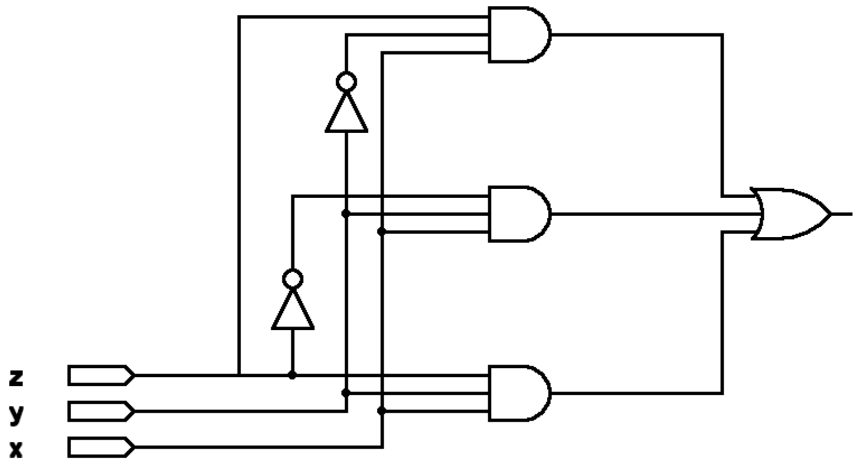
ECE380 Digital Logic

Introduction to Logic Circuits:
Synthesis using AND, OR, and NOT gates

Example logic circuit design

- Assume we want to design a logic circuit with three inputs *x*, *y*, and *z*
- The circuit output should be 1 only when *x*=1 and either *y* or *z* (or both) is 1
  - Three possible combinations
    - x=1, y=0, z=1 => xy'z
    - x=1, y=1, z=0 => xyz'
    - x=1, y=1, z=1 => xyz
- The function could be written as
  - f(x,y,z)=xy'z+xyz'+xyz
    - sum of products form

Example logic circuit design



Example logic circuit design

- Implements f correctly, but is not the simplest such network

$f(x,y,z)=xy'z+xyz'+xyz$

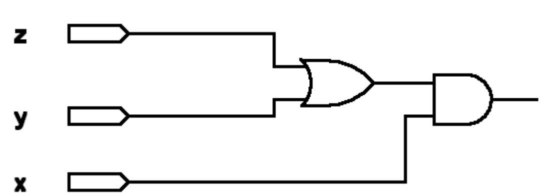
x	y	z	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

$xy'z+xyz'+xyz$

$xy'z+xy$ from 14a

$x(y'z+y)$ from 12a

$x(y+z)$ from 16a



Example logic circuit design

- Obviously, the cost (in terms of gates and connections) of this network is much less than the initial network
- The process of generating a circuit from a stated desired functional behavior is called ***synthesis***
- Generation of AND-OR style networks from a truth table is one of many types of synthesis techniques that we will cover

Logic synthesis

- If a function *f* is described in a truth table, then an expression that generates *f* can by obtained (synthesized) by
  - Considering all rows in the table where $f=1$, or
  - By considering all rows in the table where $f=0$
- This will be an application of the principal of duality

Minterms

- For a function of *n* variables f(a,b,c,...n)
  - A minterm of *f* is a product of *n* literals (variables) in which each variable appears once in either true or complemented form, but not both
    - f(a,b,c) -- minterm examples: abc, a'bc, abc'
    - f(a,b,c) -- invalid examples: ab, c', a'c
  - An *n* variable function has 2^n valid minterms

Minterms

Row number	*x*	*y*	*z*	*Minterm*
0	0	0	0	m_0=x'y'z'
1	0	0	1	m_1=x'y'z
2	0	1	0	m_2=x'yz'
3	0	1	1	m_3=x'yz
4	1	0	0	m_4=xy'z'
5	1	0	1	m_5=xy'z
6	1	1	0	m_6=xyz'
7	1	1	1	m_7=xyz

- Each row of a truth row corresponds to a single minterm
- When a function is written as a sum of minterms, the form is called a standard (or canonical) ***sum-of-products***

