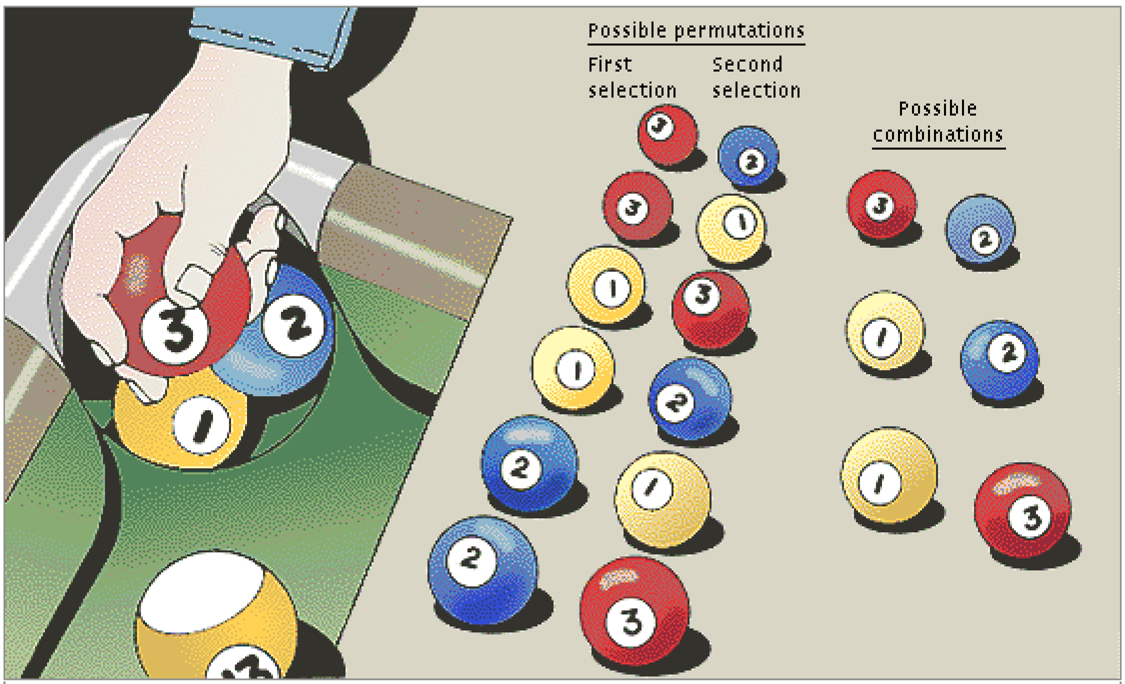
Permutations and Combinations

I INTRODUCTION

Permutations and Combinations, in mathematics, arrangements of objects taken from a given set of objects. Permutations and combinations are important in many branches of mathematics. For example, in the use of the binomial expansion and in probability theory and statistics, where they can be used to count the number of possible arrangements of a system. A new branch of mathematics, called *combinatorics,* is founded on the formulas for permutations and combinations, and it has important applications to the design and operation of computers, as well as to the physical and social sciences. Indeed, in any area where the possible arrangements of a finite number of elements play a role, the theory of permutations and combinations is useful.

II PERMUTATIONS



Permutations and Combinations

Permutations and combinations are two different ways of arranging objects. If a person removes two balls, one at a time, from a billiard pocket containing three balls, then there are six possible permutations. The order of arrangement is important in determining the number of permutations; choosing a red ball then a blue ball is a different permutation from choosing a blue ball then a red ball. When determining the number of possible combinations, however, ordering is not important; choosing a red ball then a blue ball results in the same combination as choosing a blue ball then a red one. As a result, there are only three possible combinations.

Permutations are *sequenced* arrangements of objects. Consider a small pack of cards consisting of just the Jack, Queen, King, and Ace of Hearts. The order in which the cards are dealt would be a permutation. There are 24 such combinations: JQKA JQAK JKAQ JKQA JAQK JAKQ QJKA QJAK QKAJ

QKJA QAJK QAKJ KQJA KQAJ KJAQ KJQA KAQJ KAJQ AQKJ AQJK AKJQ AKQJ AJQK AJKQ The number of different permutations can be worked out by considering what happens when the cards are being dealt. The first card dealt and turned over could be 1 of 4 possible cards. The second card could be 1 of 3 remaining possibilities, the third card 1 of 2 and there would be only one choice of fourth card. This gives the total permutations as $4 \times 3 \times 2 \times 1 = 24$, which can also be written 4!—read “four factorial”. In general there are *n*! permutations of choosing, or arranging, *n* objects in sequence, where *n*! represents the product of all the positive integers from 1 to *n*, and 0! is defined as 1.

The above example represents the permutation of four objects taken four at a time; the number of such permutations is denoted by the symbol ${}_4P_4$. The number of permutations of dealing four cards from a full pack of 52 cards would be written ${}_{52}P_4$. The value of ${}_nP_k$ is given by the general formula

$${}_nP_k = \frac{n!}{(n-k)!}$$

III COMBINATIONS

Combinations are arrangements of objects with *no* consideration of sequence. Continuing with the above example of dealing cards, we are normally more concerned with the combination of cards, or hand, we are dealt rather than with the order in which the cards are received. The number of possible combinations of hands dealt is equal to the number of ways that a hand *could* be dealt divided by the number of ways of re-ordering one hand. For example, there are ${}_{52}P_4$ ways of dealing 4 cards from a pack, ${}_4P_4$ ways of re-ordering those 4 cards and therefore ${}_{52}P_4/{}_4P_4$ possible hands. In general the number of combinations of taking k objects from a set of n objects, denoted as ${}_nC_k$ or $\binom{n}{k}$, is given by the general formula

$${}_nC_k = \frac{{}_nP_k}{{}_kP_k} = \frac{n!}{k!(n-k)!}$$