Analytic Geometry

Analytic Geometry, branch of geometry in which straight lines, curves, and geometric figures are represented by numerical and algebraic expressions using a set of axes and coordinates. Any point in a plane may be located with respect to a pair of perpendicular axes by specifying the distance of the point from each of these axes. In figure 1, point a is 1 unit from the vertical y-axis and 4 units from the horizontal x-axis. The coordinates of point a are 1 and 4, and the point is located by the statements $x = 1$, $y = 4$. Positive x values are located to the right of the y-axis and negative values to the left; positive y values are above the x-axis and negative y values below. Thus, point b in figure 1 has the coordinates $x = 5$, $y = 0$. Points in three-dimensional space can be similarly located with respect to three axes, of which the third, usually called the z-axis, is perpendicular to the other two at their point of intersection, which is called the *origin.*

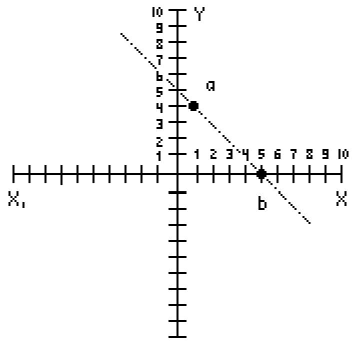

Figure 1

In general, a straight line can always be represented by a linear equation in two variables, x and y, in the form $ax + by + c = 0$. In the same way, equations can be derived for the circle, ellipse, and other conic sections and regular curves. The problems treated in analytic geometry are of two classic kinds. The first is: given a geometric description of a set of points, determine the algebraic equation that is satisfied by these points. In the above example, the collection of points that lies on the straight line passing through the points a and b satisfies the linear equation $x + y = 5$. In general, $ax + by = c$. The second kind of problem is: given an algebraic statement, describe the locus of the points that satisfy the statement in geometric terms. For example, a circle of radius 3 and with its centre at the origin is the locus of points that satisfy the equation $x^2 + y^2 = 9$.

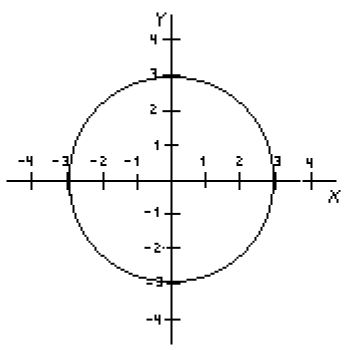

Figure 2

From equations such as these it is possible to solve algebraically such geometrical construction problems as bisecting a given line or angle, constructing a perpendicular to a given line at a given point, or drawing a circle that will pass through three given points not on the same straight line.

Analytic geometry has been of great value in the development of mathematics because it has unified the concepts of analysis (number relationships) and geometry (space relationships). The study of non-Euclidean geometry and the geometries of spaces that have more than three dimensions would not have been possible without the analytic approach. Similarly, the techniques of analytic geometry, which made possible the representation of numbers and of algebraic expressions in geometric terms, have cast new light on calculus, the theory of functions, and other problems in higher mathematics.