Solving Differential Equations with OPAMP based circuits

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Design Flow

- Take the Laplace transform of the differential equation you wish to model.
- Draw the circuit with summers, and integrators, and multipliers generically
- Use behavioral sources in Ltspice to model ideal behavior
- If a solution to the differential equation exists match behavioral circuit against ideal circuit
- Implement with inverting and non inverting gain, summer and integrator stages.
- Estimate the input vectors that the circuit will work under
- Simulate
- Design Build Test

Case Study: Nuclear Decay

N is the number of atoms to be decayed
λ is the decay rate.

$$\frac{dN}{dt} = -\lambda N$$

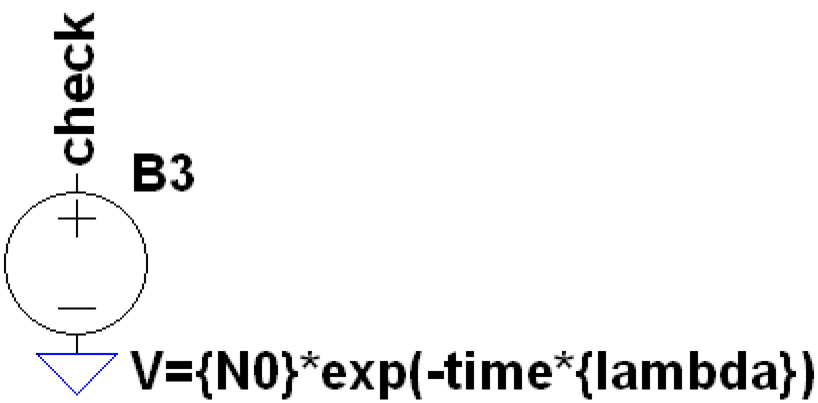
$$sN(s) - N_0 + \lambda N(s) = 0$$

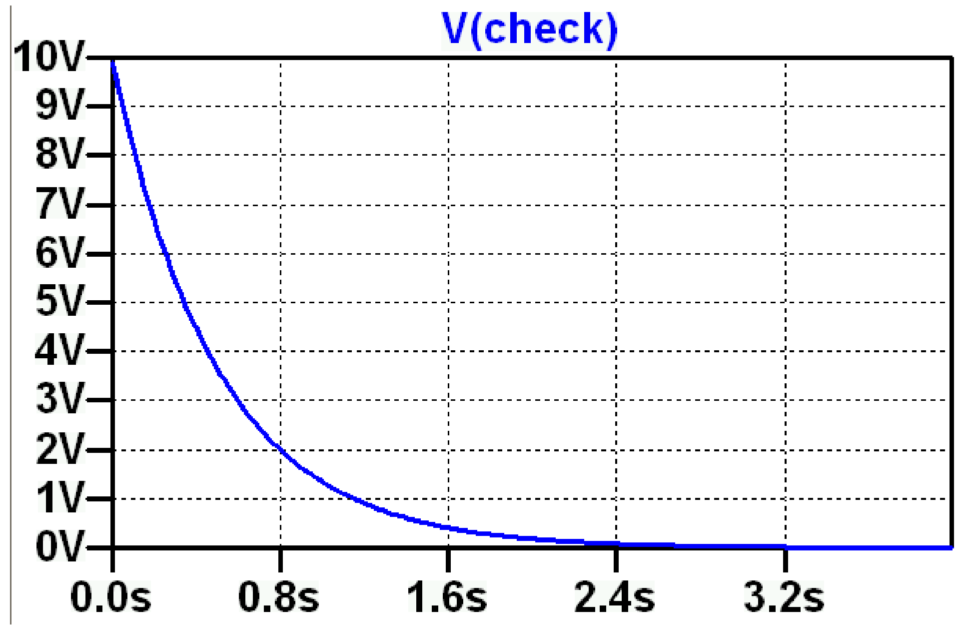
$$N(s) = -\frac{sN(s)}{\lambda} + \frac{N_0}{\lambda}$$

$$N(t) = N_0 \times e^{-\lambda t}$$

If a solution to the differential equations exists program it into Ltspice with a behavioral voltage source.

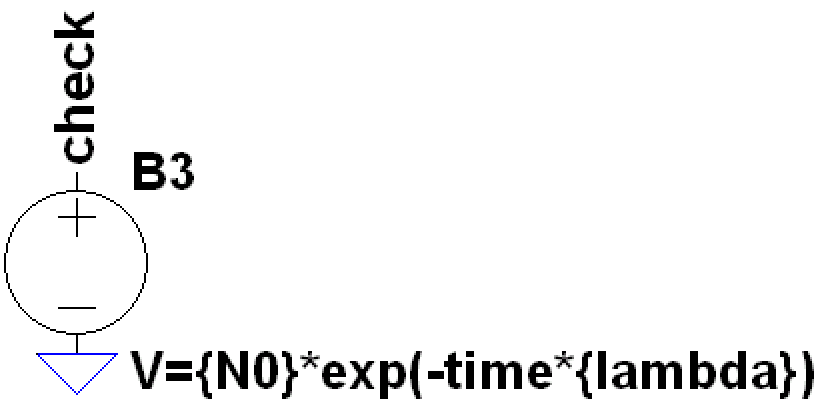
- N0=10, λ=2

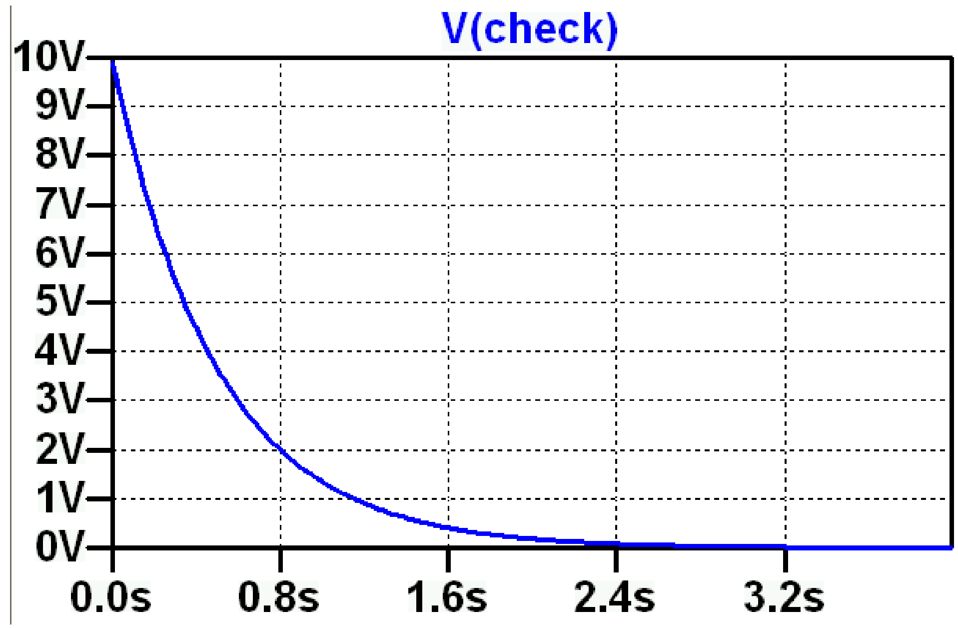




Take the Laplace transform of the equation.

