Principles of Cryptography-I

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Credits: *Adapted from Kurose and Ross textbook on Computer Networking and based on slides from Stefan Savage, Steven Bellovin and a host of others and Internet sources*

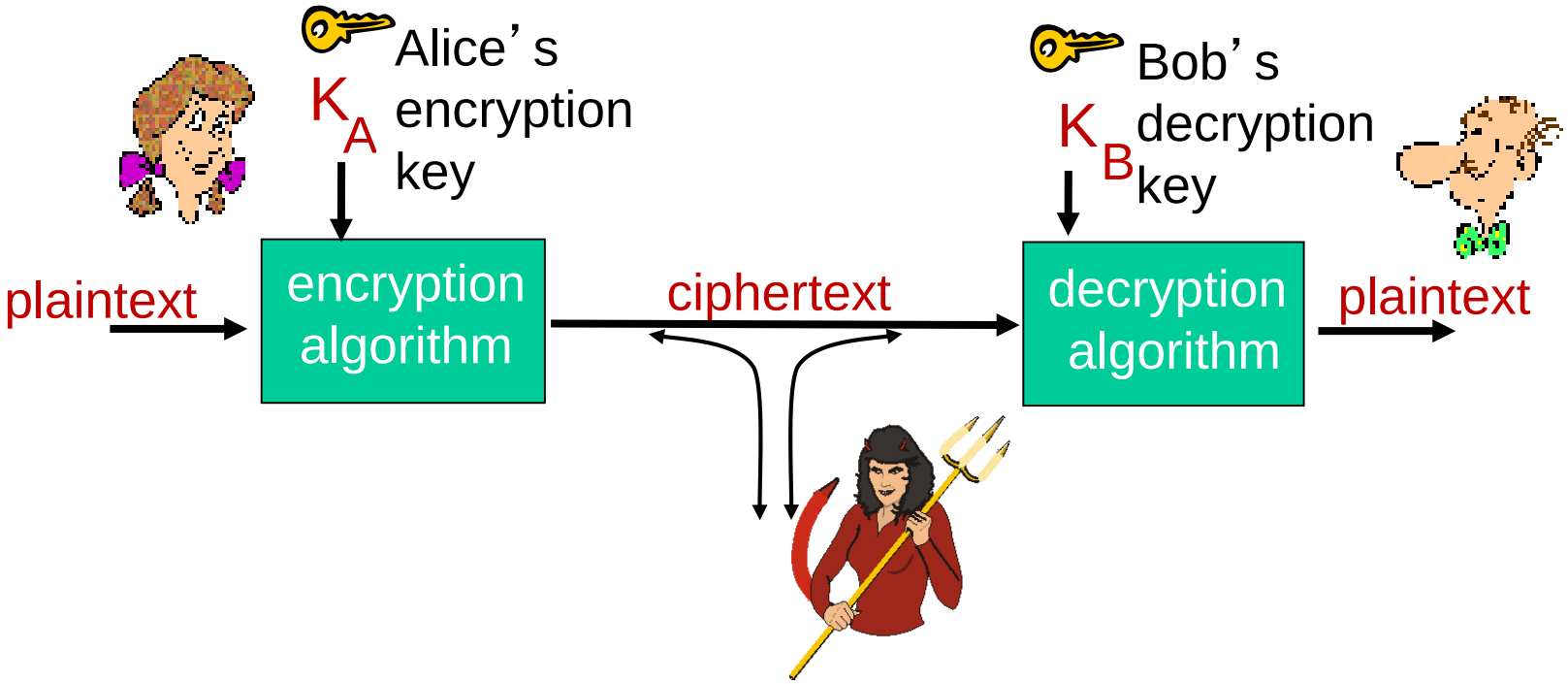
Cryptography?

- Art of writing or solving (secret) codes
- Modern cryptography provides mechanisms for confidentiality, integrity, authentication, non-repudiation, privacy, …
- Very broad subject
- We focus primarily on using it as a (black box) tool for secure communication

Encryption & Decryption

- Plaintext (*m*): unencrypted message to be sent by Alice
  - Binary string of arbitrary length
- Ciphertext (*c*): encrypted version of message by using encryption function E
  - $c = E(m)$
  - *c* is also a binary string (may not be same length as plaintext)
- Bob decrypts c by using decryption function D
  - $m = D(c)$
- A **cipher** is an algorithm for transforming plaintext to/from ciphertext
  - Encryption and decryption functions should be parameterized by a key
  - Only the key is secret, but ciphers are considered public knowledge!

