

1. Let us consider the collection  $F$  contains  $k$  elements

$$\text{i.e. } F = \{\sigma_1, \sigma_2, \sigma_3, \dots, \sigma_k\}$$

We can always ~~the~~ say that  $k \leq n$

Proof: If we consider  $v_i$  to be the last vertex of a linear ordering, then it satisfies  $\forall v_j$  such that  $(v_j, v_i) \notin E$ . As every vertex has to be considered for a maximum solution we require  $n$  linear orderings

$$\text{i.e. } |F| \leq n \Rightarrow O(n)$$

By another intuition we can show that the no. of linear orderings is less than  $n$ . In the above scenario we have only considered the last vertex in the ordering. But if we considered to the permutation before it and arrange them in some optimal manner we can achieve it in  $O(\Delta \log^n)$ .