- N0=10, λ=2





$$\frac{dN}{dt} = -\lambda N$$

$$sN = -\lambda N$$

sN means integrate everything on the right hand side

$$N = -\lambda \int N$$

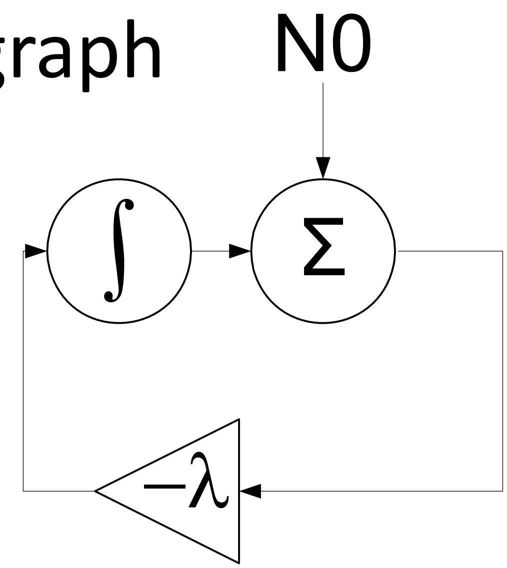
$$N = N0 - \lambda \int N$$

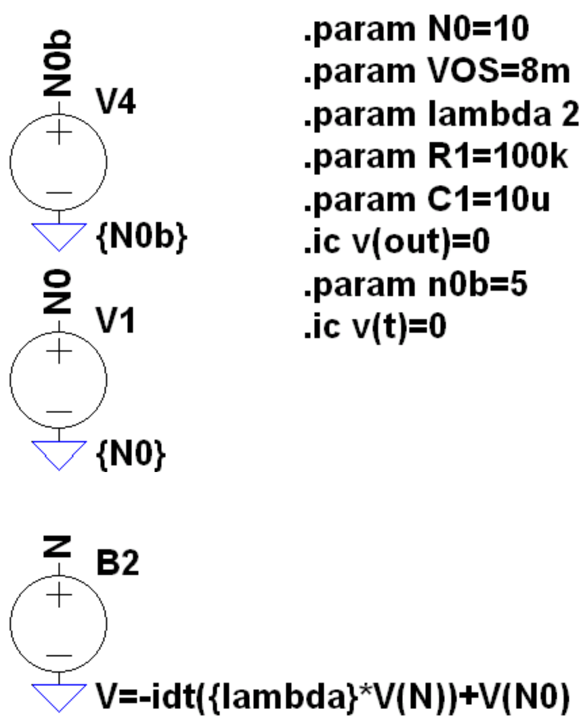
It is easier to add the initial condition after.

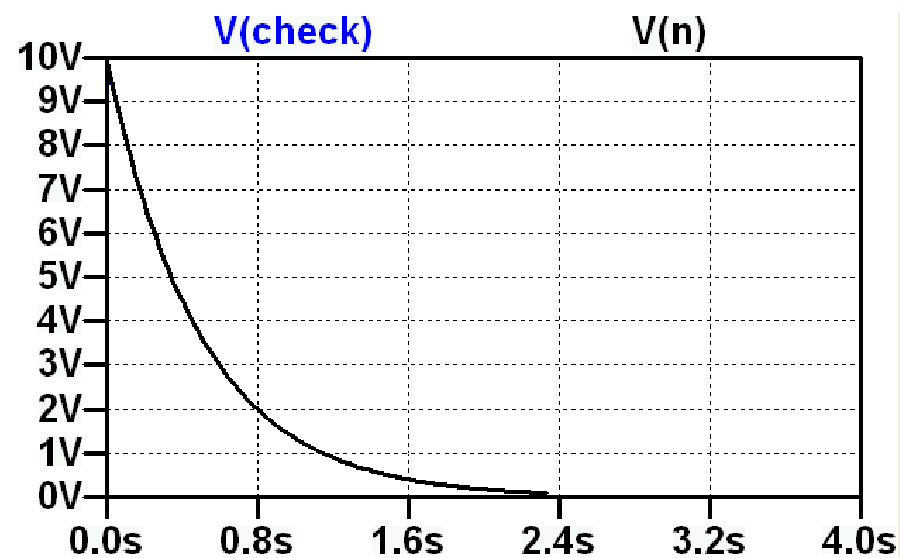
Convert equation to signal flow graph

- N0=10, λ=2

Program into LTspice





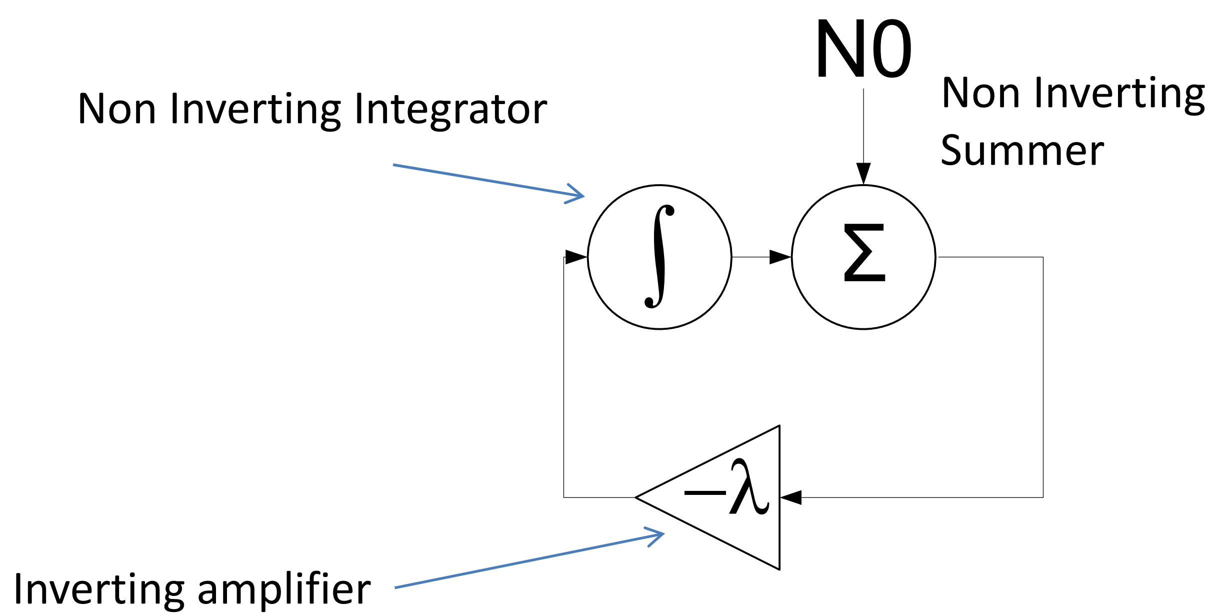


Check against solution!

