29/10/2019

Magnetic Circuits

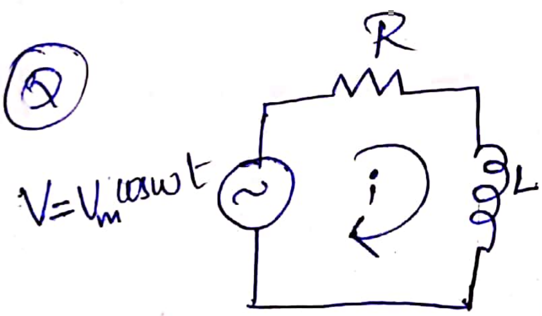
1) sinusodial Analysis
2) AC. & DC power
3) Magnetic coupling → self inductance & mutual inductance
4) Transformers
5) Resonance

Resistor → active (dissipates energy)

~~Induct~~

Inductor(L) ← passive
capacitor ← store energy element

Q



$$V_m\cos\omega t = Ri + L\frac{di}{dt}$$

$$I = I_1\cos\omega t + I_2\sin\omega t$$

$$V_m\cos\omega t = R(I_1\cos\omega t) + RI_2\sin\omega t + L(I_1)(-\sin\omega t)(\omega) + L(I_2)(\cos\omega t)(\omega)$$

$$\Rightarrow V_m\cos\omega t = [RI_1 + LI_2(\omega)]\cos\omega t + [RI_2 - LI_1(\omega)]\sin\omega t$$

$$I_1 = \frac{RV_m}{R^2+\omega^2L^2} \qquad I_2 = \frac{\omega L(V_m)}{R^2+\omega^2L^2}$$

$$i(t) = \overbrace{\frac{RV_m}{R^2+\omega^2L^2}}^{\cos\theta}\cos\omega t + \overbrace{\frac{\omega L(V_m)}{R^2+\omega^2L^2}}^{\sin\theta}\sin\omega t$$

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$v(t) = V_m \cos\omega t$

$i(t) = I_m \cos(\omega t - \theta)$

$I_m = \dfrac{V_m}{\sqrt{R^2 + \omega^2 L^2}}$ $\quad \theta = \tan^{-1}\left(\dfrac{\omega L}{R}\right)$

Assosiativity + Homogenity $\rightarrow$ linearity

$v_1(t) + j v_2(t) \rightarrow i_1(t) + j i_2(t)$

$V_m\left[\cos(\omega t + \theta) + j \sin(\omega t + \theta)\right] \rightarrow$

$I_m\left[\cos(\omega t + \phi) + j \sin(\omega t + \phi)\right]$

$V_m e^{j(\omega t + \theta)} \rightarrow I_m e^{j(\omega t + \phi)}$

$V_m e^{j\omega t} \cdot e^{j\theta} \rightarrow I_m e^{j\omega t} \cdot e^{j\phi}$

$\mathrm{Re}\left[V_m e^{j\theta}\right] \rightarrow \mathrm{Re}\left[I_m e^{j\phi}\right]$

$\rightarrow v(t) = V_m \cos\omega t \rightarrow \vec{V} = V_m \angle 0^\circ \rightarrow V_m \cos\omega t = \mathrm{Re}\left[V_m \angle 0^\circ\right]$

$v(t) = R\, i(t)$

$\vec{I} = I_m \angle 0^\circ$

$i(t) = \dfrac{V_m}{R} \cos\omega t$

$\dfrac{V(t)}{i(t)} = R$

→ Impedance - Restriction

→ equivalent impedance of Resistor is Z

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(2) $i(t) = I_m \cos\omega t$

$$v(t) = L\frac{d(i(t))}{dt}$$

$$= L\left(I_m e^{j\omega t} j\omega\right)$$

$$= L j\omega(\vec{I})$$

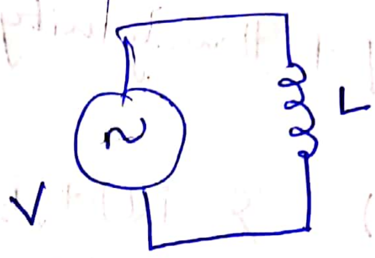
$$\vec{V} = j\omega L \vec{I}$$

$$Z = \frac{\vec{V}}{\vec{I}} = j\omega L$$

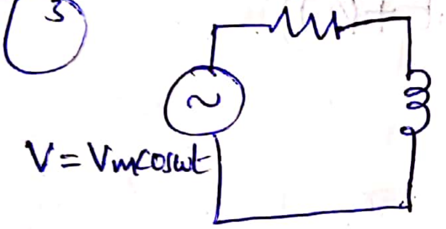
$$\vec{I} = I_m \angle 0^\circ$$

$$= I_m e^{j\omega t}$$

$$\frac{di}{dt} = I_m e^{j\omega t}(j\omega)$$



(3)

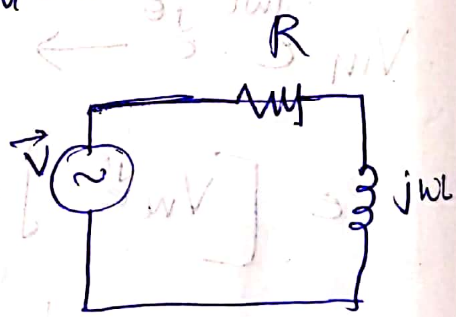


$$\vec{V} = V\angle 0$$

$$= Re\left[Ve^{j\omega t}\right]$$

$$= V_m \cos\omega t$$

$$\vec{I} = \frac{\vec{V}}{R + j\omega L}$$



$$\vec{I} = \frac{V_m \angle 0}{Z_m \angle \phi}$$

$$\vec{I} = \frac{V_m}{Z_m}\angle 0 - \phi$$

1/11/19

1) phasor notation
2) concept of lead & lag
3) concept of impedance
4) Re-collection of theorems - Thevinon & Norton

