DEPARTMENT OF MATHEMATICS
IIT HYDERABAD

End Semester Date: 10-10-2018

Course name: **MA 2120** Max marks: 40 Max. time 2 hrs

Answer the following questions. Each question carry 4 marks.

(1) Find the Laplace transform of $f(t) = u_\pi \cos(t-\pi)$ and the inverse Laplace transform of $\mathcal{F}(s) = \log(\frac{s^2+a^2}{s^2+b^2})$, $s > 0$

(2) Define exponential order and piecewise continuity of a real valued function defined on $[0,\infty)$. Show that if a function satisfy these two properties, then the Laplace transform exists.

(3) Solve $y'' + \lambda^2 y = \cos(\lambda t)$ with the conditions $y(0) = 1 = y(\frac{\pi}{2\lambda})$.

(4) Define the convolution of two functions defined on $[0,\infty)$. Find the convolution of the functions $f(t) = \sin(t)$ and $g(t) = t^2$ for $t \in [0,\infty)$. Find the Laplace transform of $f * g$.

(5) Solve the integral equation

$$te^{-at} = \int_0^t x(\tau)x(t-\tau)d\tau.$$

(6) Let $k > 0$ be a constant. Find the Fourier sine and cosine integral of

$$f(x) = \begin{cases} e^{-kx}, & x > 0, \\ 0, & x < 0. \end{cases}$$

(7) Let $a > 0$ be a constant. Find the inverse Fourier transform of $F(w) = e^{-aw^2}, w \in \mathbb{R}$.

(8) Using the Fourier transform method find the convolution of $f(x) = e^{-ax^2}$ and $g(x) = xe^{-x^2}$ for $x \in \mathbb{R}$, where $a > 0$ is a constant.

(9) Find the Fourier cosine transform of

$$f(x) = \begin{cases} e^{-ax}, & x > 0 \\ 0, & x < 0. \end{cases}$$

(10) Find the Fourier integral representation of

$$f(x) = \begin{cases} \sin(\pi x), & 0 < x < 1, \\ 0, & \text{else.} \end{cases}$$