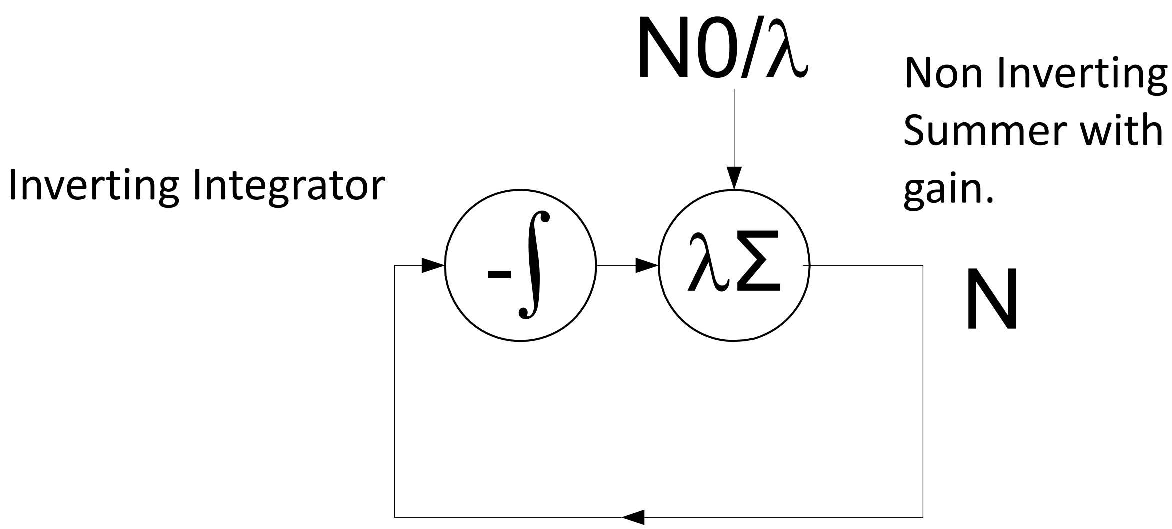
Convert Signal Flow graph into “real” parts such as:

- Non inverting amplifiers
- Inverting amplifiers
- Non inverting integrators
- Inverting integrators
- Non inverting summers
- Inverting summers

Note there are summer/integrator combinations or amplifier/summer combinations!



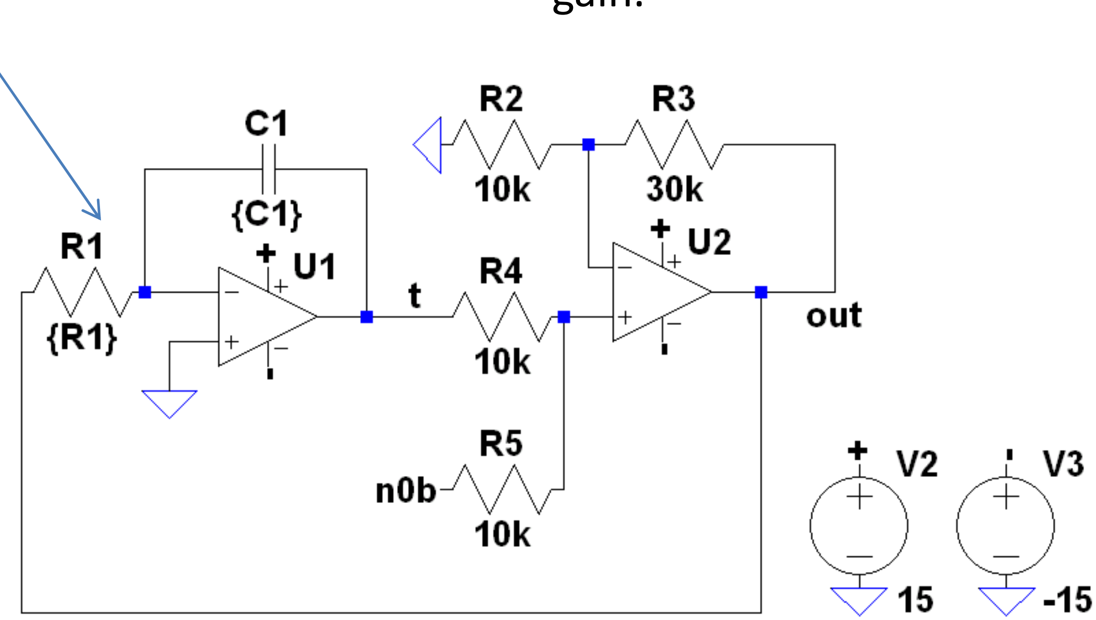
Investigate if functionality can be combined.



Draw Schematic with OPAMPS, Resistors and Capacitors.

