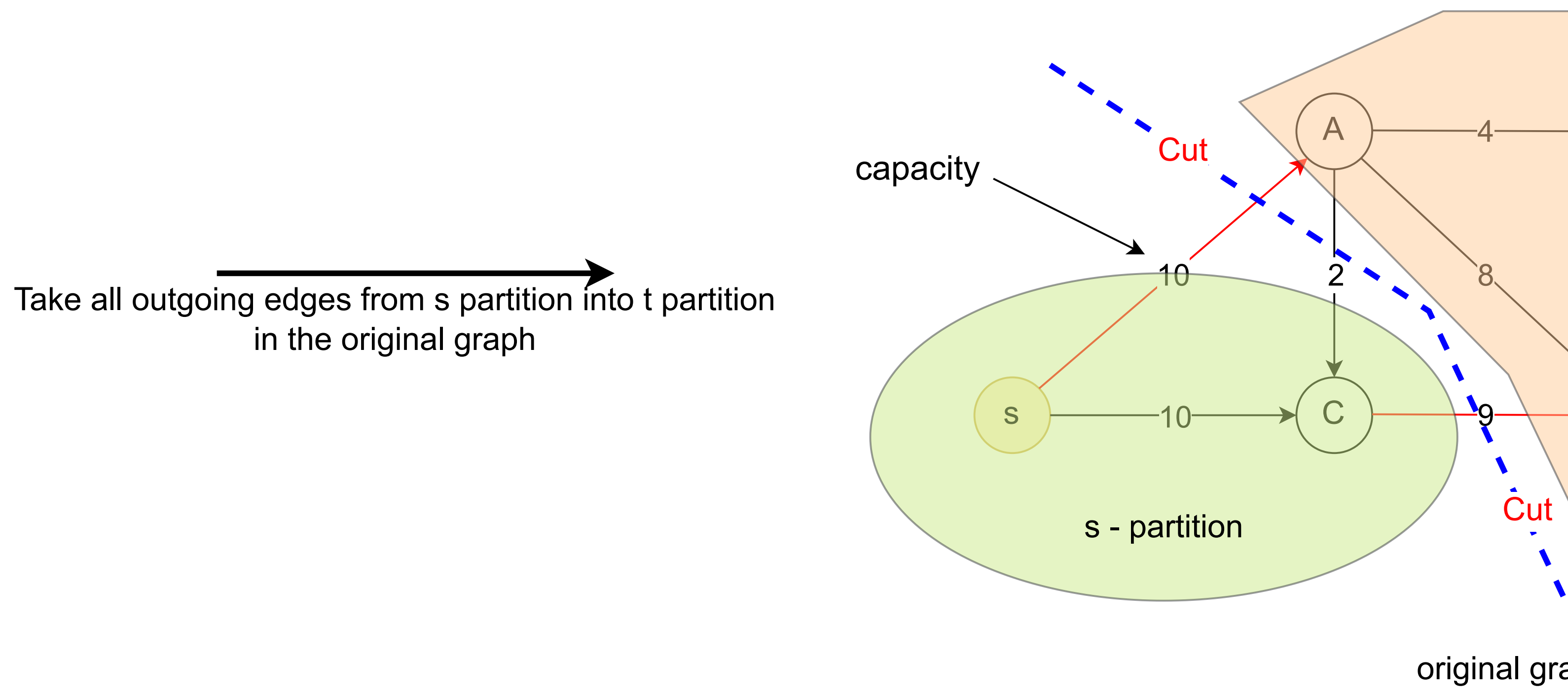
flow at t : 4 + 9 + 1 + 5 = 19
so Max flow in this network is 10

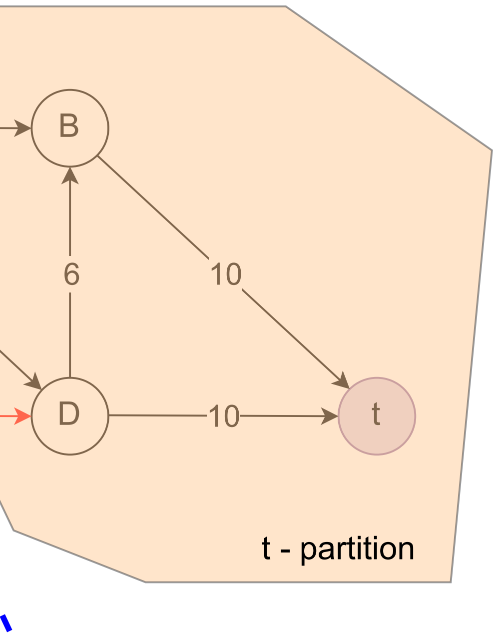
= 19

forms one partition

forms another partition




Take all outgoing edges from s partition into t partition
in the original graph
capacity
Cut
A
4
10
2
8
s
10
C
9
s - partition
Cut
original gra



aph

Max-flow min-cut theorem

- capacity of outgoing edges in min cut is 10+9=19
- Max flow we computed using ford fulkerson Algo is 19