CS6350: Topics in Combinatorics
Assignment 4

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1. **An "independent set" in a graph is a set of vertices such that no two of them have an edge between them.**
Let G be a graph on n vertices and let d denote the average degree of a vertex in G (i.e., d = (sum of degrees of all the vertices in G)/n). Then, show that G contains an independent set of size at least n/(d+1).

A. Let us consider that the vertices in the graph G with n vertices and average degree of vertex as d are arranged in a random permutation
$V = (v_1, v_2, v_3, ..., v_n)$.
Let us define an independent set I will contain a vertex v_i, if it appears before all its $N(v_i)$ neighbouring vertices in the considered permutation. As all the $N(v_i) + 1$(neighbouring vertices of v and the vertex v itself) has equal probability to occupy the front position, the probability that v would be in front position of these vertices is $\dfrac{1}{1 + N(v_i)}$.
So the expected number of vertices in the set I is

$$E = \sum_{i=1}^{n} \frac{1}{1 + N(v_i)}$$

Now consider,

$$AM \geq HM$$

$$\frac{\displaystyle\sum_{i=1}^{n} \frac{1}{1 + N(v_i)}}{n} \geq \frac{n}{\displaystyle\sum_{i=1}^{n} (1 + N(v_i))}$$

$$\frac{E}{n} \geq \frac{n}{n + \sum_{i=1}^{n} N(v_i)}$$

$$E \geq \frac{n}{1 + \left(\frac{\sum_{i=1}^{n} N(v_i)}{n} \right)}$$

$$E \geq \frac{n}{1+d} \qquad \left(\because d = \frac{\sum_{i=1}^{n} N(v_i)}{n} \right)$$

Hence it is proved that there exits an independent set atleast of size $\frac{n}{d+1}$.