$v(t) = V_m \cos\omega t$ any circuit having L & C

→ Differential equations $\xleftrightarrow{\text{Transform}}$ Linear set of equation

$v(t) = V_m \cos\omega t + j V_m \sin\omega t$

$= V_m e^{j\omega t}$

Time Domain | Phasor Domain

$V_m \cos\omega t \rightarrow Re\left[V_m e^{j\omega t}\right] \rightarrow V_m \angle 0^\circ$

$V_m \cos(\omega t + \phi) \rightarrow Re(\cancel{V}. \rightarrow V_m \angle \phi$

$V_m \sin\omega t \rightarrow V_m \angle -90^\circ \rightarrow V_m \cos(\omega t - 90)$

$V_m \sin(\omega t + \phi) \rightarrow$ $V_m \angle \phi - 90^\circ$

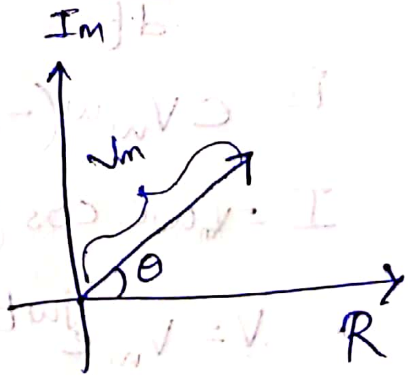
→ $\sin\omega t$ is lagging $\cos\omega t$ by $\pi/2$

$\vec{V}\angle\theta = V_m \cos(\omega t + \theta)$

R

$I = \frac{V_m}{R} \cos(\omega t + \theta)$

$= \frac{\vec{V}\angle\theta}{R} = \vec{I}_m \angle\theta$



$V = L\frac{di}{dt}$ → cause

effect

$I_m \angle 0^\circ = I_m \cos\omega t$

$V = L \frac{d\, I_m \cos\omega t}{dt} = L(\omega) I_m (-\sin\omega t)$

$= I_m(\omega L)\cos(\omega t + 90)$

$\boxed{\vec{V} = j\omega L (\vec{I}}$

$I_m \cancel{V_m} \cos\omega t \longrightarrow \omega L I_m \cos(\omega t + 90^\circ)$

$\vec{V} = L\frac{di}{dt} = L\left(\frac{d I_m e^{j\omega t}}{dt}\right)$

$= L(\omega) j I_m e^{\omega t}$

$V = L\frac{di}{dt} \Rightarrow I = I_m e^{j\omega t}$

In pure Inductor

$\cancel{I = I_m \cos\omega t \angle 0^\circ}$

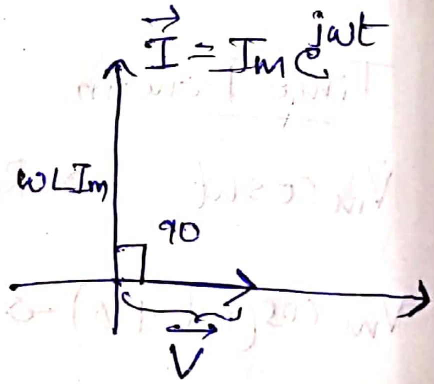
$\cancel{V = \omega L I_m} \sin\omega t$

$V = L I_m (j\omega) e^{j\omega t}$

$= j\omega L \cdot (I_m e^{j\omega t})$

$= j\omega L \cdot \vec{I}$

$\vec{V} = (\omega L) \vec{I} \cdot \angle 90^\circ$



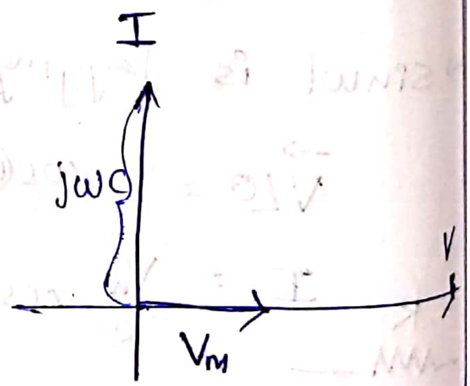
voltage leads by 90°

For capacitor

$i = C\frac{dV}{dt}$ $\quad V = V_m \cos\omega t$

$i = C V_m \omega(-\sin\omega t)$

$I = V_m \omega C \cos(\omega t + 90^\circ)$



$V = V_m e^{j\omega t}$ $\quad \boxed{\vec{I} = j\omega C \vec{V}}$

$\vec{I} = \frac{-j}{\omega L} \vec{V}$

$\vec{V} = \frac{1}{j\omega C}\vec{I} = \frac{-j}{\omega C}\vec{I}$

$$V = L\frac{d\,I_m\cos\omega t}{dt} = -\omega L I_m \sin\omega t$$

$$= \omega L I_m \cos(\omega t + 90)$$

In Resistance - both V, I are in same phase

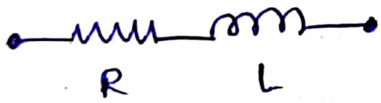
In capacitor - voltage lags current by 90°

