29/11/19

Practical - 01

Limits & Continuity

① $\lim_{x \to a} \left[\dfrac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right]$

② $\lim_{y \to 0} \left[\dfrac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \right]$

③ $\lim_{x \to \frac{\pi}{6}} \left[\dfrac{\cos x - \sqrt{3}\sin x}{\pi - 6x} \right]$

④ $\lim_{x \to \infty} \left[\dfrac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \right]$

⑤ Examine the continuity of the following function at given points.

(i) $f(x) = \begin{cases} \dfrac{\sin 2x}{\sqrt{1-\cos 2x}}, & \text{for } 0 < x \le \dfrac{\pi}{2} \\ \dfrac{\cos x}{\pi - 2x}; & \text{for } \dfrac{\pi}{2} < x < \pi \end{cases}$ at $x = \dfrac{\pi}{2}$

(ii) $f(x) = \begin{cases} \dfrac{x^2-9}{x-3} & 0 < x < 3 \\ x+3 & 3 \le x < 6 \\ \dfrac{x^2-9}{x+3} & 6 \le x < 9 \end{cases}$ at $x = 3$ & $x = 6$

⑥ Find value of k, so that the function $f(x)$ is continous at the indicated point.

(i) $f(x) = \begin{cases} \dfrac{1-\cos 4x}{x^2} & x < 0 \\ k & x = 0 \end{cases}$ at $x = 0$

(ii) $f(x) = \begin{cases} (\sec^2 x)^{\cot^2 x} & x \neq 0 \\ k & x = 0 \end{cases}$ at $x = 0$

(iii) $f(x) = \dfrac{\sqrt{3} - \tan x}{\pi - 3x}$ $\quad x \neq \frac{\pi}{3}$
$= K$ $\quad x = \frac{\pi}{3}$ $\Big\}$ at $x = \frac{\pi}{3}$

(7) Discuss the continuity of the following function and which of these functions have a removable discontinuity? Redefine the function so as to remove the discontinuity.

(i) $f(x) = \dfrac{1-\cos x}{x \tan x}$ $\quad x \neq 0$
$= 9$ $\quad x = 0$ $\Big\}$ at $x = 0$

(ii) $f(x) = \dfrac{(e^{3x}-1)\sin x^\circ}{x^2}$ $\quad x \neq 0$
$= \dfrac{\pi}{60}$ $\quad x = 0$ $\Big\}$ at $x = 0$

(8) If $f(x) = \dfrac{e^{x^2} - \cos x}{x^2}$ for $x \neq 0$ is continous at $x = 0$ find $f(0)$

(9) If $f(x) = \dfrac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$. for $x \neq \frac{\pi}{2}$ is continous at $x = \frac{\pi}{2}$ find $f\left(\frac{\pi}{2}\right)$.

Solution

29 ① $\lim_{x \to a}\left[\dfrac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\right]$

$= \lim_{x \to a}\left[\dfrac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\right] \times \left[\dfrac{\sqrt{a+2x}+\sqrt{3x}}{\sqrt{a+2x}+\sqrt{3x}}\right] \times \left[\dfrac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{3a+x}+2\sqrt{x}}\right]$

$= \lim_{x \to a}\left[\dfrac{\sqrt{a+2x}(\sqrt{a+2x}+\sqrt{3x})-\sqrt{3x}(\sqrt{a+2x}+\sqrt{3x})}{\sqrt{a+2x}+\sqrt{3x}(\sqrt{3a+x}-2\sqrt{x})}\right] \times \left[\dfrac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{3a+x}+2\sqrt{x}}\right]$

$= \lim_{x \to a}\left[\dfrac{a+2x-3x}{3a+x-4x}\right] \times \dfrac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{a+2x}+\sqrt{3x}}$

$= \lim_{x \to a}\left[\dfrac{a-x}{3a-3x}\right] \times \dfrac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{a+2x}+\sqrt{3x}}$

$= \lim_{x \to a}\left[\dfrac{a-x}{3(a-x)}\right] \times \dfrac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{a+2x}+\sqrt{3x}}$

$= \dfrac{1}{3}\lim_{x \to a}\dfrac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{a+2x}+\sqrt{3x}}$

$= \dfrac{1}{3}\left(\dfrac{\sqrt{3a+a}+2\sqrt{a}}{\sqrt{a+2a}+\sqrt{3a}}\right)$

$= \dfrac{1}{3}\left(\dfrac{\sqrt{4a}+2\sqrt{a}}{\sqrt{3a}+\sqrt{3a}}\right)$

$= \dfrac{1}{3} \times \dfrac{2\sqrt{a}+2\sqrt{a}}{2\sqrt{3a}}$

$= \dfrac{1}{3} \times \dfrac{4\sqrt{a}}{2\sqrt{3a}}$

$= \dfrac{2\sqrt{a}}{3\sqrt{3a}}$

(2) $\lim_{y \to 0} \left[\frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \right]$

$= \lim_{y \to 0} \frac{(\sqrt{a+y}) - \sqrt{a})}{y\sqrt{a+y}} \times \frac{(\sqrt{a+y} + \sqrt{a})}{(\sqrt{a+y} + \sqrt{a})}$

$= \lim_{y \to 0} \frac{a+y-a}{y(\sqrt{a+y})(\sqrt{a+y} + \sqrt{a})}$

$= \lim_{y \to 0} \frac{y}{y(\sqrt{a+y})(\sqrt{a+y} + \sqrt{a})}$

$= \lim_{y \to 0} \frac{1}{(\sqrt{a+y})(\sqrt{a+y} + \sqrt{a})}$

$= \frac{1}{(\sqrt{a+0})(\sqrt{a+0} + \sqrt{a})}$

$= \frac{1}{(\sqrt{a})(2\sqrt{a})}$

$= \frac{1}{2a}$

③ $$\lim_{x \to \pi/6} \frac{\cos x - \sqrt{3}\sin x}{\pi - 6x}$$

Put $x - \frac{\pi}{6} = h$. then $x = h + \frac{\pi}{6}$

as $x \to \frac{\pi}{6}$, $h \to 0$

$$= \lim_{h \to 0} \frac{\cos(h + \pi/6) - \sqrt{3}\sin(h + \pi/6)}{\pi - 6(h + \pi/6)}$$

$$= \lim_{h \to 0} \frac{(\sqrt{3}/2 \cos h - 1/2 \sin h) - \sqrt{3}(\sqrt{3}/2 \sin h + 1/2 \cos h)}{-6h}$$

$$= \lim_{h \to 0} \frac{-4 \sin h}{-12 h} \qquad \frac{\sin h}{h} = 1$$

$$= \frac{1}{3}$$

④ $$\lim_{x \to \infty} \frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}}$$

$$= \lim_{x \to \infty} \frac{(\sqrt{x^2+5} - \sqrt{x^2-3}) \cdot (\sqrt{x^2+3} + \sqrt{x^2+1})}{(\sqrt{x^2+3} - \sqrt{x^2+1})(\sqrt{x^2+3} + \sqrt{x^2+1})}$$

$$= \lim_{x \to \infty} \frac{(\sqrt{x^2+3} + \sqrt{x^2+1})\, x^2+5-x^2+3}{2(\sqrt{x^2+5} + \sqrt{x^2-3})}$$

$$= \lim_{x \to \infty} \frac{\sqrt{x^2(1+\frac{3}{x^2})} + \sqrt{x^2(1+\frac{1}{x^2})}}{\sqrt{x^2(1+\frac{5}{x^2})} + \sqrt{x^2(1-\frac{3}{x^2})}}$$

$$= 4 \cdot \lim_{x \to \infty} \frac{x}{x} \times \frac{\sqrt{1+\frac{3}{x^2}} + \sqrt{1+\frac{1}{x^2}}}{\sqrt{1+\frac{5}{x^2}} + \sqrt{1-\frac{3}{x^2}}}$$

$$= 4 \times \frac{\sqrt{1} + \sqrt{1}}{\sqrt{1} + \sqrt{1}}$$

$$= 4$$

(3) Examine the continuity of the following function at given points.

(i) $f(x) = \frac{\sin 2x}{\sqrt{1-\cos 2x}}$ for $0 < x \le \frac{\pi}{2}$ $\Big\}$ at $x = \pi/2$

$= \frac{\cos x}{\pi - 2x}$ for $\frac{\pi}{2} < x < \pi$

$$f(\pi/2) = \frac{\sin 2(\pi/2)}{\sqrt{1-\cos(\pi/2)}}$$

$f(\pi/2) = 0$

$\therefore$ f at $x = \pi/2$ define.

(2) $\lim_{x \to \frac{\pi}{2}^+} f(x) = \lim_{x \to \frac{\pi}{2}^+} \frac{\cos x}{\pi - 2x}$

put $x - \frac{\pi}{2} = h$, $x = \frac{\pi}{2} + h$

$x \to \frac{\pi}{2}$, $h \to 0^+$

$$= \lim_{h \to 0^+} \frac{\cos(\frac{\pi}{2}+h)}{\pi - 2(\frac{\pi}{2}+h)} = \lim_{h \to 0^+} \frac{-\sin h}{\pi - \pi - 2h} = \lim_{h \to 0^+} \frac{-\sin h}{-2h}$$

$$= \frac{1}{2}$$

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(3) $\lim_{x \to \frac{\pi}{2}^-} f(x) = \lim_{x \to \frac{\pi}{2}^-} \frac{\sin 2x}{\sqrt{1-\cos 2x}}$

$= \lim_{x \to \frac{\pi}{2}^-} \frac{2 \sin x \cos x}{\sqrt{2 \sin^2 x}}$

$= \lim_{x \to \frac{\pi}{2}^-} \frac{2 \sin x \cos x}{\sqrt{2} \sin x}$

$= 0$

$\therefore$ LHL $\neq$ RHL

$\therefore$ f is not continous at $x = \frac{\pi}{2}$

ii) $f(x) = \left.\begin{array}{ll} \frac{x^2-9}{x-3} & 0 < x < 3 \\ = x+3 & 3 \leq x < 6 \\ = \frac{x^2-9}{x+3} & 6 \leq x < 9 \end{array}\right\}$ at $x = 3$ & $x = 6$

For $x = 3$

(1) $f(3) = 3+3 = 6 \rightarrow (1)$

$\therefore$ f is define at $x = 3$.

