CSE-5301 : Data Analysis and Modeling Techniques [Homework 2 Sample Solutions]

3.2 Let X be the number of network blackouts, and Y be the loss. Then $Y = 500X$. Compute

$$\begin{aligned}
\mathbf{E}(X) &= \sum_x xP(x) = (0)(0.7) + (1)(0.2) + (2)(0.1) = 0.4; \\
\mathrm{Var}(X) &= \sum_x (x - 0.4)^2 P(x) \\
&= (0 - 0.4)^2(0.7) + (1 - 0.4)^2(0.2) + (2 - 0.4)^2(0.1) = 0.44.
\end{aligned}$$

Hence,

$$\mathbf{E}(Y) = 500\,\mathbf{E}(X) = (500)(0.4) = \boxed{\text{200 dollars}}$$

and

$$\mathrm{Var}(Y) = 500^2\,\mathrm{Var}(X) = (250,000)(0.44) = \boxed{\text{110,000 squared dollars}}$$

3.10 *Solution 1.*

Let X be the number of accidents on Thursday, and Y be the number of accidents on Friday. Using independence, compute the joint distribution $P(x, y) = P(x)P(y)$,

$P(x,y)$		$x=0$	$x=1$	$x=2$
y	0	.36	.12	.12
	1	.12	.04	.04
	2	.12	.04	.04

Then

$$\boldsymbol{P}\{X < Y\} = P(0,1) + P(0,2) + P(1,2) = .12 + .12 + .04 = \boxed{0.28}$$

Solution 2.

Since X and Y have the same distribution, events $\{X < Y\}$ and $\{X > Y\}$ occur with the same probability. The only other case is $X = Y$ and the probability of this is

$$\boldsymbol{P}\{X = Y\} = P(0,0) + P(1,1) + P(2,2) = (0.6)^2 + (0.2)^2 + (0.2)^2 = 0.44.$$

Then

$$\boldsymbol{P}\{X < Y\} = \frac{1 - 0.44}{2} = \boxed{0.28}$$

3.15 (a) $1 - P_{(X,Y)}(0,0) = 1 - 0.52 = 0.48$.

(b) Compute the marginal distributions of X and Y,

$$P_X(x) = \sum_{y=0}^{2} P(x,y) \text{ and } P_Y(y) = \sum_{x=0}^{2} P(x,y)$$

$P(x,y)$		$x=0$	$x=1$	$x=2$	$P_Y(y)$
y	0	0.52	0.20	0.04	0.76
	1	0.14	0.02	0.01	0.17
	2	0.06	0.01	0	0.07
$P_X(x)$		0.72	0.23	0.05	

Variables X and Y are dependent. For example, $P_{(X,Y)}(2,2) \neq P_X(2)P_Y(2)$ because $(0.05)(0.07) \neq 0$.

3.17 Let X be the profit made on 1 share of A, and Y be the profit made on 1 share of B. Each of these variables has the distribution

x	$P(x)$
-1	0.5
1	0.5

Then

$$E(X) = E(Y) = (-1)(0.5) + (1)(0.5) = 0$$

and

$$Var(X) = Var(Y) = E(X-0)^2 = (-1)^2(0.5) + (1)^2(0.5) = 1$$

The risk of each portfolio is measured by the variance of its return Z:

(a) $Z = 100X;\ \mathrm{Var}(Z) = 100^2\,\mathrm{Var}(X) = 10,000.$

(b) $Z = 50X + 10Y;\ \mathrm{Var}(Z) = 50^2\,\mathrm{Var}(X) + 10^2\,\mathrm{Var}(Y) = 2,600.$

(c) $Z = 40X + 12Y;\ \mathrm{Var}(Z) = 40^2\,\mathrm{Var}(X) + 12^2\,\mathrm{Var}(Y) = 1,744.$

Thus, the *third portfolio* has the lowest risk.

