

ECE380 Digital Logic

Optimized Implementation of Logic Functions:

Strategy for Minimization, Minimum Product-of-Sums Forms, Incompletely Specified Functions

Electrical & Computer Engineering

Dr. D. J. Jackson Lecture 8-1



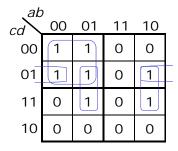
Terminology

- For a given term, each appearance of a variable (in true or complemented form) is called a *literal*
 - xyz' => three literals
 - abc'd => four literals
- Any '1' or group of '1's that can be combined on a K-map represents an *implicant* of a function
- An implicant is a *prime implicant* if it cannot be combined with another implicant to remove a variable
- A collection of implicants that account of all valuations for which a given function is '1' is called a cover of that function
- *Cost* is the number of gates plus the total number of inputs to all gates in the circuit

Electrical & Computer Engineering



Terminology example



 $f(a,b,c,d) = \Sigma m(0,1,4,5,7,9,11)$

Example Implicants: all single '1's, a'c', a'b'c', a'bd, ab'd

Prime Implicants: a'c', a'bd, ab'd, b'c'd

 $f(a,b,c,d)_{min}$: a'c'+a'bd+ab'd

Thus, a minimum SOP form contains <u>only</u> (but not necessarily all) prime implicants.

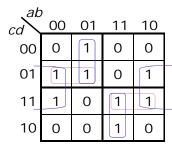
Electrical & Computer Engineering

Dr. D. J. Jackson Lecture 8-3



Prime implicants distinctions

- **Essential**: needed to form a minimum solution
- Nonessential: not <u>necessarily</u> needed to form a minimum solution



All prime implicants: b'd, a'bc', abc a'c'd, acd

Essential primes: b'd, a'bc', abc

Nonessential primes: a'c'd, acd

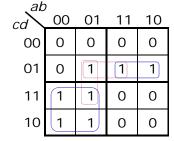
f(a,b,c,d)_{min}: b'd+a'bc'+abc

Minimum contains all essential and possibly some nonessential primes

Electrical & Computer Engineering



Prime implicants example



Essential primes: a'c, ac'd

Nonessential primes: a'bd, bc'd

One of these must be included to form a minimum solution

$$f(a,b,c,d)_{min}$$
: $a'c+ac'd+\left\{ egin{align*} a'bd \\ bc'd \end{array}
ight\}$

Electrical & Computer Engineering

Dr. D. J. Jackson Lecture 8-5



Prime implicants example

Identify all prime implicants for the given truth table. Which are essential and which are nonessential? What is a minimum SOP expression for this function?

Electrical & Computer Engineering



Minimization of POS expressions

- POS minimization using K-maps proceeds exactly as does SOP form except that groupings of '0's in the K-map are used to form POS terms.
- K-map can be constructed directly from ΠM expression for a function
- Place '0's in the K-map for every maxterm in the ΠM expression

Electrical & Computer Engineering

Dr. D. J. Jackson Lecture 8-7



Minimization of POS example

$$f(a,b,c) = (a+b'+c')(a'+b+c')(a'+b'+c)(a'+b'+c')$$

$$f(a,b,c) = \Pi M(3,5,6,7)$$

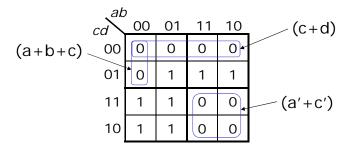
$$f=(a'+b')(b'+c')(a'+c')$$

Electrical & Computer Engineering



Minimization of POS example

$$f(a,b,c,d) = \Pi M(0,1,4,8,10-12,14,15)$$



$$f(a,b,c,d)_{min} = (a+b+c)(a'+c')(c+d)$$

Electrical & Computer Engineering

Dr. D. J. Jackson Lecture 8-9



K-map groupings example

- Draw the K-map and give the minimized POS logic expression for the following.
 - $f(a,b,c) = \Pi M(0,2,3,5-7)$
- Show the groupings made in the K-map

Electrical & Computer Engineering



Incompletely specified functions

- In digital systems it often happens that some input conditions (i.e. some input valuations) can never happen
- An input combination that can never happen is referred to as a don't care condition
- As a circuit is designed, a don't care condition can be ignored (i.e. the output for that condition can be treated as 0 or 1 in the truth table)
- A function that has don't care condition(s) is said to be *incompletely specified*

Electrical & Computer Engineering

Dr. D. J. Jackson Lecture 8-11



Example function with don't cares

Х	У	Z	f
0	0	0	1
0	0	1	1
0	1	0	d
0	1	1	d
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

Assume for a three variable function f(x,y,z) that the input combination xy=01 never occurs, otherwise the function is $\Sigma m(0,1,4,5)$

$$f(x,y,z) = \Sigma m(0,1,4,5) + D(2,3)$$

Or

$$f(x,y,z) = \Pi M(6,7) \cdot D(2,3)$$

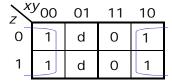
Electrical & Computer Engineering



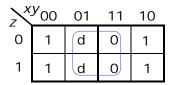
Example function with don't cares

 $f(x,y,z) = \Sigma m(0,1,4,5) + D(2,3)$

 $f(x,y,z) = \Pi M(6,7) \cdot D(2,3)$



$$f(x,y,z) = y'$$



$$f(x,y,z) = y'$$

Electrical & Computer Engineering

Dr. D. J. Jackson Lecture 8-13



Minimum SOP form

- 1. Choose a minterm (a '1' in the K-map) which is not yet covered (don't consider d's).
- 2. Find all adjacent '1's and 'd's (check the n adjacent cells for an n-variable K-map).
- 3. If a single term (i.e. a single looping) covers the '1' and all adjacent '1's and 'd's then the looping forms an essential prime implicant. Loop the essential prime.
- 4. Repeat steps 1-3 until all essential prime implicants are located.
- 5. Find a minimum set of nonessential prime implicants to cover (loop) the remaining '1's. If more than 1 set is possible, choose the set with the minimum number of literals (the largest grouping).

Electrical & Computer Engineering



Minimum POS form

- 1. Choose a maxterm (a '0' in the K-map) which is not yet covered (don't consider d's).
- 2. Find all adjacent '0's and 'd's (check the n adjacent cells for an n-variable K-map).
- 3. If a single term (i.e. a single looping) covers the '0' and all adjacent '0's and 'd's then the looping forms an essential prime implicant. Loop the essential prime.
- 4. Repeat steps 1-3 until all essential prime implicants are located.
- 5. Find a minimum set of nonessential prime implicants to cover (loop) the remaining '0's. If more than 1 set is possible, choose the set with the minimum number of literals (the largest grouping).

Electrical & Computer Engineering