AI1001: Introduction to Modern AI
Homework Assignment 4
Due Date: 28 August 2019

1. Consider the Markov process with the state transition matrix

$$A = \begin{bmatrix} 0.1 & 0.4 & 0.3 & 0.2 & 0 \\ 0.1 & 0 & 0.4 & 0.3 & 0.2 \\ 0 & 0.3 & 0 & 0.5 & 0.2 \\ 0 & 0.1 & 0.3 & 0.2 & 0.4 \\ 0 & 0.2 & 0.3 & 0.4 & 0.1 \end{bmatrix}.$$

   Number the states consecutively from 1 to 5. What are the absorbing states, if any?

2. Consider the Markov process with the state transition matrix

$$A = \begin{bmatrix} 0.1 & 0.4 & 0.3 & 0.2 & 0 \\ 0.1 & 0 & 0.4 & 0.3 & 0.2 \\ 0 & 0.3 & 0 & 0.5 & 0.2 \\ 0 & 0.1 & 0.3 & 0.2 & 0.4 \\ 0 & 0 & 0 & 0 & 1.0 \end{bmatrix}.$$

   Number the states consecutively from 1 to 5.

   - Find the absorbing state(s) if any.
   - Compute the average number of time steps needed to reach the absorbing state, starting from each of the other states.

3. Now consider the Markov process with the following state transition matrix

$$A = \begin{bmatrix} 0.1 & 0.4 & 0.3 & 0.2 & 0 & 0 \\ 0.1 & 0 & 0.4 & 0.3 & 0.1 & 0.1 \\ 0 & 0.3 & 0 & 0.4 & 0.1 & 0.2 \\ 0 & 0.1 & 0.3 & 0.3 & 0.2 & 0.1 \\ 0 & 0 & 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

   - What are the absorbing states?
   - For each of the nonabsorbing states, compute the probability of hitting each of the absorbing states.

4. Recall Bellman's optimality equation for Markov Decision Processes: Define $V^*(x)$ to be the **Highest possible reward** that can be achieved by any "policy" (method of selecting actions for each state), when starting at state x. Then V^* satisfies

$$V^*(x) = \max_{u \in U} \left(R(x,u) + \gamma \sum_{y \in \mathbb{N}} A^u_{x,y} V^*(y) \right).$$

   Justify the validity of the Bellman equation by interpreting each term on the right side of the equation.