Homework 3

1. Prove sampling theorem without using impulse train.

2. Consider a sinusoidal signal

$$x(t) = 3cos(1000\pi + 0.1\pi) \tag{1}$$

Let us sample it at a frequency Fs = 2kHz.

a) Determine an expression for the sampled sequence x[n] = x(nTs) and determine its Discrete Time Fourier Transform

$$X(j\omega) = DTFTx[n]. \tag{2}$$

b) Determine

$$X(j\Omega) = FTx(t). \tag{3}$$

c) Recompute $X(j\omega)$ from the $X(j\Omega)$and verify that you obtain the same expression as in a).

3. For each $X(j\Omega)$= FTx(t) shown, determine

$$X(j\omega) = DTFTx[n]. \tag{4}$$

where x[n] = x(nTs) is the sampled sequence. The Sampling frequency Fs is given for each case.
{rect(t) is a function which produces a rectangular-shaped pulse with a width of 1 centered at t = 0 and has a height 1. rect((t-X)/Y) is centered at X and has width Y which also has a height 1.}

$$a)X(j\Omega) = 2\pi\delta(\Omega - 1000), Fs = 3000kHz. \tag{5}$$

$$b)X(j\Omega) = 2\pi\delta(\Omega - 500) + 2\pi\delta(\Omega + 500), Fs = 1200Hz. \tag{6}$$

$$c)X(j\Omega) = 6\pi rect(\Omega/1000), Fs = 2000Hz. \tag{7}$$

$$d)X(j\Omega) = 6\pi rect(\Omega/1000), Fs = 1000Hz. \tag{8}$$

$$e)X(j\Omega) = 2\pi rect((\Omega - 3000)/1000) + 2\pi rect((\Omega + 3000)/1000), Fs = 3000Hz. \tag{9}$$

4. The frequency response of a discrete time system is given by
$H(j\omega) = 1; for|w| < \pi/4$
$= 0; for\ \pi/4 < |w| < \pi$
a)Find the impulse response(h(n)) of the system.
b)Find the overall impulse response(h(n)) at n=3,if the above system is cascaded by a second system with unit sample response $\delta[n-1]$.

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5. Using the definition determine the DTFT of the following sequences. If it does not exist say why.
   $a) x[n] = 0.5^n u[n]$
   $b) x[n] = 0.5^n u[-n]$
   $c) x[n] = 2^n u[n]$
   $d) x[n] = 0.5^{|n|}$
   $e) x[n] = 2^{|n|}$
   $f) x[n] = 3(0.8)^{|n|} cos(0.1\pi n)$

6. A sequence has the DTFT

$$X(j\omega) = \frac{1-a^2}{(1-ae^{-j\omega})(1-ae^{j\omega})} \quad , |a| < 1 \tag{10}$$

   a) Find the sequence x[n].
   b) Calculate

$$1/(2\pi) \int_{-\pi}^{\pi} X(j\omega) \cos\omega \, d\omega$$

7. Let x[n] be a periodic signal with one period given by [1,-2,3,-4,5,-6] with the zero index at '3'.
   Determine the frequency response of the system and one period of the output sequence y[n] if x[n] is the input to a LTI system with an impulse response h[n]=$0.8^{|n|}$