Double Integrals

1. Find the values of the following double integrals:
   (a) $\int_{-1}^{1}\int_{3}^{5}(1+4x+5y)\,dxdy.$
   (b) $\int_{0}^{2}\int_{-1}^{5}(p^3x^2-5px^2+3px-1)\,dpdx.$
   (c) $\int_{-2}^{2}\int_{0}^{x}(x^2y+9xy^3-10x)\,dydx.$
   (d) $\int_{0}^{\Pi}\int_{0}^{2\Pi}(\sin x+\cos y)\,dxdy.$
2. Find the Area under the Rectangle which is centred at origin and having a length of 8 and breadth of 4 using double integration. (Hint: verify your answer with formula ab).
3. Find area under the rhombus centered around origin with intersecting x and y axis at 10 and 2 respectively in I quadrant and similarly at -10 and -2 at negative sides using Double integration method.
4. Evaluate the integral of $f=x^2+y^2$over the triangular region. The triangle is a equilaeral triangle with side 5 and one whole side lies on x-axis with one of the vertices as (0,5). Provide the solution for any two possible answers among all possibilities.
5. Evaluate the double integral $\int\int(x^2y-xy^2)\,dA$ over $R: 0<=x<=5, -2<=y<=1$.
6. Evaluate the integral $f=e^x ln(y)$ over the region bounded by $y=e-x, y=1$ and $x=1, x=10$.
7. Evaluate $\int\int xy\,dxdy$ over the area in the first quadrant of $x^2+y^2=a^2$.
8. Change the order of integration and evaluate $\int_{0}^{a}\int_{\frac{x}{a}}^{\sqrt{\frac{x}{a}}}(x^2+y^2)\,dydx.$
9. Sketch the region of integration and write the equivalent double integral with order of integration reversed for the following:
   (a) $\int_{0}^{1}\int_{2}^{4-2x}(xy)\,dxdy.$
   (b) $\int_{0}^{1}\int_{y}^{\sqrt{y}}(xy^2)\,dxdy.$
   (c) $\int_{0}^{1.5}\int_{0}^{9-4x^2}(16x)\,dydx.$
   (d) $\int_{0}^{1.3}\int_{0}^{\sin x}1.\,dydx.$
10. Evaluate the integral $\int_{0}^{\Pi}\int_{x}^{\Pi}\frac{\sin y}{y}\,dydx.$

11. Evaluate the integral of $\int_0^{2\sqrt{\ln 3}} \int_{\frac{y}{2}}^{\sqrt{ln 3}} e^{x^2}\, dxdy$.

12. Calculate the volume under the region $Z = 4 - x + y$. in the region $R : -2 <= x <= 2, 0 <= y <= 1$.

13. Evaluate the $\int\int (4xy - 6x^2y^3)\, dA$ over the area bounded by the curves $y^2 = 4x$ and $x + y = 1$.

14. Evaluate the integral $\int_0^b \int_0^{\frac{b}{a}\sqrt{b^2-y^2}} (xy)\, dxdy$.

15. By using double integration determine the area of region bounded by $y^2 = 4ax$, $x + y = 3a$, $y = 0$.

16. Find the volume common to the cylinders $x^2 + y^2 = a^2$ and the $x^2 + z^2 = a^2$ using the double integrals.

17. By using double integrals find the volume bounded by cylinder $x^2 + y^2 = 4$, $y + z = 4$ and $z = 0$.

Hints and Solutions

1. (a) Take the double integral for each term and then solve integral for each term and proceed.
   $\int\int 1\, dxdy \int\int 4x\, dxdy \int\int 5y\, dxdy$
   Compute the integral and find the answer.
   Answer: 68

   (b) Apply integral for the each term and continue with solving x and y part separately and then proceed.
   $\int p^3\, dp \int x^2\, dx - 5\int p\, dp \int x^2\, dx + 3\int p\, dp \int xdx - \int dp \int dx$
   Apply the limits after the integration. Answer: 316

   (c) Answer: -18.133

   (d) Approach as mentioned in (a) part
   Answer: 0

2. $\int_{-2}^{2}\int_{-4}^{4} dxdy$
   Answer: 32

3. As the rhombus has equal area in 4 quadrants take area in 1st quadrant and multiply with 4 times and can be also done by integrating first x from 0 to $10(1-(\frac{y}{2}))$ and then y from 0 to 2. And vive-versa as follows :
   $A = 4\int_0^1 0 \int_0^{2(1-\frac{x}{10})} dydx$
   Answer: 40

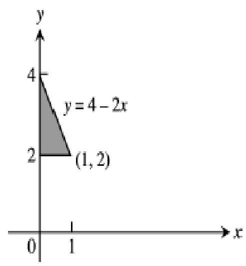
4. $\displaystyle\int_0^{\frac{5}{2}}\int_0^{\sqrt{3}x}(x^2+y^2)\, dydx + \int_{\frac{5}{2}}^{5}\int_0^{\sqrt{3}(x-5)} dydx$
   Considering $(0,0)$ as another vertex.
   Upon Solving
   Answer : $\sqrt{3}(\frac{5}{2})^4 + 5^4\frac{\sqrt{3}}{12}$

