CS6380 - Topics in Combinatorics

Assignment 1

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Erdös-Ko-Rado theorem:
Let F be an intersecting family of k-element subsets of $[n]$. If $2k \leq n$ then the number of members in F is atmost $\binom{n-1}{k-1}$.

i.e., $|F| \leq \binom{n-1}{k-1}$

Problem 7.3:
Given that $A_1, A_2, \ldots A_m$ are k-element subsets of $[n]$ and $A_i \cup A_j \neq [n]$ $\forall$ i,j.
Now Let us consider $\bar{A}_1, \bar{A}_2, \ldots \bar{A}_m$ to be the k'-element subsets of $[n]$ where $\bar{A}_i$ represents the compliment subset of A_i

i.e., $\bar{A}_i = [n] - A_i$

and $k' = n - k$.

By applying De-morgan's Law to the given condition

$$\overline{(A_i \cup A_j)} \neq \overline{[n]}$$

$$(\bar{A}_i) \cap (\bar{A}_j) \neq \phi$$

The result obtained is of the same form of requirement condition of Erdös-Ko-Rado theorem.

So according to the theorem

$$m \leq \binom{n-1}{k'-1}$$

$$\leq \binom{n-1}{n-k-1} \leq \binom{n-1}{(n-1)-k} \leq \binom{n-1}{k} = \frac{(n-1)!}{k!\,(n-1-k)!} \times \frac{n-k}{n} \times \frac{n}{n-k}$$

$$\leq \frac{(n-1)! \times n}{k!\,(n-1-k) \times (n-k)} \times \left(1 - \frac{k}{n}\right) = \binom{n}{k}\left(1 - \frac{k}{n}\right)$$

Hence proved.