Lecture 2

Instructor: Subrahmanyam Kalyanasundaram

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Runway Reservation System

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- Reservation requests for future landings
  - Need to land at time t
- We can approve landing request if no other landing within k minutes
- Once approved, we can add t to the set R of landing times
- Remove t from the set after plane lands

Runway Reservation System

- Let $|R| = n$
- Ideally, all the operations to be done in $O(\log n)$ time

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- Sorted List:
- Insertion is $O(1)$, but search is $O(n)$

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Fast insertion into a sorted array

Trees

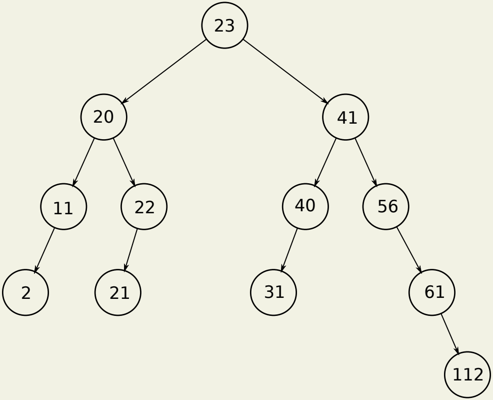
- Root
- Parent, Child
- Ancestor, Descendant
- Sibling
- Leaves, Internal Nodes
- Depth, Height

Trees

- Organization Structure
- File System
- Family Tree

Binary Trees

A binary tree is an ordered tree in which every node has at most 2 children.

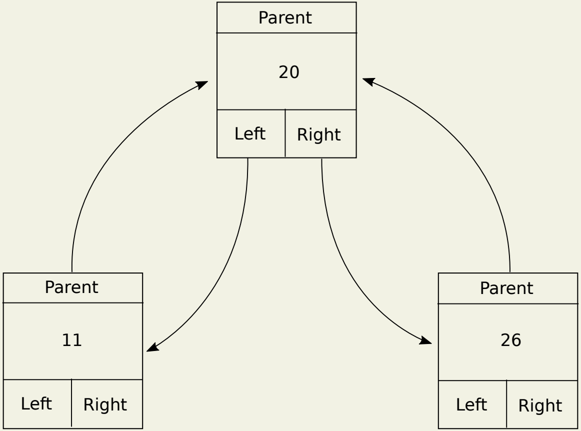


Implementation

Similar to a node in a linked list, each node in a Binary Tree has the following:

- int *val* – holds the data/value of the node.
- Left child pointer.
- Right child pointer.
- Parent node pointer.

Data Structure



Questions

1. What is the maximum height of a Binary Tree with n nodes?
2. What is the minimum height of a Binary Tree with n nodes?

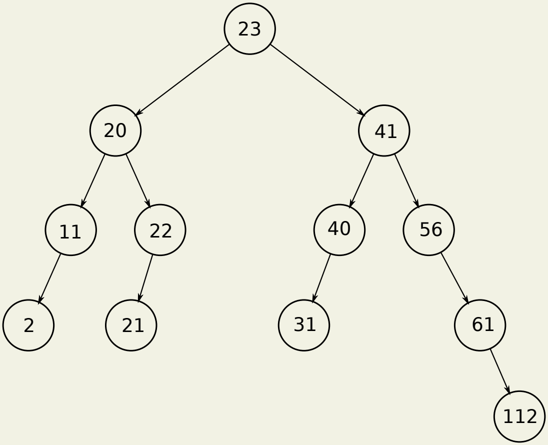
Binary Search Tree

A Binary Search Tree (BST) is a tree that satisfies the following:

- For every node X in the BST, we have:

  $$\text{Values in left subtree} \leq \text{value}(X) \leq \text{Value in right subtree}$$

Example BST



Abstract Data Type - BST

A BST supports the following functions:

- INSERT(*node*, *val*) – Inserts *val* into the BST rooted at *node*.
- SEARCH(*node*, *val*) – Returns True of *val* exists in the BST rooted at *node*. False otherwise.
- SUCC(*val*) – Returns the smallest element greater than *val* in the BST.
- PRED(*val*) – Returns the largest element lesser than *val* in the BST.
- DELETE(*val*) – Deletes *val* from the BST.

Example BST

The order in which elements are inserted makes a difference!
Consider two different sequences of values:

- **Sequence A**: 23, 11, 20, 21, 2, 56, 40, 41
- **Sequence B**: 2, 11, 20, 21, 23, 40, 41

(See whiteboard).

Insert procedure

The $\textsc{Insert}(node, x)$ procedure:

- If $node = \text{NULL}$, create new node with x and attach to parent.
- Else If $x < \text{value}(node)$,
  - $\textsc{Insert}(node \rightarrow left, x)$
- Else If $x > \text{value}(node)$ Then,
  - $\textsc{Insert}(node \rightarrow right, x)$

Binary Search Trees

Recall that a Binary Search Tree (BST) has the following crucial property:
For every node X in the BST, we have:

- Every node in the left subtree of X contains a value smaller than that of X.
- Every node in the right subtree of X contains a value larger than that of X.

Questions

- Write the SEARCH($node$, x) procedure.

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- If $node = \text{NULL}$, then return NULL

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- Else If *x* = value(*node*), then return *node*

Questions

- Write the $\textsc{Search}(node, x)$ procedure.

- $\textsc{Search}(node, x)$:
- If $node = \text{NULL}$, then return NULL
- Else If $x = \text{value}(node)$, then return *node*
- Else If $x < \text{value}(node)$, then
  - Return $\textsc{Search}(node \to left, x)$

Questions

- Write the $\textsc{Search}(node, x)$ procedure.

- $\textsc{Search}(node, x)$:
- If $node = \text{NULL}$, then return NULL
- Else If $x = \text{value}(node)$, then return *node*
- Else If $x < \text{value}(node)$, then
  - Return $\textsc{Search}(node \rightarrow left, x)$
- Else
  - Return $\textsc{Search}(node \rightarrow right.x)$