# Probability

Probability, also theory of probability, branch of mathematics that deals with measuring or determining quantitatively the likelihood that an event or experiment will have a particular outcome. Probability is based on the study of permutations and combinations and is the necessary foundation for statistics.

The foundation of probability is usually ascribed to the 17th-century French mathematicians Blaise Pascal and Pierre de Fermat, but mathematicians as early as Gerolamo Cardano, in the 16th century, had made important contributions to its development. Mathematical probability began in an attempt to answer certain questions arising in games of chance, such as how many times a pair of dice must be thrown if there is to be a 50-50 chance that a 6 will have appeared by then.

The probability of an outcome is represented by a number between 0 and 1, inclusive. If the number of possible outcomes is finite, probability 0 indicates that an outcome will never occur and probability 1 indicates that it will always occur. The simplest problems are concerned with the probability of a specified favourable result of an event that has a finite number of equally likely outcomes. If an event has $n$ equally likely outcomes and $f$ of them are termed favourable, the probability of a favourable outcome is $f/n$. For example, a fair die can be cast in six equally likely ways; therefore, the probability of throwing a 5 or a 6 is 2/6. More involved problems are concerned with events in which the various possible outcomes are not equally likely. For example, in finding the probability of throwing a 5 or 6 with a pair of dice, the various outcomes (2, 3, ... 12) are not all equally likely. An event may even have infinitely many possible outcomes, as when a point is chosen at random on a circle. In such cases an outcome may occur and yet have probability 0.

Problems involving repeated trials form one of the connections between probability and statistics. To illustrate, what is the probability that exactly five 3s and at least four 6s will occur in 50 tosses of a fair die? Or, a person tossing a fair coin takes a step forwards for heads and backwards for tails. What is the probability that at the end of 50 steps the person will be within 10 steps of the starting point?

In probability problems, two outcomes of an event are mutually exclusive if the probability of their joint occurrence is zero; two outcomes are independent if the probability of their joint occurrence is given as the product of the probability of their separate occurrences. Two outcomes are mutually exclusive if the occurrence of one precludes the occurrence of the other; two outcomes are independent if the occurrence or nonoccurrence of one does not alter the probability that the other will or will not occur. Compound probability is the probability of several outcomes occurring jointly. Conditional probability is the probability of an outcome when it is known that some other outcome has occurred or will occur.

If the probability that an outcome will occur is $p$, the probability that it will not occur is $q = 1 - p$. The odds in favour of the occurrence are given by the ratio $p:q$, and the odds against the occurrence are given by the ratio $q:p$. If the probabilities of two mutually exclusive outcomes $X$ and $Y$ are $p$ and $P$, respectively, the odds in favour of $X$ and against $Y$ are $p$ to $P$. If an event must result in one of the mutually exclusive outcomes $O_1$, $O_2$, ..., $O_n$, with probabilities $p_1$, $p_2$, ..., $p_n$, respectively, and if $v_1$, $v_2$, ..., $v_n$ are numerical values attached to the respective outcomes, the expected value, or mean

value, of the event is $E = p_1v_1 + p_2v_2... + p_nv_n$. For example, a person throws a die and wins 4 cakes if it falls 1, 2, or 3; 3 cakes for 4 or 5; but loses 12 if it falls 6. The expected value on a single throw is 3/6 × 4 + 2/6 × 3 - 1/6 × 12 = 1, or a gain of one cake (on average).

The most common interpretation of probability is used in statistical analysis. For example, the probability of throwing a 7 in one throw of two dice is 1/6, and this answer is interpreted to mean that if two fair dice are randomly thrown a very large number of times, about one sixth of the throws will be 7s. This concept is frequently used to determine statistically the probability of an outcome that cannot readily be tested or is impossible to obtain. Thus, if long-range statistics show that out of every 100 people between 20 and 30 years of age, 42 will be alive at age 70, the assumption is that a person between those ages has a 42 per cent probability of surviving to the age of 70.

Mathematical probability is widely used in the physical, biological, and social sciences and in industry and commerce. It is applied in such diverse areas as genetics, quantum mechanics, and insurance. It also involves deep and important theoretical problems in pure mathematics and has strong connections with the theory, known as mathematical analysis, that developed out of calculus.