Why dummy indices cannot be appear more that two times

In class I mentioned that if a dummy index appears more than twice it will lead to inconsistency/erroneous expressions. This can be demonstrated simply, even in the case of Cartesian tensors in 2D. Let's say we have an expression, $(\mathbf{a}\cdot\mathbf{b})^2$, which is the square of $(\mathbf{a}\cdot\mathbf{b})$, the scalar product and is a real (or more generally complex) number. Now we can expand,

$$(\mathbf{a}\cdot\mathbf{b})^2 = (a_1b_1 + a_2b_2)^2 = a_1^2b_1^2 + a_2^2b_2^2 + 2a_1a_2b_1b_2. \tag{1}$$

Lets try to reproduce this result from Einstein summation convention. According to this convention any repeated index must be summed over values 1,2. E.g.$\mathbf{a}\cdot\mathbf{b} = a_ib_i = a_1b_1 + a_2b_2$. Then we can have for our example, two possible choices,

$$(\mathbf{a}\cdot\mathbf{b})^2 = (\mathbf{a}\cdot\mathbf{b})\,(\mathbf{a}\cdot\mathbf{b}) = \begin{cases} (a_ib_i)\ (a_ib_i) = a_ia_ib_ib_i, \\ (a_ib_i)\ (a_jb_j) = a_ia_jb_ib_j. \end{cases}$$

In the first choice we have repeated i, four times while in the second case we have not repeated any index more than twice. Now let's see which one reproduces the correct result (1).

$$\begin{aligned} a_ia_ib_ib_i &= a_1a_1b_1b_1 + a_2a_2b_2b_2 \\ &= a_1^2b_1^2 + a_2^2b_2^2. \end{aligned}$$

Thus the expression with dummy index, i repeated more than twice misses the cross term, $2a_1a_2b_1b_2$ in (1). So this must be incorrect. On the other hand, the second choice,

$$\begin{aligned} a_ia_jb_ib_j &= a_1a_jb_1b_j + a_2a_jb_2b_j \\ &= a_1a_1b_1b_1 + a_1a_2b_1b_2 + a_2a_1b_2b_1 + a_2a_2b_2b_2 \\ &= a_1^2b_1^2 + a_2^2b_2^2 + 2a_1a_2b_1b_2. \end{aligned}$$

This is indeed the correct expression. Thus as a general rule, a repeated/dummy index cannot be appear more than two times.