CS5820-GPU

HW: Assignment 1

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-CS18BTECH11001

Given linear equations are

$$c_x x_0 + c_y y_0 + c_k = q_0 \quad -①$$

$$c_x x_1 + c_y y_1 + c_k = q_1 \quad -②$$

$$c_x x_2 + c_y y_2 + c_k = q_2 \quad -③$$

$$c_x x + c_y y + c_k = q \quad -④$$

If we consider ①, ②, ③ in matrix form

$$[c_x \; c_y \; c_k] \times \begin{bmatrix} x_0 & x_1 & x_2 \\ y_0 & y_1 & y_2 \\ 1 & 1 & 1 \end{bmatrix} = [q_0 \; q_1 \; q_2]$$

If $M = \begin{bmatrix} x_0 & x_1 & x_2 \\ y_0 & y_1 & y_2 \\ 1 & 1 & 1 \end{bmatrix}$ then

$$[c_x \; c_y \; c_k] \times M = [q_0 \; q_1 \; q_2]$$

Multipling with M^{-1} we get

$$[c_x \; c_y \; c_k] \times M \times M^{-1} = [q_0 \; q_1 \; q_2] \times M^{-1}$$

$$\boxed{\Rightarrow [c_x \; c_y \; c_k] = [q_0 \; q_1 \; q_2] \times M^{-1}}$$

Now consider matrix $M = \begin{bmatrix} x_0 & x_1 & x_2 \\ y_0 & y_1 & y_2 \\ 1 & 1 & 1 \end{bmatrix}$

$$M^{-1} = \frac{\mathrm{adj}(M)}{\det(M)}.$$

So $\boxed{\det(M) = x_0(y_1 - y_2) - x_1(y_0 - y_2) + x_2(y_0 - y_1)}$

$$\mathrm{adj}(M) = (\mathrm{cofac}(M))^T$$

$$= \begin{bmatrix} y_1 - y_2 & y_2 - y_0 & y_0 - y_1 \\ x_2 - x_1 & x_0 - x_2 & x_1 - x_0 \\ x_1y_2 - x_2y_1 & x_2y_0 - y_2x_0 & x_0y_1 - x_1y_0 \end{bmatrix}^T$$

$$\boxed{\mathrm{adj}(M) = \begin{bmatrix} y_1 - y_2 & x_2 - x_1 & x_1y_2 - x_2y_1 \\ y_2 - y_0 & x_0 - x_2 & x_2y_0 - y_2x_0 \\ y_0 - y_1 & x_1 - x_0 & x_0y_1 - x_1y_0 \end{bmatrix}}$$

$$\boxed{\therefore M^{-1} = \frac{1}{x_0(y_1 - y_2) - x_1(y_0 - y_2) + x_2(y_0 - y_1)} \begin{bmatrix} y_1 - y_2 & x_2 - x_1 & x_1y_2 - x_2y_1 \\ y_2 - y_0 & x_0 - x_2 & x_2y_0 - y_2x_0 \\ y_0 - y_1 & x_1 - x_0 & x_0y_1 - x_1y_0 \end{bmatrix}}$$

So $[c_x \; c_y \; c_k] = [q_0 \; q_1 \; q_2] \times M^{-1}$

$$= \frac{1}{x_0(y_1-y_2) - x_1(y_0-y_2) + x_2(y_0-y_1)} [q_0 \; q_1 \; q_2] \times \begin{bmatrix} y_1-y_2 & x_2-x_1 & x_1y_2-x_2y_1 \\ y_2-y_0 & x_0-x_2 & x_2y_0-x_0y_2 \\ y_0-y_1 & x_1-x_0 & x_0y_1-x_1y_0 \end{bmatrix}$$

By comparing

$$\boxed{c_x = \frac{(y_1-y_2)q_0 + (y_2-y_0)q_1 + (y_0-y_1)q_2}{(y_1-y_2)x_0 + (y_2-y_0)x_1 + (y_0-y_1)x_2}}$$

$$\boxed{c_y = \frac{(x_2-x_1)q_0 + (x_0-x_2)q_1 + (x_1-x_0)q_2}{(y_1-y_2)x_0 + (y_2-y_0)x_1 + (y_0-y_1)x_2}}$$

$$\boxed{c_k = \frac{(x_1y_2-x_2y_1)q_0 + (x_2y_0-x_0y_2)q_1 + (x_0y_1-x_1y_0)q_2}{(y_1-y_2)x_0 + (y_2-y_0)x_1 + (y_0-y_1)x_2}}$$

Now consider equation (4)

$$c_x x + c_y y + c_k = q$$

By transforming into interpolation equation

$$c_y y = (q - c_k) + c_x x$$

$$y = \left(\frac{q-c_k}{c_y}\right) + x\left(\frac{c_x}{c_y}\right)$$

So by substituting the values

$$y = \frac{\left(q - \left(\dfrac{(x_1y_2 - y_1x_2)q_0 + (x_2y_0 - x_0y_2)q_1 + (x_0y_1 - x_1y_0)q_2}{(y_1-y_2)x_0 + (y_2-y_0)x_1 + (y_0-y_1)x_2}\right)\right)}{\dfrac{(x_2-x_1)q_0 + (x_0-x_2)q_1 + (x_1-x_0)q_2}{(y_1-y_2)x_0 + (y_2-y_0)x_1 + (y_0-y_1)x_2}}$$

$$+ x\left(\frac{\dfrac{(y_1-y_2)q_0 + (y_2-y_0)q_1 + (y_0-y_1)q_2}{(y_1-y_2)x_0 + (y_2-y_0)x_1 + (y_0-y_1)x_2}}{\dfrac{(x_2-x_1)q_0 + (x_0-x_2)q_1 + (x_1-x_0)q_2}{(y_1-y_2)x_0 + (y_2-y_0)x_1 + (y_0-y_1)x_2}}\right)$$

The interpolation equation:

$$\boxed{\therefore y = \left(\frac{(x_1y_2 - y_1x_2)(q-q_0) + (x_2y_0 - x_0y_2)(q-q_1) + (x_0y_1 - x_1y_0)(q-q_2)}{(x_2-x_1)q_0 + (x_2-x_0)q_1 + (x_1-x_0)q_2}\right) + x\left(\frac{(y_1-y_2)q_0 + (y_2-y_0)q_1 + (y_0-y_1)q_2}{(x_2-x_1)q_0 + (x_0-x_2)q_1 + (x_1-x_0)q_2}\right)}$$