Z - Impedance

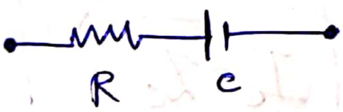
$$\text{Reactance} = \frac{V}{I}$$

$$\vec{V} = \vec{Z}\vec{I} \qquad \vec{I} = \vec{Y}\vec{V}$$

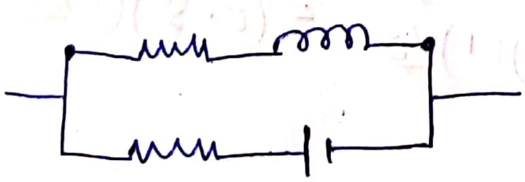
	Z	Y
R	R	$1/R$
L	$j\omega L$	$-j/\omega L$
C	$\frac{-j}{\omega C}$	$j\omega C$



$$Z_1 = R + j\omega L$$

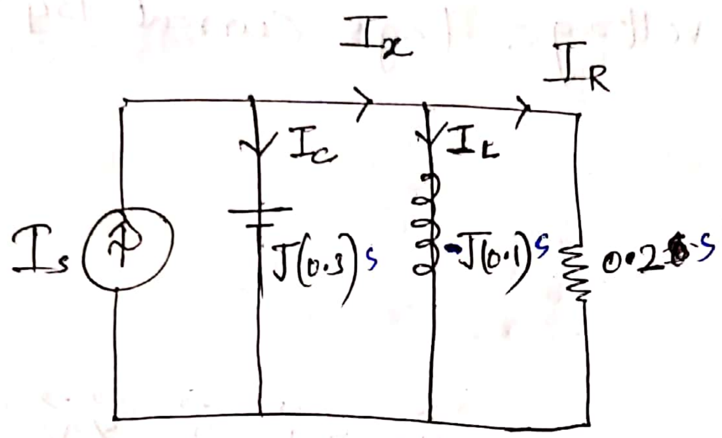


$$Z_2 = R - \frac{j}{\omega C}$$



$$Z = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

① Construct phasor diagram showing IR, IL, IC for the below circuit. Determine the angle by which IS leads IR, IC and I_x.



$V = I_m \cos\omega t$

$V = 1\angle 0^\circ$

$\vec{V} = \vec{Z}\vec{I}$

$\Rightarrow I_s = I_x + I_c$

$I_x = I_R + I_L$

$\Rightarrow \vec{V} = \frac{1}{j\omega C}\vec{I_c}$

$= \frac{\vec{V}}{0.2} + \frac{\vec{V}}{\omega L}$

$\vec{V} = \omega L \vec{I_L}$

$I_s = \frac{\vec{V}}{0.2} + \frac{\vec{V}}{\omega L} + j\omega C(V)$

$\vec{V} = (0.2) I_R$

sol.

$\boxed{I_R = GV}$ $\boxed{I_L = \frac{1}{Z_L}V}$ $\boxed{I_c = \frac{1}{Z_c}(V)}$

$= (0.2)(1)\angle 0$ $= (0.1)(1)\angle 0$ $= (0.3)(1)\angle 0^\circ$

$= 0.2\angle 0^\circ$

$I_x = \cancel{0.2} + 0.1$

$\cancel{= 0.3}$

$I_s = I_x + I_c$

$\frac{\omega t - \pi/2}{\omega t}$

$= 1 - \frac{(\pi/2)}{\omega t}$

$= 0.2\angle 0^\circ + 0.1\angle -90^\circ$

$= 0.3 + 0.3$

$= 0.2 V_m \cos\omega t + 0.1 (1)\cos(\omega t - 90)$

$= 0.6\angle 0^\circ$

$= \sqrt{(0.2)^2 + 0.1^2}\left(\tan^{-1}\left(\frac{b}{a}\right)\right)$

$= 0.6 \cos\omega t$

$I_L = -J(0.1)\,1(\angle 0) = 0.1 \angle -90$ $= 0.1 \sin\omega t = 0.1\angle -90$ $= 0.1\cos(\omega t - 90)$

$I_C = J(0.3)\,1\angle 0 = 0.3 \angle 90$ $0.3\cos(\omega t + 90)$

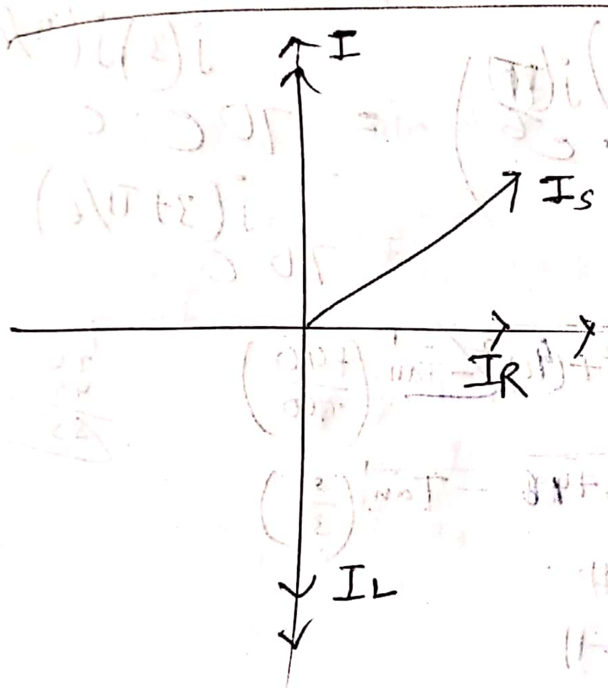
$I_S = I_c + I_x = 0.2 + 0.2J = 0.283 \angle 45^\circ$

$I_x = 0.224 \angle -26.6^\circ = 1\angle 0^\circ I_R + I_L(\angle -90)J = 0.2\angle 0^\circ + 0.1\angle -90$

