

2.16 Organize the data. Let $D = \{\text{defective part}\}$. We are given:

$$\begin{array}{l|l} P\{S1\} = 0.5 & P\{D|S1\} = 0.05 \\ P\{S2\} = 0.2 & P\{D|S2\} = 0.03 \\ P\{S3\} = 0.3 & P\{D|S3\} = 0.06 \end{array}$$

We need to find $P\{S1|D\}$.

(a) By the Law of Total Probability:

$$\begin{aligned} P\{D\} &= P\{D|S1\}P\{S1\} + P\{D|S2\}P\{S2\} + P\{D|S3\}P\{S3\} \\ &= (0.5)(0.05) + (0.2)(0.03) + (0.3)(0.06) = \boxed{0.049} \end{aligned}$$

(b) Bayes Rule:

$$P\{S1|D\} = \frac{P\{D|S1\}P\{S1\}}{P\{D\}} = \frac{(0.5)(0.05)}{0.49} = \boxed{25/49 \text{ or } 0.5102}$$

2.18 Let $C = \{\text{correct}\}$, $G = \{\text{guessing}\}$. It is given that:

$$P\{\bar{G}\} = 0.75, \quad P\{C | \bar{G}\} = 0.9, \quad P\{C | G\} = 1/4 = 0.25.$$

Also, $P\{G\} = 1 - 0.75 = 0.25$.

Then, by the Bayes Rule,

$$\begin{aligned} P\{G | C\} &= \frac{P\{C | G\}P\{G\}}{P\{C | G\}P\{G\} + P\{C | \bar{G}\}P\{\bar{G}\}} \\ &= \frac{(0.25)(0.25)}{(0.25)(0.25) + (0.9)(0.75)} = \boxed{0.0847} \end{aligned}$$

- 2.28** The sample space consists of sequences of 6 answers where each answer is one of 4 possible answers, say, A, B, C, or D. Then a sequence of 6 answers is a 6-letter word written with letters A, B, C, and D with replacement. The student guesses, therefore, all outcomes are equally likely.

The total number of outcomes is

$$\mathcal{N}_T = P_r(4, 6) = 4^6 = 4096.$$

Favorable outcomes occur when the student guesses at least 3 answers correctly. This includes 3, 4, 5, and 6 correct answers. The correctly answered questions are chosen at random from 6 questions. Then, a correct answer is given to each of the chosen questions. Also, an incorrect answer to each remaining question is chosen out of 3 possible incorrect answers. Altogether, the number of favorable outcomes is

$$\begin{aligned}\mathcal{N}_F &= C(6, 3)(3^3) + C(6, 4)(3^2) + C(6, 5)(3^1) + C(6, 6)(3^0) \\ &= \frac{(6)(5)(4)}{(3)(2)(1)}(27) + \frac{(6)(5)}{(2)(1)}(9) + (6)(3) + 1 = 694.\end{aligned}$$

$$P\{\text{he will pass}\} = \frac{\mathcal{N}_F}{\mathcal{N}_T} = \frac{694}{4096} = \boxed{0.1694}$$

One can also use the complement rule for a little shorter solution.

Note: You can also use Binomial Distribution

- 2.29** Outcomes are sets of four databases selected from nine. Favorable outcomes are such sets where at least 2 databases have a keyword, out of 5 such databases (and the remaining ones don't have a keyword, so they come from the remaining 4 databases). Then

$$\mathcal{N}_T = C(9, 4) = \frac{(9)(8)(7)(6)}{(4)(3)(2)(1)} = 126,$$

$$\begin{aligned}\mathcal{N}_F &= C(5, 2)C(4, 2) + C(5, 3)C(4, 1) + C(5, 4)C(4, 0) \\ &= (10)(6) + (10)(4) + (5)(1) = 105,\end{aligned}$$

and

$$P\{\text{at least two have the keyword}\} = \frac{\mathcal{N}_F}{\mathcal{N}_T} = \frac{105}{126} = \boxed{\frac{5}{6} \text{ or } 0.8333}$$

Note: You can also use Binomial Distribution

- 3.20** (a) We need to compute $\mathbf{P}\{X = 3\}$, where X is the number of defective computers (“successes”) in a shipment of 20 (“trials”). It has Binomial distribution with parameters $n = 20$ and $p = 0.05$. From Table A2,

$$P(X = 3) = P(X \leq 3) - P(X \leq 2) = .9841 - .9245 = \boxed{0.0596}$$

- (b) Let Y be the number of defective computers among the first four. It has Binomial distribution with $n = 4$ and $p = 0.05$. From Table A2,

$$\begin{aligned} \mathbf{P}\{\text{at least 5 computers are tested until 2 defective ones are found}\} \\ &= \mathbf{P}\{\text{among the first 4 computers, at most 1 is defective}\} \\ &= \mathbf{P}\{Y \leq 1\} = \boxed{0.9860} \end{aligned}$$

