Chapter 2 Particle properties of waves

Electronics: particles $\longrightarrow$ charge , mass Wave?

Electromagnetic wave: wave $\longrightarrow$ diffraction, interference,

Polarization particle?

Wave-particle Duality

2.1Emwaves

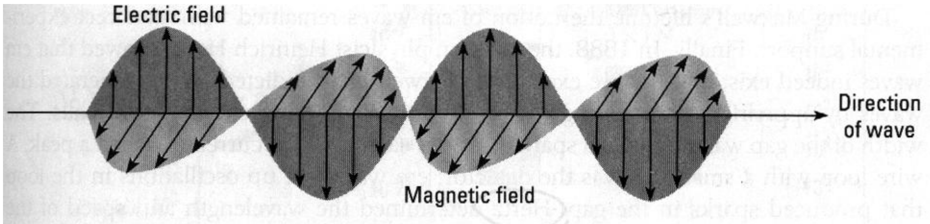


Figure 2.1 The electric and magnetic fields in an electromagnetic wave vary together. The fields are perpendicular to each other and to the direction of the wave.

Changing magnetic field $\longrightarrow$ current (or voltage)

Maxwell proposed: changing electric field $\longrightarrow$ magnetic field

Hertz created EM waves and determined the wavelength and speed of the wave, and showed that they both have E and B component, and that they could be reflected, refracted, and diffracted. $\longrightarrow$ Wave characteristic.

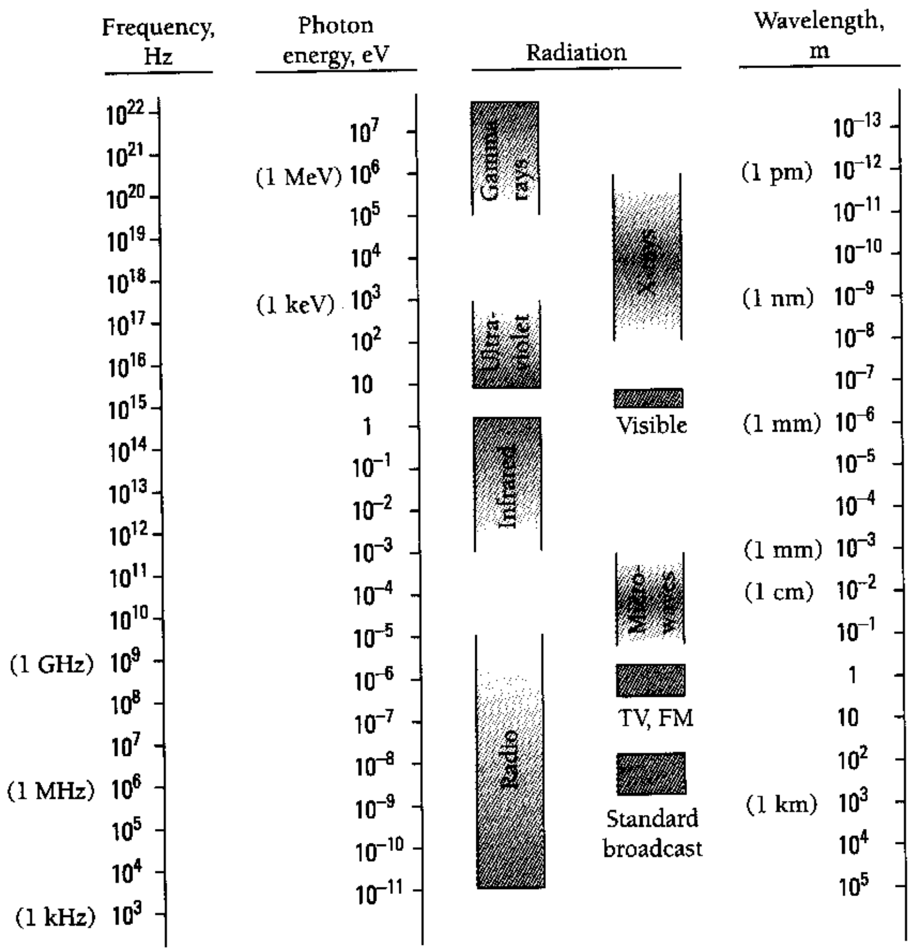


Figure 2.2 The spectrum of electromagnetic radiation.

Principle of Superposition:

When two or more waves of the same nature travel past a point at the same time, the instantaneous amplitude is the sum of the instantaneous amplitude of the individual waves.

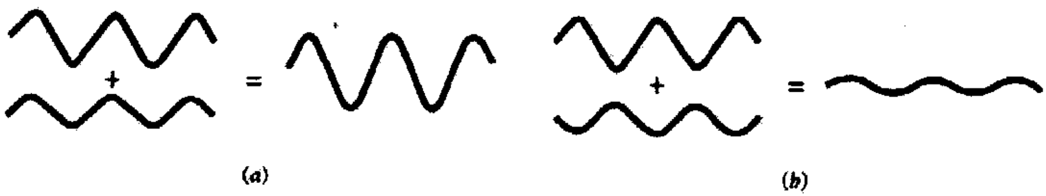


Figure 2.3 (*a*) In constructive interference, superposed waves in phase reinforce each other. (*b*) In destructive interference, waves out of phase partially or completely cancel each other.

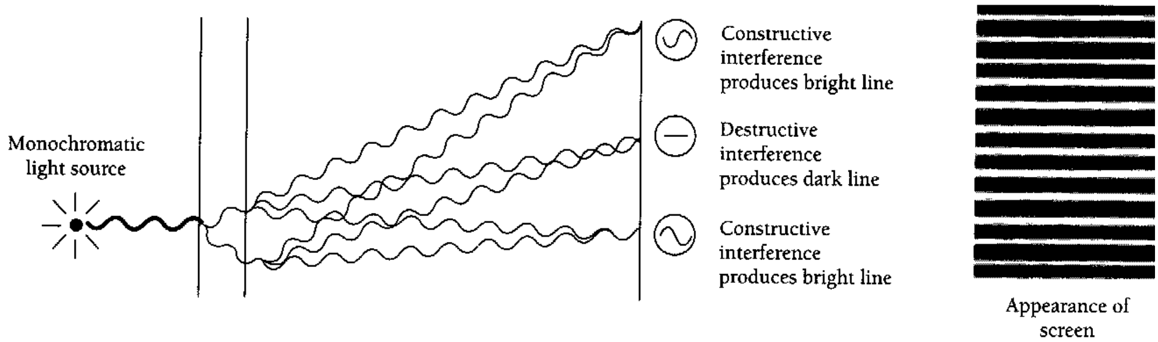
Constructive interference $\longrightarrow$ same phase, greater amplitude

Destructive interference $\longrightarrow$ different phase, partial or completely cancellation of waves

Interference $\longrightarrow$ wave characteristic

Young's diffraction experiments:

Figure 2.4 Origin of the interference pattern in Young's experiment. Constructive interference occur where the difference in path lengths from the slits to the screen is 0, λ, 2λ, ... Destructive interfer ence occurs where the path difference is $\lambda/2$, $3\lambda/2$, $5\lambda/2$, ...



diffraction $\longrightarrow$ wave characteristic

2.2Blackbody radiation

Is light only consistent of waves? Amiss: understand the origin of the radiation emitted by bodies of matter.

Blackbody: a body that absorbs all radiation incident upon it, regardless of frequency.

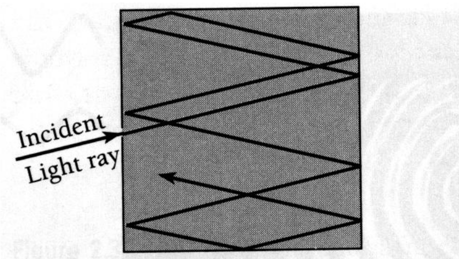
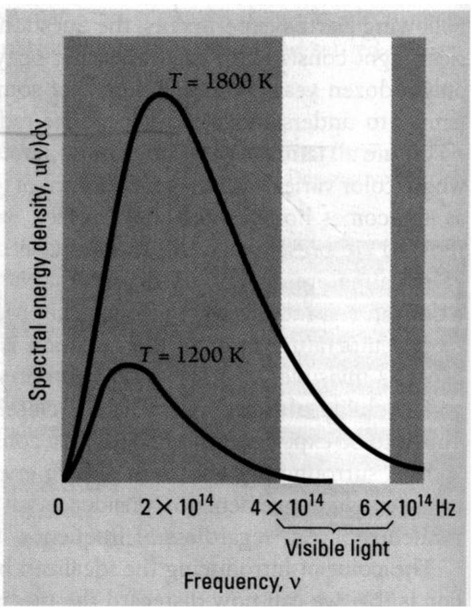


Figure 2.5 A hole in the wall of a hollow object is an excellent approximation of a blackbody.

A blackbody radiates more when it is hot than it is cold, and the spectrum of a hot blackbody has its peak at a higher frequency than that of a cooler one.

Figure 2.6 Blackbody spectra. The spectral distribution of energy in the radiation depends only on the temperature of the body.