$0.3\cos(\omega t + 90)$

$= 0.3$

$$\boxed{a - Jb = \sqrt{a^2+b^2} \angle -\tan^{-1}\left(\frac{b}{a}\right)}$$

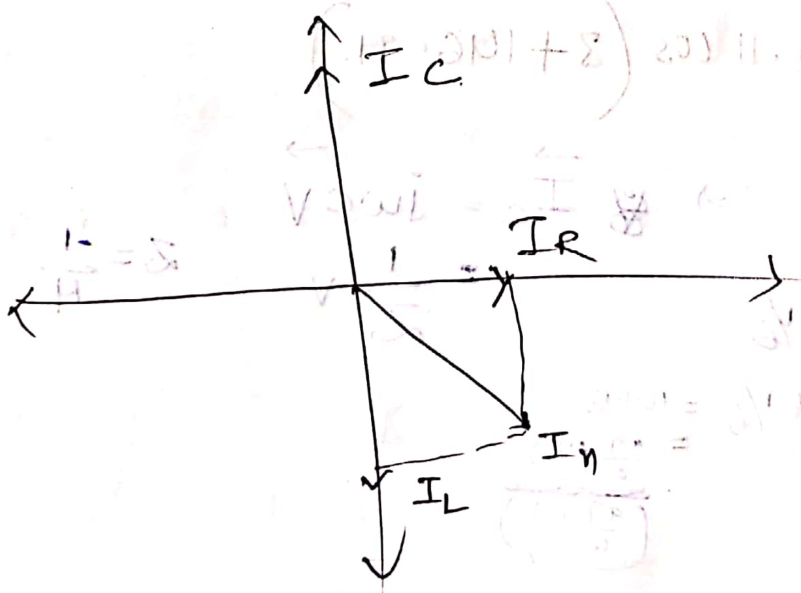


$0.2 - J(0.1) =$

$\Rightarrow I_s$ leads I_R by 45

$\Rightarrow I_c$ by -45°

$\Rightarrow I_x$ by $45 - (-26.6)$
$= 71.6$



$\vec{V} = \vec{I}R$ $\vec{I}_L = \frac{-j}{\omega L}(\vec{V})$

$\vec{V} = j\omega L \vec{I}_L$

$\boxed{\vec{I}_c = j\omega C \vec{V}}$

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Q) If $\omega = 600$ rad/s, $t = 5$ msec Find inst value

(a) $70\angle 30^\circ$ (b) $-60 + 40J$

$$V(t) = V_m \cos\omega t = Re\left[V_m e^{j\omega t}\right]$$

$$= 70\left[e^{j(\omega t+30)}\right] = 70\cos(\omega t + 30)$$

$$= 70\left(e^{j(600\times5\times10^{-3})} \cdot e^{j\left(\frac{\pi}{6}\right)}\right) \quad = 70\cos\left(3 + \frac{\pi}{6}\right) = -64.95V$$

$$= 70\left(e^{j(3)} \cdot e^{j\left(\frac{\pi}{6}\right)}\right) = 70\, e^{j(3)} \cdot e^{j(\pi/6)}$$

$$= 70\, e^{j(3+\pi/6)}$$

(b) $-60 + 40J = \sqrt{(60)^2 + (40)^2}\angle -\tan^{-1}\left(\frac{+40}{+60}\right)$

$a - Jb$

$a = -60$

$b = -40$

$$= \cancel{10\sqrt{36+49}} - \tan^{-1}\left(\frac{2}{3}\right)$$

$= \cancel{72.11}$

$= \cancel{72.11}$

$$\begin{array}{r} 36 \\ 49 \\ \hline 85 \end{array}$$

$$V_r = 72.11\cos(3 + 146.31)$$

$$I_L = \frac{-j}{\omega C}\vec{V} \Rightarrow \vec{I}_C = j\omega C\vec{V}$$

$$= \frac{1}{Z}V \qquad Z = \frac{-1}{4}$$

$= 2 + 1/4 \qquad = 8 + 1/6$

$=$

$8 + 8 + 1/6 = 16 + 1/6$

$$= \frac{\frac{97}{6}(1/3)}{\left(\frac{97}{6} + \frac{1}{7}\right)}$$

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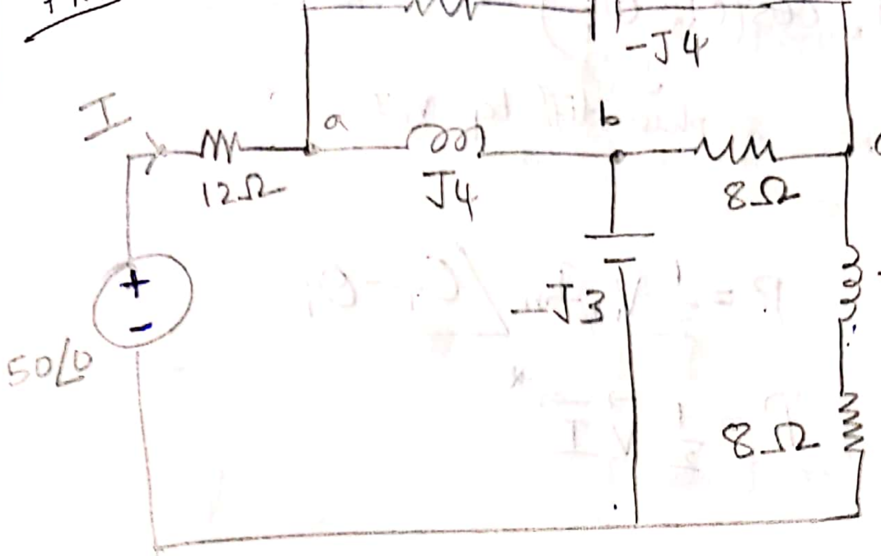
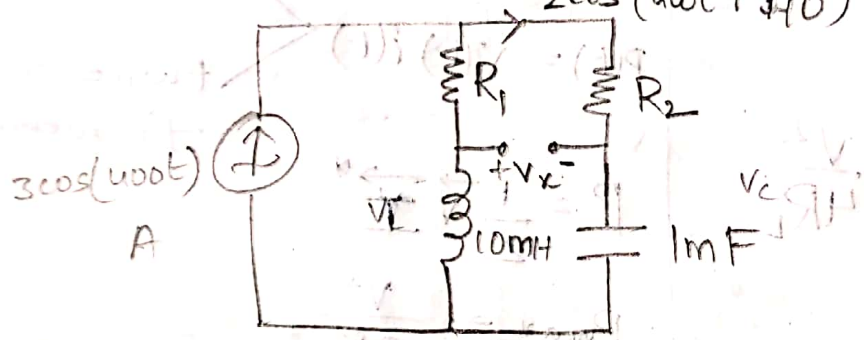
Find I



