Assignment

September 20, 2015

Problem 1. *Find the mass of the 3D region B given by* $x^2 + y^2 + z^2 \leq 4$, $x \geq 0, y \geq 0, z \geq 0$,*if the density is equal to xyz.*

Problem 2. *Sketch the region B whose volume is given by the triple integral*

$$\int_0^4 \int_0^{(4-x)/2} \int_0^{(12-3x-6y)/4} 1\, dz\, dy\, dx$$

Rewrite and evaluate the triple integral using the order of integration $dV = dy$ dx dz.

Problem 3. *Evaluate the following integrals*

$$a) \int_0^2 \int_0^1 \int_y^1 sinh(z^2)\, dz\, dy\, dx$$

$$b) \int_0^2 \int_0^4 \int_z^2 yze^{x^3}\, dz\, dy\, dx$$

Problem 4. *Find the coordinates of the center of gravity of the solid S with indicated mass density* $\delta = \delta(x, y, z)$. $S : x^2 + y^2 \leq a^2, \frac{b}{a}\sqrt{x^2 + y^2} \leq z \leq b$ *for constants* $b > 0, a > 0$ *and* $\delta = x^2 + y^2 + z^2$.

Problem 5. *Evaluate* $\iiint\limits_D \frac{z}{(x^2+y^2+z^2)^{3/2}}\, dV$, $D = (x, y, z) : x^2 + y^2 + z^2 \leq 4a^2, z \geq a$

Problem 6. *Find the volume of the region bounded above by the sphere* $x^2 + y^2 + z^2 = 10$ *and bounded below by the cone* $z^2 = x^2 + y^2$.

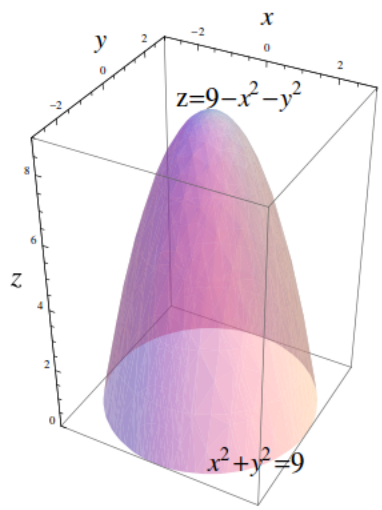
Problem 7. *Evaluate*

$$\iiint\limits_D log\sqrt{x^2 + y^2 + z^2}\, dx\, dy\, dz$$

where D is the region in R^3 *which lies between* $x^2+y^2+z^2 = 1$ *and* $x^2+y^2+z^2 = 4$ *and above xy-plane.*

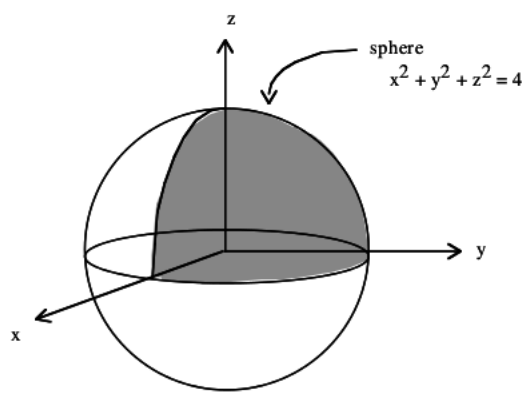
Problem 8. *Evaluate*

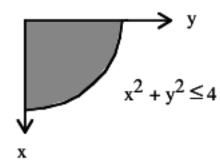
$$\int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_{0}^{9-x^2-y^2} x^2 \, dz \, dy \, dx$$



HINTS:

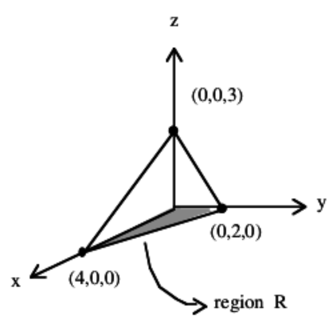
Problem 1:



