EP 1027: Maxwell's Equations and Electromagnetic Waves

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Lecture 14

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Agenda

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- Review of Radiation emitted from point charges: Retarded potential, Liénard-Wiechert Potential, Radiation fields and emitted power pattern

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- Half-wave antenna

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- Half-wave antenna
- Quiz 3

References/Readings

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- Griffiths, D.J., **Introduction to Electrodynamics, Ch. 10, 11**

Radiation from a point charge: Retarded potential

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- When point charge is at rest or uniform motion, **E** and **B** fields are dragged along as if rigidly attached to the charge

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- When a point charge accelerates, **E** and **B** fields can't keep up and pieces of EM fields "break away" in the form of spherical wave-pulse (radiation)

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- Retarded potentials: Solution to Maxwell equations for charge distribution w/ arbitrary motion
- Notation: Combine $\Phi, \mathbf{A}$ into a four-component vector $A^\mu = (A^0, A^1, A^2, A^3) = \left(\frac{\Phi}{c}, \mathbf{A}\right)$ and combine $\rho, \mathbf{j}$ into $j^\mu = (\rho c, \mathbf{j})$

Retarded potentials for a general charge-current distribution

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- In Lorenz gauge

$$\frac{1}{c^2}\frac{\partial \Phi}{\partial t} + \boldsymbol{\nabla} \cdot \mathbf{A} = \partial_\mu A^\mu = 0,$$

the Maxwell equations in potential formulation look like,

$$\Box A^\mu(t, \mathbf{x}) = \frac{j^\mu(t, \mathbf{x})}{c^2 \varepsilon_0}, \quad \Box = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \boldsymbol{\nabla}^2$$

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- Solution: *Retarded* potentials
$$A^\mu(t,\mathbf{x}) = \frac{1}{4\pi\varepsilon_0}\frac{1}{c^2}\int d^3\mathbf{x}'\frac{j^\mu(t',\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}, t' = t - \frac{|\mathbf{x}-\mathbf{x}'|}{c}.$$
$t' = t - \frac{|\mathbf{x}-\mathbf{x}'|}{c}$ is called the *retarded* time.

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$t' = t - \frac{|\mathbf{x}-\mathbf{x}'|}{c}$ is called the *retarded* time.
- Not the unique solution: There are *advanced* potentials with $t' = t + \frac{|\mathbf{x}-\mathbf{x}'|}{c}$. But these advanced solutions are unphysical; violate causality.

Retarded potentials for a point charge: Liénard-Wiechert Potential

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- For a point charge,

$$\rho(t, \mathbf{x}) = q\,\delta^3(\mathbf{x} - \zeta(t)),$$
$$\mathbf{j}(t, \mathbf{x}) = q\,\mathbf{v}(t)\,\delta^3(\mathbf{x} - \zeta(t)),\ \mathbf{v}(t) = \dot{\zeta}(t).$$

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- Retarded potentials: Liénard-Wiechert potentials,

$$A^\mu(t,\mathbf{x}) = \frac{q}{4\pi\varepsilon_0}\frac{v^\mu(t')}{c^2}\frac{1}{1-\frac{\hat{\mathbf{n}}\cdot\mathbf{v}}{c}}\left.\frac{1}{|\mathbf{x}-\mathbf{x}'|}\right|_{\mathbf{x}'=\zeta(t')},$$

$$v^\mu(t') = \left(c,\dot{\zeta}(t')\right),\quad \hat{\mathbf{n}}(t) = \left.\frac{\mathbf{x}-\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|}\right|_{\mathbf{x}'=\zeta(t')}$$

Note here, $t' = t - \frac{|\mathbf{x}-\mathbf{x}'|}{c}$. (Refer to supplementary material for lecture 13 for derivation).

Radiation from a point charge: Radiation fields

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- EM fields,

$$\mathbf{E} = -\nabla\Phi - \frac{\partial \mathbf{A}}{\partial t}, \quad \mathbf{B} = \nabla \times \mathbf{A}$$

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$$\mathbf{E} = -\boldsymbol{\nabla}\Phi - \frac{\partial \mathbf{A}}{\partial t}, \quad \mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A}$$

- We need,

$$\frac{\partial t'}{\partial t} = \frac{1}{1 - \frac{\hat{\mathbf{n}} \cdot \mathbf{v}(t')}{c}}, \qquad \nabla t' = -\frac{\hat{\mathbf{n}}/c}{1 - \frac{\hat{\mathbf{n}} \cdot \mathbf{v}(t')}{c}}.$$

