CSE-5301, HW1 Solutions

Homework 1 Sample Solutions

2.2 Denote the events:

$$\begin{aligned} M &= \{ \text{ problems with a motherboard } \} \\ H &= \{ \text{ problems with a hard drive } \} \end{aligned}$$

We have:

$$\boldsymbol{P}\{M\} = 0.4,\ \boldsymbol{P}\{H\} = 0.3, \quad \text{and} \quad \boldsymbol{P}\{M \cap H\} = 0.15.$$

Hence,

$$\boldsymbol{P}\{M \cup H\} = \boldsymbol{P}\{M\} + \boldsymbol{P}\{H\} - \boldsymbol{P}\{M \cap H\} = 0.4 + 0.3 - 0.15 = 0.55,$$

and

$$\boldsymbol{P}\{\text{fully functioning MB and HD}\} = 1 - \boldsymbol{P}\{M \cup H\} = \boxed{0.45}$$

2.4 Denote the events,

$$C = \{ \text{ knows C/C++ } \}, \quad F = \{ \text{ knows Fortran } \}.$$

Then

(a) $\boldsymbol{P}\{\bar{F}\} = 1 - \boldsymbol{P}\{F\} = 1 - 0.6 = \boxed{0.4}$

(b) $$\begin{aligned} \boldsymbol{P}\{\bar{F} \cap \bar{C}\} &= 1 - \boldsymbol{P}\{F \cup C\} = 1 - (\boldsymbol{P}\{F\} + \boldsymbol{P}\{C\} - \boldsymbol{P}\{F \cap C\}) \\ &= 1 - (0.7 + 0.6 - 0.5) = 1 - 0.8 = \boxed{0.2} \end{aligned}$$

(c) $\boldsymbol{P}\{C \backslash F\} = \boldsymbol{P}\{C\} - \boldsymbol{P}\{F \cap C\} = 0.7 - 0.5 = \boxed{0.2}$

(d) $\boldsymbol{P}\{F \backslash C\} = \boldsymbol{P}\{F\} - \boldsymbol{P}\{F \cap C\} = 0.6 - 0.5 = \boxed{0.1}$

(e) $\boldsymbol{P}\{C \mid F\} = \dfrac{\boldsymbol{P}\{C \cap F\}}{\boldsymbol{P}\{F\}} = \dfrac{0.5}{0.6} = \boxed{0.8333}$

(f) $\boldsymbol{P}\{F \mid C\} = \dfrac{\boldsymbol{P}\{C \cap F\}}{\boldsymbol{P}\{C\}} = \dfrac{0.5}{0.7} = \boxed{0.7143}$

2.6 Let $A = \{\text{arrive on time}\}$, $W = \{\text{good weather}\}$. We have

$$\boldsymbol{P}\{A \mid W\} = 0.8,\ \boldsymbol{P}\{A \mid \bar{W}\} = 0.3,\ \boldsymbol{P}\{W\} = 0.6$$

By the Law of Total Probability,

$$\begin{aligned} \boldsymbol{P}\{A\} &= \boldsymbol{P}\{A \mid W\}\boldsymbol{P}\{W\} + \boldsymbol{P}\{A \mid \bar{W}\}\boldsymbol{P}\{\bar{W}\} \\ &= (0.8)(0.6) + (0.3)(0.4) = \boxed{0.60} \end{aligned}$$

2.8 Let $A_1 = \{\text{1st device fails}\}$, $A_2 = \{\text{2nd device fails}\}$, $A_3 = \{\text{3rd device fails}\}$.

$$\begin{aligned} \boldsymbol{P}\{\text{ on time }\} &= \boldsymbol{P}\{\text{ all function }\} \\ &= \boldsymbol{P}\{\overline{A}_1 \cap \overline{A}_2 \cap \overline{A}_3\} \\ &= \boldsymbol{P}\{\overline{A}_1\}\boldsymbol{P}\{\overline{A}_2\}\boldsymbol{P}\{\overline{A}_3\} && \text{(independence)} \\ &= (1-0.01)(1-0.02)(1-0.02) && \text{(complement rule)} \\ &= \boxed{0.9508} \end{aligned}$$

2.10
$$\begin{aligned} \boldsymbol{P}\{A \cup B \cup C\} &= 1 - \boldsymbol{P}\{\bar{A} \cap \bar{B} \cap \bar{C}\} = 1 - \boldsymbol{P}\{\bar{A}\}\boldsymbol{P}\{\bar{B}\}\boldsymbol{P}\{\bar{C}\} \\ &= 1 - (1-0.4)(1-0.5)(1-0.2) = \boxed{0.76} \end{aligned}$$

2.15 Let $A = \{\text{Error in the 1st block}\}$ and $B = \{\text{Error in the 2nd block}\}$. Then $P\{A\} = 0.2$, $P\{B\} = 0.3$, and $P\{A \cap B\} = 0.06$ by independence;
$P\{\text{ error in program }\} = P\{A \cup B\} = 0.2 + 0.3 - 0.06 = 0.44$.

