2. from the above result

$$|N(v) \cap V_i| = O(\log \Delta), \ \forall v \in V(G), \ \forall i \in \{1, 2, 3, \ldots r\}$$

Now consider

$$(N(v) \cap V_1) \cup (N(v) \cap V_2) \cup (N(v) \cap V_3) \cdots \cup (N(v) \cap V_r)$$

as these are disjoint sets (since V_i are disjoint)

$$|N(v)| = \log \Delta + \log \Delta + \cdots \log \Delta$$

$$\sum_{i=1}^{r} (N(v) \cap V_i) \quad |N(v) \cap V_1| \quad |N(v) \cap V_2| \quad |N(v) \cap V_r|$$

$$\Delta \leq r \log \Delta \implies r \geq \Delta / \log \Delta$$

atmax Δ (max neighbourhood) $\boxed{r = O(\Delta / \log \Delta)}$

1. $Pr(v_1 \in V_1) = \frac{1}{r}$ $\ldots$ $Pr(v_n \in V_1) = \frac{1}{r}$

$Pr(v_1 \in V_2) = \frac{1}{r}$ $\ldots$ $Pr(v_n \in V_2) = \frac{1}{r}$

$\vdots$ $\vdots$

$Pr(v_1 \in V_r) = \frac{1}{r}$ $\ldots$ $Pr(v_n \in V_r) = \frac{1}{r}$

Now Consider

$$E(N(v) \cap V_1) = Pr(v_{i_1} \in V_1) + Pr(v_{i_2} \in V_1) + \ldots Pr(v_{i_{N(v)}} \in V_1)$$

where $N(v) = \{v_{i_1}, v_{i_2}, \ldots v_{i_{N(v)}}\}$

$$= \frac{1}{r} + \frac{1}{r} + \ldots \frac{1}{r}$$

$$= \frac{\Delta}{r}$$

from before proof $r = O(\Delta/\log\Delta)$

So $E(N(v) \cap V_1) = \frac{\Delta}{\Delta/\log\Delta}$

$$E(N(v) \cap V_1) = O(\log\Delta)$$

$$\boxed{\therefore |N(v) \cap V| = O(\log\Delta)}$$