(2) $\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (x+3) = 3+3 = 6 \rightarrow (2)$

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(3) $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} \frac{x^2-9}{x-3} = \lim_{x\to 3^-} \frac{(x+3)(x-3)}{(x-3)}$

$= 3+3 = 6 \rightarrow (3)$

From (2) and (3)
LHL = RHL
f is continuous at $x=3$

For $x=6$

(1) $f(6) = \frac{6^2-9}{6+3} = \frac{36-9}{9} = \frac{27}{9} = 3 \rightarrow (1)$

(2) $\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} \frac{x^2-9}{x+3} = \lim_{x\to 3^+} \frac{(x+3)(x-3)}{(x+3)} = 6-3 = 3 \rightarrow (2)$

(3) $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} x+3 = 6+3 = 9 \rightarrow (3)$

From (2) and (3)
LHL $\neq$ RHL
f is not continuous at $x=6$.

6) Find value of k, so that function $f(x)$ is continuous at the indicated point.

$$f(x) = \frac{1-\cos 4x}{x^2} \quad x<0 \\ \quad = k \qquad \Bigg\} \text{ at } x=0$$

f is continuous at $x=0$

$\therefore \lim\limits_{x\to 0} f(x) = f(0)$

$\therefore \lim\limits_{x\to 0} \frac{1-\cos 4x}{x^2} = k$

$\therefore \lim\limits_{x\to 0} \frac{2\sin^2 x}{x^2} = k$

$\therefore \lim\limits_{x\to 0} 4\times 2 \frac{\sin^2 2x}{4\times x^2} = k$

$8 = k$

(ii) $f(x) = (\sec^2 x)^{\cot^2 x} \quad x \neq 0$
$\quad = k \quad x = 0$ } at $x=0$

iii) $f(x) = \frac{\sqrt{3} - \tan x}{\pi - 3x} \quad x \neq \frac{\pi}{3}$

$\quad = k \quad x = \frac{\pi}{3}$ } at $x = \frac{\pi}{3}$

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solution:

$x - \frac{\pi}{3} = h$, $x = \frac{\pi}{3} + h$

as $x \to \frac{\pi}{3}$, $h \to 0$.

$$\lim_{h \to 0} f(h) = \lim_{h \to 0} \frac{\sqrt{3} - \tan\left(\frac{\pi}{3} + h\right)}{\pi - 3\left(\frac{\pi}{3} + h\right)}$$

$$= \lim_{h \to 0} \frac{\sqrt{3} - 3\tan h - \sqrt{3} - \tan h}{-3h\left(1 - \sqrt{3}\tan h\right)}$$

$$= \frac{4}{3} \lim_{h \to 0} \frac{\tan h}{h} \times \frac{1}{1 - \sqrt{3}\tan h}$$

$$= \frac{4}{3}$$

(7) Discuss the continuity of the following function which of these functions have a removable discontinuity? Redefine.

i) $f(x) = \left.\begin{array}{ll} \dfrac{1 - \cos x}{x \tan x} & x \neq 0 \\ = 9 & x = 0 \end{array}\right\}$ at $x = 0$

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{1-\cos 3x}{x \tan x}$$

$$= \lim_{x\to 0} \frac{2\sin^2 \frac{3x}{2} \times x}{x \times x \tan x}$$

$$= \frac{9}{4} \times 2$$

$$= \frac{9}{2}$$

$$\lim_{x\to 0} f(x) = \frac{9}{2} \ne 9 \ne f(0)$$

$\therefore f$ is not continous at $x=0$

Redefine function:

$$f(x) = \frac{1-\cos 3x}{x \cdot \tan x} \quad x \ne 0.$$

$$= \frac{9}{2}$$

Now $\lim_{x\to 0} f(x) = f(0)$

$\therefore f$ has removable discontinuity at $x=0$

ii). $$f(x) = \left.\begin{cases} \dfrac{(e^{3x}-1)\sin x^{\circ}}{x^2}, & x \ne 0 \\ \dfrac{\pi}{60}, & x = 0 \end{cases}\right\} \text{ at } x=0$$

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{(e^{3x}-1)\sin x^\circ}{x^2}$$

$$= \lim_{x\to 0} \frac{(e^{3x}-1)\sin\frac{\pi x}{180}}{x^2}$$

$$= 3\log e \times \frac{\pi}{180}$$

$$= \frac{\pi}{60}$$

$$= f(0)$$

f is continuous at $x=0$
It does not have removable discontinuity

8) If $f(x) = \frac{e^{x^2} - \cos x}{x^2}$ for $x \neq 0$ is continuous at $x=0$ find $f(0)$

f is continuous at $x=0$.

$$\lim_{x\to 0} f(x) = f(0)$$

$$\lim_{x\to 0} \frac{e^{x^2} - \cos x}{x^2} = f(0)$$

$$\lim_{x\to 0} \left(\frac{e^{x^2}-1}{x^2}\right) + \frac{2\sin^2 x/2}{\frac{1}{4}x^2} \times \frac{1}{4}$$

$$= \log e + 2 \times \frac{1}{4} \quad \therefore f(0) = \frac{3}{2}$$

6) Find the value of K, so that function $f(x)$ is continuous at the indicated point :

ii) $f(x) = (\sec^2 x)^{\cot^2 x}$ $\quad x \neq 0$
$\quad = K$ $\quad x = 0$ $\Big\}$ at $x = 0$

solution :- f is continuous at $x = 0$

$f(x) = (\sec^2 x)^{\cot^2 x}$

using $\tan^2 x - \sec^2 x = 1$

$\therefore \sec^2 x = 1 + \tan^2 x$

& $\cot^2 x = \frac{1}{\tan 2x}$

$\therefore \lim_{x \to 0} (\sec^2 x)^{\cot 2x}$

$\lim_{x \to 0} (1 + \tan 2x)^{1/\tan^2 x}$

$\lim_{x \to 0} (1 + px)^{1/px} = e$

$\lim_{x \to 0} f(x) = f(0)$

$e = k$

$k = e$

AV
6/12/19

Prac 2

Q.1 · Show that function defined from R to R are differentiable.

(i) $\cot x$

$f(x) = \cot x$

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\frac{1}{\tan x} - \frac{1}{\tan a}}{x - a}$$

$$= \lim_{x \to a} \frac{\tan a - \tan x}{(x-a)\tan x \tan a}$$

Put $x - a = h$

$x = a + h$

as $x \to a, h \to 0$

$$Df(h) = \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{(a+h-a)\tan(a+h)\tan a}$$

$$= \lim_{h \to 0} \frac{\tan(a-a-h) - (1 + \tan A + \tan(a+h))}{h \times \tan(a+h)\tan a}$$

$$= \frac{-1 \times \tan^2 a}{\tan^2 a}$$

$$= \frac{-\sec^2 a}{\tan^2 a}$$

$= -\operatorname{cosec}^2 a$

$\therefore Df(a) = -\operatorname{cosec}^2 a$

$\therefore f$ is differentiable $\forall a \in R$.

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ii) cosec x

$f(x) = \operatorname{cosec} x$

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\frac{1}{\sin x} - \frac{1}{\sin a}}{x - a}$$

Put $x - a = h$

$x = a + h$

as $x \to a$, $h \to 0$

$$Df(h) = \lim_{h \to 0} \frac{\sin a - \sin(a+h)}{(a+h-a)\sin a \cdot \sin(a+h)}$$

$$= \lim_{h \to 0} \frac{-\sin \frac{h}{2}}{h/2} \times \frac{1}{2} \times \frac{2\cos\left(\frac{2a+h}{2}\right)}{\sin a \sin(a+h)}$$

$$= -\frac{1}{2} \times \frac{2\cos\left(\frac{2a+0}{2}\right)}{\sin(a+0)}$$

$$= \frac{-\cos a}{\sin^2 a} = -\cot a \operatorname{cosec} a.$$

iii) sec x

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\frac{1}{\cos x} - \frac{1}{\cos a}}{x - a}.$$

$$= \lim_{x \to a} \frac{\cos a - \cos x}{(x - a)\cos a \cos x}$$

Put $x - a = h$

$x = a + h$

as $x \to a$, $h \to 0$

$$Df(h) = \lim_{h \to 0} \frac{\cos a - \cos(a+h)}{h \times \cos a \cos(a+h)}$$

$$= \lim_{h \to 0} \frac{-2\sin\left(\frac{a+a+h}{2}\right)\sin\left(\frac{a-a-h}{2}\right)}{h \times \cos a \cdot \cos(a+h)}$$

$$= \frac{-1}{2} \times \frac{-2\sin\left((2a+0)/2\right)}{\cos a \cos(a+0)}$$

$= \tan a \sec a.$

Q.2. If $f(x) = 4x + 1$, $x \leq 2$
$= x^2 + 5$. $x > 0$ at $x = 2$

$$Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{4x + 1 - (4 \times 2 + 1)}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{4x - 8}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{4(x-2)}{(x-2)}$$

$= 4$

$Df(2^-) = 4$

RHD:

$$Df(2^+) = \lim_{x\to 2^+} \frac{x^2+5-9}{x-2}$$

$$= \lim_{x\to 2^+} \frac{x^2-4}{x-2}$$

$$= \lim_{x\to 2^+} \frac{(x+2)(x-2)}{(x-2)}$$

$Df(2^+) = 4$

f is differentiable at $x=2$

Q.3 If $f(x) = 4x+7$ at $x=2$
$= x^2+3x+1$ $x \geq 3$ at $x=3$ then
Find f is differentiable or not.