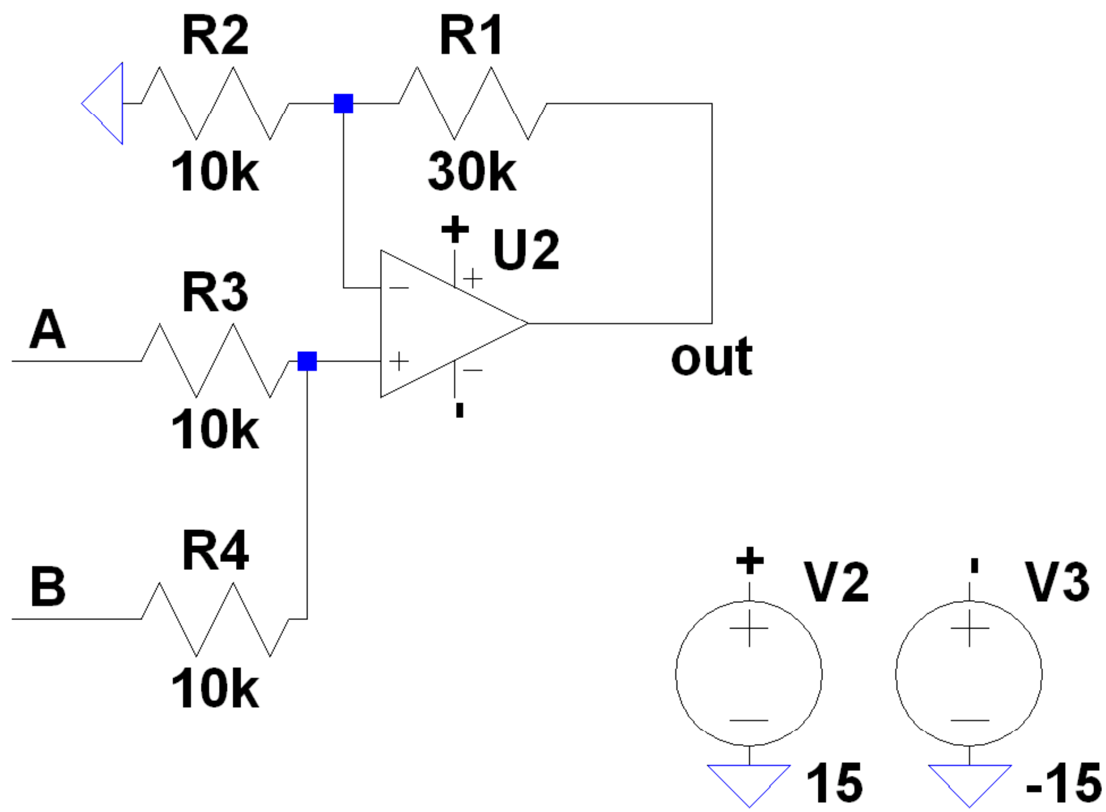
Non Inverting Summer with gain.

Inverting Integrator



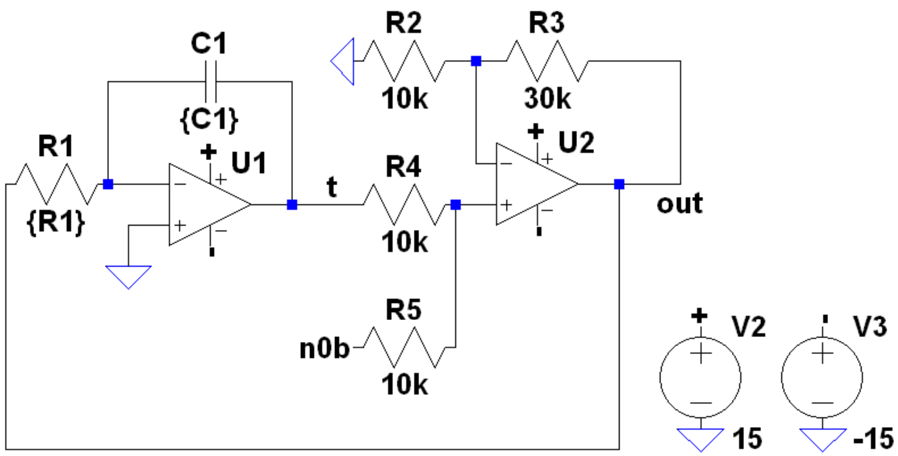
Gain for a Non inverting summer

Vout=(Va*R4/(R3+R4)+Vb*R3/(R3+R4))*(1+R1/R2)



$$V_{out} = \left(V_A \frac{R_4}{R_3 + R_4} + V_B \frac{R_3}{R_3 + R_4} \right)\left(1 + \frac{R_1}{R_2} \right)$$

Real integrators have an 1/RC component



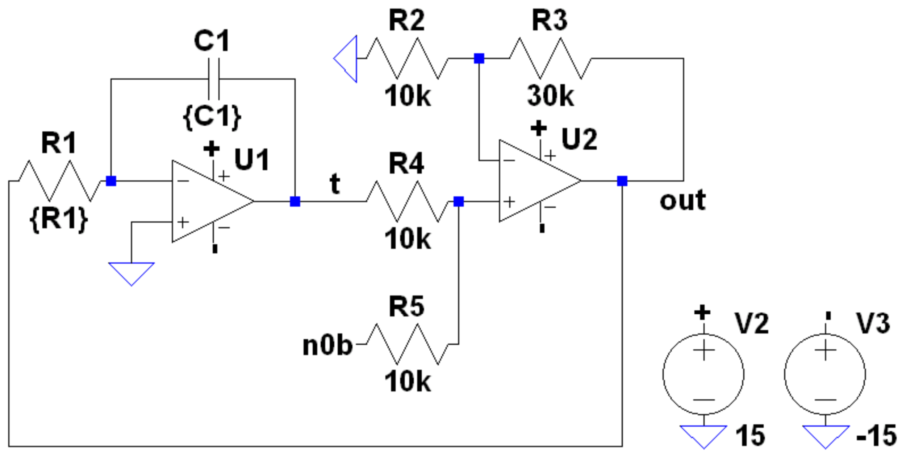
We can choose
R1=100kΩ and C1=10μF
so RC=1, Set R4=R5

$$N=\left(n_{ob}\times\frac{R_4}{R_4+R_5}-\frac{1}{R_1C_1}\times\frac{R_5}{R_4+R_5}\int N\right)\left(1+\frac{R_3}{R_2}\right)$$

$$N=\left(n_{ob}\times\frac{1}{2}-\frac{1}{1}\times\frac{1}{2}\int N\right)\left(1+\frac{R_3}{R_2}\right)=\left(\frac{1}{2}\times n_{ob}-\frac{1}{2}\int N\right)\left(1+\frac{R_3}{R_2}\right)$$

More Math

- If we want N0=10, λ=2, we set n0b to N0/λ and 1+R3/R2 to 4.

