Prime Numbers

Prime Numbers, integers (whole numbers) greater than 1 that cannot be products of smaller positive integers. For example, 2, 3, 5, and 7 are prime, whereas 4 (= 2 × 2), 6 (= 2 × 3), 8 (= 2 × 4), and 9 (= 3 × 3) are not.

The importance of prime numbers arises from the Fundamental Theorem of Arithmetic: each integer *n* > 1 is a product of one or more prime numbers (possibly with repetitions), and this factorization is unique (apart from reordering factors). For instance, 12 = 2 × 2 × 3 = 2 × 3 × 2 = 3 × 2 × 2. This theorem reduces many problems about integers to problems about prime numbers.

Euclid proved that there are infinitely many prime numbers. These occur very irregularly among the integers, however, with no simple way of describing them all. The Prime Number Theorem (Hadamard and de la Vallée Poussin, 1896) shows that large integers are less likely to be prime than small ones. More precisely, the proportion of primes among the integers between 1 and x is approximately 1/l*n*(x) which approaches 0 as *x* increases (here ln(*x*) denotes the natural logarithm of x).

Unsolved problems concerning prime numbers include Goldbach's Conjecture that every even integer *n* > 2 is the sum of two primes, and the conjecture that there are infinitely many twin primes (primes differing by 2).

In recent years, computers have been applied extensively to the problems of finding new primes, testing whether a given integer is prime, and factorizing a given integer, all requiring very laborious calculations. Effective secret codes (*see*Cryptography: *Computer Ciphers*) have been based on the computational difficulty of factorizing very large integers.