CS6350: Topics in Combinatorics
Assignment 5

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1. **Let G = (V,E) be a bipartite graph with n vertices and a list S(v) of more than $\log_2 n$ colors associated with each vertex v $\in$ V . Prove that there is a proper coloring of G assigning to each vertex v a color from its list S(v).**

A. Let the bipartitions for the graph be V_1 *and* V_2.
Let S = $\bigcup_{v \in V} S(v)$.
Now distribute each element of S into S_{V_1} and S_{V_2} with equal probability.
For each $v \in V_1$, define $S_{V_1}(v) = S_{V_1} \cap S_v$
Similarly, for each $v \in V_2$, define $S_{V_2}(v) = S_{V_2} \cap S_v$
Then for all $v \in V_1, P(S_{V_1}(v) = \phi) = \left(\frac{1}{2}\right)^{|S(v)|} < \left(\frac{1}{2}\right)^{\log_2 n} = \frac{1}{n}$
Similarly, for all $v \in V_2, P(S_{V_2}(v) = \phi) = \frac{1}{n}$
So,

$$
\begin{aligned}
P(\bigcup_{v \in V_1}(S_{V_1}(v) = \phi)) \cup P(\bigcup_{v \in V_2}(S_{V_2}(v) = \phi)) &= \sum_{v \in V_1} P(S_{V_1}(v) = \phi) + \sum_{v \in V_2} P(S_{V_2}(v) = \phi) \\
&< \sum_{v \in V_1} \frac{1}{n} + \sum_{v \in V_2} \frac{1}{n} \\
&= \sum_{v \in V} \frac{1}{n} \\
&= 1
\end{aligned}
$$

So, we can say that there always exists a vertex coloring where vertices of V_1 only use colors of S_{V_1} and vertices of V_2 only use colors of S_{V_2}. As V_1 and V_2 are bipartities of graph G we can make a proper coloring.
Hence Proved.