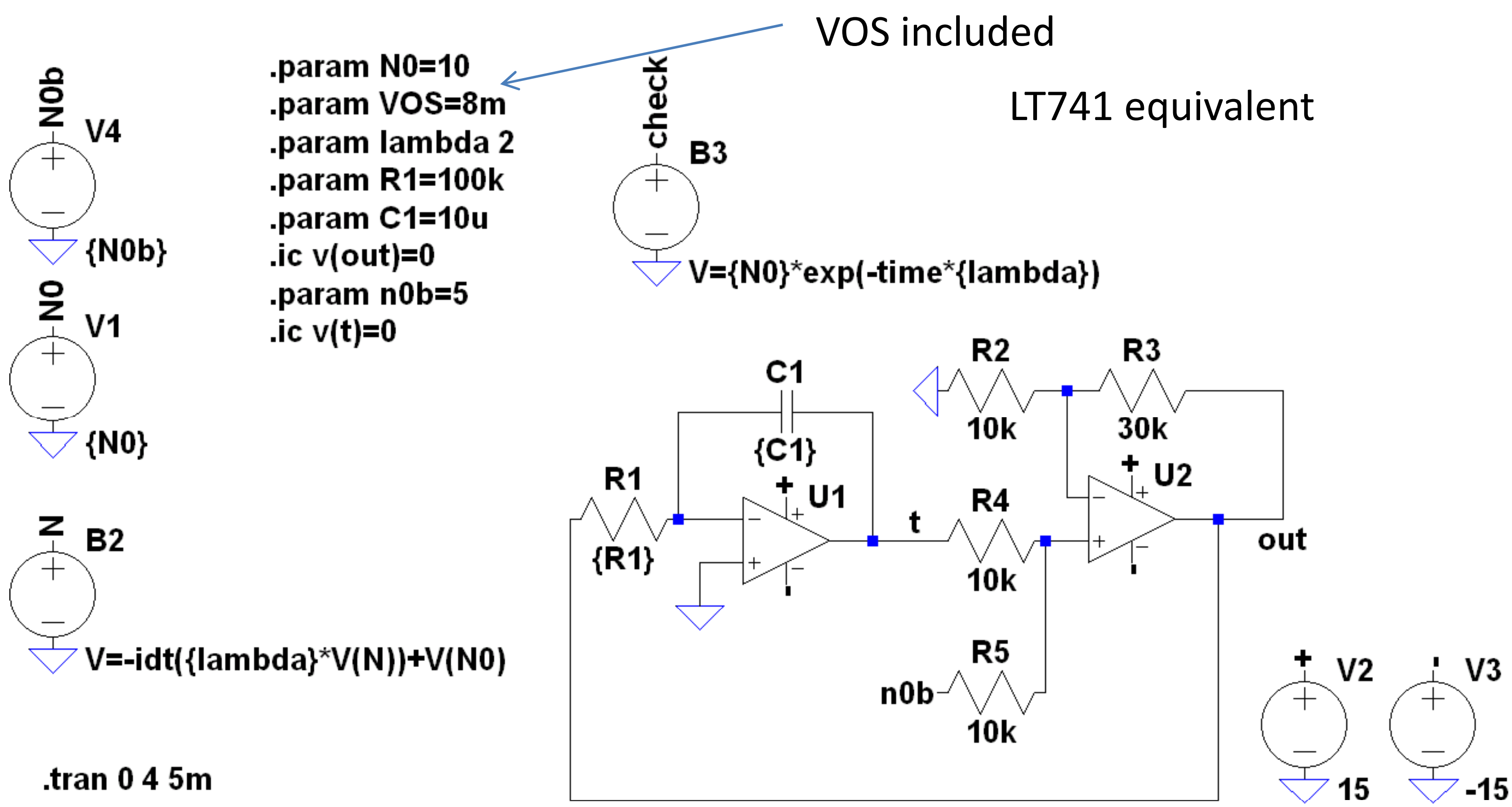


$$N = \left(n_{ob} - \int N\right)\left(1 + \frac{R_3}{R_2}\right) \times \frac{1}{2}$$

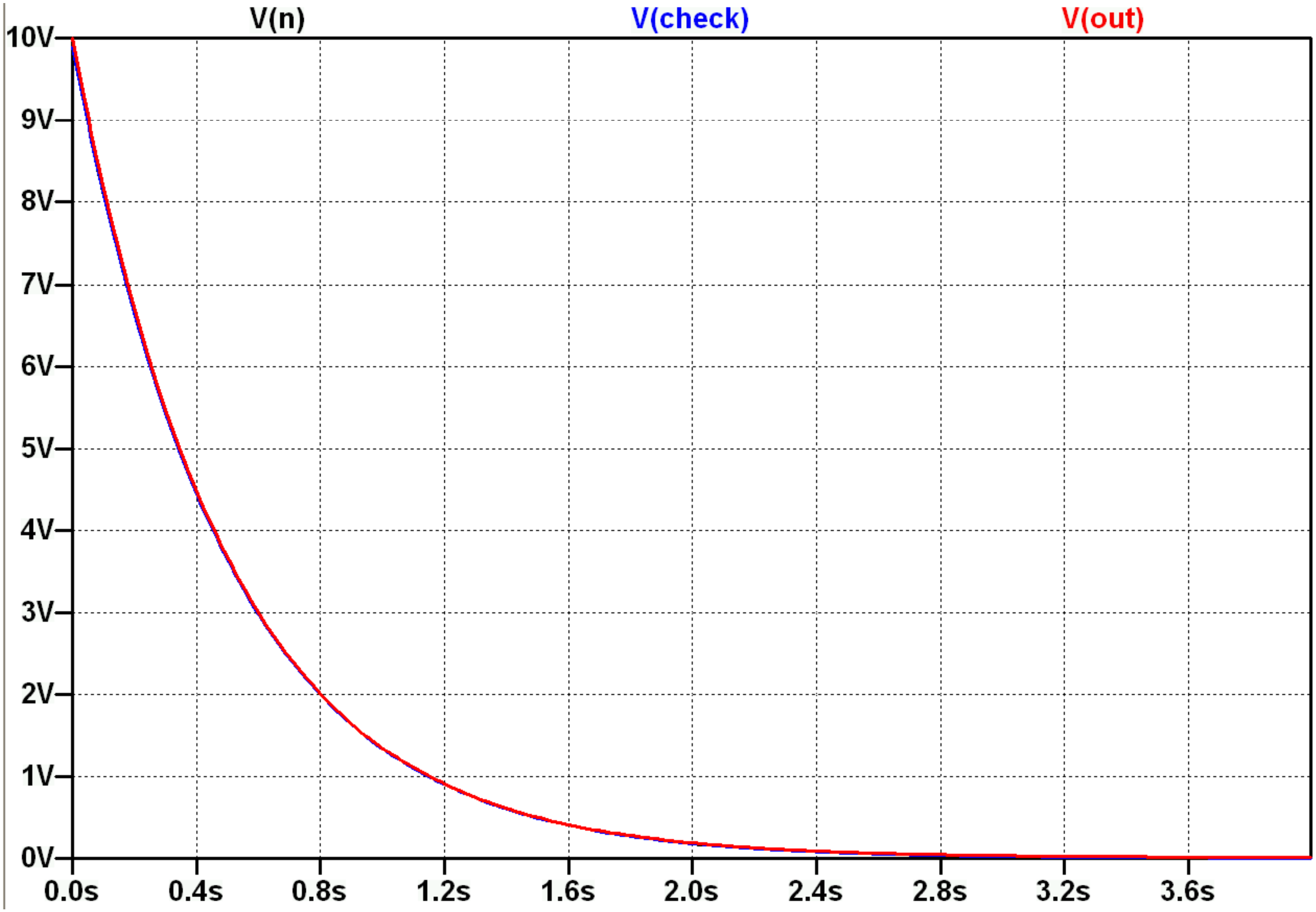
Since the integrator slope is 1V/s slew rate limiting is not problem.

