Homework Assignments III

MA1130 VECTOR CALCULUS

January 21, 2019

1. find the volume under the surface $z = f(x, y)$ over the domain R, where
   1. $f(x, y) = x^4 + xy + y^3, R = [1, 2] \times [0, 2]$.
   2. $f(x, y) = \sin x \cos(y - \pi), R = [1, \pi/2] \times [0, \pi]$.
   3. $f(x, y) = xy \cos(x^2 y), R = [0, 1] \times [0, \pi/2]$.
   4. $f(x, y) = \cos x \sin y, R = [0, y] \times [0, \pi/2]$.
   5. $f(x, y) = 24x^2 y, R = [0, 1] \times [\sqrt{x}, 1]$.
2. Find the volume V of the solid bounded by the three coordinate planes and the plane $2x + y + 4z = 4$. (Hint: Use double integral)
3. Prove that the volume of a tetrahedron with mutually perpendicular adjacent sides of lengths a, b, and c, is $\frac{abc}{6}$.
4. Calculate the integrals for the functions $f(x, y, z)$ over the domain R, where
   1. $f(x, y, z) = x^2 \sin z, R = [0, \pi] \times [0, x] \times [0, xy]$.
   2. $f(x, y, z) = ze^{y^2}, R = [0, y] \times [0, z] \times [0, 1]$.
5. show that
$$\int_a^b \int_a^z \int_a^y f(x)\, dx\, dy\, dz = \int_a^b \frac{(b - x)^2}{2} f(x)\, dx$$
.
6. Show that
$$\iiint_{\mathbb{R}^3} f(x, y, z)\, dx\, dy\, dz = \int_0^\infty \int_0^\pi \int_0^{2\pi} f(\rho \sin\phi \cos\theta, \rho \sin\phi \sin\theta, \rho \cos\phi)\rho^2 \sin\phi\, d\theta d\phi d\rho.$$
Where $x = \rho \sin\phi \cos\theta, y = \rho \sin\phi \sin\theta, z = \rho \cos\phi$ is the polar co-ordinate representation of $\mathbb{R}^3$

7. Find the volume V of the solid inside the intersection of the sphere $x^2 + y^2 + z^2 = 4$ and the cylinder $x^2 + y^2 = 1$.

8. Evaluate
$$\iint_R \sin(\frac{x+y}{2})\cos(\frac{x-y}{2})dA,$$
where R is the triangle with vertices $(0,0), (2,0)$ and $(1,1)$.

9. Show that the Beta function, defined by
$$B(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}dt, \text{ for } x > 0, y > 0,$$
satisfies the relation $B(y,x) = B(x,y)$ for $x > 0, y > 0$.