RHD

$$Df(3^+) = \lim_{x\to 3^+} \frac{f(x)-f(3)}{x-3}$$

$$= \lim_{x\to 3^+} = \frac{x^2+3x-18}{x-3}$$

$$= \lim_{x\to 3^+} = \frac{x^2+6x-3x-18}{x-3}$$

$$= \lim_{x\to 3^+} \frac{(x-3)(x+6)}{(x-3)} = 3+6 = 9$$

$Df(3^+) = 9$

Practical - 3

AOD

1) Find function is increasing or decreasing

(i) $f(x) = x^3 - 5x - 11$

(ii) $f(x) = x^2 - 4x$

(iii) $f(x) = 2x^3 + x^2 - 20x + 4$

(iv) $f(x) = x^3 - 27x + 5$

(v) $f(x) = 69 - 24x - 9x^2 + 2x^3$

2) Find the intervals in which function is concave upwards and downwards.

(1) $y = 3x^2 - 2x^3$

(2) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

(3) $y = x^3 - 27x + 5$

(4) $y = 69 - 24x - 9x^2 + 2x^3$

(5) $y = 2x^3 + x^2 - 20x + 4$

Solution

Q.1

(1) $f(x) = x^3 - 5x - 11$

$\therefore f'(x) = 3x^2 - 5$

$\therefore$ f is increasing iff $f(x) > 0$

$3x^2 - 5 > 0$

$3(x^2 - 5/3) > 0$

$(x - \sqrt{5/3})(x + \sqrt{5/3}) > 0$

$x \in (-\infty, -\sqrt{5/3}) \cup (\sqrt{5/3}, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 3(x^2 - 5/3) < 0$

$\therefore (x - \sqrt{5/3})(x + \sqrt{5/3}) < 0$

(2) $f(x) = x^2 - 4x$

$f'(x) = 2x - 4$

$\therefore f(x)$ is increasing iff $f'(x) > 0$

$\therefore 2x - 4 > 0$

$\therefore 2(x - 2) > 0$

$x - 2 > 0$

$x \in (2, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 2(x - 2) < 0$

$\therefore x - 2 < 0$

$x \in (-\infty, 2)$

(3) $f(x) = 2x^3 + x^2 - 20x + 4$

$\therefore f'(x) = 6x^2 + 2x - 20$

$\therefore$ f is increasing iff $f'(x) > 0$

$\therefore 2(3x^2 + x - 10) > 0$

$\therefore 3x(x+2) - 5(x+2) > 0$

$\therefore (x+2)(3x-5) > 0$

$x \in (-\infty, -2) \cup (5/3, \infty)$.

and f is decreasing iff $f'(x) < 0$

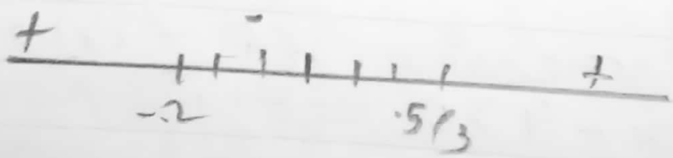
$\therefore 6x^2 + 2x - 20 < 0$

$\therefore 2(3x^2 + x - 10) < 0$

$\therefore 3x^2 + x - 10 < 0$

$\therefore 3x(x+2) - 5(x+2) < 0$

$\therefore (x+2)(3x-5) < 0$



$x \in (-2, 5/3)$

(4) $f(x) = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$\therefore 3(x^2 - 9) > 0$

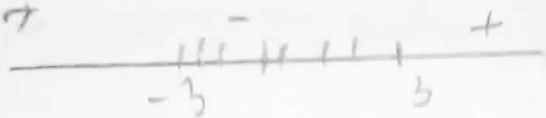
$\therefore (x-3)(x+3) > 0$

$\therefore x \in (-\infty, -3) \cup (3, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 3(x^2 - 9) < 0$

$\therefore (x-3)(x+3) < 0$



(5) $f(x) = 2x^3 - 9x^2 - 24x + 69$

$f'(x) = 6x^2 - 18x - 24$

$\therefore f$ is increasing iff $f'(x) > 0$

$\therefore 6x^2 - 18x - 24 > 0$

$6(x^2 - 3x - 4) > 0$

$\therefore x^2 - 4x + x - 4 > 0$

$\therefore (x-4)(x+1) > 0$

$\therefore x \in (-\infty, -1) \cup (4, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 6x^2 - 18x - 24 < 0$

$\therefore 6(x^2 - 3x - 4) < 0$

$\therefore x(x-4) + 1(x-4) < 0$

$\therefore (x-4)(x+1) < 0$

$\therefore x \in (-1, 4)$

Q.2 ① $Y = 3x^2 - 2x^3$

$\therefore f'(x) = 6x - 6x^2$

$f''(x) = 6x - 12x$

f is concave upward if $f''(x) > 0$

$\therefore (6 - 12x) > 0$

$\therefore x - 1/2 > 0$

$x > 1/2$

$f''(x) > 0 \quad \therefore x \in (1/2, \infty)$

$f''(x)$ is concave downward iff $f''(x) < 0$

$6(1 - 2x) < 0$

$1 - 2x < 0$

$2x > 1$

$x > 1/2 \quad x \in (1/2, \infty)$

② $Y = x^4 - 6x^3 + 12x^2 + 5x + 7$

$f'(x) = 4x^3 - 18x^2 + 24x + 5$

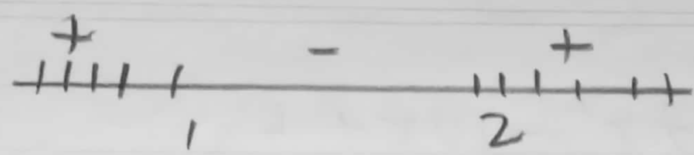
$f''(x) = 12x^2 - 36x + 24$

f is concave upward iff $f''(x) > 0$

$\therefore 12(x^2 - 3x + 2) > 0$

$\therefore x^2 - 2x - x + 2 > 0$

$\therefore (x - 2)(x - 1) > 0$.



$\therefore x \in (-\infty, 1) \cup (2, \infty)$.

(3) $y = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$f''(x) = 6x$

f is concave upward iff $f''(x) > 0$

$\therefore 6x > 0$

$\therefore x > 0$

$\therefore x \in (0, \infty]$

(4) $y = 69 - 24x - 9x^2 + 2x^3$

$f(x) = 2x^3 - 9x^2 - 24x + 69$

$f'(x) = 6x^2 - 18x - 24$

$f''(x) = 12x - 18$.

f is concave upward iff $f''(x) > 0$

$\therefore 12x - 18 > 0$

$\therefore 12(x - 18/12) > 0$

$\therefore x - 3/2 > 0$

$\therefore x > 3/2$

$\therefore x \in (3/2, \infty)$

$f''(x)$ is concave downward iff $f''(x) < 0$

$-18x + 12x < 0$

$12x < 18$

$x \in (-\infty, 3/2)$.

5) $y = 2x^3 + x^2 - 20x + 4$

$f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$f''(x) = 12x + 2$

f is concave upward iff $f''(x) > 0$

$\therefore f''(x) > 0$

$\therefore 12x + 2 > 0$

$\therefore 12(x + 2/12) > 0$

$\therefore x < -1/6$

$\therefore f''(x) \not> 0$

$\therefore$ There exists no interval

$f''(x)$ is concave downward iff

$f''(x) < 0$

$2(6x+1) < 0$

$6x < -1$

$x < -1/6$

$x \in (-\infty, -1/6)$.

A1

2/12/19

Practical No. 4

Application of Derivative & Newton's Method

Q1] Find maximum and minimum value of following functions:

① $f(x) = x^2 + \frac{16}{x^2}$ ② $f(x) = 3 - 5x^3 + 3x^5$

③ $f(x) = x^3 - 3x^2 + 1$ in $[\frac{-1}{2}, 4]$

④ $f(x) = 2x^3 - 3x^2 - 12x + 1$ in $[-2, 3]$

Q.2 Find the root of following equation by Newton's Method (Take 4 iteration only). Correct upto 4 decimal.