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- Sample computation,

$$\begin{aligned}
\frac{\partial \mathbf{A}}{\partial t} &= \frac{\partial \mathbf{A}}{\partial t'}\frac{\partial t'}{\partial t} \\
&= \frac{\partial}{\partial t'}\left[\frac{q}{4\pi\varepsilon_0}\frac{\mathbf{v}(t')}{c^2}\frac{1}{1 - \frac{\hat{\mathbf{n}}\cdot\mathbf{v}}{c}}\frac{1}{|\mathbf{x} - \mathbf{x}'|}\bigg|_{\mathbf{x}'=\zeta(t')}\right]\left(\frac{1}{1 - \frac{\hat{\mathbf{n}}\cdot\mathbf{v}(t')}{c}}\right) \\
&= \frac{q}{4\pi\varepsilon_0 c^2}\frac{1}{|\mathbf{x} - \mathbf{x}'|}\left[\frac{\mathbf{a}(t')}{\left(1 - \frac{\hat{\mathbf{n}}\cdot\mathbf{v}}{c}\right)^2} + \mathbf{v}(t')\frac{\hat{\mathbf{n}}\cdot\mathbf{a}(t')/c^2}{\left(1 - \frac{\hat{\mathbf{n}}\cdot\mathbf{v}}{c}\right)^3}\right] + O\left(\frac{1}{|\mathbf{x} - \mathbf{x}'|^2}\right)
\end{aligned}$$

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- Similarly,

$$\begin{aligned}
\nabla\Phi &= \nabla\left(\frac{q}{4\pi\varepsilon_0}\frac{1}{1-\frac{\hat{\mathbf{n}}\cdot\mathbf{v}}{c}}\left.\frac{1}{|\mathbf{x}-\mathbf{x}'|}\right|_{\mathbf{x}'=\zeta(t')}\right) \\
&= \frac{q}{4\pi\varepsilon_0}\frac{1}{|\mathbf{x}-\mathbf{x}'|}\nabla\left(\frac{1}{1-\frac{\hat{\mathbf{n}}\cdot\mathbf{v}}{c}}\right) + O\left(\frac{1}{|\mathbf{x}-\mathbf{x}'|^2}\right) \\
&= -\frac{q}{4\pi\varepsilon_0 c^2}\frac{\hat{\mathbf{n}}\cdot\mathbf{a}(t')}{\left(1-\frac{\hat{\mathbf{n}}\cdot\mathbf{v}}{c}\right)^3|\mathbf{x}-\mathbf{x}'|}\hat{\mathbf{n}} + O\left(\frac{1}{|\mathbf{x}-\mathbf{x}'|^2}\right)
\end{aligned}$$

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- Similarly,

$$
\begin{aligned}
\boldsymbol{\nabla}\Phi &= \boldsymbol{\nabla}\left(\frac{q}{4\pi\varepsilon_0}\frac{1}{1-\frac{\hat{\mathbf{n}}\cdot\mathbf{v}}{c}}\;\frac{1}{|\mathbf{x}-\mathbf{x}'|}\bigg|_{\mathbf{x}'=\zeta(t')}\right) \\
&= \frac{q}{4\pi\varepsilon_0}\frac{1}{|\mathbf{x}-\mathbf{x}'|}\boldsymbol{\nabla}\left(\frac{1}{1-\frac{\hat{\mathbf{n}}\cdot\mathbf{v}}{c}}\right)+O\left(\frac{1}{|\mathbf{x}-\mathbf{x}'|^2}\right) \\
&= -\frac{q}{4\pi\varepsilon_0 c^2}\frac{\hat{\mathbf{n}}\cdot\mathbf{a}(t')}{\left(1-\frac{\hat{\mathbf{n}}\cdot\mathbf{v}}{c}\right)^3|\mathbf{x}-\mathbf{x}'|}\hat{\mathbf{n}}+O\left(\frac{1}{|\mathbf{x}-\mathbf{x}'|^2}\right)
\end{aligned}
$$

- Thus,

$$
\begin{aligned}
\mathbf{E}_{\mathrm{rad}} &= \frac{q}{4\pi\varepsilon_0 c^2}\frac{1}{|\mathbf{x}-\mathbf{x}'|}\left[\frac{\hat{\mathbf{n}}\cdot\mathbf{a}(t')}{\left(1-\frac{\hat{\mathbf{n}}\cdot\mathbf{v}}{c}\right)^3}\hat{\mathbf{n}}-\frac{\mathbf{a}(t')}{\left(1-\frac{\hat{\mathbf{n}}\cdot\mathbf{v}}{c}\right)^2}-\mathbf{v}(t')\frac{\hat{\mathbf{n}}\cdot\mathbf{a}(t')/c^2}{\left(1-\frac{\hat{\mathbf{n}}\cdot\mathbf{v}}{c}\right)^3}\right] \\
&= \frac{q}{4\pi\varepsilon_0 c^2}\frac{1}{|\mathbf{x}-\mathbf{x}'|}\left[\frac{\hat{\mathbf{n}}\times\left\{\left(\hat{\mathbf{n}}-\frac{\mathbf{v}(t')}{c}\right)\times\mathbf{a}(t')\right\}}{\left(1-\frac{\hat{\mathbf{n}}\cdot\mathbf{v}}{c}\right)^3}\right].
\end{aligned}
$$

