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$[n] := \{1, 2, \ldots, n\}$

2^n, 0

<u>Intersecting family</u>: A family $\mathcal{F}$ of subsets of $[n]$ is an intersecting family if $\forall A, B \in \mathcal{F}$, $A \cap B \neq \emptyset$.

<u>Eg</u>: $\mathcal{F} = \{\{1,2,3\}, \{1,2\}, \{2,3\}, \{1,3\}\}$

How large can an intersecting family of subsets of $[n]$ be?

$\{\{1\}, \{1,2\}, \{1,3\}, \ldots, \{1,n\},$
$\{1,2,3\}, \ldots \{1, \ldots\}\}$

2^{n-1}

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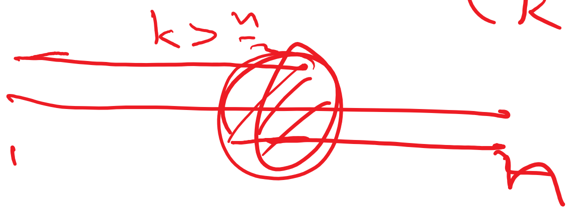
$2^{n-1} \leq$ Size of a largest intersecting family of subsets of $[n]$ $\leq 2^{n-1}$ ✓

$\underbrace{\{1\}, \{2,3,\ldots,n\} \quad \{2\}, \{1,3,4,5,\ldots,n\} \quad \ldots \quad A, \bar{A} \quad \ldots}_{2^{n-1}}$

size of a largest <u>k-uniform</u> family of subsets of $[n]$ that is <u>intersecting</u>

If $k > \frac{n}{2}$, $= \binom{n}{k}$



<u>Suppose $k \le \frac{n}{2}$,</u>

$\binom{n-1}{k-1} \le$ size of largest k-uniform family that is <u>intersecting</u> $\le \binom{n-1}{k-1}$

Erdos-Ko-Rado

<u>Example</u> $k=3$

$\mathcal{F} = \{\{1,2,3\},\{1,2,4\},\ldots\ \ldots\{1,n-1,n\}\}$

Theorem [Erdős-Ko-Rado, 1960s] Let $\mathcal{F}$ be a k-uniform family of subsets of $[n]$ that is intersecting. Further $k \leq \frac{n}{2}$. Then, $|\mathcal{F}| \leq \binom{n-1}{k-1}$

Proof. [Katona, 1970s]

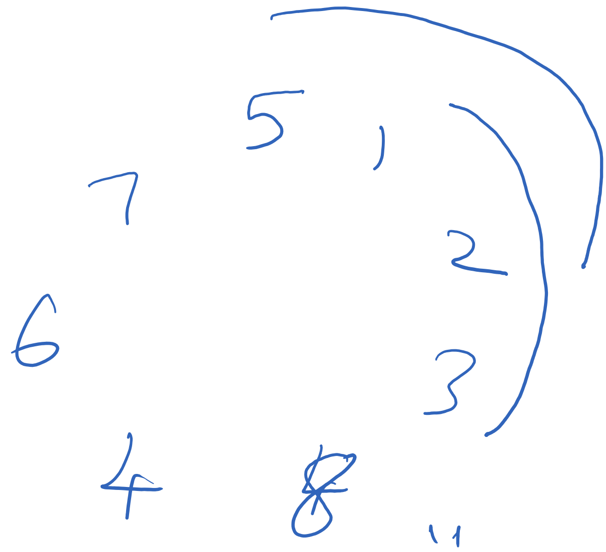
$\hookrightarrow$ Proof from the book

Proof: $\longrightarrow$ Suppose you are given a a k-uniform intersecting family $\mathcal{F}$.

$k = 3$, $\mathcal{F} = \{\{1,2,3\}, \{2,3,4\}, \{1,5,2\}, \{2,3,5\}\}$

$n = 8$

σ: be a circular permutation of $[n]$



A set $S \in \mathcal{F}$ is "present" in σ if its elements are present contiguously or together in the permutation.

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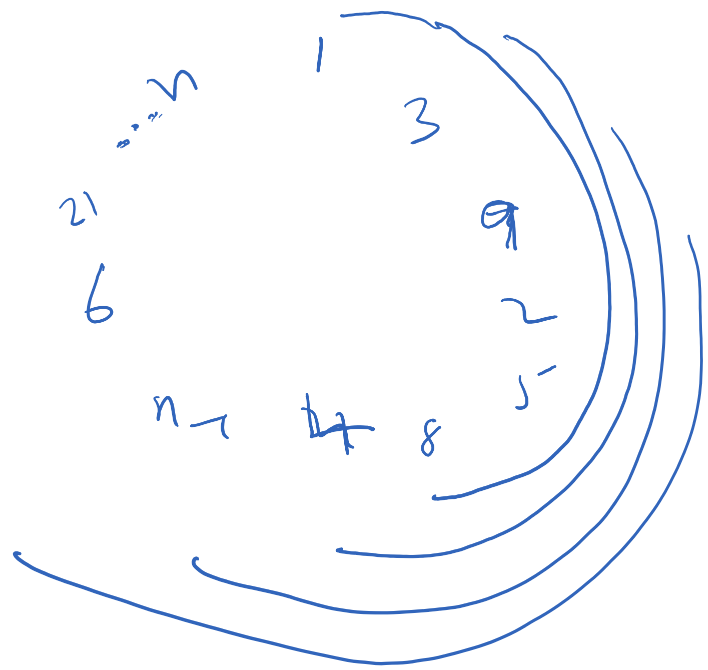
$\mathcal{F}$ — intersecting, k-uniform

σ :



$\leq k$

$$\mathcal{S} = \left\{ (S, \sigma) : S \in \mathcal{F} \text{ is present in the circular perm } \sigma \right\}$$

$$|\mathcal{F}|\, k!\, (n-k)! = |\mathcal{S}| \le (n-1)!\, k$$

Rearranging the terms,

$$|\mathcal{F}| \le \frac{(n-1)!\, k}{k!\, (n-k)!}$$

$$= \binom{n-1}{\ }$$

$$= \binom{n-1}{k-1}$$

$\square$