All questions are objective type.

Throughout X, Y, D etc will denote **discrete** random variables.

Question 1. *A man plays a gamble with probability of head being* p. *The rules of the gamble are - at any round he bets* $1\$$. *If he wins then he will get* $1\$$ *extra. If he loses then he will lose his* $1\$$. *His strategy is this - he bets on head in the first round and if he wins then he quits the gamble. If he loses in the first round, then he bets on head for next two consecutive rounds. His expected earning (in simplest possible form) is* _______ *Marks:* 2

Question 2. *True or False:* *Marks:* 1×3

(1) *For any* $a < b$, *we have* $P(X < b \,|\, a < X) = P(a < X < b)$.
(2) *Let* $F : \mathbb{R} \to \mathbb{R}$ *be any function and let* $F(X) : \Omega \to \mathbb{R}$ *be the composed random variable. For any* $a \in \mathbb{R}$, $F^{-1}(a)$ *is a countable set.*
(3) *Suppose* X *and* Y *are independent random variables. Then* $F(X)$ *and* $G(Y)$ *are also independent (do NOT assume any special properties about* F *or* G *but you can use* (2) *above.)*

Question 3. *Fill in the blanks:* *Marks:* 1×3

(1) *Let* $E[X] = \mu$. *Define* $f(a) = E[(X - a)^2]$. *The value of* a *for which this function achieves its minimum is* _______
(2) *Let* $X_i \sim Poi(\lambda_i)$ *be a sequence of independent random variables for* $i = 1, 2, \cdots, n$. *Define* $Y = X_1 + X_2 + \cdots + X_n$. *Then* $E[Y^2]$ *is* _______
(3) *An experiment consists of a random number generator outputting digits*

$$\{0, 1, 2, \cdots, 9\}$$

(with equal probability). The experiment is run till we get all the 10 digits. Let N *denotes the number of times experiment was run then* $E[N]$ *is* _______

Question 4. *A fair coin is tossed* n *times. Let* D *denotes the number of heads minus the number of tails. Suppose* k *be any integer. Then* *Marks:* 1×2

(1) *the values of* k *for which* $P(D = k) \neq 0$ *are* _______
(2) *For any such* k *as above,* $P(D = k)$ *is* _______