Radiation from a point charge: Radiation fields, Poynting vector, Power Emitted

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- Similarly,

$$\mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A}$$
$$= \underbrace{\frac{q}{4\pi\varepsilon_0 c^2} \frac{1}{|\mathbf{x} - \mathbf{x}'|} \left[-\frac{\hat{\mathbf{n}} \times \mathbf{a}(t')/c}{\left(1 - \frac{\hat{\mathbf{n}}\cdot\mathbf{v}}{c}\right)^2} - \frac{\hat{\mathbf{n}} \times \mathbf{v}(t')}{\left(1 - \frac{\hat{\mathbf{n}}\cdot\mathbf{v}}{c}\right)^3} \frac{\hat{\mathbf{n}} \cdot \mathbf{a}(t')}{c} \right]}_{\mathbf{B}_{\text{rad}}} + O\left(\frac{1}{|\mathbf{x} - \mathbf{x}'|^2} \right.$$

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$$\mathbf{B}_{\text{rad}} = \hat{\mathbf{n}} \times \mathbf{E}_{\text{rad}}$$

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- Thus

$$\mathbf{B}_{\mathbf{rad}} = \hat{\mathbf{n}} \times \mathbf{E}_{\mathbf{rad}}$$

- Poynting vector

$$\mathbf{S} = \frac{1}{\mu_0}\left(\mathbf{E}_{\mathbf{rad}} \times \mathbf{B}_{\mathbf{rad}}\right).$$

$$\mathbf{S} = \hat{\mathbf{n}}\frac{E_{rad}^2}{\mu_0} \propto \frac{q^2 a'^2 \sin^2\theta\,\hat{\mathbf{n}}}{|\mathbf{x}-\mathbf{x}'(t)|^2}, |\mathbf{v}| \ll c.$$

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Radiation fields in general

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- Back to generic expression

$$A^\mu(t,\mathbf{x}) = \frac{1}{4\pi\varepsilon_0}\frac{1}{c^2}\int d^3\mathbf{x}'\frac{j^\mu(t',\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}, t' = t - \frac{|\mathbf{x}-\mathbf{x}'|}{c}.$$

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- Far zone approximation: $|\mathbf{x}-\mathbf{x}'| \approx |\mathbf{x}| - \hat{\mathbf{x}}\cdot\mathbf{x}'$

$$A^\mu(t,\mathbf{x}) \approx \frac{1}{4\pi\varepsilon_0 c^2}\frac{1}{|\mathbf{x}|}\int d^3\mathbf{x}' j^\mu(t - \frac{|\mathbf{x}|}{c} + \frac{\hat{\mathbf{x}}\cdot\mathbf{x}'}{c}, \mathbf{x}'), \quad t' = t - \frac{|\mathbf{x}-\mathbf{x}'|}{c}$$

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- From this,

$$\mathbf{B} = \nabla\times\mathbf{A} \approx \frac{1}{4\pi\varepsilon_0 c^2}\left[\cancelto{\textit{falls off}}{\nabla\frac{1}{|\mathbf{x}|}} \times \int d^3\mathbf{x}'\mathbf{j} + \frac{\int d^3\mathbf{x}'\,\boldsymbol{\nabla}\times\mathbf{j}(t - \frac{|\mathbf{x}|}{c} + \frac{\hat{\mathbf{x}}\cdot\mathbf{x}'}{c},\mathbf{x}')}{|\mathbf{x}|}\right]$$

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- Need to use,

$$\frac{\partial}{\partial x}[\mathbf{j}(t-\frac{|\mathbf{x}|}{c}+\frac{\hat{\mathbf{x}}\cdot\mathbf{x}'}{c},\mathbf{x}')]=\frac{1}{c}\frac{\partial\mathbf{j}}{\partial t}\frac{\partial\left(|\mathbf{x}|-\hat{x}\cdot\mathbf{x}'\right)}{\partial x}$$

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- Far zone approximation:

$$\mathbf{B}\approx-\hat{\mathbf{x}}\times\frac{\partial\mathbf{A}}{c\partial t};\quad \mathbf{E}\approx-\frac{1}{c}\frac{\partial\mathbf{A}}{\partial t}-\hat{\mathbf{x}}\frac{\partial\Phi}{c\partial t}.$$

