

MODULE <i>HermesRMWs</i>	
EXTENDS	<i>Hermes</i>
VARIABLES	<i>Rmsgs</i> , <i>nodeFlagRMW</i> , <i>committedRMWs</i> , <i>committedWrites</i>
	all <i>Hermes</i> (+ environment, + <i>RMW</i>) variables
<i>hrvars</i>	$\triangleq \langle \textit{msgs}, \textit{nodeTS}, \textit{nodeState}, \textit{nodeRcvdAcks}, \textit{nodeLastWriter},$ $\textit{nodeLastWriteTS}, \textit{nodeWriteEpochID}, \textit{aliveNodes}, \textit{epochID},$ $\textit{Rmsgs}, \textit{nodeFlagRMW}, \textit{committedRMWs}, \textit{committedWrites} \rangle$
<i>HRMessage</i>	\triangleq Invalidation <i>msgs</i> exchanged by the <i>Hermes</i> Protocol w/ <i>RMWs</i> $[\textit{type} : \{ \text{"RINV"} \}, \quad \textit{flagRMW} : \{0, 1\}, \quad \textit{RMW change}$ $\quad \quad \quad \textit{epochID} : 0 \dots (\textit{Cardinality}(\textit{H_NODES}) - 1),$ $\quad \quad \quad \textit{sender} : \textit{H_NODES},$ $\quad \quad \quad \textit{version} : 0 \dots \textit{H_MAX_VERSION},$ $\quad \quad \quad \textit{tieBreaker} : \textit{H_NODES}]$
<i>HRTs</i>	$\triangleq [\textit{version} : 0 \dots \textit{H_MAX_VERSION},$ $\quad \quad \textit{tieBreaker} : \textit{H_NODES}]$
<i>HRTYPEOK</i>	\triangleq The type correctness invariant $\wedge \textit{HTypeOK}$ $\wedge \textit{Rmsgs} \subseteq \textit{HRMessage}$ $\wedge \textit{nodeFlagRMW} \in [\textit{H_NODES} \rightarrow \{0, 1\}]$ $\wedge \textit{committedRMWs} \subseteq \textit{HRTs}$ $\wedge \textit{committedWrites} \subseteq \textit{HRTs}$
<i>HRSemanticsRMW</i>	\triangleq The invariant that an we cannot have two operations committed with same versions (<i>i.e.</i> , that read the same value unless they are both writes) $\wedge \forall x \in \textit{committedRMWs} :$ $\quad \quad \quad \forall y \in \textit{committedWrites} : \wedge x.\textit{version} \neq y.\textit{version}$ $\quad \quad \quad \wedge x.\textit{version} \neq y.\textit{version} - 1$ $\wedge \forall x, y \in \textit{committedRMWs} : \wedge x.\textit{version} \neq y.\textit{version}$ $\quad \quad \quad \wedge x.\textit{tieBreaker} = y.\textit{tieBreaker}$
<i>HRInit</i>	\triangleq The initial predicate $\wedge \textit{HInit}$ $\wedge \textit{Rmsgs} = \{ \}$ $\wedge \textit{committedRMWs} = \{ \}$ $\wedge \textit{committedWrites} = \{ \}$ $\wedge \textit{nodeFlagRMW} = [n \in \textit{H_NODES} \mapsto 0] \quad \textit{RMW change}$

A buffer maintaining all Invalidation messages. Messages are only appended to this variable (not removed once delivered) intentionally to check protocols tolerance in duplicates and reorderings

$$HRsend(m) \triangleq Rmsgs' = Rmsgs \cup \{m\}$$

$$\begin{aligned} hr_upd_nothing &\triangleq \\ &\wedge \text{UNCHANGED } \langle nodeFlagRMW, Rmsgs, committedRMWs, committedWrites \rangle \\ hr_completeWrite(ver, tieB) &\triangleq \\ &\wedge committedWrites' = committedWrites \cup \{[version \mapsto ver, tieBreaker \mapsto tieB]\} \\ &\wedge \text{UNCHANGED } \langle Rmsgs, nodeFlagRMW, committedRMWs \rangle \\ hr_completeRMW(ver, tieB) &\triangleq \\ &\wedge committedRMWs' = committedRMWs \cup \{[version \mapsto ver, tieBreaker \mapsto tieB]\} \\ &\wedge \text{UNCHANGED } \langle Rmsgs, nodeFlagRMW, committedWrites \rangle \end{aligned}$$