$I = 3.66 \angle 4.2\ A$

Find 'V_x' for following circuit



$I\ J_x L$

$I\ \frac{-1}{J_x}C$

$V_x = V_L - V_c$

$V_x = 9.892 \angle 78.76°$

6/11/19

observation time $\quad t \to \infty$ steady state

$v(t) = V_m \cos(\omega t + \phi)$

$\vec{v} = V_m \angle \phi$

$\rightarrow P = I^2 R$
$= I(V)$

$p(t) = I(t) \cdot V(t)$

$v(t) = V_m \cos(\omega t + \theta_v)$

$i(t) = I_m \cos(\omega t + \theta_i)$

$$P(t) = \frac{1}{2} V_m I_m \left[\cos(\theta_v - \theta_i)\right] + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

$P_{avg} = \frac{1}{T} \int_0^T p(t)\,dt$ the avg of cosine fun over its period $= 0$

$$P_{avg} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

$\downarrow$ phase diff by v, I

$$\vec{V} = V_m \angle \theta_v$$

$$\vec{I} = I_m \angle \theta_i \qquad P = \frac{1}{2} V_m I_m \angle \theta_v - \theta_i$$

$$\vec{I}^* = I_m \angle -\theta_i \qquad P = \frac{1}{2} \vec{V} \vec{I}^*$$

↳ complex conjugate

DC	AC
$P = VI$	$p(t) = v(t)\, i(t)$
$P_{max} = \frac{V^2}{4R_L}$	$P = \frac{1}{2} \vec{V} \vec{I}^*$
	$P_{max} = \frac{V^2}{8R_L}$

independent frequency

twice the frequency

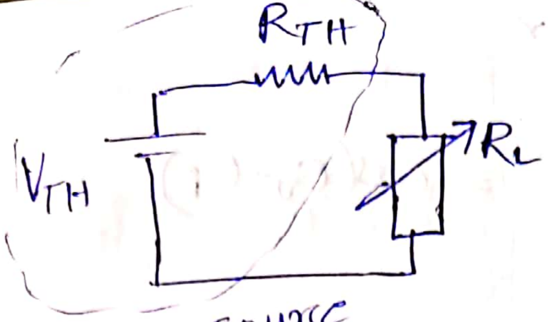
$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

In Resistor $\theta_v = \theta_i$

$P = \frac{1}{2} V_m I_m = \frac{1}{2} I_m^2 R$ more power dissipated

In pure inductor (or) Capacitor $P = 0$

→ using sinusodial analysis we can convert it into any arbitary signal

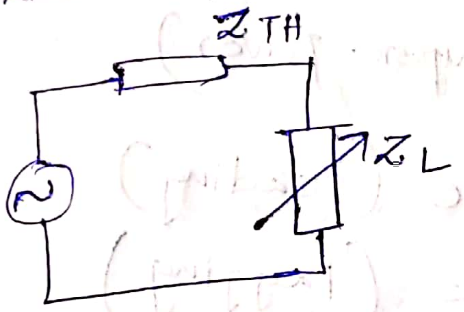


$R_{TH} = R_L$

thevinon Resistance = loaded Resistance

$$I = \frac{V_{TH}}{R_L + R_{TH}}$$

→ maximum power transfers when $Z_{TH} = Z_L^*$



$$\frac{\partial P}{\partial R_L} = 0 \qquad \frac{\partial P}{\partial X_L} = 0$$

$P = I^2 R$

$$P = \frac{1}{T}\int_0^T I^2 R\,dt = \frac{R}{T}\int_0^T I^2\,dt = R \cdot I_{eff}^2$$

for any periodic function

$$I_{eff} = \sqrt{\frac{1}{T}\int i^2\,dt} = I_{rms}$$

$i = I_m \cos\omega t$

$$I_{rms} = \frac{I_m}{\sqrt{2}}, \qquad V_{rms} = \frac{V_m}{\sqrt{2}}$$

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

$$= \frac{1}{2} \frac{V_{rms}(\sqrt{2})}{\cancel{\sqrt{2}}} \cdot \frac{I_{rms}(\sqrt{2})}{\cancel{\sqrt{2}}} \cos(\theta_v - \theta_i) = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$$

$$P = V_{rms} I_{rms} \cos(\theta_v - \theta_i) = S\cos(\theta_v - \theta_i)$$

$S \rightarrow$ Apparent power

$$\text{powerfactor } (P_f) = \frac{P}{S} = \cos(\theta_v - \theta_i)$$

$$\vec{S} = \vec{P} + j\vec{Q}$$

$$\vec{P} = S\cos(\theta_v - \theta_i)\;;\; \vec{Q} = S\sin(\theta_v - \theta_i)$$

