EP 1027: Maxwell's Equations and Electromagnetic waves

Instructor: Shubho Roy[1]
(Dept. of Physics)

Lecture 1

March 26, 2019

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Office hours By email appointment or walk in

Rough Plan

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- Vector Calculus: Notations and algebra of Cartesian Vectors, Vector Differential operators, Integration of vectors, Curvilinear coordinates

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- Electrodynamics (general case): Maxwell's equations

- Electromagnetic (EM) Waves in vacuum, in media (reflection, refraction, absorption, dispersion), Guided Waves .

References

References

- Griffiths, D.J. **Introduction to Electrodynamics**.
- Purcell, E. **Electricity and Magnetism**
- **Feynman lectures on Physics**, Vol. 2
- Reitz, Milford and Christy: **Foundations of Electromagnetic Theory**
- Schaum's Outline of Electromagnetics (Edminister, J)
- ** Advanced books: Jackson, Panofsky & Phillips, Schwartz, Schwinger

Omissions?

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- Boundary Value problems: Solving Laplace or Poisson's equations (Method of Images, Separation of variables method, Green's function method, Computational approaches)

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- Multipole Expansions
- DC and AC Circuits
- Special Relativistic formulation of Electrodynamics: Spacetime tensor notation
- Radiation from accelerating charges: Cerenkov, Synchrotron, ... only if we have time

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- Assessment: Final (100 points, 3 hrs.) $+$ 3(4) Quizzes (10 points, 10 min.)

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- Tentative final date (May 4, 10 AM - 1 PM), Review of grades (May 7)

Review of Cartesian Vectors: Notation and Terminology

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- Position vector: 3-component object (triplet) giving the location from a origin of Cartesian coordinate system: row vector (x, y, z) or a column vector $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ or $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$.

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- Notation: Call the components of the vector: x_k, k can take values from 1 to 3. Whole vector is denoted in boldface, $\mathbf{x}$ or, with an arrow overhead, $\vec{x}$.

Cartesian Vectors: Notation and Terminology

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- Rule for addition of 2 vectors: Add the respective components

$$\mathbf{x} + \mathbf{y} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \equiv \begin{pmatrix} x_1 + y_1 \\ x_2 + y_2 \\ x_3 + y_3 \end{pmatrix}$$

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- Rule for multiplication with a number, λ : Multiply each component by λ

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- Once we choose the vector to be a column, then to denote the row vector, we will use the transpose

$$\mathbf{x}^T = (x_1, x_2, x_3)$$

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$$
\begin{aligned}
\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} &= x_1 \underbrace{\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}}_{\hat{\mathbf{e}}_1} + x_2 \underbrace{\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}}_{\hat{\mathbf{e}}_2} + x_3 \underbrace{\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}}_{\hat{\mathbf{e}}_3} \\
&= x_1\,\hat{\mathbf{e}}_1 + x_2\,\hat{\mathbf{e}}_2 + x_3\,\hat{\mathbf{e}}_3 \\
&= \sum_{k=1}^{3} x_k\,\hat{\mathbf{e}}_k .
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- "Dot product/ inner product/ scalar product" of $\mathbf{x}$ and $\mathbf{y}$:

$$\mathbf{x} \cdot \mathbf{y} \equiv \mathbf{x}^T \mathbf{y} = x_1 y_1 + x_2 y_2 + x_3 y_3 = \sum_{k=1}^{3} x_k y_k .$$

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- Rule 1) Repeating an index $\Rightarrow$it is summed over all values $1, 2, 3$. E.g.

$$\mathbf{x} = \sum_{i=1}^{3} x_i\, \hat{\mathbf{e}}_i = x_i\, \hat{\mathbf{e}}_i, \qquad \mathbf{x}\cdot\mathbf{y} = \sum_{i=1}^{3} x_i\, y_i = x_i y_i.$$

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- Rule 2) Non-repeated indices are "free" to take any values, should not be summed,
- Subrule 3) Repeated index cannot appear more than twice, $x_k x_k y_k$ is illegal/inconsistent
- Subrule 4) A repeated index can be relabeled anytime (dummy). E.g. $\mathbf{x} \cdot \mathbf{y} = x_i y_i = x_l y_l = x_m y_m$.

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- Vector product/Cross product

$$\mathbf{x} \times \mathbf{y} \equiv (\epsilon_{ijk} x_i y_j)\,\hat{\mathbf{e}}_k,$$

where

$$\begin{aligned} \epsilon_{123} &= \epsilon_{231} = \epsilon_{312} = 1, \\ \epsilon_{132} &= \epsilon_{213} = \epsilon_{321} = -1, \\ \epsilon_{112} &= \epsilon_{121} = \epsilon_{223} \dots = 0. \end{aligned}$$

ϵ_{ijk} = "Levi Civita Symbol or Completely antisymmetric symbol"

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- e.g. Let's compute the 1st component of $\mathbf{x} \times \mathbf{y}$:

$$(\mathbf{x} \times \mathbf{y})_1 = \epsilon_{ij1} x_i y_j ,$$

i and j are both repeated $\implies$ we have to sum over all values of i and j. Sum over i first,

$$\begin{aligned} (\mathbf{x} \times \mathbf{y})_1 = \epsilon_{ij1} x_i y_j &= \underbrace{\epsilon_{1j1}}_{=0} x_1 y_j + \epsilon_{2j1} x_2 y_j + \epsilon_{3j1} x_3 y_j . \\ &= \epsilon_{2j1} x_2 y_j + \epsilon_{3j1} x_3 y_j . \end{aligned}$$

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- Next, sum over index j. The first term,

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- Similarly, can show the second term: $\epsilon_{3j1} x_3 y_j = -x_3 y_2$.
- Thus we have derived: $(\mathbf{x} \times \mathbf{y})_1 = x_2 y_3 - x_3 y_2$.

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- Vector transformation rule under rotation of coordinate axes: Implemented through Matrix operations, $\mathbf{O}$

$$\mathbf{x}' = \mathbf{O}\mathbf{x},$$

$$x_i' = O_{ij}\, x_j = \begin{pmatrix} O_{11} & O_{12} & O_{13} \\ O_{21} & & \\ O_{22} & & \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}.$$

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- Component notation: $O_{ij} O_{ik} = \delta_{jk}.$

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- Summation convention applied to simplify dot products and cross products.
- Two new symbols (indexed objects) introduced, namely, the Levi Civita (antisymmetric) ϵ_{ijk} and the Kronecker (symmetric) δ_{ij}.
- Effect of Rotation of coordinate axis accomplished through orthogonal transformations on the vector.