Considering the radiation inside a cavity of absolute temperature T whose walls are perfect reflectors to be a series standing EM waves.

$$L = n * \lambda / 2$$

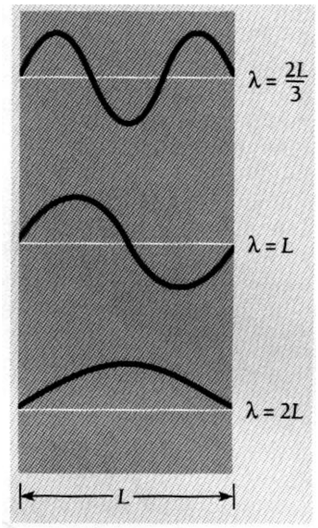


Figure 2.7 Em radiation in a cavity whose walls are perfect reflectors consists of standing waves that have nodes at the walls, which restricts their possible wavelengths. Shown are three possible wavelengths when the distance between opposite walls is *L*.

Density of standing waves in cavity

$$G(\nu)d\nu = 8\pi\nu^2 d\nu / c^3$$

The higher ν, the shorter the wavelength, and the greater the number of possible standing waves.

The average energy per degree of freedom of an entity that is a member of a system of such entities in thermal equilibrium at T is 1/2kT. K is Boltzmann's constant=$1.381*10^{-23}$J/k

An idea gas molecular has three degree of freedom: kinetic energy in three independent directions $\longrightarrow$ 3/2kT

One dimensional harmonic oscillator has two degree of freedom: kinetic energy and potential energy.

Each standing wave in a cavity originates in an oscillating electric charge in the cavity wall. $\longrightarrow$ Two degree of freedom.

Classic average energy per standing wave $\varepsilon = kT$

Total energy per unit volume in the cavity in ν and ν+dν

$u(\nu)d\nu = \varepsilon G(\nu)d\nu = (8\pi kT/c^3)\nu^2 d\nu$ **Rayleigh-Jeans formula**

ν increase $\longrightarrow$ energy density increase with ν^2.

In the limit of infinitely high frequencies, u(ν)dν goes to infinity. In reality, the energy density(and the radiation rate)falls to 0 as ν goes to infinity. $\longrightarrow$ Ultraviolet catastrophe

Plank Radiation Formula

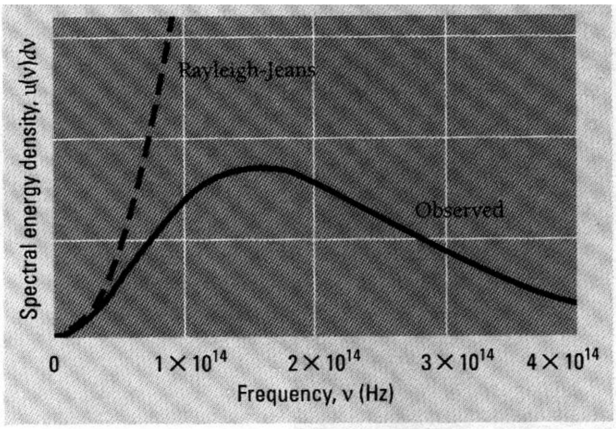


Figure 2.8 Comparison of the Rayleigh-Jeans formula for the spectrum of the radiation from a blackbody at 1500 K with the observed spectrum. The discrepancy is known as the ultraviolet catastrophe because it increases with increasing frequency. This failure of classical physics led Planck to the discovery that radiation is emitted in quanta whose energy is $h\nu$.

$u(\nu)d\nu=(8\pi h/c^3)(\nu^3 d\nu)/(e^{h\nu/kT}-1)$

h is planck's constant=$6.626*10^{-34}$Js

$h\nu >> kT \longrightarrow e^{h\nu/kT} \longrightarrow \infty \longrightarrow u(\nu) \longrightarrow 0$

No ultraviolet catastrophe.

In general, $e^x=1+x+x^2/2+\ldots\ldots\ldots$

When $h\nu << kT$, $1/(e^{h\nu/kT}-1)\sim 1/((1+(h\nu/kT)-1)\sim kT/h\nu$

$u(\nu)d\nu \sim (8\pi h/c^3)(\nu^3 d\nu)/(kT/h\nu)\sim(8\pi kT/c^3)\nu^2 d\nu$

which is Rayleigh-Jeans formula.

How to justify the Plank radiation formula

The oscillators in the cavity walls could not have a continuous Distribution of possible energy ε but must have only specific energies $\boldsymbol{\varepsilon_n = nh\nu}$ **n=0,1,2……**

An ocillator emits radiation of frequency ν when it drops from one energy state to the next lower one, and it jumpsto the next higher state when it absorbs radiation of ν. Each discrete bundle of energy $h\nu$ is called a quantum.

With oscillator energies limited to $nh\nu$, the average energy per oscillator in the cavity walls turn out to be not kT as for a continuous distribution of oscillator energies, but

$\boldsymbol{\varepsilon = h\nu/(e^{h\nu/kT}-1)}$ average energy per standing wave

2.3 Photoelectric effect

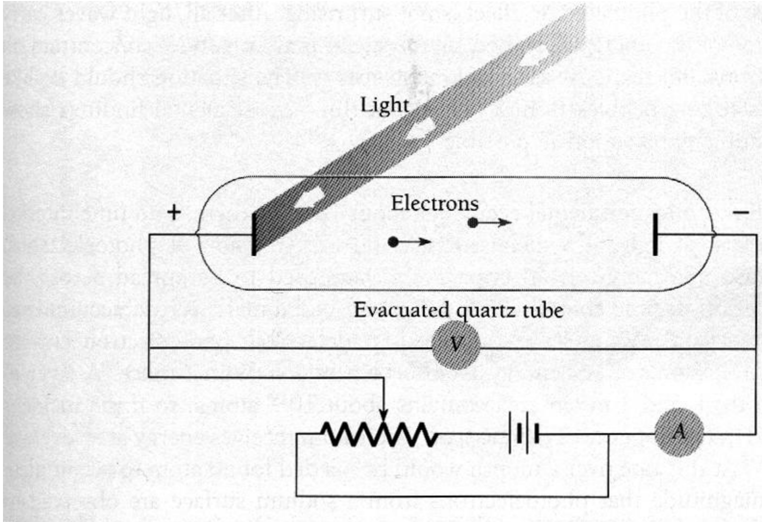


Figure 2.9 Experimental observation of the photoelectric effect.

Some of photoelectrons that emerge from the metal surface have enough energy to reach the cathode despite its negative polarity

$\longrightarrow$ Current

When V is increased to a certain value V_0, no more photoelectrons arrive. V_0 correspond to the max photoelectron kinetic energy.

Three experimental finding:

(1)No delay between the arrival of the light at the metal surface and the emission of photoelectrons.

(2) A bright light yields more photoelectrons than a dim one, but highest electron energy remain the same.

(3)The higher the frequency of the light, the more energy the photoelectrons have. At the frequencies smaller than ν_0, which is a characteristic of the specific metal, no more electrons are emitted.

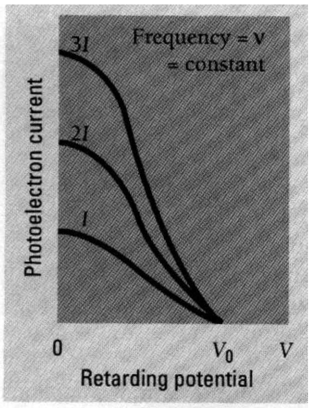


Figure 2.10 Photoelectron current is proportional to light intensity I for all retarding voltages. The extinction voltage v_0, which corresponds to the maximum photoelectron energy, is the same for all intensities of light of the same frequency ν.

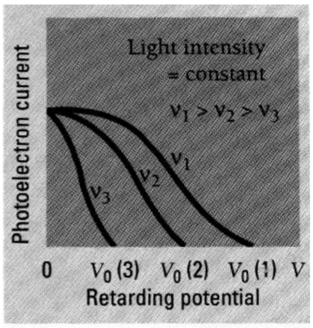


Figure 2.11 The extinction voltage V_0, and hence the maximum photoelectron energy, depends on the frequency of the light. When the retarding potential is $V = 0$, the photoelectron current is the same for light of a given intensity regardless of its frequency.

Quantum theory of light

Einstein proposed **Photons.** The energy in light is not spread out, but is concentrated in small packets. Each photon of light of frequency ν has the energy hν.

Einstein proposed that energy was not only given to em waves in separate quanta but was also carried by the waves in separate quanta.

Explanation of experiments:

(1)Since em wave energy is concentrated in photons and not spread out, there should be no delay in the emission.

(2)All photons of frequency ν have the same energy $h\nu$. Changing the intensity of light only change the number of photoelectrons but not their energy.

(3)The higher ν, the greater photon energy and so the more energy the photoelectrons have.

ν_o corresponds to the min energy Φ for the electron to escape from the metal surface. This energy is called **work function**.

$$\mathbf{\Phi=h\nu_o}$$

Photoelectric effect $\mathbf{h\nu=kE_{max}+\Phi}$

$$h\nu = kE_{max}+h\nu_o$$

$$kE_{max}=h(\nu-\nu_o).$$

Photo energy

$E=(6.626*10^{-34}Js/1.602*10^{-19}J/eV\ )\nu=(4.136*10^{-15})\nu eVs$

$\nu=c/\lambda$ $E=1.24*10^{-6}eVm/\lambda$

Example 2.2

Ultraviolet light of wavelength 350 nm and intensity 1.00 W/m^2 is directed at a potassium surface. (*a*) Find the maximum KE of the photoelectrons. (*b*) If 0.50 percent of the incident photons produce photoelectrons, how many are emitted per second if the potassium surface has an area of 1.00 cm^2?