The language of cryptography



m plaintext message

$K_A(m)$ ciphertext, encrypted with key K_A

$m = K_B(K_A(m))$

Simple encryption scheme

substitution cipher: substituting one thing for another

- **monoalphabetic cipher**: substitute one letter for another

```
plaintext:   abcdefghijklmnopqrstuvwxyz
                ↓                     ↓
ciphertext:  mnbvcxzasdfghjklpoiuytrewq
```

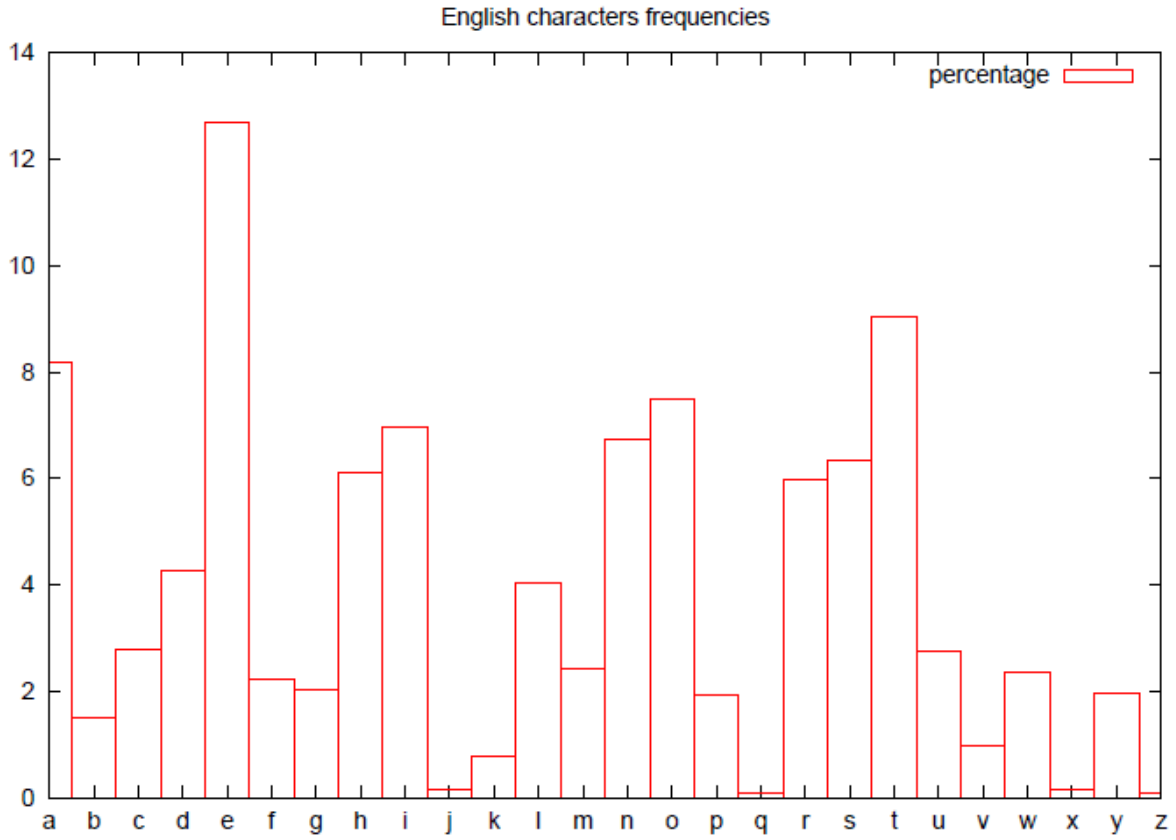
e.g.:

```
Plaintext: bob. i love you. alice
ciphertext: nkn. s gktc wky. mgsbc
```

Encryption key: mapping from set of 26 letters to set of 26 letters

Attacks on monoalphabetic cipher

- Easy to learn patterns
- Frequency analysis



A more sophisticated encryption approach

- n substitution ciphers, $M_1, M_2, \ldots, M_n$
- example, n=4 with cycling pattern of length 5
  - M_1, M_3, M_4, M_3, M_2;
  - for each new plaintext symbol, use subsequent subsitution pattern in cyclic pattern
  - drink: d from M_1, r from M_3, i from M_4, n from M_3, k from M_2

