A Multimodal Extension for MFD-based Modelling Framework.

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MFD-based models

- Public transport and private cars have different affect on network level traffic dynamics (Boyac and Geroliminis, 2011, Chiabaut *et al.*, 2014).
- Multimodality extended to existing MFD framework (Geroliminis *et al.*, 2014).
- Loder *et al.* (2017) calibrated a 3D-MFD based on empirical data of Zurich city network.
- Huang *et al.* (2019) estimated a bi-modal MFD based on taxi and private car data for city of Shenzhen, China.

Objectives

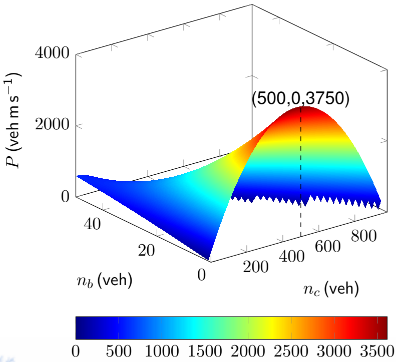
- Extend the existing accumulation and trip-based models to account for multimodality.
- Revision of accumulation-based model with outflow delay proposed in link flow dynamics (Friesz *et al.*, 1989).
- In addition, a continuum space-time MFD model (Leclercq *et al.*, 2015) based on hyperbolic conservation equations is proposed.
- Investigate the appropriate inflow and outflow functions.
- Compare the results of different models on various benchmark scenarios.
- Verify the models using results from micro-simulations.

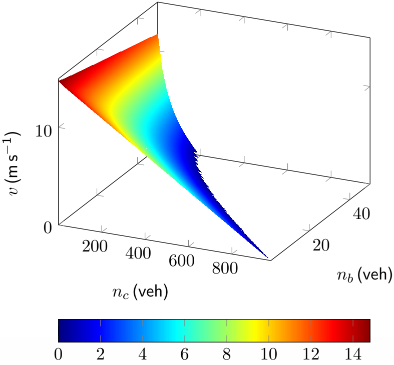
Functional form of 3D vMFD and pMFD

- Functional form proposed by Loder *et al.* (2017) is used in the present work.

$$
\begin{aligned}
v_c(n_c, n_{pt}) &= \beta_{c,0} + \beta_{c,c}\, n_c + \beta_{pt,c}\, n_{pt} \\
v_b(n_c, n_{pt}) &= \beta_{pt,0} + \beta_{c,pt}\, n_c + \beta_{pt,pt}\, n_{pt} \\
v(n_c, n_{pt}) &= v_c\, \frac{n_c}{n_c + n_{pt}} + v_{pt}\, \frac{n_{pt}}{n_c + n_{pt}} \\
P(n_c, n_{pt}) &= v_c\, n_c + v_{pt}\, n_{pt}
\end{aligned}
$$

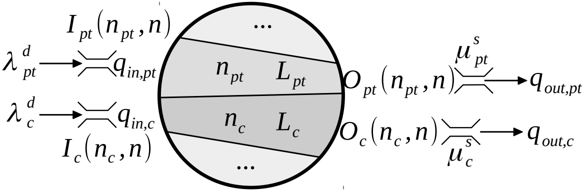
Graphical 3D-pMFD and-vMFD





Accumulation-based MFD model

Based on vehicle conservation equation:

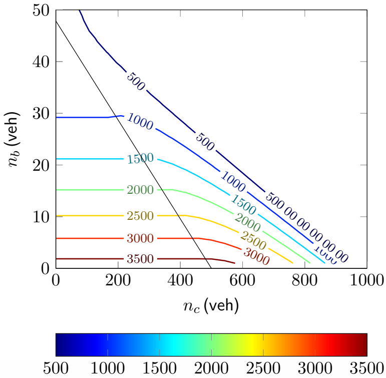


$$\frac{\mathrm{d}n_m}{\mathrm{d}t} = q_{in,m}(t) - q_{out,m}(t) \quad \text{where } i = c, pt$$