Helper functions

$$\begin{aligned} hr_upd_state(n, newVersion, newTieBreaker, newState, newAcks, flagRMW) &\triangleq \\ &\wedge nodeFlagRMW' = [nodeFlagRMW \text{ EXCEPT } ![n] = flagRMW] \quad \text{RMW change} \\ &\wedge h_upd_state(n, newVersion, newTieBreaker, newState, newAcks) \end{aligned}$$

$$\begin{aligned} hr_send_inv(n, newVersion, newTieBreaker, flagRMW) &\triangleq \\ &\wedge HRsend([type \mapsto \text{"RINV"}, \\ &\quad epochID \mapsto epochID, \quad \text{we always use the latest } epochID \\ &\quad flagRMW \mapsto flagRMW, \quad \text{RMW change} \\ &\quad sender \mapsto n, \\ &\quad version \mapsto newVersion, \\ &\quad tieBreaker \mapsto newTieBreaker]) \end{aligned}$$

$$\begin{aligned} hr_actions_for_upd(n, newVersion, newTieBreaker, newState, newAcks, flagRMW) &\triangleq \quad \text{Execute a write} \\ &\wedge hr_upd_state(n, newVersion, newTieBreaker, newState, newAcks, flagRMW) \\ &\wedge hr_send_inv(n, newVersion, newTieBreaker, flagRMW) \\ &\wedge \text{UNCHANGED } \langle aliveNodes, epochID, msgs, committedRMWs, committedWrites \rangle \end{aligned}$$

$$\begin{aligned} hr_actions_for_upd_replay(n, acks) &\triangleq \quad \text{Apply a write-replay using same } TS \text{ (version, Tie Breaker)} \\ &\quad \text{and either reset } acks \text{ or keep already gathered } acks \\ &\wedge hr_actions_for_upd(n, nodeTS[n].version, nodeTS[n].tieBreaker, \text{"replay"}, acks, nodeFlagRMW[n]) \end{aligned}$$

Coordinator functions

$$\begin{aligned} HRWrite(n) &\triangleq \quad \text{Execute a write} \\ &\wedge nodeState[n] \in \{\text{"valid"}, \text{"invalid"}\} \\ &\quad \text{writes in invalid state are also supported as an optimization} \\ &\wedge nodeState[n] = \text{"valid"} \\ &\wedge nodeTS[n].version + 2 \leq H_MAX_VERSION \quad \text{Only to configurably terminate the model checking} \end{aligned}$$

$$\begin{aligned}
& \wedge \text{hr_actions_for_upd}(n, \text{nodeTS}[n].\text{version} + 2, n, \text{"write"}, \{\}, 0) \\
\text{HRRMW}(n) & \triangleq \text{Execute an RMW} \\
& \wedge \text{nodeState}[n] = \text{"valid"} \\
& \wedge \text{nodeTS}[n].\text{version} + 1 \leq H_MAX_VERSION \quad \text{Only to configurably terminate the model checking} \\
& \wedge \text{hr_actions_for_upd}(n, \text{nodeTS}[n].\text{version} + 1, n, \text{"write"}, \{\}, 1) \\
\text{HRWriteReplay}(n) & \triangleq \text{Execute a write-replay} \\
& \wedge \text{nodeState}[n] \in \{\text{"write"}, \text{"replay"}\} \\
& \wedge \text{nodeWriteEpochID}[n] < \text{epochID} \\
& \wedge \neg \text{receivedAllAcks}(n) \quad \text{optimization to not replay when we have gathered acks from all alive} \\
& \wedge \text{nodeFlagRMW}[n] = 0 \\
& \wedge \text{hr_actions_for_upd_replay}(n, \text{nodeRcvdAcks}[n]) \\
\text{HRRMWReplay}(n) & \triangleq \text{Execute an RMW-replay} \\
& \wedge \text{nodeState}[n] \in \{\text{"write"}, \text{"replay"}\} \\
& \wedge \text{nodeWriteEpochID}[n] < \text{epochID} \\
& \wedge \neg \text{receivedAllAcks}(n) \quad \text{optimization to not replay when we have gathered acks from all alive} \\
& \wedge \text{nodeFlagRMW}[n] = 1 \\
& \wedge \text{hr_actions_for_upd_replay}(n, \{\}) \\
\text{Keep the } H\text{Read}, H\text{RcvAck} \text{ and } H\text{SendVals} \text{ the same as } \textit{Hermes} \text{ w/o RMWs} \\
H\text{Read}(n) & \triangleq \\
& \wedge H\text{Read}(n) \\
& \wedge \text{hr_upd_nothing} \\
H\text{RcvAck}(n) & \triangleq \\
& \wedge H\text{RcvAck}(n) \\
& \wedge \text{hr_upd_nothing} \\
H\text{SendValsRMW}(n) & \triangleq \\
& \wedge \text{nodeFlagRMW}[n] = 1 \\
& \wedge H\text{SendVals}(n) \\
& \wedge \text{hr_completeRMW}(\text{nodeTS}[n].\text{version}, \text{nodeTS}[n].\text{tieBreaker}) \\
H\text{SendValsWrite}(n) & \triangleq \\
& \wedge \text{nodeFlagRMW}[n] = 0 \\
& \wedge H\text{SendVals}(n) \\
& \wedge \text{hr_completeWrite}(\text{nodeTS}[n].\text{version}, \text{nodeTS}[n].\text{tieBreaker}) \\
H\text{CoordinatorActions}(n) & \triangleq \text{Actions of a read/write/RMW coordinator} \\
& \vee H\text{Read}(n) \\
& \vee H\text{RRMWReplay}(n) \\
& \vee H\text{RWriteReplay}(n) \\
& \vee H\text{RWrite}(n) \\
& \vee H\text{RRMW}(n) \\
& \vee H\text{RcvAck}(n)
\end{aligned}$$