Solution

(*a*) From Eq. (2.11) the energy of the photons is, since 1 nm = 1 nanometer = 10^{-9} m,

$$E_p = \frac{1.24 \times 10^{-6}\ \text{eV}\cdot\text{m}}{(350\ \text{nm})(10^{-9}\ \text{m/nm})} = 3.5\ \text{eV}$$

Table 2.1 gives the work function of potassium as 2.2 eV, so

$$\text{KE}_{\text{max}} = h\nu - \phi = 3.5\ \text{eV} - 2.2\ \text{eV} = 1.3\ \text{eV}$$

(*b*) The photon energy in joules is 5.68×10^{-19} J. Hence the number of photons that reach the surface per second is

$$n_p = \frac{E/t}{E_p} = \frac{(P/A)(A)}{E_p} = \frac{(1.00\ \text{W/m}^2)(1.00 \times 10^{-4}\ \text{m}^2)}{5.68 \times 10^{-19}\ \text{J/photon}} = 1.76 \times 10^{14}\ \text{photons/s}$$

The rate at which photoelectrons are emitted is therefore

$$n_e = (0.0050)n_p = 8.8 \times 10^{11}\ \text{photoelectrons/s}$$

- **What is light**

Wave model: light intensity $\propto \overline{E^2}$

Particle model: light intensity $\propto$ N(#of photons/sec.area)

$$N \propto \overline{E^2}$$

.N is large $\longrightarrow$ interference pattern

N is small $\longrightarrow$ a series of random flashes

.if keep track of flashes for long time

$\longrightarrow$ same as large N

$\longrightarrow$ intensity of wave at a given place on the specific space $\propto$ the probability of finding photons.

Wave & quantum theory complement each other.

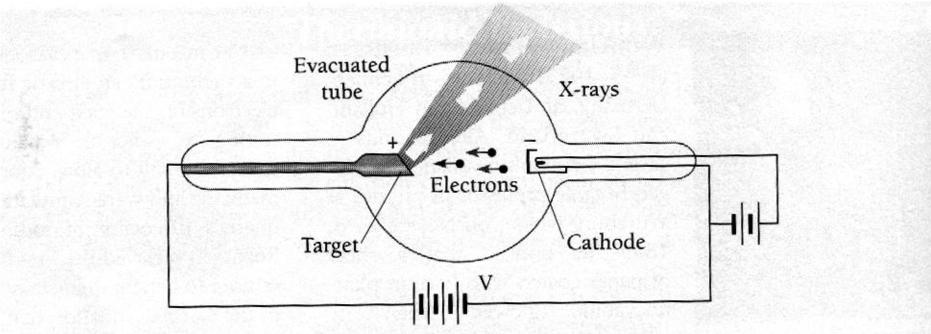


Figure 2.15 An x-ray tube. The higher the accelerating voltage V, the faster the electrons and the shorter the wavelengths of the x-rays.

photoelectric effect : $E_{photons} \longrightarrow E_{e'}$

yes $\longrightarrow$ x-ray

faster e′ $\longrightarrow$ more x-ray

# of e′ increase $\longrightarrow$ Intensity of x-ray increase $\nearrow$

.for given accelerating V $\longrightarrow \lambda_{min}$

V $\uparrow \longrightarrow \lambda_{min} \downarrow$

.most of e′ $\longrightarrow$ heat

A few e′ lose E in single collisions $\longrightarrow$ x-ray

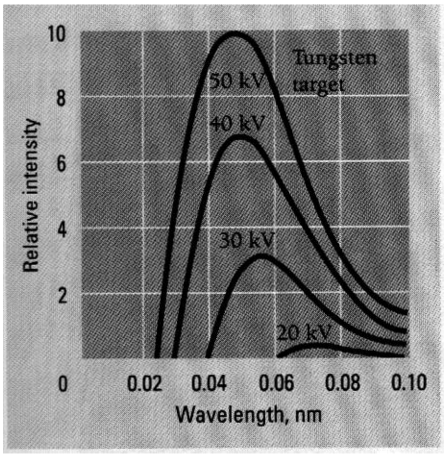


Figure 2.16 X-ray spectra of tungsten at various accelerating potentials.

- .x-ray are em waves

EM theory predicts that an accelerated electric charge will radiate em waves, and a rapidly moving e′ suddenly brought to rest is certainly accelerated $\longrightarrow$ Bremsstranlung ("braking radiation")

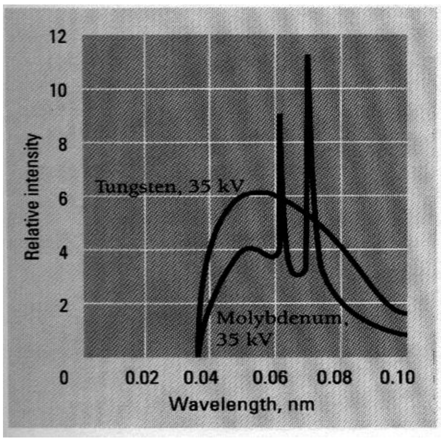


Figure 2.17 X-ray spectra of tungsten and molybdenum at 35 kV accelerating potential.

x-ray at specific $\lambda \longrightarrow$ nonclassical

* different targets give different characteristic x-ray

* for the same V, λ_{min} is the same for different materials

$$\lambda_{min}=(1.24\times10^{-6})/V(m)$$

$h\nu_{max} = Ve = hc/\lambda_{min} \longrightarrow \lambda_{min} = hc/Ve = (1.24\times10^{-6})/V$

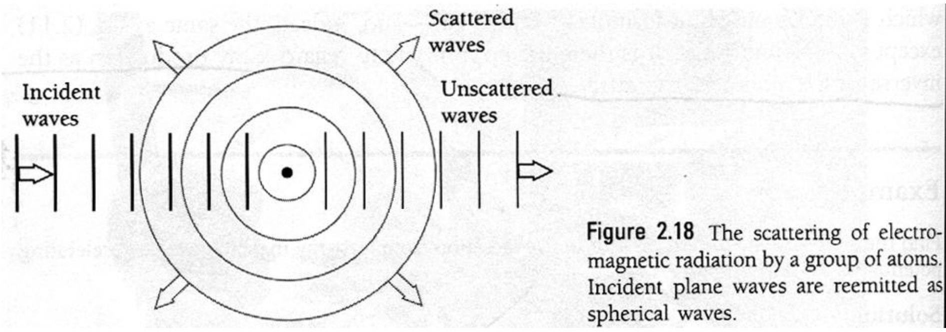


Figure 2.18 The scattering of electromagnetic radiation by a group of atoms. Incident plane waves are reemitted as spherical waves.

Scattering by an atom (wave model)

atom in E $\longrightarrow$ polarized $\longrightarrow$ distorted charge distribution

$\longrightarrow$ electric dipole

em wave with ν on atom $\longrightarrow$ polarization charge with ν

$\longrightarrow$ oscillating electric dipole $\longrightarrow$ radiate em wave

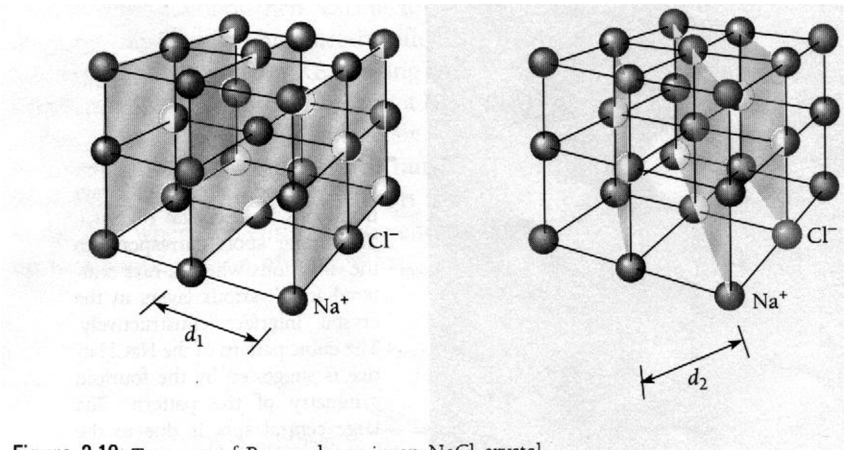


Figure 2.19 Two sets of Bragg planes in an NaCl crystal.

x-ray falls on a crystal will be scattered in all directions because of regular arrangement of atoms $\longrightarrow$ constructively interference

$\longrightarrow$ Bragg's condition ($2d\sin\theta=\lambda$)

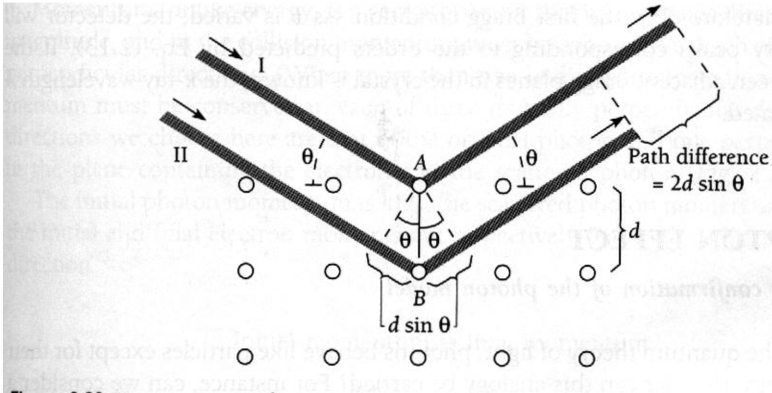


Figure 2.20 X-ray scattering from a cubic crystal.