(i) $f(x) = x^3 - 3x^2 - 55x + 9.5$ (take $x_0 = 0$)

(ii) $f(x) = x^3 - 4x - 9$ in $[2, 3]$

(iii) $f(x) = x^3 - 1.8x^2 - 10x + 17$ in $[1, 2]$

Answers

Q.1

(i) $f(x) = x^2 + \frac{16}{x^2}$

$f'(x) = 2x - \frac{32}{x^3}$

Now, consider,

$f'(x) = 0$

$2x - \frac{32}{x^3} = 0$

$2x = \frac{32}{x^3}$

$x^4 = \frac{32}{2}$

$x = \pm 2$

$f''(x) = 2 + \frac{96}{x^4}$

$f''(2) = 2 + \frac{96}{16}$

$= 2 + 6$

$= 8 > 0$

$\therefore$ f has minimum value at $x = 2$

$\therefore f(2) = 2^2 + \frac{16}{2^2}$

$= 4 + 4$

$= 8$

$f''(-2) = 2 + \frac{96}{-2^4}$

$= 2 + 6$

$= 8 > 0$

$\therefore$ f has minimum value at $x = -2$

$\therefore$ Function reaches minimum value at $x = 2$ and $x = -2$

(iii) $f(x) = 3 - 5x^3 + 3x^5$

$\therefore f'(x) = -15x^2 + 15x^4$

consider;

$f'(x) = 0$

$-15x^2 + 15x^4 = 0$

$15x^4 = 15x^2$

$x^2 = 1$

$x = \pm 1$

$f''(x) = -30x + 60x^3$

$f(1) = -30 + 60$

$= 30 > 0$

$\therefore$ f has minimum value at $x = 1$

$\therefore f(1) = 3 - 5(1)^3 + 3(1)^5$

$= 6 - 5$

$= 1$

$f''(-1) = -30(-1) + 60(-1)^3$

$= 30 - 60$

$= -30 < 0$

$\therefore$ f has maximum value at $x = -1$

$\therefore f(-1) = 3 - 5(-1)^3 + 3(-1)^5$

$= 3 + 5 - 3$

$= 5$

$\therefore$ f has the maximum value 5 at $x = -1$ and has the minimum value 1 at $x = 1$

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(iii) $f(x) = x^3 - 3x^2 + 1$

$\therefore f'(x) = 3x^2 - 6x$

consider

$f'(x) = 0$

$\therefore 3x^2 - 6x = 0$

$3x(x-2) = 0$

$\therefore 3x = 0$ or $x - 2 = 0$

$\therefore x = 0$ or $x = 2$

$f''(x) = 6x - 6$

$f''(0) = 6(0) - 6$

$= -6 < 0$

$\therefore$ f has maximum value at $x = 0$

$\therefore f(0) = 0^3 - 3(0)^2 + 1$

$= 1$

$f''(2) = 6(2) - 6$

$= 12 - 6$

$= 6 > 0$

$\therefore$ f has minimum value at $x = 2$

$f(2) = (2)^3 - 3(2)^2 + 1$

$= 8 - 3(4) + 1$

$= 9 - 12$

$= -3$

$\therefore$ f has maximum value 1 at $x = 0$ and f has minimum value -3 at $x = 2$.

(iii) $f(x) = x^3 - 3x^2 + 1$

$\therefore f'(x) = 3x^2 - 6x$

consider

$f'(x) = 0$

$\therefore 3x^2 - 6x = 0$

$3x(x-2) = 0$

$\therefore 3x = 0$ or $x - 2 = 0$

$\therefore x = 0$ or $x = 2$

$f''(x) = 6x - 6$

$f''(0) = 6(0) - 6$

$= -6 < 0$

$\therefore f$ has maximum value at $x = 0$

$\therefore f(0) = 0^3 - 3(0)^2 + 1$

$= 1$

$f''(2) = 6(2) - 6$

$= 12 - 6$

$= 6 > 0$

$\therefore f$ has minimum value at $x = 2$

$f(2) = (2)^3 - 3(2)^2 + 1$

$= 8 - 3(4) + 1$

$= 9 - 12$

$= -3$

$\therefore f$ has maximum value 1 at $x = 0$ and f has minimum value -3 at $x = 2$.

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(iv) $f(x) = 2x^3 - 3x^2 - 12x + 1$

$\therefore f'(x) = 6x^2 - 6x - 12$

consider.

$f'(x) = 0$

$\therefore 6x^2 - 6x - 12 = 0$

$6(x^2 - \cancel{x} - 2) = 0$

$x^2 - \cancel{x} - 2 = 0$

$x^2 + x - 2x - 2 = 0$

$x(x+1) - 2(x+1) = 0$

$(x-2)(x+1) = 0$

$x = 2$ or $x = -1$

$\therefore f''(x) = 12x - 6$

$f''(2) = 12(2) - 6$

$= 24 - 6$

$= 18 > 0$

$\therefore$ f has minimum value at $x = 2$.

$f(2) = 2(2)^3 - 3(2)^2 - 12(2) + 1$

$= 2(8) - 3(4) - 24 + 1$

$= 16 - 12 - 24 + 1$

$= -19$

$f''(-1) = 12(-1) - 6$

$= -12 - 6$

$= -18 < 0$

$\therefore$ f has maximum value at $x = -1$

$\therefore f(-1) = 2(-1)^3 - 3(-1)^2 - 12(-1) + 1$

$= -2 - 3 + 12 + 1$

$= 8$

$\therefore$ f has maximum value 8 at $x = -1$ and f has minimum value -19 at $x = 2$.

Q.2 :

① $f(x) = x^3 - 3x^2 - 55x + 9.5$ $\quad x_0 = 0$ (given)

$f'(x) = 3x^2 - 6x - 55$

By Newton's method.

$x_{n+1} = x_n - f(x_n) / f'(x_n)$

$\therefore x_1 = 0 + 9.5/55$

$x_1 = 0.1727$

$f(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 9.$

$= -0.0829$

$f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$

$= -55.9467$

$x_2 = x_1 - f(x_1)/f'(x_1)$

$= 0.1727 - 0.0829/55.9467$

$= 0.1712$

$f(x_2) = (0.1712)^3 - 3(0.1712)^2 - 55(0.1712) + 9.5$

$= 0.0011$

$f'(x_2) = 3(0.1712)^2 - 6(0.1712) - 55$

$= -55.9393$

$x_3 = x_2 - f(x_2) / f'(x_2)$

$= 0.1712 + 0.0011/55.9393$

$= 0.1712$

$\therefore$ The root of the equation is 0.1712

iii) $f(x) = x^3 - 4x - 9$ $[2,3]$

$f'(x) = 3x^2 - 4$

$f(2) = 23 - 4(2) - 9$

$= 8 - 8 - 9$

$= -9$

$f(3) = 33 - 4(3) - 9$

$= 27 - 12 - 9$

$= 6$

Let $x_0 = 3$ be the initial approximation

$\therefore$ By Newton's Method

$x_{n+1} = x_n - f(x_n)/f'(x_n)$

$x_1 = x_0 - f(x_0)/f'(x_0)$

$= 3 - 6/23$

$= 2.7392$

$f(x_1) = (2.7392)^3 - 4(2.7392) - 9$

$= 0.596$

$f'(x_1) = 3(2.7392)^2 - 4$

$= 18.5096$

$x_2 = x_1 - f(x_1)/f'(x_1)$

$= 2.7392 - 0.5961/18.5096$

$= 2.7071$

$f(x_2) = (2.7071)^3 - 4(2.7071)$

$= 0.0102$

$f'(x_2) = 3(2.7071)^2 - 4$

$= 17.9851$

$= 2.7071 - \frac{0.0102}{17.9851}$

$= 2.7071 - 0.0056 = 2.7015$

$f(x_3) = (2.7015)^3 - 4(2.7015) - 9$

$= -0.0901$

$f'(x_3) = 3(2.7015)^2 - 4$

$= 17.8943$

$x_4 = 2.7015 + 0.0901/17.8943 = 2.7065$

(3) $f(x) = x^3 - 1.8x^2 - 10x + 17$ $[1, 2]$

$f'(x) = 3x^2 - 3.6x - 10$

$f(1) = (1)^3 - 1.8(1)^2 - 10(1) + 17$

$= -1.8 - 10 + 17$

$= 6.2$

$f(2) = (2)^3 - 1.8(2)^2 - 10(2) + 17$

$= -2.2$

Let $x_0 = 2$ be initial approximation.

By Newton's Method

$x_{n+1} = x_n - f(x_n)/f'(x_n)$

$x_1 = x_0 - f(x_2)/f'(x_2)$

$= 2 - 2.2/5.2$

$= 2 - 0.4230 = 1.577$

$f(x_1) = (1.577)^3 - 1.8(1.577)^2 - 10(1.577) + 17$

$= 0.6755$

$f'(x_1) = 3(1.577)^2 - 3.6(1.577) - 10$

$= 7.4608 - 5.6772 - 10$

$= -8.2164$

$x_2 = x_1 - f(x_1)/f'(x_1)$

$= 1.577 + 0.6755/8.2164 = 1.6592$

$$f(x_2) = (1.6592)^3 - 1.8(1.6592)^2 - 10(1.6592) + 17$$
$$= 0.0204$$
$$f'(x_2) = 3(1.6592)^2 - 3.6(1.6592) - 10$$
$$= -7.7143$$
$$x_3 = x_2 - f(x_2)/f'(x_2)$$
$$= 1.6592 + 0.0204/7.7143$$
$$= 1.6618$$
$$f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618) + 17$$
$$= 0.0004$$
$$f'(x_3) = 3(1.6618)^2 - 3.6(1.6618) - 10$$
$$= -7.6977$$
$$x_4 = x_3 - f(x_3)/f'(x_3)$$
$$= 1.8618 + \frac{0.0004}{7.6977}$$
$$= 1.6618$$