3.20 (a) We need to compute $\boldsymbol{P}\{X = 3\}$, where X is the number of defective computers ("successes") in a shipment of 20 ("trials"). It has Binomial distribution with parameters $n = 20$ and $p = 0.05$. From Table A2,

$$P(X=3) = P(X \le 3) - P(X \le 2) = .9841 - .9245 = \boxed{0.0596}$$

(b) Let Y be the number of defective computers among the first four. It has Binomial distribution with $n = 4$ and $p = 0.05$. From Table A2,

$$\begin{aligned} &\boldsymbol{P}\{\text{at least 5 computers are tested until 2 defective ones are found}\} \\ =\ &\boldsymbol{P}\{\text{among the first 4 computers, at most 1 is defective}\} \\ =\ &\boldsymbol{P}\{Y \le 1\} = \boxed{0.9860} \end{aligned}$$

The problem can also be solved directly, but computing $\boldsymbol{P}\{W \ge 5\}$, where W is the Negative Binomial ($k = 2$, $p = 0.05$) number of computers the engineer has to test in order to find 2 defective computers:

$$\begin{aligned} \boldsymbol{P}\{W \ge 5\} &= 1 - P(2) - P(3) - P(4) \\ &= 1 - 0.05^2 - (2)(0.05)^2(0.95) - (3)(0.05)^2(0.95)^2 \\ &= \boxed{0.9860} \end{aligned}$$

3.22 We need $\boldsymbol{P}\{X > 3\}$, where X is the number of defective computers in this sample. It is the number of successes in $n = 16$ Bernoulli trials, where each computer is a "trial" (defective or not) and a defective computer is a "success". Therefore, X has Binomial distribution with $n = 16$, $p = 0.05$. From Table A2,

$$\boldsymbol{P}\{X > 3\} = 1 - F(3) = 1 - 0.9930 = \boxed{0.0070}$$

3.25 (a) Let X be the number of users who don't close Windows properly before someone does. It is the number of trials before the first success, and therefore, $(X+1)$ has Geometric distribution with $p = 1 - 0.1 = 0.9$. Compute the expectation,

$$\mathbf{E}(X) = \mathbf{E}(X+1) - 1 = \frac{1}{0.9} - 1 = \boxed{0.1111}$$

(b) Let Y be the number of users (out of the next ten) who close Windows properly. It has Binomial distribution with $n = 10$ and $p = 0.9$. From Table A2,

$$\boldsymbol{P}\{Y = 8\} = F(8) - F(7) = 0.2639 - 0.0702 = \boxed{0.1937}$$

3.26 (a) Let X be the number of damaged files. This is the number of "successes" (damaged files) out of 20 "trials" (files), thus, it has Binomial distribution with $n = 20$, $p = 0.2$. From Table A2,

$$\boldsymbol{P}\{X \geq 5\} = 1 - F(4) = 1 - 0.6296 = \boxed{0.3704}$$

(b) The number of files to be checked has Negative Binomial distribution with parameters $k = 3$ (successes, undamaged files) and $p = 0.8$ (probability of an undamaged file). However, we can use Table A2 as follows:

$$\begin{aligned} &\boldsymbol{P}\{\text{need to check at least 6 files}\} \\ &= \boldsymbol{P}\{\text{ 5 files are not enough }\} \\ &= \boldsymbol{P}\{\text{ there at most 2 undamaged files among the first 5}\} \\ &= F_X(2) = \boxed{0.0579}, \end{aligned}$$

where X, the number of undamaged files among the first five, has Binomial distribution with $n = 5$ and $p = 0.8$.

3.27 We need to find $\boldsymbol{P}\{X \geq 5\}$ and $\boldsymbol{P}\{X = 5\}$, where X is the number of arrived messages during the next hour. This is the number of "rare" events, which can be any nonnegative integer number. Also, their distribution has only one parameter, the arrival rate $\lambda = 9$. Therefore, X has Poisson distribution with parameter $\lambda = 9$. From Table A3,

(a) $\boldsymbol{P}\{X \geq 5\} = 1 - \boldsymbol{P}\{X \leq 4\} = 1 - .055 = \boxed{.945}$

