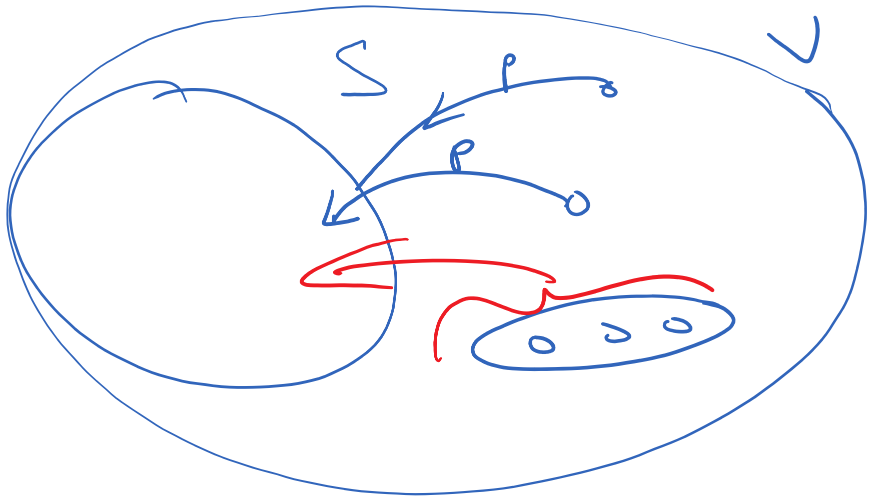
n vertices, 3-uniform hypergraph

$m \geq \frac{n}{3}$ hyperedges.

To show: Independent set of size $\geq \frac{2n^{3/2}}{3\sqrt{3}\sqrt{m}}$

Proof:



r.v. X_S: size of S.

$$E[X_S] = np. \qquad \text{—— (1)}$$

For each edge e,

$$\Pr\left(\begin{array}{c}\text{all points in } e\\ \text{gets chosen}\end{array}\right) = p^3$$

r.v. $Y_e = \begin{cases} 1, & \text{if all points in } e \text{ are chosen into } S. \\ 0, & \text{o/w} \end{cases}$

r.v. Y: no of edges present

Y_S : no of edges present fully inside S.

$$Y_S = \sum_e Y_e$$

$$E[Y_S] = \sum_e E[Y_e]$$

$$= \underline{\underline{mp^3}} \quad \text{—— (2)}$$

$$Z = X - Y$$

$$E[Z] = E[X] - E(Y)$$

$$= np - mp^3$$

when,

$$p = \frac{\sqrt{n}}{\sqrt{3}\sqrt{m}}$$

$$n = 3mp^2$$

$$p^2 = \frac{n}{3m}$$

$\sqrt{3}\sqrt{m}$

$$G(2) = \frac{n\sqrt{n}}{\sqrt{3m}} - \frac{mn\sqrt{n}}{3m\sqrt{3m}}$$

$P = \frac{n}{3m}$

$P = \frac{\sqrt{n}}{\sqrt{3}\sqrt{m}}$

$$= \frac{3mn\sqrt{n} - mn\sqrt{n}}{3m\sqrt{3m}}$$

$$= \frac{2\cancel{m}n\sqrt{n}}{3\cancel{m}\sqrt{3m}}$$

$$= \frac{2n^{3/2}}{3\sqrt{3m}}$$

$\square$