Encryption key: n substitution ciphers, and cyclic pattern

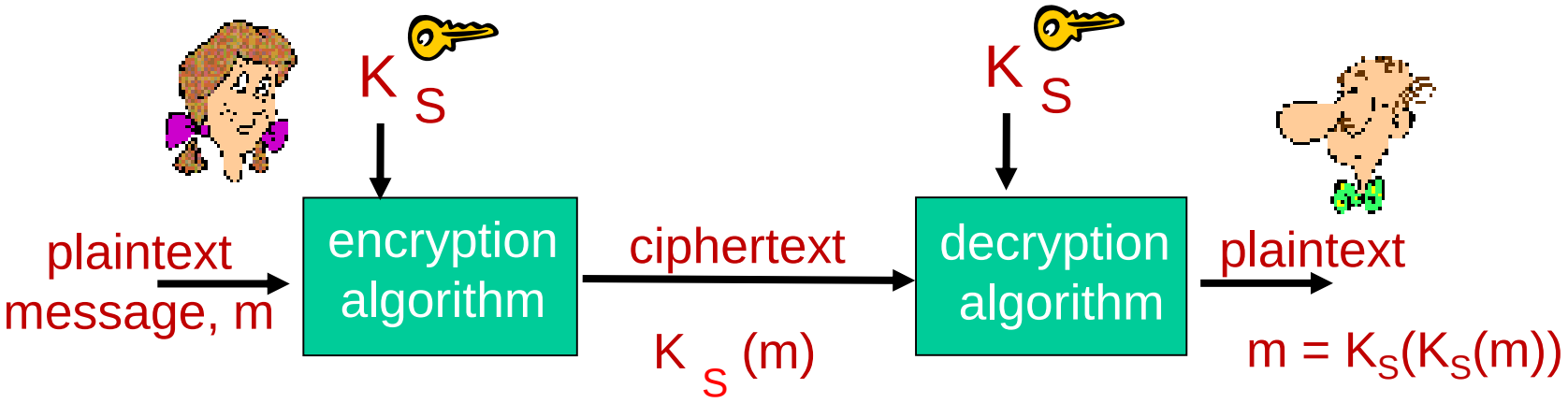
- key need not be just n-bit pattern

Cryptanalysis: Breaking an encryption scheme

- cipher-text only attack:
  Trudy has ciphertext she can analyze
- two approaches:
  - brute force: search through all keys
  - statistical analysis

- known-plaintext attack:
  Trudy has plaintext corresponding to ciphertext
  - e.g., in monoalphabetic cipher, Trudy determines pairings for a,l,i,c,e,b,o,
- chosen-plaintext attack:
  Trudy can get ciphertext for a chosen plaintext

Symmetric key cryptography



symmetric key crypto: Bob and Alice share same (symmetric) key: K_S

- e.g., key is knowing substitution pattern in monoalphabetic substitution cipher

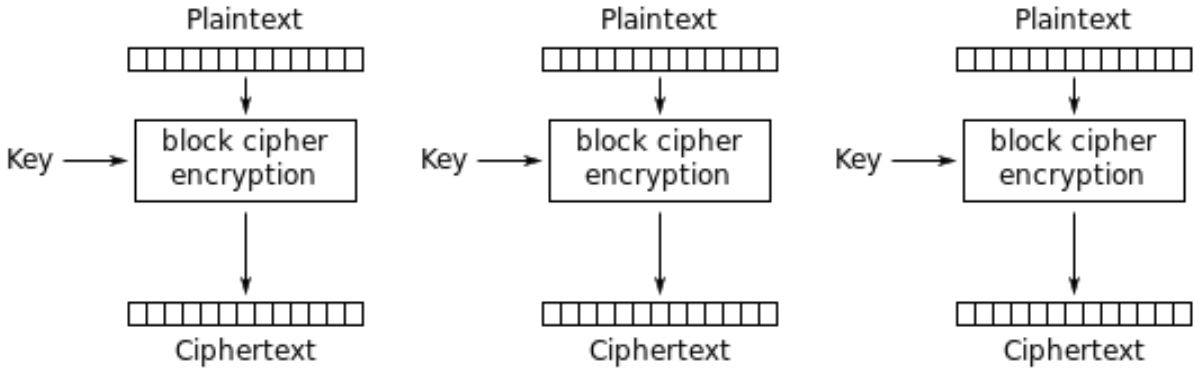
Q: how do Bob and Alice agree on key value?

