EP 1027: Maxwell's Equations and Electromagnetic Waves

Instructor: Shubho Roy[1]
(Dept. of Physics)

Lecture 9 & 10

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[1]Email: sroy@iith.ac.in, Office: C 313 D, Office hrs: Walk in/ By Email Appointment

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- Wave equation basics: d'Alembert's solution, Bernoulli's solution, superposition

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- EM waves in matter
- Laws of reflection and refraction for waves
- Fresnel Equations

References/Readings

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- Griffiths, D.J., **Introduction to Electrodynamics, Ch. 9**

Wave mechanics : Basics

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- Generic Wave equation (in 1D),

$$\left(\frac{\partial^2}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) f(x,t) = 0,$$

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$$f(x,t) = g(x-ct) + h(x+ct).$$

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- Bernoulli's solution: Sinusoidal

$$f(x,t) = A\cos(kx-\omega t+\Delta_1)+B\cos(kx+\omega t+\Delta_2), \omega = kc.$$

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- c is the speed of light in vacuum, light is an EM wave!

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- Wave front is planar & perpendicular to direction of propagation

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  k is the wave vector, normal to wave fronts.

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- Faraday law ($\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$) implies,

$$\mathbf{B} = \hat{\mathbf{k}} \times \frac{\mathbf{E}}{c}.$$

EM waves in vacuum: Energy and Momentum

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- Using, $\mathbf{B} = \hat{\mathbf{k}} \times \frac{\mathbf{E}}{c}$, can show,

$$\frac{1}{2\mu_0}\mathbf{B}^2 = \frac{\epsilon_0}{2}\mathbf{E}^2, u_{\mathbf{E}} = u_{\mathbf{B}},$$

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- Energy flux density (per unit cross-sectional area, per unit time), also called intensity

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- Momentum density, $\boldsymbol{\pi}_{EM}$

$$\boldsymbol{\pi}_{EM} = \frac{1}{c^2}\mathbf{S}, \boldsymbol{\pi}_{EM} = \frac{u}{c}.$$

EM waves in vacuum: Pressure of radiation

[2] Note that $\langle \mathbf{E}^2 \rangle = \frac{1}{2}\mathbf{E}_0^2$, since $\langle \sin^2\theta \rangle = \frac{1}{2}$.

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- Absorption of EM waves: Momentum transferred to the absorber,

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- Microscopic level, **E** and **B** push/pull on charges in the wall.

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EM waves in matter

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- Maxwell Equations in absence of sources in matter,

$$\boldsymbol{\nabla} \cdot \mathbf{D} = 0, \qquad \boldsymbol{\nabla} \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0,$$

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- Linear homogenous media: Constitutive relations,

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- Speed of EM waves,

$$v = \frac{1}{\sqrt{\mu\epsilon}} = \frac{c}{n},$$

n is called the refractive index.

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- Energy density in media,

$$
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u &= \frac{1}{2}\mathbf{D}\cdot\mathbf{H} + \frac{1}{2}\mathbf{B}\cdot\mathbf{H} \\
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- Boundary Conditions at the interface of media:

$$\epsilon^1 E^1_\perp = \epsilon^2 E^2_\perp, \qquad \mathbf{E}^1_\parallel = \mathbf{E}^2_\parallel,$$

$$B^1_\perp = B^2_\perp, \qquad \frac{1}{\mu_1}\mathbf{B}^1_\parallel = \frac{1}{\mu_2}\mathbf{B}^2_\parallel.$$

Reflection and Transmission of EM waves

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- Monochromatic waves incident on the interface from left (fig. 9.14 of Griffiths' text)

$$\mathbf{E}_I = \tilde{\mathbf{E}}_{0I} e^{i(\mathbf{k}_I \cdot \mathbf{x} - \omega t)}, \ \mathbf{B}_I = \hat{\mathbf{k}}_I \times \frac{\mathbf{E}_I}{v_1}.$$