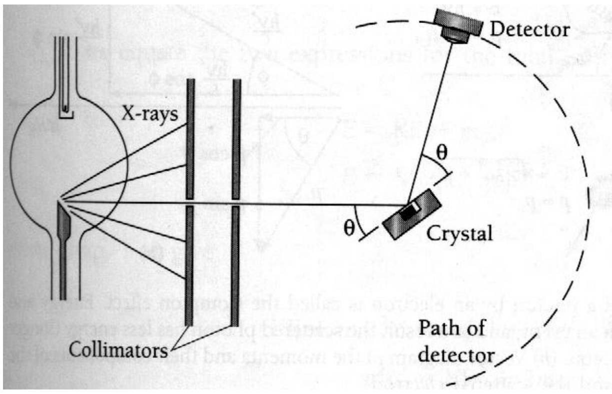


Figure 2.21 X-ray spectrometer.

- **Compton effect**

Loss in photon energy=gain in e′ energy

$$h\nu - h\nu' = kE$$

for massless particle E= Pc (P=momentum)

⟶ photon momentum P =E/c = hν/c

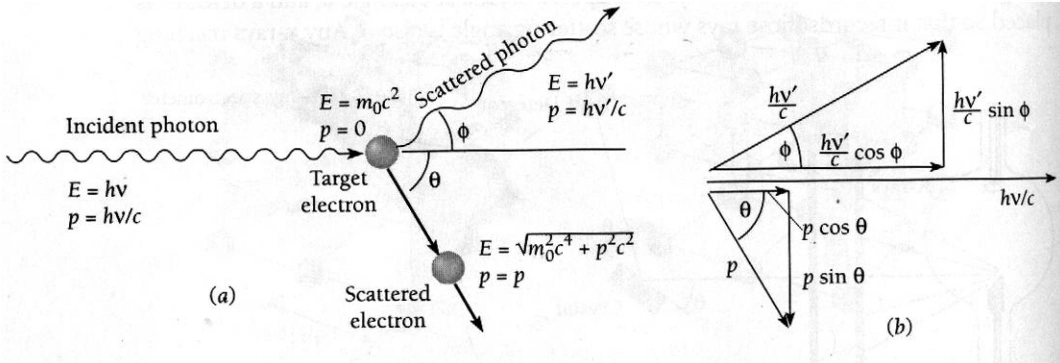


Figure 2.22 (a) The scattering of a photon by an electron is called the Compton effect. Energy and momentum are conserved in such an event, and as a result the scattered photon has less energy (longer wavelength) than the incident photon. (b) Vector diagram of the momenta and their components of the incident and scattered photons and the scattered electron.

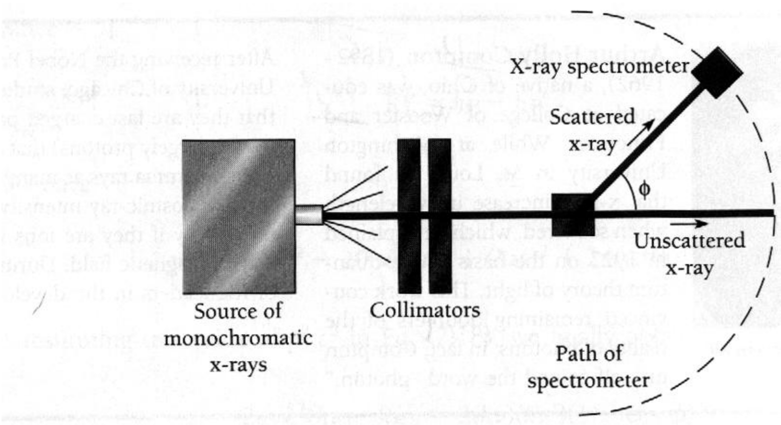


Figure 2.23 Experimental demonstration of the Compton effect.

$h\nu/c = (h\nu'/c)\cos\Phi + P\cos\theta$ (parallel)……..(1)

$0 = (h\nu'/c)\sin\Phi - P\sin\theta$ () ……………….(2)

(1)&(2)x c

$Pc(\cos\theta) = h\nu - h\nu'\cos\Phi$

$Pc(\sin\theta) = (h\nu')\sin\Phi$

$P^2c^2 = (h\nu)^2 - 2(h\nu)(h\nu')\cos\Phi + (h\nu')^2$

& $E = KE + m_oc^2$ $E = \sqrt{m_o^2c^4 + P^2c^2}$

$\longrightarrow (KE + m_oc^2)^2 = m_o^2c^4 + P^2c^2$

$P^2c^2 = KE^2 + 2m_oc^2KE$

Because $KE = h\nu - h\nu'$

$\longrightarrow P^2c^2 = (h\nu - h\nu')^2 + 2m_oc^2KE$

$2m_oc^2(h\nu - h\nu') = 2(h\nu)(h\nu')(1 - \cos\Phi)$……(3)

(3)/$2h^2c^2$ $m_oc/h(\nu/c - \nu'/c) = (\nu/c)(\nu'/c)(1 - \cos\Phi)$

$\longrightarrow m_oc/h(1/\lambda - 1/\lambda') = (1 - \cos\Phi)/(\lambda\lambda')$

$\longrightarrow \lambda' - \lambda = (h/m_oc)(1 - \cos\Phi) = \lambda c(1 - \cos\Phi)$

λ=Compton wavelength

- **Relativistic formulas**

Total energy $E=\dfrac{m_o c^2}{\sqrt{1-\dfrac{v^2}{c^2}}}$ m_o= rest mass

Relativistic momentum $P = \dfrac{m_o v}{\sqrt{1-v^2/c^2}}$

When $m_o = 0$ (massless particle) & $v<c$ $\longrightarrow$ $E=P=0$

How about $v=c$, $m_o= 0$ $\longrightarrow$ $E=0/0$, $P=0/0$ (any values)

$E^2=\dfrac{m_o{}^2c^4}{1-v^2/c^2}$, $P^2=\dfrac{m_o{}^2v^2}{1-\dfrac{v^2}{c^2}}$ $\longrightarrow$ $P^2c^2 = \dfrac{m_o{}^2v^2}{1-\dfrac{v^2}{c^2}}c^2$

$\longrightarrow$ $E^2 - P^2c^2 = \dfrac{m_o{}^2c^4}{1-v^2/c^2}(1-v^2/c^2) = m_o{}^2c^4$

$\longrightarrow$ $E^2 = m_o{}^2c^4 + P^2c^2$

$\longrightarrow$ **For all particles** $\mathbf{E}=\sqrt{m_0{}^2c^4+p^2c^2} = \sqrt{E_o{}^2+p^2c^2}$

Restriction of massless particles : E = Pc (m_o=0)

Total energy $\mathbf{mc^2=m_oc^2+KE} = \dfrac{m_oc^2}{\sqrt{1-v^2/c^2}}$

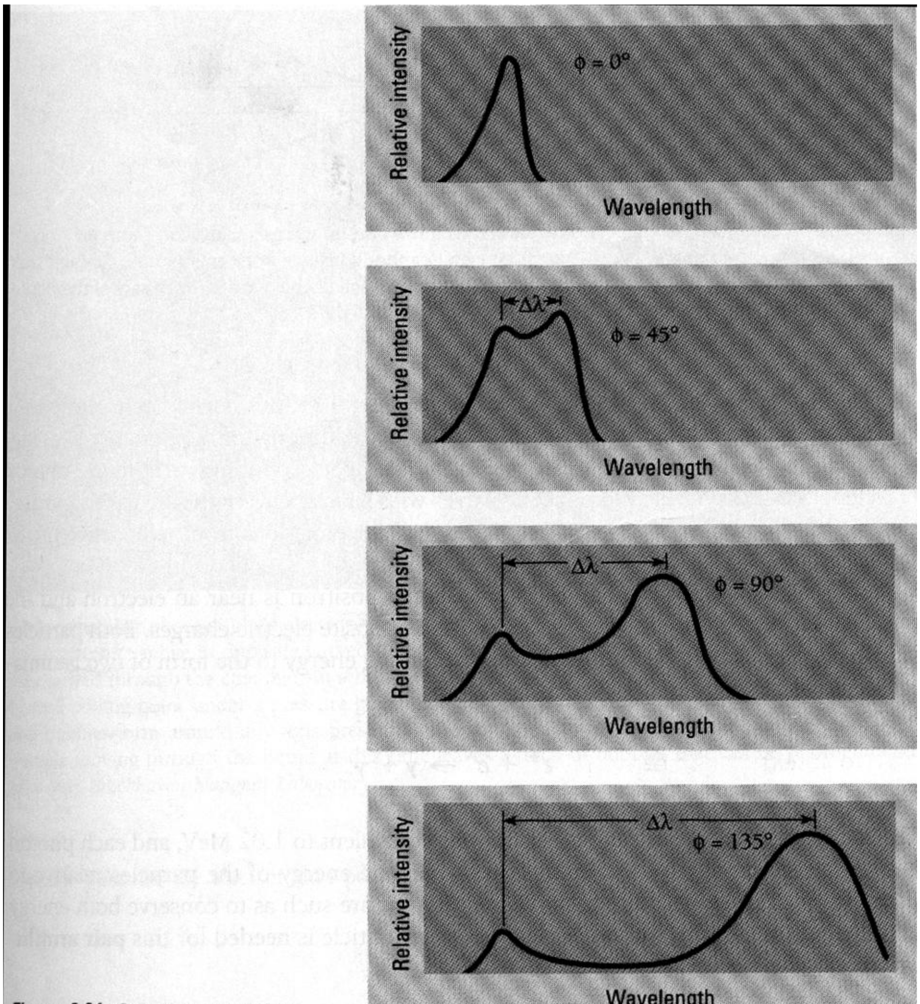


Figure 2.24 Compton scattering.