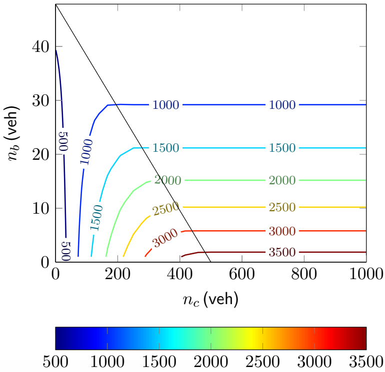
$$I_m(n_m, n) = \begin{cases} \frac{n_m}{n}\frac{P_{cr}(n_m,n)}{L_m} & \text{if } n \leq n_{cr}(n_m, n) \\ \frac{n_m}{n}\frac{P(n_m,n)}{L_m} & \text{otherwise} \end{cases}$$

$$O_m(n_m, n) = \begin{cases} \frac{n_m}{n}\frac{P(n_m,n)}{L_m} & \text{if } n \leq n_{cr}(n_m, n) \\ \frac{n_m}{n}\frac{P_{cr}(n_m,n)}{L_m} & \text{otherwise} \end{cases}$$

Conventional Inflow and Outflow Functions



Inflow Function



Outflow Function

Trip-based model

Explicit formulation based on trip length (Arnott, 2013, Lamotte and Geroliminis, 2016):

$$L = \int_{t-\tau(t)}^{t} V(n(s))\,\mathrm{d}s = \int_{t-\tau(t)}^{t} \frac{P(n(s))}{n(s)}\,\mathrm{d}s$$

Congestion spill-backs are modelled using inflow function:

$$t_{m,\text{entry supply}}^{N} = t_{m,\text{entry}}^{N-1} + \frac{1}{I_m(n_m, n)} \qquad \forall m$$

Delay accumulation-based model

- It is a variant of accumulation-based model, where outflow is delayed in the order of travel time.
- Introduced by Friesz *et al.* (1989), Ran *et al.* (1993) in the context of link level traffic flow dynamics.

Mathematically, it can be expressed as:

$$\int_{-\infty}^{t} q_{m,\text{in}}(s)\,\mathrm{d}s = \int_{-\infty}^{t+\tau_m(t)} q_{m,\text{out}}(s)\,\mathrm{d}s$$

Differentiating yields,

$$q_{m,\text{out}}(t+\tau_m(t)) = \frac{q_{m,\text{in}}(t)}{1+\dfrac{\mathrm{d}\tau_m(t)}{\mathrm{d}t}}$$

Delay accumulation-based model

The term $\dfrac{\mathrm{d}\tau_m(t)}{\mathrm{d}t}$ can be computed using chain rule:

$$\frac{\mathrm{d}\tau_m(t)}{\mathrm{d}t} = \sum_{i=\{c,pt\}} \frac{\partial \tau_m}{\partial n_i} \frac{\mathrm{d}n_i(t)}{\mathrm{d}t} \equiv \sum_{i=\{c,pt\}} \frac{\partial \tau_m}{\partial n_i} \left(q_{i,\text{in}}(t) - q_{i,\text{out}}(t)\right)$$

where Travel Time Function, τ_m, is defined using velocity MFD as:

$$\tau_m(n_m, n) = \frac{L_m}{v(n_m, n)}$$

In conventional accumulation-based model, production MFD is used as input whereas, in delay accumulation-based model, velocity MFD is embedded in the travel time function.

Delay accumulation-based model: Limitation

- Consider a single mode network:

$$q_{\text{out}}(t+\tau(t)) = \frac{q_{\text{in}}(t)}{1+\dfrac{\mathrm{d}\tau(t)}{\mathrm{d}t}} = \frac{q_{\text{in}}(t)}{1+\dfrac{\mathrm{d}\tau}{\mathrm{d}n}\left(q_{\text{in}}(t)-q_{\text{out}}(t)\right)}$$

- Model is stable if and only if $1+\dfrac{\mathrm{d}\tau}{\mathrm{d}n}\left(q_{\text{in}}(t)-q_{\text{out}}(t)\right) > 0$.
- Travel time function is always increasing and hence, $\dfrac{\mathrm{d}\tau}{\mathrm{d}n} > 0$ for all n. However, $q_{\text{in}}(t)-q_{\text{out}}(t) < 0$ during unloading.
- Physically, when $1+\dfrac{\mathrm{d}\tau(t)}{\mathrm{d}t} < 0$, FIFO rule is violated and hence, model fails. This limitation is addressed by proposing a *weak FIFO discipline* in this work.