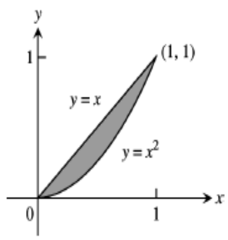
5. $\displaystyle\int_0^5 x^2\, dx \int_{-2}^{1} y\, dy - \int_0^5 x\, dx \int_{-2}^{-1} y^2\, dy$
   Answer: -100

6. $\displaystyle\int_1^1 0 \int_{e^{-x}}^{1} e^x lny\, dydx$ First integrate w.r.t y and then apply limits and the apply integration to x term and evaluate.
   Answer: 22083.748

7. $\displaystyle\int_0^a \int_0^{(\sqrt{a^2-x^2})} xy\, dydx$
   Answer: $(\frac{a^3}{3})$

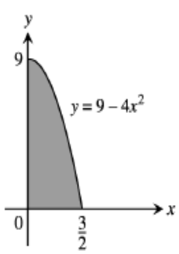
8. (a) $\displaystyle\int_2^4 \int_0^{(\frac{4-y2}{})} xy\, dxdy$





(b) $\int_0^1 \int_{x^2}^{x} xy^2 \, dydx$

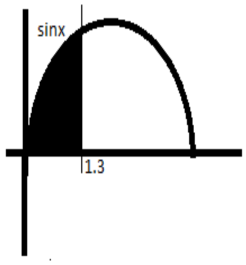
(c) $\int_0^a \int_0^{(\frac{\sqrt{9-y}2}{)}} 16x \, dxdy$



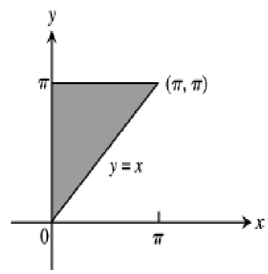
(d) $\int_0^{0.963} \int_{sin^{-1}x}^{1.3} dxdy$

9. $\int_0^1 \int_{ay^2}^{ay} x^2 + y^2 \, dxdy$
   Answer: $(\frac{a^3}{28})$

10. The answer is:

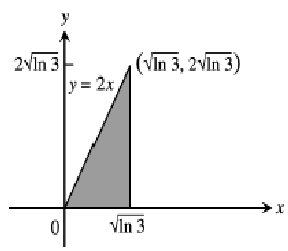


$$\int_0^{\pi}\int_x^{\pi}\frac{\sin y}{y}\,dy\,dx=\int_0^{\pi}\int_0^{y}\frac{\sin y}{y}\,dx\,dy=\int_0^{\pi}\sin y\,dy=2$$



11. The answer is:

$$\int_0^{2\sqrt{\ln 3}}\int_{y/2}^{\sqrt{\ln 3}}e^{x^2}\,dx\,dy=\int_0^{\sqrt{\ln 3}}\int_0^{2x}e^{x^2}\,dy\,dx$$
$$=\int_0^{\sqrt{\ln 3}}2xe^{x^2}\,dx=\left[e^{x^2}\right]_0^{\sqrt{\ln 3}}=e^{\ln 3}-1=2$$

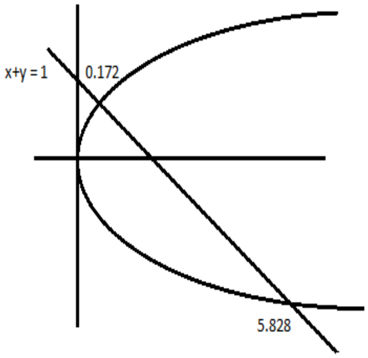


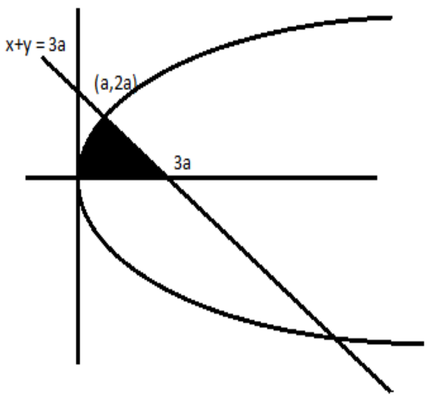
12. $\int_{-2}^{2}\int_0^1 4-x+y\,dydx$
Answer: 18

13. $\int_0^{3-2\sqrt{2}}\int_{-2\sqrt{x}}^{2\sqrt{x}}4xy-6x^2y^3\,dydx+\int_{3-2\sqrt{2}}^{3+2\sqrt{2}}\int_{-2\sqrt{x}}^{1-x}4xy-6x^2y^3\,dydx$

14. Answer: $(\frac{b^6}{8a^2})$

15. $\int_0^{a}\int_0^{2\sqrt{ax}}\,dydx+\int_a^{3}a\int_0^{3a-x}\,dydx$





16. $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$
    $-\sqrt{1-x^2} <= z <= \sqrt{1-x^2}$
    $V = 2\displaystyle\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} (\sqrt{1-x^2})\, dydx$
    $= \displaystyle\int_{-1}^{1} 4(1-x^2)\, dx = \frac{16}{3}$
17. Change of Variable
    Change of Cylinderical Coordinates $x = p\cos\theta, y = p\sin\theta, z = z$
    $p^2 + z^2 = a^2$
    $z = \sqrt{a^2 - p^2}$
    $V = 2\int_0^{\Pi}\int_0^{a\cos\theta}\int_0^{\sqrt{a^2-p^2}} dzdpd\theta$
    Answer $= 2a^3\frac{(3\Pi-4)}{9}$