- **Pair production**

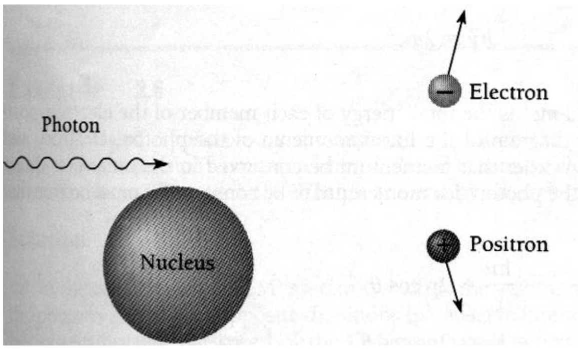


Figure 2.25 In the process of pair production, a photon of sufficient energy materializes into an electron and a positron.

A photon give an e′ all of its energy $\longrightarrow$ photoelectric

part of its energy $\longrightarrow$ compton

a photon materialize into an e′ & position

(momentum is conserved with the help of the nucleus which carries away enough photon momentum)

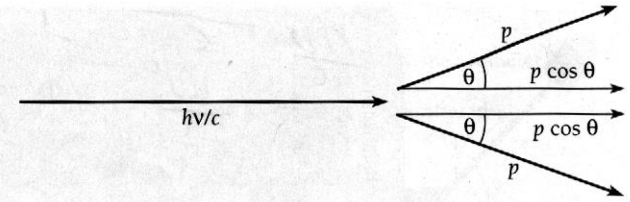


Figure 2.26 Vector diagram of the momenta involved if a photon were to materialize into an electron-positron pair in empty space. Because such an event cannot conserve both energy and momentum, it does not occur. Pair production always involves an atomic nucleus that carries away part of the photon momentum.

rest energy m_oc^2 of electron or position is 0.51Mev $\longrightarrow$ pair production requires a photon energy $\geq$ 1.02Mev

- **pair production cannot occur in empty space**

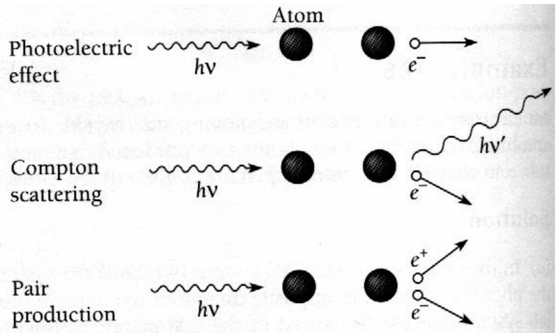
concervation of energy $h\nu = 2mc^2$

momentum concervation $h\nu/c = 2P\cos\theta \longrightarrow h\nu = 2Pc(\cos\theta)$

$P = mv \longrightarrow h\nu = 2mc^2(v/c)\cos\theta$

$v/c < 1$ & $\cos\theta \leq 1 \longrightarrow h\nu < 2mc^2$

Figure 2.27 X- and gamma rays interact with matter chiefly through the photoelectric effect, Compton scattering, and pair production. Pair production requires a photon energy of at least 1.02 MeV.



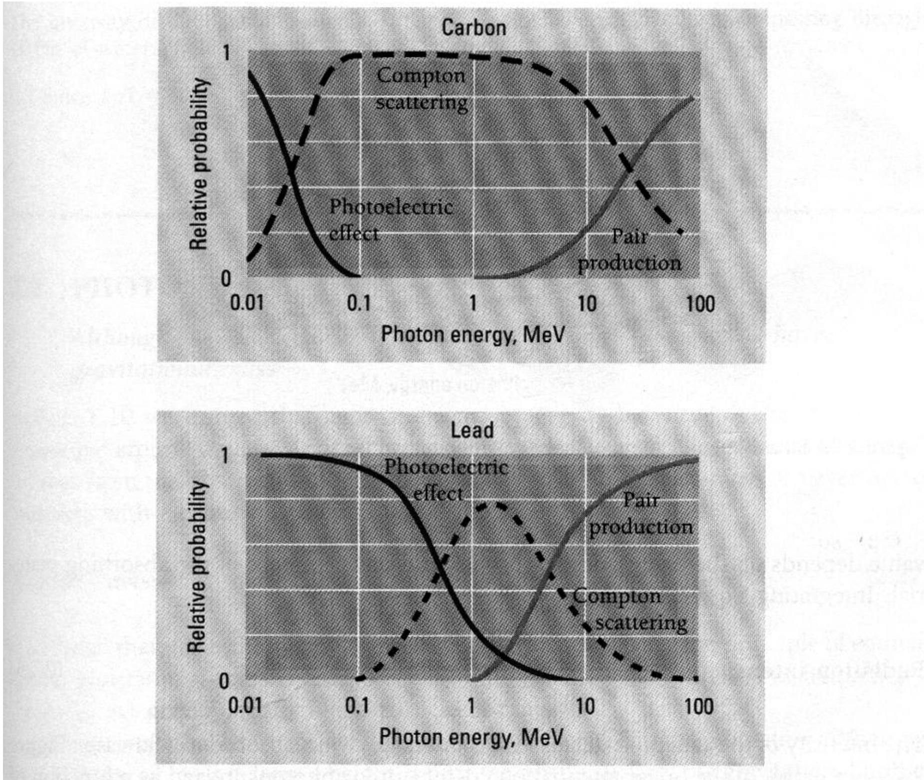


Figure 2.28 The relative probabilities of the photoelectric effect, Compton scattering, and pair production as functions of energy in carbon (a light element) and lead (a heavy element).

linear attenuation coefficient

$$\frac{-dI}{I} = udx \longrightarrow I = I_o \exp(-ux)$$

absolute thickness $$x = \frac{\ln(I_o/I)}{u}$$

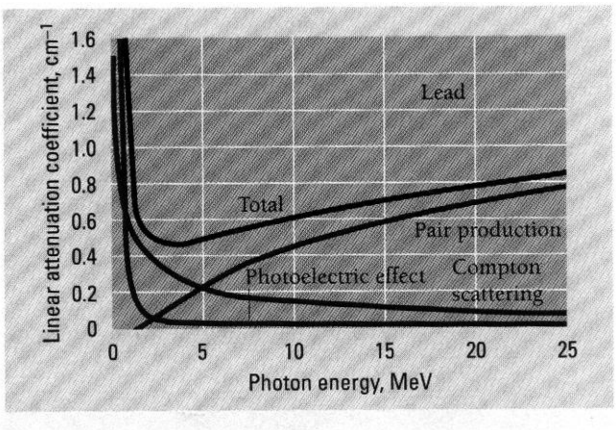


Figure 2.29 Linear attentuation coefficients for photons in lead.

Wave properties of particles

【Chapter 3 **Wave Properties of Particles**】

.De Broglie waves

A moving body behaves in certain ways as though it has a wave nature.

* for photon

$$P = h\nu/c = h/\lambda$$

Photon wavelength $\longrightarrow$ $\lambda = h/P$………(3.1)

De Broglie Suggested (3.1) is general one that applies to **material particles** as well as to photons.

$\longrightarrow$ De Broglie wavelength

$$\lambda = h/P = h/mv$$

$$(m = \frac{m_o}{\sqrt{1 - v^2/c^2}})$$

Example 3.1

Find the de Brogli wavelengths of

(a) 46-g golf ball with a v = 30 m/s

(b) e' with a v = 10^7m/s

(1) v<<c $\longrightarrow$ m =m_o

λ= h/mv =6.63x10^{-34}Js/(0.046kg)(30m/s)=4.8x10^{-34}m

wavelength is very small

λ= h/mv =6.63x10^{-34}Js/(9.1x10^{-31}kg)(10^7m/s)=7.3x10^{-11}m

=0.73Å

the radius of H atom = 5.3x10^{-11}m=0.53 Å

wave character of moving e' is the key to understand atomic

structure behavior

【3.2 Waves of probability】

Water wave $\longrightarrow$ (varing quantity) height of water surface

Light wave $\longrightarrow$ E& H fields

How about matter waves

$\longrightarrow$ Wave function Ψ

The value of wave function associated with a moving body at the particular point x , y, z at time t is related to the likehood of finding the body there at the time.

*Ψ has no direct physical significance

$0 \le$ probability ≤ 1

but the amplitude of wave am be positive or negative

$\longrightarrow$ no negative probability

$\Longrightarrow$ $|\varphi|^2$:squae of the absolute value of wave function

$\longrightarrow$ probability density

** The probability of experimentally finding the body described by the wave functionΨat the point x , y , z at time t is proportional to $|\varphi|^2$ there at t.

wave functionΨthat described a particle is spread out is spall, but it does not mean that the particle itself is spread out.

