

26) Let us consider the analog sinusoid as

$$x(t) = \cos(\Omega_0 t)$$

The fourier transform of $x(t)$ will be

$$\begin{aligned} X(\Omega) &= \int_{-\infty}^{\infty} e^{-j\Omega t} \cos(\Omega_0 t) dt \\ &= \int_{-\infty}^{\infty} e^{-j\Omega t} \left(\frac{e^{j\Omega_0 t} + e^{-j\Omega_0 t}}{2} \right) dt \\ &= \frac{1}{2} \int_{-\infty}^{\infty} \left[e^{-j(\Omega - \Omega_0)t} + e^{-j(\Omega + \Omega_0)t} \right] dt \end{aligned}$$

$$X(\Omega) = \pi \left[\delta(\Omega - \Omega_0) + \delta(\Omega + \Omega_0) \right]$$

$$\begin{aligned} \text{Now consider } x[n] &= x(nT_s) = \cos\left(\Omega_0 n \cdot \frac{2\pi}{\Omega_s}\right) \\ &= \cos\left(2\pi \frac{\Omega_0}{\Omega_s} n\right) \end{aligned}$$

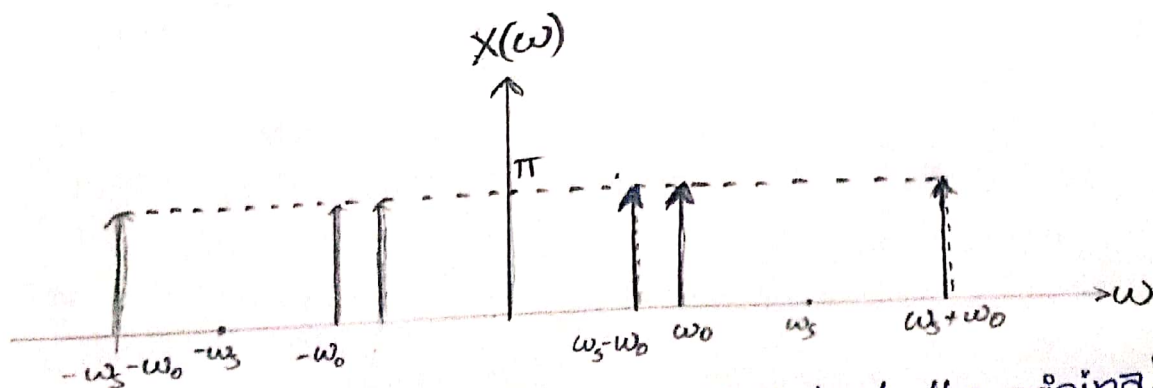
$$x[n] = \cos(\omega_0 n)$$

The fourier transform of $x[n]$ will be

$$X(\omega) = \sum_{k=-\infty}^{\infty} \pi \left[\delta(\omega - \omega_0 + \omega_s k) + \delta(\omega + \omega_0 + \omega_s k) \right] \approx \sum_{n=-\infty}^{\infty} X(-\Omega) \delta(\Omega - n\Omega_s)$$

So it is periodic with period ω_s .

if we consider $\omega_0 > \frac{\omega_s}{2} \Rightarrow \omega_0 - \omega_s < \omega_0$ (so there will be aliasing)



As there is aliasing we cannot construct back the original signal