Department of Electrical Engineering
IIT Hyderabad
ID 1370 – Digital Signal Processing

भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology Hyderabad

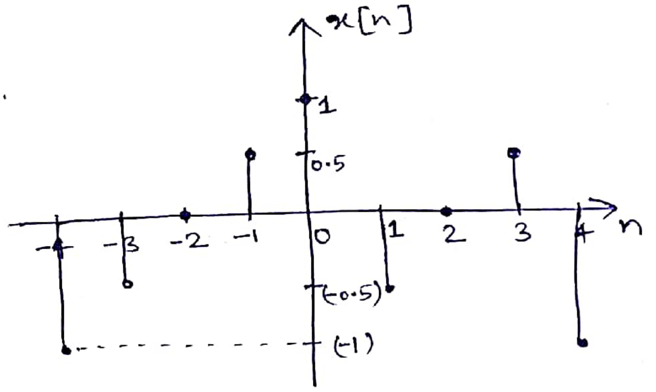
Class Test - 1 | Name: | Jan. 14, 2019
Max: 25 Marks | Roll Number: | 1 hour

1. A discrete-time signal is defined as

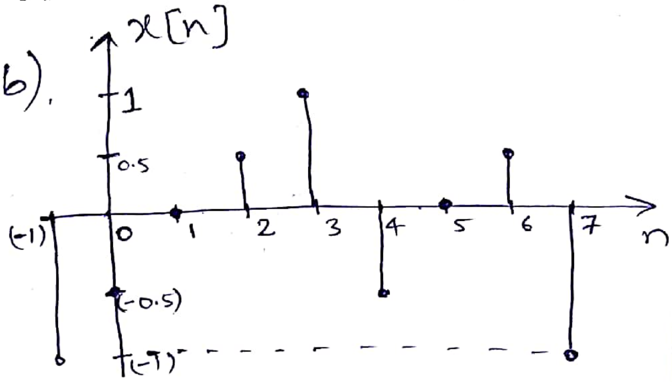
$$x[n] = \begin{cases} 1 + \frac{n}{2}, & -4 \le n \le -1 \\ (-1)^n \left(1 - \frac{n}{2}\right), & 0 \le n \le 4 \\ 0, & \text{otherwise} \end{cases}$$

Sketch the following signals: (a) $x[n]$, (b) $x[n-3]$, (c)$x[3-n]$, (d) $x[2n]$, (e) $x[n-1]\delta[n-3]$ (5)
(f) Any arbitrary signal $x[n]$ can be expressed as a sum of its even component $x_e[n]$ and its odd component $x_o[n]$, where $x_e[n] = \frac{1}{2}(x[n] + x[-n])$, and $x_o[n] = \frac{1}{2}(x[n] - x[-n])$. Determine and sketch the even and odd components of the above signal $x[n]$. (5)

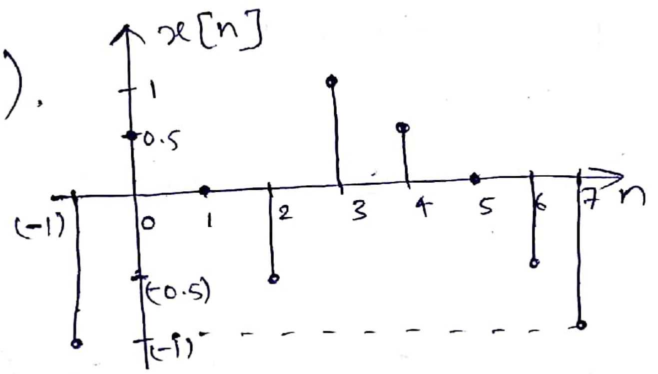
a).