A: Diffie-Hellman Symmetric Key Exchange Protocol, Public key crypto

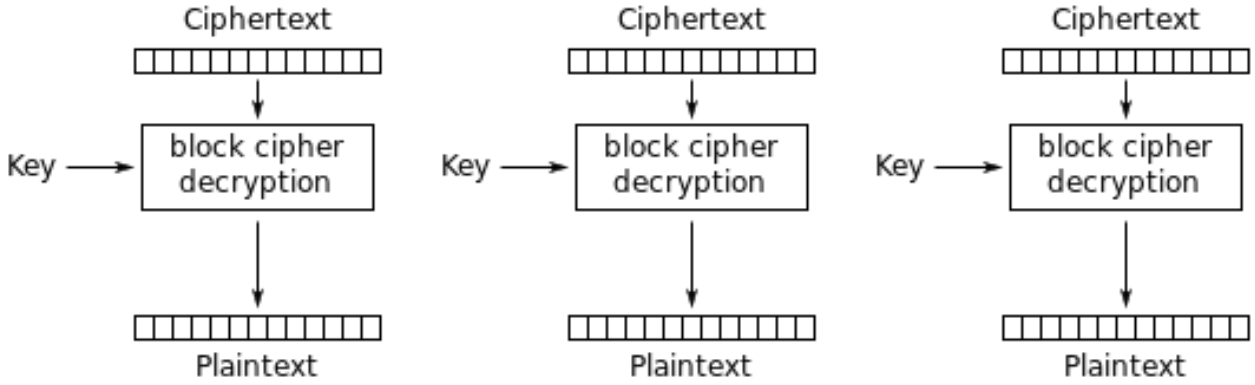
AES: Advanced Encryption Standard

- symmetric-key NIST standard, replaced DES which used 56-bit keys (Nov 2001)
- processes data in 128 bit blocks (Block Cipher)
- 128, 192, or 256 bit keys
- brute force decryption (try each key) taking 1 sec on DES, takes 149 trillion years for AES!

AES: Electronic Codebook (ECB) Mode



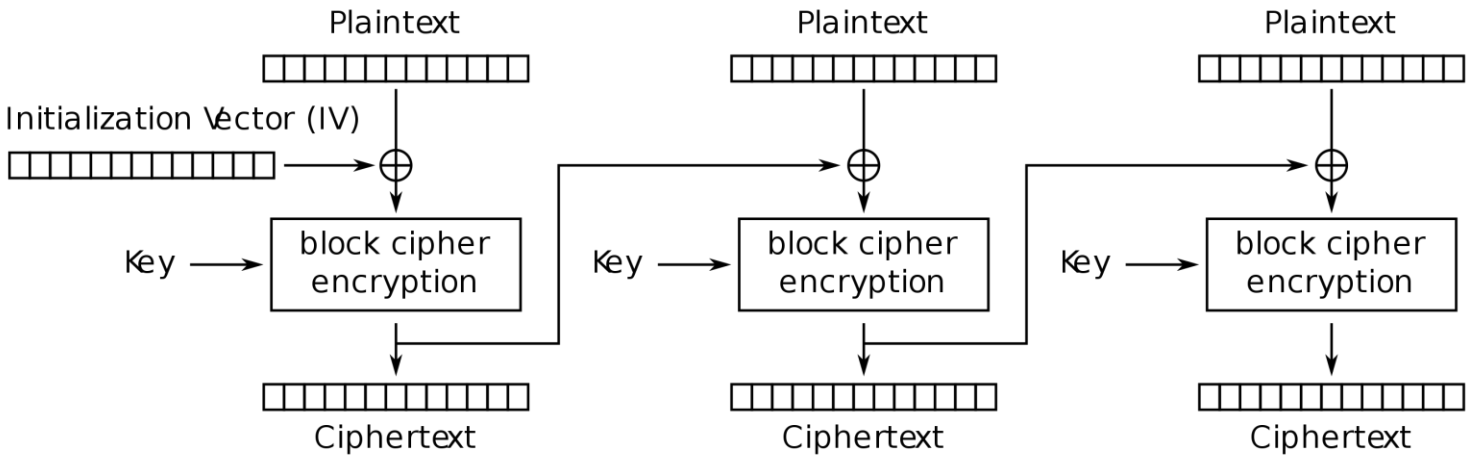
Electronic Codebook (ECB) mode encryption



