Homework Assignments IV

MA1130 VECTOR CALCULUS

January 26, 2019

1. Let $\mathbf{f}(x,y) = P(x,y)\hat{\boldsymbol{\imath}} + Q(x,y)\hat{\boldsymbol{\jmath}}$ defined over the region $R = \{(x,y) : 0 < x^2 + y^2 \leq 1\}$ where
$$P(x,y) = \frac{-y}{x^2+y^2} \text{ and } Q(x,y) = \frac{x}{x^2+y^2}.$$
Show that the Green's Theorem does not hold for this setup. Explain why.

2. Is there a potential $F(x,y)$ for $\mathbf{f}(x,y) = (x^3\cos(xy) + 2x\sin(xy))\hat{\boldsymbol{\imath}} + x^2y\cos(xy)\hat{\boldsymbol{\jmath}}$? If so, find one.

3. Show that for any constants a, b and any closed simple curve C, $\oint_C a dx + b dy = 0$.

4. Evaluate $\oint_C e^x \sin y dx + (y^3 + e^x \cos y)dy$, where C is the boundary of the rectangle with vertices $(1,-1), (1,1), (-1,1)$ and $(-1,-1)$, traversed counterclockwise.

5. For a region R bounded by a simple closed curve C, show that the area A of R is
$$A = \oint_C y dx = \oint_C x dy = 1/2 \oint_C x dy - y dx,$$
where C is traversed so that R is always on the left.

6. Evaluate the surface integral $\iint_\Sigma \mathbf{f} \cdot d\sigma$, where $\boldsymbol{f}(x,y,z) = yz\hat{\boldsymbol{\imath}} + xz\hat{\boldsymbol{\jmath}} + xy\hat{\boldsymbol{k}}$ and Σ is the part of the plane $x + y + z = 1$ with $x \geq 0, y \geq 0$, and $z \geq 0$, with the outward unit normal n pointing in the positive z direction.

7. Use a surface integral to show that the surface area of a sphere of radius r is $4\pi r^2$.

8. Use a surface integral to show that the surface area of a right circular cone of radius R and height h is $\pi R\sqrt{h^2 + R^2}$

9. prove that the surface area S over a region R in R^2 of a surface $z = f(x, y)$ is given by the formula

$$S = \iint\limits_R \sqrt{1 + \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}\, dx\, dy$$

(Hint: Think of the parametrization of the surface.)