Continuum space-time model

Based on scalar conservation equation

$$\frac{\partial k_m}{dt} + \frac{\partial k_m v}{\partial x_m} = s_m(x_m, t)$$

Can also be expressed as:

$$\frac{\partial \boldsymbol{K}}{dt} + \mathbf{A}(\boldsymbol{K}) \frac{\partial \boldsymbol{K}}{\partial x_m} = s_m(x_m, t)$$

The Jacobian matrix $\mathbf{A}$ can be expressed:

$$\mathbf{A} = \begin{bmatrix} v + k_c \frac{\partial v_c}{\partial k_c} & k_{pt} \frac{\partial v_c}{\partial k_c} \\ k_c \frac{\partial v_c}{\partial k_{pt}} & v + k_{pt} \frac{\partial v_c}{\partial k_{pt}} \end{bmatrix}$$

For m $\leq$ 2, strict hyperbolicity can be easily proved when $\frac{\partial v_c}{\partial k_c}, \frac{\partial v_c}{\partial k_{pt}} < 0$.

Continuum space-time model: time and space discretization

Riemann problem is defined as:

$$\begin{aligned} \text{PDEs: } & \boldsymbol{K}_t + \boldsymbol{F}(\boldsymbol{K})_x = \mathbf{0} \\ \text{ICs: } & \boldsymbol{K}(x,0) = \boldsymbol{K}_0(x) \\ \text{BCs: } & \boldsymbol{K}(0,t) = \boldsymbol{K}_1(t), \boldsymbol{K}(L,t) = \boldsymbol{K}_L(t) \end{aligned}$$

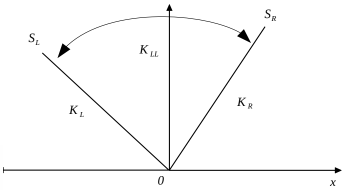
Discretized using Forward Euler (FE) in time

$$\frac{\boldsymbol{K}_i^{n+1} - \boldsymbol{K}_i^n}{\Delta t} + \frac{\boldsymbol{F}_{i-\frac{1}{2}} - \boldsymbol{F}_{i+\frac{1}{2}}}{\Delta x} = \mathbf{0}$$

Numerical flux $\boldsymbol{F}_{i-\frac{1}{2}}$ is yet to be defined.

Approximated Riemann solver proposed by Harten, Lax and van Leer (HLL) is used.

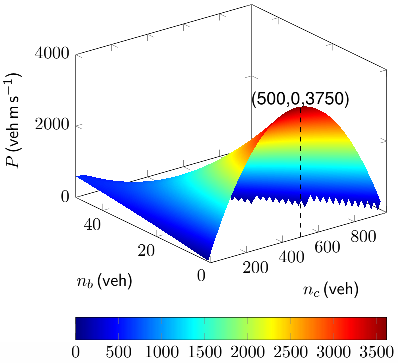
$$\boldsymbol{F}_{i+\frac{1}{2}}^{HLL} = \begin{cases} \boldsymbol{F}_L & \text{if } 0 \leq S_L \\ \dfrac{S_R \boldsymbol{F}_L - S_L \boldsymbol{F}_R + S_L S_R (\boldsymbol{K}_R - \boldsymbol{K}_L)}{S_R - S_L} & \text{if } S_L \leq 0 \leq S_R \\ \boldsymbol{F}_R & \text{if } 0 \geq S_R \end{cases}$$



Wave speeds S_L, S_R are computed based on the Eigen values of the each control volume.