Practical - 5

Integration

Solve the following integration

① $\frac{dx}{\sqrt{x^2+2x-3}}$

② $\int (4e^{3x}+1)\,dx$

③ $\int (2x^2-3\sin x+5\sqrt{x})\,dx$

④ $\int \frac{x^3+3x+4}{\sqrt{x}}\,dx$

⑤ $\int t^7 \sin(2t^4)\,dt$

⑥ $\int \sqrt{x}\,(x^2-1)\,dx$

⑦ $\int \frac{1}{x^3}\sin\left(\frac{1}{x^2}\right)dx$

⑧ $\int \frac{\cos x}{\sqrt[3]{\sin^2 x}}\,dx$

⑨ $\int e^{\cos^2 x}\sin 2x\,dx$

⑩ $\int \left(\frac{x^2-2x}{x^3-3x^2+1}\right)dx$

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① $\int \frac{1}{\sqrt{x^2+2x+1-4}} \, dx$

$= \int \frac{1}{\sqrt{(x+1)^2-4}} \, dx$

Subsitute put $x+1 = t$
where $t = 1$ $t = x+1$

$\int \frac{1}{\sqrt{t^2-4}} \, dt$

$= \ln(|t+\sqrt{t^2-4}|)$

$t = x+1$

$= \ln(|x+1+\sqrt{(x+1)^2-4}\,|)$

$= \ln(|x+1+\sqrt{x^2+2x-3}|)$

$= \ln(|x+1+\sqrt{x^2+2x-3}|)+C$

② $\int (4e^{3x}+1)\,dx$

$= \int 4e^{3x}dx + \int 1\,dx$

$= 4\int e^{3x}dx + \int 1\,dx$

$= \frac{4e^{3x}}{3} + x + C$

③ $\int 2x^2 - 3\sin(x) + 5\sqrt{x}\,dx$

$= \int 2x^2 - 3\sin(x) + 5x^{1/2}\,dx$

$= \int 2x^2 dx - \int 3\sin(x)\,dx + \int 5x^{1/2}dx$

$= \frac{2x^3 + \cancel{10x\sqrt{2}}}{3} + 3\cos x + C$

4. $\int \frac{x^3+3x+4}{\sqrt{x}}\,dx$

$= \int \frac{x^3+3x+4}{x^{1/2}}\,dx$

$= \int x^{5/2} + 3x^{1/2} + \frac{4}{x^{1/2}}\,dx$

$= \cancel{\frac{x^{5/2+1}}{5/2+1}}$

$= \frac{2x^3\sqrt{x}}{7} + 2x\sqrt{x} + 8\sqrt{x} + c$

5. $\int t^7 \times \sin(2t^4)\,dt$

put $u = 2t^4$

$du = 2\times 4t^3$

$= \int t^7 \times \sin(2t^4) \times \frac{1}{2\times 4t^3}\,du$

$= \int t^4 \sin(2t^4) \times \frac{1}{8}\,du$

$= \cancel{\frac{t^4 \times \sin(2t^4)}{8}}\,du$

Substitute t^4 with $u/2$

$$= \int \frac{4 \times \sin(4)}{2} / 8 \, d4$$

$$= \frac{1}{16} \int 4 \times \sin(4) d4$$

where $u = 4$

$dv = \sin(4) \times d4$

$v = -\cos(4)$

$$= \frac{1}{16} \times (4(-\cos 4)) + \int \cos(4) \, d4$$

$$= 1/16 \times (2t^4 \times (-\cos(2t^4)) + \sin(2t^4))$$

$$= -\frac{t^4 \times \cos(2t^4)}{8} + \frac{\sin(2t^4)}{16} + C$$

7 $\int \sqrt{x}\,(x^2 - 1)\, dx$

$$= \int x^{1/2} \times x^2 - x^{1/2} \, dx$$

$$= \int x^{5/2} dx - \int x^{1/2} dx$$

$$I_1 = \frac{x^{5/2+1}}{5/2+1} = \frac{x^{7/2}}{7/2} = \frac{2x^{7/2}}{7} = \frac{2\sqrt{x^7}}{7} = \frac{2x^3\sqrt{x}}{7}$$

$$I_2 = \frac{x^{1/2+1}}{1/2+1} = \frac{2x^{3/2}}{\cancel{3/2}} = \frac{2\sqrt{x^3}}{3}$$

$$= \frac{2x^3\sqrt{x}}{7} + \frac{2\sqrt{x^3}}{3} + C$$

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(8) $\int \frac{\cos x}{\sqrt[3]{\sin(x)^2}} dx$

$= \int \frac{\cos x}{\sin x^{2/3}} dx$

Put $t = \sin(x)$

$t = \cos x$

$= \int \frac{\cos(x)}{\sin(x)^{3/2}} \times \frac{1}{\cos(x)\, dt}$

$= \frac{1}{\sin x^{3/2}} dt$

$= \frac{1}{t^{2/3}} dt.$

$I = \int \frac{1}{t^{2/3}}$

$= \frac{1}{1/3\, t^{-1/3}}$

$= \frac{t^{1/3}}{1/3}.$

$= 3t^{1/3}$

$= 3\sqrt[3]{t}.$

$= 3\sqrt[3]{\sin(x)} + c.$

⑩ $\int \frac{x^2-2x}{x^3-3x^2+1}\,dx$

Put $x^3-3x^2+1 = dt$

$= \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3x^2-3\times 2x}\,dt$

$= \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3(x^2-2x)}\,dt$

$= \int \frac{1}{x^3-3x^2+1} \times \frac{1}{3}\,dt$

$= \int \frac{1}{3t}\,dt$

$= \frac{1}{3}\int \frac{1}{t}\,dt$

$= \frac{1}{3} \times \ln|t| + c$

$= \frac{1}{3} \times \ln(|x^3-3x^2+1|) + c$

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03/01/2020

Practical - 6

Application of integration & Numerical Integra

Q.1] Find the length of the following curve

1) $x = t\sin t$, $y = 1-\cos t$ $t \in [0, 2\pi]$

for t belong to $[0, 2\pi]$

Sol] $L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$x = t - \sin t$

$\frac{dx}{dt} = t - \cos t$

$y = 1 - \cos t$

$\frac{dy}{dt} = 0 - (-\sin t)$

$\frac{dy}{dt} = \sin t$

$\cdot L = \int_0^{2\pi} \sqrt{(t - \cos t + \sin t)^2 + (\sin t)^2}\, dt$

$\int_0^{2\pi} \sqrt{1 - 2\cos t + 1}\, dt$

$\int_0^{2\pi} \sqrt{2 - 2\cos t}\, dt$

$\int_0^{2\pi} 2 \sin\frac{t}{2}\, dt.$

$= 4 + 4 = 8$

2) $y = \sqrt{4-x^2}$ $\quad x \in [-2,2]$

$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$\frac{dy}{dx} = 2 \int_0^2 \sqrt{1 + \left(\frac{-x}{\sqrt{4-x^2}}\right)^2}\, dx$

$= 4 \int_0^2 \frac{1}{\sqrt{4-x^2}}\, dx$

$= 4 \left(\sin^{-1}\left(\frac{x}{2}\right)\right)_0^2$

$= 2\pi$

3) $y = x^{3/2}$ in $[0,4]$

$f'(x) = \frac{3}{2} x^{1/2}$

$[f'(x)]^2 = 9/4\, x$

$L = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$

$= \int_0^x \sqrt{1 + \frac{3}{4}x}\, dx$

Put $u = 1 + \frac{9}{4}x$, $du = \frac{9}{4}\, dx$

$$L = \int_{1}^{1+9/4\,x} \frac{4}{9}\sqrt{u}\,du$$

$$= \frac{8}{27}\left[\left(1+\frac{9x}{4}\right) - 1\right]$$

9) $x = 3\sin t$, $y = 3\cos t$

$$\frac{dx}{dt} = 3\cos t$$

$$\frac{dy}{dt} = -3\sin t$$

$$L = \int_{0}^{2\pi} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\,dt$$

$$= \int_{0}^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\,dt$$

$$= \int_{0}^{2\pi} 3\sqrt{x}\,dt$$

$$= 3\left[x\right]_0^{2\pi}$$

$$= 3(2\pi - 0)$$

$L = 6\pi$ units.

(5) $x = \frac{1}{6}y^3 + \frac{1}{2y}$ on $y = [1, 2]$

$$\frac{dx}{dy} = \frac{y^2}{2} - \frac{1}{2y^2}$$

$$\frac{dx}{dy} = \frac{y^4 - 1}{2y^2}$$

$$L = \int_1^2 \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$= \int_1^2 \sqrt{\frac{(y^4+1)^2}{(2y)^2}}\, dy$$

$$= \int_1^2 \frac{y^4+1}{2y^2}\, dy$$

$$= \frac{1}{2}\int_1^2 y^2\, dy + \frac{1}{2}\int_1^2 y^{-2}\, dy$$

$$= \frac{1}{2}\left[\frac{8}{3} - \frac{1}{2} - \frac{1}{3} + 1\right]$$

$$= \frac{1}{2}\left[\frac{7}{3} - \frac{1}{2}\right]$$

$= \frac{17}{12}$ units.

