Topics in Combinatorics 2020

Rogers Mathew
(rogers@cse.iith.ac.in)

Combinatorics

- Oxford Dictionary says:
  - “The branch of mathematics dealing with combinations of objects belonging to a finite set in accordance with certain constraints, such as those of graph theory.”
- Collins dictionary says:
  - “the branch of mathematics concerned with the theory of enumeration, or combinations and permutations, in order to solve problems about the possibility of constructing arrangements of objects which satisfy specified conditions.”
- Wikipedia says:
  - “a branch of mathematics concerning the study of finite or countable discrete structures.”

Great combinatorialists speak

- **G. Pólya**, **R.E. Tarjan** and **D.R. Woods**, *Notes on Introductory Combinatorics*, 1983.

  Combinatorics can be classified into three types: enumerative, existential, and constructive. Enumerative combinatorics deals with the counting of combinatorial objects. Existential combinatorics studies the existence or nonexistence of combinatorial configurations. Constructive combinatorics deals with methods for actually finding specific configurations (as opposed to merely demonstrating their existence theoretically). [..] In constructive combinatorics, the problem is usually one of finding a solution *efficiently*, [..] using a reasonable length of time.

Great combinatorialists speak

- **Noga Alon**, Non-Constructive Proofs in Combinatorics, in *Proc. ICM Kyoto*, 1990.

  One of the main reasons for the fast development of Combinatorics during the recent years is certainly the widely used application of combinatorial methods in the study and the development of efficient algorithms. It is therefore somewhat surprising that many results proved by applying some of the modern combinatorial techniques, including Topological methods, Algebraic methods, and Probabilistic methods, merely supply existence proofs and do not yield efficient (deterministic or randomized) algorithms for the corresponding problems.

Great combinatorialists speak

- **Joel Spencer**, Probabilistic methods, in *Handbook of Combinatorics*, vol. 2, Elsevier, Amsterdam, 1995.

  Does the heart of mathematics lie in the building of structures or in the solving of individual problems? Not an either – or question, to be sure, but one that is particularly effective in splitting the ranks of combinatorialists. Use of algebraic structure to explain discrete phenomena will be central to some, to others grotesque. A clever argument is beautiful to the problem-solver, a curiosity to a structuralist. The very term "combinatorial methods", has to this author, an oxymoronic character. It is the brilliant proofs, those that expand and/or transcend known technologies, which express the soul of the subject.

Great combinatorialists speak

- **W.T. Gowers**, The two cultures of mathematics, in *Mathematics: frontiers and perspectives*, AMS, 2000, 65–78.

  Combinatorics appears to many to consist of a large number of isolated problems and results, and therefore to be at a disadvantage in this respect. Each result individually may well require enormous ingenuity, but ingenious people exist, especially in Hungary, and future generations of combinatorialists will not have the time or inclination to read and admire more than a tiny fraction of their output.

  Let me attempt to answer this criticism. It is certainly rare in combinatorics for somebody to and a very general statement which suddenly places a large number of existing results in their proper context. It is also true that many of the results proved by combinatorialists are somewhat isolated and will be completely forgotten (but this does not distinguish combinatorics from any other branch of mathematics). However, it is not true that there is no structure at all to the subject. The reason it appears to many mathematicians as though combinatorics is just a miscellaneous collection of individual problems and results is that the organizing principles are less explicit.

Course outline (tentative syllabus)

- **THE CLASSICS: Counting:** The binomial theorem, selections with repetitions, partitions, double counting, the averaging principles. **Advanced counting:** Bounds on intersection size, Zarankiewicz's problem, Density of 0-1 matrices. **Inclusion and Exclusion principle:** The number of derangements. **The pigeon-hole principle:** The Erdos-Szekeres theorem, Mantel's theorem, Turan's theorem, Dirichlet's theorem.

[Planning to start with EXTREMAL SET THEORY]

- **EXTREMAL SET THEORY: Intersecting families:** The sunflower lemma, The Erdos-Ko-Rado theorem. **Chains and antichains**: Dilworth's theorem, Sperner's theorem, Bollobas's theorem.
- **THE LINEAR ALGEBRA METHOD:** A short introduction to some basic concepts of linear algebra. **The basic method (using linear independence):** Fisher's inequality, polynomial technique, Frankl-Wilson theorem, another proof of Bollobas's theorem. **Orthogonality and rank arguments:** Orthogonal coding, a bribery party, balanced families.
- **THE PROBABILISTIC METHOD: Basic tools. Counting sieve:** Ramsey numbers, Van der Waerden's theorem, Tournaments, Kleitman-Spencer theorem. **The Lovasz sieve:** The local lemma and some of its applications.

Books

- Extremal combinatorics, by Stasys Jukna.
- The probabilistic method, by Noga Alon and Joel Spencer.
- Linear algebra methods in combinatorics with applications in geometry and computer science, by Laszlo Babai and Peter Frankl.

Grading/Evaluation Policy

- 40 marks – assignments
  - Assignments every week. Each assignment is made of say 1 or 2 questions and it won't be lengthy. You are expected to solve it yourself. If you are unable to solve an assignment Q even after spending considerable effort, then you may seek help from your friends. You can discuss with them. But, write the answer in your own language.
  - Only 4 assignments (chosen arbitrarily) will be evaluated. We will always choose those assignments which you have not submitted in this 4.
  - Each assignment carries 10 marks
- 30 marks – make one video or one pdf (in latex) on a topic
- 30 marks – 3 one-hour duration exams each out of 10 marks
- Grading will be relative
- **Auditing students**: They need to submit all assignments. Should get at least 50% marks for assignments (that is, 20 out of 40)

Class timings

- Mon: 12:00 to 12:55 (no live class)
- Tue 9:00 to 9:55 (no live class)
- **Fri: 11:00 to 11:55 (live doubt clearance. Will meet on 4 Sep. Will discuss grading policy then.)**
- **Watch videos uploaded in youtube (3 x 1 hour videos per week. Will mail you whenever I upload new videos)**

THANK YOU