(b) $\boldsymbol{P}\{X = 5\} = \boldsymbol{P}\{X \leq 5\} - \boldsymbol{P}\{X \leq 4\} = .116 - .055 = \boxed{.061}$

3.32 Let X be the number of crashed computers. This is the number of "successes" (crashed computers) out of 4,000 "trials" (computers), with the probability of success 1/800. Thus, it has Binomial distribution with parameters $n = 4,000$ (large) and $p = 1/800$ (small), that is approximately Poisson with $\lambda = np = 5$.
Using Table A3 with parameter 5,

(a) $\boldsymbol{P}\{X < 10\} = F(9) = \boxed{0.968}$

(b) $\boldsymbol{P}\{X = 10\} = F(10) - F(9) = 0.986 - 0.968 = \boxed{0.018}$

3.34 The number of files X affected by the virus has Binomial distribution with $n = 250$ (large) and $p = 0.032$ (small), which is approximately Poisson with

$$\lambda = np = 8.$$

From Table A3,

$$\boldsymbol{P}\{X > 7\} = 1 - F(7) = \boxed{0.547}$$

3.36 At any time, the number of terminals X that are ready to transmit has Binomial distribution with $n = 10$ and $p = 0.7$. From Table A2,

$$\boldsymbol{P}\{X = 6\} = F(6) - F(5) = 0.3504 - 0.1503 = \boxed{0.2001}$$

3.37 We need $\boldsymbol{P}\{X > 4\}$, where X is the number of breakdowns during 21 weeks. This is the number of rare events, averaging 1 per 3 weeks, or 7 per 21 weeks. Thus, X is Poisson with $\lambda = 7$, and from Table A3,

$$\boldsymbol{P}\{X > 4\} = 1 - F(4) = 1 - 0.173 = \boxed{0.827}$$

4.5 (a) Find C from the condition $\iint f(x,y)dx\,dy = 1$:

$$\iint f(x,y)dx\,dy = \int_{y=0}^{1}\int_{x=-1}^{1} C(x^2+y)dx\,dy = C\int_{y=0}^{1}\left(\frac{x^3}{3}+xy\right)\Big|_{x=-1}^{1} dy$$

$$= C\int_{y=0}^{1}\left(\frac{2}{3}+2y\right)dy = C\left(\frac{2}{3}y+y^2\right)\Big|_{y=0}^{1} = \frac{5C}{3} = 1.$$

Hence, $\boxed{C = 3/5 \text{ or } 0.6}$.

(b) $$\boldsymbol{P}\{Y < 0.6\} = \int_{y=0}^{0.6}\int_{x=-1}^{1} 0.6(x^2+y)dx\,dy \qquad \text{[similarly to (a)]}$$

$$= 0.6\int_{y=0}^{0.6}\left(\frac{2}{3}+2y\right)dy = 0.6\left(\frac{2}{3}y+y^2\right)\Big|_{y=0}^{0.6} = 0.6(0.4+0.6^2) = \boxed{0.456}$$

Computing the conditional probability $P\{Y < 0.6 \mid X = 0.5\}$ directly by formula (2.7) yields an indeterminate expression

$$P\{Y < 0.6 \mid X = 0.5\} = \frac{P\{Y < 0.6 \ \cap \ X = 0.5\}}{\boldsymbol{P}\{X = 0.5\}} = \frac{0}{0} = \ldots ?$$

because X is a continuous variable.

We can resolve this by using the L'Hospital Rule,

$$P\{Y < 0.6 \mid X = 0.5\} = \lim_{\varepsilon\to 0} \boldsymbol{P}\{Y < 0.6 \mid 0.5 \le X \le 0.5+\varepsilon\}$$

$$= \lim_{\varepsilon\to 0}\frac{\boldsymbol{P}\{Y < 0.6 \ \cap \ 0.5 \le X \le 0.5+\varepsilon\}}{\boldsymbol{P}\{0.5 \le X \le 0.5+\varepsilon\}}$$