Then, by the definition of conditional probability,

$$P\{A \cap B \mid A \cup B\} = \frac{P\{A \cap B\}}{P\{A \cup B\}} = \frac{0.06}{0.44} = \boxed{0.1364}$$

Or, by the Bayes Rule,

$$\begin{aligned} P\{A \cap B \mid A \cup B\} &= \frac{P\{A \cup B \mid A \cap B\} P\{A \cap B\}}{P\{A \cup B\}} \\ &= \frac{(1)(0.06)}{0.44} = \boxed{0.1364} \end{aligned}$$

2.19 Let $D = \{\text{defective part}\}$ and $I = \{\text{inspected electronically}\}$. By the Bayes Rule,

$$\begin{aligned} \boldsymbol{P}\{I|D\} &= \frac{\boldsymbol{P}\{D|I\}\boldsymbol{P}\{I\}}{\boldsymbol{P}\{D|I\}\boldsymbol{P}\{I\} + \boldsymbol{P}\{D|\bar{I}\}\boldsymbol{P}\{\bar{I}\}} \\ &= \frac{(1-0.95)(0.20)}{(1-0.95)(0.20) + (1-0.7)(1-0.20)} = \boxed{0.0400} \end{aligned}$$

2.21 At least one of the first three components works with probability

$$1 - \boldsymbol{P}\{\text{all three fail}\} = 1 - (0.3)^3 = 0.973.$$

At least one of the last two components works with probability

$$1 - \boldsymbol{P}\{\text{both fail}\} = 1 - (0.3)^2 = 0.91.$$

Hence, the system operates with probability $(0.973)(0.91) = \boxed{0.8854}$

2.24 A customer is unaware of defects, so he buys 6 random laptops. The outcomes are equally likely, so each probability can be computed as

$$\frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$$

(a) $$\boldsymbol{P}\{\text{exactly } 2\} = \frac{\binom{5}{2}\binom{5}{4}}{\binom{10}{6}} = \frac{\frac{5\cdot 4}{2}\cdot 5}{\frac{10\cdot 9\cdot 8\cdot 7}{4\cdot 3\cdot 2\cdot 1}} = \boxed{\frac{5}{21} \text{ or } 0.238}$$

(b) This is a conditional probability because $\{X \geq 2\}$ is given. We need

$$\boldsymbol{P}\{X = 2 \mid X \geq 2\} = \frac{\boldsymbol{P}\{X = 2 \cap X \geq 2\}}{\boldsymbol{P}\{X \geq 2\}} = \frac{\boldsymbol{P}\{X = 2\}}{\boldsymbol{P}\{X \geq 2\}}$$

where $\boldsymbol{P}\{X = 2\} = 5/21$ is already computed in (a), and

$$\boldsymbol{P}\{x \geq 2\} = 1 - P(X = 1) = 1 - \frac{\binom{5}{1}\binom{5}{5}}{\binom{10}{6}} = 1 - \frac{5\cdot 1}{\frac{10\cdot 9\cdot 8\cdot 7}{4\cdot 3\cdot 2\cdot 1}} = \frac{41}{42}$$

Notice that $\boldsymbol{P}\{X = 0\} = 0$ because there are only 5 good computers, so among the purchased 6 computers there has to be at least 1 defective. So,

$$\boldsymbol{P}\{X = 2 \mid X \geq 2\} = \frac{\boldsymbol{P}\{X = 2\}}{\boldsymbol{P}\{X \geq 2\}} = \frac{5/21}{41/42} = \boxed{10/41 \text{ or } 0.244}.$$

2.27 The sample space consists of all unordered sets of five computers selected from 18 computers in the store. Favorable outcomes are sets of five non-defective computers (that come from a subset of $18 - 6 = 12$. Then

$$\mathcal{N}_T = C(18,5) = \frac{(18)(17)(16)(15)(14)}{(5)(4)(3)(2)(1)} \quad \text{and} \quad \mathcal{N}_F = C(12,5) = \frac{(12)(11)(10)(9)(8)}{(5)(4)(3)(2)(1)};$$

therefore,

$$\boldsymbol{P}\left\{\begin{array}{c}\text{five computers}\\ \text{without defects}\end{array}\right\} = \frac{\mathcal{N}_F}{\mathcal{N}_T} = \frac{(12)(11)(10)(9)(8)}{(18)(17)(16)(15)(14)} = \boxed{\frac{11}{119} \quad \text{or} \quad 0.0924}$$