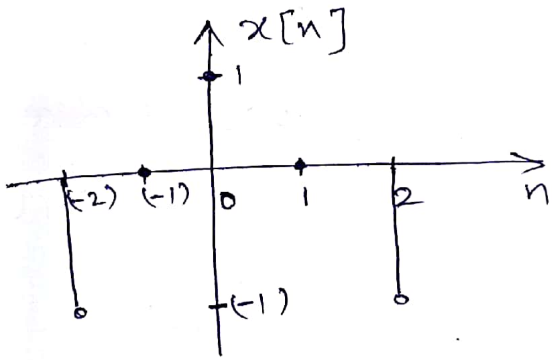
b).



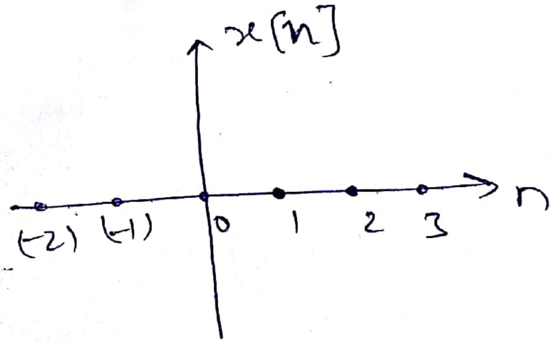
c).



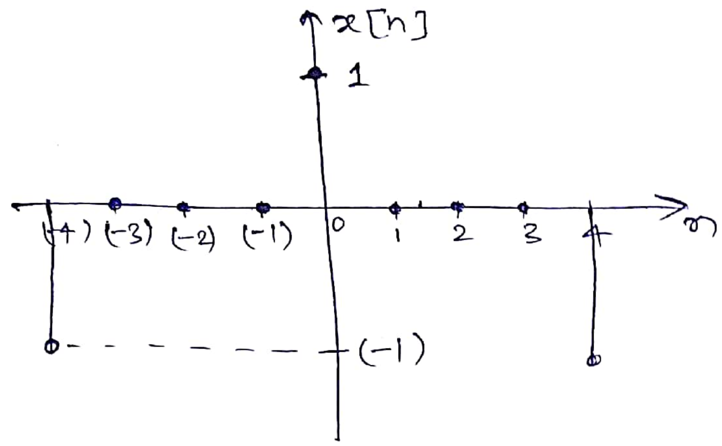
d)



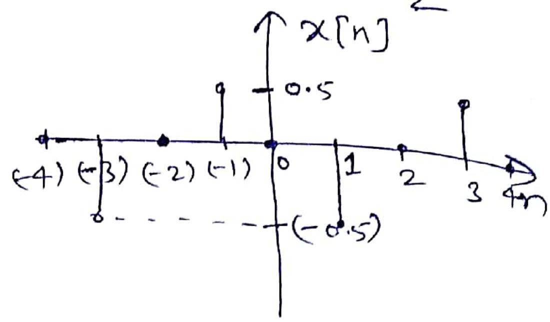
e). $x[n] = 0 \ \forall n$



f). $x_e[n] = -\delta[n+4] + \delta[n] - \delta[n-4]$



$x_o[n] = \frac{-\delta[n+3]}{2} + \frac{\delta[n+1]}{2} + \frac{\delta[n-3]}{2} - \frac{\delta[n-1]}{2}$



2. The following input-output pairs have been observed during the operation of a time-invarinat system:

$$x_1[n] = \{\underset{\uparrow}{1}, 0, 2\} \longrightarrow y_1[n] = \{0, \underset{\uparrow}{1}, 2\}$$
$$x_2[n] = \{\underset{\uparrow}{0}, 0, 3\} \longrightarrow y_2[n] = \{\underset{\uparrow}{0}, 1, 0, 2\}$$
$$x_3[n] = \{\underset{\uparrow}{0}, 0, 0, 1\} \longrightarrow y_3[n] = \{1, \underset{\uparrow}{2}, 1\}$$

where ↑ denotes the 0^{th} time-index. Can you draw any conclusions regarding the linearity of the system? What is the unit sample response of the system? (5)

Above system would be linear if it follows →

$$\left(a x_1[n] + b x_2[n] + c x_3[n] \longrightarrow a y_1[n] + b y_2[n] + c y_3[n] \right)$$

From the set of expressions in the ques., we have →

$$x_1[n] - x_3[n+3] + \frac{2}{3} x_2[n] = 0$$

But,

$$y_1[n] - y_3[n+3] + \frac{2}{3} y_2[n] \neq 0$$

For example, at $n=1$ above expression (from the equat) simplifies to $(5/3)$.