$\vee \text{HRSendValsRMW}(n)$
 $\vee \text{HRSendValsWrite}(n)$

Followers functions

$\text{hr_upd_state_greater_inv}(n) \triangleq$
 IF $\text{nodeState}[n] \in \{\text{"valid"}, \text{"invalid"}, \text{"replay"}\}$
 THEN
 $\text{nodeState}' = [\text{nodeState} \text{ EXCEPT } ![n] = \text{"invalid"}]$
 ELSE IF $\text{nodeState}[n] \in \{\text{"write"}, \text{"invalid_write"}\} \wedge \text{nodeFlagRMW}[n] = 0$
 THEN
 $\text{nodeState}' = [\text{nodeState} \text{ EXCEPT } ![n] = \text{"invalid_write"}]$
 ELSE $\text{nodeState}[n] \in \{\text{"write"}\} \wedge \text{nodeFlagRMW}[n] = 1$
 $\text{nodeState}' = [\text{nodeState} \text{ EXCEPT } ![n] = \text{"invalid"}]$

$\text{HRRcvWriteInv}(n) \triangleq$ **Process a received invalidation for a write**
 $\exists m \in \text{Rmsgs} :$
 $\wedge m.\text{type} = \text{"RINV"}$
 $\wedge m.\text{epochID} = \text{epochID}$
 $\wedge m.\text{sender} \neq n$
 $\wedge m.\text{flagRMW} = 0$ **RMW change**
always acknowledge a received invalidation (irrelevant to the timestamp)
 $\wedge h.\text{send_inv_or_ack}(n, m.\text{version}, m.\text{tieBreaker}, \text{"ACK"})$
 $\wedge \text{IF } \text{greaterTS}(m.\text{version}, m.\text{tieBreaker},$
 $\quad \text{nodeTS}[n].\text{version}, \text{nodeTS}[n].\text{tieBreaker})$
 THEN
 $\wedge \text{nodeLastWriter}' = [\text{nodeLastWriter} \text{ EXCEPT } ![n] = m.\text{sender}]$
 $\wedge \text{nodeFlagRMW}' = [\text{nodeFlagRMW} \text{ EXCEPT } ![n] = m.\text{flagRMW}]$ **RMW change**
 $\wedge \text{nodeTS}' = [\text{nodeTS} \text{ EXCEPT } ![n].\text{version} = m.\text{version},$
 $\quad ![n].\text{tieBreaker} = m.\text{tieBreaker}]$
 $\wedge \text{hr_upd_state_greater_inv}(n)$
 ELSE
 $\wedge \text{UNCHANGED } \langle \text{nodeState}, \text{nodeTS}, \text{nodeLastWriter}, \text{nodeFlagRMW} \rangle$
 $\wedge \text{UNCHANGED } \langle \text{nodeLastWriteTS}, \text{aliveNodes}, \text{nodeRcvdAcks}, \text{Rmsgs},$
 $\quad \text{epochID}, \text{nodeWriteEpochID}, \text{committedRMWs}, \text{committedWrites} \rangle$