$$= \lim_{\varepsilon\to 0}\frac{\int_{y=0}^{0.6}\int_{x=0.5}^{0.5+\varepsilon} f(x,y)dx\,dy}{\int_{y=0}^{1}\int_{x=0.5}^{0.5+\varepsilon} f(x,y)dx\,dy} = \lim_{\varepsilon\to 0}\frac{\frac{d}{d\varepsilon}\int_{y=0}^{0.6}\int_{x=0.5}^{0.5+\varepsilon} f(x,y)dx\,dy}{\frac{d}{d\varepsilon}\int_{y=0}^{1}\int_{x=0.5}^{0.5+\varepsilon} f(x,y)dx\,dy}$$

$$= \lim_{\varepsilon\to 0}\frac{\int_{y=0}^{0.6} f(x,y)\Big|_{x=0.5+\varepsilon} dy}{\int_{y=0}^{1} f(x,y)\Big|_{x=0.5+\varepsilon} dy} = \frac{\int_{y=0}^{0.6} f(x,y)dy}{\int_{y=0}^{1} f(x,y)dy}\Bigg|_{x=0.5}$$

$$= \frac{\int_{y=0}^{0.6} C(0.5^2+y)dy}{\int_{y=0}^{1} C(0.5^2+y)dy} = \frac{0.5^2\cdot 0.6 + 0.6^2/2}{0.5^2\cdot 1 + 1^2/2} = \frac{0.33}{0.75} = \boxed{0.44}$$

By the way, $\frac{f(x,y)}{\int_0^1 f(x,y)dy} = \frac{f(x,y)}{f_X(x)} = f(y|x)$ is the *conditional density* of Y given $X = x$. Thus, to find the conditional probability of $Y < 0.6$, we integrated the conditional density.

4.6 Denote Exponential(λ) times for the 3 blocks by X_1, X_2, and X_3, and let

$$X = \max_i X_i$$

be the time it takes to compile the whole program. Find the cdf, then the pdf of X and use the latter to compute the expectation $\mathbf{E}(X)$.

For an Exponential (λ) time X_i,

$$\mathbf{E}(X_i) = 1/\lambda = 5 \text{ min},$$

therefore, $\lambda = 0.2 \text{ min}^{-1}$.

Compute the cdf of X,

$$\begin{aligned} F_X(x) &= \boldsymbol{P}\left\{\max_i X_i \le x\right\} = \boldsymbol{P}\left\{\bigcap_{i=1}^{3}(X_i \le x)\right\} \\ &= \prod_{i=1}^{3} \boldsymbol{P}\left\{X_i \le x\right\} = \left(1 - e^{-0.2x}\right)^3, \quad x > 0. \end{aligned}$$

Differentiate it to find the pdf,

$$f_X(x) = F'_X(x) = 0.6\left(1 - e^{-0.2x}\right)^2 e^{-0.2x}, \quad x > 0.$$

Now we can compute $\mathbf{E}(X)$ as

$$\begin{aligned} \mathbf{E}(X) &= \int x\, f_X(x) dx = 0.6 \int_0^\infty x\left(1 - e^{-0.2x}\right)^2 e^{-0.2x} dx \\ &= 0.6 \int_0^\infty \left(xe^{-0.2x} - 2xe^{-0.4x} + xe^{-0.6x}\right) dx \\ &= 0.6\left(\frac{\Gamma(2)}{0.2^2} - \frac{2\Gamma(2)}{0.4^2} + \frac{\Gamma(2)}{0.6^2}\right) \\ &= 15 - 7.5 + 1\frac{2}{3} = \boxed{\frac{55}{6} \text{ or } 9.17 \text{ minutes}} \end{aligned}$$

We used the gamma-function ($\Gamma(2) = 1! = 1$), but one can also take the three integrals by parts.

4.7 The time it takes to process print 1 job is Exponential(λ). It has expectation $\mathbf{E}(X) = 1/\lambda = 12$ seconds. Hence, $\lambda = 1/12 \text{ sec}^{-1}$.

The time T until the job is finished is the sum of 3 Exponential times (the remaining time of the currently active job is also Exponential because of the memoryless property). Thus, T has Gamma distribution with parameters $\alpha = 3$ and $\lambda = 1/12$.

