CS 6160 Cryptology Lecture 15: Key Management, Public-Key Exchange & Security Definitions for PKC

Maria Francis

November 3, 2020

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- But limited memory and cannot store many keys.

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  - Storing and managing large number of secret keys
  - Inapplicability of SKC to open systems.

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- Or generate keys on demand, online.
  - KDC shares a key with each employee.
  - Alice wants to talk to Bob, she sends that request to KDC.
  - KDC chooses a new random key called session key and sends this key to Alice (encrypted using k_A) and Bob (using k_B).
  - Once Alice and Bob is done with the conversation the session key is erased and for next one KDC has to be contacted again.

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  The KDC can be kept in a secure location and be given the highest possible protection against network attacks.
- When an employee joins only one key need to generated and no communication with other employees are needed.

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- But there are drawbacks:
  - Single point of attack. KDCs become a high value target and vulnerable to external and internal attacks.
  - Single point of failure. if the KDC is down, secure communication is temporarily impossible.
- Solution that is often done: Replicate the KDC.
  - More points of attack possible.
  - More updates needed to add new employees.

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  - Alice can re-initiate the conversation with Bob without KDC by resending the ticket.

Key Distribution

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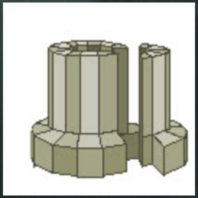
KDC



Bob

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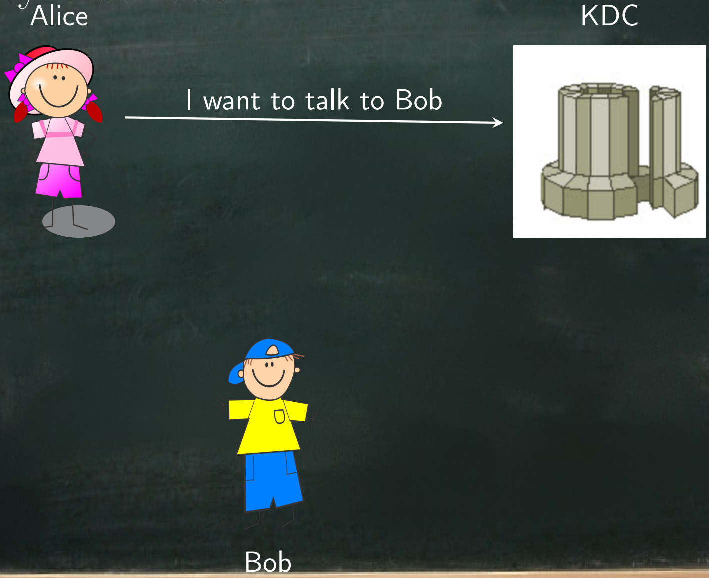
Key Distribution



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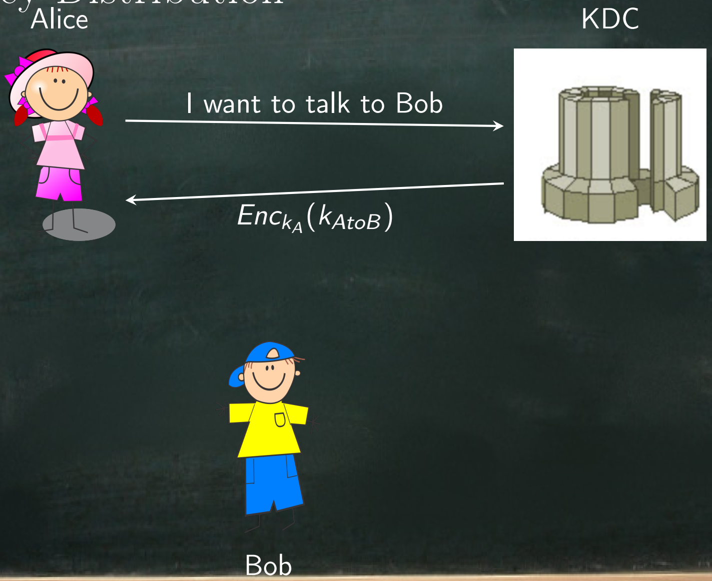