Hence, system is not linear

$$h[n] = \{1, 2, 1, 0, \underset{\uparrow}{0}\}$$

3. A continuous-time signal

$$x(t) = \sin(20\pi t) + \cos(40\pi t)$$

is sampled with a sampling period T_s to obtain the discrete-time signal (5)

$$x[n] = \sin\left(\frac{\pi n}{5}\right) + \cos\left(\frac{2\pi n}{5}\right)$$

(a) Determine a choice for T_s consistent with this information?

(b) Is your choice of T_s in Part (a) unique? If so, explain why. If not, specify another choice of T_s consistent with the information given.

a). $20\pi t \Big|_{t=nT_s} = \frac{\pi n}{5}$

$\Longrightarrow \quad 20\,T_s = \frac{1}{5}$

$\Longrightarrow \quad \boxed{T_s = \frac{1}{100} \text{ sec}}$

b). T_s — Not unique

$\Longrightarrow \quad \sin\left(\frac{\pi n}{5}\right) = \sin\left(2K\pi n + \frac{\pi n}{5}\right)$ [where $K \in \mathbb{Z}$]

$\therefore \quad 20\,T_s = 2K + \frac{1}{5} = \frac{10K+1}{5}$

$\Longrightarrow \quad T_s = \frac{10K+1}{100}$ sec.

Eg: $T_s = \frac{11}{100}$ sec.

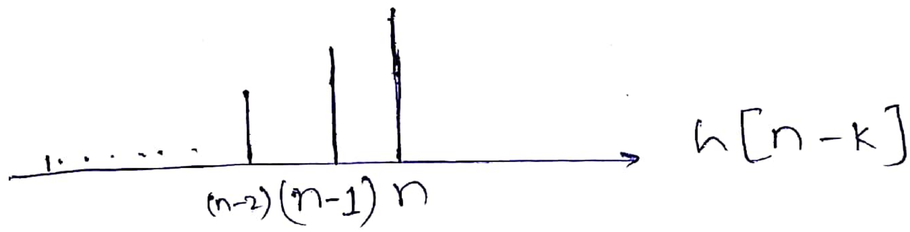
4. Determine the output of an initially relaxed LTI discrete-time system characterized by unit sample response,

$$h[n] = \begin{cases} \left(\frac{1}{2}\right)^n, & n \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

to the input signal

$$x[n] = \begin{cases} 1, & 0 \leq n \leq 10 \\ 0, & \text{otherwise} \end{cases}$$

(5)



$$y[n] = x[n] * h[n]$$

$$= \sum_{K=-\infty}^{\infty} x[K] \cdot h[n-K]$$

$(0 \leq n < 10)$

$$\sum_{K=0}^{n} (1) \cdot \left(\frac{1}{2}\right)^{n-K}$$

$$= \left(\frac{1}{2}\right)^n + \left(\frac{1}{2}\right)^{n-1} + \dots + \left(\frac{1}{2}\right)^0$$

$$= 2\left(1 - \left(\frac{1}{2}\right)^{n+1}\right)$$

$n \geq 10$

$$\sum_{K=0}^{10} \left(\frac{1}{2}\right)^{n-K}$$

$$= \left(\frac{1}{2}\right)^n + \left(\frac{1}{2}\right)^{n-1} + \dots + \left(\frac{1}{2}\right)^{n-10}$$

$$= \left(\frac{1}{2}\right)^n \left(2^{11} - 1\right)$$

Also, $y[n] = 0 \ \forall\ n < 0$