By the Gamma-Poisson formula,

$$\boldsymbol{P}\left\{T < 60 \text{ seconds}\right\} = \boldsymbol{P}\left\{X \ge 3\right\},$$

where X is a Poisson variable with parameter $\lambda t = (1/12)(60) = 5$. Then

$$\boldsymbol{P}\left\{T < 60\right\} = \boldsymbol{P}\left\{X \ge 3\right\} = 1 - F(2) = 1 - 0.125 = \boxed{0.875}$$

from Table A3 with parameter 5.

4.8 Notice that 6 months = 1/2 years. Using the Gamma-Poisson formula with a Gamma(2,2) variable X and a Poisson($2 \cdot \frac{1}{2} = 1$) variable Y,

$$\boldsymbol{P}\{X < 1/2\} = \boldsymbol{P}\{Y \geq 2\} = 1 - F(1) = 1 - e^{-1} - 1e^{-1} = 1 - 0.736 = \boxed{0.264},$$

by the formula of Poisson distribution or by Table A3.

Alternatively, one can take an integral,

$$\boldsymbol{P}\{X < 1/2\} = \int_0^{1/2} f(x)dx = \int_0^{1/2} 4xe^{-2x}dx = 1 - e^{-1} - e^{-1}$$

by parts.

4.10 Denote the events:

$$\begin{aligned} A &= \{ \text{ first specialist processes the order } \} \\ B &= \{ \text{ second specialist processes the order } \} \\ C &= \{ \text{ the order takes more than 30 minutes (1/2 hr)} \} \end{aligned}$$

Using Exponential distributions with parameters $\lambda_1 = 3$ hrs^{-1} and $\lambda_2 = 2$ hrs^{-1},

$$\boldsymbol{P}\{C \mid A\} = e^{-(3)(1/2)} = e^{-1.5} \text{ and } \boldsymbol{P}\{C \mid B\} = e^{-(2)(1/2)} = e^{-1}$$

Also, $\boldsymbol{P}\{A\} = 0.6$ and $\boldsymbol{P}\{B\} = 0.4$. By the Bayes Rule,

$$\boldsymbol{P}\{A \mid C\} = \frac{0.6\boldsymbol{P}\{C \mid A\}}{0.6\boldsymbol{P}\{C \mid A\} + 0.4\boldsymbol{P}\{C \mid B\}} = \frac{0.6e^{-1.5}}{0.6e^{-1.5} + 0.4e^{-1}} = \boxed{0.4764}$$

4.11 Both lifetimes T_A and T_B have Exponential distribution with parameter $\lambda = 1/\mathbf{E}(T) = 0.1$ years^{-1}.

(a) $\boldsymbol{P}\{T_A > 10 \cup T_B > 10\} = 1 - \boldsymbol{P}\{T_A \leq 10 \cap T_B \leq 10\}$

$$= 1 - \left(1 - e^{-(10)(0.1)}\right)\left(1 - e^{-(10)(0.1)}\right) = \boxed{0.600}$$

(b) The lifetime of the satellite $T = \max\{T_A, T_B\}$ has a cdf

$$\begin{aligned} F_T(t) &= \boldsymbol{P}\{\max\{T_A, T_B\} \leq t\} = F_{T_A}(t)F_{T_B}(t) \\ &= \left(1 - e^{-0.1t}\right)^2 = 1 - 2e^{-0.1t} + e^{-0.2t} \end{aligned}$$

Differentiating, we find the pdf

$$f_T(t) = F_T'(t) = 0.2e^{-0.1t} - 0.2e^{-0.2t}$$

The expected lifetime is

$$\begin{aligned} \mathbf{E}(T) &= \int tf_T(t)dt = \int_0^{\infty} \left(0.2te^{-0.1t} - 0.2te^{-0.2t}\right) dt \\ &= 0.2\left(\frac{\Gamma(2)}{0.1^2} - \frac{\Gamma(2)}{0.2^2}\right) = 0.2(100 - 25) = \boxed{15 \text{ years}} \end{aligned}$$