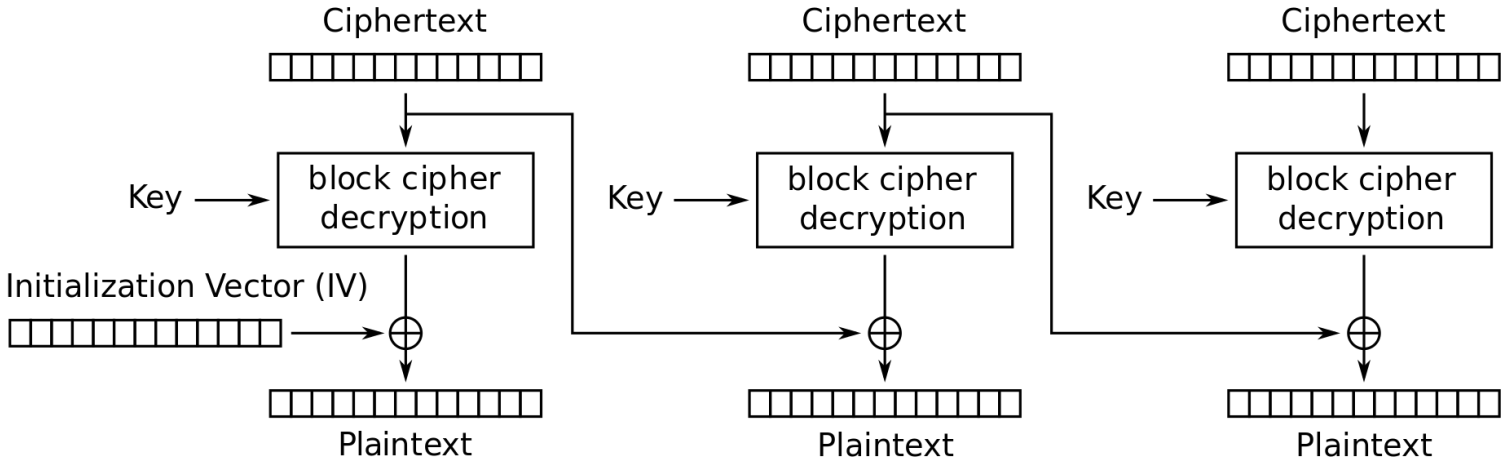
Electronic Codebook (ECB) mode decryption

ECB encrypts identical plaintext blocks into identical ciphertext blocks

AES: Cipher Block Chaining (CBC) Mode



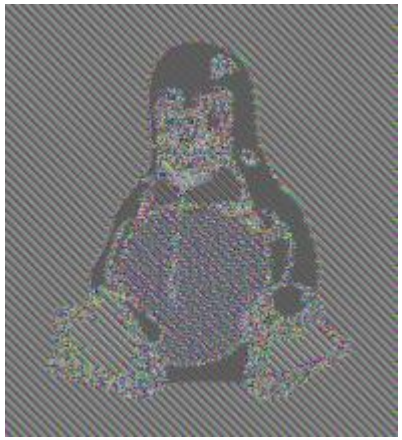
Cipher Block Chaining (CBC) mode encryption



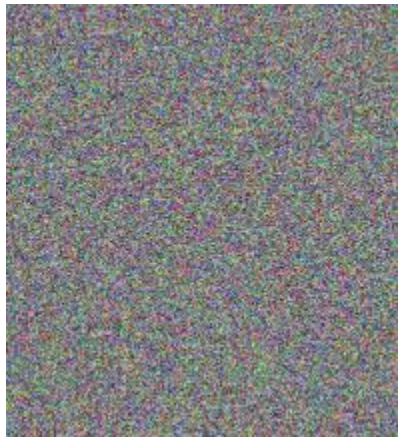
Cipher Block Chaining (CBC) mode decryption

AES, in pictures!