The output of the integrator due to VOS is VOS(1+time) (since RC=1), Even LM741 is OK.

Ready to Fabricate and Test!



Results Match

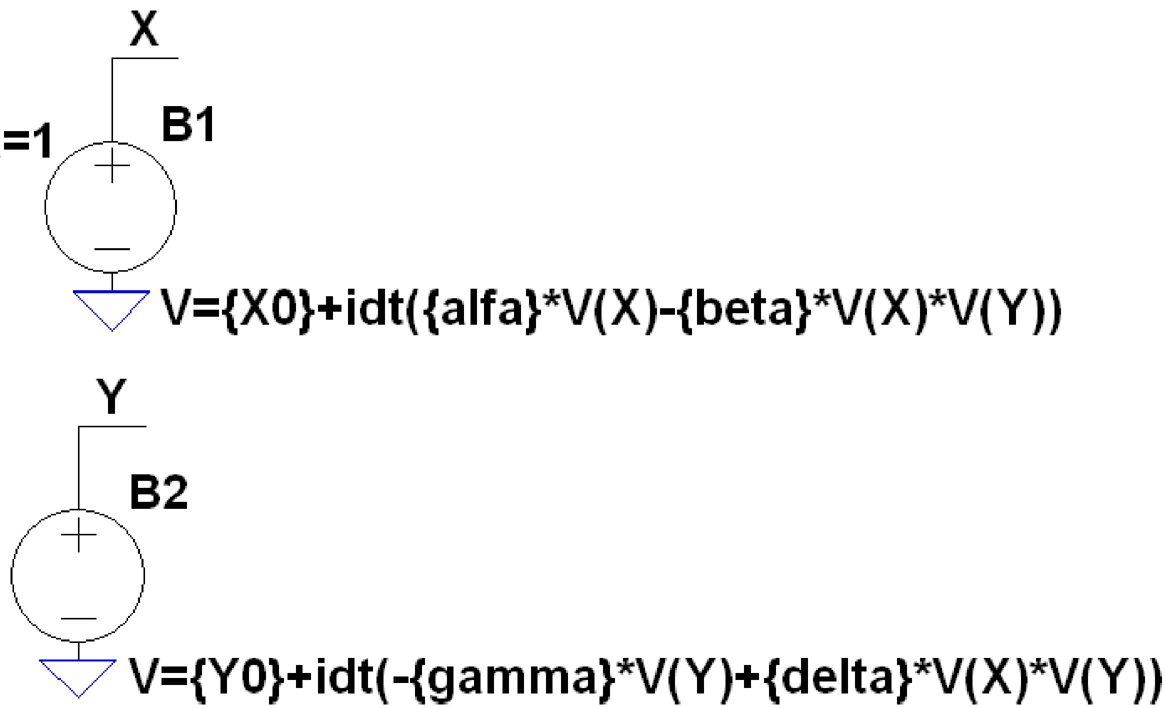


Improvements?

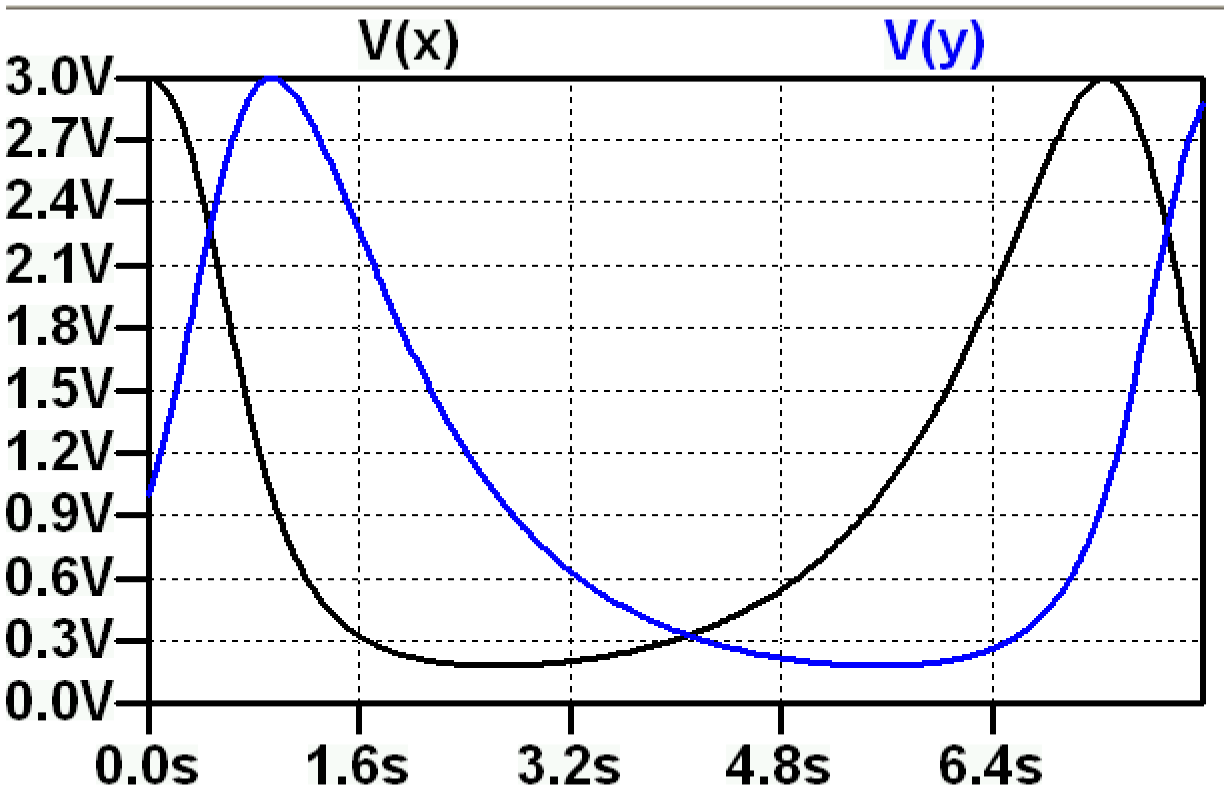
- Circuit could be scaled in voltage and time to get answers faster and for less power.
- This involves changing R values.