Radiation fields in general

- Need to use,

$$\frac{\partial}{\partial x}[\mathbf{j}(t - \frac{|\mathbf{x}|}{c} + \frac{\hat{\mathbf{x}} \cdot \mathbf{x}'}{c}, \mathbf{x}')] = \frac{1}{c}\frac{\partial \mathbf{j}}{\partial t}\frac{\partial\left(|\mathbf{x}| - \hat{x} \cdot \mathbf{x}'\right)}{\partial x}$$

- Far zone approximation:

$$\mathbf{B} \approx -\hat{\mathbf{x}} \times \frac{\partial \mathbf{A}}{c\partial t}; \quad \mathbf{E} \approx -\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} - \hat{\mathbf{x}}\frac{\partial \Phi}{c\partial t}.$$

- Use Lorenz gauge,

$$\mathbf{E} = \hat{\mathbf{x}} \times \left(\hat{\mathbf{x}} \times \frac{\partial \mathbf{A}}{\partial \mathbf{t}}\right) = -\hat{\mathbf{x}} \times \mathbf{B}$$

Half-wave antenna

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- $$I = I_0 \cos \frac{2\pi z}{\lambda} \cos \omega t$$

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- $$I = I_0 \cos \frac{2\pi z}{\lambda} \cos \omega t$$
- $$A_z = \frac{2I_0}{|\mathbf{x}|\omega} \cos(\omega t - r\omega/c) \cos\left(\frac{\frac{1}{2}\pi \cos\theta}{\sin^2\theta}\right),$$
$$B = \frac{2I_0}{|\mathbf{x}|c} \sin(\omega t - r\omega/c)...$$

Half-wave antenna

- Size is half the wavelength of the radio waves
- $$I = I_0 \cos\frac{2\pi z}{\lambda}\cos\omega t$$
- $$A_z = \frac{2I_0}{|\mathbf{x}|\omega}\cos(\omega t - r\omega/c)\cos\left(\frac{\frac{1}{2}\pi\cos\theta}{\sin^2\theta}\right),$$
  $$B = \frac{2I_0}{|\mathbf{x}|c}\sin(\omega t - r\omega/c)\ldots$$
- Poynting vector,
  $$\mathbf{S}(\theta) = \frac{I_0}{2\pi c|\mathbf{x}|^2}\cos^2\left(\frac{\frac{1}{2}\pi\cos\theta}{\sin^2\theta}\right)\hat{\mathbf{x}}$$

Quiz:

Quiz:

1. Express the wave $f(t,x) = A\sin(\omega t - kx)$ in complex notation.
2. In vacuum, EM waves are transverse with $\mathbf{E}$ and $\mathbf{B}$ perpendicular to each other. This can be shown from
   (a) Gauss law, (b) Ampere's law, (c) $\nabla \cdot \mathbf{B} = 0$, (d) Faraday-Lenz law
3. Pressure applied by EM waves incident on a surface is arises due to forces applied by the oscillating EM fields of the EM wave on electrons in the wall material. This pressure is along which force component:
   (a) $q\mathbf{E}$, (b) $\frac{q}{c}\mathbf{v} \times \mathbf{B}$, (c) $q\mathbf{E} + \frac{q}{c}\mathbf{v} \times \mathbf{B}$, (d) $\mathbf{I} \times \mathbf{B}$
4. For an Ohmic conductor, on which timescale any injected volume charge dissipates i.e. flows out to the surface/edges:
   (a) 0, (b) $\frac{\varepsilon}{\sigma}$, (c) $\frac{\mu\epsilon}{\sigma}$, (d) $\frac{1}{\omega}$
5. Skin Effect: As we increase ω, the EM wave incident on a conductor, penetrates :
   (a) deeper, (b) lesser, (c) about the same

Quiz:

Quiz:

6. Cauchy dispersion formula holds for
(a) Normal dispersion (b) Anomalous dispersion (c) General Case

7. Which frequency is not allowed in a rectangular wave guide:
(a)ω_{00}(b)ω_{10}(c)ω_{01} (d) ω_{11}

8. For reflection at conducting surface, the Fresnel equations although formally identical to the dielectric, are different due to...

9. Poynting vector in a dielectric is
(a) $\frac{1}{\mu_0}\mathbf{E}\times\mathbf{B}$ (b) $\frac{1}{c^2}\mathbf{D}\times\mathbf{B}$ (c) $\mathbf{E}\times\mathbf{H}$ (d) $\mathbf{D}\times\mathbf{H}$