4.14 (a) For this Gamma distribution, $\mathbf{E}(X) = \alpha/\lambda = 20$ and $\mathrm{Var}(X) = \alpha/\lambda^2 = 10^2 = 100$. Solving these equations for α and λ, we get

$$\alpha = \mathbf{E}^2(X)/\mathrm{Var}(X) = 20^2/100 = \boxed{4}$$

and

$$\lambda = \mathbf{E}(X)/\mathrm{Var}(X) = 20/100 = \boxed{0.2}$$

(b) Use the Poisson-Gamma formula with a Poisson variable Y that has a parameter $\lambda(15) = 3$. From Table A3,

$$\boldsymbol{P}\{X < 15\} = \boldsymbol{P}\{Y \geq 4\} = 1 - \boldsymbol{P}\{Y \leq 3\} = 1 - 0.647 = \boxed{0.353}$$

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4.16 From Table A4,

(a) $\boldsymbol{P}(Z < 1.25) = \Phi(1.25) = \boxed{0.8944}$

(b) $\boldsymbol{P}(Z \le 1.25) = \Phi(1.25) = \boxed{0.8944}$

(c) $\boldsymbol{P}(Z > 1.25) = 1 - \Phi(1.25) = 1 - 0.8944 = \boxed{0.1056}$

(d) $\boldsymbol{P}(|Z| \le 1.25) = \boldsymbol{P}(-1.25 \le Z \le 1.25) = \Phi(1.25) - \Phi(-1.25) = 0.8944 - 0.1056 = \boxed{0.7888}$

The value $z = 6.0$ is outside Table A4 because it is too large. Therefore, the next two probabilities are either near 0 or near 1. To be rigorous, $z = 6.0$ can be compared with the largest value in the Table, $z = 3.99$.

(e) $\boldsymbol{P}(Z < 6.0) > P(Z < 3.99) = \boxed{1.0}$

(f) $\boldsymbol{P}(Z > 6.0) < P(Z > 3.99) = \boxed{0.0}$

(g) Solve the equation

$$\Phi(z) = 0.75$$

for z. In Table A4, find the value of z corresponding to the probability 0.8. The closest value is $\Phi(0.84) = 0.7995$. Therefore, $z \approx \boxed{0.84}$.

4.17 From Table A4,

(a) $\boldsymbol{P}(Z \ge 0.99) = 1 - \Phi(0.99) = 1 - 0.8389 = \boxed{0.1611}$

(b) $\boldsymbol{P}(Z \le -0.99) = \Phi(-0.99) = \boxed{0.1611}$ (also follows from [a] by symmetry)

(c) $\boldsymbol{P}(Z < 0.99) = \Phi(0.99) = \boxed{0.8389}$

(d) $\boldsymbol{P}(|Z| > 0.99) = \boldsymbol{P}(Z < -0.99) + \boldsymbol{P}(Z > 0.99) = 2(0.1611) = \boxed{0.3222}$

The value $z = 10.0$ is outside Table A4 because it is too large. Comparing with the largest value in the Table, $z = 3.99$, we obtain

(e) $\boldsymbol{P}(Z < 10.0) > \boldsymbol{P}(Z < 3.99) = \boxed{1.0}$

(f) $\boldsymbol{P}(Z > 10.0) < \boldsymbol{P}(Z > 3.99) = \boxed{0.0}$

(g) Solve the equation

$$\Phi(z) = 0.9$$

for z. In Table A4, find the value of z corresponding to the probability 0.9. The closest value is $\Phi(1.28) = 0.8997$. Therefore, $z \approx \boxed{1.28}$.

