

2 a) Given $f_s = 8 \text{ KHz}$

for $f_0 = 1 \text{ KHz}$ $x[n] = A \cos(2\pi \cdot \frac{1}{8} \cdot n) = A \cos(\frac{\pi n}{4})$

for $f_0 = 2 \text{ KHz}$ $x[n] = A \cos(2\pi \cdot \frac{2}{8} \cdot n) = A \cos(\frac{\pi n}{2})$

for $f_0 = 3 \text{ KHz}$ $x[n] = A \cos(2\pi \cdot \frac{3}{8} \cdot n) = A \cos(\frac{3\pi n}{4})$

for $f_0 = 3.5 \text{ KHz}$ $x[n] = A \cos(2\pi \cdot \frac{3.5}{8} \cdot n) = A \cos(\frac{7\pi n}{8})$

for $f_0 = 4 \text{ KHz}$ $x[n] = A \cos(2\pi \cdot \frac{4}{8} \cdot n) = A \cos(\pi n)$

for $f_0 = 5 \text{ KHz}$ $x[n] = A \cos(2\pi \cdot \frac{5}{8} \cdot n) = A \cos(\frac{5\pi n}{4})$

for $f_0 = 6 \text{ KHz}$ $x[n] = A \cos(2\pi \cdot \frac{6}{8} \cdot n) = A \cos(\frac{3\pi n}{2})$

for $f_0 = 7 \text{ KHz}$ $x[n] = A \cos(2\pi \cdot \frac{7}{8} \cdot n) = A \cos(\frac{7\pi n}{4})$

Now consider $f_0 = 7 \text{ KHz}$

$$\begin{aligned} x[n] &= A \cos\left(\frac{7\pi n}{4}\right) = A \cos\left(\left(\frac{8\pi - \pi}{4}\right)n\right) \\ &= A \cos\left(2\pi n - \left(\frac{\pi n}{4}\right)\right) \end{aligned}$$

As $n \in \mathbb{Z}$ $\cos(2\pi n - \theta) = \cos \theta$

$$x[n] = A \cos\left(\frac{\pi n}{4}\right)$$

$$= x[n] \text{ for } f_0 = 1 \text{ KHz}$$

similarly graph for $f_0 = 6 \text{ KHz} \approx f_0 = 2 \text{ KHz}$

$$f_0 = 5 \text{ KHz} \approx f_0 = 3 \text{ KHz}$$

So the similarity of plots is due to cos difference identity [i.e. $\cos(2\pi - \theta) = \cos \theta$]

The frequency of analog sinusoids = f_0

But the frequency of sampled sinusoids = $\frac{f_0}{f_s} \neq f_0$

So the sampled sinusoids do not reflect the true frequency.