



$G$  — bipartite with  $n$  vertices

The diagram shows two large blue ovals labeled A and B. Inside oval A, there is a point labeled  $c_5$  and a set  $L(u) = \{c_1, c_3, c_5, c_{10}\}$ . Inside oval B, there is a point labeled  $c_6$  and a set  $L(v) = \{c_2, c_5, c_6, \dots\}$ . Red arrows point from  $c_5$  in A to  $c_5$  in B, and from  $c_6$  in B to  $c_5$  in A. Below the ovals, there are two boxes representing sets  $S_A$  and  $S_B$ .  $S_A$  contains  $c_2, c_3, c_7, c_8, \dots$  and  $S_B$  contains  $c_1, c_3, c_4, c_6, \dots$ . Red lines connect  $c_5$  in A to  $c_5$  in  $S_A$  and  $c_6$  in B to  $c_6$  in  $S_B$ . At the bottom, there is a large blue oval labeled  $X$  containing the union of  $S_A$  and  $S_B$ . A red arrow points from  $c_5$  in A to  $c_5$  in  $X$ , and another red arrow points from  $c_6$  in B to  $c_6$  in  $X$ . The set  $L(v)$  is also shown as  $L(v) = \{c_1, c_2, \dots, c_p\}$ .

$E_u$ : the <sup>bad</sup> event that  $u$  is not colored

$$\Pr[E_n] = \Pr[\text{none of the colors of } L(u) \text{ is present in } S_A]$$

$$= \frac{1}{2^{|L(u)|}} \rightarrow \underbrace{\frac{1}{2} \times \frac{1}{2} \times \dots \times \frac{1}{2}}_{|L(u)| \text{ times}}$$

$$2^{(1-\epsilon)n}$$

Since  $|L(u)|$  times  
 $> \log n$

$$< \frac{1}{2^{\log n}}$$

$$= \frac{1}{n} \quad \text{--- (A)}$$

$$\Pr \left[ \bigvee_{u \in V(G)} E_u \right] \leq \sum_{u \in V(G)} \Pr[E_u]$$

$$< n \cdot \frac{1}{n} \\ = 1$$

□