Numerical details

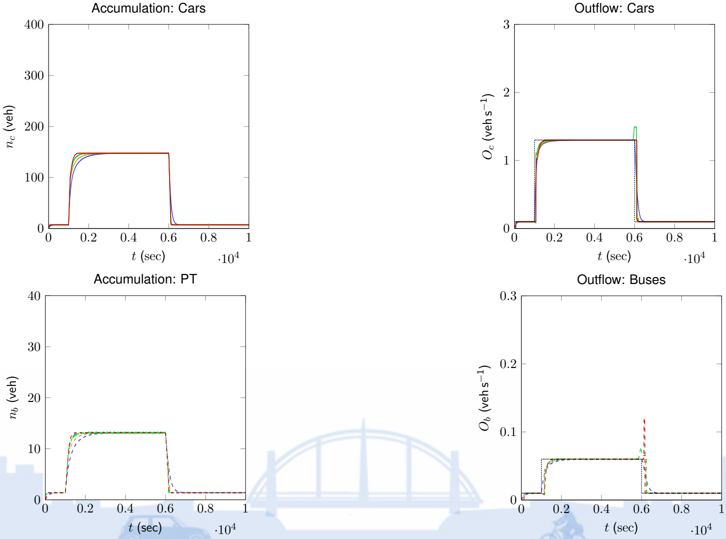
- Two different cases are considered: a reference low demand and high demand case with congestion spill-backs.
- Free flow speeds: $v_c = v_{pt} = 15\text{m sec}^{-1}$, n_{cr} for $npt = 0$ is 500.
- Trip lengths: $[L_c, L_{pt}] = [1000\,\text{m}, 2000\,\text{m}]$.
- Time step, $\Delta t = 1\,\text{sec}$ for accumulation-based model.
- Adaptive time stepping is used for continuum space-time model with $CFL = 0.5$.



pMFD

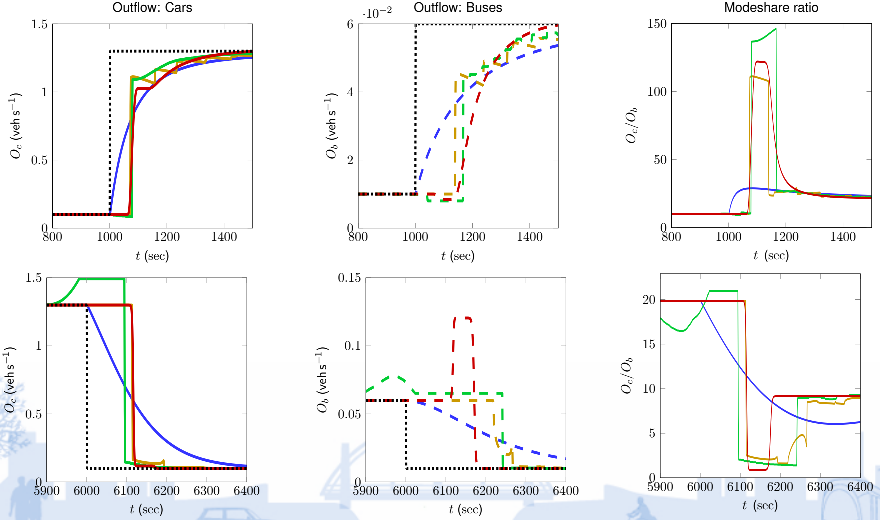
Case 1: low demand

— Acc-based, — Delay acc-based, — Trip-based, — CSM.



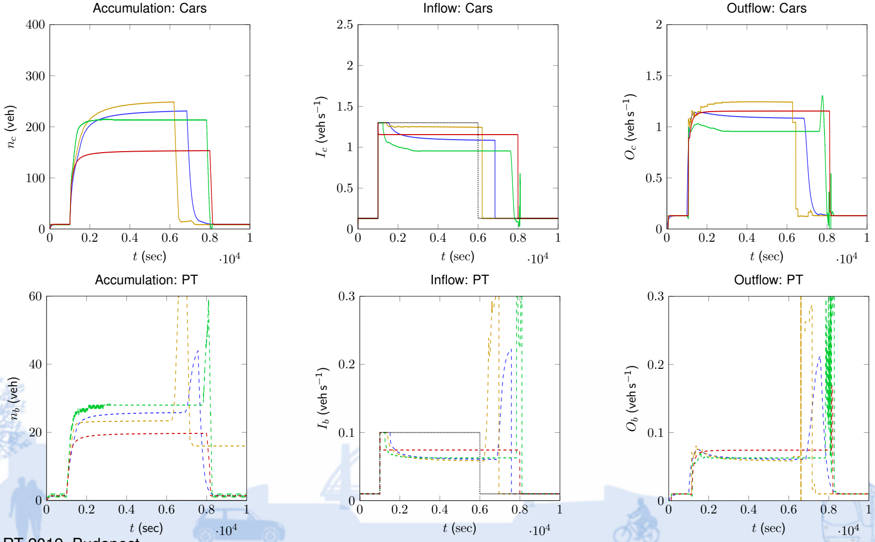
Case 1: Transition period

— Acc-based, — Delay acc-based, — Trip-based, — CSM.