$\text{HRRcvRMWInv}(n) \triangleq$ **Process a received invalidation for a write**
 $\exists m \in \text{Rmsgs} :$
 $\wedge m.\text{type} = \text{"RINV"}$
 $\wedge m.\text{epochID} = \text{epochID}$
 $\wedge m.\text{sender} \neq n$
 $\wedge m.\text{flagRMW} = 1$
 $\wedge \text{IF } \text{greaterTS}(m.\text{version}, m.\text{tieBreaker},$
 $\quad \text{nodeTS}[n].\text{version}, \text{nodeTS}[n].\text{tieBreaker})$
 THEN

$$\begin{aligned}
& \wedge \text{nodeLastWriter}' = [\text{nodeLastWriter} \text{ EXCEPT } ![n] = m.\text{sender}] \\
& \wedge \text{nodeFlagRMW}' = [\text{nodeFlagRMW} \text{ EXCEPT } ![n] = m.\text{flagRMW}] \text{ RMW change} \\
& \wedge \text{nodeTS}' = [\text{nodeTS} \text{ EXCEPT } ![n].\text{version} = m.\text{version}, \\
& \quad \quad \quad ![n].\text{tieBreaker} = m.\text{tieBreaker}] \\
& \quad \text{acknowledge a received invalidation (w/ greater timestamp)} \\
& \wedge h_send_inv_or_ack(n, m.\text{version}, m.\text{tieBreaker}, \text{"ACK"}) \\
& \wedge hr_upd_state_greater_inv(n) \\
& \wedge \text{UNCHANGED } \langle Rmsgs \rangle \\
& \text{ELSE IF } equalTS(m.\text{version}, m.\text{tieBreaker}, \\
& \quad \quad \quad \text{nodeTS}[n].\text{version}, \text{nodeTS}[n].\text{tieBreaker}) \\
& \text{THEN} \\
& \quad \text{acknowledge a received invalidation (w/ equal timestamp)} \\
& \wedge h_send_inv_or_ack(n, m.\text{version}, m.\text{tieBreaker}, \text{"ACK"}) \\
& \wedge \text{UNCHANGED } \langle \text{nodeState}, \text{nodeTS}, \text{nodeLastWriter}, \text{nodeFlagRMW}, Rmsgs \rangle \\
& \text{ELSE smaller TS} \\
& \wedge hr_send_inv(n, \text{nodeTS}[n].\text{version}, \text{nodeTS}[n].\text{tieBreaker}, \text{nodeFlagRMW}[n]) \\
& \wedge \text{UNCHANGED } \langle \text{nodeState}, \text{nodeTS}, \text{nodeLastWriter}, \text{nodeFlagRMW}, msgs \rangle \\
& \wedge \text{UNCHANGED } \langle \text{nodeLastWriteTS}, \text{aliveNodes}, \text{nodeRcvdAcks}, \text{epochID}, \\
& \quad \quad \quad \text{nodeWriteEpochID}, \text{committedRMWs}, \text{committedWrites} \rangle
\end{aligned}$$

Keep the $HRcvVals$ the same as *Hermes* w/o *RMWs*

$$\begin{aligned}
HRRcvVal(n) & \triangleq \\
& \wedge HRcvVal(n) \\
& \wedge hr_upd_nothing
\end{aligned}$$

$HRFollowerWriteReplay(n) \triangleq$ Execute a write-replay when coordinator failed

$$\begin{aligned}
& \wedge \text{nodeState}[n] = \text{"invalid"} \\
& \wedge \neg isAlive(\text{nodeLastWriter}[n]) \\
& \wedge hr_actions_for_upd_replay(n, \{\})
\end{aligned}$$

$HRFollowerActions(n) \triangleq$ Actions of a write follower

$$\begin{aligned}
& \vee HRFollowerWriteReplay(n) \\
& \vee HRRcvWriteInv(n) \\
& \vee HRRcvRMWInv(n) \\
& \vee HRRcvVal(n)
\end{aligned}$$

$HRNodeFailure(n) \triangleq$

$$\begin{aligned}
& \wedge \text{nodeFailure}(n) \\
& \wedge hr_upd_nothing
\end{aligned}$$

$HRNext \triangleq$ *Hermes* (read,write *RMWs*) protocol (Coordinator and Follower actions) + failures

$$\begin{aligned}
& \exists n \in \text{aliveNodes} : \\
& \quad \vee HRFollowerActions(n)
\end{aligned}$$

$$\begin{aligned} &\vee HRCoordinatorActions(n) \\ &\vee HRNodeFailure(n) \end{aligned}$$

$$HRSpec \triangleq HRIInit \wedge \Box[HRNext]_{hrvars}$$

$$\text{THEOREM } HRSpec \Rightarrow (\Box HRTYPEOK) \wedge (\Box HCONSISTENT) \wedge (\Box HRSEMANTICS RMW)$$