【3.3 Describing a wave】

de Broglie wave velocity v_p

$v_p = \nu\lambda (\lambda = h/mv)$

$h\nu = mc^2 \longrightarrow \nu = mc^2/h$

De Broglie phase velocity $v_p = \nu\lambda = (mc^2/h)(h/mv) = c^2/v$ (v = particle velocity)

Because $V<C$

$\longrightarrow$ de Broglie waves always travel faster than light !!

$\longrightarrow$ Phase velocity, group velocity.

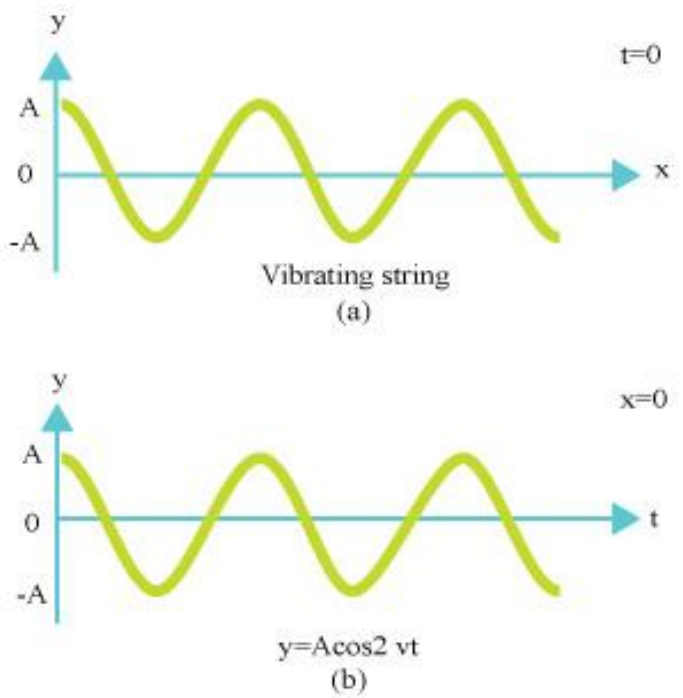


Figure3.1 (a) The appearance of a wave in a stretched string at a certain time.

(b) How the displacement of a point on the string varies with time.

At x=0, y=Acos($2\pi\nu$ t)   for time=t

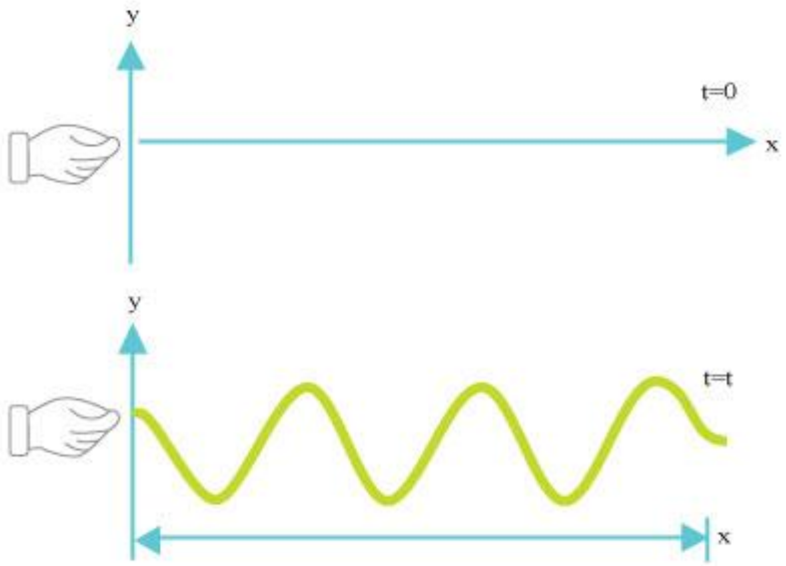


Figure3.2   Wave propagation.

$x = v_p t$, $t = x/v_p$

y=Acos$2\pi\nu(t - x/v_p)$

the amplitude for $y(x,t) = y(0,t-x/v_p)$

$y = A\cos 2\pi(\nu t - \frac{v_x}{v_p})$ $v_p = \nu\lambda$

⟶ y =Acos$2\pi(\nu t - x/\lambda)$

angular freguency $\omega = 2\pi\nu$ wave number $k = 2\pi/\lambda = \omega/v_p$

⟶ $y = A\cos(\omega t - kx)$

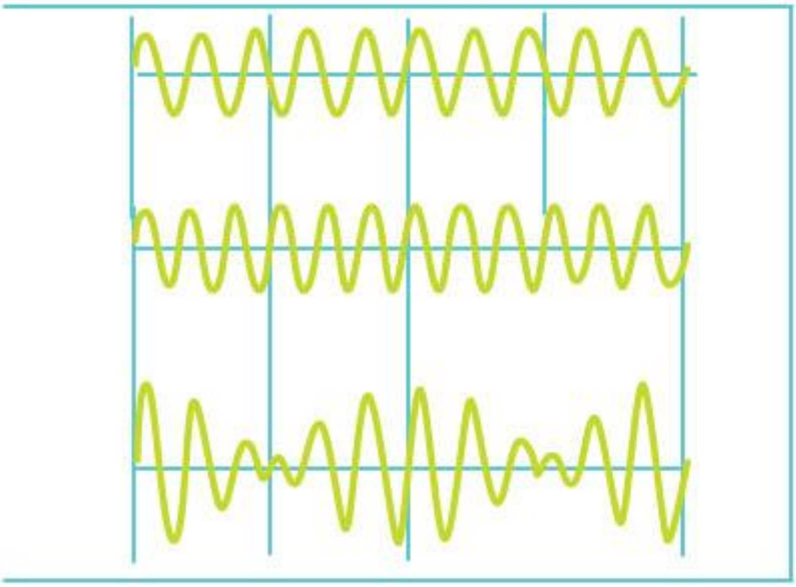


Figure3.3　A wave group.

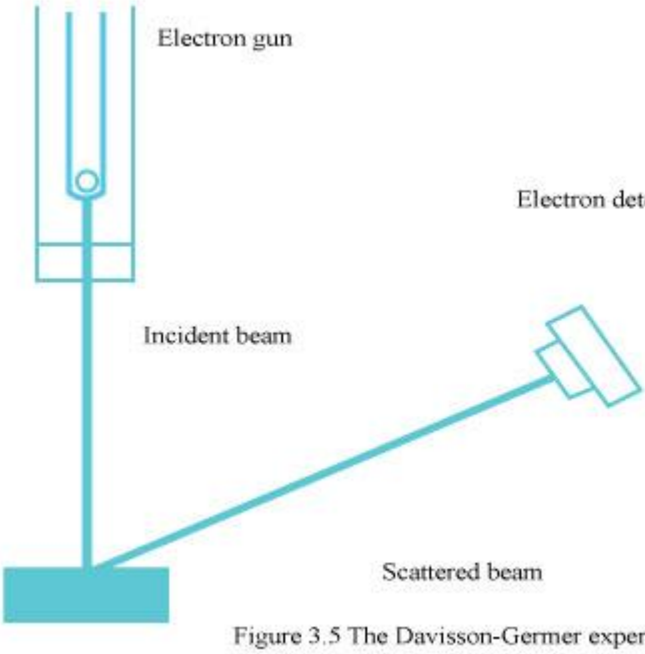


Figure 3.5 The Davisson-Germer experiment

Figure3.4　Beats are produced by the superposition of two waves with different frequencies.

The amplitude of de Broglie waves → probability

De Broglie wave can not be represented by y=Acos(wt-kx)

. wave representation of a moving body → wave packet

wave group

. An example is a beat. (two sound waves of the same amplitude but slightly different frequencies)

original 440, 442 Hz → hear fluctuating sound of 441 Hz with 2 beats/s

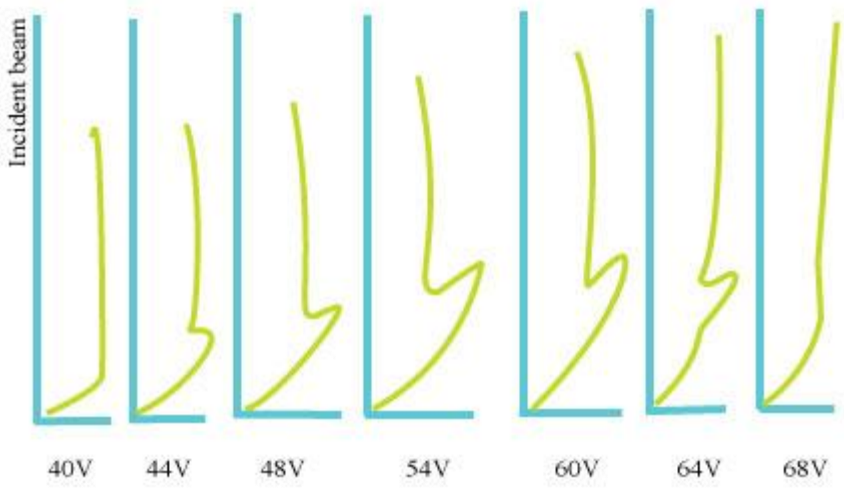


Figure3.5 The Davisson-Germer experiment.