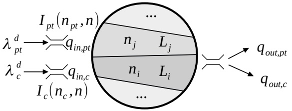


Case 2: high demand

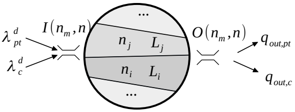
— Acc-based, — Delay acc-based, — Trip-based, — CSM.



FIFO-based entry flow for trip-based model

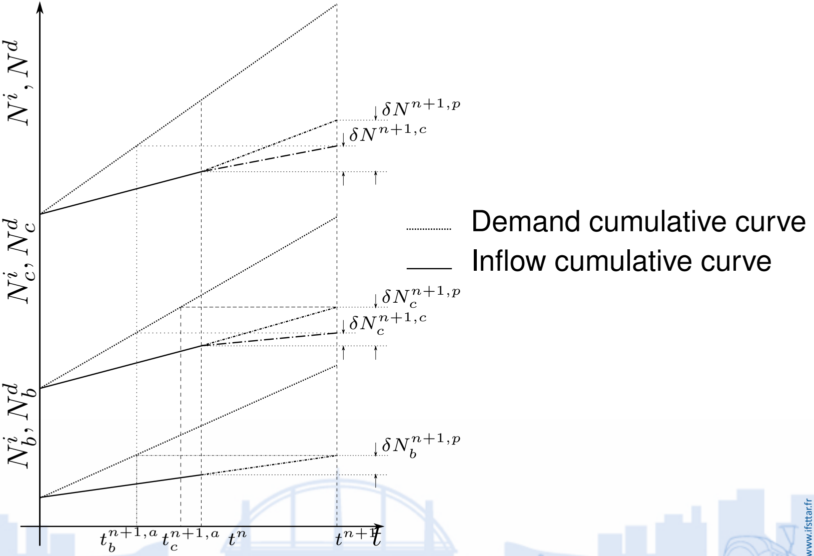


$$I_m(n_m, n) = \begin{cases} \dfrac{n_m}{n} \dfrac{P_{cr}(n_m, n)}{L_m} & \text{if } n \leq n_{cr}(n_m, n \\ \dfrac{n_m}{n} \dfrac{P(n_m, n)}{L_m} & \text{otherwise} \end{cases}$$



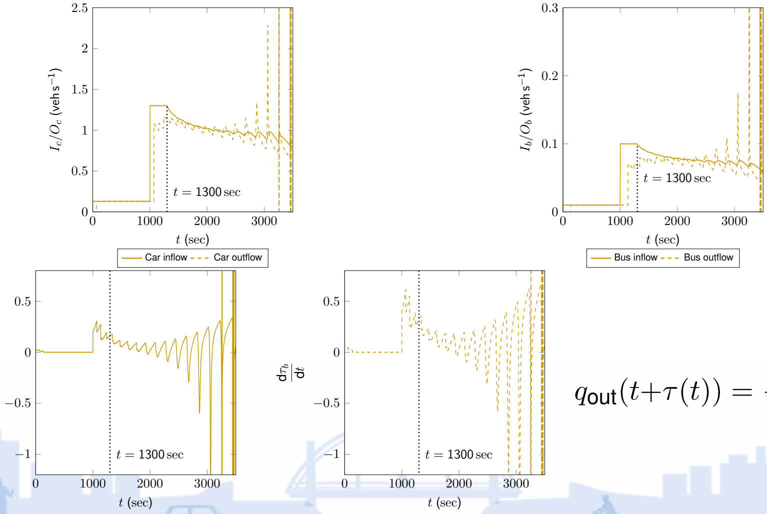
$$I(n_m, n) = \begin{cases} \dfrac{P_{cr}(n_m, n)}{\bar{L}_m} & \text{if } n \leq n_{cr}(n_m, n) \\ \dfrac{P(n_m, n)}{\bar{L}_m} & \text{otherwise} \end{cases}$$