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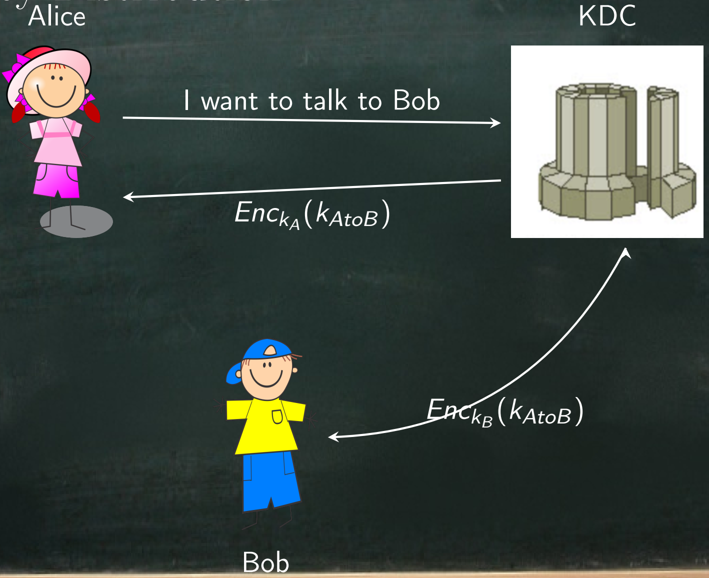

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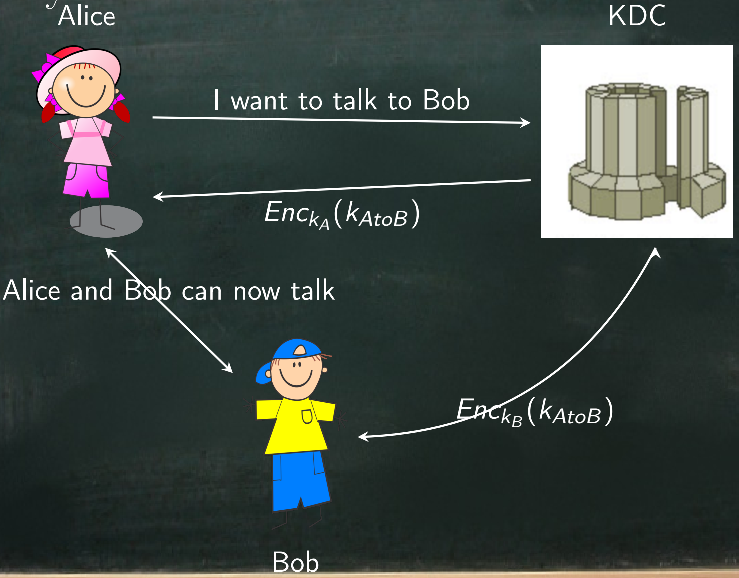

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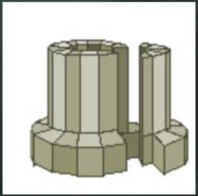

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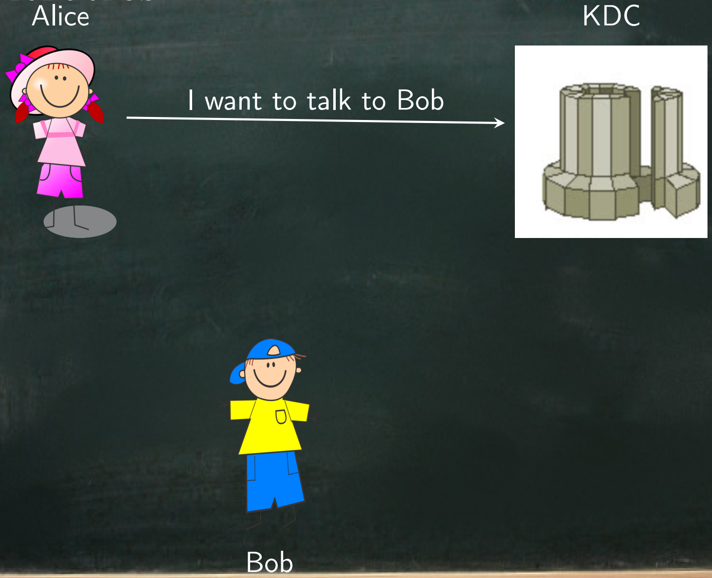

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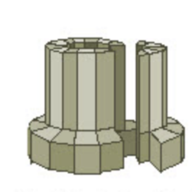

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$Enc_{k_A}(k_{AtoB}),\ Enc_{k_B}(k_{AtoB})$