Minterm notation

- An equation may be written in terms of m-notation

$f(a,b,c)=m_0+m_1+m_2+m_4$

$f(a,b,c)=\underbrace{a'b'c'}_{\substack{000\\0}}+\underbrace{a'b'c}_{\substack{001\\1}}+\underbrace{a'bc'}_{\substack{010\\2}}+\underbrace{ab'c'}_{\substack{100\\4}}$

$f(a,b,c)=\Sigma m(0,1,2,4)$

a	*b*	*c*	*f*
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

Minterm notation examples

- What is the minterm notation for the following function?
  - $f(a,b,c)=abc+a'bc+abc'+a'b'c$

- What is the function (in terms of variables) if the minterm notation is the following?
  - $f(a,b,c)=\Sigma m(1,5,6,7)$

Logic synthesis

- Duality suggests that:
  - If it is possible to synthesize a function *f* by considering the truth table rows where *f*=1, then it should also be possible to synthesize *f* by considering the rows for which *f*=0.
- This approach uses the complement of minterms, which are called ***maxterms***

Maxterms

Row number	*x*	*y*	*z*	*Maxterm*
0	0	0	0	$M_0=x+y+z$
1	0	0	1	$M_1=x+y+z'$
2	0	1	0	$M_2=x+y'+z$
3	0	1	1	$M_3=x+y'+z'$
4	1	0	0	$M_4=x'+y+z$
5	1	0	1	$M_5=x'+y+z'$
6	1	1	0	$M_6=x'+y'+z$
7	1	1	1	$M_7=x'+y'+z'$

- Each row of a truth row corresponds to a single maxterm
- When a function is written as a product of maxterms, the form is called a standard (or canonical) ***product-of-sums***

Maxterm notation

- An equation may be written in terms of M-notation

$f(a,b,c)=M_3 \cdot M_5 \cdot M_6 \cdot M_7$

$f(a,b,c)=(a+b'+c')(a'+b+c')(a'+b'+c)(a'+b'+c')$

$$\underbrace{0\ 1\ 1}_{3} \quad \underbrace{1\ 0\ 1}_{5} \quad \underbrace{1\ 1\ 0}_{6} \quad \underbrace{1\ 1\ 1}_{7}$$

$f(a,b,c)=\Pi M(3,5,6,7)$

a	*b*	*c*	*f*
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

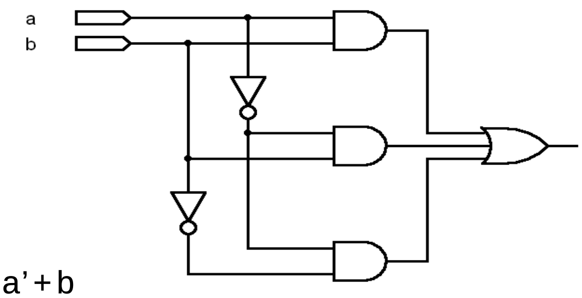
Maxterm notation examples

- What is the maxterm notation for the following function?
  - $f(a,b,c)=(a+b+c)(a'+b+c)(a+b+c')(a'+b'+c)$
- What is the function (in terms of variables) if the maxterm notation is the following?
  - $f(a,b,c)= \Pi M(1,5,6,7)$

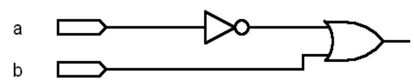
Sum-of-products and minimality

- A function expressed in standard sum-of-products (or product-of-sums) form may not be minimal

a	*b*	*f*
0	0	1
0	1	1
1	0	0
1	1	1



$f(a,b) = a'b' + a'b + ab = a' + b$



Form conversion

- If a function *f* is given in Σm or ΠM form, it is easy to find *f* or *f'* in Σm or ΠM form
- Use the following form conversion table

GIVEN FORM	DESIRED FORM			
	$f=\Sigma m$	$f=\Pi M$	$f'=\Sigma m$	$f'=\Pi M$
$f=\Sigma m$ (0,2,5,7)	--	Use numbers not on minterm list (1,3,4,6)	Use numbers not on minterm list (1,3,4,6)	Use numbers on minterm list (0,2,5,7)
$f=\Pi M$ (1,3,4,6)	Use numbers not on maxterm list (0,2,5,7)	--	Use numbers on maxterm list (1,3,4,6)	Use numbers not on maxterm list (0,2,5,7)