Original image



Encrypted using ECB mode



Modes other than ECB like CBC result in pseudo-randomness

Source: Wikipedia

Public Key Cryptography

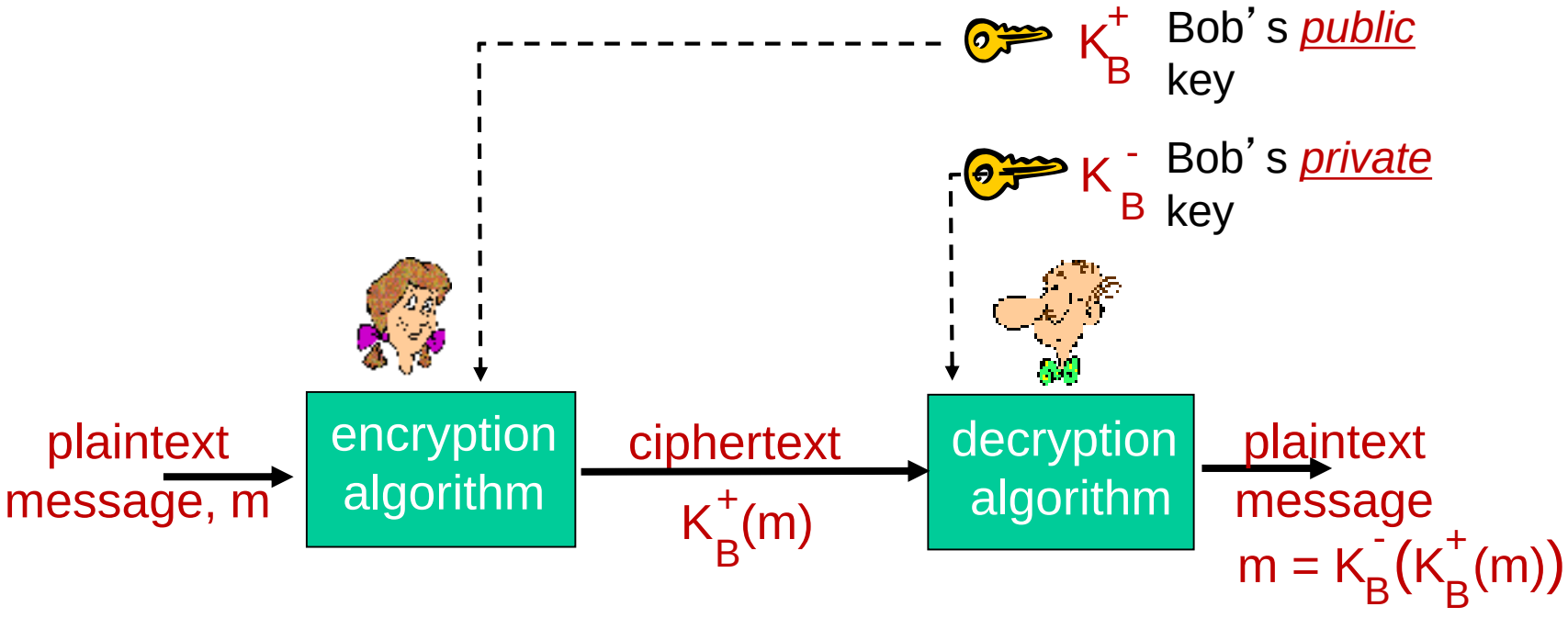
symmetric key crypto

- requires sender, receiver know shared secret key
- Q: how to agree on key in first place (particularly if never "met")?

public key crypto

- radically different approach [Diffie-Hellman76, RSA78]
- sender, receiver do *not* share secret key
- *public* encryption key known to *all*
- *private* decryption key known only to receiver

Public Key Cryptography



Wow - public key cryptography revolutionized 2000-year-old (previously only symmetric key) cryptography!

- similar ideas emerged at roughly same time, independently in US and UK (classified)

Public key encryption algorithms

requirements:

1. need $K_B^+(\cdot)$ and $K_B^-(\cdot)$ such that

$$K_B^-(K_B^+(m)) = m$$

2. given public key K_B^+, it should be impossible to compute private key K_B^-

RSA: Rivest, Shamir, Adelson algorithm

RSA: Creating public/private key pair

1. choose two large prime numbers p, q. (e.g., 1024 bits each)
2. compute $n = pq$, $z = (p-1)(q-1)$
3. choose e (with $e<n$) that has no common factors with z (e, z are "relatively prime").
4. choose d such that $ed-1$ is exactly divisible by z. (in other words: ed mod $z = 1$).
5. *public* key is (n,e). *private* key is (n,d).

