INDIAN INSTITUTE OF TECHNOLOGY HYDERABAD

DEPARTMENT OF MATHEMATICS

Problem Sheet-I

Date : 01.10.2018 MA 2120 : Transform Techniques

1. Define the following:
   (a) Let $g(t) = te^{t^2}\sin(e^{t^2}),\ t \in [0,\infty)$. Is g continuous on $[0,\infty)$. Does g have exponential order? Show that $\mathcal{L}(g)$ exists?
   (b) Give examples of functions on $[0,\infty)$ which are piecewise continuous.
2. Without actually determining it, show that the following functions possess Laplace transform;
   (a) $\dfrac{\sin(t)}{t},\ t > 0$
   (b) $\dfrac{1-\cos(t)}{t},\ t > 0$
   (c) $t^2\sinh(t),\ t \geq 0.$
3. Compute the Laplace transform of the following functions;
   (a) $\cosh^2 w(t)$
   (b) $\sinh^2(wt)$
4. Compute the Laplace transform of the following functions;
   (a) $f(t) = \sin^3 w(t),\ t \geq 0$
   (b) $f(t) = \cos^3 w(t),\ t \geq 0$
5. Can $\mathcal{F}(s) = \dfrac{s}{\log(s)},\ s > 0$ be the Laplace transform of some function?
6. Find the inverse Laplace transform of the following funcions
   (a) $\dfrac{e^{-\pi s}}{s^2-2},\ s > \sqrt{2}$
   (b) $\dfrac{s}{s^2+6s+1},\ s > 0$
   (c) $\dfrac{s}{(s^2+a^2)(s^2+b^2)},\ a \neq b$
   (d) $\dfrac{s}{(s-1)(s+2)(s-3)},\ s > 3$
   (e) $\dfrac{1}{s^2(s^2+1)}$
   (f) $\dfrac{1}{s^2(s+4)^2}.$
7. Solve
$$y''(t) + y(t) = cu_a(t),\ y(0) = 0,\ y'(0) = 1,$$
where c is constant and $a > 0$.

8. Solve $y''(t) + \lambda^2 y(t) = \cos(\lambda t),\ y(0) = 1,\ y(\frac{\pi}{2\lambda}) = 1.$

9. Solve $y'''(t) + 5y''(t) + 2y'(t) - 8y(t) = \sin(t),\ y(0) = 0 = y'(0).$

10. Solve $y''(t) + y'(t) = f(t), y(0) = 1,\ y'(0) = -1,$
    where $f(t) = \begin{cases} 1,\ 0 \leq t \leq 1 \\ 0,\ t > 1. \end{cases}$

11. Solve $y''(t) + y(t) = \begin{cases} \cos(t),\ 0 \leq t \leq \pi \\ 0,\ t > \pi. \end{cases}$

12. Solve the following boundary value problems using Laplace transform;
    (a) $y''(t) + \lambda^2 y(t) = \sin(\lambda t),\ y(0) = 1,\ y(\frac{\pi}{2\lambda}) = \pi.$
    (b) $y''(t) + \lambda^2 y(t) = t,\ y(0) = 1,\ y(\frac{\pi}{2}) = -1.$

13. Solve the following integral equations;
    (a) $\int_0^t \cos(\tau)\sin(t-\tau)d\tau = \frac{1}{2}t\sin(t)$
    (b) $x(t) = 1 + \int_0^1 \cos(t-\tau)x(\tau)d\tau$
    (c) $te^{-at} = 1 + \int_0^t x(\tau)x(t-\tau)d\tau.$

14. Compute the following;
    (a) $\cos(t) * \sin(t)$
    (b) $\sin(t) * t^2.$