Equation

Equation, statement of an equality between two expressions, used in almost all branches of pure and applied mathematics and in the physical, biological, and social sciences. An equation usually involves one or more unknown quantities, called variables or indeterminates. These are commonly denoted by letters or other symbols, as in the equations $x^2 + x - 4 = 8$, $y = \sin x + x$, and $3y = \log x$. An equation is described as being in one, two, three, or more variables, according to the number of variables it contains.

An equation is said to be satisfied, or to be true, for certain values of the variables if, when the variables are replaced by these values, the expression on the left side of the equals sign is equal to that on the right side. For example, the equation $2x + 5 = 13$ is satisfied when $x = 4$. If one or more values of the variable fail to satisfy the equation, the equation is called conditional. The equation just mentioned, $2x + 5 = 13$, is conditional because it is not satisfied by, for example, $x = 1$ (and an infinite number of other values). An equation is called an identity if it is satisfied by all possible values of the variables. For example, the equations $(x + y)^2 = x^2 + 2xy + y^2$ and $\sin^2 x + \cos^2 x = 1$ are identities because they are both true for all possible values of the unknowns. A solution of a conditional equation is a value of the variable, or a set of values of the variables, that satisfies the equation; thus, 3 is a solution of the equation $x^2 - 2x = 3$ (there is another solution, namely, $x = -1$); and $x = 2$, $y = 4$ is a solution of the equation $3x^2 + 4y = 28$ (there are infinitely many other pairs of values of x and y that satisfy the equation). A solution of an equation in one variable is commonly called a root of the equation.

A polynomial equation has the form

$$a_0 + a_1x^1 + a_2x^2 + \ldots + a_nx^n = 0$$

in which the coefficients a_0, a_1, ..., a_n are constants and n is a positive integer. The greatest exponent n is the degree of the equation. Equations of the first, second, third, fourth, and fifth degrees are often called, respectively, linear, quadratic, cubic, biquadratic or quartic, and quintic equations.

Other important types of equations are algebraic, as in $\sqrt{x} + \sqrt{x+7} = 7$; trigonometric, as in $\sin x + \cos 2x = 1$; logarithmic, as in $\log x + 2 \log (x + 1) = 8$; and exponential, as in $3^x + 2x - 5 = 0$.

Diophantine equations are equations in one or more unknowns and integral coefficients, for which integral solutions are sought (*see* Diophantine Analysis; Equation, Indeterminate). In calculus, differential and integral equations involve derivatives or differentials, and integrals.

A system of simultaneous equations is a set of two or more equations in two or more unknowns. A solution of such a system is a set of values of the unknowns that satisfies every equation of the set simultaneously.