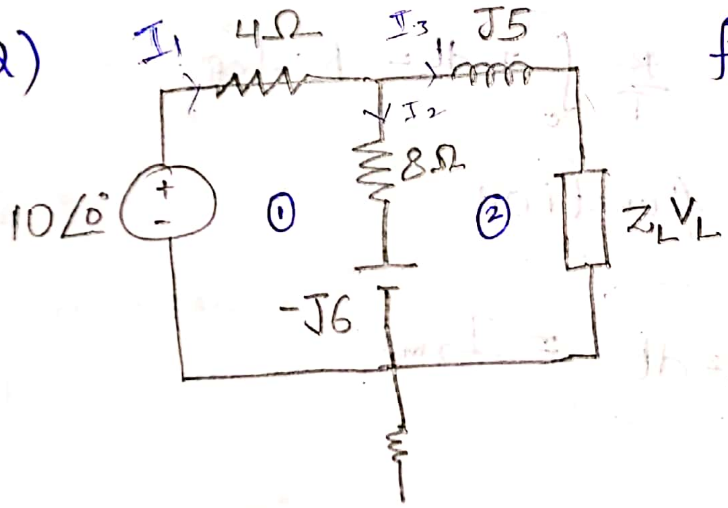
$S \rightarrow VA$ $\qquad \theta_v = \theta_i \qquad S = P$, $Q = 0$

$P \rightarrow W$

$Q \rightarrow VAP$ (voltage. Ampere. power)

$\theta_v < \theta_i \qquad Q \rightarrow -ve$ (leading)

$\theta_v > \theta_i \qquad Q \rightarrow +ve$ (lagging)

(Q) 

find P_{max} delivered to 'Z_L'

$$P_{max} = \frac{V_L^2}{8R_L}$$

$\Rightarrow$ $\vec{V} = j\omega L \vec{I}$

$\vec{V} = Z_L I$

$\vec{V} \Rightarrow \vec{V_L} = 5 I_L$

$\vec{V} = I_3(5 + Z_L)$

$$I_3 = \frac{\vec{V}}{5 + Z_L}$$

$\vec{V} = I_2(8 -$

loop 1:

$\rightarrow 10\angle 0^\circ - \left(I_1(4) + I_2(8) + I_2(-J6)\right) = 0$

$\rightarrow$ loop 2:

$I_3(J5) + (-I_2 8) - I_2(-J6) + Z_L(I_3) = 0$

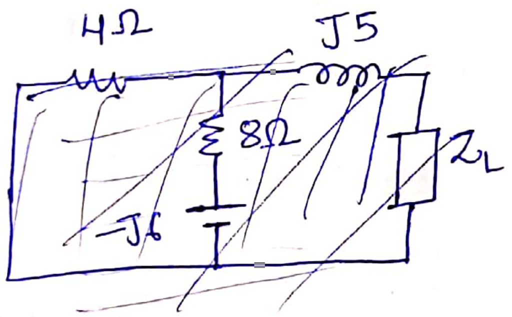
loop3 :

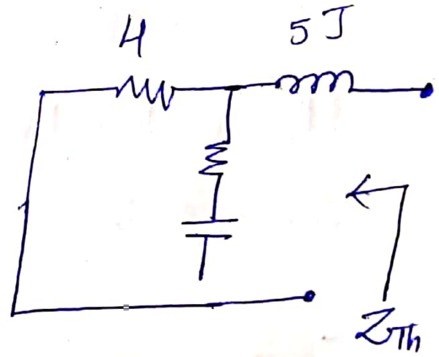
$I_1(4) + I_3(J5) + Z_L(I_3) - 10\angle 0^\circ = 0$

$\rightarrow 10\cos\omega t - 4I_1 - 8I_2 + I_2(J6) = 0$

$\Rightarrow 10\cos\omega t - 4I_1 + I_2(6J - 8) = 0$

$5JI_3 - 8I_2 + 6JI_2 + Z_L I_3 = 0$





$Z_{TH} = J5 + 4 \,||\, (8 - J6)$

$P_{max} = \dfrac{V_{Th}^2}{8R_L}$

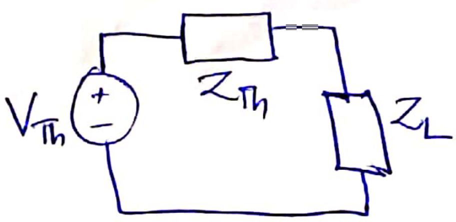
$Z_L = Z_s^* = Z_{Th}^* = R_L - JX_L$

$V_{Th} = 10\left(\dfrac{8-J6}{8-J6+4}\right)$

$V_L = 7.454\angle -10.3$

$Z_L = 2.933 - j4.467\,\Omega$

$P_{max} = 2.368\,W$



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$$8 - J6 = \sqrt{8^2+6^2}\angle \tan^{-1}\left(\frac{6}{8}\right)$$

