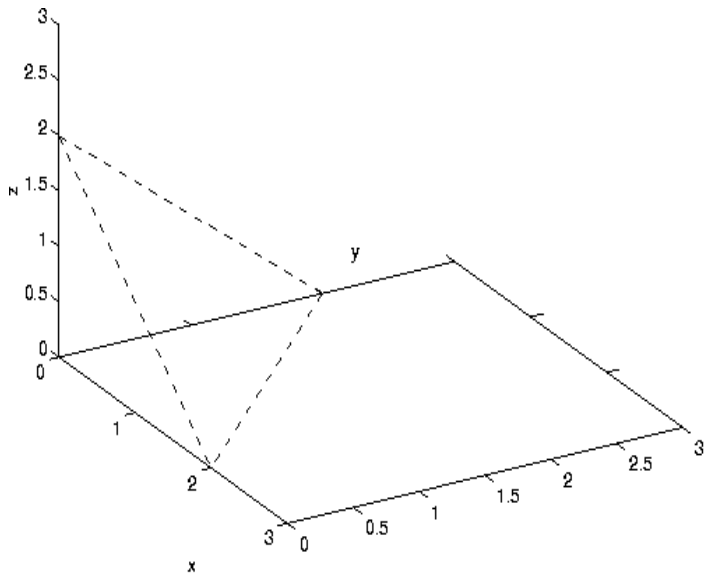
Triple Integrals

1. Evaluate $\iiint_E \sqrt{3x^2+3z^2}\text{dV}$ where E is the solid bounded by $y = 2x^2 + 2z^2$and the plane y=8.

2. Evaluate $\iiint_E ydV$ where E is the region that lies below the plane z=x+2 above the xy-plane and between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

3. A cube has sides of length 4.let one corner be at the origin and the adjacent corners be at the positive x, y and z axes. If the cubes density is proportional to the distance from the xy-plane, find its mass.

4. Consider the triple integral $\iiint_R x+2y\text{dV}$ where R is the tetrahedral region bounded by the planes x=0,y=0,z=0 and x+y+z=2.



5. Find the volume below the surface $z = x^2 + y^2$, above the plane z=0 and inside the cylinder $x^2 + y^2 = 2y$.

6. Find the mass of the 3D region B given by $x^2 + y^2 + z^2 \leq 4$,$x \leq 0$,$y \leq 0$,$z \leq 0$ if the density is equal to xyz.

7. Let E be the solid bounded by the z+x=1, z+y=1 and the first octant. Find $\iiint_E 2x^2yz\text{dV}$.

8. Find the volume that lies inside the sphere $x^2 + y^2 + z^2 = 2$ and outside the cone $x^2 + y^2 = z^2$.

Hints and Answers

1. Hint: $x = rcos\theta$ and $z = rsin\theta$ Ans: $\frac{256\sqrt{3}\pi}{15}$
2. Hint: $0 \leq z \leq x + 2 => 0 \leq z \leq rcos\theta + 2$ Ans: 0
3. Hint: $\iiint_W kz\text{dV}$ Ans: 128k
4. Hint: $0 \leq z \leq 2 - x - y$, $0 \leq x \leq 2 - y$ and $0 \leq y \leq 2$ Ans: $\frac{2}{3}$
5. Hint: $x = rcos\theta$, $y = rsin\theta$ from $x^2 + y^2 = 2y$, $r = 2sin\theta$ $0 \leq r \leq 2sin\theta$ and $0 \leq \theta \leq \pi$ Ans:$\frac{3\pi}{2}$
6. Hint: Spherical coordinates Ans: $\frac{4}{3}$
7. Hint: $0 \leq x \leq 1 - z$, $0 \leq z \leq 1 - y$, $0 \leq y \leq 1$ Ans: $\frac{1}{126}$
8. Hint: Spherical coordinates $\frac{\pi}{4} \leq \phi \leq \frac{3\pi}{4}$ Ans: $\frac{8\pi}{3}$