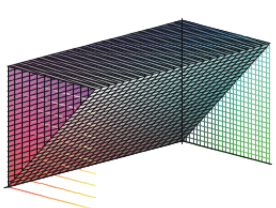


Problem 2:

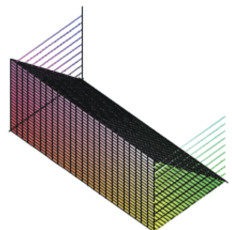
$$0 \leq z \leq \frac{12 - 3x - 6y}{4}, R : 0 \leq y \leq \frac{4 - x}{2} 0 \leq x \leq 4$$



Problem 3:



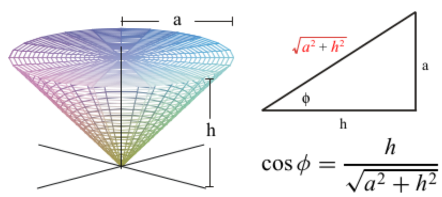
The region indicated by the integral is bounded by $z = y$, $y = 0$, $z = 1$, $x = 0$, and $x = 2$ which is indicated by the figure above. The difficulty with integrating the original triple integral is that to easily integrate $\sinh(z^2)$, we need a zdz rather than just dz. Note that if we switch the dz and dy, we might get a z where we need it.



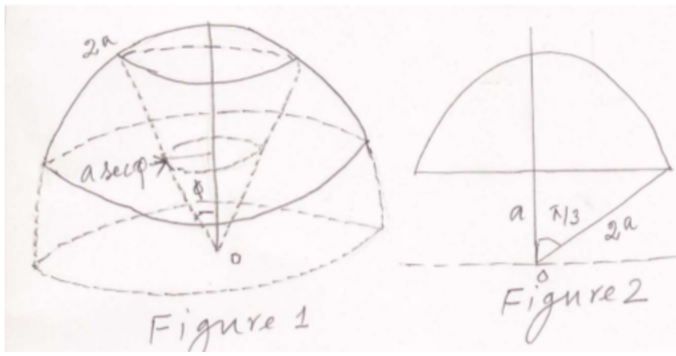
The region described by the integral is bounded by $y = 0$, $y = 4$, $z = 0$, $z = x$, and $x = 2$. A picture of the region is indicated above. In the original integral, if we try to integrate $e^{x^3}dx$ we have a problems. We can easily integrate $x^2e^{x^3}$, so this suggests switching dx and dz.

Figure 1: Left to right 3(a) and 3(b)

Problem 4:
$tan\phi = \frac{a}{h}$,so $\phi = arctan(\frac{a}{h})$; also $z = \rho cos\phi = h$ gives $\rho = \frac{h}{cos\phi}$.



Problem 5:



Problem 6:

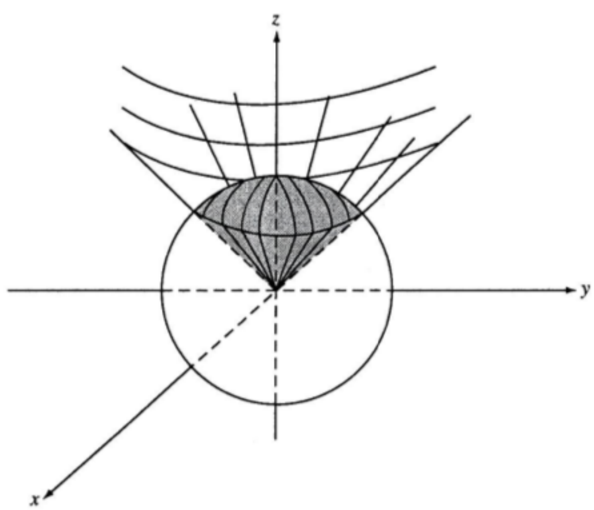


Figure 2: Region R bounded below by the upper half of the cone $z^2 = x^2 + y^2$ and bounded above by the sphere $x^2 + y^2 + z^2 = 8$

Problem 8:
Convert to cylindrical co-ordinates.