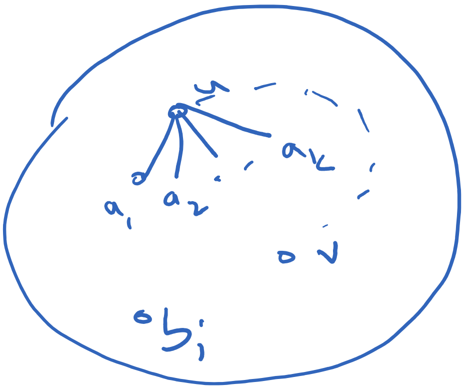

σ : $\underbrace{a_1 b_i u \cdot a_3 a_2 \cdots a_k}$ v

For any $\{u,v\} \notin E(G)$, (u,v) is separated in σ when $N[u]$ come/appear before v in σ.

Δ : max degree of G

$n \rightarrow$ # vertices.

$O(\Delta \log n)$ linear order one can separate all (u,v) pairs, when $\{u,v\} \notin E(G)$.

Independently and uniformly at random, take r linear orders $\sigma_1, \sigma_2, \ldots, \sigma_r$.

$\{u,v\} \notin E(G)$ $\quad u \Leftarrow \} \deg(u) \le \Delta$

$= \{u\} \cup N(u)$

$\{u,v\} \notin \ldots$ $\quad u \leftarrow \ldots \deg(u) \leq \Delta$ $\quad , \sigma_r.$

σ_i :

$$\underbrace{\underbrace{\overbrace{N[u] = \{u\} \cup N(u)}}_{\deg(u)+1} \quad v}_{\underbrace{\deg(u)+2} \text{ vertices}}$$

$$\Pr\left[\begin{array}{l}(u,v) \text{ is} \\ \text{separated} \\ \text{in } \sigma_i\end{array}\right] = \frac{(\deg(u)+1)!}{(\deg(u)+2)!}$$

$$= \frac{1}{\deg(u)+2}$$

$$\geq \underline{\underline{\frac{1}{\Delta+2}}} \quad \text{—— } (7)$$

$E(u,v,\sigma)$: the bad event that (u,v) pair is not separated in σ.

$$\Pr[E(u,v,\sigma)] \leq 1 - \frac{1}{\Delta+2}$$

$$\le e^{-\frac{1}{\Delta+2}} \qquad 1+x \le e^{x}$$

$$\geq \frac{1}{e^{\frac{1}{\Delta+2}}} \quad \text{—— } (2)$$

$E((u,v))$: bad event that (u,v) not separated in any of $\sigma_1, \sigma_2, \ldots, \sigma_r$.

$$\Pr[E(u,v)] \geq \Pr[E(u,v,\sigma_1) \wedge E(u,v,\sigma_2) \wedge \cdots \wedge E(u,v,\sigma_r)]$$

$$= \prod_{i=1}^{r} \Pr(E(u,v,\sigma_i))$$

$$\le \frac{1}{e^{\frac{r}{\Delta+2}}} \quad \text{—— } (3)$$

$$\Pr\Bigg[\bigvee_{\substack{(u,v) \text{ pair:} \\ \{u,v\} \notin E(G)}} E(u,v)\Bigg] \le \sum_{\substack{(u,v) \text{ pair:} \\ \{u,v\} \notin E(G)}} \Pr[E(u,v)]$$

$$\le \frac{n^2}{e^{\frac{r}{\Delta+2}}} \quad A$$

If $A < 1$, then there is a non zero prob that every pair is separated in some linear order.

So we want,

$$\frac{n^2}{e^{\frac{r}{\Delta+2}}} < 1$$

Putting, $r = 100\Delta \log n$, we can see that LHS < 1.

$$\frac{n^2}{e^{\frac{\cancel{100}^{50}\,\cancel{\Delta}\log n}{\cancel{\Delta+2}}}}$$

$$\le \frac{n^2}{e^{\log n^{50}}}$$

$$= n^2$$

$$\overline{}$$

$$n^{50}$$

$$< 1$$