a wave group: superposition of individual waves of different λ which interference with one another

→ variation in amplitude → define the group shape

(1) If the velocities of the waves are the same $\longrightarrow$ the velocity of wave group is common phase velocity

(1)If the phase velocity varies with λ

$\longrightarrow$ an effect called dispertion

$\longrightarrow$ individual waves do not proceed together

$\longrightarrow$ wave group has a velocity different from the phase velocities

$\longrightarrow$ the case of de Broglie wave

- **group velocity**

$y_1 = A\cos[(\omega t - ks)]$

$y_2 = A\cos[(\omega+\Delta\omega)t - (k+\Delta k)x]$

$\longrightarrow$ $y = y_1+y_2$

$= 2A\cos 1/2[(2\omega+\Delta\omega)t - (2k+\Delta k)x]\cos 1/2(\Delta\omega t - \Delta kx)$

because $\Delta\omega \ll \omega \longrightarrow 2\omega+\Delta\omega \approx 2\omega$

$\Delta k \ll k \longrightarrow 2k+\Delta k \approx 2k$

$\longrightarrow$ $Y = 2A\cos(\omega t - kx)\cos[(\Delta\omega/2)t - (\Delta k/2)x]$

A wave of angular frequency ω & wave number k that has superimposed upon it a modulation of angular frequency $1/2\Delta\omega$

& of wave number $1/2\Delta k$

Modulation produce wave group

$v_p = \frac{\omega}{k} = \frac{2\pi\nu}{2\pi/\lambda} = \nu\lambda$ phase velocity

$v_g = \Delta\omega/\Delta k = d\omega/dk$ group velocity

for de Broglie waves

$$\varpi = 2\pi\nu = \frac{2\pi\nu mc^2}{h} = \frac{2\pi m_o c^2}{h\sqrt{1-v^2/c^2}}$$ (because $h\nu = mc^2$)

$$k = \frac{2\pi}{\lambda} = \frac{2\pi m v}{h} = \frac{2\pi m_o v}{h\sqrt{1-v^2/c^2}}$$ (because $\lambda = h/mv$)

* both ω&k are functions of body's v $v_g = d\omega/dk = \frac{d\omega/dv}{dk/dv}$

$$\frac{d\omega}{dv} = \frac{2\pi m_o v}{h\left(1-v^2/c^2\right)^{\frac{2}{3}}} \quad , \qquad \frac{dk}{dv} = \frac{2\pi m_o}{h\left(1-v^2/c^2\right)^{\frac{3}{2}}}$$

$\longrightarrow$ $v_g = v$ (de Broglie group velocity)

De Broglie wave group associated with a moving body travels with the same velocity as the body.

De Broglie phase velocity $v_p = \omega/k = c^2/v$

$v_p >$ velocity of the body $v > c$

($\because$ it is not the motion of the body)

Ex 3.3 :

An e' has a de Broglie wavelength of 2pm=$2x10^{-12}$m.Find its kinetic energy & the phase & group velocity of its de Broglie waves.

(a) $E = E_o + kE \longrightarrow \quad kE = E - E_o = \sqrt{E_o{}^2 + p^2c^2} - E_o$

$pc = hc/\lambda = (4.136x10^{-15} ev.s)(3x10^8 m/s)/(2x10^{-12}) =$

$6.2x10^5$ev=620kv

the rest energy of e' is E_o=511kv

$\longrightarrow$ kE=$\sqrt{(511)^2 + (620)^2} - 511 = 292$ kev

(b) e' velocity

$$E = \frac{E_o}{\sqrt{1 - v^2/c^2}} \longrightarrow \quad v = c\sqrt{1 - E_o{}^2/E^2} = 0.771c$$

$\therefore\ v_p = c^2/v = 1.3c \quad , \quad v_g = v = 0.771c$

3.5 particle diffraction ⟶ e'-beam diffraction

⟶ confirm de Broglie waves

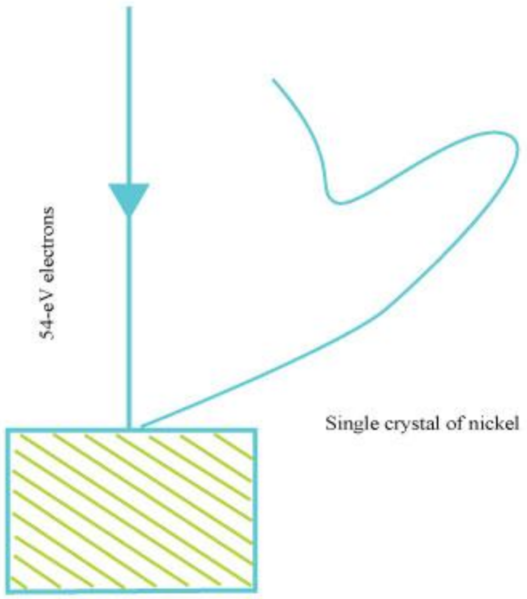


Figure3.6 Results of the Davisson-Germer experiment.

The method of plotting is such that the intensity at any angle is propotional to the distance of the curve at the angle from the point of scattering.

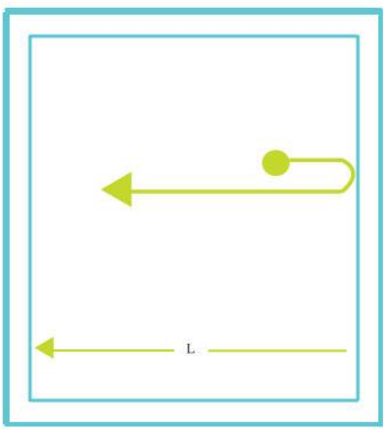


Figure3.7 The diffraction of the de Broglie waves by the target responsible for the results of Davisson and Germer.

$$n\lambda=2d\sin\theta \longrightarrow \lambda=2d\sin\theta=0.165\text{nm} \quad \lambda=h/mv=0.166\text{nm}$$

Figure3.8 Because the wavelengths of the fast electrons in an electron microscope are shorter than those of the light waves in an optical microscope can produce sharp images at higher magnifications. The electron beam in an electron microscope is focused by magnetic fields.

Figure3.9 A particle confined to a box of width L.

【3.6 particle in a box】

a prticle trapped in a box = a standing wave.

Ψmust be zero at the walls

$\longrightarrow$ $\lambda_n = 2L/n \quad n=1,2,3\ldots\ldots$

De Broglie wavelength of trapped particles.

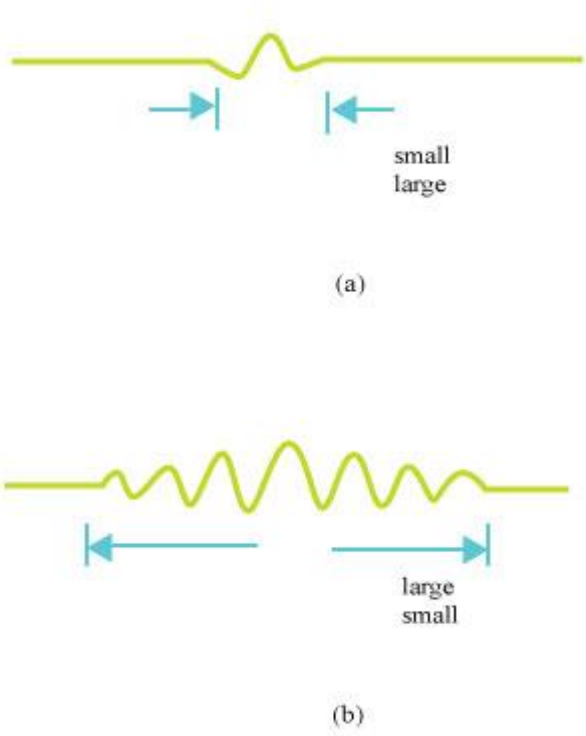


Figure3.10　Wave functions of a particle trapped in a box L wide.

$KE=1/2(mv^2)=(mv)^2/2m=h^2/(\lambda^2 2m)$

$\because \lambda_n=2L/n \longrightarrow KE+v=E_n$ the energy for the particle in a box

$$\mathbf{E_n=n^2h^2/8mL^2} \rightarrow n=1,2,3\ldots\ldots$$

Each permitted energy is called an energy level.(n=quantum number)

This can be applied to any particle confined to a certain region of space.

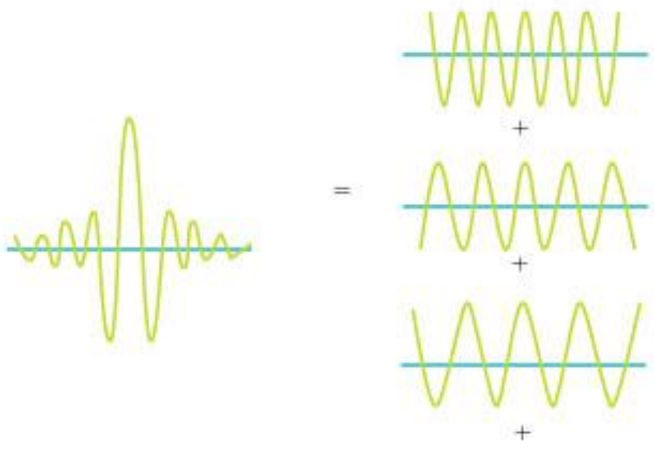
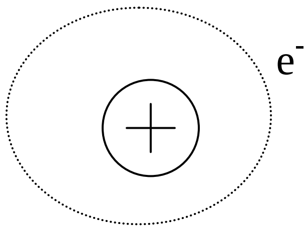


Figure3.11 Energy levels of an electron confined to a box 0.1nm wide.