Predator Prey

```
.param X0=3
.param Y0=1
.param alfa=1
.param beta =1
.param gamma=1
.param delta=1
.ic v(X)=X0
.ic v(y)=Y0
.ic v(XR)=0
.ic v(yR)=0
.ic v(t1)=0
.ic v(t2)=0
.ic v(t3)=0
.ic v(t4)=0
.ic v(t5)=0
.ic v(t6)=0
```

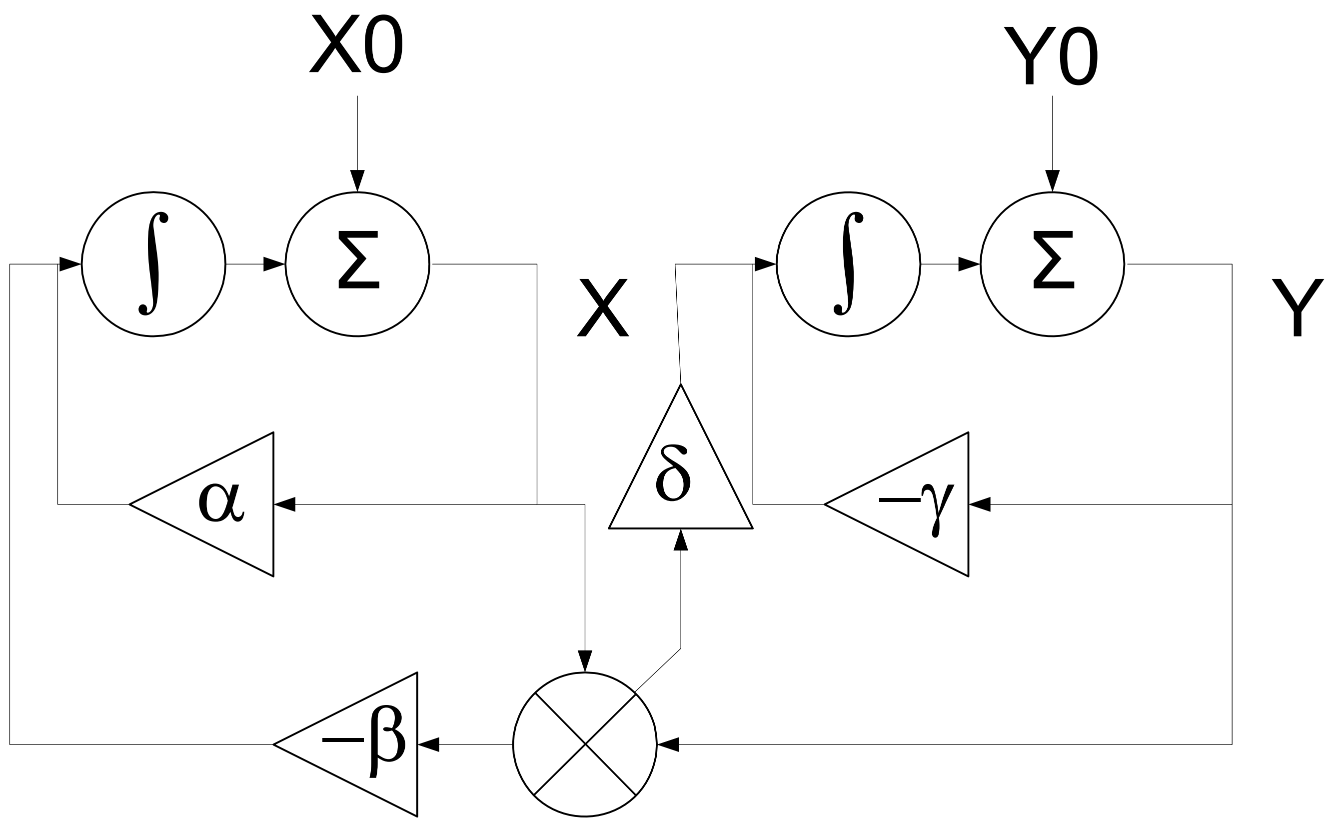


Predator Prey

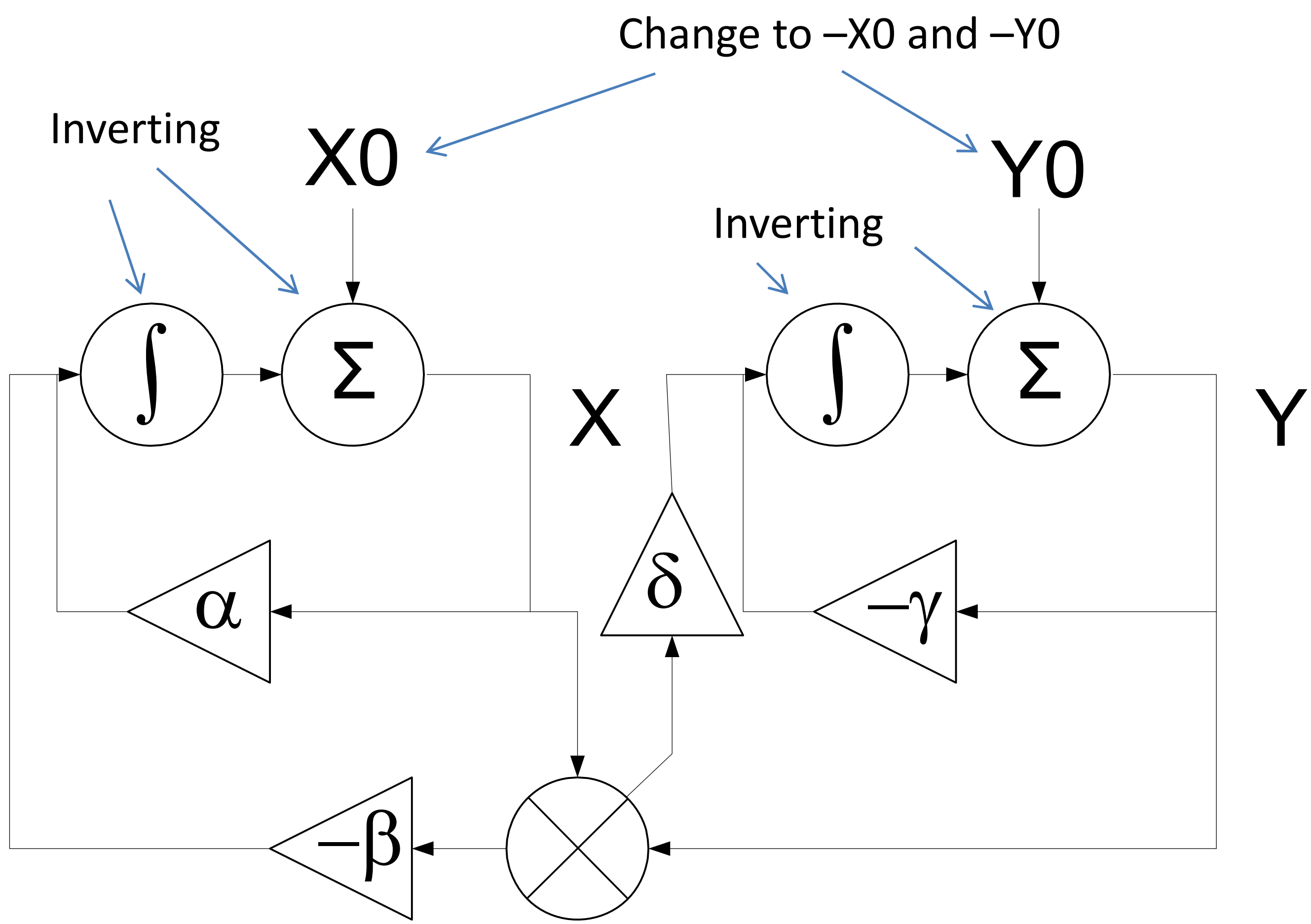


There is no solution! I had to verify my results from the mathematica website.

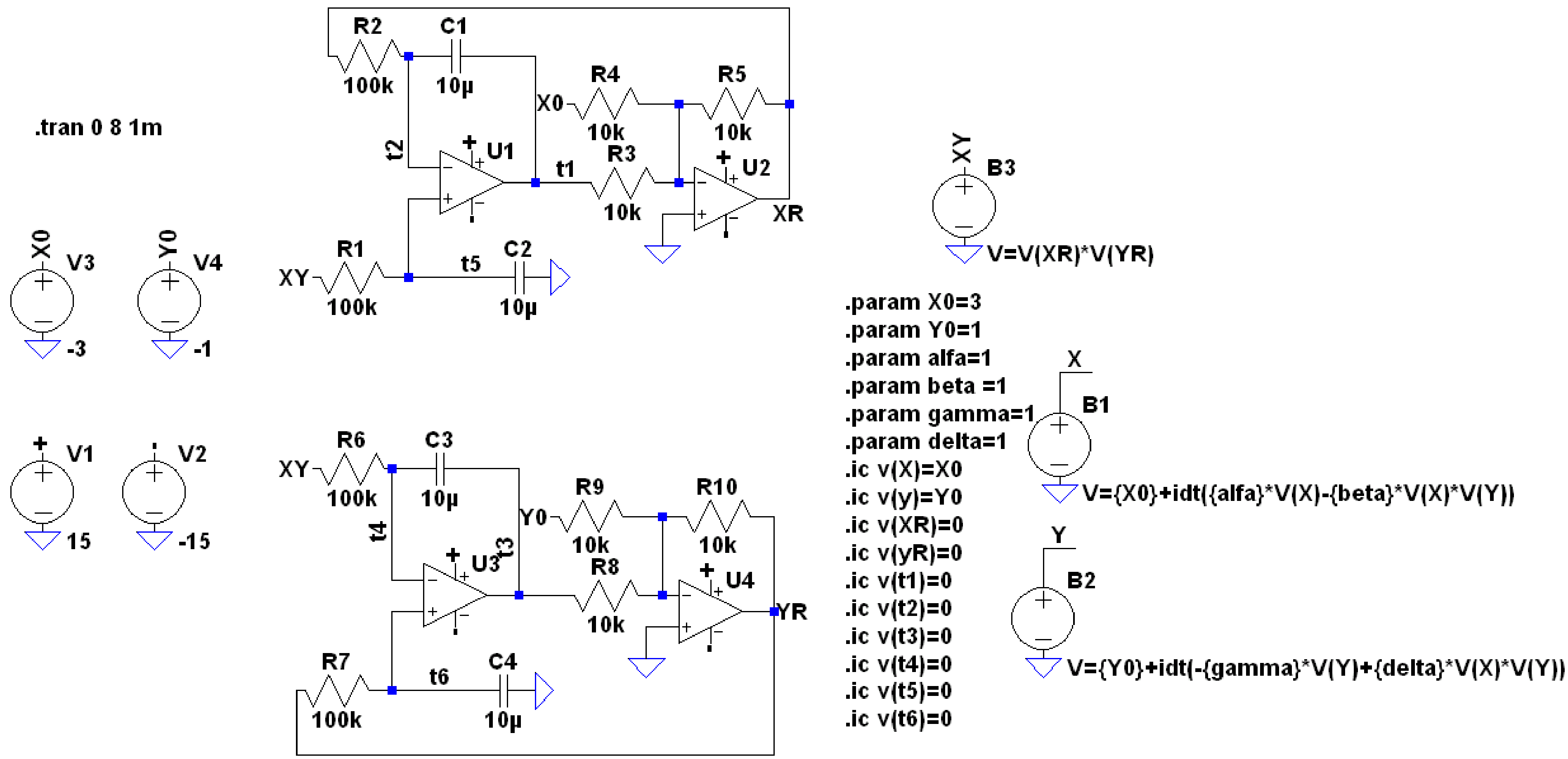
First Pass



Second Pass



Final Pass



.tran 0 8 1m
R2
100k
C1
10µ
R4
10k
R5
10k
X0
t2
U1
t1
R3
10k
U2
XR
R1
100k
XY
t5
C2
10µ
X0
V3
-3
Y0
V4
-1
V1
15
V2
-15
XY
B3
V=V(XR)*V(YR)
R6
100k
XY
C3
10µ
R9
10k
R10
10k
Y0
t4
t3
U3
R8
10k
U4
YR
R7
100k
t6
C4
10µ
.param X0=3
.param Y0=1
.param alfa=1
.param beta =1
.param gamma=1
.param delta=1
.ic v(X)=X0
.ic v(y)=Y0
.ic v(XR)=0
.ic v(yR)=0
.ic v(t1)=0
.ic v(t2)=0
.ic v(t3)=0
.ic v(t4)=0
.ic v(t5)=0
.ic v(t6)=0
X
B1
V={X0}+idt({alfa}*V(X)-{beta}*V(X)*V(Y))
Y
B2
V={Y0}+idt(-{gamma}*V(Y)+{delta}*V(X)*V(Y))