Topics in Combinatorics

Exam III (out of 10 marks)
(Date: 14 Dec 2020. Timing: 12:00 to 13:05 hours)

1. Let $f \in \mathbb{F}[x_1, \dots, x_n]$ be a polynomial and $S_1, \dots, S_n$ be non-empty subsets of $\mathbb{F}$, for some field $\mathbb{F}$. Let $(s_1, s_2, \dots, s_n)$ be a point in $S_1 \times S_2 \times \cdots \times S_n$. It is given that, $\forall (a_1, a_2, \dots, a_n) \in S_1 \times S_2 \times \cdots \times S_n$, $f(a_1, a_2, \dots, a_n) \neq 0$ if and only if $(a_1, a_2, \dots, a_n) = (s_1, s_2, \dots, s_n)$. That is, f vanishes on all but one point (, which is $(s_1, \dots, s_n)$,) in $S_1 \times \cdots \times S_n$. Show that $\deg(f) \geq \sum_{i=1}^{n}(|S_i| - 1)$. **10 marks**