CS6350: Topics in Combinatorics
Assignment 9

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1. Problem: Let G = (V, E) be a graph whose vertex set $V(G) = \{s_1, s_2, ..., s_m, t_1, t_2, ..., t_m\}$ where

(a) s_i and t_i are adjacent for every $i \in \{1, 2, ..., m\}$;

(b) s_i and t_j are not adjacent for i < j (1 ≤ i, j ≤ m).

Then, show that $\Pi(G) \geq \log_2 m$.

A. Let the Prague dimension of G be k (i.e., $\Pi(G) = k$) and the k-tuple of a vertex $v \in V$ be

$$f(v) = (f_1(v), f_2(v), ..., f_k(v))$$

We say that two vertices $v_1, v_2 \in V$ are adjacent, if

$$f_i(v_1) \neq f_i(v_2) \ \forall i \in [1, k]$$

Now let us relate this with a polynomial function with k variables

$$p_v(x_1, x_2, x_3, ..., x_k) = \prod_{i=1}^{d}(x_i - f_i(v))$$

So, for the given problem we can summarize as

$$p_{s_i}(f(t_j)) = \begin{cases} \neq 0, & \text{if } i = j \\ = 0, & \text{if } i < j \end{cases} \tag{1}$$

Triangular Criterion: For i = 1, . . . , m, let $f_i : \Omega \to F$ be functions and $a_i \in \Omega$ elements such that

$$f_i(a_j) = \begin{cases} \neq 0, & \text{if } i = j \\ = 0, & \text{if } i < j \end{cases} \tag{2}$$

Then $f_1, ..., f_m$ are linearly independent members of the space F^Ω.

From Triangular Criterion we can say that each polynomial $p_v(x_1, x_2, x_3, ..., x_k)$ is linear in all k variables and therefore making 2^k monomials span the space.
Therefore, we conclude that the number of linearly independent polynomials(m) that can be found in the space is less than the number of vectors that span the space($\leq 2^k$).

$$\Rightarrow m \leq 2^k$$
$$\Rightarrow k \geq \log_2 m$$

$$\boxed{\because \Pi(G) \geq \log_2 m}$$

Hence Proved.