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- Frequency same,

$$\omega = |\mathbf{k}|v,$$

$$\Longrightarrow \ |\mathbf{k}_I| = |\mathbf{k}_R| = \frac{\omega}{v_1}, \qquad |\mathbf{k}_T| = \frac{\omega}{v_2} = \frac{v_1}{v_2}\,|\mathbf{k}_I| = \frac{n_2}{n_1}\,|\mathbf{k}_I|\,.$$

Reflection and Transmission of waves at an interface

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- We take the interface of the media to be the xy-plane i.e. $z = 0$. Then,

$$\mathbf{k}\cdot\mathbf{x}\big|_{z=0} = \left(k_x\,x + k_y\,y + k_z\,z\right)_{z=0} = k_x\,x + k_y\,y = \mathbf{k}^{\|}\cdot\mathbf{x}$$

  where we have introduced $\mathbf{k}^{\|}$ as the part of the wave-vector parallel to the interface

$$\mathbf{k}^{\|} = \mathbf{k} - k_z\hat{\mathbf{z}} = k_x\hat{\mathbf{x}} + k_y\hat{\mathbf{y}}.$$

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$$(\ldots)\,e^{i(\mathbf{k}_I\cdot\mathbf{x}-\omega t)} + (\ldots)\,e^{i(\mathbf{k}_R\cdot\mathbf{x}-\omega t)}\Big|_{z=0} = (\ldots)\,e^{i(\mathbf{k}_T\cdot\mathbf{x}-\omega t)}\Big|_{z=0}\ \forall\ x, y, t,$$

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- First law: All three wave vectors lie in a plane (plane of incidence containing $\mathbf{k}_I$ and $\hat{\mathbf{z}}$).

$$|\mathbf{k}_I| \sin \theta_I = |\mathbf{k}_R| \sin \theta_R = |\mathbf{k}_T| \sin \theta_T.$$

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- Second law: Angle of reflection = Angle of incidence,
$$\theta_I = \theta_R .$$

Reflection and Transmission of waves at an interface

- From last slide
$$\Rightarrow \mathbf{k}_I^{\parallel} = \mathbf{k}_R^{\parallel} = \mathbf{k}_T^{\parallel}$$
- First law: All three wave vectors lie in a plane (plane of incidence containing $\mathbf{k}_I$ and $\hat{\mathbf{z}}$).
$$|\mathbf{k}_I| \sin\theta_I = |\mathbf{k}_R| \sin\theta_R = |\mathbf{k}_T| \sin\theta_T.$$
- Second law: Angle of reflection = Angle of incidence,
$$\theta_I = \theta_R.$$
- Third law: Snell's law of Refraction
$$\frac{\sin\theta_I}{\sin\theta_T} = \frac{|\mathbf{k}_T|}{|\mathbf{k}_I|} = \frac{v_1}{v_2} = \frac{n_2}{n_1}$$

Satisfying boundary conditions

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- Boundary conditions relates amplitudes

$$\epsilon^1 \left(\tilde{E}^3_{0I} + \tilde{E}^3_{0R} \right) = \epsilon^2 \tilde{E}^3_{0T},$$

$$\tilde{B}^3_{0I} + \tilde{B}^3_{0R} = \tilde{B}^3_{0T},$$
$$\tilde{E}^{1,2}_{0I} + \tilde{E}^{1,2}_{0R} = \tilde{E}^{1,2}_{0T},$$
$$\frac{1}{\mu_1} \left(\tilde{B}^{1,2}_{0I} + \tilde{B}^{1,2}_{0R} \right) = \frac{1}{\mu_2} \tilde{B}^{1,2}_{0T}.$$