FIFO-based entry for accumulation-based models



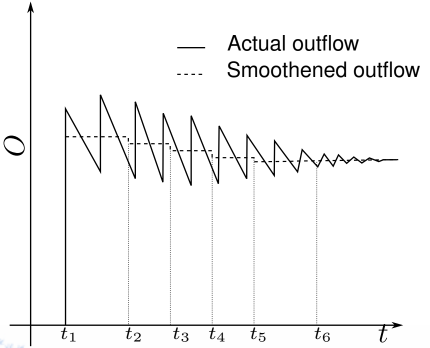
Proposed in Chevallier and Leclercq (2008)

High demand: Inflow and outflow for delay accumulation-based using FIFO-entry

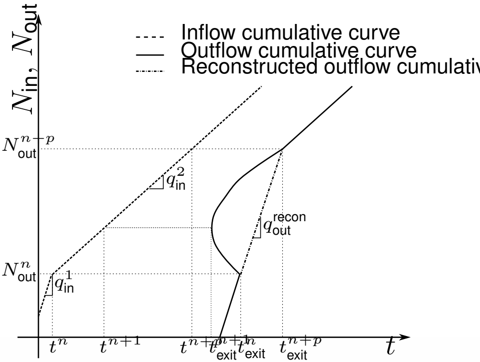


$$q_{\text{out}}(t+\tau(t)) = \frac{q_{\text{in}}(t)}{1 + \frac{d\tau(t)}{dt}}$$

Outflow stabilisation and weak FIFO discipline in delay-accumulation-based



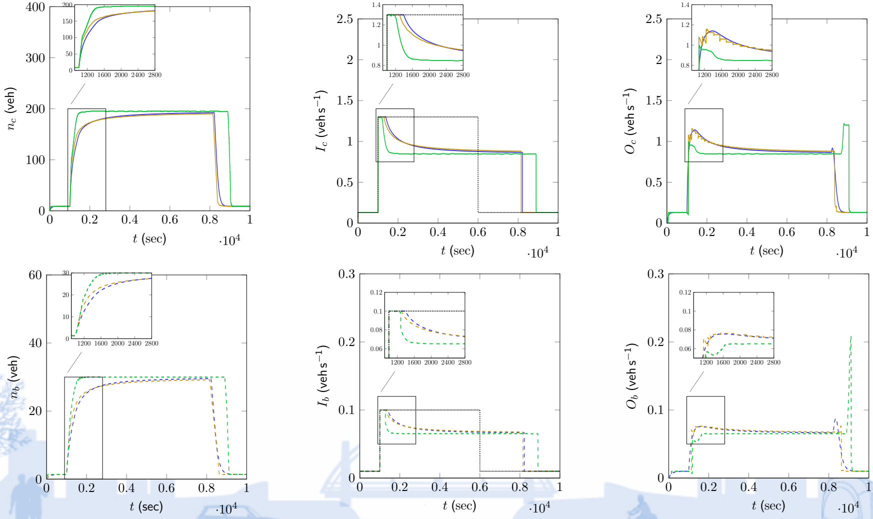
Outflow Stabilisation.



Weak FIFO discipline.

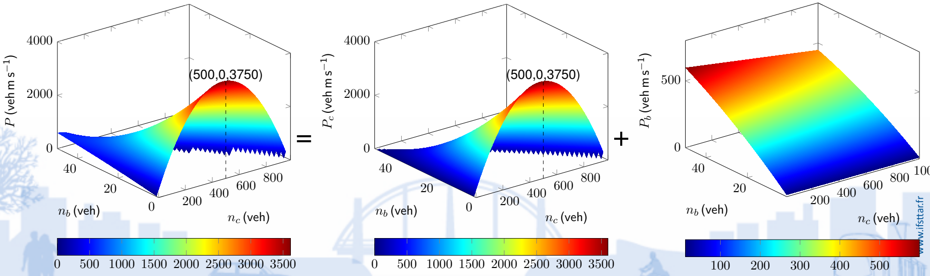
Case 2: High demand after stabilisation

— Acc-based, — Delay acc-based, — Trip-based.