4.18 Standardize and use Table A4.

(a) $\boldsymbol{P}(X \le 2.39) = \boldsymbol{P}\left(Z \le \frac{2.39-(-3)}{2.7}\right) = \Phi(2.00) = \boxed{0.9772}$

(b) $\boldsymbol{P}(Z \ge -2.39) = \boldsymbol{P}\left(Z \ge \frac{-2.39-(-3)}{2.7}\right) = 1 - \Phi(0.23) = 1 - 0.5910 = \boxed{0.4090}$

(c) $\boldsymbol{P}(|X| \ge 2.39) = \boldsymbol{P}(X \le -2.39) + \boldsymbol{P}(X \ge 2.39) = (1 - 0.4090) + (1 - 0.9772) = \boxed{0.6138}$ using answers from (a) and (b)

(d) $\boldsymbol{P}(|X+3| \ge 2.39) = P(X \le -2.39 - 3) + P(X \ge 2.39 - 3) = \boldsymbol{P}\left(\frac{|X+3|}{2.7} \ge \frac{2.39}{2.7}\right) = \boldsymbol{P}(|Z| \ge 0.89) = 2\Phi(-0.89) = 2(0.1867) = \boxed{0.3734}$

(e) $\boldsymbol{P}(X < 5) = \boldsymbol{P}\left(Z < \frac{5-(-3)}{2.7}\right) = \Phi(2.96) = \boxed{0.9985}$

(f) $\boldsymbol{P}(|X| < 5) = \boldsymbol{P}\left(\frac{-5-(-3)}{2.7} < Z < \frac{5-(-3)}{2.7}\right) = \Phi(0.89) - \Phi(-0.74) = 0.8133 - 0.2296 = \boxed{0.5837}$

(g) Solve the equation

$$\boldsymbol{P}(X > x) = 0.33$$

for x. We have

$$\boldsymbol{P}(X > x) = \boldsymbol{P}\left(Z > \frac{x+3}{2.7}\right) = 1 - \Phi\left(\frac{x+3}{2.7}\right) = 0.33,$$

so that $\Phi\left(\frac{x+3}{2.7}\right) = 0.67$.

From Table A4, we find that $\Phi(0.44) = 0.67$. Therefore (unstandardizing 0.44),

$$\frac{x+3}{2.7} = 0.67 \quad \Rightarrow \quad x = \boxed{-1.19}$$

4.21 The height X has Normal distribution with $\mu = 79''$ and $\sigma = 3.89''$. Using Table A4,

(a) $\boldsymbol{P}(X > 84'') = \boldsymbol{P}\left(Z > \frac{84-79}{3.89}\right) = P(Z > 1.29) = 1 - \Phi(1.29) = 1 - 0.9015 = \boxed{0.0985 \text{ or } 9.85\%}$

(b) Solve the equation

$$\boldsymbol{P}(X > x) = 0.20$$

for x. We have

$$\boldsymbol{P}(X > x) = \boldsymbol{P}\left(Z > \frac{x-79}{3.89}\right) = 1 - \Phi\left(\frac{x-79}{3.89}\right) = 0.20,$$

so that $\Phi\left(\frac{x-79}{3.89}\right) = 0.80$.
From Table A4, we find that $\Phi(0.84) \approx 0.80$. Therefore (unstandardizing 0.84),

$$\frac{x-79}{3.89} = 0.84 \quad \Rightarrow \quad x = \boxed{82.3 \text{ in or } 6 \text{ ft } 10.3 \text{ in}}$$

So, the height of your favorite player can be 6'10.3" or more.

4.22 The income X has Normal distribution with $\mu = 900$ and $\sigma = 200$.

(a) $\boldsymbol{P}(X < 640) = \boldsymbol{P}\left(Z < \frac{640-900}{200}\right) = \Phi(-1.3) = \boxed{0.0968 \text{ or } 9.68\%}$

(b) Solve the equation

$$\boldsymbol{P}(X < x) = 0.05$$

for x. We have

$$\boldsymbol{P}(X < x) = \boldsymbol{P}\left(Z < \frac{x-900}{200}\right) = \Phi\left(\frac{x-900}{200}\right) = 0.05.$$

From Table A4, we find that $\Phi(-1.645) \approx 0.05$. Therefore,

$$\frac{x-900}{200} = -1.645 \quad \Rightarrow \quad x = \boxed{571 \text{ coins}}$$