$$= 10 - \angle \tan^{-1}(3/4) = 10(-36.8)$$

① complex power $(S) = P + JQ = V_{rms} \cdot I^*_{rms}$

② Apparent power $(S) =$ ~~mag~~ of $V_{rms} \cdot I_{rms} = |S|$ (magnitude

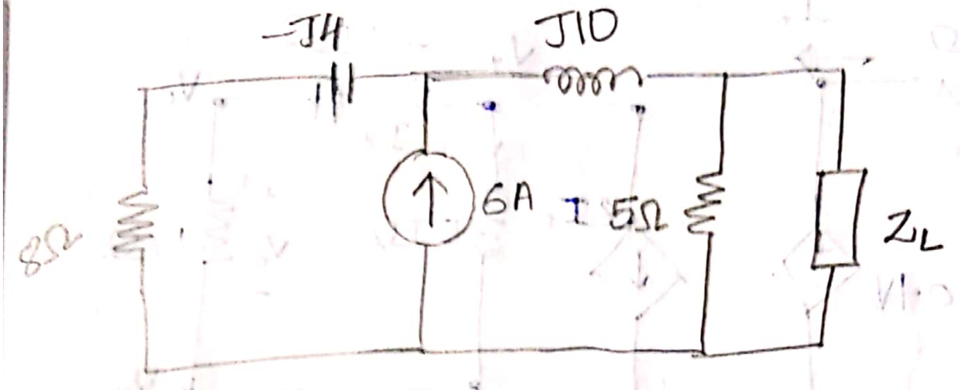
③ Real power $(P) = Rel(S)$

④ Reactive Power $(Q) = Img(S)$

⑤ power factor $= P/S = \cos(\theta_v - \theta_i)$

$$Z_{TH} = 4 + (-368.6) + 5J = 5J - 364.6$$

$=$



Find Z_L & max avg power (P_{max})

(2) The voltage across a load is V_t (instantenous) $= 60 \cdot \cos(\omega t - 10)$ and the current through element in direction of the voltage drop is $i_t = 1.5\cos(\omega t + 50)$. Find the complex and apparent power. The real and reactive power, power factor and load impedance.

(sol) $\Rightarrow$ complex power $(S) = 45\angle -60^\circ$

$\Rightarrow$ Apparent power $|S|$

$P(t) = V(t) \cdot I(t) = 60\cos(\omega t - 10)\, 1.5\cos(\omega t + 50)$

$P_{avg} = V_{rms} \cdot I_{rms} \cos(\theta_v - \theta_i)$

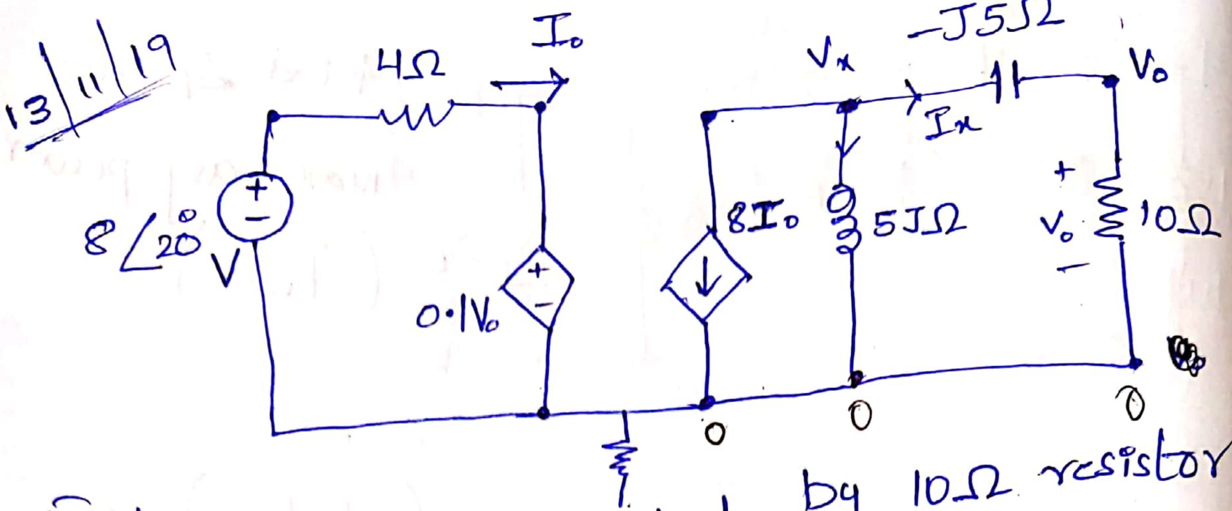
$= \frac{60}{\sqrt{2}} \cdot \frac{1.5}{\sqrt{2}} \cos(-10 - 50)$

Real power $= \frac{60}{\sqrt{2}}$.

$= \frac{\overset{30}{\cancel{60}} \times 1.5}{\cancel{2}} \cos(-60)$

$= 45\cos(-60)$

13/11/19



Find avg power absorbed by 10Ω resistor

$$P_{avg} = I(t)\, v(t) = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

$$\Rightarrow \quad 10 - 5J \Rightarrow \sqrt{(10)^2 + (5)^2} \angle -\tan^{-1}\left(\frac{5}{10}\right)$$

$$\Rightarrow \quad = \sqrt{125}$$

$$= 11.18 \angle$$

$$\Rightarrow \quad 4(I_o) + 0.1V_o - 8\angle 20^\circ = 0$$

$$\Rightarrow \quad 4(I_o) + 0.1V_o - 8\cos(\omega t + 20) = 0 \longrightarrow (1)$$