② ① $\int_0^2 e^{x^2}\,dx$ with $n=4$

Sol^n $\int_0^2 e^{x^2}\,dx = 16.4526$

In each case the width of the subinterva

be $\Delta x = \frac{2-0}{4} = \frac{1}{2}$

So the sub intervals will be [0, 0.5] [0.5

[1, 1.5] . [1.5, 2]

By Simpson rule.

$$\int_0^2 e^{x^2}\,dx = \frac{1/2}{3}\left(y_0 + 4y_1 + 2y_2 + 4y_3 + y_4\right)$$

$$\approx 17.3536$$

② $\int_0^4 x^2\,dx$ $n=4$

$\Delta x = \frac{4-0}{4} = 1$

$$\int_a^b f(x)\,dx = \frac{\Delta x}{3}\left[y_0 + 4y_1 + 2y_2 + 4y_3 + y_4\right]$$

$$= \frac{1}{3}\left[0^2 + 4(1)^2 + 2(2)^2 + 4(3)^2 + 4^2\right]$$

$$= \frac{64}{3}$$

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3. $\int_0^{\pi/3} \sqrt{\sin x}\, dx \quad n = 6$

$\Delta x = \frac{b-a}{n} = \frac{\pi/3 - 0}{6} = \frac{\pi}{18}$

x	0	$\pi/8$	$2\pi/18$	$3\pi/18$	$4\pi/18$	$5\pi/18$
y	0	0.4167	0.584	0.707	0.801	0.87
	y_0	y_1	y_2	y_3	y_4	y_5

$\int_0^{\pi/3} \sqrt{\sin x}\, dx \approx \frac{\Delta x}{3}\left(y_0 + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4) +\right.$

$= \frac{\pi/18}{3}(0 + 4(0.4167 + 0.707 + 0.875) + 930$

$\approx 0.681.$

Practical 7

Differential Equation

Q.1]

1] $x \frac{dy}{dx} + y = e^x$

$$\frac{dy}{dx} + \frac{1}{x} y = \frac{e^x}{x}$$

$$P(x) = \frac{1}{x} \quad . \quad Q = (x) = \frac{e^x}{x}$$

$$I.F = e^{\int 1/x \, dx}$$

$$= e^{\ln x}$$

$$I.F = x$$

$$y (I.F) = \int Q(x)(I.F) dx + C$$

$$= \int \frac{e^x}{x} \cdot x \, dx + C$$

$$xy = e^x + C$$

$$\frac{0.5}{3}[(1+54.5982) + 5(1.284 + 9.487) + 2.238$$

$$\int_0^2 e^{x^2} dx = 17.3535$$

2] $e^{x}\dfrac{dy}{dx} + 2e^{x}y = 1$

$\dfrac{dy}{dx} + 2\dfrac{e^{x}}{e^{x}}y = \dfrac{1}{e^{x}}$ $(\div \text{ by } e^{x})$

$\dfrac{dy}{dx} + 2y = \dfrac{1}{e^{x}}$

$\dfrac{dy}{dx} + 2y = \dfrac{1}{e^{x}}$

$\dfrac{dy}{dx} + 2y = e^{-x}$

$P(x) = 2, \quad Q(x) = e^{-x}$

$\int P(x)\,dx$

$IF = e^{\int 2\,dx}$

$= e^{2x}$

$y \cdot e^{2x} \int e^{-x} + 2x \, dx + c$

$= \int e^{x}\,dx + c$

$y \cdot e^{2x} = e^{x} + c$

3] $x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$\therefore \frac{dy}{dx} + \frac{2y}{x} = \frac{\cos x}{x^2}$

$P(x) = 2(x)$ & $Q(x) = \frac{\cos x}{x^2}$

$I.F = e\int P(x)\,dx$

$= e\int 2/x\,dx$

$= e^{2/3}x$

$= \ln x^2$

$I.F = x^2$

$y(I.F) = \int Q(x)(I.F)\,dx + c$

$= \int \frac{\cos x}{x^3} \cdot x^2\,dx + c$

$= \int \cos x + c$

$x^2 y = \sin x + c$

4] $x\frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

$\frac{dy}{dx} + \frac{3y}{x} = \frac{\sin x}{x^3}$

$IF = x^3$

$$y(I.F) = \int Q(x)\,(I.F)\,dx + c$$

$$= \int \frac{\sin x}{x^3} \cdot x^3\,dx + c$$

$$= \int \sin x\,dx + c$$

$$x^3 y = -\cos x + c$$

5] $e^{2x}\frac{dy}{dx} + 2e^{2x}y = 2x$

$$\frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$$

$P(x) = 2$, $Q(x) = 2x/e^{2x} = 2x\,e^{-2x}$

$$IF = e^{\int p(x)\,dx}$$

$$= e^{\int 2\,dx}$$

$$= e^{2x}.$$

$$y(IF) = \int Q(x)\,(IF)\,dx + c$$

$$= \int 2x\,e^{-2x} \cdot e^{2x}\,dx + c$$

$$y e^{2x} = x^2 + c.$$

6] $\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$

$\sec^2 x \tan y \, dx = -\sec^2 y \tan x \, dy$

$\dfrac{\sec^2 x \, dx}{\tan x} = \dfrac{-\sec^2 y \, dy}{\tan y}$

$\displaystyle\int \frac{\sec^2 x \, dx}{\tan x} = -\int \frac{\sec^2 y \, dy}{\tan y}$

$\log|\tan x| = -\log|\tan y| + c$

$\log|\tan x \cdot \tan y| = c$

$\tan x \cdot \tan y = e^c$

7] $\dfrac{dy}{dx} = \sin^2(x - y + 1)$

Put $x - y + 1 = v$

D w.r.t. x on both side:

$x - y + 1 = v$

$1 - \dfrac{dy}{dx} = \dfrac{dv}{dx}$

$1 - \dfrac{dv}{dx} = \dfrac{dy}{dx}$ $\qquad$ $1 - \dfrac{dv}{dx} = \sin^2 v$

$$\frac{dv}{dx} = 1 - \sin^2 v$$

$$\frac{dv}{dx} = \cos^2 v$$

$$\frac{dv}{\cos^2 v} = dx$$

$$\int \sec^2 v \, dv = \int dx$$

$$\tan v = x + c$$

$$\tan(x + y - 1) = x + c$$

(8) $$\frac{dy}{dx} = \frac{2x + 3y - 1}{6x + 9y + 6}$$

Put $2x + 3y = v$

$$2 + 3\frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx} - 2\right)$$

$$\frac{1}{3}\left(\frac{dv}{dx} - 2\right) = \frac{1}{3}\frac{(v-1)}{(v+2)}$$

$$\frac{dv}{dx} = \frac{v-1}{v+2} + 2$$

$$\frac{dv}{dx} = \frac{v - 1 + 2v + 4}{v + 2}$$

$$= \frac{3v + 3}{v+2} = \frac{3(v+1)}{v+2}$$

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$$\int \left(\frac{v+2}{v+1}\right) dv = 3\,dx$$

$$= \int \frac{v+1}{v}\,dx + \int \frac{1}{v+1}\,dv = 3x$$

$$v + \log|x| = 3x + C$$

$$2x + 3y + \log|2x+3y+1| = 3x + C$$

$$3y = x - \log|2x+3y+1| + C.$$

10/01/2020

Practical No. 8

Euler's Method

① $\frac{dy}{dx} = y + e^x - 2$ $\quad y(0) = 2,\ h = 0.5$, find $Y(2)$

② $\frac{dy}{dx} = 1 + y^2$ $\quad y(0) = 0,\ h = 0.2$. find $Y(1)$

③ $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$ $\quad y(0) = 1,\ h = 0.2$, find $Y(1)$

4] $\frac{dy}{dx} = 3x^2 + 1$ $\quad y(1) = 2$, find $Y(2)$ for $h = 0.5$, $h = 0.25$

5] $\frac{dy}{dx} = \sqrt{xy} + 2$ $\quad y(1) = 1$ find $y(1.2)$ with $h = 0.2$

① $\frac{dy}{dx} = y + e^x - 2$

$f(x, y) = y + e^x - 2$, $y_0 = 2$, $x_0 = 0$, $h = 0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.487	3.57435
2	1	3.5743	4.2925	5.3615
~~3~~				

$$y_{n+1} = y_n + n\, f(x_n, y_n)$$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
3	1.5	5.3615	7.8431	9.28305
4	2	4.2831		

By Euler's formula,

$y(2) = 9.2831$

(2) $\frac{dy}{dx} = 1 + y^2$

$f(x, y) = 1 + y^2$, $y_0 = 0$, $x_0 = 0$, $h = 0.2$

Using Eulers iteration formula

$y_{n+1} = y_n + n\, f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	0	1	0.2
1	0.2	0.2	1.04	0.408
2	0.4	0.408	1.1665	0.6413
3	0.6	0.6413	1.4113	0.9236
4	0.8	0.9236	1.8530	1.2942
5	1	1.2942		

By Euler's formula
$y(1) = 1.2942.$

(3) $\frac{dx}{dy} = \sqrt{\frac{x}{y}}$ $\cdot y(0) = 1, x_0 = 0, h = 0.2$

Using Euler's iteration formula

$y_{n+1} = y_n + n f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	0	0
1	0.2	0		
2	0.4			
3	0.6			
4	0.8			
5	1			

(4) $\frac{dy}{dx} = 3x^2 + 1$ $\cdot$ $y_0 = 2, x_0 = 1, h = 0$ for $h = 0.5$

Using Euler's iteration Formula.

$y_{n+1} = y_n + h f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	4
1	1.5	4	49	28.5
2	2	28.5		

By Eulers formula

$y(2) = 28.6$

For $h = 0.25$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	3
1	1.25	3	5.6875	4.4219
2	1.5	4.4219	7.75	6.3594
3	1.75	6.3594	10.1815	8.9048
4	2	8.9048		

By Euler's Formula

$y(x) = 8.9048$

(5) $\frac{dy}{dx} = \sqrt{xy} + 2$ $\quad y_0 = 1,\ x_0 = 1,\ h = 0.2$

Using Eulers iteration formula

$y_{n+1} = y_n + h \cdot f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	1	3	1.6
1	1.2	1.6		

By Euler's formula.