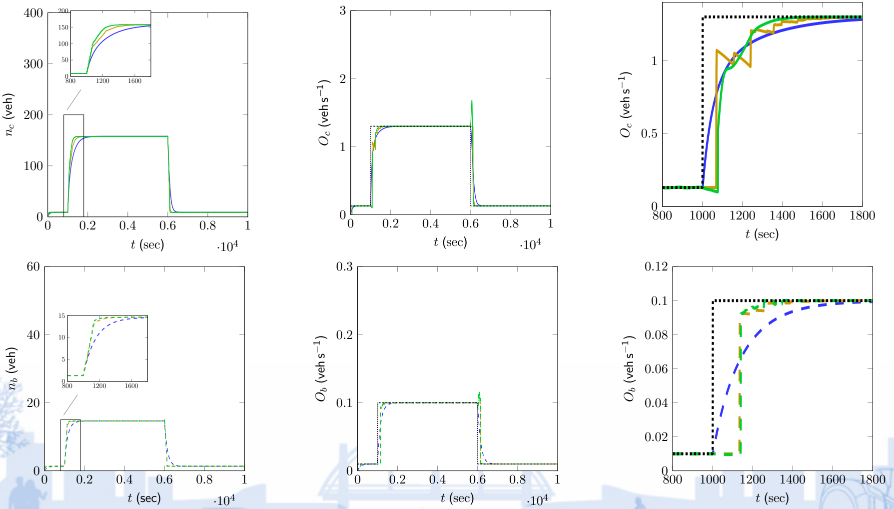
High demand: Segregated 3D-MFDs

$$v_c(n_c, n_{pt}) = \beta_{c,0} + \beta_{c,c}\, n_c + \beta_{pt,c}\, n_{pt}$$
$$v_b(n_c, n_{pt}) = \beta_{pt,0} + \beta_{c,pt}\, n_c + \beta_{pt,pt}\, n_{pt}$$
$$v(n_c, n_{pt}) = v_c \frac{n_c}{n_c + n_{pt}} + v_{pt} \frac{n_{pt}}{n_c + n_{pt}}$$
$$P(n_c, n_{pt}) = v_c\, n_c + v_{pt}\, n_{pt}$$
$$P_c = v_c\, n_c; P_{pt} = v_{pt}\, n_{pt}$$

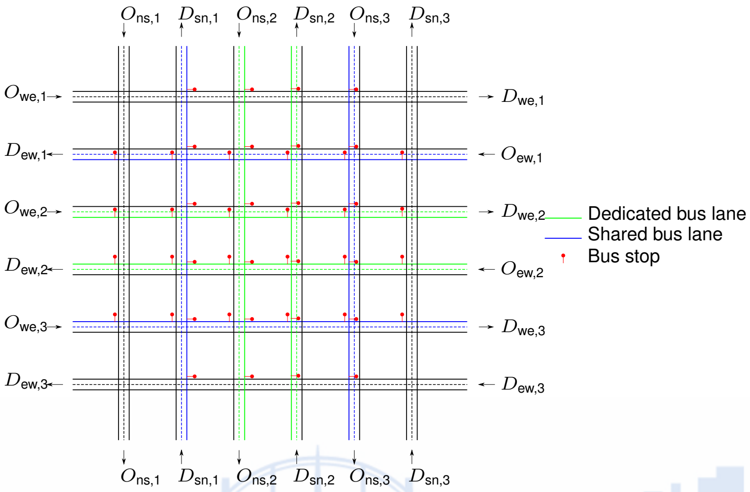


High demand: Segregated 3D-MFDs results

— Acc-based, — Delay acc-based, — Trip-based.