$\Rightarrow$ -

$$V = J\omega L(I$$

$$8I_o + \frac{V_x}{5J} + \frac{V_x}{10-5J} = 0 \rightarrow (2)$$

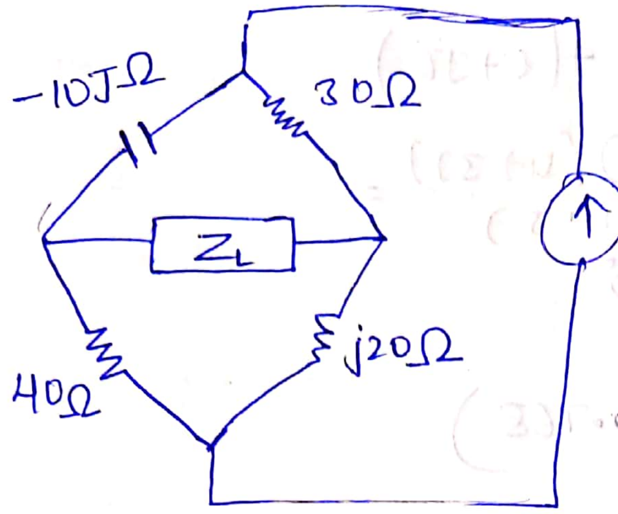
$$I_x = \frac{V_o}{10}$$

$$V_o = V_x\left(\frac{10}{10-5J}\right)$$

$$V_o = \frac{80\angle 20^\circ}{(1+J)}$$

$$P_{avg} = I_{rms}^2 R_L$$

$$160\,\text{W} = \frac{1}{2} |I_x|^2 R_L$$

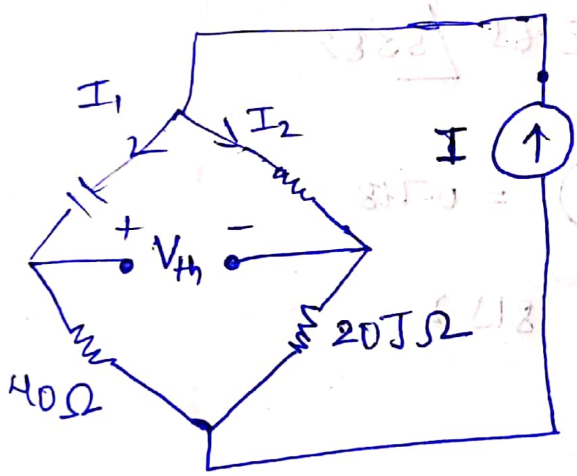


Calcultae Z_L =

max Avg power across Z_L

$Z_L \rightarrow$

$(Z_{Th})^* = (Z_L^*)^*, Z_L = Z_{Th}^* = 20\,\Omega$



$V_{Th} = 30I_2 + 10JI_1$

$I_2 = 2J\left(\dfrac{40-10J}{40-10J+30+20J}\right)$

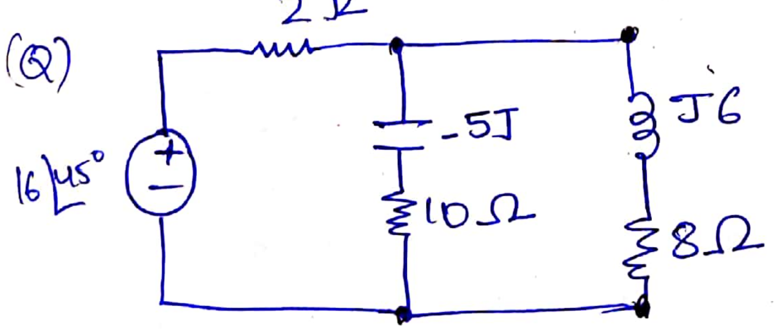
$I_1 = 2J\left(\dfrac{30+20J}{70-10J}\right)$

⇒ calculate short circuit source, open circuit Z_{Th}

Z_{Th}

$a+bJ$

(Q)



$6J+8$

$= \sqrt{36+64}$

$= 10 \tan^{-1}\left(\frac{6}{8}\right)$

$\Rightarrow \dfrac{(10-5J)\cdot(8+6J)}{(10-5J+8+6J)} = \dfrac{(10-5J)(8+6J)}{(18+1J)} + 2$

$R =$

$$Z_T = 2 + (10 - J5) \parallel (8 + J6)$$

$$= 2 + \frac{10(2-J)(4+3J)}{(18+J)} =$$

$$= 8188 \angle 5382^\circ$$

$$= 8.152 + J(0.768)$$

$$PF = \cos(5382) = 0.9956$$

$$avg = \frac{1}{2} \frac{|V|^2}{Z^*} = 15.63 \angle 5382$$

Reactive power $(Q) = 0.768$

Real power $= 8152$

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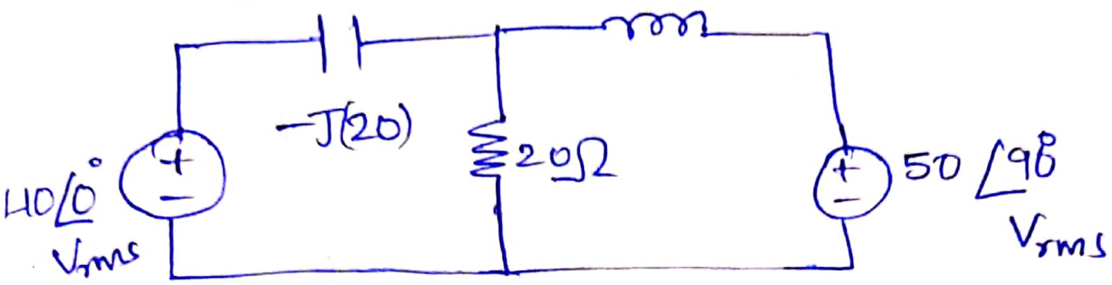
(Q) find complex power absorbed by each element



60
360
6

→ Generator consist of stator (stationary) and rotor (motion)

→ 3-phase output

→ 3-phase voltage sources (a) Y- connected source
(b) Δ - connected

phase voltage - line voltage
phase current - line current