9.7 Given $n = 100$, $\bar{x} = 37.7$ $s = 9.2$

(a) As n is large (greater than 30). s can be taken as σ and t as Z.

$$\text{Confidence Interval} = \bar{x} \pm Z_{\alpha/2} * \frac{\sigma}{\sqrt{n}}$$

$$= \bar{x} \pm Z_{\alpha/2} * \frac{s}{\sqrt{n}}$$

90% of confidence interval $= 37.7 \pm (1.645) * \frac{9.2}{\sqrt{100}}$

($\alpha = 10\% = 0.1$

$\alpha/2 = 0.05$

$Z_{0.05} = 1.645$)

$= 37.7 \pm 1.5134$

$= \boxed{[36.19, 39.21]}$

(b) We have to test the null hypothesis.

$H_0 : \mu = 35$ against the alternative hypothesis

$H_1 : \mu > 35$ at level $\alpha = 0.01$

As n is large, we will do a large sample Z test. The rejection region is $Z > Z_{\alpha} = 2.33$.

using the normal table.

$$Z = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{37.7 - 35}{9.2/\sqrt{100}} = 2.93$$

Since $Z = 2.93 > 2.33$. $\boxed{H_0 \text{ is rejected}}$

Thus, there is significant evidence at 1% significance level that the mean number of concurrent users is greater than 35.

9.9 $n=3$ 50, 30, 70

(a) Mean $= \frac{30+50+70}{3} = 50$.

$$\text{Variance} = s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2$$

$$= \frac{1}{2}\left[(30-50)^2+(50-50)^2+(70-50)^2\right]$$

$$s^2 = 400 \Rightarrow s \Rightarrow 20.$$

Confidence Interval $\Rightarrow \bar{x} \pm t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$

$$\Rightarrow 50 \pm \frac{20}{\sqrt{3}} \times 2.920$$

$$\Rightarrow 50 \pm 33.7$$

$$\Rightarrow \boxed{[16.3,\ 83.7]}$$

(b) H_0: null hypothesis $\mu = 80$.

H_1: Alternative hypothesis $\mu \neq 80$.

level of significant $\alpha = 0.1$.

Rejection region $t < -t_{\alpha/2}$ (or) $t > t_{\alpha/2}$

With the 90% confidence, we get at 2 degrees of freedom critical value $t_{0.05/2} = 2.92$

Rejection region: $t < -2.92$ (or) $t > 2.92$

Here our test statistics lies in the acceptance region. So we fail to reject our null hypothesis at $\alpha = 0.1$. We can conclude that there is statistically insignificant that the average salary of all employee level computer engineers is different from \$80,000.

9.9(c)

Sample standard deviation $s = 20$

Sample size $n = 3$.

Significant level $\alpha = 1 - 0.9 = 0.1$

Degree of freedom $DF = n - 1 = 2$.

Since $\frac{(n-1)s^2}{\sigma}$ follows chi-squared with $(n-1)$ DF

$$\Rightarrow P\left(\chi^2_{1-\alpha/2} \le \frac{(n-1)s^2}{\sigma^2} \le \chi^2_{\alpha/2}\right) = (1-\alpha)$$

90% CI is $\sqrt{\frac{(n-1)s^2}{\chi^2_{\alpha/2}}} \le \sigma \le \sqrt{\frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}}$

$$\chi_1^2 = \chi^2_{1-\alpha/2} = \chi^2_{1-0.1/2}(d_f = n-1) = \chi^2_{0.95}(df = 2)$$

$$= 0.103 \text{ (from } \chi^2 \text{ table)}$$

$$\chi_R^2 = \chi^2_{\alpha/2} = \chi^2_{0.1/2}(df = 2) = \chi^2_{0.05} = 5.991$$

(from χ^2 table)

$$\sqrt{\frac{(n-1)s^2}{\chi^2_{\alpha/2}}} = \sqrt{\frac{(3-1)400}{5.991}} \approx 11.556 = 11.6$$

$$\sqrt{\frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}} = \sqrt{\frac{(3-1)\,400}{0.103}} = 89.4$$

90% CI for population std deviation is

$[11.6,\ 89.4]$.

9.10.