$y(1.2) = 1.6$

AV
17/01/2020

Practical - 9
Limits in Partial Order derivative.

Q.1)

i. $\lim_{(x,y)\to(-4,-1)} \frac{x^3-3y+y^2-1}{xy+5}$

at $(-4,-1)$, Denominator $\neq 0$

$\therefore$ By applying limit

$= \frac{(-4)^3-3(-1)+(-1)^2-1}{-4(-1)+5}$

$= \frac{-64+3+1-1}{4+5}$

$= \frac{-61}{9}$

(ii) $\lim_{(x,y)\to(2,0)} \frac{(y+1)(x^2+y^2-4x)}{x+3y}$

at $(2,0)$, Denominator $\neq 0$

$\therefore$ By applying limit,

$= \frac{(0+1)(2^2+0-4(2))}{2+0}$

$= \frac{1.(4+0-8)}{2}$

$= \frac{-4}{2} = -2$

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(3) $$\lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2z^2}{x^3-x^2yz}$$

at $(1,1,1)$, Denominator $= 0$

$$\therefore \lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2z^2}{x^3-x^2yz}$$

$$= \lim_{(x,y,z)\to(1,1,1)} \frac{(x-yz)(x+yz)}{x^2(x-yz)}$$

$$= \lim_{(x,y,z)\to(1,1,1)} \frac{x+yz}{x^2}$$

On applying limit

$$= \frac{1+1(1)}{(1)^2} = 2$$

Q. 2.

(i) $f(x,y) = xy\,e^{x^2+y^2}$

$$\therefore f_x = \frac{\partial}{\partial x}(f(x,y))$$

$$= \frac{\partial}{\partial x}\left(xy\,e^{x^2+y^2}\right)$$

$$= y\,e^{x^2+y^2}(2x)$$

$$\therefore f_x = 2xy\,e^{x^2+y^2}$$

$$f_y = \frac{\partial}{\partial y}(f(x,y)) = \frac{\partial}{\partial y}\left(xy\,e^{x^2+y^2}\right) = x\,e^{x^2+y^2}(2y$$

$$f_y = 2yx\,e^{x^2+y^2}$$

(ii) $f(x,y) = e^{x}\cos y$

$fx = \frac{\partial}{\partial x}(f(x,y))$

$= \frac{\partial}{\partial x}(e^{x}\cos y)$

$\therefore fx = e^{x}\cos y$

$fy = \frac{\partial}{\partial y}(f(x,y))$

$= \frac{\partial}{\partial y}(e^{x}\cos y)$

$fy = -e^{x}\sin y.$

(iii) $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

$fx = \frac{\partial}{\partial x}(f(x,y))$

$= \frac{\partial}{\partial x}(x^3y^2 - 3x^2y + y^3 + 1)$

$f(x) = 3x^2y^2 - 6xy$

$fy = \frac{\partial}{\partial y}(f(x,y))$

$= \frac{\partial}{\partial y}(x^3y^2 - 3x^2y + y^3 + 1)$

$\therefore Fy = 2x^3y - 3x^2 + 3y^2$

Q.3]

(i) $f(x,y) = \frac{2x}{1+y^2}$

$fx = \frac{\partial}{\partial x}\left(\frac{2x}{1+y^2}\right)$

$= \frac{1+y^2 \cdot \frac{\partial}{\partial x}(2x) - 2x\frac{\partial}{\partial x}(1+y^2)}{(1+y^2)^2}$

$= 2+2y^2$

$$= \frac{2(1+y^2)}{(1+y^2)(1+y)^2}$$

$$= \frac{2}{(1+y)^2}$$

at $(0,0)$

$$= \frac{2}{1+0}$$

$$= 2$$

$$f_y = \frac{\partial}{\partial y}\left(\frac{2x}{1+y^2}\right)$$

$$= \frac{1+y^2 \frac{\partial}{\partial y}(2x) - 2x \frac{\partial}{\partial y}(1+y^2)}{(1+y^2)^2}$$

$$= \frac{1+y^2(0) - 2x(2y)}{(1+y^2)^2}$$

$$= \frac{-4xy}{(1+y^2)^2}$$

at $(0,0)$

$$= \frac{-4(0)(0)}{(1+0)^2}$$

$$= 0$$

(1) $f(x,y) = \dfrac{y^2 - xy}{x^2}$

$f_x = \dfrac{x^2(-y) - (y^2 - xy)(2x)}{x^4}$

$= \dfrac{-x^2y - 2x\,(y^2 - xy)}{x^4}$

$f_y = \dfrac{2y - x}{x^2}$

$f_{xx} = \dfrac{x^4(-2xy - 2y^2 + 4xy) - 4x^3(-x^2y - 2xy + 2x^2y)}{x^6}$

$f_{yy} = \dfrac{2}{x^2}$

$f_{xy} = \dfrac{-x^2 - 4xy + 2x^2}{x^4}$

$f_{yx} = \dfrac{-x^2 - 4xy + 2x^2}{x^4}$

$f_{xy} = f_{yx}$

(2) $f(x,y) = x^3 + 3x^2y^2 - \log(x^2+1)$

$f_x = 3x^2 + 6xy^2 - \dfrac{2x}{x^2+1}$. $f_y = 6x^2y$

$f_{xx} = 6x + 6y^2 - \left(\dfrac{2(x^2+1) - 4x^2}{(x^2+1)^2}\right)$

$f_{yy} = 6x^2$

$f_{xy} = 12xy$

$f_{yx} = 12xy$

$f_{xy} = f_{yx}$

(iii) $f(x,y) = \sin(xy) + e^{x+y}$

$f_x = y\cos(xy) + e^{x+y}$ $f_y = x\cos(xy) + e^{x+y}$

$f_{xx} = -y^2 \sin(xy) + e^{x+y}$

$f_{yy} = -x^2 \sin(xy) + e^{x+y}$

$f_{xy} = -y^2 \sin(xy) + \cos(xy) + e^{x+y}$

$f_{yx} = -x^2 \sin(xy) + \cos(xy) + e^{x+y}$

$f_{xy} \neq f_{yx}$

Q.5 $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

$f(1,1) = \sqrt{2}$

$f(x) = \dfrac{x}{\sqrt{x^2+y^2}}$ $f(y) = \dfrac{y}{\sqrt{x^2+y^2}}$

f_x at $(1,1) = \dfrac{1}{\sqrt{2}}$ f_y at $(1,1) = \dfrac{1}{\sqrt{2}}$

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$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1+y-1)$

$= \frac{x+y}{\sqrt{2}}$

② $f(x,y) = 1 - x + y\sin x$ at $(\frac{\pi}{2}, 0)$

$f(\frac{\pi}{2}, 0) = 1 - \frac{\pi}{2} + 0 = 1 - \frac{\pi}{2}$

$f_x = 0 - 1 + y\cos x$ $\quad f_y = \sin x$

f_x at $(\frac{\pi}{2}, 0) = -1$ $\quad f_y$ at $(\frac{\pi}{2}, 0) = \sin\frac{\pi}{2} = 1$

$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= 1 - x + y$.

③ $f(x,y) = \log x + \log y$ at $(1,1)$

$f(1,1) = \log(1) + \log(1)$

$= 0$

$f_x = \frac{1}{x}$

$f_y = \frac{1}{y}$

f_x at $(1,1) = 1$

f_y at $(1, 1) = 1$

$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= x - 1 + y - 1$

$= x + y - 2$

24/01/2020

1) Find the directional derivative of the following function at given points and in the direction of given vector.

i) $f(x, y) = x + 2y - 3 \quad a = (1, -1) \quad u = 3i - j$

Here $u = 3i - j$ is not a unit vector

$$|\bar{u}| = \sqrt{(3)^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}$$

Unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{10}}(3, -1)$

$$= \left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$$

$$f(a+h) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$$

$$f(a) = f(1,-1) = (1) + 2(-1) - 3 = 1 - 2 - 3 = -4$$

$$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$$

$$\cancel{= f\left(1 + \frac{3}{\sqrt{10}}\right), \left(-1 - \frac{h}{\sqrt{10}}\right)}$$

$$f(a+hu) = \left(1 + \frac{3}{\sqrt{10}}\right) + 2\left(-1 - \frac{h}{\sqrt{10}}\right) - 3$$

$$= 1 + \frac{3}{\sqrt{10}} - 2 - \frac{2h}{\sqrt{10}} - 3$$

$$f(a+hu) = -4 + \frac{h}{\sqrt{10}}$$

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x:a

$$D_u f(a) = \lim_{h \to 0} \frac{f(a+hu) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{-4 + h\sqrt{10} + 4}{h}$$

$$D_u f(a) = \frac{1}{\sqrt{10}}$$

(2) $f(x) = y^2 - 4x + 1 \quad a = (3, 4) \quad u = i + 5j$

Here $u = i + 5j$ is not a unit vector

$$|\bar{u}| = \sqrt{(1)^2 + (5)^2} = \sqrt{26}$$

Unit Vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{26}}(1, 5)$

$$= \left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$$

$$f(a) = f(3, 4) = (4)^2 - 4(3) + 1 = 5$$

$$f(a+hu) = f(3,4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$$

$$= f\left(3 + \frac{h}{\sqrt{26}}; 4 + \frac{5h}{\sqrt{26}}\right)$$

$$\underset{(x,y)}{f(a+hu)} = \left(4 + \frac{5h}{\sqrt{26}}\right)^2 - 4\left(3 + \frac{h}{\sqrt{26}}\right) + 1$$

