Simultaneous Equations

Simultaneous Equations, sets of equations that must be satisfied together: that is, values must be found for all the variables in them that will make the equations true. The simplest examples are of this type:

$$5x + 2y = 16 \qquad (1)$$
$$2x + 3y = 13 \qquad (2)$$

If we multiply these equations by 3 and 2 (the coefficients of *y*) respectively, giving

$$15x + 6y = 48 \qquad (1')$$
$$4x + 6y = 26 \qquad (2')$$

we can then subtract (2') from (1') to eliminate *y;* this gives $11x = 22$, so $x = 2$. Substituting this in equation (1), we have $10 + 2y = 16$, so $y = (16-10)/2 = 3$. (Alternatively, we could multiply (1) by 2 and (2) by 5, and then subtract, thus eliminating *x* rather than *y.*) More generally, if the equations are

$$ax + by = p \qquad (3)$$
$$cx + dy = q \qquad (4)$$

where *a, b, c, d, p,* and *q* are constants, then multiplying (3) by *d* and (4) by *b,* we have

$$adx + bdy = dp \qquad (3')$$
$$bcx + bdy = bq \qquad (4')$$

Subtracting (4') from (3'), we eliminate *y:*

$$(ad - bc)x = dp - bq$$

so that

$$x = \frac{dp-bq}{ad-bc}$$

provided $ad - bc$ is not 0. Substituting this value for *x* in (3), we get a single equation for *y,* with solution

$$y = \frac{aq-cp}{ad-bc}$$

The geometric interpretation (*see* Analytic Geometry) is that the graphs of (3) and (4) are straight lines, and the solutions *x* and *y* are the coordinates of the point (x,y) where they meet. Thus the lines $5x + 2y = 16$ and $2x + 3y = 13$ meet at (2,3).

Exceptional cases occur when $ad - bc = 0$. For instance, the equations

$$5x + 2y = 16$$

$$5x + 2y = 13$$

corresponding to two parallel lines, have no solutions. And

$$5x + 2y = 16$$

$$>10x + 4y = 32$$

describe a single straight line, so there are infinitely many solutions:

$$y = \frac{16 - 5x}{2}$$

Sets of more than two simultaneous equations involving more than two variables can be dealt with similarly, but a more efficient method here is to use matrix theory.

Non-linear equations are those that contain terms such as x^2, xy, x^3y^5, and so on. They represent curved rather than straight lines in graphs, and these may intersect each other many times or not at all, producing many solutions or none. For example, in the simultaneous equations

$$x + y = 7 \qquad (5)$$

$$xy = 12 \qquad (6)$$

(5) is linear, but (6) is not (its graph is a hyperbola). Writing (5) as $y = 7 - x$ and then substituting for y in (6), we have

$$x(7 - x) = 12$$

This is equivalent to the quadratic equation

$$x^2 - 7x + 12 = 0$$

which has solutions $x = 3$ and $x = 4$. Since $y = 12/x$, we find that $y = 4$ and $y = 3$ respectively, so the graphs meet at two points (3,4) and (4,3), corresponding to two solutions $x = 3$, $y = 4$ and $x = 4$, $y = 3$.

See Also Equations, Theory of.

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