The problem can also be solved directly, but computing $\mathbf{P}\{W \geq 5\}$, where W is the Negative Binomial ($k = 2$, $p = 0.05$) number of computers the engineer has to test in order to find 2 defective computers:

$$\begin{aligned} \mathbf{P}\{W \geq 5\} &= 1 - P(2) - P(3) - P(4) \\ &= 1 - 0.05^2 - (2)(0.05)^2(0.95) - (3)(0.05)^2(0.95)^2 \\ &= \boxed{0.9860} \end{aligned}$$

- 3.33** (a) The number X of computer shutdowns during one year (12 months) averages

$$\lambda = (0.25)(12) = 3 \text{ shutdowns per year.}$$

From Table A3 with $\lambda = 3$,

$$\mathbf{P}\{X \geq 3\} = 1 - F_X(2) = 1 - 0.423 = \boxed{0.577}$$

- (b) Let Y be the number of months with exactly 1 computer shutdown. Each month is a Bernoulli trial because it either has exactly 1 shutdown or not. Hence, Y has Binomial distribution with $n = 12$ and

$$\begin{aligned} p &= \mathbf{P}\{1 \text{ shutdown in one month}\} = e^{-0.25} \frac{0.25^1}{1!} \\ &= 0.25 e^{-0.25} = 0.1947. \end{aligned}$$

Then,

$$\begin{aligned} \mathbf{P}\{Y \geq 3\} &= 1 - P_Y(0) - P_Y(1) - P_Y(2) \\ &= 1 - (1 - 0.1947)^{12} - (12)(0.1947)(1 - 0.1947)^{11} \\ &\quad - \frac{(12)(11)}{2}(0.1947)^2(1 - 0.1947)^{10} = \boxed{0.4228} \end{aligned}$$

4.18 Standardize and use Table A4.

$$(a) \mathbf{P}(X \leq 2.39) = \mathbf{P}\left(Z \leq \frac{2.39 - (-3)}{2.7}\right) = \Phi(2.00) = \boxed{0.9772}$$

$$(b) \mathbf{P}(Z \geq -2.39) = \mathbf{P}\left(Z \geq \frac{-2.39 - (-3)}{2.7}\right) = 1 - \Phi(0.23) = 1 - 0.5910 = \boxed{0.4090}$$

$$(c) \mathbf{P}(|X| \geq 2.39) = \mathbf{P}(X \leq -2.39) + \mathbf{P}(X \geq 2.39) = (1 - 0.4090) + (1 - 0.9772) = \boxed{0.6138} \text{ using answers from (a) and (b)}$$

$$(d) \mathbf{P}(|X + 3| \geq 2.39) = P(X \leq -2.39 - 3) + P(X \geq 2.39 - 3) = \mathbf{P}\left(\frac{|X+3|}{2.7} \geq \frac{2.39}{2.7}\right) = \mathbf{P}(|Z| \geq 0.89) = 2\Phi(-0.89) = 2(0.1867) = \boxed{0.3734}$$

$$(e) \mathbf{P}(X < 5) = \mathbf{P}\left(Z < \frac{5 - (-3)}{2.7}\right) = \Phi(2.96) = \boxed{0.9985}$$

$$(f) \mathbf{P}(|X| < 5) = \mathbf{P}\left(\frac{-5 - (-3)}{2.7} < Z < \frac{5 - (-3)}{2.7}\right) = \Phi(0.89) - \Phi(-0.74) = 0.8133 - 0.2296 = \boxed{0.5837}$$

(g) Solve the equation

$$\mathbf{P}(X > x) = 0.33$$

for x . We have

$$\mathbf{P}(X > x) = \mathbf{P}\left(Z > \frac{x + 3}{2.7}\right) = 1 - \Phi\left(\frac{x + 3}{2.7}\right) = 0.33,$$

so that $\Phi\left(\frac{x+3}{2.7}\right) = 0.67$.

From Table A4, we find that $\Phi(0.44) = 0.67$. Therefore (unstandardizing 0.44),

$$\frac{x + 3}{2.7} = 0.67 \quad \Rightarrow \quad x = \boxed{-1.19}$$

4.21 The height X has Normal distribution with $\mu = 79''$ and $\sigma = 3.89''$. Using Table A4,

$$(a) \mathbf{P}(X > 84'') = \mathbf{P}\left(Z > \frac{84 - 79}{3.89}\right) = P(Z > 1.29) = 1 - \Phi(1.29) = 1 - 0.9015 = \boxed{0.0985 \text{ or } 9.85\%}$$

(b) Solve the equation

$$\mathbf{P}(X > x) = 0.20$$

for x . We have

$$\mathbf{P}(X > x) = \mathbf{P}\left(Z > \frac{x - 79}{3.89}\right) = 1 - \Phi\left(\frac{x - 79}{3.89}\right) = 0.20,$$

so that $\Phi\left(\frac{x-79}{3.89}\right) = 0.80$.

From Table A4, we find that $\Phi(0.84) \approx 0.80$. Therefore (unstandardizing 0.84),

$$\frac{x - 79}{3.89} = 0.84 \quad \Rightarrow \quad x = \boxed{82.3 \text{ in or } 6 \text{ ft } 10.3 \text{ in}}$$

So, the height of your favorite player can be 6'10.3" or more.