MA2120 Transform Techniques: Fourier Integral

IIT Hyderabad

1.Find the fourier integral representation of the given function

$$f(x) = \begin{cases} 0, & \text{if } x < 0 \text{ and } x > 2; \\ 1, & \text{if } 0 \leq x \leq 2; \end{cases}$$

2.Find the fourier integral of the function

$$f(x) = \begin{cases} 0 & \text{if } x < 0; \\ \frac{1}{2} & \text{if } x = 0; \\ e^{-x} & \text{if } x > 0; \end{cases}$$

Verify the the representation directly at point x=0.

3.Using fourier representation method, show that

$$f(x) = \begin{cases} 0 & \text{if } x < 0; \\ \pi & \text{if } x = 0; \\ \pi e^{-x} & \text{if } x > 0; \end{cases}$$

where

$$f(x) = \int_0^\infty \frac{\cos wx + w \sin wx}{1 + w^2} dw$$

4.Find the solution of the integral equation

$$\int_0^\infty f(x) \cos axdx = e^{-a}$$

where 'a' is constant.
5.Using fourier integral represntation, show that

$$f(x) = \begin{cases} \frac{\pi \cos x}{2} & \text{if } |x| \leq \frac{\pi}{2}; \\ 0 & \text{if } |x| > 0; \end{cases}$$

where

$$f(x) = \int_0^\infty \frac{\cos \frac{\alpha\pi}{2} + \cos \alpha x}{1 - \alpha^2} d\alpha$$

6.Given:

$$f(x) = \begin{cases} -1 & \text{if -a} < x < 0; \\ 1 & \text{if 0<x < a;} \end{cases}$$

Show that f(x) has fourier representation

$$f(x) \approx \frac{2}{\pi} \int_0^\infty \frac{[1 - \cos a\alpha] sinx\alpha}{\alpha} d\alpha$$

7.Consider the function

$$f(x) = \begin{cases} 0, & x < 0; \\ \cos x, & 0 < x < \pi; \\ 0, & \pi < \text{x}; \end{cases}$$

(a) Show that f(x) has fourier integral representation

$$f(x) \approx \frac{1}{\pi}\int_0^\infty \frac{\alpha[sin(\pi - x) - sinx\alpha]}{1-\alpha^2}d\alpha$$

(b) When x=0, show that

$$\int_0^\infty \frac{\alpha \sin x\alpha}{1-\alpha^2}d\alpha = \pi/2$$

8.Consider the function

$$f(x) = \begin{cases} 0 & \text{if } x < 0; \\ x & \text{if 0<x} < 1; \\ 0 & \text{if 1<x;} \end{cases}$$

(a) Drwa the graph of f(x)
(b) Find the fourier integral formula for f(x)
(c) Determine the convergence of fourier integral in part (b) at x=1.
9.Find the fourier sin and cosine integral representation of

$$f(x) = e^{-kx}$$

$$(k > 0, x > 0)$$

10.Using the result of the problem (9), find the fourier cosine integral representation of

$$f(x) = \frac{1}{1+x^2}$$

$$(x > 0)$$

11.Express the function f(x) as the fourier integral.

$$f(x) = \begin{cases} 1, & |x| \le 1; \\ 0, & |x| > 1; \end{cases}$$

Hence evaluate

$$\int_0^\infty \frac{\sin \lambda cos\lambda x}{2}d\lambda$$

and find the value of

$$\int_0^\infty \frac{\sin \lambda}{\lambda}d\lambda$$

12.Using fourier integral formula, prove that

$$e^{-ax} - e^{-bx} = \frac{2(b^2-a^2)}{\pi}\int_0^\infty \frac{x \sin xu}{[u^2+a^2][u^2+a^2]}du$$

13.Find the fourier cosine integral representation of the following function

$$f(x) = \begin{cases} x, & 0 \le x \le 5; \\ 0, & x > 5; \end{cases}$$

14.Find the fourier sine integral representation of the following function

$$f(x) = \begin{cases} \pi - x, & 0 < x < \pi; \\ 0, & x > \pi; \end{cases}$$

15.Using fourier integral formula, prove that

$$e^{-x}\cos x = \frac{2}{\pi}\int_0^\infty \frac{[\lambda^2+2]\cos x\lambda}{\lambda^2+4}d\lambda$$