$\Rightarrow$ For example



1. Atraped particle cannot have an arbitrary energy, as a free particle can .

Confinement leads to restriction on its wave function that alloy the particle to have certain energies.

2. A trapped particle cannot have zero energy.

$\because$ de Broglie wavelength $\lambda = h/mv$ If $v = 0 \longrightarrow \lambda = \infty$

$\longrightarrow$ it can not be a trapped particle.

3. $\because$ $h = 6.63 \times 10^{-34}$Js very small

$\therefore$ only if m & L are very small, or we are not aware of energy quantization in our own experience.

Ex 3.4

An e' is in a box 0.1nm across, which is the order of magnitude of atomic distance, find its permitted energy.

$m = 9.1 \times 10^{-31}$kg & $L = 0.1\text{nm} = 10^{-10}$m

$E_n=n^2(6.63x10^{-34})/8x(9.1x10^{-31})(10^{-10})^2=6x10^{-18}n^2J=38n^2ev$

When n=1 $\longrightarrow$ 38 ev

n=2 $\longrightarrow$ 152ev see fig 3.11

n=3 $\longrightarrow$ 342 ev

Ex 3.5

A long marble is in a box 10 cm across, find its permitted energies

$E_n = 5.5x10^{-64}n^2J$ n=1 $E=5.5x10^{-64}J \longrightarrow$ $v=3.3x10^{-31}$ m/s

Which can not be experimentally distinguished from a stationary marble.

For a reasonable speed 1/3 m/s $\longrightarrow$ $n=10^{30}$!!

Energy levels are very close $\longrightarrow$ quantum effects are imperceptible

- Uncertainty principle

  * wave group narrower $\longrightarrow$ particles position precise.

  However, λof waves in a narrow packet is not well defined ∵λ=h/mv ∴ P is not precise

  * A wide wave group $\longrightarrow$ clearly definedλ but position is not certain

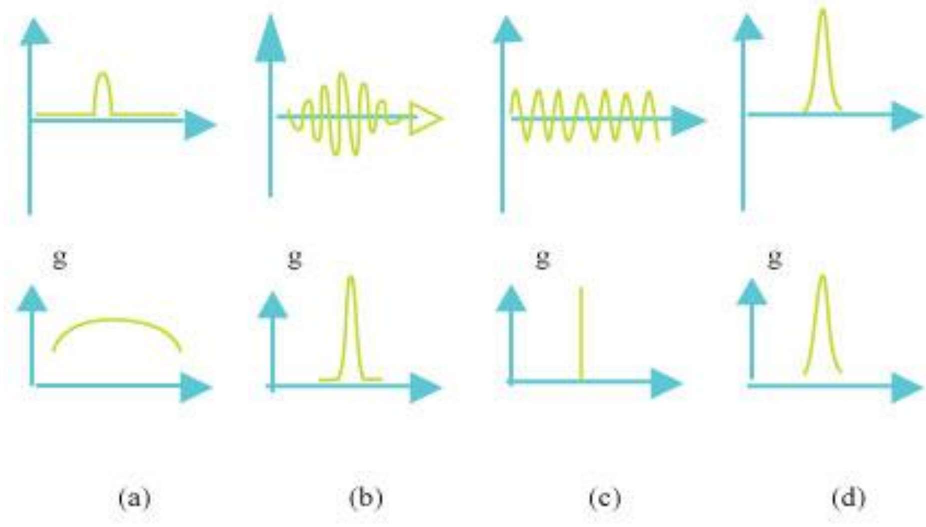


Figure3.12 (a) A narrow de Broglie wave group. The position of the particle can be precisely determined, but the wavelength (and hence the particle's momentum) cannot be established because there are not enough waves to measure accurately. (b) A wide wave group. Now the wavelength can be precisely determined but not the position of the particle.

uncertainty principle:

It is impossible to know both the exact position & exact momentum of an object at the same time.

Figure3.13 An isolated wave group is the result of superposing an infinite number of waves with different wavelengths. The narrower the wave group, the greater the range of wavelengths involves. A narrow de Broglie wave group thus means a well-defined position (Δx smaller) but a poorly defined wavelength and a large uncertainty Δp in the momentum of the particle the group represents. A wide wave group means a more precise momentum but a less precise position.

An infinite # of wave trains with different frequencies wave numbers and amplitude is required for an isolated group of arbitrary shape.

$\varphi(x) = \int_0^\infty g(k)\cos kx\,dk$ Fourier integral

g(k): amplitude of the waves varying with k , furrier transform of $\varphi(x)$

Figure3.14 The wave functions and Fourier transforms for (a) a pulse, (b) a wave group, (c) an wave train, and (d) a gaussian distribution. A brief disturbance needs a broader range of frequencies to describe it than a disturbance of greater duration. The Fourier transform of a gaussian function is also a gaussian function.

* wave numbers needed to represent a wave group extend from k=0 to k=∞, but for a group which length Δx is finite → waves which amplitudes g(k) are appreciable have wave number that lie within a finite interval Δk the shorter the group, the broader the range of wave numbers needed.

Figure3.15 A gaussian distribution. The probability of finding a value of x is given by the gaussian function f(x). The mean value of x is x_0, and the total width of the curve at half its maximum value is 2.35σ, whereσis the standard deviation of the distribution. The total probability of finding a value of x within a standard deviation of x_0 is equal to the shaded area and is 68.3 percent.

*Gaussian function: $f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-(x-x_o)^2/2\sigma^2}$

Standard deviation $\sigma=\sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i-x_o)^2}$ (square-root-mean)

Width of a gaussian curve at half its max is 2.35σ

$$p_{x_o\pm\sigma}=\int_{x_o-\sigma}^{x_o+\sigma}f(x)dx=0.683$$

- Min ΔxΔk occur for Gaussian function

Take Δx,Δk as standard deviation ofφ(x)& g(x) → ΔxΔk=1/2 ∴

in general ΔxΔk ≥ 1/2

∵ $k=2\pi/\lambda=2\pi P/h$ → $P=hk/2\pi$ → $\Delta P=h\Delta k/2\pi$

$\because \Delta x \Delta k \geq 1/2 \qquad \Delta k \geq 1/2\Delta x$

$\longrightarrow \Delta x \Delta p \geq h/4\pi \ (\because \Delta x(h\Delta k/2\pi) \geq h/4\pi)$

$\longrightarrow \Delta x \Delta p \geq \frac{\hbar}{2} \qquad [\hbar = h/2\pi]$

Ex 3.6

A measurement establishes the position of a proton with an accuracy of $\pm$ 1.00x10^{-11}m. Find the uncertainty in the proton's position 1.00s later. Assume v<<c

Sol: At time t=o, uncertainty in positionΔx_o= 1.00x10^{-11}m

$\longrightarrow$ The uncertainty in P at this time $\geq \frac{\hbar}{2\Delta x_o}$

$\because \Delta P = m_o \Delta v \longrightarrow \Delta v = \Delta P/m_o \geq \frac{\hbar}{2m_o\Delta x_o}$

$\Delta x = t\Delta v \geq \frac{\hbar t}{2m_o\Delta x_o} = 3.15\text{x}10^3\text{m} \quad (\because \Delta x \ \alpha \ 1/\Delta x_o)$

*the more we know at t=0, the less we know at t=t *

Figure3.16 An electron cannot be observed without changing its momentum.

look at e' light of wavelengthλ $\longrightarrow$ $P=h/\lambda$ $\longrightarrow$ when one of three photons bounces off the e' $\longrightarrow$ e' momentum is changed.

The exact P cannot be predicted, but $\Delta P \sim h/\lambda$ (the order of magnitude as P) $\Delta x \sim \lambda$

ie if we use shorterλ $\longrightarrow$ increase accuracy of position

$\longrightarrow$ higher photon momentum disturb e' motion more

$\longrightarrow$ accuracy of the momentum measurement deceasing

$\longrightarrow$ $\Delta x \Delta P \geqq h$ (consist with $\Delta x \Delta P \geqq \hbar/2$)

(1) If the energy is in the form of em waves, the limited time available restricts the accuracy with which we can determine the frequencyν.

(2) Assume the min uncertainty in the number of waves we count in a wave group is one wave.

$\because$ Frequency of wave = # of wave/time interval $\longrightarrow$ $\Delta\nu \geqq 1/\Delta t$

$\because E=h\Delta\nu$ $\longrightarrow$ $\Delta E \geqq h/\Delta t$ or $\Delta E \Delta t \geqq h$

more precise calculation $\longrightarrow$ $\Delta E \Delta t \geqq \hbar/2$

ex 3.9

An "excited" atom gives up its excess energy by emitting a photon of characteristic frequency. The average period that elapses between the excitation of an atom & the time it radiates is 1.0×10^{-9}s. find the uncertainty in the frequency of the photon.

$$\Delta E \geqq \hbar / 2\Delta t = 5.3 \times 10^{-27} \text{J}$$

$$\Delta \nu = \Delta E / h = 8 \times 10^{6} \text{Hz}$$