

Indian Institute of Technology Hyderabad

CALCULUS OF SEVERAL VARIABLES:MA1220

Date : 05.06.2019

Marks : 30

Time : 2 hours

Answer all the questions. Maximum you can score is 30.

- (1) Discuss the differentiability of $f(x, y)$ at $(x, y) = (0, 0)$, where $f(x, y) = \frac{3x^2y - y^3}{x^2 + y^2}$ when $(x, y) \neq (0, 0)$ and 0 otherwise. [3]
- (2) Define differentiability of a function of n variable at a given point. Show that a function which is continuous may not be differentiable. [2+3]
- (3) Let $f(x, y) = 6 - x^2 - 4y^2$, find a vector which is perpendicular to the surface $z = f(x, y)$ at the point $(1, 1, 1)$. [3]
- (4) Find the extreme values of $z = \cos^2(x) + \cos^2(y)$ subject to the condition $x - y = \pi/4$. [3]
- (5) Use the method of Lagrange Multiplier to find the greatest and least distances of point on the Ellipse $x^2 + 4y^2 = 1$ from the straight line. $x + 2y = 2$ [3+3]
- (6) Let $f : [0, 1] \rightarrow \mathbb{R}$ be a continuous function satisfying $|f(x)| \leq \int_0^x f(t)dt$ for all $x \in [0, 1]$. Show that $f = 0$ on $[0, 1]$. [3]
- (7) Let $f : [0, 1] \rightarrow \mathbb{R}$ be a continuous function. Find the value of $\lim_{n \rightarrow \infty} \int_0^1 x^n f(x)dx$. [5]
- (8) Find the volume of the ellipsoid $\{(x, y, z) : \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1\}$, using cartesian coordinate. [4]
- (9) Find the volume of the solid bounded by the cylinders $\{(x, y, z) : (x + 1)^2 + y^2 = \frac{9}{4}\}$ and $\{(x, y, z) : (x - 1)^2 + y^2 = \frac{9}{4}\}$. [5]