(a) Sample proportion $24/200 = 0.12$

alpha $\alpha = 1 - 0.96 = 0.04$

$z_{\alpha/2} = z_{0.02} = 2.054$.

$$CI = 0.12 \pm 2.054 * \sqrt{\frac{0.12(1-0.12)}{200}}$$

$$= 0.12 \pm 0.047$$

$$= [0.073, 0.167]$$

(b) Null hypothesis $H_0: p = 0.1$

alternate hypothesis $H_a: p < 0.1$

Disapproving the manufactures claim means rejecting H_0 in favour of H_a.

The observed test statistic is

$$z = (0.12 - 0.1) \sqrt{\frac{0.12(1-0.12)}{200}} = 0.8704$$

The critical values, from the table of Normal distribution are $z_{0.04} = 1.75$ $z_{0.15} = 1.04$

$z_{0.20} = 0.84$

Therefore, we do not have a significant Evidence at 4% and 15% levels to disprove the manufacture's claim. However, since $z > 0.84$ belongs to the rejection region for the 20% level, we do have a significance evidence against the manufacturer's claim at the 20% level of significance.

9.15

Town A : $x = 42$, $n = 70$, $\hat{p}_A = \frac{42}{70} = 0.16$

Town B : $x = 59$, $n = 100$, $\hat{p}_B = \frac{59}{100} = 0.59$

H_0 : null hypothesis $\hat{p}_1 - \hat{p}_2 = 0$.

$H_A = H_1$: $\hat{p}_1 - \hat{p}_2 \neq 0$.

α = significant level = 0.05.

$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}} = \frac{0.6 - 0.59}{\sqrt{\frac{0.6(1-0.6)}{70} + \frac{0.59(1-0.59)}{100}}}$$

$$= 0.13$$

$$\text{p-value} = 2P(Z > Z_0)$$

$$= 2(1 - P(Z \leq 0.13))$$

$$= 2(1 - 0.551)$$

$$= 0.448$$

$$= \boxed{0.896}$$

P value is greater than significant level, so we fail to reject H_0 & conclude there is no significant difference between town A & town B at 5% significant level.

The p value is 0.896 and it is greater than the level of significance then we fail to reject the null hypothesis there is no significance difference.

9.18 Before firewall 56, 47, 49, 37, 38, 60, 50, 43, 43.

(a) 59, 50, 56, 54, 58

After firewall 53, 21, 32, 49, 45, 38, 44, 33, 32, 43,
53, 46, 36, 48, 39, 35, 37, 36, 39, 45.

$\overline{x_1} = \frac{700}{14} = 50 \qquad \overline{x_2} = \frac{804}{20} = 40.2$

$s_1 = 7.62 \qquad s_2 = 7.96.$

$S_p = \sqrt{\frac{13 * (7.62)^2 + 19(7.96)^2}{14 + 20 - 2}}$

$= \sqrt{61.209} = 7.82$

$CI = \bar{x} - \bar{y} \pm t_{\alpha/2} \, S_p \sqrt{\frac{1}{n} + \frac{1}{m}}$

$= 50 - 40.2 \pm 2.037 (7.82) \left(\sqrt{\frac{1}{14} + \frac{1}{20}}\right)$

$= 9.8 \pm 5.54341032 = [4.25658968, 15.3434]$

$\approx \boxed{[4.25, 15.35]}$

(b) Null Hypothesis $H_0 : \mu_B = \mu_A$

Altanate hypothesis: $H_A = \mu_B \neq \mu_A$

test statistics $t = \frac{50 - 40.2}{2.7244} = 3.597$

$\boxed{\text{p-value} = 0.00055}$

$\alpha = 0.05$

so p value $< \alpha$, By rejection rule, it can be concluded that there is evidence to reject H_0 at $\alpha = 0.05$

$\therefore$ Hence there is significance difference between average no. of intrusion attempts per day before and after change of firewall settings.

(ii) $H_0: \mu_B = \mu_A$ (Assume unequal variance)

$H_A: \mu_B \neq \mu_A$

$$t = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{s_x^2}{m} + \frac{s_y^2}{n}}} = \frac{50 - 40.2}{\sqrt{\frac{7.62^2}{14} + \frac{7.46}{20}}} = \frac{9.8}{2.704} = 3.62$$

$\boxed{\text{p value} = 0.001}$

p value $< \alpha$

It can be concluded that there is evidence to reject H_0 at $\alpha = 0.05$

Hence, there is significant difference between average no. of intrusion attempts per day before and after change of firewall settings.

Based on these assumptions, there is significant Reduction.