$$= 16 + \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1$$

$$= \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5$$

$$D_uf(a) = \lim_{h \to 0} \frac{\frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5 - 5}{h}$$

$$= \frac{h\left(\frac{25h}{26} + \frac{36}{\sqrt{26}}\right)}{h}$$

$$D_uf(u) = \frac{25h}{26} + \frac{36}{\sqrt{26}}$$

(3) $2x+3y \quad a=(1,2), \; u=(3i+4j)$

Here, $u = 3i+4j$ is not a unit vector

$|\bar{u}| = \sqrt{(3)^2+(4)^2} = \sqrt{25} = 5$

Unit vector along u is $\frac{u}{|u|} = \frac{1}{5}(3,4)$

$$= \left(\frac{3}{5}, \frac{4}{5}\right)$$

$f(a) = f(1,2) = 2(1) + 3(2) = 8$

$f(a+hu) = f(1,2) + h\left(\frac{3}{5}, \frac{4}{5}\right)$

$$= f\left(1+\frac{3h}{5}, 2+\frac{4}{5}\right)$$

$f(a+hu) = 2\left(1+\frac{3h}{5}\right) + 3\left(2+\frac{4h}{5}\right)$

$$= \frac{18h}{5} + 8$$

$$D_uf(a) = \lim_{h \to 0} \frac{\frac{18h}{5} + 8 - 8}{h}$$

$$= \frac{18}{5}$$

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Q. 2) Find gradient vector for the following function at given points.

① $f(x,y) = x^y + y^x \quad . \quad a = (1,1)$

$f_x = y \cdot x^{y-1} + y^x \log y$

$f_y = x^y \log x + x y^{x-1}$

$\nabla f(x,y) = (f_x, f_y)$

$= (y x^{y-1} + y^2 \log y, \ x^y \log x + x y^{x-1})$

$f(1,1) = (1+0, 1+0)$

$= (1,1)$

② $f(x,y) = (\tan^{-1} x) \cdot y^2 \quad a = (1,-1)$

$f_x = \frac{1}{1+x^2} \cdot y^2$

$f_y = 2y \tan^{-1} x$

$\nabla f(x,y) = (f_x, f_y)$

$= \left(\frac{y^2}{1+x^2}, 2y \tan^{-1} x \right)$

$f(1,-1) = \left(\frac{1}{2}, \frac{\pi}{2} \right)$

$f(x, y, z) = xyz - e^{x+y+z}$ $\quad a = (1, -1, 0)$

$f_x = yz - e^{x+y+z}$

$f_y = xz - e^{x+y+z}$

$f_z = xy - e^{x+y+z}$

$\nabla f(x, y, z) = f_x, f_y, f_z$

$= yz - e^{x+y+z}, xz - e^{x+y+z}, xy - e^{x+y+z}$

$f(1, -1, 0) = \cancel{f(-1)(0) - e}$

$= (0 - e^0, 0 - e^0, -1 - e^0)$

$= (-1, -1, -2)$

Q3. Find the equation of tangent and normal to each of the following using curves at given points.

(i) $x^2 \cos y + e^{xy} = 2$ at $(1, 0)$

$f_x = \cos y \, 2x + e^{xy} y$

$f_y = x^2(-\sin y) + e^{xy} \cdot x$

$(x_0, y_0) = (1, 0)$. $x_0 = 1, y_0 = 0$

Eq^n Of tangent:

$$f_x(x-x_0)+f_y(y-y_0)=0$$

$$f_x(x_0,y_0)=\cos 0\, 2(1)+e^0 0$$
$$=1(2)+0$$
$$=2$$

$$f_y(x_0,y_0)=(1)^2(-\sin 0)+e^0\cdot 1$$
$$=0+1\cdot 1$$
$$=1$$

$$2(x-1)+1(y-0)=0$$
$$2x-2+y=0$$
$$2x+y-2=0$$

Required Equation Of tangent.

Equation Of Normal

$$=ax+by+c=0$$
$$=bx+ay+d=0$$

$$1(1)+2(y)+d=0$$
$$1+2y+d=0 \quad \text{at } (1,0)$$
$$d=-1$$

$x^2+y^2-2x+3y+2=0$ at $(2,-2)$

$f_x = 2x-2$

$f_y = 2y+3$

$(x_0, y_0) = (2,-2)$

$\therefore x_0 = 2, y_0 = -2$

$f_x(x_0, y_0) = 2$

$f_y(y_0, x_0) = -1$

Eq^n Of tangent

$f_x(x-x_0) + f_y(y-y_0) = 0$

$2x-2-y-2=0$

$2x-y-4=0$

Required Eq^n Of tangent.

Eq^n Of Normal.

$= ax+by+c=0$

$bx+ay+d=0$

$d=6$

Q.4] Find the equation of tangent and normal line to each of the following surface.

(i) $x^2 - 2yz + 3y + xz = 7$ at $(2, 1, 0)$

$f_x = 2x + z$

$f_y = 2z + 3$

$f_z = -2y + x$

$(x_0, y_0, z_0) = (2, 1, 0) \quad \therefore x_0 = 2, y_0 = 1, z_0 = 0$

$f_x(x_0, y_0, z_0) = 2(2) + 0 = 4$

$f_y(x_0, y_0, z_0) = 3$

$f_z(x_0, y_0, z_0) = 0$

Equation of tangent

$f_x(x - x_0) + f_y(y - y_0) + f_z(z - z_0) = 0$

$= 4x - 8 + 3y - 3 = 0$

$4x + 3y - 11 = 0$

Required Equation of tangent.

Equation of normal at $(4, 3, -11)$

$$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}$$

$$\frac{x - 2}{4} = \frac{y - 1}{3} = \frac{z + 11}{0}$$

① $3xy - x - y + z = -4$ at $(1, -1, 2)$

$3xyz - x - y + z + 4 = 0$ at $(1, -1, 2)$

$f_x = 3yz - 1$

$f_y = 3xz - 1$

$f_z = 3xy + 1$

$(x_0, y_0, z_0) = (1, -1, 2)$ $x_0 = 1, y_0 = -1, z_0 = 2$

$f_x(x_0, y_0, z_0) = -7$

$f_y(x_0, y_0, z_0) = 5$

$f_z(x_0, y_0, z_0) = -2$

Equation of tangent:

$-7(x - 1) + 5(y + 1) - 2(z - 2) = 0$

$-7x + 5y - 2z + 16 = 0$

Required equation of tangent.

Equation of normal at $(-7, 5, -2)$

$$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}$$

$$\frac{x - 1}{-7} = \frac{y + 1}{5} = \frac{z - 2}{-2}$$

Q. 5] Find the local maxima & minima for the following function

(i) $f(x,y) = 3x^2 + y^2 - 3xy + 6x - 4y$

$f_x = 6x - 3y + 6$

$f_y = 2y - 3x - 4$

$f_x = 0$

$2x - y = -2 \rightarrow (1)$

$f_y = 0$

$2y - 3x - 4 = 0$

$2y - 3x = 4 \rightarrow (2)$

~~Multiply~~ Adding eqn 1 ~~and~~ with 2

$4x - \cancel{2y} = -4$

$\cancel{2y} - 3x = 4$

$x = 0$

Subsitute value of x in eqn (1)

$y = 2$

Critical points are $(0, 2)$

Scanned with CamScanner

$r = f_{xx} = 6$

$t = f_{yy} = 2$

$s = f_{xy} = -3$

Here $r > 0$

$\Rightarrow rt - s^2$

$= 3 > 0$

$\therefore$ f has maximum at $(0,2)$

$3x^2 + y^2 - 3xy + 6x - 4y$ at $(0,2)$

$= -4$

$f(x,y) = 2x^4 + 3x^2y - y^2$

$f_x = 8x^3 + 6xy$

$f_y = 3x^2 - 2y$

$f_x = 0$

$8x^3 + 6xy = 0$

$2x(4x^2 + 3y) = 0$

$4x^2 + 3y = 0 \rightarrow (1)$

$f_y = 0$

$3x^2 - 2y = 0$

$y = 0$

Substitute value of y in eq^n ①

$4x^2 + 3(0) = 0$

$4x^2 = 0$

$x = 0$

Critical point is $(0,0)$

$r = f_{xx} = 24x^2 + 6x$

$t = f_{yy} = -2$

$s = f_{xy} = 6x - 0 = 6(0) = 0$

r at $(0,0)$

$\therefore r = 0.$

$rt - s^2 = 0$

$r = 0$ & $rt - s^2 = 0.$

(nothing to say)

$f(x,y)$ at $(0,0)$

$2(0)^4 + 3(0)^2 = 0$

$= 0$

Q. 3

(iii) . $f(x,y) = x^2 - y^2 + 2x + 8y - 70$

$f_x = 2x + 2$

$f_y = -2y + 8$

$f_x = 0$

$\therefore x = -1$

$f_y = 0$

$y = 4$ $\therefore$ Critical point is $(-1,4)$

$r = f_{xx} = 2$
$t = f_{yy} = -2$
$s = f_{xy} = 0$

$r > 0$

$rt - s^2 = 2(-2) - (0)^2$
$= -4 < 0$

$f(x, y)$ at $(-1, 4)$

$(-1)^2 - (4)^2 + 2(-1) + 8(4) - 70$
$= 17 + 30 - 70$
$\cancel{= 37 - 70}$
$\cancel{= 30}$
$= 33$