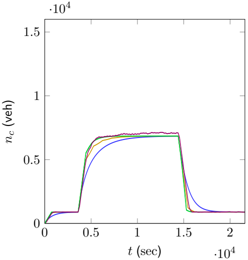


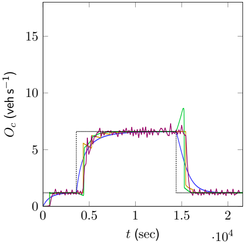
Verification with micro-simulation

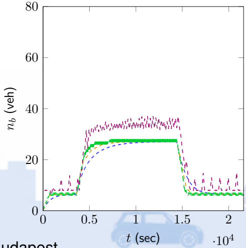


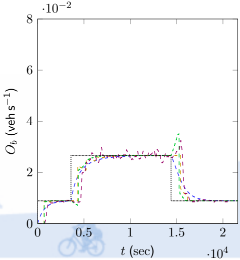
Verification with micro-sim. with SINGLE 3D-MFD, WITHOUT dedicated bus lanes

— Acc-based, — Delay acc-based, — Trip-based, — micro-sim.



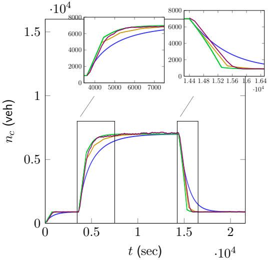


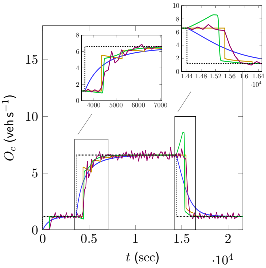


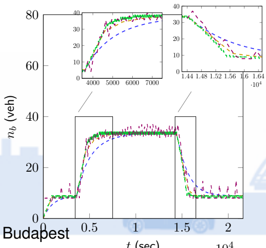


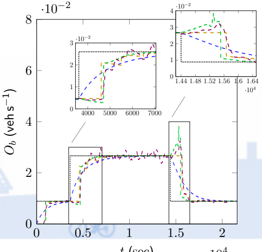
Verification with micro-sim. with SEGREGATED 3D-MFDs, WITHOUT dedicated bus lanes

— Acc-based, — Delay acc-based, — Trip-based, — micro-sim.



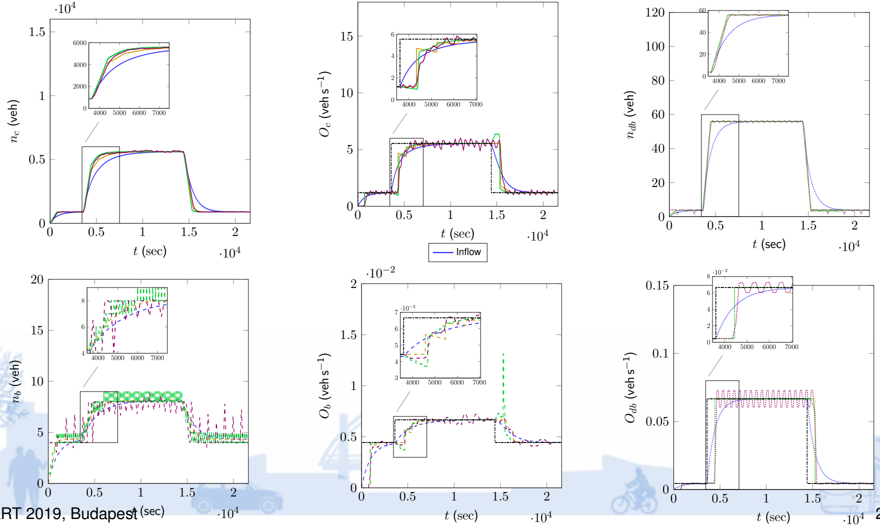






Verification with micro-sim. with SEGREGATED 3D-MFDs, WITH dedicated bus lanes

— Acc-based, — Delay acc-based, — Trip-based, — micro-sim.



Conclusions & Future work

- Transition dynamics are well captured by trip-based model followed by delay accumulation-based.
- FIFO-based entry supply function is more robust compared to conventional one used in literature.
- Stabilization techniques proposed in this work address the shortcomings of delay accumulation-based model.
- Verify the model with results from micro-simulation for the real network of Lyon city.
- Extend the model to multi reservoir settings and validate using the empirical data.

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Thank you for your attention.

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