EP 1027: Quiz II Solutions

April 12, 2019

1. Point out where rationalized units are not used

$$(a)\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{|\mathbf{x}|^3}\mathbf{x},\ (b)\mathbf{B} = \frac{\mu_0}{4\pi}\oint\frac{Id\mathbf{x}' \times (\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^3}\ (c)\boldsymbol{\nabla}\cdot\mathbf{E} = 4\pi\rho\ (d)\nabla\times\mathbf{B} = \frac{4\pi}{c}\mathbf{j}$$

Answer: (c) and (d)

2 points

2. Direction of $\mathbf{E}(\mathbf{x})$ of point charge at origin in spherical polar coordinates is along

$$(a)\hat{\mathbf{r}},\ (b)\hat{\boldsymbol{\theta}},\ (c)\hat{\boldsymbol{\phi}},\ (d)\hat{\boldsymbol{\rho}}$$

Answer: (a)

1 point

3. Direction of $\mathbf{B}$ of infinitely long current carrying wire along z-axis in cylindrical coordinates is along

$$(a)\hat{\boldsymbol{\rho}},\quad (b)\hat{\boldsymbol{\phi}},\quad (c)\hat{\boldsymbol{z}},\quad (d)\rho\hat{\boldsymbol{\rho}} + z\hat{\mathbf{z}}$$

Answer: (b)

1 point

4. Magnetic field $\mathbf{B}(\mathbf{x})$ due to a point charge, q at $\mathbf{x}'$ moving with velocity, $\mathbf{v}$

$$(a)\text{absent},\ (b)\text{approx} \propto \frac{\mathbf{v}}{c}\times\frac{\mathbf{x}}{|\mathbf{x}-\mathbf{v}t|^3},\ (c)\text{approx} \propto q\frac{\mathbf{v}}{c}\times\frac{\mathbf{x}-\mathbf{v}t}{|\mathbf{x}-\mathbf{v}t|^3},\ (d)\text{approx} \propto q\frac{\mathbf{v}}{c}\times\mathbf{E}(\mathbf{x})$$

Answer: (b) and (c)

2 points

5. Inside a conductor, in general

$$(a)\mathbf{E} = 0,\ (b)\rho = 0,\ (c)\mathbf{j} = \sigma\mathbf{E}\ (d)\mathbf{E} = \sigma\mathbf{j}$$

Answer: (b) and (c)

2 points

6. At the boundary of a pair of dielectrics cum paramegnetics which of the following is not continuous

$$(a)\mathbf{E}_{\parallel}, \quad (b)\mathbf{H}_{\parallel} \quad (c)D_{\perp} \quad (d)B_{\perp}$$

Answer: (b) and (c)

2 points

7. 7. If **E**, **B** and $\phi, \mathbf{A}$ contain same amount of information, why are they different in number of components?

5 points

Answer: Not all components of **E** and **B** or even $\phi, \mathbf{A}$ are independent (1 point). Total number of independent degrees of freedom is actually **2** (1 point).

Counting the degrees of freedom**:**

To start with $\mathbf{E}, \mathbf{B}$ have 6 total number of components. But there are 4 constraint equations (1 vector constraint plus 1 scalar constraint),

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0, \qquad \nabla \cdot \mathbf{B} = 0.$$

Thus the total number of *independent* components is 2. (1 point)

Same conclusion can be arrived at considering the potential formulation (showing this is worth 2 points). Φ and **A** have 4 degrees of freedom in all, but we know there is gauge freedom (redundancy), and the potentials are ambiguous up to,

$$\mathbf{A} \to \mathbf{A}' = \mathbf{A} + \nabla\chi, \qquad \Phi \to \Phi' = \Phi - \frac{\partial \chi}{\partial t}$$

(The scalar field χ is called a gauge transformation parameter). One can try to fix this redundancy by imposing an extra condition on the potentials, say for example the Lorenz condition

$$\dot{\Phi} + \nabla \cdot \mathbf{A} = 0.$$

This is a scalar equation and hence it removes just one out of the 4 components of Φ and **A** combined. However this still does not determine the potentials uniquely and one can still do further gauge transformations which are compatible with the Lorenz gauge condition as long as the gauge transformation satisfies the wave equation, i.e.

$$\Box \chi = 0,$$

where $\Box \equiv \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2$ is the wave operator. So still one degree of freedom is a gauge redundancy. Removing (fixing) all the redundancies leaves us with $4 - 1 - 1 = 2$ independent degrees of freedom.