CS5820: GPU

HW: Assignment 2

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Given the point on the cylinder = (x, y, z)

Let the unit vector $V_1 = (V_{11}, V_{12}, V_{13})$

$V_2 = (V_{21}, V_{22}, V_{23})$

1. Rotate point on cylinder around V_1:-

To do this we consider the following steps

1.1 Translate given point to origin

1.2 Align V_1 with z-axis

1.3 Rotate about aligned z-axis

1.4 Realign V_1

1.5 Translate back to given point

So Now consider

1.1) Translate given point to origin

Transformation matrix

$$T_{P,O} = \begin{bmatrix} 1 & 0 & 0 & -x \\ 0 & 1 & 0 & -y \\ 0 & 0 & 1 & -z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

1.2) Align V_1 with z-axis

To do this we follow the following steps

1.2.1) project V_1 onto x-z-plane

1.2.2) project along z-axis

1.2.1) Project V_1 onto xz-plane

By using directional cosines we get

Transformation matrix

$$T_{V_1 xz} = \begin{bmatrix} \frac{V_{11}}{\sqrt{V_{11}^2+V_{12}^2}} & \frac{V_{12}}{\sqrt{V_{11}^2+V_{12}^2}} & 0 & 0 \\ -\frac{V_{12}}{\sqrt{V_{11}^2+V_{12}^2}} & \frac{V_{11}}{\sqrt{V_{11}^2+V_{12}^2}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

1.2.2) project along z-axis

Transformation matrix

$$T_{V_1 z} = \begin{bmatrix} \frac{V_{13}}{\sqrt{V_{11}^2+V_{12}^2+V_{13}^2}} & 0 & \frac{-\sqrt{V_{11}^2+V_{12}^2}}{\sqrt{V_{11}^2+V_{12}^2+V_{13}^2}} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{\sqrt{V_{11}^2+V_{12}^2}}{\sqrt{V_{11}^2+V_{12}^2+V_{13}^2}} & 0 & \frac{-V_{13}}{\sqrt{V_{11}^2+V_{12}^2+V_{13}^2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

1.3) Rotation about Z-axis :-

Let us consider the angle of rotation as α. Then Transformation matrix would be

$$R_{\alpha Z} = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 & 0 \\ \sin\alpha & \cos\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

1.4), 1.5) are simple the inverse of 1.2) & 1.1) and so we can consider these transformation matrix inverses.

Therefoeafter applying the required transformations the new point (x', y', z') can be given by

$$\boxed{\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = T^{-1}_{P,O}\, T^{-1}_{V,XZ}\, T^{-1}_{V,Z}\, R_{\alpha Z}\, T_{V,Z}\, T_{V,XZ}\, T_{P,O} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}}$$

2. Translate the new point along Y-axis

for doing this we first translate it to origin, translate it along y-axis retranslate it back to old index system.

2.1) Translate new point to origin

Translation matrix:

$$T_{P_2O} = \begin{bmatrix} 1 & 0 & 0 & -x' \\ 0 & 1 & 0 & -y' \\ 0 & 0 & 1 & -z' \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2.2) Translate new point along Y-axis

Let us consider the distance of translation as γ units. Then

Transformation Matrix

$$T_{P_2 y} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \gamma \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Therefore after applying 2nd Transformation

$$\boxed{\begin{bmatrix} x'' \\ y'' \\ z'' \\ 1 \end{bmatrix} = T^{-1}_{P_2O}\, T_{P_2y}\, T_{P_2O}\, T^{-1}_{P_1O}\, T^{-1}_{V_1xz}\, T^{-1}_{V_1z}\, R_{\alpha z}\, T_{V_1z}\, T_{V_1xz}\, T_{P_1O} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}}$$

3. Rotate the point around V_2

This is similar to that of First part with the few considerations that using $V_2 = [v_{21}, v_{22}, v_{23}]$ instead of $V_1 = [v_{11}, v_{12}, v_{13}]$, $P_3 = (x'', y'', z'')$ instead of $P_1 = (x, y, z)$ and an angle of rotation β instead of α.

So the final Matrix will be otained by

$$\boxed{\begin{bmatrix} x''' \\ y''' \\ z''' \\ 1 \end{bmatrix} = T^{-1}_{P_3O} \, T^{-1}_{V_2XZ} \, T^{-1}_{V_2Z} \, R_{\beta Z} \, T_{V_2Z} \, T_{V_2XZ} \, T_{P_3O} \, T^{-1}_{P_2O} \, T_{P_2Y} \, T^{-1}_{P_2O} \, T^{-1}_{P_1O} \, T^{-1}_{V_1XZ} \, R_{\alpha Z} \, T_{V_1Z} \, T_{V_1XZ} \, T_{P_1O} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}}$$

∴ The final transformation Matrix is

$$\boxed{T = T^{-1}_{P_3O} \, T^{-1}_{V_2XZ} \, T^{-1}_{V_2Z} \, R_{\beta Z} \, T_{V_2Z} \, T_{V_2XZ} \, T_{P_3O} \, T^{-1}_{P_2O} \, T_{P_2Y} \, T_{P_2O} \, T^{-1}_{P_1O} \, T^{-1}_{V_1XZ} \, R_{\alpha Z} \, T_{V_1Z} \, T_{V_1XZ} \, T_{P_1O}}$$