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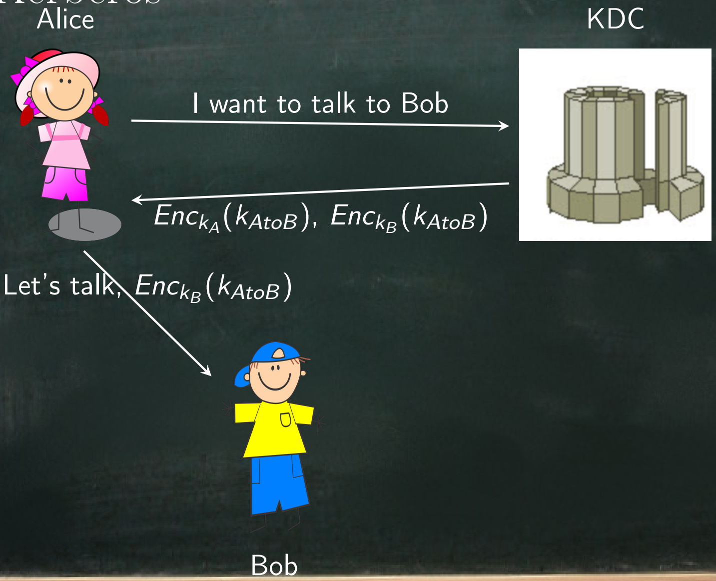
Kerberos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- Diffie and Hellman used to derive interactive protocols for secure key exchange.
- They indeed created a revolution with the first steps into PKC.

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- Stronger notion that saying unable to compute k exactly, since this is the key used for SKC.

Key-exchange experiment

$KE^{eav}_{\mathcal{A},\Pi}(1^n)$

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Why give $\mathcal{A}$ the transcript? To capture that $\mathcal{A}$ can eavesdrop the entire interaction.

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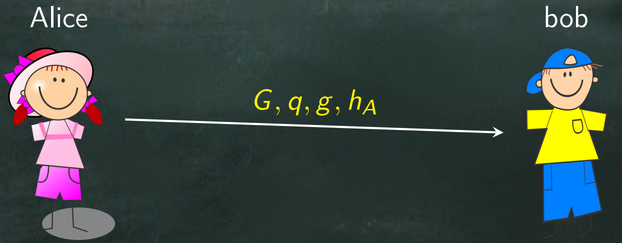

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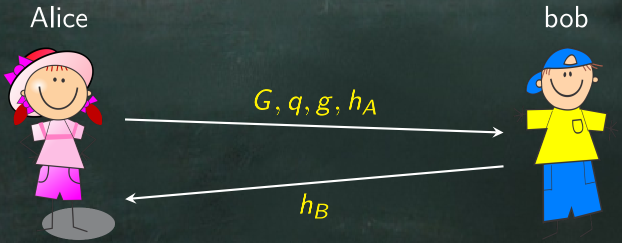

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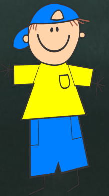

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- This is not going to work in practice since group elements are not useful as keys typically and the representation of a uniform group element is not in general uniform bit-string.
- But we assume $\overline{KE}^{eav}_{\mathcal{A},\Pi}(1^n)$ denotes a modified experiment where if $b = 1$ $\mathcal{A}$ is given k' chosen uniformly from G instead of uniform n-bit string.

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- In any case, it was the first step to asymmetric techniques and therefore is very important.

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- Thus we have key distribution for open environments!

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- We are not considering active attacks by letting other mechanisms take care of it.

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- In fact, given pk, c we can compute m with probability 1 – Practice q!

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  - Just encrypt all the grades and then compare with ciphertext!

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- Indistinguishable encryptions in the presence of an eav, CPA-security, and indistinguishable multiple encryptions are all equivalent.
- CPA-secure PKE for fixed-length messages implies PKE for arbitrary-length messages satisfying the same notion of security.

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- Second class of attacks happens only in PKE.

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  - Then $\mathcal{A}$ submits highest bid always and wins.

CCA indistinguishability experiment

$PubK^{cca}_{\mathcal{A},\Pi}(1^n)$

1. (pk, sk) is generated by running $Gen(1^n)$.
2. $\mathcal{A}$ is given pk and $Dec_k(\cdot)$.

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Difference is $\mathcal{A}$ is given pk which means no need separate access to Enc oracle.

CCA-secure

Now that we have an indistinguishability experiment, we can have the security definition.

Definition

A PKE Π has indistinguishable encryptions under a chosen-ciphertext attack or is CCA-secure if for PPT adversaries $\mathcal{A}$ there is a negligible function negl such that:

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CCA-secure for fixed-length messages do NOT hold for arbitrary-length messages.