9.20 $\sigma = 5$, $S = 6.2$, $n = 40$

$H_0: \sigma^2 = 5^2$

$H_A: \sigma^2 \neq 5^2$.

$\alpha = 0.05$

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{39(6.2)^2}{5^2} = \frac{39 \times 38.44}{25}$$

$$= \frac{1499.16}{25} = 59.9664$$

$df = 40-1 = 39$

pvalue $= 0.017043$.

Conclusion:

pvalue is less than given significance level so we reject the null hypothesis and conclude that significantly different from assumed value of 5 min.

9.22 We have $n=20$, $m=30$.

$s_x = 0.6$ $s_y = 1.2$

(a) $H_0: \sigma^2_x = \sigma^2_y$

$H_A: \sigma^2_x \neq \sigma^2_y$

F statistics

$$F_{obs} = \frac{s^2_x}{s^2_y} = \frac{0.6^2}{1.2^2} = 0.25$$

$P = 2\min\left(P\{F > 0.25\},\ P\{F < 0.25\}\right)$

$= 0.0026$

using F distribution using $n-1 = 19$ and $m-1 = 29$ dF gives $\boxed{P \cong 0.001}$

Thus, there is significant Evidence that Variance are unequal we are using Satterthwaite approxiamation for two sample t test comparing two population mean.

(b) $F_{0.025}(19.29) = 2.23$ and $F_{0.025}(29, 19) = 2.40$

$= [0.6^2/1.2^2 * 2.23,\ 0.6^2 * 2.40/1.2^2]$

$\boxed{= [0.11,\ 0.60]}$

$F_{0.025}(19, 29) \cong 2.2$ and $F_{0.025}(29, 19) \cong 2.35$

The approxiamate 95% interval is

$[0.6^2/1.2^2 * 2.2,\ 0.6^2 * 2.35/1.2^2]$

$= [0.11,\ 0.59]$

9.23 From the Table, $n = 6$.

Mean of Anthony scores $\bar{x}_a = \frac{510}{6} = 85$.

Variance $s_x^2 = \sqrt{\frac{1}{n-1} \Sigma (x_i - \bar{x})^2}$

$= \sqrt{\frac{1}{6-1} [(85-85)^2 + (92-85)^2 + (97-85)^2 + (65-85)^2 + (75-85)^2 + (96-85)^2]}$

$= \sqrt{\frac{1}{6-1} [814]} = 12.7594$

Mean of Eric scores $\bar{x}_e = \frac{480}{6} = 80$

Variance $s_y^2 = \sqrt{\frac{1}{n-1} \Sigma (x_i - \bar{x})^2}$

$= \sqrt{\frac{1}{6-1} [(81-80)^2 + (79-80)^2 + (76-80)^2 + (84-80)^2 + (83-80)^2 + (77-80)^2]}$

$= \sqrt{\frac{1}{6-1} [52]}$

$= 3.2249$

(a) H_0: Null Hypothesis
there is no significant difference between means

H_1: Mean of Anthony is higher than Eric mean

T-test for difference of mean with Equality of variance.

$$t = \frac{\bar{x} - \bar{y}}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t_{n_1+n_2-2}$$

$$s^2 = \frac{(n_1+1)s_x^2 + (n_2-1)s_y^2}{m+n_2-2}$$

$= 86.6008$

$s = 9.306$

$$t = \frac{85-80}{9.306\sqrt{\frac{1}{6}+\frac{1}{6}}} = \boxed{0.9305}$$

t critical value at 5% (generally we take 5%) with $(n_1+n_2-2) = 10$ $d_f = 1.812$

Since $t < t_{0.05,10}$ So we accept H_0.

$\therefore$ we fail to reject the hypothesis. There is no significant difference b/w Averages.

(b) Null hypothesis $H_0 : \sigma_1^2 = \sigma_2^2$

Eric's claim : stable variance.

Alternate hypothesis : $H_a : \sigma_1^2 > \sigma_2^2$

Test is right tailed.

following is the output for F test.

Anthony	Eric
Mean = 85	Mean = 80
Variance = 162.8	Variance = 10.4
Observations = 6	6
df = 5	5

$F = 15.65384615$

P value one tail $\boxed{0.004494529}$

F critical one tail 5.050329058

Since p value < 0.05, we reject null hypothesis

that is there is evidence to support Eric claim.