CS6350 : Topics in Combinatorics

Exam III

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1. Consider the Combinatorial Nullstellensatz theorem:

Theorem: Let F be an arbitary field and let $f = f(x_1, x_2, \dots x_n)$ be a polynomial in $F[x_1, \dots x_n]$. Suppose the degree $deg(f)$ of f is $\sum_{i=1}^{n} t_i$, where each t_i is a non negative integer, and suppose the coefficient of $\prod_{i=1}^{n} x_i^{t_i}$ in f is nonzero. Then if $S_1, \dots, S_n$ are subsets of F with $|S_i| > t_i$, then are $s_1 \in S_1, s_2 \in S_2, \dots s_n \in S$ so that

$$f(s_1, \dots, s_n) \neq 0$$

In the present question it is given that

$$f(a_1, a_2 \dots a_n) = \begin{cases} \neq 0 & \text{if } (a_1, a_2 \dots a_n) = (s_1, s_2 \dots s_n) \\ = 0 & \text{otherwise} \end{cases}$$

as the converse condition of the theorem is satisfied, we can relate that

$$deg(f) = \sum_{i=1}^{n} t_i$$

But as given $|s_i| > t_i \Rightarrow |s_i| \geq t_i + 1$

As it is a converse $\Rightarrow t_i \geq |s_i| - 1$

$$\therefore deg(f) = \sum_{i=1}^{n} t_i$$

$$\boxed{\therefore deg(f) \geq \sum_{i=1}^{n} (|s_i| - 1)}$$

Hence proved.