K_B^+ K_B^-

RSA: encryption, decryption

0. given (n,e) and (n,d) as computed above

1. to encrypt message m ($<n$), compute

$$c = m^e \bmod n$$

2. to decrypt received bit pattern, c, compute

$$m = c^d \bmod n$$

magic happens!

$$m = \underbrace{(m^e \bmod n)}_{c}{}^d \bmod n$$

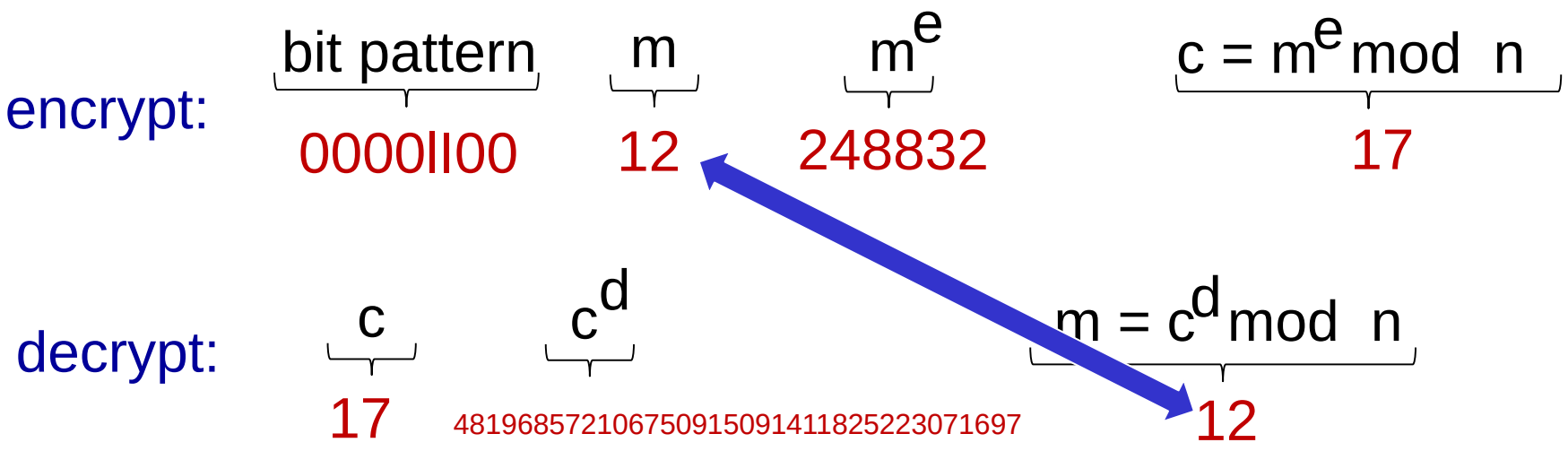
RSA example:

Bob chooses *p=5*, *q=7*. Then *n=35*, *z=24*.

e=5 (so *e*, *z* relatively prime).

d=29 (so *ed-1* exactly divisible by z).

encrypting 8-bit messages.



http://fringe.davesource.com/Fringe/Crypt/RSA/Example.html

Why is RSA secure?

- suppose you know Bob's public key (n,e). How hard is it to determine d in his private key (n,d)?
- essentially need to find factors of n without knowing the two factors p and q
  - n=p*q is easy to calculate but hard to reverse
    - fact: factoring a large number is really hard
    - So, uses relatively large keys (1024 bits) and relies on the high computational cost of factoring large numbers

RSA in practice: session keys

- exponentiation in RSA is computationally intensive
- DES (sym key crypto) is at least 100 times faster than RSA (asym key crypto)
- use public key cryto to establish secure connection, then establish second key – symmetric session key – for encrypting application data faster

session key, K_S

- Bob and Alice use RSA to exchange a symmetric key K_S
- once both have K_S, they use symmetric key cryptography

References

- Crypto Book: http://web.cs.ucdavis.edu/~rogaway/classes/227/spring05/book/main.pdf
- Cryptographic Standards and Guidelines | CSRC (nist.gov)
- Latacora - Cryptographic Right Answers
- A Few Thoughts on Cryptographic Engineering by Matthew Green: https://blog.cryptographyengineering.com/