<u>**All questions are objective type.**</u>

Throughout X, Y etc will denote **discrete** random variables and $m, n, \beta \in \mathbb{N}$.

Question 1. *In India there are β cases of a certain disease per million population per month (assume that $\beta << 10^6$). The probability of more than 4 cases of that disease (in a month) in a town having half a million population is* _____   *Marks:* 2

Question 2. *In a survey N married couples are observed (i.e. N men and N women). Few decades later, in a follow up survey, it was discovered that k individuals, among those who were surveyed, have died. Assuming that the deaths were random, the expected number of couple who survive is* _____   *Marks:* 2

Question 3. *Fix a rational $r = m/n$ (where m, n are coprimes). Let X, Y be independent random variables taking values in positive integers with probability mass function*

$$P(X = k) = P(Y = k) = \frac{1}{2^k}, \quad k = 1, 2, 3, \cdots$$

The probability of the event that $X = rY$ is given as _____   *Marks:* 2

Question 4. *True or False:*   *Marks:* 1×2

   (1) *Let $Y = \max(0, X)$. Then $E[Y] \geq E[X]$.*
   (2) *Let $m \geq n$ and let $X \sim Bin(m, p)$, $Y \sim Bin(n, p)$ then $X - Y \sim Bin(m - n, p)$.*

Question 5. *Let X be a positive discrete random variable satisfying*   *Marks:* 2

$$P(X > m + n \,|\, X > n) = P(X > m) \qquad m, n \geq 1$$

Denote $P(X = 1) = p$. Find the probability mass function of X in terms of p.