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- Case I: "In plane" polarization,

$$\epsilon^1 \left(- \left| \tilde{\mathbf{E}}_{0I} \right| \sin\theta_I + \left| \tilde{\mathbf{E}}_{0R} \right| \sin\theta_R \right) = -\epsilon^2 \left| \tilde{\mathbf{E}}_{0T} \right| \sin\theta_T$$

$$\left| \tilde{\mathbf{E}}_{0I} \right| \cos\theta_I + \left| \tilde{\mathbf{E}}_{0R} \right| \cos\theta_R = \left| \tilde{\mathbf{E}}_{0T} \right| \cos\theta_T$$

$$\frac{1}{\mu_1 v_1} \left(\left| \tilde{\mathbf{E}}_{0I} \right| - \left| \tilde{\mathbf{E}}_{0R} \right| \right) = \frac{1}{\mu_2 v_2} \left| \tilde{\mathbf{E}}_{0T} \right|$$

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$$\left|\tilde{\mathbf{E}}_{0I}\right| - \left|\tilde{\mathbf{E}}_{0R}\right| = \beta \left|\tilde{\mathbf{E}}_{0T}\right|, \beta = \frac{\mu_1 v_1}{\mu_2 v_2} = \frac{\mu_1 n_2}{\mu_2 n_1}$$

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- Fresnel's Equations for "in plane" component of polarization

$$\frac{\left|\tilde{\mathbf{E}}_{0R}\right|}{\left|\tilde{\mathbf{E}}_{0I}\right|} = \frac{\alpha - \beta}{\alpha + \beta},$$

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$$\alpha = \beta,$$

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- Next class: EM waves incident on conductors

Quiz

1. Point out where rationalized units are not used

   $(a)\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{|\mathbf{x}|^3}\mathbf{x}, \quad (b)\mathbf{B} = \frac{\mu_0}{4\pi}\oint\frac{Id\mathbf{x}'\times(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^3}, \quad (c)\boldsymbol{\nabla}\cdot\mathbf{E} = 4\pi\rho, \quad (d)\nabla\times\mathbf{B} = \frac{4\pi}{c}\mathbf{j}$

2. Direction of **E**of point charge at origin in spherical polar coordinates is along

   $$(a)\hat{\mathbf{r}}, \qquad (b)\hat{\boldsymbol{\theta}}, \qquad (c)\hat{\boldsymbol{\phi}}, \qquad (d)\hat{\boldsymbol{\rho}}$$

3. Direction of **B** of infinitely long current carrying wire along z-axis in cylindrical coordinates is along

   $$(a)\hat{\boldsymbol{\rho}}, \qquad (b)\hat{\boldsymbol{\phi}}, \qquad (c)\hat{\boldsymbol{z}}, \qquad (d) \quad \rho\hat{\boldsymbol{\rho}} + z\hat{\mathbf{z}}$$

4. Magnetic field $\mathbf{B}(\mathbf{x})$ due to a point charge, q at $\mathbf{x}'$ moving with vel, $\mathbf{v}$

   $$(a)\text{absent}, (b)\text{approx} \propto q\frac{\mathbf{v}}{c}\times\frac{\mathbf{x}}{|\mathbf{x}-\mathbf{v}t|^3}, \ (c)\text{approx} \propto q\frac{\mathbf{v}}{c}\times\frac{\mathbf{x}-\mathbf{v}t}{|\mathbf{x}-\mathbf{v}t|^3}, (d)\text{approx} \propto q\frac{\mathbf{v}}{c}\times\mathbf{E}$$

5. Inside a conductor, in general

   $$(a)\mathbf{E} = 0, \quad (b)\rho = 0, \quad (c)\mathbf{j} = \sigma\mathbf{E} \quad (d)\mathbf{E} = \sigma\mathbf{j}$$

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6. At the boundary of a pair of dielectrics cum paramegnetics which of the following is not continuous

$$(a)\mathbf{E}_{\parallel}, \quad (b)\mathbf{H}_{\parallel} \quad (c)D_{\perp} \quad (d)B_{\perp}$$

7. If **E**, **B** and $\phi, \mathbf{A}$ contain same amount of information, why are they different in number of components?

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