All questions are objective type.

Throughout X, Y etc will denote random variables.

Question 1. *Let $X \sim \mathcal{N}(0,1)$ and $F(x) = P(X \le x)$ be the cumulative distribution function. State whether the following assertions are true or false:* *Marks:2*

(1) *For any x, $F(x) + F(-x) = 1$.*
(2) *$E[X^k] = 0$ where k is an odd integer.*

Question 2. *The amount of time that a mobile call lasts is an exponential random variable with mean 5 minutes. Suppose A calls B and finds B's mobile busy. Find the probability that after 10 minutes, B is still on the same mobile call.* *Marks:2*

Question 3. *The probability density function of a continuous random variable X is given as* *Marks:2*

$$f(x) = \begin{cases} \lambda \cdot (x - [x]), & x \in [0,2] \\ 0, & \text{otherwise} \end{cases}$$

where λ is some constant and $[x]$ denotes the largest integer not greater than x.

(1) *The value of λ is* _______
(2) *If $A = \left[\frac{1}{2}, 2\right]$ then $P(X \in A) =$* _______

Question 4. *Let X and Y be independent random variables with uniform distribution over $(0,1)$. Let $f_{X+Y}(x)$ be the probability density function of $X+Y$.* *Marks:2*

(1) *For $0 \le x \le 1$, $f_{X+Y}(x) =$* _______
(2) *For $1 \le x \le 2$, $f_{X+Y}(x) =$* _______

[Verify your answer by checking that the area $\int_{-\infty}^{\infty} f_{X+Y}(x)\,dx$ is indeed 1. This example also shows that even if X and Y have identical and independent distribution, $X+Y$ may <u>not</u> have the same distribution.]

Question 5. *Let X and Y have the joint probability mass function* *Marks:2*

$$P_{X,Y}(m,n) = \begin{cases} \frac{1}{2^{m+1}}, & \text{if } m \ge n \\ 0, & \text{otherwise} \end{cases}$$

for $m, n = 1, 2, 3 \cdots$.

(1) *Find the probability mass function of X.*
(2) *Find the probability mass function of Y.*